

Study of the $\gamma\gamma \rightarrow \tau^+\tau^-$ process including $\tau^+\tau^-$ spin information in Pb-Pb ultraperipheral collisions and at a lepton collider

Peng-Cheng Lu^{✉,*}, Zong-Guo Si^{✉,†}, Han Zhang^{✉,‡}, and Xin-Yi Zhang^{✉,§}

School of Physics, Shandong University, Jinan, Shandong 250100, People's Republic of China



(Received 30 November 2025; accepted 14 January 2026; published 23 February 2026)

We study the $\gamma\gamma \rightarrow \tau^+\tau^-$ process including full $\tau^+\tau^-$ spin information in Pb-Pb ultraperipheral collisions and at lepton colliders. We present the predictions for the corresponding cross sections and spin correlations at next-to-leading order (NLO) electroweak precision, and find that the NLO electroweak contributions are numerically small for the observables considered. Additionally, we use the spin correlations obtained in this paper to analyze the quantum entanglement in the $\tau^+\tau^-$ system of the $\gamma\gamma \rightarrow \tau^+\tau^-$ process. Our results show that there is a genuine entangled configuration near the $\tau^+\tau^-$ invariant mass threshold. This work is helpful for studying the τ -pair production induced by photon-photon collision at high-energy colliders.

DOI: [10.1103/2g5j-k778](https://doi.org/10.1103/2g5j-k778)

Introduction. The heaviest charged lepton in the Standard Model (SM) is the τ lepton, with a mass of $\sim 1.777 \text{ GeV}/c^2$ and a lifetime of $\sim 2.9 \times 10^{-13} \text{ s}$ [1], exhibiting both leptonic and hadronic decay modes with precisely measured branching ratios [2,3], making it a powerful probe to study electroweak (EW) couplings and to search for physics beyond the SM. Since τ lepton discovery, its properties and interactions have been studied extensively not only at lepton colliders (LCs) [4–7], but also at hadron colliders [8–10]. More recently, both A Toroidal LHC Apparatus (ATLAS) and Compact Muon Solenoid (CMS) reported the observation of photon induced τ -pair production process in ultraperipheral Pb-Pb collisions (UPCs) at LHC, which facilitated tighter constraints on beyond SM physics and more precise understanding of τ lepton properties [11,12]. Among the phenomena associated with τ lepton, the spin polarization and correlation effects reflect in detail the interactions involved in τ lepton production and decay, the study of them plays an important role.

The spin density matrix can be reconstructed using either decay method [13] or kinematic method [14]. The spin-dependent observables are sensitive to anomalous couplings [15], CP -violating effects [16], and other new physics signals [17,18], which have been studied in

multiple processes [9,19–22]. Furthermore, recent studies explored the quantum entanglement of the spin system and proposed observables that could be used to test Bell inequalities [23] and the concurrence [24], which provide a measure of bipartite quantum correlations at colliders. The first system that is used to study quantum entanglement at the LHC is the $t\bar{t}$ system [25,26]. This was extended to the $\tau^+\tau^-$ system and others [27,28]. References [29–32] demonstrated that $\tau^+\tau^-$ systems exhibit entanglement over a wide range of energies in the future e^+e^- colliders and at the LHC. In this paper, we extend the analysis to the process $\gamma\gamma \rightarrow \tau^+\tau^-$ both in UPC collisions at the LHC and at future LCs, where photon induced τ -pair production can be studied with high precision.

In UPCs, the Lorentz-boosted electromagnetic fields of heavy nuclei serve as intense sources of quasireal photons, with fluxes scaling as Z^4 in nucleus-nucleus collisions, where Z represents the nuclear charge. Such processes are described by the equivalent photon approximation (EPA) [33,34], offering a clean environment with very low hadronic backgrounds and pileup for multi-TeV $\gamma\gamma$ studies [35]. At LCs, the same EPA framework—often referred to as the Weizsäcker-Williams approximation (WWA) [36]—is applied to initial-state photons. These machines combine high luminosities and tunable energies with clean final states, enabling precise reconstruction of τ decays and detailed polarization studies. Future facilities [37–40] will further extend this reach with higher luminosities and energies. In this context, the next-to-leading order (NLO) quantum electrodynamics corrections for $\gamma\gamma \rightarrow \tau^+\tau^-$ in UPCs were obtained in Ref. [41]. Following this, we studied the NLO EW corrections for the unpolarized τ -pair production process [42]. Further developments include the creation of automated software for NLO-accurate predictions of $\gamma\gamma \rightarrow \tau^+\tau^-$ in UPCs [43], as well

*Contact author: pclu@sdu.edu.cn

†Contact author: zgzi@sdu.edu.cn

‡Contact author: han.zhang@mail.sdu.edu.cn

§Contact author: xinyizhang@mail.sdu.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

as the computation of the leptonic decays of spin-correlated τ leptons, including NLO EW corrections in Pb-Pb UPCs [44]. Parallel investigations have explored the $\gamma\gamma \rightarrow \tau^+\tau^-$ process as a probe for new physics effects at LCs [45–48]. Building on our previous study, this paper extends the analysis by incorporating the full $\tau^+\tau^-$ spin information. The spin formalism we used is based on Ref. [49], allowing the extraction of polarization and correlation observables relevant for experimental analyses.

The structure of this paper is as follows. In section “Theoretical framework and spin-correlations effect”, we introduce the theoretical framework for $\gamma\gamma \rightarrow \tau^+\tau^-$ in Pb-Pb UPCs and at LCs, reviewing the spin density matrix of τ -pair production and the associated spin correlations. In section “Numerical results and analysis”, we present the numerical results for the cross section as well as spin correlation effects corresponding to different collider setups. We also discuss the prospects of probing quantum entanglement through this process. Section “Summary” is reserved for the summary.

Theoretical framework and spin-correlations effect. This section presents the calculation framework for photon-induced τ lepton pair production at LCs and in Pb-Pb UPCs, incorporating NLO EW corrections and the effects of τ lepton spin. The total cross section for this process can be described as the convolution of the two-photon flux luminosity with the partonic cross section of the subprocess $\gamma\gamma \rightarrow \tau^+\tau^-$, denoted as $\hat{\sigma}(\gamma\gamma \rightarrow \tau^+\tau^-)$. It is given by

$$\sigma_{\text{UPC/LC}} = \int dx_1 dx_2 n(x_1) n(x_2) \hat{\sigma}(\gamma\gamma \rightarrow \tau^+\tau^-). \quad (1)$$

For UPC events, we adopt the equivalent photon approach based on the electric dipole form factor [50]. In this framework, the photon distribution function (PDF) can be formulated as

$$n(x_i) = \frac{2Z^2\alpha}{x_i\pi} \left\{ \bar{x}_i K_0(\bar{x}_i) K_1(\bar{x}_i) - \frac{\bar{x}_i^2}{2} [K_1^2(\bar{x}_i) - K_0^2(\bar{x}_i)] \right\}. \quad (2)$$

Here, $n(x_i)$ is the effective photon spectrum integrated over impact parameter b [51,52], charge number $Z = 82$ for Pb, $x_i = E_i/E_{\text{beam}}$ is the ratio of the energy E_i of the emitted photons from ion i to the beam energy E_{beam} , $\bar{x}_i = x_i m_N b_{\text{min}}$, m_N is the mass of the nucleon $m_N = 0.9315$ GeV, the minimum of impact parameter b_{min} is set to be the nuclear radius $b_{\text{min}} = R_A \approx 1.2A^{1/3}$ fm = $6.09A^{1/3}$ GeV $^{-1}$ with $A = 208$ for Pb, and K_0, K_1 are the modified Bessel functions of the second kind of zero and first order, respectively. Where γ_L is the Lorentz factor $\gamma_L = E_{\text{beam}}/m_N$ [53].

At an LC, the initial photons are emitted from the incoming leptons through bremsstrahlung, and their energy distributions can be described using the WWA [54]. Within

this framework, the photon luminosity can be derived from the convolution of the PDF associated with the emitting leptons. The equivalent PDF of an electron or muon under the WWA is given by [54]

$$n(x_i) = \frac{\alpha}{2\pi} \left[\frac{1 + (1 - x_i)^2}{x_i} \ln\left(\frac{Q_{\text{max}}^2}{Q_{\text{min}}^2}\right) + \frac{2m_f^2 x_i}{Q_{\text{max}}^2} - \frac{2m_f^2 x_i}{Q_{\text{min}}^2} \right], \quad (3)$$

where $x_i = E_i/E_f$ is the fraction of the energy carried by the photon, α is the fine-structure constant, and m_f is the mass of incoming lepton, $m_f = m_e (m_\mu)$ for e^+e^- (muon) colliders. The photon virtuality bounds Q_{min}^2 and Q_{max}^2 are determined by

$$Q_{\text{min}}^2 = \frac{m_f^2 x_i^2}{1 - x_i}, \quad Q_{\text{max}}^2 = \frac{(E_f \theta_c)^2}{1 - x_i} + Q_{\text{min}}^2, \quad (4)$$

where θ_c denotes the maximal electron scattering angle allowed by the detector acceptance, with a typical value of $\theta_c = 32$ mrad used at e^+e^- colliders [55] and muon colliders. Given the clean initial state and high-precision environment of LCs, the $f^+f^- \xrightarrow{\gamma\gamma} f^+f^- \tau^+\tau^-$ process, with $f^\pm = e^\pm$ or $\mu^\pm$, provides a promising channel for studying the properties of the τ lepton.

We consider NLO EW effects and spin correlations in the subprocess

$$\gamma(p_1, \lambda_1) + \gamma(p_2, \lambda_2) \rightarrow \tau^-(k_1, s_1) + \tau^+(k_2, s_2), \quad (5)$$

where $p_{1,2}$ and $k_{1,2}$ denote the 4-momenta of the incoming photons and outgoing τ leptons, respectively. $\lambda_{1,2} = \pm 1$ are the photon helicities. The spin states of the τ^- and τ^+ are represented by the spin 4-vectors $s_{1,2}$, respectively, and $k_1 \cdot s_1 = k_2 \cdot s_2 = 0$. The leading order (LO) squared amplitudes for this subprocess have the following four terms: with

$$|M_{\text{LO}}|^2 = \frac{64\alpha^2\pi^2}{(1 - \beta^2 y^2)^2 m^2} [A_0 + B_0(p_1 \cdot s_2)(p_2 \cdot s_1) + C_0(p_1 \cdot s_1)(p_2 \cdot s_2) + D_0(s_1 \cdot s_2)], \quad (6)$$

with

$$\begin{aligned} A_0 &= m^2(1 - \beta^2[2(y^2 - 1) + \beta^2(2 - 2y^2 + y^4)]), \\ B_0 &= \beta^2(\beta^2 - 1)(y^2 - 1)(-1 + \beta y), \\ C_0 &= -\beta^2(\beta^2 - 1)(y^2 - 1)(1 + \beta y), \\ D_0 &= m^2(1 - 2\beta^2 + \beta^4(2 - 2y^2 + y^4)), \end{aligned} \quad (7)$$

where $\beta = \sqrt{1 - 4m^2/s_{\gamma\gamma}}$ denotes the velocity of the τ , and $s_{\gamma\gamma} = (p_1 + p_2)^2$ is the squared c.m. energy of two photons. The variable $y = \cos \theta$ with θ being the scattering angle between p_1 and k_1 in this frame.

The NLO EW results are rather lengthy and therefore not shown here. The production density matrix for $\gamma\gamma \rightarrow \tau^+\tau^-$ process is defined by

$$R_{\alpha_1\alpha_2,\beta_1\beta_2} = \sum_{\lambda_1,\lambda_2} \langle \tau^-(k_1, \alpha_2), \tau^+(k_2, \beta_2) | T | \gamma(p_1, \lambda_1), \gamma(p_2, \lambda_2) \rangle^* \times \langle \tau^-(k_1, \alpha_1), \tau^+(k_2, \beta_1) | T | \gamma(p_1, \lambda_1), \gamma(p_2, \lambda_2) \rangle, \quad (8)$$

where the bar denotes averaging over the initial photon polarizations, and T is the transition operator. The indices α and β are spin labels referring to τ^- and τ^+ , respectively. The decomposition of the matrix R in the spin spaces of τ^- and τ^+ is given by

$$R = \tilde{A} \mathbb{1} \otimes \mathbb{1} + \tilde{B}_i^+ \sigma^i \otimes \mathbb{1} + \tilde{B}_i^- \mathbb{1} \otimes \sigma^i + \tilde{C}_{ij} \sigma^i \otimes \sigma^j. \quad (9)$$

The first factor in each tensor products refers to the spin space of τ^- , and the second factor corresponds to that of τ^+ . The scalar coefficient $\tilde{A}$ represents the unpolarized production rate, while the vectors $\tilde{B}_i^+$ and $\tilde{B}_i^-$ correspond to the single spin polarization of τ^- and τ^+ , respectively. At LO in quantum field theory, the parity symmetry of the electromagnetic interaction ensures that the $\tilde{B}_i^+$ and $\tilde{B}_i^-$ vanish. Although weak corrections introduce parity-violating contributions, their numerical impact is negligible in comparison to τ -pair spin correlations coefficients and is neglected in this paper. The tensor $\tilde{C}_{ij}$ captures the spin correlations between the two τ leptons, and under the aforementioned approximation, the production density matrix is fully characterized by the coefficients $\tilde{A}$ and $\tilde{C}_{ij}$.

The tensor $\tilde{C}_{ij}$ can be further decomposed by introducing an orthonormal basis, which is defined as follows. In what follows, $\hat{k}$ denotes the flight direction of the τ^- in the $\tau^+\tau^-$ zero-momentum frame, and the unit vector $\hat{p} = \hat{p}_1$ specifies the direction of one of the incoming photons in this frame. The set of vectors introduced below then forms a right-handed orthonormal basis [49]:

$$\{\hat{r}, \hat{k}, \hat{n}\}: \quad \hat{r} = \frac{1}{r}(\hat{p} - y\hat{k}), \quad \hat{n} = \frac{1}{r}(\hat{p} \times \hat{k}), \\ y = \hat{k} \cdot \hat{p}, \quad r = \sqrt{1 - y^2}. \quad (10)$$

We expand the tensor $\tilde{C}_{ij}$ with respect¹ to the orthogonal basis in Eq. (10), utilizing rotational invariance, as follows:

$$\tilde{C}_{ij} = c_{rr}\hat{r}_i\hat{r}_j + c_{kk}\hat{k}_i\hat{k}_j + c_{nn}\hat{n}_i\hat{n}_j + c_{rk}(\hat{r}_i\hat{k}_j + \hat{k}_i\hat{r}_j) \\ + c_{kn}(\hat{k}_i\hat{n}_j + \hat{n}_i\hat{k}_j) + c_{rn}(\hat{r}_i\hat{n}_j + \hat{n}_i\hat{r}_j) \\ + c_{ijl}(c_r\hat{r}_l + c_k\hat{k}_l + c_n\hat{n}_l). \quad (11)$$

¹The expansion of $\tilde{C}_{ij}$ in a chosen basis is not unique because $\delta_{ij} = \hat{n}_i\hat{n}_j + \hat{r}_i\hat{r}_j + \hat{k}_i\hat{k}_j$.

The coefficients $c_{vv'}$ ($v, v' = r, k, n$) are functions of $s_{\gamma\gamma}$ and y .

The production density matrix R fully describes the $\tau^+\tau^-$ spin state. Connecting it to experimentally measurable quantities requires defining the spin correlation observables [56]

$$C_{ab} = 4\langle (\hat{\mathbf{a}} \cdot \vec{s}_1)(\hat{\mathbf{b}} \cdot \vec{s}_2) \rangle = \frac{\int \phi a_i \tilde{C}_{ij} b_j d\Pi_{\text{LIPS}}}{\int d\sigma}. \quad (12)$$

Here, $\vec{s}_1$ and $\vec{s}_2$ are the spin operators for τ^- and τ^+ respectively, the flux factor $\phi = 1/(2s_{\gamma\gamma})$, and a_i and b_j are the i th and j th components of $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$, respectively, where the $\hat{\mathbf{a}}$ ($\hat{\mathbf{b}}$) are the chosen spin quantization axes for τ^- (τ^+). In this paper, they are the axes defined in Eq. (10). Π_{LIPS} is the corresponding Lorentz invariant phase space. Equation (12) is defined with respect to the matrix elements for the production of the τ pair, which is related to the more familiar double spin asymmetries [56,57]

$$C_{ab} = \frac{\sigma(\uparrow\uparrow) + \sigma(\downarrow\downarrow) - \sigma(\uparrow\downarrow) - \sigma(\downarrow\uparrow)}{\sigma(\uparrow\uparrow) + \sigma(\downarrow\downarrow) + \sigma(\uparrow\downarrow) + \sigma(\downarrow\uparrow)}, \quad (13)$$

where the $\uparrow$ and $\downarrow$ refer to the spin orientations of the τ^+ and τ^- along their respective spin quantization axis, and the first (second) arrow refers to the spin state of the τ^+ (τ^-).

Numerical results and analysis. In this section, we adopt an $\alpha(0)$ scheme and present numerical results for the NLO EW corrections to spin correlation observables in the process $\gamma\gamma \rightarrow \tau^+\tau^-$ and to the total cross section in Eq. (1), considering both Pb-Pb UPCs and LC environments, and then discuss the quantum entanglement of this system.

The input parameters adopted in our calculations are specified as follows [1]:

$$m_W = 80.369 \text{ GeV}, \quad m_Z = 91.188 \text{ GeV}, \quad m_H = 125.20 \text{ GeV}, \\ m_t = 172.4 \text{ GeV}, \quad m_b = 4.183 \text{ GeV}, \quad m_c = 1.273 \text{ GeV}, \\ m_e = 0.511 \text{ MeV}, \quad m_\mu = 0.106 \text{ GeV}, \quad m = 1.777 \text{ GeV}, \\ \alpha^{-1}(0) = 137.036. \quad (14)$$

Total cross section and spin observables. We first discuss the inclusive cross section of the process and the spin observables. In Table I, we show the cross section at various nucleon-nucleon c.m. energies $\sqrt{s_{NN}}$, as available for Pb-Pb UPCs at the LHC [58,59]. The σ_{LO} , $\delta\sigma_{\text{NLO}}$, and σ_{NLO} denote the LO, pure NLO correction, and total NLO cross section, respectively, and the factor $\delta = \frac{\sigma_{\text{NLO}}}{\sigma_{\text{LO}}} - 1$ in the last column denotes the relative correction. These numerical results are given in the $\alpha(0)$ scheme, and for a comparison, results in the G_μ scheme can be found in our previous work [42]. Both results are consistent with Ref. [44], with minor discrepancies arising from difference in parametrization of photon flux and survival probability for the heavy ions. For LCs, we summarized the cross sections for e^+e^- and $\mu^+\mu^-$

TABLE I. Cross sections for $\gamma\gamma \rightarrow \tau^+\tau^-$ in Pb-Pb UPCs at different nucleon-nucleon c.m. energies $\sqrt{s_{NN}}$.

$\sqrt{s_{NN}}$ [TeV]	σ_{LO} [mb]	$\delta\sigma_{NLO}$ [mb]	σ_{NLO} [mb]	δ [%]
5.02	0.940	8.55×10^{-3}	0.949	0.909
5.36	1.003	9.44×10^{-3}	1.012	0.941
5.52	1.030	9.93×10^{-3}	1.040	0.964

TABLE II. Cross sections for $\gamma\gamma \rightarrow \tau^+\tau^-$ at e^+e^- colliders at different e^+e^- c.m. energies $\sqrt{s}$.

$\sqrt{s}$ [GeV]	σ_{LO} [nb]	$\delta\sigma_{NLO}$ [nb]	σ_{NLO} [nb]	δ [%]
250	0.5090	4.182×10^{-3}	0.513	0.821
380	0.5350	4.178×10^{-3}	0.539	0.781
1500	1.028	5.613×10^{-3}	1.034	0.546
3000	1.340	3.499×10^{-3}	1.344	0.261

initial states in Tables II and III at various benchmark c.m. energies, respectively, following the same notation as Table I.

For the UPC setup, as shown in Table I, the total cross section at NLO EW is of the order of about 1 mb, with EW correction contributing approximately 9 permille to LO at $\sqrt{s_{NN}} = 5.02$ TeV, 5.36 TeV, and 5.52 TeV. The relative correction factor δ in the last column increases with $\sqrt{s_{NN}}$, which means the EW effects become more significant at higher collision energies. On the contrary, for e^+e^- colliders, as presented in Table II, the relative contribution of NLO EW corrections decreases with c.m. energy. This trend shows that NLO EW corrections are more relevant at lower energies, while the impact diminishes in the high-energy regime considered. The total cross sections at NLO increase from 0.513 nb at $\sqrt{s} = 250$ GeV to 1.344 nb at $\sqrt{s} = 3$ TeV with the NLO EW correction also at about the permille level. For muon colliders, as shown in Table III, the total cross section is 0.501 nb at $\sqrt{s} = 3$ TeV, corresponding to a 3.62 permille relative NLO contribution. The cross section difference from Table II at $\sqrt{s} = 3$ TeV originates from the mass dependence of the PDF, resulting in a larger and more observable cross section for e^+e^- colliders than for muon colliders. At $\sqrt{s} = 10$ TeV, both the pure NLO correction and δ factor becomes negative, where the negative weak corrections dominate.

We turn to discuss spin observables, evaluated with respect to defined set of reference axes. The inclusive LO

TABLE III. Cross sections for $\gamma\gamma \rightarrow \tau^+\tau^-$ at $\mu^+\mu^-$ collider at different $\mu^+\mu^-$ c.m. energies $\sqrt{s}$.

$\sqrt{s}$ [TeV]	σ_{LO} [nb]	$\delta\sigma_{NLO}$ [nb]	σ_{NLO} [nb]	δ [%]
3	0.4990	1.807×10^{-3}	0.501	0.362
10	0.8290	-3.656×10^{-3}	0.826	-0.441

TABLE IV. Spin correlation coefficients C_{ij} in Pb-Pb UPCs at LO and NLO EW at different nucleon-nucleon c.m. energies $\sqrt{s_{NN}}$.

$\sqrt{s_{NN}}$ [TeV]		C_{ab}	C_{pp}	C_{nn}	C_{rr}	C_{kk}
5.02	LO	-0.0213	-0.3281	0.0199	-0.2998	
	NLO	-0.0214	-0.3230	0.0195	-0.2956	
5.36	LO	-0.0192	-0.3273	0.0214	-0.2968	
	NLO	-0.0193	-0.3223	0.0210	-0.2926	
5.52	LO	-0.0183	-0.3270	0.0221	-0.2955	
	NLO	-0.0184	-0.3220	0.0216	-0.2913	

TABLE V. Spin correlation coefficients C_{ij} for e^+e^- colliders at LO and NLO EW at different e^+e^- c.m. energies $\sqrt{s}$.

$\sqrt{s}$ [GeV]		C_{ab}	C_{pp}	C_{nn}	C_{rr}	C_{kk}
250	LO	0.0335	-0.3090	0.0580	-0.2183	
	NLO	0.0334	-0.3070	0.0579	-0.2167	
380	LO	0.0514	-0.3030	0.0695	-0.1909	
	NLO	0.0513	-0.3012	0.0694	-0.1893	
1500	LO	0.0705	-0.2973	0.0796	-0.1604	
	NLO	0.0703	-0.2956	0.0795	-0.1590	
3000	LO	0.0763	-0.2956	0.0825	-0.1512	
	NLO	0.0756	-0.2938	0.0824	-0.1506	

and NLO results for the spin correlation coefficients C_{ab} are presented in Table IV for UPCs, and in Tables V and VI for e^+e^- colliders and muon colliders, respectively. In these tables, we show the P-even and CP-even correlation coefficients such as C_{rr} , C_{nn} , C_{kk} and C_{pp} . The P-odd or CP-odd spin correlation coefficients, which originate from parity-violating weak contributions, are not included as their contributions are numerically small and can be neglected safely.

One can see from these tables, within the SM, the spin correlation coefficients C_{ab} exhibit a characteristic pattern that can be qualitatively understood from angular momentum conservation and the dominance of specific partial waves in different energy regimes. The behavior of C_{rr} and C_{kk} is particularly instructive. Near the $\tau^+\tau^-$ invariant mass threshold, the $\tau^+\tau^-$ system produced via photon fusion tends to be a 1S_0 state, a configuration which will be shown

TABLE VI. Spin correlation coefficients C_{ij} for muon colliders at LO and NLO EW at different $\mu^+\mu^-$ c.m. energies $\sqrt{s}$.

$\sqrt{s}$ [TeV]		C_{ab}	C_{pp}	C_{nn}	C_{rr}	C_{kk}
3	LO	0.0743	-0.2905	0.0799	-0.1515	
	NLO	0.0737	-0.2944	0.0814	-0.1536	
10	LO	0.0821	-0.2938	0.0853	-0.1419	
	NLO	0.0799	-0.2921	0.0852	-0.1533	

in the following subsection to yield antiparallel spins along any common quantization axis. When projected onto the individual $\hat{r}$ -axes or $\hat{k}$ -axes (the helicity axes), this antiparallel alignment manifests preferentially as $\uparrow\downarrow$ or $\downarrow\uparrow$ configurations, leading to negative values for both C_{rr} and C_{kk} in the threshold region. As the c.m. energy increases, the τ leptons become relativistic, and helicity conservation becomes the dominant constraint, driving a transition toward a 3S_1 -like state. Consequently, the spin projections along the $\hat{r}$ - and helicity axes shift towards $\uparrow\uparrow$ or $\downarrow\downarrow$ configurations. This leads to an increase in the values of C_{rr} and C_{kk} at higher energies, although the increase differs between collider environments due to variations in the photon spectrum and polarization. The rather large value of C_{nn} in the SM can be understood from total angular momentum conservation. The consistently small magnitude of C_{pp} across different energies and colliders suggests that spin correlations along the photon beam direction are generally suppressed.

The NLO EW corrections introduce only tiny modifications, primarily a slight rescaling of the correlation amplitudes without altering the dominant signs or energy-dependence trends. This provides a theoretically consistent and stable baseline for high-precision SM calculations.

Quantum entanglement. It is natural to investigate whether these correlations arise solely from classical spin alignment or contain genuine quantum entanglement features, with the establishment of the spin correlation coefficients characterizing the joint spin configurations of the $\tau^+\tau^-$ system. The extent of quantum entanglement can be quantified by the concurrence [24].

For the process we considered, in the nonrelativistic limit, the concurrence can be written as

$$\mathcal{C}(\rho) = -\frac{1}{2}(1 + C_{11} + C_{22} + C_{33}) = -\frac{1}{2}(1 + 3D), \quad (15)$$

where the scalar quantity

$$D = \frac{C_{11} + C_{22} + C_{33}}{3} = \frac{\text{tr}[C]}{3} \quad (16)$$

serves as a compact, rotation invariant criterion for entanglement, and C_{11} , C_{22} , and C_{33} represent the spin correlation coefficients corresponding to any set of orthogonal vectors. The sufficient condition for an entangled $\tau^+\tau^-$ state is $-1 \leq D < -\frac{1}{3}$, with $D = -1$ corresponding to maximal entanglement. Thus, quantum entanglement can be probed through a single spin correlation observable, as has been demonstrated in $t\bar{t}$ analyses at the LHC [25,26]. D is invariant under rotation and can therefore be evaluated in any orthonormal basis, such as $\{\hat{n}, \hat{r}, \hat{k}\}$ defined in Eq. (10). Our simulation across the inclusive phase space shows that the average value of quantum entanglement criterion D , is always greater than $-1/3$, thereby failing to satisfy the entanglement condition. In realistic analyses, NLO EW

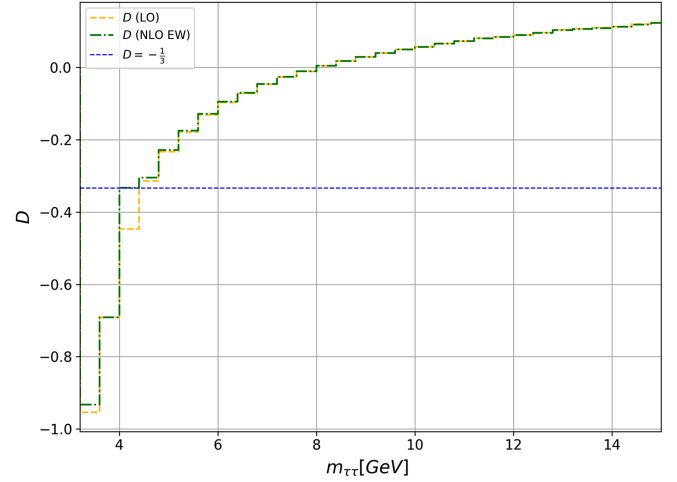


FIG. 1. Differential entanglement observable D is shown as a function of the τ -pair invariant mass $m_{\tau\tau}$ for $\sqrt{s_{NN}} = 5.02$ TeV in UPCs. The representative mass interval displayed is $m_{\tau\tau} \in [2m, 15]$ GeV. The yellow dashed and green dash-dotted curves represent the LO and NLO EW results, respectively. The horizontal blue dashed line indicates the entanglement threshold at $D = -1/3$. Negative values near $\tau^+\tau^-$ invariant mass threshold reflect the dominance of spin singlet configurations, while the transition toward positive D at higher masses signals the onset of spin-triplet states and reduced entanglement.

corrections to τ decays do influence its distributions, and potentially affect the spin-correlation observables such as D . However, this influence is minor, amounting to only a few percent as shown in Ref. [44].

To further explore quantum entanglement event by event, the observable D can be studied as a function of the invariant mass of the produced τ pair, defined as $m_{\tau\tau}$. Figure 1 plots the value of D for each bin as a function of the $m_{\tau\tau}$ in the interval $[2m, 15]$ GeV. The yellow dashed line shows the LO result, and the green dash-dotted line includes NLO EW corrections. The horizontal blue dashed line marks the entanglement threshold $D = -1/3$. Near $\tau^+\tau^-$ invariant mass threshold, both curves exhibit large negative values of D , well below the entanglement criterion, which originates from the dominance of low- β configurations. In this regime, the $\tau^+\tau^-$ system produced via photon fusion is predominantly in a spin singlet state 1S_0 , resulting in antiparallel spins along any common quantization axis and representing a maximally quantum entangled state. The NLO EW corrections induce modest upward shift while preserving the overall shape, slightly reducing the degree of entanglement near $\tau^+\tau^-$ invariant mass threshold.

Summary. In this paper, we present a comprehensive analysis of the process $\gamma\gamma \rightarrow \tau^+\tau^-$ including $\tau^+\tau^-$ spin information at NLO EW accuracy both in Pb-Pb UPCs and at LCs. We provide the predictions for the corresponding cross section and the spin correlation coefficients C_{ab} . For Pb-Pb UPCs, the NLO EW corrected cross sections are

found to be of the order of 1 mb with positive NLO EW corrections at the level of a few permille in the energy range we considered. For LCs, the e^+e^- collider has superior observability over the muon collider at equal c.m. energy. For the spin correlation coefficients, C_{rr} and C_{kk} tend to take negative values close to the $\tau^+\tau^-$ invariant mass threshold where antiparallel spin configurations dominate, and C_{nn} remains large in magnitude due to total angular momentum conservation, whereas C_{pp} stays consistently small. The NLO EW corrections produce small numerical shifts and do not alter these qualitative features. Using the available spin coefficients C_{nn} , C_{rr} , and C_{kk} , we investigate quantum entanglement in the $\tau^+\tau^-$ system by evaluating the rotation invariant entanglement criterion D . The inclusive value of D remains above the entanglement threshold

$D = -1/3$, indicating no net entanglement after full phase space integration. However, a differential study as a function of the invariant mass $m_{\tau\tau}$ reveals that $D < -1/3$ near the threshold region where the total spin configuration of $\tau^+\tau^-$ tends to be a 1S_0 state, so that a genuinely entangled quantum state can be confirmed. The study in this paper can provide a helpful analysis template to study the τ -pair production induced by $\gamma\gamma$ collision at high-energy colliders.

Acknowledgments. This work is supported in part by National Natural Science Foundation of China under Grants No. 12321005, No. 12235008, and No. 12405121.

Data availability. The data are not publicly available. The data are available from the authors upon reasonable request.

-
- [1] https://pdg.lbl.gov/2024/reviews/contents_sports.html.
 - [2] S. Schael *et al.* (ALEPH Collaboration), *Phys. Rep.* **421**, 191 (2005).
 - [3] S. Navas *et al.* (Particle Data Group Collaboration), *Phys. Rev. D* **110**, 030001 (2024).
 - [4] A. Heister *et al.* (ALEPH Collaboration), *Eur. Phys. J. C* **20**, 401 (2001).
 - [5] J. Abdallah *et al.* (DELPHI Collaboration), *Eur. Phys. J. C* **36**, 283 (2004).
 - [6] K. Abe *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **99**, 011801 (2007).
 - [7] J. Y. Zhang (BESIII Collaboration), *SciPost Phys. Proc.* **1**, 004 (2019).
 - [8] M. Aaboud *et al.* (ATLAS Collaboration), *Phys. Rev. D* **99**, 072001 (2019).
 - [9] A. Hayrapetyan *et al.* (CMS Collaboration), *Rep. Prog. Phys.* **87**, 107801 (2024).
 - [10] G. Aad *et al.* (ATLAS Collaboration), *Eur. Phys. J. C* **83**, 563 (2023).
 - [11] G. Aad *et al.* (ATLAS Collaboration), *Phys. Rev. Lett.* **131**, 151802 (2023).
 - [12] A. Tumasyan *et al.* (CMS Collaboration), *Phys. Rev. Lett.* **131**, 151803 (2023).
 - [13] J. H. Kuhn and A. Santamaria, *Z. Phys. C* **48**, 445 (1990).
 - [14] K. Cheng, T. Han, and M. Low, *Phys. Lett. B* **868**, 139675 (2025).
 - [15] J. Bernabeu, G. A. Gonzalez-Sprinberg, J. Papavassiliou, and J. Vidal, *Nucl. Phys. B* **790**, 160 (2008).
 - [16] X. Chen and Y. Wu, *J. High Energy Phys.* **10** (2019) 089.
 - [17] S. Bar-Shalom, G. Eilam, and A. Soni, *Phys. Rev. Lett.* **80**, 4629 (1998).
 - [18] B. K. Bullock, K. Hagiwara, and A. D. Martin, *Nucl. Phys. B* **395**, 499 (1993).
 - [19] D. Bodrov, *Int. J. Mod. Phys. A* **39**, 2442006 (2024).
 - [20] P.-C. Lu, Z.-G. Si, and H. Zhang, *Phys. Rev. D* **112**, 075039 (2025).
 - [21] T. Han, M. Low, and Y. Su, [arXiv:2501.04801](https://arxiv.org/abs/2501.04801).
 - [22] X. Sun, Y. Wu, and X. Zhou, *Chin. Phys.* **49**, 113001 (2025).
 - [23] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, *Phys. Rev. Lett.* **23**, 880 (1969).
 - [24] W. K. Wootters, *Phys. Rev. Lett.* **80**, 2245 (1998).
 - [25] G. Aad *et al.* (ATLAS Collaboration), *Nature (London)* **633**, 542 (2024).
 - [26] A. Hayrapetyan *et al.* (CMS Collaboration), *Rep. Prog. Phys.* **87**, 117801 (2024).
 - [27] R. Ashby-Pickering, A. J. Barr, and A. Wierzychucka, *J. High Energy Phys.* **05** (2023) 020.
 - [28] Y. Wu, R. Jiang, A. Ruzi, Y. Ban, X. Yan, and Q. Li, *Phys. Rev. D* **111**, 036008 (2025).
 - [29] T. Han, M. Low, and Y. Su, *J. High Energy Phys.* **10** (2025) 217.
 - [30] M. Fabbrichesi and L. Marzola, *Phys. Rev. D* **110**, 076004 (2024).
 - [31] K. Ehatäht, M. Fabbrichesi, L. Marzola, and C. Veelken, *Phys. Rev. D* **109**, 032005 (2024).
 - [32] Y. Zhang, B.-H. Zhou, Q.-B. Liu, S. Li, S.-C. Hsu, T. Han, M. Low, and T. A. Wu, [arXiv:2504.01496](https://arxiv.org/abs/2504.01496).
 - [33] C. F. von Weizsacker, *Z. Phys.* **88**, 612 (1934).
 - [34] E. J. Williams, *Phys. Rev.* **45**, 729 (1934).
 - [35] M. Dyndal (ALICE, ATLAS, CMS, LHCb, MoEDAL Collaborations), *Proc. Sci., LHCP2022* (**2023**) 215.
 - [36] S. Frixione, M. L. Mangano, P. Nason, and G. Ridolfi, *Phys. Lett. B* **319**, 339 (1993).
 - [37] M. Dong *et al.* (CEPC Study Group Collaboration), [arXiv:1811.10545](https://arxiv.org/abs/1811.10545).
 - [38] J. de Blas *et al.* (CLIC Collaboration), *CERN Yellow Rep. Monogr.* **3**, 1 (2018).
 - [39] J. P. Delahaye, M. Diemoz, K. Long, B. Mansoulié, N. Pastrone, L. Rivkin, D. Schulte, A. Skrinsky, and A. Wulzer, [arXiv:1901.06150](https://arxiv.org/abs/1901.06150).
 - [40] N. Bartosik *et al.* (Muon Collider Collaboration), [arXiv:2203.07964](https://arxiv.org/abs/2203.07964).
 - [41] H.-S. Shao and D. d'Enterria, *J. High Energy Phys.* **02** (2025) 023.

- [42] J. Jiang, P.-C. Lu, Z.-G. Si, H. Zhang, and X.-Y. Zhang, *Phys. Rev. D* **111**, 036023 (2025).
- [43] H.-S. Shao and L. Simon, *J. High Energy Phys.* **07** (2025) 020.
- [44] S. Dittmaier, T. Engel, J. L. H. Ariza, and M. Pellen, *J. High Energy Phys.* **08** (2025) 051.
- [45] A. A. Billur and M. Koksals, *Phys. Rev. D* **89**, 037301 (2014).
- [46] A. Rajaraman, J. N. Howard, R. Riley, and T. M. P. Tait, *Lett. High Energy Phys.* **2**, 5 (2019).
- [47] M. Koksals, *J. Phys. G* **46**, 065003 (2019).
- [48] Z. Wang, *Proc. Sci., ICHEP2024* (**2025**) 327 [arXiv: 2410.12663].
- [49] W. Bernreuther, D. Heisler, and Z.-G. Si, *J. High Energy Phys.* **12** (2015) 026.
- [50] R. N. Cahn and J. D. Jackson, *Phys. Rev. D* **42**, 3690 (1990).
- [51] H.-S. Shao and D. d’Enterria, *J. High Energy Phys.* **09** (2022) 248.
- [52] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1998).
- [53] G. Baur, K. Hencken, D. Trautmann, S. Sadovsky, and Y. Kharlov, *Phys. Rep.* **364**, 359 (2002).
- [54] V. M. Budnev, I. F. Ginzburg, G. V. Meledin, and V. G. Serbo, *Phys. Rep.* **15**, 181 (1975).
- [55] M. Klasen, B. A. Kniehl, L. N. Mihaila, and M. Steinhauser, *Phys. Rev. Lett.* **89**, 032001 (2002).
- [56] W. Bernreuther, A. Brandenburg, Z. G. Si, and P. Uwer, *Nucl. Phys.* **B690**, 81 (2004).
- [57] W. Bernreuther, A. Brandenburg, Z. G. Si, and P. Uwer, *Phys. Rev. Lett.* **87**, 242002 (2001).
- [58] R. Bruce *et al.*, *J. Phys. G* **47**, 060501 (2020).
- [59] D. d’Enterria *et al.*, *J. Phys. G* **50**, 050501 (2023).