

Recoil corrections to double heavy and hidden heavy molecular pentaquark states with an SU(3) antitriplet baryon

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Recoil corrections, which appear at order $\mathcal{O}(\frac{1}{M})$, turn out to be crucial for the pentaquark molecules with heavy flavor. In the past, such corrections were typically regarded as negligible for the formation of molecular states. However, our research manifests their important impacts on the spectra of possible baryon-meson bound states. In certain cases, the binding energy sharply decreases after considering the recoil corrections. Using the one-boson-exchange model, we have studied a series of double heavy and hidden heavy pentaquark states with an SU(3) antitriplet baryon, including $\Xi_c \bar{D}^{(*)}$, $\Lambda_c \bar{D}^{(*)}$, $\Xi_c D^{(*)}$, $\Lambda_c D^{(*)}$, along with their bottom analogues. Both the S - D wave mixing effect and the recoil corrections are incorporated in the investigation. Our results indicate that such corrections, working against the stability of bound-state solutions in certain isospin-singlet channels, cannot be ignored. For the $\Xi_c \bar{D}$ and $\Xi_c D$ systems with $I(J^P) = 0(\frac{1}{2}^-)$, the $\Xi_c \bar{D}^*$ and $\Xi_c D^*$ systems, as well as their bottom counterparts, with $I(J^P) = 0(\frac{1}{2}^-)$ and $0(\frac{3}{2}^-)$, and the $\Lambda_b \bar{B}^*$ system with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ and $\frac{1}{2}(\frac{3}{2}^-)$, although the bound-state solution can be found both with and without considering the recoil corrections, the inclusion of them weakens the attractive interaction of the molecular structures, as evidenced by smaller binding energies and larger root-mean-square radii in the refined results. This phenomenon is especially prominent in the $\Xi_c \bar{D}^*$ and $\Xi_c D^*$ systems.

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I. INTRODUCTION

Given that quantum chromodynamics (QCD), the first principle of strong interaction, is nonperturbative in the low-energy region, physicists have developed various approaches to explore the nature of hadrons. One of the earliest and most influential frameworks is the traditional quark model [1,2], in which hadrons are classified into baryons and mesons, consisting of three quarks (qqq) and a quark-antiquark pair ($q\bar{q}$), respectively. Although the traditional quark model has successfully described and predicted many experimental observations throughout the 20th century, it is widely believed that exotic states, which do not easily fit into the conventional hadron spectrum, may exist.

With the advances in technology and the accumulation of experimental data, a series of exotic candidates with heavy flavor including XYZ states [3–8], P_c states [9,10], and T_{cc}

states [11] have been emerging over the past few years, making their theoretical study a topic of considerable interest.

The spectra and dynamical behavior of the observed exotic states have been elucidated through various theoretical interpretations, such as hadronic molecule [12–15], compact multiquark [16–18], hybrid [19–22], and glueball [23–25].

Among these, the molecular picture is prominent and widely adopted, largely because most experimental candidates lie close to the threshold of a hadron pair. Let us take some well-known examples. $X(3872)$ and $Y(4260)$ are often interpreted as $D\bar{D}^*$ and $D\bar{D}_{(1)}$ molecular tetraquarks, respectively [26–31]. $P_c(4380)$, $P_c(4440)$, and $P_c(4457)$ are commonly regarded as $\Sigma_c \bar{D}^*$ or $\Sigma_c^* \bar{D}$ molecular pentaquarks [32–42]. Moreover, this picture demonstrates powerful predictive capabilities in previous works, which provides valuable guidance for future experimental investigations. In 2013, DD^* system with the quantum numbers of $I(J^P) = 0(1^+)$ was theoretically believed to be a possible molecular state candidate, which was later confirmed by the LHCb observation of $T_{cc}^+(3875)$ in 2021 [43,44]. Recently, the molecular picture has also been widely adopted to predict the pentaquark states. Researchers predicted a dynamically generated pentaquark state strongly coupled to $\Omega_c \bar{D}$ and $\Xi_c' \bar{D}_s$ channels [45,46].

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A series of hidden charm strange molecular baryon candidates were also proposed by theoretical physicists [47].

Inspired by Yukawa's research on nuclear forces in 1935 [48], the potential model emerged and developed as a powerful tool for describing the effective interactions of hadronic molecules. This approach postulates that the strong interaction between the constituent hadrons can be equivalently treated as being mediated by meson exchanges. Its simplest and earliest version is the one-pion-exchange model, which includes only the contribution from π exchange. The one-boson-exchange (OBE) model was later proposed to tackle the intermediate- and long-range forces, in which the π , η , σ , ρ , and ω exchanges are simultaneously taken into account. Over the following decades, various dynamical effects including the S - D wave mixing effect, the coupled-channel effect, the isospin breaking effect, and the recoil corrections have been incorporated into the OBE model to promote its accuracy and predictive power. Among them, the recoil corrections are generally perceived insignificant in the heavy flavor systems because they correspond to higher-order terms suppressed by the large masses of constituent hadrons. Nevertheless, our recent study on the $D_1 D_1$ and the $D_1 \bar{D}_1$ molecules indicates that, in certain cases, the impact of recoil corrections has been severely underestimated, thus calling for more systematic and in-depth investigations [49].

Recently, LHCb Collaboration reported robust evidence for the $P_{cs}(4338)$ and $P_{cs}(4459)$ states [10,50], which has attracted significant theoretical attention. Given that their spectra are close to the threshold of $\Xi_c \bar{D}$ and $\Xi_c \bar{D}^*$, respectively, some previous research investigated them within the hadronic molecular picture [42,51–61]. Additionally, a latest study indicates that the $P_{cs}(4338)$ are more likely to be interpreted as a $\Xi_c \bar{D}$ molecule rather than a compact state [62]. However, several aspects remain unexplored. Within the flavor SU(3) framework, Ξ_c and Λ_c both belong to the $\bar{\mathbf{3}}$ representation, naturally motivating extensive studies to the systems containing Λ_c . Furthermore, as the hidden charm molecular states are the possible candidates of P_{cs} states, their open charm counterparts also represent promising double charm T_{cc} candidates. Additionally, the bottom partners of all such systems are of equal interest. Consequently, a comprehensive investigation of double heavy and hidden heavy molecular pentaquark states with an antitriplet baryon is clearly warranted.

This work is dedicated to the ground state spectra of possible double heavy and hidden heavy molecular pentaquarks with an antitriplet baryon including $\Xi_c \bar{D}^{(*)}$, $\Lambda_c \bar{D}^{(*)}$, $\Xi_c D^{(*)}$, $\Lambda_c D^{(*)}$, and their bottom analogues. We employ the OBE model and consider the S - D wave mixing effect. Our primary objective is to examine the influence of recoil corrections on the spectra of molecular state

solutions. The momentum-related terms are preserved up to $\mathcal{O}(1/M^2)$, where M denotes the mass of one of the components. In order to elucidate the impacts of recoil corrections, other higher-order effects, such as the coupled-channel effect, are not included. This allows us to clearly attribute any spectral changes to the recoil effect itself.

The paper is organized as follows. After the Introduction, the formalism of the discussed systems will be shown in Sec. II. With the obtained effective potentials, we will present and analyze the computational results for the mass spectrum of each system in Sec. III. Finally, we will give the overall discussion and conclusion in Sec. IV. Some intermediate results will be listed in the Appendix.

II. FORMALISM

In the following section, we will construct the ground state wave functions and the relativistic Lagrangian, and derive the effective potentials for the hidden charm systems. Additionally, the derived potentials are then extended to open-charm systems via the G -parity rule. Apart from the difference in the heavy flavor hadron masses, the formalism of their bottom analogies is identical.

A. Wave functions

The wave function of a baryon-antimeson system is the product of its isospin, spatial, and spin components. The isospin wave function yields an isospin factor in the effective potential, whereas the spatial and wave functions collectively govern the structure of the ground state

$$\psi^{\text{tot}} = \psi^I \otimes \psi^J = \psi^I \otimes \psi^L \otimes \psi^S. \quad (1)$$

First, we discuss the isospin wave function ψ^I . The Ξ_c baryon and the $D^{(*)}$ meson each carry isospin $I = \frac{1}{2}$, while the Λ_c baryon is an isospin singlet ($I = 0$). Therefore, the $\Xi_c \bar{D}^{(*)}$ system can couple to both $I = 1$ and $I = 0$ channels, while the $\Lambda_c \bar{D}^{(*)}$ system is restricted to $I = \frac{1}{2}$ channel. We list the explicit isospin wave functions for these two systems in Table I and Table II, respectively. Due to isospin conservation in the strong interaction, the effective potentials only depend on the total isospin I , and are independent of its third component I_3 .

TABLE I. The isospin wave functions for the $\Xi_c \bar{D}^{(*)}$ system.

$ I, I_3\rangle$	$\Xi_c \bar{D}^{(*)}$ system
$ 1, 1\rangle$	$ \Xi_c^+ \bar{D}^{(*)0}\rangle$
$ 1, 0\rangle$	$\frac{1}{\sqrt{2}}(\Xi_c^+ \bar{D}^{(*)-}\rangle + \Xi_c^0 \bar{D}^{(*)0}\rangle)$
$ 1, -1\rangle$	$ \Xi_c^0 \bar{D}^{(*)-}\rangle$
$ 0, 0\rangle$	$\frac{1}{\sqrt{2}}(\Xi_c^+ \bar{D}^{(*)-}\rangle - \Xi_c^0 \bar{D}^{(*)0}\rangle)$

TABLE II. The isospin wave functions for the $\Lambda_c \bar{D}^{(*)}$ system.

$ I, I_3\rangle$	$\Lambda_c \bar{D}^{(*)}$ system
$ \frac{1}{2}, \frac{1}{2}\rangle$	$ \Lambda_c^+ \bar{D}^{(*)0}\rangle$
$ \frac{1}{2}, -\frac{1}{2}\rangle$	$ \Lambda_c^+ \bar{D}^{(*)-}\rangle$

Then, we consider the spatial and the spin wave functions together. Due to the centrifugal barrier, a bound state solution is more likely to appear in the ground states, and that is why the higher partial waves are excluded in this research. The ground state of the systems without tensor force interaction is the pure S wave, whereas the ground state of the systems with tensor force interaction is the superposition of the S and the D wave for such interaction leads to the S - D wave mixing.

For the $\Xi_c \bar{D}$ and $\Lambda_c \bar{D}$ systems, the total angular momentum only couple to $J = \frac{1}{2}$, and the ground state reads

$$|\psi^{J=\frac{1}{2}}\rangle = R(r)|^2 S_{\frac{1}{2}}. \quad (2)$$

For the $\Xi_c \bar{D}^*$ and $\Lambda_c \bar{D}^*$ systems, both $J = \frac{1}{2}$ and $J = \frac{3}{2}$ channels are accessible, and the ground states read

$$|\psi^{J=\frac{1}{2}}\rangle = R_s(r)|^2 S_{\frac{1}{2}} + R_d(r)|^4 D_{\frac{1}{2}}, \quad (3)$$

$$|\psi^{J=\frac{3}{2}}\rangle = R_{s1}(r)|^4 S_{\frac{3}{2}} + R_{d1}(r)|^2 D_{\frac{3}{2}} + R_{d2}(r)|^4 D_{\frac{3}{2}}. \quad (4)$$

The D wave coefficients R_d , R_{d1} , and R_{d2} are nonzero only when recoil corrections are included. This is because the tensor force related terms emerge in the effective potentials only if the momentum-related terms are considered in our cases.

Based on the above analysis, all possible channels of the hidden charm systems are summarized as follows. For the $\Xi_c \bar{D}$ system, channels with $I(J^P) = 0(\frac{1}{2}^-), 1(\frac{1}{2}^-)$ are allowed; for the $\Lambda_c \bar{D}$ system, only the channel with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ is accessible; for the $\Xi_c \bar{D}^*$ system, channels with $I(J^P) = 0(\frac{1}{2}^-), 0(\frac{3}{2}^-), 1(\frac{1}{2}^-), 1(\frac{3}{2}^-)$ are possible; for the $\Lambda_c \bar{D}^*$ system, channels with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-), \frac{1}{2}(\frac{3}{2}^-)$ are supported.

B. Relativistic effective Lagrangians and effective potentials

The relativistic effective Lagrangians are supposed to include all possible structures satisfying the symmetries of the strong interaction—namely, parity, charge conjugation, and isospin conservation. Besides, they should degenerate to its nonrelativistic form after applying the heavy quark limit. Therefore, the coupling constants in this work are extracted from previous research that did not consider the recoil corrections. Within the OBE model, pseudoscalar, scalar, and vector meson exchanges contribute to the interaction. However, previous phenomenological analyses have shown that the coupling constants of the vertices

between baryons in the $\bar{\mathbf{3}}$ representation and the pseudo-scalar mesons are equal to zero [63].

We introduce the following notations to represent the constituent hadrons and the exchanged light mesons

$$\Lambda_c = \Lambda_c^+, \quad \Xi_c = \begin{pmatrix} \Xi_c^+ \\ \Xi_c^0 \end{pmatrix}, \quad \bar{D}^{(*)} = \begin{pmatrix} \bar{D}^{(*)0} \\ \bar{D}^{(*)-} \end{pmatrix},$$

$$\sigma = \sigma, \quad V_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \end{pmatrix}_\mu. \quad (5)$$

The Lagrangians for interactions between $\bar{\mathbf{3}}$ baryons and the exchanged light mesons read

$$\mathcal{L}_{\Xi_c \Xi_c \sigma}^R = g_{\Xi_c \Xi_c \sigma} \bar{\Xi}_c \sigma \Xi_c, \quad (6)$$

$$\mathcal{L}_{\Xi_c \Xi_c V}^R = g_{\Xi_c \Xi_c V} (\bar{\Xi}_c \gamma^\mu \Xi_c) V_\mu + \frac{1}{4M_{\Xi_c}} f_{\Xi_c \Xi_c V} (\bar{\Xi}_c \sigma^{\mu\nu} \Xi_c) (\partial_\mu V_\nu - \partial_\nu V_\mu), \quad (7)$$

$$\mathcal{L}_{\Lambda_c \Lambda_c \sigma}^R = g_{\Lambda_c \Lambda_c \sigma} \bar{\Lambda}_c \sigma \Lambda_c, \quad (8)$$

$$\mathcal{L}_{\Lambda_c \Lambda_c V}^R = g_{\Lambda_c \Lambda_c V} (\bar{\Lambda}_c \gamma^\mu \Lambda_c) V_\mu + \frac{1}{4M_{\Lambda_c}} f_{\Lambda_c \Lambda_c V} (\bar{\Lambda}_c \sigma^{\mu\nu} \Lambda_c) (\partial_\mu V_\nu - \partial_\nu V_\mu). \quad (9)$$

The Lagrangians for interactions between $\bar{D}^{(*)}$ mesons and the exchanged light mesons read

$$\mathcal{L}_{\bar{D} \bar{D} \sigma}^R = -2g_\sigma M_D \bar{D} \sigma \bar{D}^\dagger, \quad (10)$$

$$\mathcal{L}_{\bar{D} \bar{D} V}^R = -i \frac{\sqrt{2}}{2} \beta g_V (\bar{D} \overleftrightarrow{\partial}^\mu \bar{D}^\dagger) V_\mu, \quad (11)$$

$$\mathcal{L}_{\bar{D}^* \bar{D}^* \sigma}^R = 2g_\sigma M_{D^*} \bar{D}^{*\mu} \sigma \bar{D}_\mu^{*\dagger}, \quad (12)$$

$$\mathcal{L}_{\bar{D}^* \bar{D}^* V}^R = i \frac{\sqrt{2}}{2} \beta g_V (\bar{D}^{*\nu} \overleftrightarrow{\partial}^\mu \bar{D}_\nu^{*\dagger}) V_\mu - i 2\sqrt{2} \lambda g_V M_{D^*} \bar{D}^{*\mu} (\partial_\mu V_\nu - \partial_\nu V_\mu) \bar{D}^{*\nu\dagger}. \quad (13)$$

In Table III, we collect the numerical values of the coupling constants and the masses of the hadrons used in our calculations.

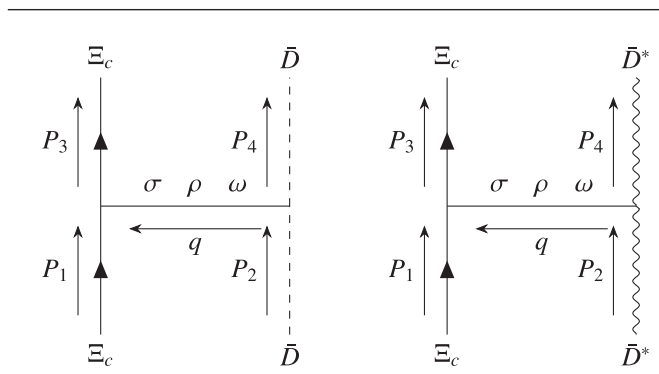
Each constituent meson and baryon provides a polarization vector ϵ and a spinor u , respectively. In the rest frame, they read

$$\epsilon = (0, \mathbf{\epsilon}), \quad u = \sqrt{M}(\xi, 0). \quad (14)$$

To derive the polarization vector ϵ and the spinor u in the laboratory frame, we make a boost to Eq. (14), and the results read

$$c^{\text{Lab}} = \left(\frac{\mathbf{p} \cdot \boldsymbol{\epsilon}}{m}, \boldsymbol{\epsilon} + \frac{\mathbf{p}(\mathbf{p} \cdot \boldsymbol{\epsilon})}{m(p_0 + m)} \right),$$

$$u^{\text{Lab}} = \sqrt{E + M} \left(\xi, \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E + M} \xi \right). \quad (15)$$



The scattering processes for the hidden charm systems are depicted schematically in the Feynman diagrams above. We firstly define the inter momentum $\mathbf{q}$ and the external momentum $\mathbf{k}$

$$\mathbf{q} = \mathbf{P}_3 - \mathbf{P}_1, \quad \mathbf{k} = \frac{1}{2}(\mathbf{P}_3 + \mathbf{P}_1), \quad (16)$$

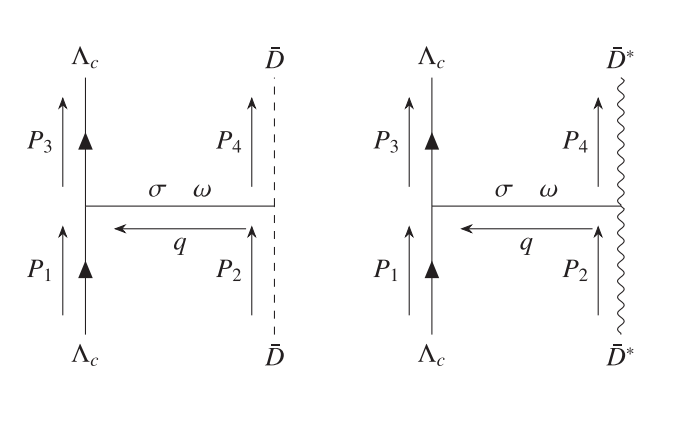
which represent the motion of the exchanged mesons and the constituent hadrons separately.

Applying the firstborn approximation, the effective potential $V(\mathbf{q}, \mathbf{k})$ can be obtained from the invariant scattering amplitude

TABLE III. The masses of the hadrons and the coupling constants used in this study. The masses are extracted from the Particle Data Group's results [64].

	Mass (MeV)	Coupling constants
Scalar	$m_\sigma = 600$	$g_{\Xi_c \Xi_c \sigma}(g_{\Lambda_c \Lambda_c \sigma}) = -3.65$ [51]
Vector	$m_\rho = 775.26$	$g_{\Xi_c \Xi_c \rho}(g_{\Lambda_c \Lambda_c \rho}) = 4.59$ [65]
	$m_\omega = 782.66$	$f_{\Xi_c \Xi_c V}(f_{\Lambda_c \Lambda_c V}) = -4.59$ [65]
		$g_\sigma = 0.761$ [66]
		$g_V = 5.83$ [67]
Heavy flavor	$m_D = 1864.8$	$\beta = 0.9$ [68]
	$m_{D^*} = 2006.9$	$\lambda = -0.56 \text{ GeV}^{-1}$ [67]
	$m_B = 5279.7$	
	$m_{B^*} = 5324.8$	
	$m_{\Xi_c} = 2470.4$	
	$m_{\Xi_b} = 5791.9$	
	$m_{\Lambda_c} = 2286.5$	
	$m_{\Lambda_b} = 5619.6$	

Using the effective Lagrangians, we extract the Feynman rules for the vertices and calculate the corresponding scattering amplitudes. The effective potentials are subsequently derived in the momentum representation, and then they are transformed into the coordinate space.



$$V(\mathbf{q}, \mathbf{k}) = T_{fi} = -\frac{\mathcal{M}}{\sqrt{2E_1}\sqrt{2E_2}\sqrt{2E_3}\sqrt{2E_4}}, \quad (17)$$

where E_1 – E_4 are energies of the initial and final state hadrons.

By means of the Fourier transformation, we can derive the effective potential in the coordinate space

$$V(\mathbf{r}, \mathbf{r}', \nabla_{\mathbf{r}}, \nabla_{\mathbf{r}}') = \frac{1}{(2\pi)^6} \iint d^3\mathbf{q} d^3\mathbf{k} \mathcal{F}(\mathbf{q})^2 V(\mathbf{q}, \mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}' - \mathbf{r}) + i\frac{1}{2}\mathbf{q} \cdot (\mathbf{r}' + \mathbf{r})}, \quad (18)$$

where $\mathcal{F}(\mathbf{q})$ is the form factor to describe the non-point-like structures attached to each vertex. With the form factor, the ultraviolet divergence can be well controlled. Here, we apply the monopole form factor

$$\mathcal{F}(\mathbf{q}) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}, \quad (19)$$

given that it achieved an exciting success in explaining the spectrum of deuteron. Λ here is an input cutoff parameter.

The effective potentials for the $\Xi_c \bar{D}$ and the $\Xi_c \bar{D}^*$ systems are given below. The effective potentials share the similar form in the $\Lambda_c \bar{D}$ and the $\Lambda_c \bar{D}^*$ systems except for the baryon masses and the isospin factors. We expand the potentials to $\frac{1}{M^2}$ order, where M represents the masses of the constituent hadrons. The recoil related terms initially emerge at the $\frac{1}{M}$ order. The results of the Fourier transformations in Eq. (18) are listed in the Appendix.

For the $\Xi_c \bar{D}$ system, the σ exchange potential reads

$$V_{\text{scalar}}^{\text{Lab}[\Xi_c \bar{D}]}(\mathbf{r}, \nabla) = C_\sigma \left[\frac{m_\sigma}{4\pi} g_{\Xi_c \Xi_c \sigma} g_\sigma H_0 - \frac{m_\sigma^3}{16\pi M_D^2} g_{\Xi_c \Xi_c \sigma} g_\sigma H_1 - \frac{m_\sigma^3}{32\pi M_{\Xi_c}^2} g_{\Xi_c \Xi_c \sigma} g_\sigma H_1 \right. \\ \left. + \frac{m_\sigma^3}{8\pi M_{\Xi_c}^2} g_{\Xi_c \Xi_c \sigma} g_\sigma H_2(\mathbf{r} \cdot \nabla) + \frac{m_\sigma^3}{8\pi M_D^2} g_{\Xi_c \Xi_c \sigma} g_\sigma H_2(\mathbf{r} \cdot \nabla) \right. \\ \left. + \frac{m_\sigma}{8\pi M_{\Xi_c}^2} g_{\Xi_c \Xi_c \sigma} g_\sigma H_0 \nabla^2 + \frac{m_\sigma}{8\pi M_D^2} g_{\Xi_c \Xi_c \sigma} g_\sigma H_0 \nabla^2 + \frac{m_\sigma^3}{16\pi M_{\Xi_c}^2} g_{\Xi_c \Xi_c \sigma} g_\sigma H_2(\boldsymbol{\sigma} \cdot \mathbf{L}) \right], \quad (20)$$

while the vector (ρ, ω) exchange potential reads

$$V_{\text{vector}}^{\text{Lab}[\Xi_c \bar{D}]}(\mathbf{r}, \nabla) = \sum_\alpha C_\alpha \left[\frac{\sqrt{2}m_v}{8\pi} \beta g_V g_{\Xi_c \Xi_c V} H_0 - \frac{\sqrt{2}m_v^3}{64\pi M_{\Xi_c}^2} \beta g_V g_{\Xi_c \Xi_c V} H_1 + \frac{\sqrt{2}m_v^3}{32\pi M_{\Xi_c} M_D} \beta g_V g_{\Xi_c \Xi_c V} H_1 \right. \\ \left. - \frac{\sqrt{2}m_v^3}{32\pi M_{\Xi_c}^2} \beta g_V f_{\Xi_c \Xi_c V} H_1 - \frac{\sqrt{2}m_v^3}{8\pi M_{\Xi_c} M_D} \beta g_V g_{\Xi_c \Xi_c V} H_2(\mathbf{r} \cdot \nabla) - \frac{\sqrt{2}m_v}{8\pi M_{\Xi_c} M_D} \beta g_V g_{\Xi_c \Xi_c V} H_0 \nabla^2 \right. \\ \left. - \frac{\sqrt{2}m_v^3}{32\pi M_{\Xi_c}^2} \beta g_V g_{\Xi_c \Xi_c V} H_2(\boldsymbol{\sigma} \cdot \mathbf{L}) - \frac{\sqrt{2}m_v^3}{16\pi M_{\Xi_c} M_D} \beta g_V g_{\Xi_c \Xi_c V} H_2(\boldsymbol{\sigma} \cdot \mathbf{L}) \right. \\ \left. - \frac{\sqrt{2}m_v^3}{16\pi M_{\Xi_c}^2} \beta g_V f_{\Xi_c \Xi_c V} H_2(\boldsymbol{\sigma} \cdot \mathbf{L}) - \frac{\sqrt{2}m_v^3}{16\pi M_{\Xi_c} M_D} \beta g_V f_{\Xi_c \Xi_c V} H_2(\boldsymbol{\sigma} \cdot \mathbf{L}) \right]. \quad (21)$$

For the $\Xi_c \bar{D}^*$ system, the σ exchange potential reads

$$V_{\text{scalar}}^{\text{Lab}[\Xi_c \bar{D}^*]}(\mathbf{r}, \nabla) = C_\sigma \left[\frac{m_\sigma}{4\pi} g_{\Xi_c \Xi_c \sigma} g_\sigma H_0(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger) - \frac{m_\sigma^3}{48\pi M_{D^*}^2} g_{\Xi_c \Xi_c \sigma} g_\sigma H_1(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger) \right. \\ \left. + \frac{m_\sigma}{8\pi M_{D^*}^2} g_{\Xi_c \Xi_c \sigma} g_\sigma H_0(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger) \nabla^2 + \frac{m_\sigma}{8\pi M_{\Xi_c}^2} g_{\Xi_c \Xi_c \sigma} g_\sigma H_0(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger) \nabla^2 \right. \\ \left. - \frac{m_\sigma^3}{32\pi M_{\Xi_c}^2} g_{\Xi_c \Xi_c \sigma} g_\sigma H_1(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger) + \frac{m_\sigma^3}{8\pi M_{D^*}^2} g_{\Xi_c \Xi_c \sigma} g_\sigma H_2(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger)(\mathbf{r} \cdot \nabla) \right. \\ \left. + \frac{m_\sigma^3}{8\pi M_{\Xi_c}^2} g_{\Xi_c \Xi_c \sigma} g_\sigma H_2(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger)(\mathbf{r} \cdot \nabla) - \frac{m_\sigma^3}{24\pi M_{D^*}^2} g_{\Xi_c \Xi_c \sigma} g_\sigma H_3(\hat{\mathbf{r}}, \boldsymbol{\epsilon}_2, \boldsymbol{\epsilon}_4^\dagger) \right. \\ \left. + \frac{m_\sigma^3}{8\pi M_{D^*}^2} g_{\Xi_c \Xi_c \sigma} g_\sigma H_2(i\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) \cdot \mathbf{L} + \frac{m_\sigma^3}{16\pi M_{\Xi_c}^2} g_{\Xi_c \Xi_c \sigma} g_\sigma H_2(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger)(\boldsymbol{\sigma} \cdot \mathbf{L}) \right], \quad (22)$$

while the vector (ρ, ω) exchange potential reads

$$V_{\text{vector}}^{\text{Lab}[\Xi_c \bar{D}^*]}(\mathbf{r}, \nabla) = \sum_\alpha C_\alpha \left\{ \frac{\sqrt{2}m_v}{8\pi} \beta g_V g_{\Xi_c \Xi_c V} H_0(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger) - \frac{\sqrt{2}m_v^3}{12\pi M_{D^*}^2} g_{\Xi_c \Xi_c V} \lambda g_V H_3(\hat{\mathbf{r}}, \boldsymbol{\epsilon}_2, \boldsymbol{\epsilon}_4^\dagger) + \frac{\sqrt{2}m_v^3}{12\pi M_{D^*}^2} g_{\Xi_c \Xi_c V} \lambda g_V H_1(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger) \right. \\ \left. + \frac{\sqrt{2}m_v^3}{4\pi M_{D^*}^2} g_{\Xi_c \Xi_c V} \lambda g_V H_2[(i\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) \cdot \mathbf{L}] + \frac{\sqrt{2}m_v^3}{4\pi M_{\Xi_c}^2} g_{\Xi_c \Xi_c V} \lambda g_V H_2[(i\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) \cdot \mathbf{L}] \right. \\ \left. + \frac{\sqrt{2}m_v^3}{24\pi M_{\Xi_c}^2} g_{\Xi_c \Xi_c V} \lambda g_V H_3(\hat{\mathbf{r}}, i\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger, \boldsymbol{\sigma}) + \frac{\sqrt{2}m_v^3}{12\pi M_{\Xi_c}^2} g_{\Xi_c \Xi_c V} \lambda g_V H_1[(i\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) \cdot \boldsymbol{\sigma}] \right. \\ \left. + \frac{\sqrt{2}m_v^3}{24\pi M_{\Xi_c}^2} f_{\Xi_c \Xi_c V} \lambda g_V H_3(\hat{\mathbf{r}}, i\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger, \boldsymbol{\sigma}) + \frac{\sqrt{2}m_v^3}{12\pi M_{\Xi_c}^2} f_{\Xi_c \Xi_c V} \lambda g_V H_1[(i\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) \cdot \boldsymbol{\sigma}] \right. \\ \left. - \frac{\sqrt{2}m_v^3}{64\pi M_{\Xi_c}^2} \beta g_V g_{\Xi_c \Xi_c V} H_1(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger) + \frac{\sqrt{2}m_v^3}{32\pi M_{\Xi_c} M_{D^*}^2} \beta g_V g_{\Xi_c \Xi_c V} H_1(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger) - \frac{\sqrt{2}m_v^3}{32\pi M_{\Xi_c}^2} \beta g_V f_{\Xi_c \Xi_c V} H_1(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger) \right\}$$

$$\begin{aligned}
& -\frac{\sqrt{2}m_v^3}{8\pi M_{\Xi_c} M_{D^*}} \beta g_V g_{\Xi_c \Xi_c V} H_2(\mathbf{e}_2 \cdot \mathbf{e}_4^\dagger)(\mathbf{r} \cdot \mathbf{V}) - \frac{\sqrt{2}m_v}{8\pi M_{\Xi_c} M_{D^*}} \beta g_V g_{\Xi_c \Xi_c V} H_0(\mathbf{e}_2 \cdot \mathbf{e}_4^\dagger) \nabla^2 \\
& - \frac{\sqrt{2}m_v^3}{32\pi M_{\Xi_c}^2} \beta g_V g_{\Xi_c \Xi_c V} H_2(\mathbf{e}_2 \cdot \mathbf{e}_4^\dagger)(\boldsymbol{\sigma} \cdot \mathbf{L}) - \frac{\sqrt{2}m_v^3}{16\pi M_{\Xi_c} M_{D^*}} \beta g_V g_{\Xi_c \Xi_c V} H_2(\mathbf{e}_2 \cdot \mathbf{e}_4^\dagger)(\boldsymbol{\sigma} \cdot \mathbf{L}) \\
& - \frac{\sqrt{2}m_v^3}{16\pi M_{\Xi_c}^2} \beta g_V f_{\Xi_c \Xi_c V} H_2(\mathbf{e}_2 \cdot \mathbf{e}_4^\dagger)(\boldsymbol{\sigma} \cdot \mathbf{L}) - \frac{\sqrt{2}m_v^3}{16\pi M_{\Xi_c} M_{D^*}} \beta g_V f_{\Xi_c \Xi_c V} H_2(\mathbf{e}_2 \cdot \mathbf{e}_4^\dagger)(\boldsymbol{\sigma} \cdot \mathbf{L}) \\
& - \frac{\sqrt{2}m_v^3}{48\pi M_{D^*}^2} \beta g_V g_{\Xi_c \Xi_c V} H_3 S(\hat{\mathbf{r}}, \mathbf{e}_2, \mathbf{e}_4^\dagger) + \frac{\sqrt{2}m_v^3}{48\pi M_{D^*}^2} \beta g_V g_{\Xi_c \Xi_c V} H_1(\mathbf{e}_2 \cdot \mathbf{e}_4^\dagger) \\
& + \frac{\sqrt{2}m_v^3}{16\pi M_{D^*}^2} \beta g_V g_{\Xi_c \Xi_c V} H_2[(i\mathbf{e}_2 \times \mathbf{e}_4^\dagger) \cdot \mathbf{L}] \Big\}, \tag{23}
\end{aligned}$$

where $\alpha = \rho, \omega$.

C_σ and C_α in Eqs. (20)–(23) are isospin factors derived from the isospin wave functions. The definition of the tensor operator $S(\hat{\mathbf{r}}, \mathbf{a}, \mathbf{b})$ and the expressions of H_0 , H_1 , H_2 , and H_3 are given in the Appendix.

G -parity rule states that the one-boson-exchange amplitude (equivalent to the effective potentials) of the AB to AB scattering process is linked with that of the $A\bar{B}$ to $A\bar{B}$ one-boson-exchange scattering process by a factor $(-)^{I_G}$, where $(-)^{I_G}$ is the G parity of the exchanged light meson [63,69]. Consequently, the effective potentials of the open charm systems read

$$\begin{aligned}
V_{\text{scalar}}^{\text{Lab}[\Xi_c D^{(*)}]} &= (-)^{I_G[\sigma]} V_{\text{scalar}}^{\text{Lab}[\Xi_c \bar{D}^{(*)}]}, \\
V_{\text{vector}}^{\text{Lab}[\Xi_c D^{(*)}]} &= (-)^{I_G[\alpha]} V_{\text{vector}}^{\text{Lab}[\Xi_c \bar{D}^{(*)}]}, \\
V_{\text{scalar}}^{\text{Lab}[\Lambda_c D^{(*)}]} &= (-)^{I_G[\sigma]} V_{\text{scalar}}^{\text{Lab}[\Lambda_c \bar{D}^{(*)}]}, \\
V_{\text{vector}}^{\text{Lab}[\Lambda_c D^{(*)}]} &= (-)^{I_G[\alpha]} V_{\text{vector}}^{\text{Lab}[\Lambda_c \bar{D}^{(*)}]}. \tag{24}
\end{aligned}$$

For practical calculations, we absorb the $(-)^{I_G}$ factors of the exchanged mesons into a redefined set of isospin factors for the open-charm systems, which ensures the isospin-independent parts of the potentials remain identical. The isospin factors used in this work are exhibited in Table IV.

When solving the coupled channel Schrödinger equation, the structures related to angular momentum will be integrated into the matrix elements, where the element

corresponds to a set of initial and final partial waves. The matrix elements are given in the Appendix.

III. NUMERICAL RESULTS

With the effective potentials derived in the previous chapter, we obtain the bound state solutions by solving the coupled channel Schrödinger equation numerically. The computed values of characteristic quantities such as the binding energy and the root-mean-square (RMS) radius are listed in the following tables. In the loosely bound picture, a plausible hadronic molecular candidate is expected to have a binding energy of no more than a few tens of MeV and a root-mean-square radius of approximately 1 fm.

Besides, the cutoff value of the form factor affects the judgment of the hadronic molecular states. Unfortunately, due to the nonperturbative nature of QCD in the low-energy region and the lack of relevant experimental data, the cutoff value still cannot be precisely determined. In practice, researchers typically fix the range of cutoff values based on the experience from the studies of the deuteron. Since the typical value of the cutoff parameter for the deuteron lies in the range of 1.2–1.5 GeV, it is generally believed that the cutoff value for other hadronic molecules should not deviate significantly from 1 GeV. In this study, we therefore adopt a cutoff value range of 0.8–2.0 GeV.

A. The mass spectra of $\Xi_c \bar{D}^{(*)}$ systems

The constraint of angular momentum conservation requires that the $0(\frac{1}{2}^-)$ channel of the $\Xi_c \bar{D}$ system in its ground state must be pure S wave, while the $0(\frac{1}{2}^-)$ and $0(\frac{3}{2}^-)$ channels of the $\Xi_c \bar{D}^*$ system gain a few D wave components in their ground states with recoil corrections.

For the isospin triplet channels, the effective potentials always provide a repulsive interaction, which leads to a bound state solution impossible. For the isospin singlet channels, we collect the numerical results of binding energy and the root-mean-square radius with different cutoff value in Table V. In order to exhibit the influence of recoil corrections to certain

TABLE IV. The isospin factors for different channels.

Channels	C_ρ	C_ω	C_σ
$[\Xi_c \bar{D}^{(*)}]_{I=1} \leftrightarrow [\Xi_c \bar{D}^{(*)}]_{I=1}$	$\frac{1}{2}$	$\frac{1}{2}$	1
$[\Xi_c \bar{D}^{(*)}]_{I=0} \leftrightarrow [\Xi_c \bar{D}^{(*)}]_{I=0}$	$-\frac{3}{2}$	$\frac{1}{2}$	1
$[\Xi_c D^{(*)}]_{I=1} \leftrightarrow [\Xi_c D^{(*)}]_{I=1}$	$\frac{1}{2}$	$-\frac{1}{2}$	1
$[\Xi_c D^{(*)}]_{I=0} \leftrightarrow [\Xi_c D^{(*)}]_{I=0}$	$-\frac{3}{2}$	$-\frac{1}{2}$	1
$[\Lambda_c^+ \bar{D}^{(*)}]_{I=\frac{1}{2}} \leftrightarrow [\Lambda_c^+ \bar{D}^{(*)}]_{I=\frac{1}{2}}$	0	$\frac{1}{2}$	1
$[\Lambda_c^+ D^{(*)}]_{I=\frac{1}{2}} \leftrightarrow [\Lambda_c^+ D^{(*)}]_{I=\frac{1}{2}}$	0	$-\frac{1}{2}$	1

TABLE V. The numerical results of the $\Xi_c \bar{D}$ and $\Xi_c \bar{D}^*$ systems in different channels.

$\Xi_c \bar{D} [I(J^P) = 0(\frac{1}{2}^-)]$								
Without recoil corrections				With recoil corrections				
Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}}$ (%)	Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}}$ (%)	
1.55	0.28	5.23	100	1.55	0.21	5.57	100	
1.60	0.84	3.55	100	1.60	0.71	3.79	100	
1.65	1.65	2.64	100	1.65	1.46	2.79	100	
1.70	2.70	2.12	100	1.70	2.42	2.23	100	
1.75	3.96	1.80	100	1.75	3.58	1.88	100	

$\Xi_c \bar{D}^* [I(J^P) = 0(\frac{1}{2}^-)]$								
Without recoil corrections				With recoil corrections				
Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}}$ (%)	Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}}$ (%)	$^4D_{\frac{3}{2}}$ (%)
1.55	0.68	3.81	100	1.55	0.22	5.50	99.99	0.01
1.60	1.47	2.73	100	1.60	0.67	3.85	99.99	0.01
1.65	2.53	2.15	100	1.65	1.31	2.89	99.98	0.02
1.70	3.83	1.80	100	1.70	2.12	2.34	99.98	0.02
1.75	5.34	1.56	100	1.75	3.08	1.99	99.97	0.03

$\Xi_c \bar{D}^* [I(J^P) = 0(\frac{3}{2}^-)]$									
Without recoil corrections				With recoil corrections					
Λ (GeV)	B.E.(MeV)	RMS(fm)	$^4S_{\frac{3}{2}}$ (%)	Λ (GeV)	B.E.(MeV)	RMS(fm)	$^4S_{\frac{3}{2}}$ (%)	$^2D_{\frac{3}{2}}$ (%)	$^4D_{\frac{3}{2}}$ (%)
1.55	0.68	3.81	100	1.55	0.22	5.50	99.99	0.00	0.01
1.60	1.47	2.73	100	1.60	0.67	3.85	99.99	0.00	0.01
1.65	2.53	2.15	100	1.65	1.31	2.89	99.98	0.01	0.01
1.70	3.83	1.80	100	1.70	2.12	2.34	99.98	0.01	0.01
1.75	5.34	1.56	100	1.75	3.07	1.99	99.97	0.01	0.02

channels, the results with and without the momentum-related terms are given separately.

The effective potentials for the $0(\frac{1}{2}^-)$ channel of the $\Xi_c \bar{D}$ system are presented in Fig. 1. Panel (a) displays the S wave potentials without recoil corrections, while Panel (b) displays those with recoil corrections. To clarify the

contribution of each meson and to offer a comprehensive perspective, we show both the individual meson contributions and the total potential in each panel.

As we can see in Table V and Fig. 1, a bound state solution appears when the cutoff parameter $\Lambda = 1.55$ GeV. The ρ and σ exchanges provide the

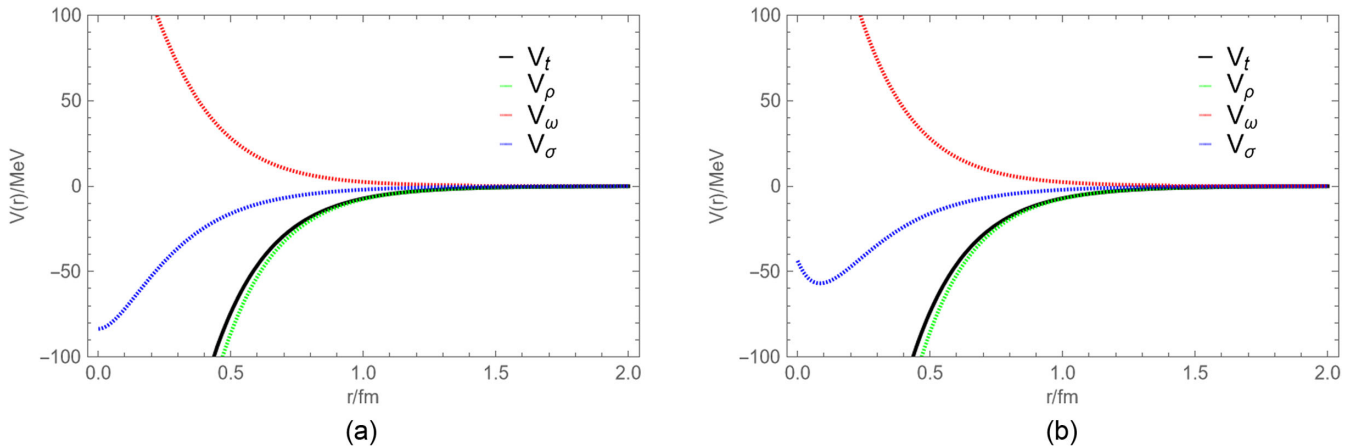


FIG. 1. The effective potentials for the $\Xi_c \bar{D}$ system in the $0(\frac{1}{2}^-)$ channel. Panel (a): contributions from σ , ρ , and ω exchange without recoil corrections. Panel (b): same with recoil corrections. The cutoff value is $\Lambda = 1.75$ GeV.

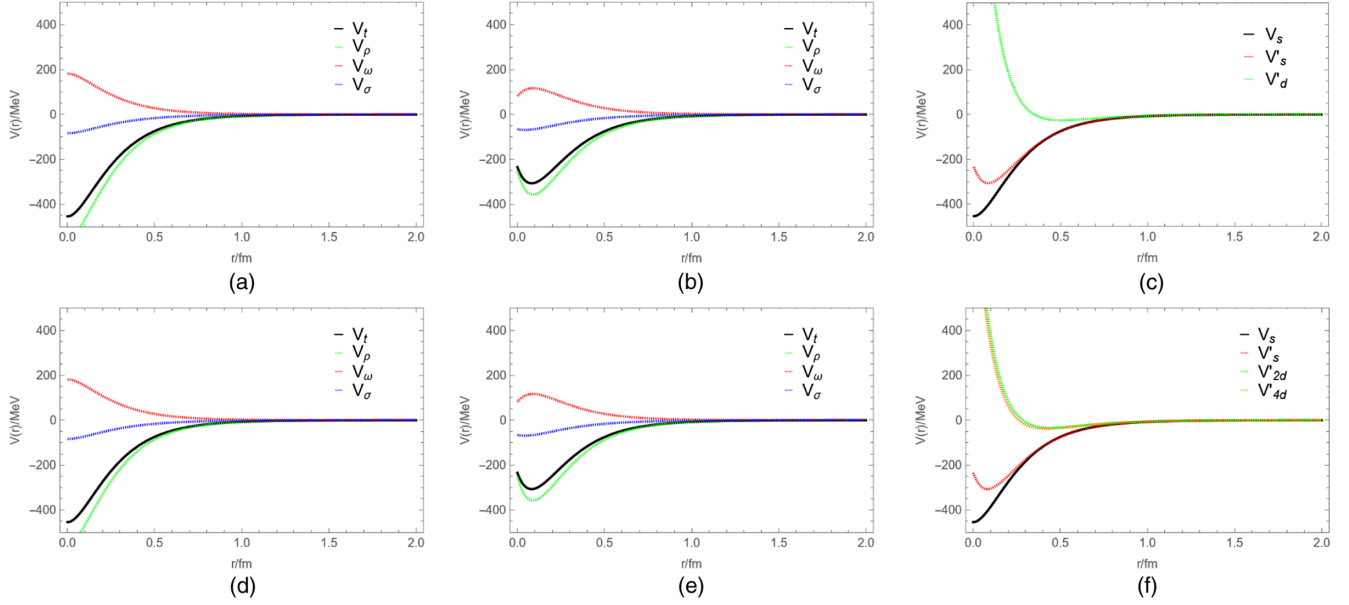


FIG. 2. The effective potentials for the $\Xi_c \bar{D}^*$ system in the $0(\frac{1}{2}^-)$ (upper row) and $0(\frac{3}{2}^-)$ (lower row) channels. Panels (a,d): contributions from σ , ρ , and ω exchange without recoil corrections. Panels (b,e): same with recoil corrections. Panels (c,f): comparison of the total potentials in the ground state with and without recoil corrections. The cutoff value is $\Lambda = 1.75$ GeV.

attractive interaction. For a given Λ , the inclusion of recoil corrections slightly reduces the binding energy and increases the RMS radius, which indicates that such corrections, though inconspicuous, make the bound state weaker and more extended.

The effective potentials for channels of the $\Xi_c \bar{D}^*$ system are presented in Fig. 2, where the upper and lower rows correspond to the $0(\frac{1}{2}^-)$ and $0(\frac{3}{2}^-)$ channels, respectively. Within each row, the first two panels [Panels (a) and (b) above; (d) and (e) below] display the same physical objects as Panels (a) and (b) in Fig. 1. Since recoil corrections are noticeable in these channels, the last panel [Panel (c) above; (f) below] compare the overall effective potentials with and without the recoil corrections.

As evident from Table V and Fig. 2, compared with the case of the $\Xi_c \bar{D}$ system, recoil corrections play a more significant role in the $\Xi_c \bar{D}^*$ system, markedly altering the spectrum of each channel. For instance, in the $0(\frac{1}{2}^-)$ channel with $\Lambda = 1.75$ GeV, a bound state with binding energy 5.34 MeV emerges without recoil corrections. However, if we take such corrections into consideration, the binding energy decreases to 3.08 MeV (just over half). Comparing Panel (a) and Panel (b) in Fig. 2, recoil corrections shallow the ρ exchange potential in the S wave, especially in the short-distance region. Now that the ρ exchange potential is the major donor for the attractive interaction, similar changes in the total S wave potential is exhibited in Panel (c). Another noteworthy phenomenon is that even if the recoil corrections introduce D wave components into the ground state, the proportion of these components are so small that the split of the $0(\frac{1}{2}^-)$ and $0(\frac{3}{2}^-)$ channels can hardly be observed.

B. The mass spectra of $\Xi_c D^{(*)}$ systems

Similar with the $\Xi_c \bar{D}^{(*)}$ systems, in the $0(\frac{1}{2}^-)$ channel of the $\Xi_c D$ system, the ground state only contains S wave; in the $0(\frac{1}{2}^-)$ and $0(\frac{3}{2}^-)$ channels of the $\Xi_c D^*$ system, recoil corrections introduce the D wave components into the ground states.

For the isospin triplet channels, the effective potentials derived from ρ and ω exchanges almost cancel out, as they possess opposite signs and comparable strengths. Since there are no pseudoscalar exchanges, the total S wave effective potential is predominantly governed by σ exchange. Though attractive, this contribution is insufficient to support the formation of a bound state. For the isospin singlet channels, we collect the numerical results with and without the recoil corrections in Table VI.

The effective potentials for the $0(\frac{1}{2}^-)$ channel of the $\Xi_c D$ system are presented in Fig. 3. Panels (a) and (b) are the same as in Fig. 1.

As we can see in Table VI and Fig. 3, each of the three exchanged light mesons offers an attractive potential. Thus, it is easier for this channel to form a stable bound state compared with the same channel in its hidden charm partner. Without the recoil corrections, a bound state solution with binding energy 1.17–17.3 MeV, and with corresponding RMS radius 3.11–1.01 fm appears when the cutoff value is chosen between 1.25 and 1.45 GeV. The recoil corrections in this channel are unfavorable to the formation of bound states. For the same range of Λ , the binding energy reduces to 1.05–16.2 MeV, while the RMS radius increases to 3.26–1.04 fm.

The effective potentials for channels of the $\Xi_c D^*$ system are presented in Fig. 4, where the upper and lower rows

TABLE VI. The numerical results of the $\Xi_c D$ and $\Xi_c D^*$ systems in different channels.

$\Xi_c D [I(J^P) = 0(\frac{1}{2}^-)]$								
Without recoil corrections				With recoil corrections				
Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}}$ (%)	Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}}$ (%)	
1.25	1.17	3.11	100	1.25	1.05	3.26	100	
1.30	3.58	1.91	100	1.30	3.30	1.98	100	
1.35	7.14	1.44	100	1.35	6.66	1.48	100	
1.40	11.7	1.18	100	1.40	11.0	1.21	100	
1.45	17.3	1.01	100	1.45	16.2	1.04	100	

$\Xi_c D^* [I(J^P) = 0(\frac{1}{2}^-)]$								
Without recoil corrections				With recoil corrections				
Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}}$ (%)	Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}}$ (%)	$^4D_{\frac{1}{2}}$ (%)
1.25	1.82	2.52	100	1.25	1.03	3.23	99.99	0.01
1.30	4.70	1.68	100	1.30	3.05	2.03	99.98	0.02
1.35	8.76	1.30	100	1.35	5.97	1.54	99.96	0.04
1.40	13.9	1.09	100	1.40	9.66	1.27	99.95	0.05
1.45	19.9	0.95	100	1.45	14.0	1.10	99.93	0.07

$\Xi_c D^* [I(J^P) = 0(\frac{3}{2}^-)]$									
Without recoil corrections				With recoil corrections					
Λ (GeV)	B.E.(MeV)	RMS(fm)	$^4S_{\frac{3}{2}}$ (%)	Λ (GeV)	B.E.(MeV)	RMS(fm)	$^4S_{\frac{3}{2}}$ (%)	$^2D_{\frac{3}{2}}$ (%)	$^4D_{\frac{3}{2}}$ (%)
1.25	1.82	2.52	100	1.25	1.03	3.23	99.99	0.00	0.01
1.30	4.70	1.68	100	1.30	3.05	2.03	99.98	0.01	0.01
1.35	8.76	1.30	100	1.35	5.97	1.54	99.96	0.02	0.02
1.40	13.9	1.09	100	1.40	9.66	1.27	99.95	0.02	0.03
1.45	19.9	0.95	100	1.45	14.0	1.10	99.93	0.03	0.04

correspond to the $0(\frac{1}{2}^-)$ and $0(\frac{3}{2}^-)$ channels, respectively. Panels (a) to (f) are the same as in Fig. 2.

Comparable to the channels of the $\Xi_c \bar{D}^*$ system, recoil corrections show striking influence on the spectrum of each channel of the $\Xi_c D^*$ system. It is exhibited in Table VI and

Fig. 4 that such corrections are also unfavorable for the formation of a bound state because the ρ and ω exchange potentials become shallower. In the $0(\frac{1}{2}^-)$ channel, when the cutoff parameter is fixed as $\Lambda = 1.45$ GeV, a bound state with binding energy 19.9 MeV emerges without recoil

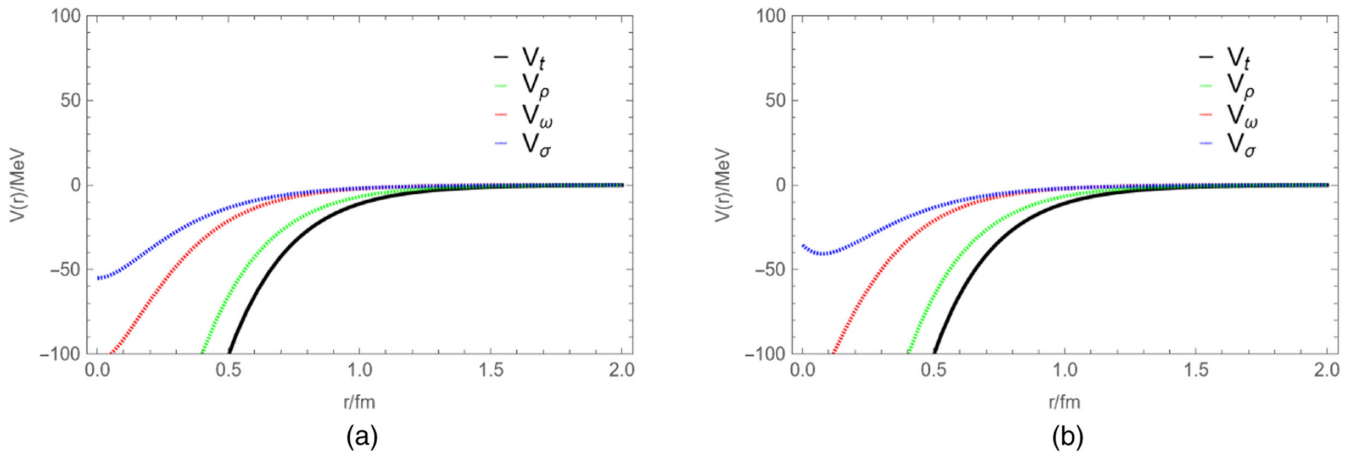


FIG. 3. The effective potentials for the $\Xi_c D$ system in the $0(\frac{1}{2}^-)$ channel. Panel (a): contributions from σ , ρ , and ω exchange without recoil corrections. Panel (b): same with recoil corrections. The cutoff value is $\Lambda = 1.45$ GeV.

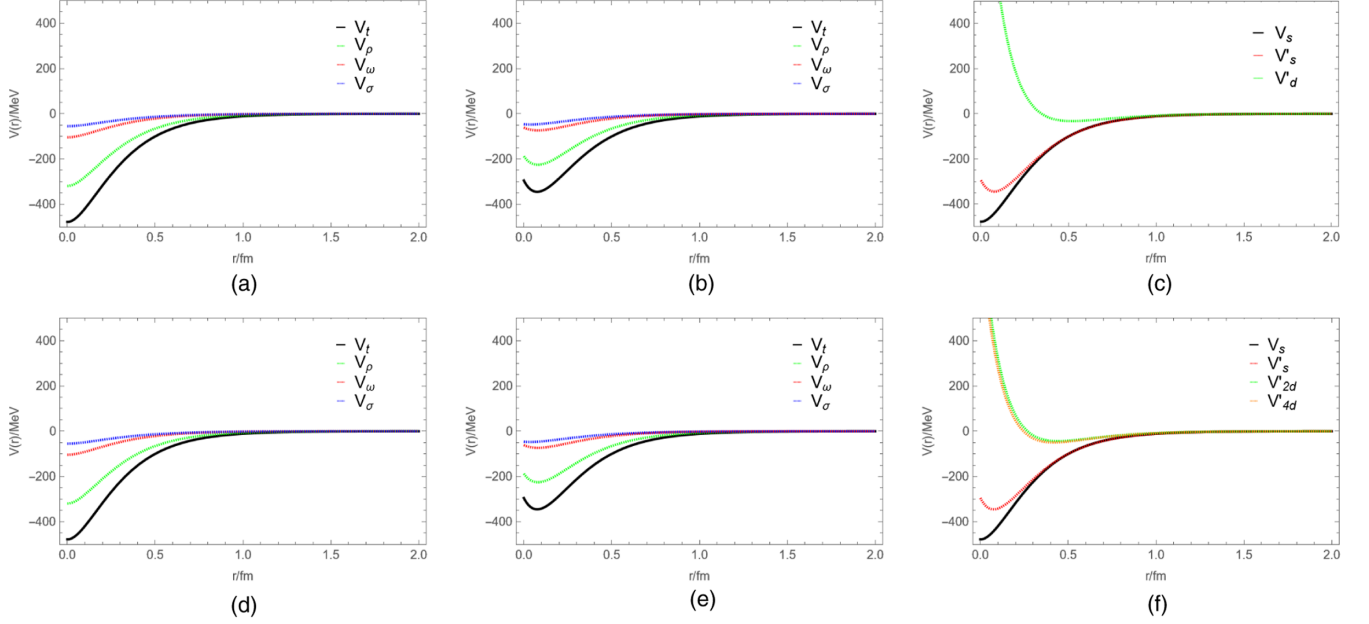


FIG. 4. The effective potentials for the $\Xi_c D^*$ system in the $0(\frac{1}{2}^-)$ (upper row) and $0(\frac{3}{2}^-)$ (lower row) channels. Panels (a, d): contributions from σ , ρ , and ω exchange without recoil corrections. Panels (b, e): same with recoil corrections. Panels (c, f): comparison of the total potentials in the ground state with and without recoil corrections. The cutoff value is $\Lambda = 1.45$ GeV.

TABLE VII. The numerical results of the $\Lambda_b \bar{B}$ and $\Lambda_b \bar{B}^*$ systems in different channels.

$\Lambda_b \bar{B} \ [I(J^P) = \frac{1}{2}(\frac{1}{2}^-)]$									
Without recoil corrections				With recoil corrections					
Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}} \ (%)$	Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}} \ (%)$		
1.30	0.55	2.87	100	1.30	0.55	2.88	100		
1.40	2.28	1.56	100	1.40	2.27	1.56	100		
1.50	4.97	1.14	100	1.50	4.95	1.14	100		
1.60	8.41	0.93	100	1.60	8.37	0.93	100		
1.70	12.4	0.80	100	1.70	12.4	0.80	100		
$\Lambda_b \bar{B}^* \ [I(J^P) = \frac{1}{2}(\frac{1}{2}^-)]$									
Without recoil corrections				With recoil corrections					
Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}} \ (%)$	Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}} \ (%)$	$^4D_{\frac{1}{2}} \ (%)$	
1.30	0.58	2.81	100	1.30	0.49	3.01	100.0	0.00	
1.40	2.34	1.54	100	1.40	2.07	1.62	100.0	0.00	
1.50	5.05	1.13	100	1.50	4.52	1.19	99.99	0.01	
1.60	8.53	0.92	100	1.60	7.62	0.97	99.99	0.01	
1.70	12.6	0.80	100	1.70	11.2	0.84	99.98	0.02	
$\Lambda_b \bar{B}^* \ [I(J^P) = \frac{1}{2}(\frac{3}{2}^-)]$									
Without recoil corrections				With recoil corrections					
Λ (GeV)	B.E.(MeV)	RMS(fm)	$^4S_{\frac{3}{2}} \ (%)$	Λ (GeV)	B.E.(MeV)	RMS(fm)	$^4S_{\frac{3}{2}} \ (%)$	$^2D_{\frac{3}{2}} \ (%)$	$^4D_{\frac{3}{2}} \ (%)$
1.30	0.58	2.81	100	1.30	0.49	3.01	100.0	0.00	0.00
1.40	2.34	1.54	100	1.40	2.07	1.62	100.0	0.00	0.00
1.50	5.05	1.13	100	1.50	4.52	1.19	99.99	0.00	0.01
1.60	8.53	0.92	100	1.60	7.62	0.97	99.99	0.00	0.01
1.70	12.6	0.80	100	1.70	11.2	0.84	99.98	0.01	0.01

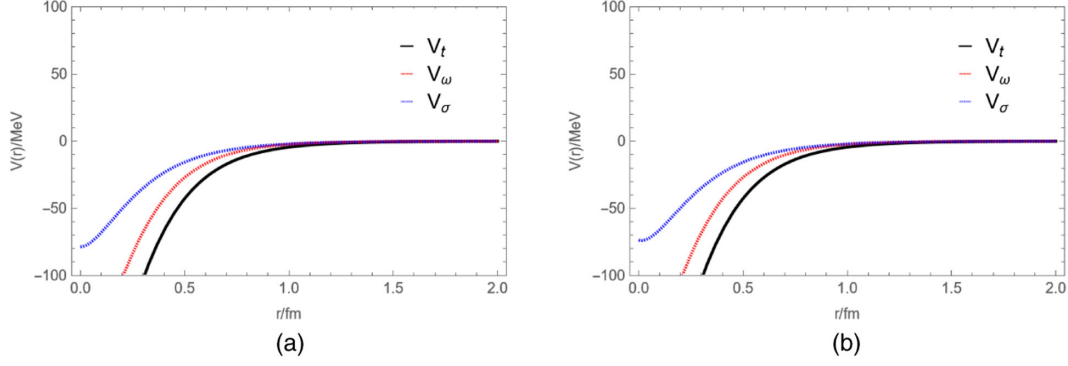


FIG. 5. The effective potentials for the $\Lambda_b \bar{B}$ system in the $\frac{1}{2}(\frac{1}{2}^-)$ channel. Panel (a): contributions from σ and ω exchange without recoil corrections. Panel (b): same with recoil corrections. The cutoff value is $\Lambda = 1.70$ GeV.

corrections, which substantially decreases to 14.0 MeV with recoil corrections.

C. The mass spectra of $\Lambda_c \bar{D}^{(*)}$ and $\Lambda_c D^{(*)}$ systems

When the cutoff value is constrained under the 2.00 GeV, we fail to find a bound state solution in any channels of the $\Lambda_c \bar{D}^{(*)}$ or $\Lambda_c D^{(*)}$ system, whether recoil corrections are considered or not.

D. The mass spectra of $\Xi_b B^{(*)}$ and $\Xi_b \bar{B}^{(*)}$ systems

The dynamics of binding in $\Xi_b B^{(*)}$ and $\Xi_b \bar{B}^{(*)}$ systems is qualitatively analogous to that of their charmed counterparts. Therefore, we present relevant results in the Appendix Table IX. Due to their larger masses, the bottom-flavor systems are more favorable for the formation

of hadronic molecules, while the influence of recoil corrections is undermined.

E. The mass spectra of $\Lambda_b B^{(*)}$ and $\Lambda_b \bar{B}^{(*)}$ systems

None of the channels of the $\Lambda_b B^{(*)}$ systems owns a bound state solution. However, unlike the $\Lambda_c D$ and $\Lambda_c D^*$ systems, there exists bound state solutions in the channels of the open bottom systems. We collect the numerical results with and without the recoil corrections in Table VII.

The effective potentials for the $\frac{1}{2}(\frac{1}{2}^-)$ channel of the $\Lambda_b \bar{B}$ system are presented in Fig. 5. Panels (a) and (b) are the same as in Fig. 1. The effective potentials for channels of the $\Lambda_b \bar{B}^*$ system are presented in Fig. 6, where the upper and lower rows correspond to the $\frac{1}{2}(\frac{1}{2}^-)$ and $\frac{1}{2}(\frac{3}{2}^-)$ channels, respectively. Panels (a) to (f) are the same as in Fig. 2.

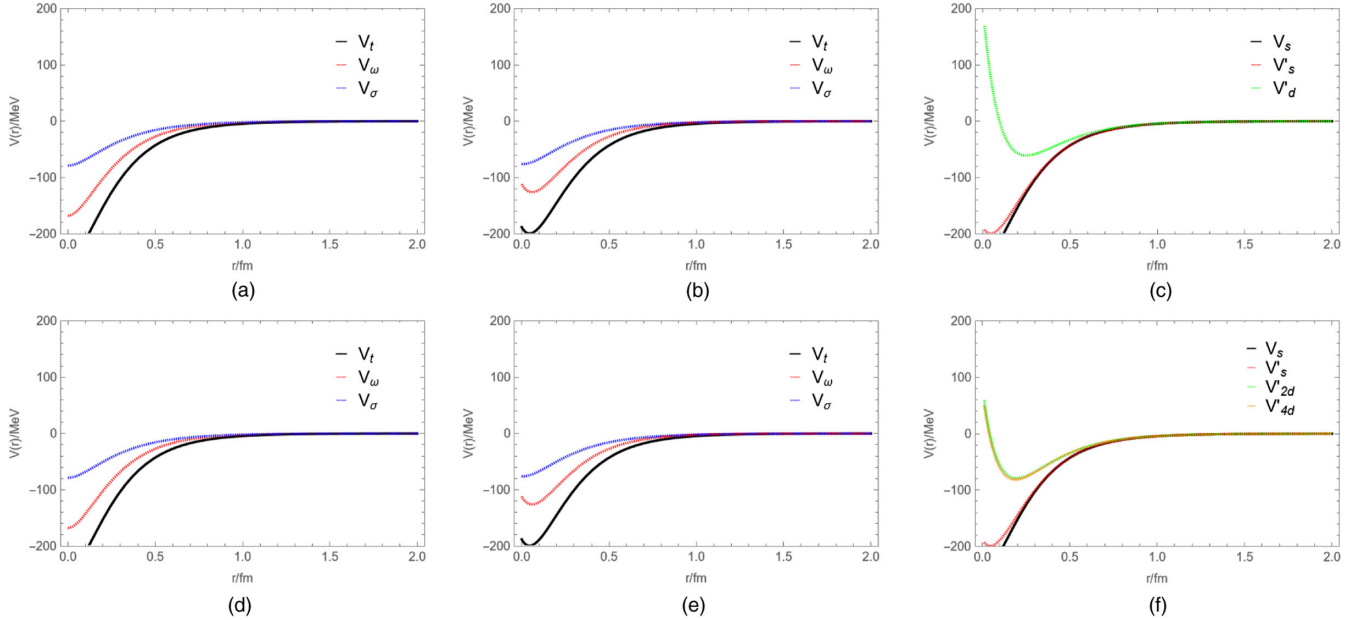


FIG. 6. The effective potentials for the $\Lambda_b \bar{B}^*$ system in the $\frac{1}{2}(\frac{1}{2}^-)$ (upper row) and $\frac{1}{2}(\frac{3}{2}^-)$ (lower row) channels. Panels (a,d): contributions from σ and ω exchange without recoil corrections. Panels (b,e): same with recoil corrections. Panels (c,f): comparison of the total potentials in the ground state with and without recoil corrections. The cutoff value is $\Lambda = 1.70$ GeV.

As is shown in Table VII, Fig. 5, and Fig. 6, the exchange potentials from both σ and ω are attractive. For the $\Lambda_b \bar{B}$ system, the impact of recoil corrections is minimal and almost negligible. In contrast, for the $\Lambda_b \bar{B}^*$ system, such corrections further weaken the binding, making the molecular state looser.

IV. SUMMARY AND DISCUSSION

Within the framework of the OBE model, we systematically study the $\Xi_c \bar{D}^{(*)}$, $\Lambda_c \bar{D}^{(*)}$, $\Xi_c D^{(*)}$, and $\Lambda_c D^{(*)}$ systems, along with their bottom analogues. We mainly focus on how recoil corrections affect the mass spectra of them. In addition, the S - D wave mixing effect is included. The recoil corrections are embodied in the effective potentials as momentum-related terms in the next and next-to-next leading order, including spin-spin interaction terms, spin-orbit force terms, tensor force terms, and the spatial derivative terms. It is worth noting that the tensor force terms would not exist unless the recoil corrections are considered. As a result, the contribution of the D wave to the ground state is quite small. The numerical results indicate that the recoil corrections, whose importance are always underestimated in previous research, can be crucial to the molecular pentaquark candidates.

The channels of the charmed systems and the bottomed systems share the same quantum numbers $I(J^P)$. For the $\Xi_c \bar{D}$ system, channels with $I(J^P) = 0(\frac{1}{2}^-), 1(\frac{1}{2}^-)$ are allowed; for the $\Lambda_c \bar{D}$ system, only the channel with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ is accessible; for the $\Xi_c \bar{D}^*$ system, channels with $I(J^P) = 0(\frac{1}{2}^-), 0(\frac{3}{2}^-), 1(\frac{1}{2}^-), 1(\frac{3}{2}^-)$ are possible; for the $\Lambda_c \bar{D}^*$ system, channels with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-), \frac{1}{2}(\frac{3}{2}^-)$ are supported.

After solving the coupled channel Schrödinger equation, we cannot find a bound state solution in the $I = 1$ channel of any systems. Besides, we also fail to discover a sign of molecular state in any channels of $\Lambda_c \bar{D}^{(*)}$ and $\Lambda_c D^{(*)}$ systems. Other channels possess bound state solutions whether or not the recoil corrections are taken into account.

For the $I(J^P) = 0(\frac{1}{2}^-)$ channel of $\Xi_c \bar{D}$ and $\Xi_c D$ systems, the $I(J^P) = 0(\frac{1}{2}^-)$ and $0(\frac{3}{2}^-)$ channels of $\Xi_c \bar{D}^*$ and $\Xi_c D^*$ systems, as well as their bottom counterparts, and for the $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ and $\frac{1}{2}(\frac{3}{2}^-)$ channels of $\Lambda_b \bar{B}^*$ system, the recoil corrections are unfavorable for the formation of bound states, as evidenced by smaller binding energies and larger root-mean-square radiuses. Particularly, with a fixed cutoff value, the binding energy nearly halved in the $\Xi_c \bar{D}^*$ and $\Xi_c D^*$ systems.

Based on the results presented above, we conclude three key findings and elaborate on them in detail. First, compared with the charmed systems, bound state solutions in the bottom systems are more readily obtained, and recoil corrections play a less significant role. This is attributed to the larger mass of the bottom systems, which decreases the kinetic energies and weakens the relativistic effects. Second, open-flavor systems

are more likely to form bound states than their hidden-flavor counterparts, since the change in isospin factors turns the repulsive ω exchange potentials into attractive ones. Last and most importantly, the recoil corrections of the isospin singlet $\Xi_c \bar{D}^*$ and $\Xi_c D^*$ systems are far more striking than the others systems. To explain this phenomenon, we call back to the formulas of the effective potentials in Eqs. (20)–(23) and the isospin factors in Table IV. From the former, we find that the recoil corrections of the systems containing a D meson emerge at $\mathcal{O}(\frac{1}{M^2})$, while such corrections of the systems containing a D^* meson emerge at $\mathcal{O}(\frac{1}{M})$ —an order of magnitude larger. From the latter, we observe that the absolute value of C_ρ in these systems is three times that of C_ω , given that ρ is an isospin triplet state while ω is an isospin singlet state. Consequently, whether the recoil corrections from ω exchange counteract or reinforce those from ρ exchange, the net diminution of the short-range attractive ρ exchange potential from the $\mathcal{O}(\frac{1}{M})$ recoil corrections dominates the linear change of the S wave.

Furthermore, we analyze the contributions to the binding energy from various physically distinct terms in the recoil corrections to clarify how such corrections affect the formation of molecular states. We find that the contributions of the spin-spin interaction terms and the spatial derivative terms are much greater than other higher-order terms. Here, we take the $\Xi_c \bar{D}^*$ system with $I(J^P) = 0(\frac{1}{2}^-)$ as an example. When the cutoff value $\Lambda = 1.75$ GeV, considering the recoil corrections of the S wave, the spin-spin interaction terms of ρ , ω , and σ exchange provide 1.45 MeV, -0.47 MeV, and 0.10 MeV to the binding energy, respectively. Under the same cutoff value, the spatial derivative terms of ρ and ω exchange provide 1.55 MeV and -0.51 MeV to the binding energy, respectively. Given that the total donation of these two kinds of corrections to the binding energy is 2.12 MeV, such corrections are unfavorable for the formation of molecular states.

We also notice that the recoil corrections have a greater impact on ρ and ω exchange than on σ exchange. This result can be attributed to two reasons. First, σ has a large width and a very short lifetime, which makes its coupling to the constituent hadrons much weaker than those of ρ and ω . Besides, unlike ρ and ω , σ being a scalar meson does not carry angular momentum. As a consequence, the effective potential for σ exchange owns less recoil terms, which further weakens the influence of recoil corrections on σ exchange.

In conclusion, this research explores the double heavy and hidden heavy molecular pentaquark states with an SU(3) antitriplet baryon, which exhibits the great importance of the recoil corrections in the molecular candidates and further explains how they work. Consequently, our study holds significant implications which shed light on future investigations. From the experimental perspective, the molecular candidates predicted by this work are waiting for experimental evidence. From the theoretical perspective, more attention and in-depth studies of recoils

corrections are needed so that people can gain a clearer understanding of the mechanism and dynamical effects of these higher-order corrections.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX

1. Fourier transformations of q and k related structures

$$\begin{aligned}\frac{1}{-q^2 - m^2 + i\epsilon} &= -\frac{m}{4\pi} H_0, \\ \frac{q^2}{-q^2 - m^2 + i\epsilon} &= -\frac{m^3}{4\pi} H_1, \\ \frac{q_i q_j}{-q^2 - m^2 + i\epsilon} &= \left(\frac{m^3}{12\pi}\right) \left[H_3 \left(\frac{3r_i r_j}{r^2} - \delta_{ij} \right) - H_1 \delta_{ij} \right], \\ \frac{i[\boldsymbol{\sigma} \cdot (\mathbf{q} \times \mathbf{k})]}{-q^2 - m^2 + i\epsilon} &= \frac{m^3}{4\pi} H_2(\boldsymbol{\sigma} \cdot \mathbf{L}), \\ \frac{k^2}{-q^2 - m^2 + i\epsilon} &= \frac{m}{4\pi} H_0 \nabla^2 - \frac{m^3}{16\pi} H_1 + \frac{m^3}{4\pi} H_2(\mathbf{r} \cdot \nabla),\end{aligned}$$

where

$$\begin{aligned}H_0 &= \frac{1}{mr} (e^{-mr} - e^{-\Lambda r}) - \frac{\Lambda^2 - m^2}{2m\Lambda} e^{-\Lambda r}, \\ H_1 &= -\frac{1}{mr} (e^{-mr} - e^{-\Lambda r}) + \Lambda \frac{\Lambda^2 - m^2}{2m^2} e^{-\Lambda r}, \\ H_2 &= -(1 + mr) \frac{1}{m^3 r^3} e^{-mr} + (1 + \Lambda r) \frac{1}{m^3 r^3} e^{-\Lambda r} + \frac{\Lambda^2 - m^2}{2m^2} \frac{1}{mr} e^{-\Lambda r}, \\ H_3 &= \left(1 + \frac{3}{mr} + \frac{3}{m^2 r^2}\right) \frac{1}{mr} e^{-mr} - \left(1 + \frac{3}{\Lambda r} + \frac{3}{\Lambda^2 r^2}\right) \frac{\Lambda^2}{m^2} \frac{1}{mr} e^{-\Lambda r} - \frac{\Lambda^2 - m^2}{2m^2} (1 + \Lambda r) \frac{1}{mr} e^{-\Lambda r}.\end{aligned}$$

2. Representations of momentum-related operators

The operators $O_1, O_2 \dots O_5$ in Table VIII read

$$\begin{aligned}O_1 &= \boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger, \\ O_2 &= S(\hat{\mathbf{r}}, \boldsymbol{\epsilon}_2, \boldsymbol{\epsilon}_4^\dagger), \\ O_3 &= S(\hat{\mathbf{r}}, i\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger, \boldsymbol{\sigma}), \\ O_4 &= i(\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) \cdot \mathbf{L}, \\ O_5 &= i(\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) \cdot \boldsymbol{\sigma}, \\ O_6 &= (\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_4^\dagger)(\boldsymbol{\sigma} \cdot \mathbf{L}),\end{aligned}$$

where $S(\hat{\mathbf{r}}, \mathbf{a}, \mathbf{b}) = 3 \frac{(\mathbf{a} \cdot \mathbf{r})(\mathbf{b} \cdot \mathbf{r})}{r^2} - \mathbf{a} \cdot \mathbf{b}$, $\mathbf{L} = \mathbf{r} \times (-i\nabla)$.

TABLE VIII. The matrix elements $\langle f|O_k|i\rangle$ for the angular momentum related operators O_k .

	O_1	O_2	O_3	O_4	O_5	O_6
$\langle f O_k i\rangle_{\frac{1}{2}}$	$Diag(1, 1)$	$\begin{pmatrix} 0 & -\sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & -\sqrt{2} \\ -\sqrt{2} & -2 \end{pmatrix}$	$Diag(0, -3)$	$Diag(-2, 1)$	$Diag(0, -3)$
$\langle f O_k i\rangle_{\frac{3}{2}}$	$Diag(1, 1, 1)$	$\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix}$	$Diag(1, -2, 1)$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & -2 & -2 \end{pmatrix}$

3. The numerical results of $\Xi_b B^{(*)}$ and $\Xi_b \bar{B}^{(*)}$ systems

TABLE IX. The numerical results of the $\Xi_b B$, $\Xi_c B^*$, $\Xi_b \bar{B}$, and $\Xi_c \bar{B}^*$ systems in different channels.

$\Xi_b B \ [I(J^P) = 0(\frac{1}{2}^-)]$									
Without recoil corrections				With recoil corrections					
Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}} \ (%)$	Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}} \ (%)$		
1.10	0.54	2.89	100	1.10	0.54	2.90	100		
1.15	2.08	1.64	100	1.15	2.07	1.64	100		
1.20	4.51	1.21	100	1.20	4.49	1.21	100		
1.25	7.75	0.98	100	1.25	7.71	0.99	100		
1.30	11.7	0.85	100	1.30	11.6	0.85	100		
$\Xi_c B^* \ [I(J^P) = 0(\frac{1}{2}^-)]$									
Without recoil corrections				With recoil corrections					
Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}} \ (%)$	Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}} \ (%)$	$^4D_{\frac{1}{2}} \ (%)$	
1.10	0.57	2.83	100	1.10	0.49	3.03	100.0	0.00	
1.15	2.13	1.62	100	1.15	1.90	1.70	100.0	0.00	
1.20	4.59	1.20	100	1.20	4.14	1.25	99.99	0.01	
1.25	7.85	0.98	100	1.25	7.12	1.02	99.99	0.01	
1.30	11.8	0.84	100	1.30	10.7	0.88	99.99	0.01	
$\Xi_c B^* \ [I(J^P) = 0(\frac{3}{2}^-)]$									
Without recoil corrections				With recoil corrections					
Λ (GeV)	B.E.(MeV)	RMS(fm)	$^4S_{\frac{3}{2}} \ (%)$	Λ (GeV)	B.E.(MeV)	RMS(fm)	$^4S_{\frac{3}{2}} \ (%)$	$^2D_{\frac{3}{2}} \ (%)$	$^4D_{\frac{3}{2}} \ (%)$
1.10	0.57	2.83	100	1.10	0.49	3.03	100.0	0.00	0.00
1.15	2.13	1.62	100	1.15	1.90	1.70	100.0	0.00	0.00
1.20	4.59	1.20	100	1.20	4.14	1.25	99.99	0.00	0.01
1.25	7.85	0.98	100	1.25	7.12	1.02	99.99	0.00	0.01
1.30	11.8	0.84	100	1.30	10.7	0.88	99.99	0.00	0.01
$\Xi_b \bar{B} \ [I(J^P) = 0(\frac{1}{2}^-)]$									
Without recoil corrections				With recoil corrections					
Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}} \ (%)$	Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}} \ (%)$		
1.00	0.69	2.63	100	1.00	0.68	2.63	100		
1.05	3.96	1.29	100	1.05	3.94	1.29	100		
1.10	9.76	0.92	100	1.10	9.71	0.93	100		
1.15	17.8	0.75	100	1.15	17.7	0.75	100		
1.20	27.9	0.64	100	1.20	27.8	0.64	100		

$\Xi_c \bar{B}^* [I(J^P) = 0(\frac{1}{2}^-)]$									
Without recoil corrections				With recoil corrections					
Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}}$ (%)	Λ (GeV)	B.E.(MeV)	RMS(fm)	$^2S_{\frac{1}{2}}$ (%)	$^4D_{\frac{1}{2}}$ (%)	
1.00	0.71	2.58	100	1.00	0.62	2.75	100.0	0.00	
1.05	4.03	1.28	100	1.05	3.66	1.33	99.99	0.01	
1.10	9.87	0.92	100	1.10	9.02	0.95	99.99	0.01	
1.15	18.0	0.74	100	1.15	16.4	0.77	99.98	0.02	
1.20	28.1	0.64	100	1.20	25.6	0.67	99.97	0.03	

$\Xi_c \bar{B}^* [I(J^P) = 0(\frac{3}{2}^-)]$									
Without recoil corrections				With recoil corrections					
Λ (GeV)	B.E.(MeV)	RMS(fm)	$^4S_{\frac{3}{2}}$ (%)	Λ (GeV)	B.E.(MeV)	RMS(fm)	$^4S_{\frac{3}{2}}$ (%)	$^2D_{\frac{3}{2}}$ (%)	$^4D_{\frac{3}{2}}$ (%)
1.00	0.71	2.58	100	1.00	0.62	2.75	100.0	0.00	0.00
1.05	4.03	1.28	100	1.05	3.66	1.33	99.99	0.00	0.01
1.10	9.87	0.92	100	1.10	9.02	0.95	99.99	0.00	0.01
1.15	18.0	0.74	100	1.15	16.4	0.77	99.98	0.01	0.01
1.20	28.1	0.64	100	1.20	25.6	0.67	99.97	0.01	0.02

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