

NRQCD reconfronts LHCb data on quarkonium production within jets

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(Received 1 August 2025; accepted 2 October 2025; published 29 October 2025)

We compare LHCb measurements of J/ψ and $\psi(2S)$ transverse-momentum distributions within jets with QCD calculations, which may be crucial in understanding the quarkonium production mechanism. Our theoretical calculations are based on the fragmenting jet function formalism, while the nonperturbative formation of quarkonia is described by the nonrelativistic QCD factorization formalism. We include the newest refinements in the perturbative calculation including resummation of threshold and Dokshitzer-Gribov-Lipatov-Altarelli-Parisi logarithms. We find that the $\psi(2S)$ data have the potential to discriminate between the different production mechanisms proposed in the literature.

DOI: [10.1103/24js-gqmg](https://doi.org/10.1103/24js-gqmg)

Introduction. Understanding the heavy quarkonium production mechanism is one of the most significant challenges in quantum chromodynamics (QCD). Existing approaches based on nonrelativistic QCD (NRQCD) descriptions of inclusive production rates as functions of the transverse momentum p_T had difficulty in pinpointing the exact production mechanism [1,2]. This happens because inclusive cross sections generally do not have enough degrees of freedom to determine a sufficient number of NRQCD long-distance matrix elements (LDMEs), which govern the nonperturbative formation of the quarkonium. Because of this, a more differential observable such as the transverse-momentum distribution of the quarkonium within a jet has been proposed a way to scrutinize the production mechanism [3–5].

The momentum distribution of J/ψ within a jet has been measured as a function of the transverse-momentum fraction z by LHCb [6], CMS [7], and STAR [8] experiments, and recently LHCb reported measurements for $\psi(2S)$ as well [9]. An initial theoretical study [10] based on the fragmenting jet function (FJF) formalism [3] offered comparisons of predictions from different LDME sets to the LHCb J/ψ data, demonstrating the advantage of this

novel approach. However, this has not led to a definitive determination of the NRQCD LDMEs and the quarkonium production formalism, because the authors of Ref. [10] considered only a limited class of LDME sets and have not considered important logarithmic corrections near the kinematical threshold, which was unknown at that time. Also, the calculation in Ref. [10] did not match exactly the kinematical conditions of the LHCb experiment. Therefore, an updated analysis of both J/ψ and $\psi(2S)$ data involving more extensive LDME sets with more refined perturbative calculations is much desired.

In this paper, we provide improved theoretical predictions for the transverse-momentum distribution of J/ψ and $\psi(2S)$ produced in jets. We employ a FJF formalism developed in Ref. [11] that can facilitate the same kinematical conditions of LHCb measurements. In the perturbative calculation of the FJF, we resum not only the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) logarithms, but also the threshold double logarithms, which is crucial in obtaining a nonsingular, positive definite distributions near the threshold $z = 1$ [12]. For the LDMEs, we choose three representative examples in Refs. [13–15], which effectively span the ranges of available LDME sets in the literature. We then compare our results with LHCb data [6,9] to assess the power of the quarkonium-in-jet measurements in resolving the quarkonium production mechanism.

NRQCD factorization and LDMEs. In the NRQCD factorization formalism, the inclusive production rate of a heavy quarkonium H is given by

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$$\sigma_H = \sum_n \sigma_{Q\bar{Q}(n)} \langle \mathcal{O}^H(n) \rangle, \quad (1)$$

where $\sigma_{Q\bar{Q}(n)}$ is the production rate of a heavy quark Q and antiquark $\bar{Q}$ pair in the state n and $\langle \mathcal{O}^H(n) \rangle$ is the LDME that describes the evolution of the $Q\bar{Q}(n)$ into the meson H . We use the spectroscopic notation $^{2S+1}L_J^{[c]}$ to denote the state n , where S and L represent the spin and orbital angular momentum, respectively, and $c = 1, 8$ denote the color. In practice, the sum in Eq. (1) is truncated to include only the most important contributions. For production of J/ψ or $\psi(2S)$, they include $n = {}^3S_1^{[1]}, {}^3S_1^{[8]}, {}^1S_0^{[8]},$ and ${}^3P_J^{[8]}$. While the $\sigma_{Q\bar{Q}(n)}$ are perturbatively calculable, it is, in general, not known how to compute the color-octet LDMEs from first principles. Hence, the three color-octet LDMEs have been determined from fits to production data. This approach has not been successful in obtaining a satisfactory description of the J/ψ or $\psi(2S)$ production mechanism, as the LDME sets obtained in this way can vary significantly depending on the choice of data and the accuracy of the perturbative calculation [1,2].

The LDME sets available in the literature can generally be classified into three categories.

Category 1. Small ${}^1S_0^{[8]}$ LDME, with ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$ having the same sign. The cross section is dominated by ${}^3S_1^{[8]} + {}^3P_J^{[8]}$. Examples include Refs. [12,16–18] and Ref. [19] (minimized ${}^1S_0^{[8]}$).

Category 2. Large ${}^1S_0^{[8]}$ LDME, with ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$ having the same sign. The cross section is dominated by ${}^1S_0^{[8]}$. Examples include Refs. [14,20,21], Table II (fit D) in Refs. [19,22] (maximized ${}^1S_0^{[8]}$).

Category 3. Small ${}^1S_0^{[8]}$, positive ${}^3S_1^{[8]}$, and negative ${}^3P_J^{[8]}$. Examples include Ref. [15] and fits A and B in Ref. [22].

Predictions within each category would be similar. Representative sets for J/ψ and $\psi(2S)$ are listed in Table I.

Fragmenting jet function. Given tensions between LDME sets, studying quarkonium distributions inside jets [3–5] provides a valuable probe. There are two variants of FJF:

one defined from exclusive n -jet processes [24–30] and the other from semi-inclusive jet processes [11,31–34]. We compute the cross section using semi-inclusive FJFs, which suit the LHCb experimental framework, in contrast to a prior analysis based on exclusive jets [10]. These FJFs depend on the jet’s transverse momentum (p_T) and radius (R), the quarkonium’s momentum fraction ($z_H = p_T^H/p_T^{\text{jet}}$), and the jet’s momentum fraction relative to the initial parton (z).

When the jet virtuality $p_T R \gg m_H$, the FJF factorizes into a convolution of quarkonium fragmentation functions (FFs) [35–37] and Wilson coefficients, up to $\mathcal{O}(m_H^2/(p_T R)^2)$ corrections:

$$\mathcal{G}_i^H(z, z_H, p_T R, \mu) = \sum_j \mathcal{J}_{ij}(z, z_H, \mu) \otimes D_j^H(z_H, \mu), \quad (2)$$

where the coefficients $\mathcal{J}_{ij}$ describe splitting of a mother parton $i = \{q, g\}$ into a daughter parton j that resides in the jet [11]. The FF D_j^H , which describes the probability of a parton j fragmenting into a quarkonium H , is further factorized into short-distance coefficients and the LDMEs. Note that the coefficients $\mathcal{J}_{ij}$ [11] in Eq. (2) are different from those for exclusive jet [3] in out-of-jet contributions. The hierarchy $m_H \ll p_T R \ll p_T$ induces large logarithms, $\ln(p_T R/m_H)$ and $\ln R$, in the cross section, introducing significant theoretical uncertainties. These uncertainties can be systematically reduced by resumming the logarithms through renormalization group evolution (RGE): from m_H to $p_T R$ for FF and from $p_T R$ to p_T for FJF. The RGE for the FF is governed by the well-known DGLAP, while for the FJF, it follows DGLAP at the leading order and becomes DGLAP-like but different at higher orders [38,39]. The FFs in ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$ channels show divergent behavior in the threshold limit $z_H \rightarrow 1$. For the first time in the FJF framework, we adopt the recently developed threshold resummation formula [12], which resums the double logarithms $\log(1 - z_H)/(1 - z_H)$ and resolves the catastrophic failure of FFs.

Numerical calculation. We perform the corresponding calculation for the LHCb measurements, J/ψ [6] and

TABLE I. Three sets of LDMEs for J/ψ and for $\psi(2S)$ in units of GeV^3 and $m_c = 1.5 \text{ GeV}$.

		$\langle \mathcal{O}({}^3S_1^{[1]}) \rangle$	$\langle \mathcal{O}({}^3S_1^{[8]}) \rangle$	$\langle \mathcal{O}({}^1S_0^{[8]}) \rangle$	$\langle \mathcal{O}({}^3P_0^{[8]}) \rangle / m_c^2$
		$\times \text{GeV}^3$	$\times 10^{-2} \text{ GeV}^3$	$\times 10^{-2} \text{ GeV}^3$	$\times 10^{-2} \text{ GeV}^3$
J/ψ	Brambilla <i>et al.</i> [13]	1.18 ± 0.35	1.40 ± 0.42	-0.63 ± 3.22	2.33 ± 0.83
	Bodwin <i>et al.</i> [14]	1.32 ± 0.20	-0.71 ± 0.36	11.0 ± 1.4	-0.31 ± 0.15
	BK [15]	1.32 ± 0.20	0.22 ± 0.06	4.97 ± 0.44	-0.72 ± 0.09
$\psi(2S)$	Brambilla <i>et al.</i> [13]	0.71 ± 0.21	0.84 ± 0.25	-0.37 ± 1.92	1.55 ± 0.49
	Bodwin <i>et al.</i> [14]	0.76 [23]	-0.16 ± 0.28	3.14 ± 0.79	-0.12 ± 0.12
	BK [22]	0.76 [23]	0.054 ± 0.003	1.00 ± 0.03	-0.217 ± 0.005

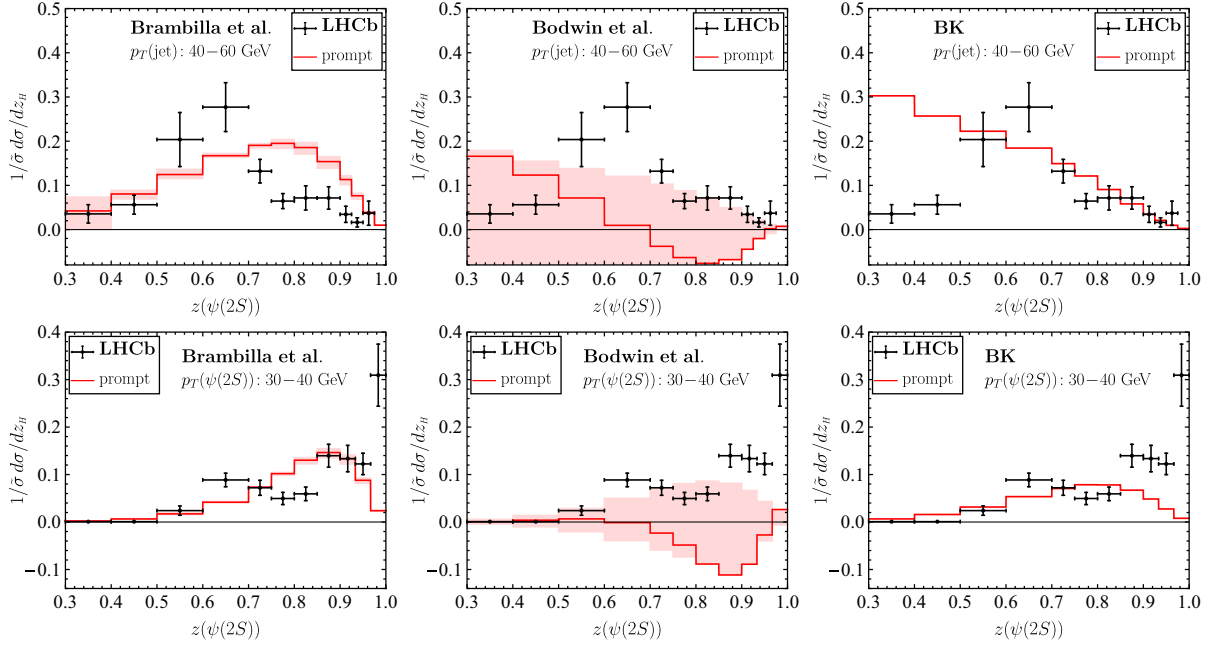


FIG. 1. Prediction for $z_{\psi(2S)}$ distributions for three sets of LDMEs in Table I and the LHCb measurements. The factor $\bar{\sigma}$ follows LHCb's normalization.

$\psi(2S)$ [9]. The LHCb analysis uses anti- k_t jets with a radius parameter $R = 0.5$ selected within the pseudorapidity range $2.5 < \eta < 4.0$. For J/ψ production, all events with jet transverse momentum $p_T > 20$ GeV are accumulated, generating a single momentum fraction z_H distribution. The $\psi(2S)$ dataset provides finer resolution through binning in both jet- p_T (seven bins spanning 8–60 GeV) and $\psi(2S)$ - p_T (six bins spanning 5–40 GeV), and we focus on comparison to the three highest- p_T bins in each category, where our predictions are accurate.

Experimentally, quarkonia are reconstructed via their muon-pair decay channels. LHCb's muon identification requires both decay products to satisfy pseudorapidity $2.0 < \eta < 4.5$ and transverse momentum $p_T > 0.5$ GeV, with additional momentum thresholds of $p_\mu > 5$ GeV for J/ψ and $p_\mu > 6$ GeV for $\psi(2S)$. These cuts preferentially suppress low- z_H events where the quarkonium carries a small fraction of the jet's momentum. Our calculations implement these selections by first modeling unpolarized quarkonium decays with isotropic $\mu^+\mu^-$ angular distributions in the quarkonium rest frame and then applying Lorentz boosts to the lab frame before enforcing the kinematic cuts on the boosted muons. This approach properly accounts for the z_H -dependent acceptance efficiency across the full kinematic range.

In our calculation, we convolute partonic cross sections at next-to-leading order (NLO) [40] with FJFs for quark and gluon channels. The FJFs incorporate NLO coefficients $\mathcal{J}_{ij}$ [11] and $\mathcal{O}(\alpha_s^2)$ FFs [41] for the four channels listed in Table I. The NLO cross section expansion reaches $\mathcal{O}(\alpha_s^4)$, with the $^3S_1^{[8]}$ channel entering from $\mathcal{O}(\alpha_s^3)$ while the other

three channels contribute at $\mathcal{O}(\alpha_s^4)$. We perform DGLAP evolution of threshold-resummed FFs from $2m_c$ to the jet scale $p_T R$ at leading-log (LL) accuracy, followed by FJF evolution from $p_T R$ to the hard scale p_T at LL. This procedure achieves an accuracy of $\text{LL}_R + \text{LL}_{\text{threshold}} + \text{NLO}$, representing the most refined perturbative calculation for this process.

Prompt- $\psi(2S)$ results. Figure 1 displays our predicted $z_{\psi(2S)}$ distributions for the three LDME sets from Table I, comparing results in both a 40–60 GeV jet- p_T bin (upper panel) and a 30–40 GeV $\psi(2S)$ - p_T bin (lower panel), with additional lower- p_T predictions provided in Figs. 3 and 4. The $z_{\psi(2S)}$ variation is performed through two complementary approaches: varying $\psi(2S)$ - p_T within a fixed jet- p_T bin and varying jet- p_T within a fixed $\psi(2S)$ - p_T bin.

While the LHCb data are normalized in $0 < z_H < 1$, our theoretical predictions are normalized to the corresponding area under the histogram within the mid- z_H region ($0.5 < z_H < 0.8$), avoiding potential biases from the less reliable low- z_H and high- z_H regimes. The theoretical uncertainties are calculated using covariance matrices for all three LDME sets. As shown in Table I, the Butenschoen and Kniehl (BK) set exhibits significantly smaller uncertainties compared to the other sets. For clarity in comparing LDME effects, we intentionally exclude perturbative uncertainties from scale variations.

The three LDME sets produce distinctive predictions, yet none fully reproduces the LHCb data. The agreement appears to be region dependent, with the Brambilla set

providing a better description at low z_H and the BK set performing better at high z_H , while a conclusion for the Bodwin set is difficult to draw owing to its large uncertainties. All cases demonstrate dominant contributions from the $^3P_J^{[8]} + ^3S_1^{[8]}$ channels, with $^3S_1^{[1]}$ being negligible and $^3S_1^{[8]}$ relatively minor compared to other octet contributions. Notably, the Bodwin set produces unphysical negative predictions across significant z_H ranges due to its large negative $^3S_1^{[8]}$ contribution overwhelming the positive $^3P_J^{[8]}$ contribution. In the upper panel, a low- z_H region receives power corrections of an order of $(m_H/p_T^H)^2$ that is not included in our results and shows significant deviations from the data, while in the lower panel we observe better agreement with the data since p_T^H is held fixed and the correction remains small in the low- z_H region.

The divergence as $z_H \rightarrow 1$ observed in [10] is cured by the threshold resummation, yielding well-behaved predictions in Fig. 1. To clearly illustrate the impact of the resummation, we provide the distributions (both resummed and unresummed) in Fig. 5. However, the LHCb data exhibit pronounced peaks at $z_H = 1$ across all p_T bins, except for two highest jet- p_T intervals, 40–60 and 30–40 GeV. These peaks grow more statistically significant at lower p_T values, which may indicate the dominance of a power-suppressed $(m_H/p_T)^2$ double-parton fragmenting process [41–46]. For example, in a process $gg \rightarrow c\bar{c}g$, where all gluons are coupled to the c -quark line, the $c\bar{c}$ pair retains most momentum, producing strong $z_H = 1$ peaking. PYTHIA simulations show similar peaks but broader due to different origins, $g \rightarrow c\bar{c}$ transitions, disagreeing with LHCb data [9].

The LHCb prompt- $\psi(2S)$ data demonstrate that binned z_H distributions in both jet- p_T and $\psi(2S)$ - p_T significantly

improve LDME discrimination and our understanding of the production mechanisms. For a comprehensive understanding of LHCb's z_H spectrum, a more quantitative analysis with improved theoretical predictions should be carried out. High $\psi(2S)$ - p_T bins, relatively free from mass corrections and the double-parton process, validate the FJFs framework. Low- z_H regions in fixed jet- p_T bins reveal mass correction effects, while the peaks at $z_H = 1$ imply a crucial role of the double-parton process at lower p_T bins. This complimentary dual binning makes FJFs ideal to test the NRQCD formalism. While Ref. [10] identified FJFs' potential as a LDME discriminator through shape differences from the J/ψ data, the $\psi(2S)$ case provides clearer insights. The J/ψ analysis is limited by reduced sensitivity to the high- p_T region due to $1/p_T^{5.5}$ scaling of the cross section, which causes the accumulated jet- p_T data dominated by the low- p_T region and less sensitive to the high- p_T region.

Prompt- J/ψ results. Figure 2 displays the predicted $z_{J/\psi}$ distributions for the LDME sets listed in Table I in two regimes: jet- $p_T > 20$ GeV (upper panel) and J/ψ - p_T in 30–40 GeV (lower panel). Both direct J/ψ production (blue curves) and prompt J/ψ production (red curves) are shown, with the latter including feeddown contributions from excited states. We note that the BK set does not explicitly account for feeddown contributions, as these effects are implicitly absorbed into the LDME values. In the other LDME sets, we incorporate feeddown through direct production of $\psi(2S)$, χ_{c1} , and χ_{c2} states, which are converted using their respective branching ratios: $\{0.615, 0.195, 0.343\}$. The LDMEs for S -wave excited states are taken from Table I, while those for P -wave

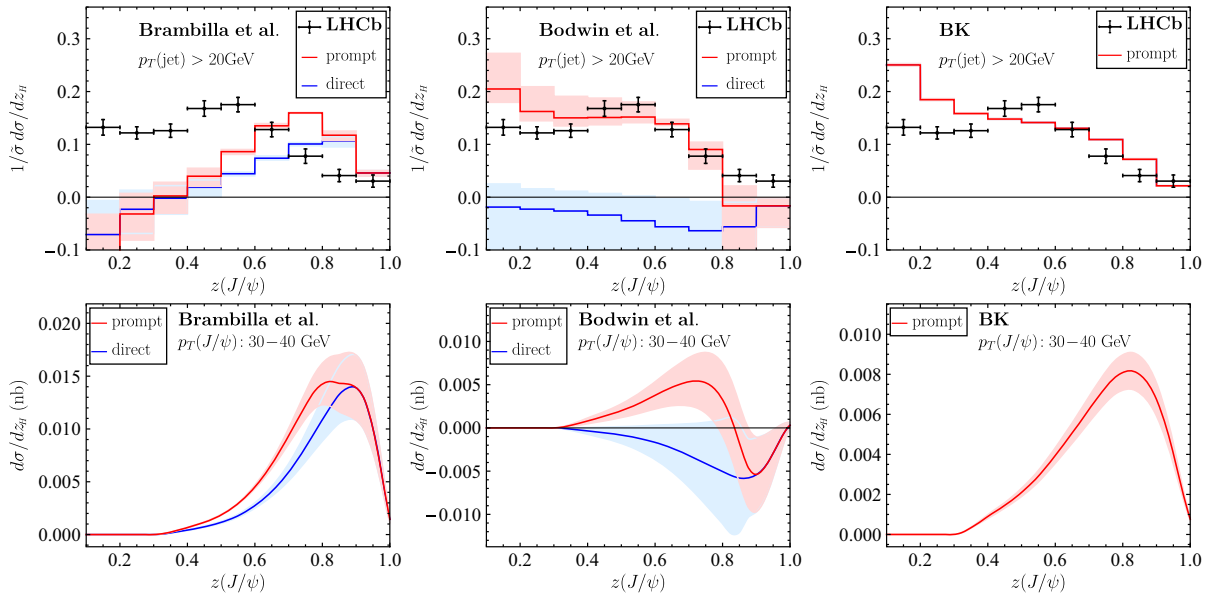


FIG. 2. Prediction for $z_{J/\psi}$ distributions for three sets of LDMEs in Table I and the LHCb measurements.

states adopt Refs. [12–14]. To account for phase-space differences between quarkonium states, their distributions are rescaled via $(m_H/m_{J/\psi})z_{J/\psi}$ as implemented in Ref. [14]. The feeddown contribution accounts for approximately 35% of the total yield in the Brambilla set, while it dominates in the Bodwin set. As observed in the $\psi(2S)$ case, the $^3P_J^{[8]} + ^3S_1^{[8]}$ channels remain the dominant production mechanisms. Notably, the Bodwin set exhibits unphysical negative values in the direct component across the entire z_H range, requiring feeddown contributions to compensate and restore physical positivity in the prompt production yield.

In the upper panel, the predictions from the Bodwin and BK sets show reasonable agreement with the LHCb data, while the Brambilla set deviates significantly at low z_H . However, this apparent success is tempered by theoretical caveats: The Bodwin set predicts unphysical negative values for direct production, and the BK set's treatment of feeddown contributions requires further refinement. Our results remain consistent with lower-order predictions in Refs. [10,47] within LDME uncertainties for corresponding LDME sets, despite differences in feeddown treatment. Unlike $\psi(2S)$ data, the $z_H = 1$ peak is not observed, likely due to coarser z_H binning in the region. Higher z_H granularity could resolve this structure.

The z_H distributions binned in J/ψ - p_T , analogous to the $\psi(2S)$ analysis, would provide complementary information by suppressing low- z_H subleading corrections and further constraining the production mechanism. Existing J/ψ data from LHCb [6] and CMS [7] could be used to extract these distributions and compare them to our predicted distribution for the 30–40 GeV bin (lower panel in Fig. 2). Notably, our predictions were intentionally left unnormalized by area, as both the absolute cross sections and the shape of the distribution offer critical constraints.

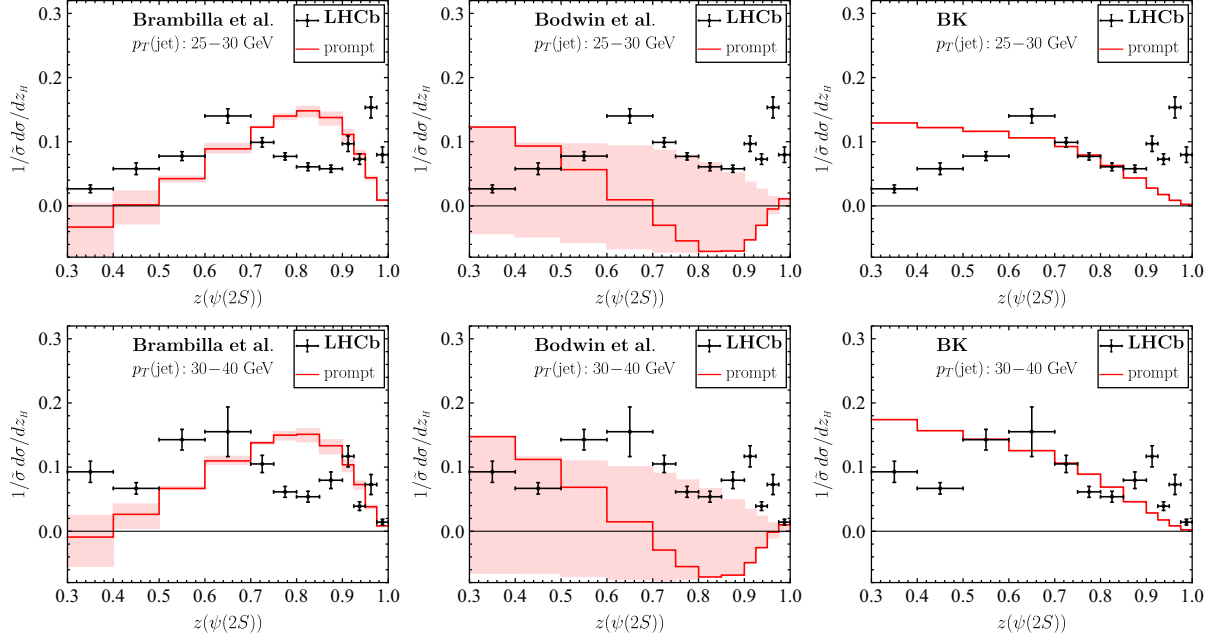
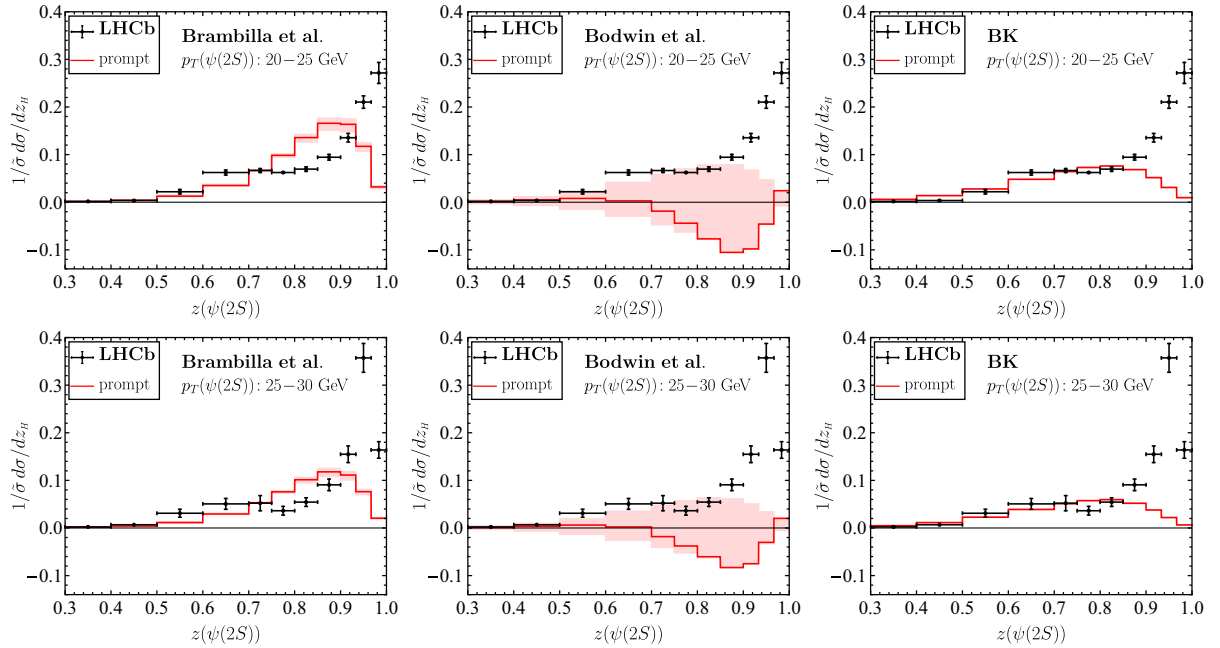
Conclusions. The LHCb $\psi(2S)$ -momentum fraction (z_H) data, binned in both jet and quarkonium p_T , provide an unprecedented level of resolution and enable discrimination between NRQCD LDME sets for $\psi(2S)$. Our LL + NLO

predictions with threshold resummation, when compared with the LHCb data, validate discriminating power of the FJF framework. Deviations in small z_H highlight the need for mass-suppressed corrections, and the observed near $z_H = 1$ excess suggests double-parton process beyond current leading-power contribution, urging theoretical refinements in the FJF framework. The p_T -integrated J/ψ distributions exhibit moderate agreement for Bodwin (category 2) and BK (category 3) sets, obscuring clear discrimination, a limitation arising from the coarser p_T resolution. This analysis establishes quarkonium production within jets as a critical probe of quarkonium-production mechanisms. The dual p_T -bin strategy for $\psi(2S)$ suppresses theoretical uncertainties and isolates LDME-dependent features, outperforming integrated J/ψ measurements. Future J/ψ analyses with analogous binning could resolve long-standing LDME tensions. These results underscore the synergy between resolved p_T measurements and effective field theory techniques in unraveling quarkonium production mechanisms.

Acknowledgments. This work was inspired by Tom Mehen, who initially suggested pursuing this direction. D. K. thanks Korea University for their hospitality during an important phase of this work and acknowledges Chul Kim for fruitful discussions on FJFs. Special thanks go to Byungsik Hong, Soohwan Lee, Bayu Putra, and Junseok Lee for valuable discussions regarding the CMS experiment. The work of D. K. and Y. W. is supported by the National Natural Science Foundation of China (NSFC) through National Key Research and Development Program under Contract No. 2024YFA1610503. The work of H. S. C. is supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (Grant No. RS-2023-00248313).

Data availability. The data that support the findings of this article are openly available [48].

APPENDIX

FIG. 3. Predicted $z(\psi(2S))$ distributions for the three sets of LDMEs and the LHCb measurements for two jet- p_T bins.FIG. 4. Predicted $z(\psi(2S))$ distributions for the three sets of LDMEs and the LHCb measurements for two $\psi(2S)$ - p_T bins.

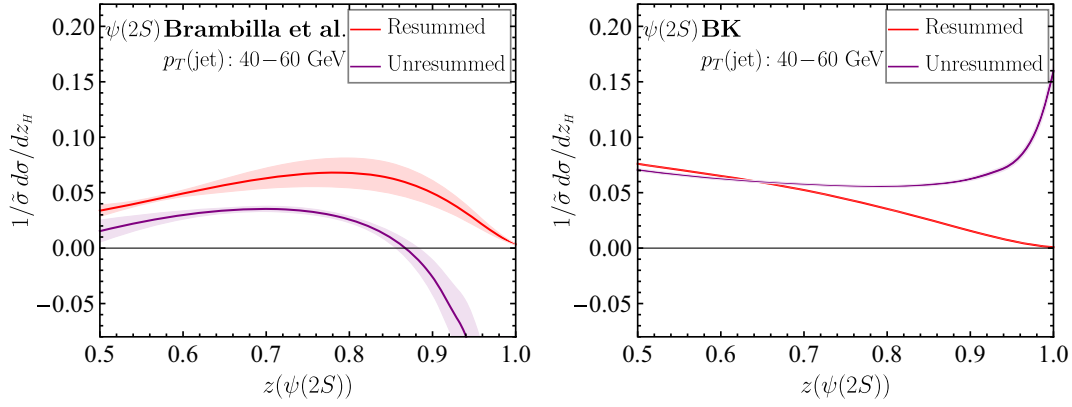


FIG. 5. z distributions for $\psi(2S)$ with and without threshold resummation (resummed and unresummed). Similar effects are observed for J/ψ .

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