

Complex scalar dark matter with effective Higgs portals beyond radiation domination

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The increasingly stringent bounds on the Higgs-portal coupling, arising from dark matter (DM) direct-detection searches, confront the minimal renormalizable complex scalar DM scenario with thermal production, where freeze-out occurs in the standard radiation-dominated era. This limitation can be alleviated by introducing a dimension-five Higgs-portal operator in the minimal renormalizable complex scalar DM model and/or by modifying the standard cosmological history of the Universe. In this article, we analyze complex scalar DM production in both the reheating and radiation-dominated epochs within an effective field theory (EFT) framework. While both scenarios exhibit sizable regions of parameter space consistent with existing constraints, freeze-out during reheating opens up additional viable regions that are otherwise ruled out by DM overabundance in the radiation-dominated scenario. Notably, the renormalizable Higgs-portal coupling is constrained by relic density and direct- and indirect-detection limits. In contrast, the EFT coupling associated with the dimension-five operator is constrained by relic density and indirect-detection bounds arising from DM semiannihilation. We further study the production cross section of complex scalar DM at hadron and lepton colliders.

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I. INTRODUCTION

Despite strong theoretical and observational [1,2] motivations for dark matter (DM), its identity and fundamental properties remain elusive. Among the myriad proposed candidates, spin-0 scalar DM [3] provides the simplest framework for exploring its dynamics and interactions. The production of scalar DM during the Standard Model (SM) radiation-dominated era, particularly when the reheating temperature is much larger than the freeze-out temperature, has been extensively studied in the literature [4–8]. Scalar DM is often overproduced once constraints from direct- and indirect-detection experiments, as well as collider searches, are taken into account. This can be evaded if the minimal renormalizable setup is being extended by additional dark-sector states [9–18] or by effective field theory (EFT) operators [19–22] involving DM and SM fields. Alternatively, a modification of the

standard cosmological history, such as entropy dilution [23–28] during the reheating era, also serves as a viable framework to overcome this constraint.

Among scalar DM candidates, the SM gauge-singlet real scalar offers the simplest realization, whereas complex scalar dark matter (CSDM) is a viable alternative. A CSDM, stabilized by a $\mathbb{Z}_3$ symmetry [29–31], and interacting with the Higgs boson through a renormalizable portal coupling, is strongly constrained by direct-detection (DD) experiments. Based on the recent LZ-2025 [32] results, the spin-independent (SI) DM-nucleon scattering cross section larger than $[10^{-46} - 10^{-48}] \text{ cm}^2$ is excluded for DM masses up to 10 TeV. This imposes severe constraint on the CSDM parameter space, except around the Higgs resonance. Even if the next-to-leading-order (NLO) corrections of the Higgs-portal coupling are included [33], the bounds remain largely unchanged. Additional constraints in this region arise from Fermi-LAT observations of DM self-annihilation [34–36]. The severe constraints in this minimal setup arise because the same portal coupling governs both the DM relic abundance and direct-detection cross section. Once constraints from direct detection are imposed, the DM relic density is overabundant in most of the parameter space.

A promising approach to enlarge the viable parameter space of the CSDM model is by introducing an additional dimension-five (dim-5) interaction term involving CSDM

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and SM Higgs boson into the theory, which does not contribute to CSDM-nucleon scattering but plays a crucial role in the DM freeze-out process, and hence, DM relic density can be satisfied. In this work, we study this $\text{dim} = 5$ EFT extension, focusing on scenarios in which DM freeze-out occurs in the radiation-dominated era or during reheating. We show that, compared to the standard instantaneous reheating scenario, a much larger parameter space is allowed by the DM relic density constraint if freeze-out occurs during reheating. We further constrain the model parameter space using γ -ray observations from Fermi-LAT, focusing on limits derived from DM self- and semiannihilation processes. Since the semiannihilation rate depends on the Wilson coefficient of the DM-EFT operator, we ensure the validity of the EFT description, which imposes constraints on the DM mass, the new-physics scale, and the maximum temperature of the thermal bath. After imposing relic-density requirements as well as direct- and indirect-detection bounds, the DM production cross sections at future colliders [37–44] are found to be very small $\sim 10^{-3}$ fb, rendering discovery through conventional cut-based analyses highly nontrivial.

The paper is organized as follows. In Sec. II, we present the CSDM framework, in which the dark sector interacts with the SM through the Higgs portal. In Sec. III, we discuss the experimental constraints on the parameter space consistent with the observed relic density, explore prospects for detection in future DM experiments, and evaluate the DM production cross section in future colliders. Finally, the summary and conclusions are given in Sec. IV.

II. THE MODEL

In this article, we extend the renormalizable Higgs-portal CSDM model by introducing a dim-5 CSDM-Higgs EFT operator, $\mathcal{O}_5: \Lambda_{\text{NP}}^{-1} \Phi^3 (H^\dagger H)$, where Λ_{NP} is the new-physics (NP) scale. In contrast, in the case of real scalar and vector boson DM, the leading gauge-invariant DM-EFT operator arises only at dim-6,¹ which is always suppressed. Moreover, for fermionic DM, the leading operator arises at dim-5 and is tightly constrained by DD and indirect-detection constraints [45]. We explore both the possibilities: standard thermal production of CSDM in the radiation-dominated era and during reheating. The latter scenario is augmented with two new parameters: the inflaton decay width and the expansion rate at the end of inflation. Throughout this work, we assumed that the inflaton decays predominantly into SM particles, while its decay into DM particles is negligible. Up to dim-5, the Lagrangian corresponds to a Higgs-portal SM gauge-singlet CSDM field (Φ), which transforms under a $\mathbb{Z}_3$ symmetry as $\Phi \rightarrow e^{i2\pi/3} \Phi$ is [29,30,46,47]

¹Assuming the dark sector contains only one type of beyond the SM particle.

$$\begin{aligned} \mathcal{L} \supset & |\partial_\mu \Phi|^2 - \mu_\Phi^2 |\Phi|^2 - \lambda_\Phi |\Phi^* \Phi|^2 \\ & - \frac{1}{2} \mu_3 [\Phi^3 + (\Phi^*)^3] - \lambda_{\Phi H} |\Phi|^2 H^\dagger H \\ & - \Lambda_{\text{NP}}^{-1} [\Phi^3 + (\Phi^*)^3] (\lambda_5 |\Phi|^2 - \lambda'_{\Phi H} H^\dagger H) \\ & + \mathcal{O}(\Lambda_{\text{NP}}^{-2}), \end{aligned} \quad (2.1)$$

where $\lambda'_{\Phi H}$ is taken to be positive, although negative values are equally viable, subject to theoretical constraints. We follow the notation of [47] in defining the Lagrangian. Higher-dimensional operators beyond dim-5 always yield subdominant contributions compared to the dim-5 operator. The dim-5 self-interaction term is relevant only for low DM masses, $m_\Phi \sim \mathcal{O}(\text{MeV})$, where it dominantly contributes to the DM relic abundance through the $3\Phi \rightarrow 2\Phi$ annihilation process [48]. Therefore, we remain agnostic about λ_5 throughout the rest of this work. The Higgs field acquires a nonzero vacuum expectation value (VEV) after spontaneous electroweak symmetry breaking (EWSB), $\langle H \rangle \rightarrow v/\sqrt{2}$ with $v = 246$ GeV, and the CSDM mass is modified as $m_\Phi^2 = \mu_\Phi^2 + \frac{1}{2} \lambda_{\Phi H} v^2$. It is important to note that inclusion of the dim-5 operator $\frac{\lambda'_{\Phi H}}{\Lambda_{\text{NP}}} \Phi^3 (H^\dagger H)$ shifts the cubic coupling after EWSB to $\mu_{\text{eff}} = \mu_3 - \frac{\lambda'_{\Phi H} v^2}{\Lambda_{\text{NP}}}$, and the vacuum stability condition is then $|\mu_{\text{eff}}| \leq 2\sqrt{\lambda_\Phi} m_\Phi$ [29]. Although λ_Φ does not affect the dark matter relic abundance or its detection, it plays a crucial role in ensuring vacuum stability, thereby constraining the trilinear coupling (μ_3) and higher-dimensional parameters ($\lambda'_{\Phi H}$ and Λ_{NP}). In this context, negative values of $\lambda'_{\Phi H}$ impose tighter constraints, while positive values alleviate them.

The search for Higgs boson invisible decays by the ATLAS detector, using 139 fb^{-1} of proton-proton collision data at a center-of-mass energy of $\sqrt{s} = 13$ TeV at the LHC, combined with results obtained at $\sqrt{s} = 7$ and $\sqrt{s} = 8$ TeV, sets an observed upper limit of 0.107 on the Higgs boson branching ratio to invisible final states at the 95% confidence level [49]. A combined analysis of $\sqrt{s} = 7, 8$, and 13 TeV proton-proton collision data at the CMS detector sets an observed 95% confidence level (CL) upper limit of 0.15 on $\mathcal{B}(h \rightarrow \text{inv})$ [50],

$$\mathcal{B}(h \rightarrow \text{invisible}) < \begin{cases} 0.107 & \text{ATLAS [95\% CL]}, \\ 0.15 & \text{CMS [95\% CL]}. \end{cases} \quad (2.2)$$

In this model framework, the Higgs boson can decay into CSDM via $h \rightarrow \Phi \Phi^*$ and $h \rightarrow 3\Phi, (3\Phi^*)$ provided that $m_h > 2m_\Phi$ and $m_h > 3m_\Phi$, respectively. The corresponding Feynman diagrams and decay width expressions for these processes are presented in the Appendix. The constraint on the Higgs-portal coupling $\lambda_{\Phi H}$ from Higgs boson invisible decays is always weaker [49] than the constraint from the spin-independent CSDM-nucleon scattering cross section

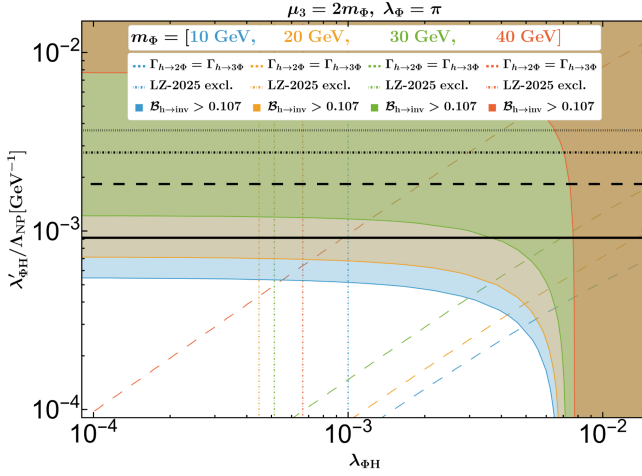


FIG. 1. The blue, orange, and olive colors correspond to $m_\Phi = 10, 20$, and 30 GeV, respectively. The shaded regions are excluded by constraints on the Higgs boson invisible decay width from LHC measurements, while the regions to the right of the dot-dashed lines are ruled out by direct-detection limits from LZ (2025). The dashed lines denote $\Gamma_{h \rightarrow 2\Phi} = \Gamma_{h \rightarrow 3\Phi}$. The region above the black lines is disfavored by the vacuum stability constraint $\mu_{\text{eff}} < 2\sqrt{\lambda_{\Phi H}} m_\Phi$, while the thick, dashed, dot-dashed, and dotted lines correspond to $m_\Phi = \{10, 20, 30, \text{ and } 40\}$ GeV, respectively.

measured by LZ-2025, if only renormalizable interactions are present.

Additionally, in the presence of a nonrenormalizable interaction term, the Higgs boson invisible decay width is modified in the kinematic regime $m_h > 3m_\Phi$. This contribution becomes dominant when $\lambda'_{\Phi H}/\Lambda_{\text{NP}} \gtrsim 5 \times 10^{-4} \text{ GeV}^{-1}$ for $m_\Phi = 10$ GeV. For larger benchmark masses, such as $m_\Phi = 20$ and 30 GeV, this constraint is relaxed, as illustrated in Fig. 1. Although this region is allowed by direct-detection constraints due to the tiny renormalizable portal coupling ($\lambda_{\Phi H} \lesssim 4.47 \times 10^{-4}$), it may still be constrained by Higgs boson invisible decay limits from the LHC, depending on the strength of the nonrenormalizable interaction ($\lambda'_{\Phi H}/\Lambda_{\text{NP}}$).

In our regime of interest ($m_\Phi \gtrsim m_h/2$), in particular, invisible decay bounds are not relevant; however, vacuum stability may become significant when $\lambda'_{\Phi H}/\Lambda_{\text{NP}} \gtrsim 10^{-2}$. For completeness, we impose all relevant theoretical and experimental constraints throughout our analysis.

III. DARK MATTER PHENOMENOLOGY

In the reheating era, the inflaton field (ϕ) decays into SM radiation with decay width Γ_ϕ , and its energy density evolution is governed by the Boltzmann equations (BEQs) [23,51],

$$\frac{d\rho_\phi}{dt} + 3\mathcal{H}\rho_\phi = -\Gamma_\phi\rho_\phi, \quad (3.1)$$

$$\frac{d\rho_r}{dt} + 4\mathcal{H}\rho_r = +\Gamma_\phi\rho_\phi. \quad (3.2)$$

Here, $\rho_\phi(\rho_r)^2$ denotes the inflaton (radiation) energy density. In terms of entropy, (3.2) can be rewritten as

$$\frac{ds}{dt} + 3\mathcal{H}s = +\frac{\Gamma_\phi\rho_\phi}{T}. \quad (3.3)$$

The entropy density (s) is defined as a function of the SM radiation temperature T ,

$$s = \frac{2\pi^2}{45} h_{\text{eff}}(T) T^3, \quad (3.4)$$

where h_{eff} is the number of relativistic degrees of freedom contributing to the entropy density [52], and the Hubble expansion rate ($\mathcal{H}$) is given by

$$\mathcal{H}^2 = \frac{8\pi\rho_\phi + \rho_R}{3m_{\text{pl}}^2}, \quad (3.5)$$

where $m_{\text{pl}} \simeq 1.2 \times 10^{19} \text{ GeV}$ is the Planck mass and ρ_R denotes the Standard Model energy density and is given by

$$\rho_R = \frac{\pi^2}{30} g_{\text{eff}}(T) T^4, \quad (3.6)$$

where g_{eff} is the number of relativistic degrees of freedom contributing to ρ_R .

The reheating temperature (T_{rh}) is defined as the temperature where $\rho_\phi(T_{\text{rh}}) = \rho_R(T_{\text{rh}})$ and estimated implicitly by the equality $\mathcal{H}(T_{\text{rh}}) = \Gamma_\phi$, which implies [23]

$$T_{\text{rh}}^2 = \frac{3}{2\pi} \sqrt{\frac{5}{\pi g_{\text{eff}}(T_{\text{rh}})}} m_{\text{pl}} \Gamma_\phi, \quad (3.7)$$

and the maximum temperature T_{max} reached during reheating by the thermal bath, which can be estimated by [53,54]

$$T_{\text{max}}^4 = \frac{15}{2\pi^3 g_{\text{eff}}(T_{\text{max}})} \left(\frac{3}{8}\right)^{8/5} m_{\text{pl}}^2 \Gamma_\phi \mathcal{H}_1, \quad (3.8)$$

where $\mathcal{H}_1$ is the expansion rate of the Universe at the end of inflation. The desired value of T_{max} , which should remain below the new-physics scale Λ_{NP} depending on the underlying theory, can be achieved through appropriate choices of Γ_ϕ and $\mathcal{H}_1$. For a reheating temperature fixed by Γ_ϕ [see Eq. (3.7)], the maximum bath temperature can be adjusted

²For a monomial potential ($V(\phi) = \lambda\phi^n$), the Boltzmann equation becomes $\frac{d\rho_\phi}{dt} + 3\mathcal{H}(1 + \omega_\phi)\rho_\phi = -\Gamma_\phi\rho_\phi$, where $\omega_\phi = p_\phi/\rho_\phi = (n-2)/(n+2)$ is the effective equation-of-state parameter of the inflaton.

by varying $\mathcal{H}_I$. However, this freedom is subject to several observational constraints, as discussed below. We have rewritten Eqs. (3.1) and (3.3) in terms of the cosmic scale factor a . To solve the BEQs, we use the initial conditions at the onset of reheating, $t = t_I$, corresponding to the scale factor $a_I \equiv a(t_I) = 1$. At this time, the radiation energy density is $\rho_R(t_I) = 0$, and the inflaton energy density is $\rho_\phi(t_I) = \frac{3}{8\pi} m_{\text{pl}}^2 \mathcal{H}_I^2 \neq 0$, where $\mathcal{H}_I$ denotes the inflationary scale. This scale is bounded from above by $2.5 \times 10^{-5} m_{\text{pl}}$ [55,56], based on the nonobservation of B-mode polarization. However, there also exist lower bounds on this inflationary parameter, derived from cosmic microwave background (CMB) observations and BBN. From the Planck measurements, the amplitude of scalar perturbations is found to be $A_s \approx 2.1 \times 10^{-9}$, while the tensor-to-scalar ratio r can, in principle, be arbitrarily small. The inflationary parameter is related to these quantities as $\mathcal{H}_I \simeq \pi m_{\text{pl}} \sqrt{\frac{r A_s}{2}}$ [57]. Therefore, inflation could occur at a much lower energy scale without contradicting the CMB data. However, for BBN to proceed normally, inflation must end early enough to allow reheating to produce the SM bath with temperature $T_{\text{rh}} \gtrsim 4$ MeV [58–60].

The evaluation of CSDM number density (n_Φ) is governed by the Boltzmann equation,

$$\frac{dn_\Phi}{dt} + 3\mathcal{H}n_\Phi = -\langle\sigma v\rangle_{\Phi\Phi^* \rightarrow \text{SM SM}}(n_\Phi^2 - n_\Phi^{\text{eq}^2}) - \frac{1}{2}\langle\sigma v\rangle_{\Phi\Phi \rightarrow \Phi^* h}(n_\Phi^2 - n_\Phi n_\Phi^{\text{eq}}), \quad (3.9)$$

where $\text{SM} = \{\text{leptons, quarks, Higgsboson, } W^\pm, \text{ and } Z\}$, $\langle\sigma v\rangle_{\Phi\Phi^* \rightarrow \text{SM SM}}$ and $\langle\sigma v\rangle_{\Phi\Phi \rightarrow \Phi^* h}$ are the self- and semi-annihilation cross sections³ of the CSDM, and n_Φ^{eq} is the equilibrium number density of the CSDM. Finally, the DM relic abundance is given by

$$\Omega_{\text{DM}} h^2 = 2.744 \times 10^8 \frac{n_\Phi}{s} \Big|_{T=T_0}, \quad (3.10)$$

where $T_0 \sim 2.35 \times 10^{-13}$ GeV [61] denotes present-day temperature of the Universe. The observed DM relic abundance reported by Planck [62] is

$$\Omega_{\text{DM}}^{\text{obs}} h^2 = 0.1200 \pm 0.0012, \quad (3.11)$$

where h is the reduced Hubble parameter $H/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ with $H = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [62] being the current Hubble constant. Notably, late-time entropy production

³The other annihilation channels, such as $3\Phi \rightarrow \Phi h$, $2\Phi h \rightarrow \Phi h$, and $3\Phi \rightarrow hh$, are not included in Eq. (3.9), as these processes remain subdominant throughout the CSDM mass range considered.

from heavy particle decays enhances the post-freeze-out plasma entropy, thereby diluting the comoving DM relic density and reducing its abundance below the standard thermal freeze-out value. This provides a minimal mechanism to reconcile an initially overabundant thermal DM relic with the observed relic density. In this article, the BEQs [Eqs. (3.1), (3.3), and (3.9)] are solved numerically, and the corresponding values of T_{rh} , T_{max} , and $\Omega_{\text{DM}} h^2$ (3.10) are evaluated using the `micrOMEGAs` package [23,63]. Importantly, T_{max} must satisfy the relation $T_{\text{max}} < \Lambda_{\text{NP}}$; otherwise, it would violate the EFT validity criterion [64].

The thermally averaged DM semiannihilation cross section can be expressed as $\langle\sigma v\rangle \sim \mathcal{O}(T/\Lambda_{\text{NP}})$. Requiring the convergence of $\langle\sigma v\rangle$ implies $\Lambda_{\text{NP}} > T_{\text{max}}$, which also forbids the on-shell production of the heavy mediators. Nevertheless, the condition $T_{\text{max}} < \Lambda_{\text{NP}}$ is not necessary [65] for thermal DM freeze-out, since the relic density is controlled by the dynamics at the DM freeze-out temperature $T_{\text{FO}} (\ll T_{\text{max}})$.

A. Relic density and direct detection

The relic abundance of DM is evaluated by solving the Boltzmann equation (3.9) at the present epoch. We have solved it for two separate scenarios: (i) thermal freeze-out during the standard radiation-dominated era assuming instantaneous reheating and (ii) freeze-out during the reheating phase, where the maximum bath temperature is governed by (3.8). In the second scenario, the newly introduced model parameters Γ_ϕ and $\mathcal{H}_I$ determine the values of T_{rh} and T_{max} , which are constrained by CMB and BBN observations, as discussed in the previous section. The parameters m_Φ , $\lambda_{\Phi H}$, μ_3 , $\lambda'_{\Phi H}$, and Λ_{NP} play a crucial role in determining the DM number density, as they govern the annihilation processes. In addition, T_{rh} (which is decided by Γ_ϕ and $\mathcal{H}_I$) influences the entropy dilution, thereby simultaneously reducing the DM number density, as illustrated in Fig. 5. The freeze-out is governed not only by DM self-annihilation but also by semiannihilation processes, as illustrated in Fig. 2. It should be noted that the new-physics parameters (Λ_{NP} , $\lambda'_{\Phi H}$) associated with the dim-5 DM-EFT operators contribute exclusively to DM semiannihilation processes, in addition to the portal ($\lambda_{\Phi H}$) and trilinear (μ_3) couplings. Notably, in this analysis of the DM number density, we consistently impose the EFT validity condition $2m_\Phi < \Lambda_{\text{NP}}$, since the s -wave contribution dominates the semiannihilation process during freeze-out. In the absence of this condition, the thermal average of the semiannihilation cross section should be evaluated by integrating over the center-of-mass energy s in the range $\max[4m_\Phi^2, (m_\Phi + m_h)^2] \leq s \leq \Lambda_{\text{NP}}^2$.

We now turn to direct-detection constraints; the CSDM can elastically scatter off detector nuclei, inducing nuclear recoils that may provide observable signatures of dark matter. Numerous experiments [66–68] are actively searching for DM; however, in this study, we focus solely on the

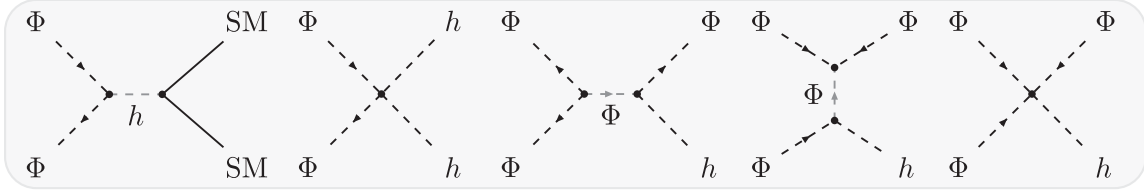


FIG. 2. The Feynman diagrams correspond to the self- and semiannihilation of DM, where $\text{SM} = \{h, W^\pm, Z, \text{quarks, and leptons}\}$.

most stringent constraint, the spin-independent DM-nucleon ($\Phi - N$) scattering cross section ($\sigma_{\Phi N}^{\text{SI}}$), currently reported by the LUX-ZEPLIN experiment [32]. In this model framework, DM interacts with nucleons through the Higgs portal interaction. The scattering cross section $\sigma_{\Phi N}^{\text{SI}}$ is primarily governed by the parameters m_Φ and $\lambda_{\Phi H}$. This complementarity—where direct-detection constraints depend mainly on $\lambda_{\Phi H}$, while the relic density is sensitive to additional parameters such as Λ_{NP} , $\lambda'_{\Phi H}$, and μ_3 —alleviates the severe constraints typically encountered in minimal renormalizable CSDM models.

In Fig. 3, we show the variation of the DM relic density ($\Omega_\Phi h^2$) and the spin-independent DM-nucleon scattering cross section ($\sigma_{\Phi N}^{\text{SI}}$) (shown on the magenta right y axis) as functions of the DM mass m_Φ , while keeping all other parameters fixed as $\Lambda_{\text{NP}} = 20$ TeV, $\lambda_\Phi = 1$, $\mu_3 = 2m_\Phi$, $\lambda_{\Phi H} = 10^{-2}$, $\lambda'_{\Phi H} = 1$, and $\mathcal{H}_1 = 10^{-4}$ GeV. The magenta dot-dashed line represents the upper limit on the spin-independent DM-nucleon scattering cross section from LZ-2025. The results are shown for three values of the inflaton decay width, $\Gamma_\phi = \{10^{-14}, 10^{-16}, 10^{-18}\}$ GeV, represented by the dashed brown, green, and orange lines, respectively. For these three values of Γ_ϕ , with $\mathcal{H}_1 = 10^{-4}$ GeV, the

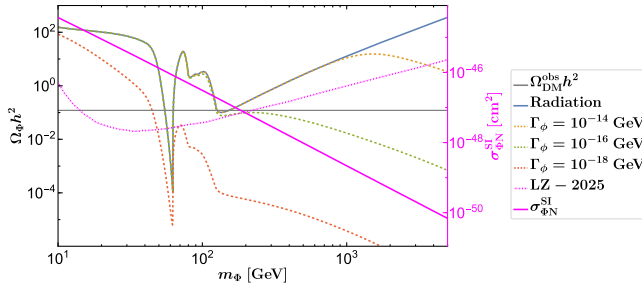


FIG. 3. In this figure, we present the variation of the CSDM relic density $\Omega_\Phi h^2$ as a function of the DM mass m_Φ for scenarios in which freeze-out occurs during the standard radiation-dominated epoch and during the reheating phase. These are represented by the thick blue and dashed (yellow, green, magenta) lines, respectively. In addition, the right y axis shows the DM-nucleon scattering cross section. The thick magenta line represents the Φ -nucleon scattering cross section. The dot-dashed magenta line indicates the LZ-2025 limit on the Φ -nucleon scattering cross section. For this figure, we have considered the following benchmark parameters: $\Lambda_{\text{NP}} = 20$ TeV, $\lambda_\Phi = 1$, $\mu_3 = 2m_\Phi$, $\lambda_{\Phi H} = 10^{-2}$, $\lambda'_{\Phi H} = 1$, and $\mathcal{H}_1 = 10^{-4}$ GeV.

corresponding reheating and maximum temperatures are $T_{\text{rh}} = \{55.44, 5.70, 0.61\}$ GeV and $T_{\text{max}} = \{16.23, 5.13, 1.62\}$ TeV, respectively. The thick blue line represents the standard radiation-dominated scenario. The green and yellow dashed lines begin to deviate from it when the DM mass is such that the reheating temperature (T_{rh}) falls below the freeze-out temperature (T_{FO}), while the deviation for the brown line has started at $m_\Phi \ll 10$ GeV. For the standard freeze-out scenario, when $m_\Phi > m_h$, the relic density increases with increasing m_Φ , as both the annihilation and semiannihilation rates of the CSDM into SM particles decreases with increasing m_Φ . For $m_\Phi \lesssim m_h$, we observe four dips in the blue relic-density curve originating from CSDM annihilation into various SM final states, which dominantly contribute to the relic density. From left to right, the four dips arise due to the dominant ($\gtrsim 10\%$) contributions to the relic density from the processes $\Phi\Phi^* \rightarrow b\bar{b}$, $\Phi\Phi^* \rightarrow W^+W^-$, $\Phi\Phi^* \rightarrow ZZ$, and $\Phi\Phi \rightarrow h\Phi^*$,⁴ respectively. A similar behavior is observed for the other three dashed relic-density curves; however, it diminishes once the effects of the reheating phase become significant in determining the DM relic abundance. We note that the DM-nucleon scattering cross section gradually decreases with the enhancement of m_Φ , and some of the parameter space is excluded by the LZ-2025 data, which are mostly overabundant. Consequently, the LZ-2025 allowed parameter space remains viable for further studies if the freeze-out is assumed to occur exclusively during the reheating epoch.

In Fig. 4, the yellow points represent parameter values in the $m_\Phi - \Lambda_{\text{NP}}$ plane that are consistent with the observed DM relic density for scenarios in which freeze-out occurs in the radiation-dominated epoch [Fig. 4(a)] and during the reheating phase [Fig. 4(b)]. As in Fig. 3, the variation of $\sigma_{\Phi N}^{\text{SI}}$ is shown on the right y axis, with the thick magenta line representing its dependence on m_Φ and the dot-dashed magenta line indicating the current LZ-2025 direct-detection limit. As Λ_{NP} gradually increases, the semiannihilation cross section correspondingly decreases, leading to an overabundance of the relic density. This overabundance can be mitigated by selecting a very small T_{rh} (i.e., Γ_ϕ). With increasing m_Φ , a decrease in Λ_{NP} during the radiation-dominated era [Fig. 4(a)] and an increase in

⁴Also, the process $\Phi\Phi^* \rightarrow hh$ can be relevant in this regime, particularly when the semiannihilation channel is comparatively suppressed [7].

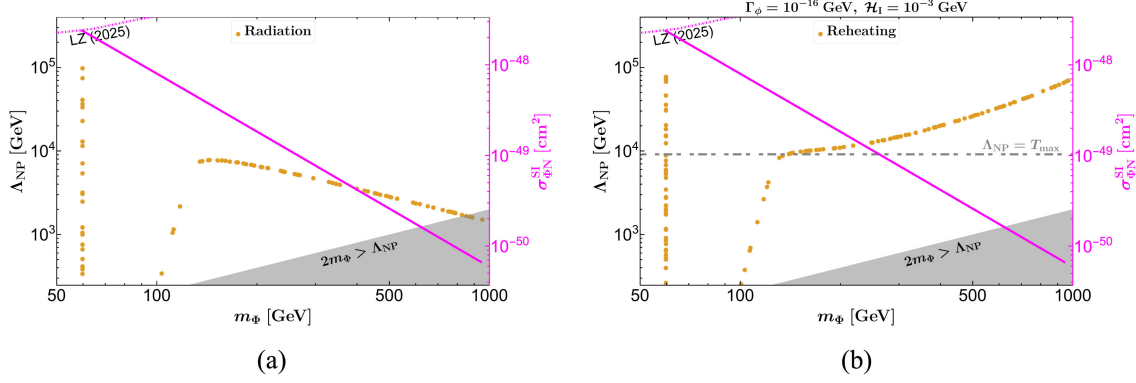


FIG. 4. The orange points in (a) and (b) represent the parameter values in the $m_\phi - \Lambda_{\text{NP}}$ plane that are consistent with the observed relic density. The magenta color-coded lines correspond to the $m_\phi - \sigma_{\Phi N}^{\text{SI}}$ plane (x axis and right y axis) in both plots. (a) These points correspond to freeze-out during the radiation-dominated epoch, while in (b) they correspond to freeze-out during the reheating epoch. In both plots, the thick magenta line represents the Φ -nucleon scattering cross section considering parameters as $\lambda_\phi = 1$, $\mu_3 = 2m_\phi$, $\lambda_{\phi H} = 10^{-3}$, and $\lambda'_{\phi H} = 1$. In addition, the right y axis in figures shows the CSDM-nucleon scattering cross section, with the dot-dashed magenta line indicating the LZ-2025 limit on DM-nucleon scattering. The gray-shaded region is excluded due to the EFT validity criterion, $\Lambda_{\text{NP}} > 2m_\phi$.

Λ_{NP} in the reheating scenario [Fig. 4(b)] are required to reproduce the observed DM relic density. It is important to ensure that the resulting T_{max} remains below Λ_{NP} for the chosen values of Γ_ϕ and $\mathcal{H}_I$. The gray dashed line in Fig. 4 (b) denotes the lower limit $\Lambda_{\text{NP}} = T_{\text{max}}$, applicable for the scenario where relic density is predominantly governed by the CSDM semiannihilation.⁵ Notably, if the correct relic density is achieved with thermal freeze-out occurring in the radiation-dominated era ($T_{\text{rh}} \gg T_{\text{FO}}$) where the processes governing the relic density are Λ_{NP} independent, $\Lambda_{\text{NP}} > T_{\text{max}}$ may be relaxed.

In the left and right panels of Fig. 5, we present the relic-density-allowed points for the radiation-dominated era and during reheating epochs, respectively. In the left panel of Fig. 5, we show the points consistent with the observed DM relic density in the $m_\phi - \lambda_{\phi H}$ plane, where different colors correspond to different values of $\lambda'_{\phi H}$. The new-physics cutoff scale is fixed to $\Lambda_{\text{NP}} = 2$ TeV, as indicated at the top of the figure. The effect of the $\lambda'_{\phi H}$ coupling is most significant near the Higgs boson mass, as it contributes solely to the DM semiannihilation cross section, $\langle \sigma v \rangle_{\Phi\Phi \rightarrow \Phi^* h} \propto \lambda'_{\phi H}$. Its impact becomes particularly pronounced when $\lambda_{\phi H}$ is comparatively small. Therefore, with the enhancement of $\lambda'_{\phi H}$, the DM semiannihilation cross section increases, and consequently, the relic density decreases. However, this effect becomes diluted in the regime $m_\phi \gg m_h$, where the behavior gradually reverts to that of the renormalizable CSDM framework. This is evident from the clustering of all color points around $\lambda_{\phi H} \simeq 0.4$ and a CSDM mass in the range 0.8–1 TeV. Notably, the CSDM masses above 1 TeV are disfavored by the EFT

validity criterion and are therefore not shown beyond this range.

In the right panel of Fig. 5, we present the DM relic-density-allowed points in the $m_\phi - \Gamma_\phi$ plane, while noting that entropy dilution can modify the relic density. We also show the variation of T_{max} , along with Γ_ϕ while $\mathcal{H}_I = 10^{-3}$ GeV, on the right y axis. In this analysis, all model parameters are fixed as specified in the figure inset, except for the NP scale Λ_{NP} . The burnt orange, mustard yellow, olive green, and steel blue points correspond to $\Lambda_{\text{NP}} = \{4, 5, 6.67, \text{ and } 10.0\}$ TeV, respectively, and the correct relic density is achieved via the freeze-out mechanism. However, in the white region within the monochromatic funnel around the Higgs boson mass, the relic density is substantially underabundant due to semiannihilation processes. Beyond the Higgs boson mass (i.e., $m_\phi \gg m_h$), this effect becomes more pronounced. As Λ_{NP} gradually increases, the cross section for semiannihilation decreases; in this scenario, the relic density can be tuned by reducing the inflaton decay width Γ_ϕ , which effectively lowers T_{rh} . As mentioned in the figure caption, the dashed lines correspond to $\Lambda_{\text{NP}} = T_{\text{max}}$, above which the EFT validity is violated. This constraint is relaxed in the resonant regions $m_\phi \sim m_h/2$ as self-annihilation processes are independent of Λ_{NP} . Notably, for $\Lambda_{\text{NP}} = 10$ TeV, the effect of reheating for obtaining the correct relic density is most pronounced. The magenta-shaded region in the bottom right of the right panel denotes the exclusion of thermally produced CSDM by the requirement $T_{\text{max}} \geq m_\phi$; otherwise, CSDM production occurs nonthermally. Additionally, black star points are excluded by the current direct-detection limits set by LZ-2025.

B. Indirect detection

An alternative approach to probing DM involves detecting SM particles, such as γ rays, neutrinos, and cosmic rays (electrons, positrons, antiprotons), produced

⁵The CSDM self-annihilation is independent of the cutoff scale Λ_{NP} .

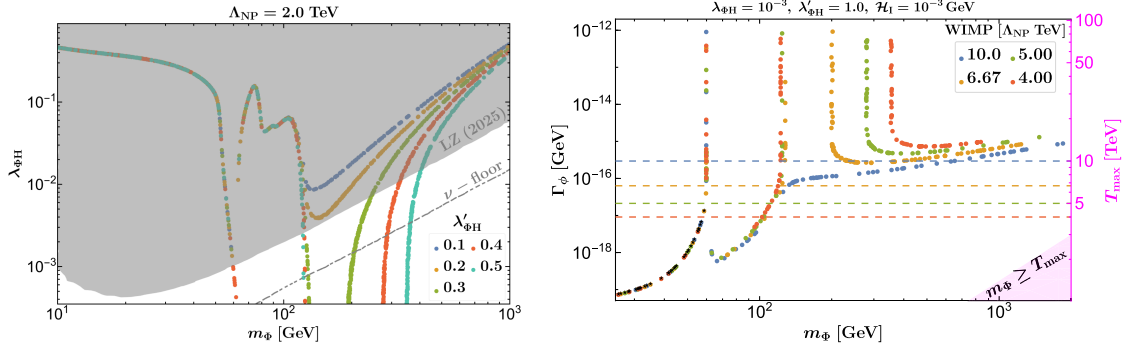


FIG. 5. The left (right) figure shows the relic-density-allowed parameter space in the radiation-dominated (reheating) epoch. Left: the different contours depict the parameter space in the $m_\Phi - \lambda_{\Phi H}$ plane that satisfy the observed relic density. The different color codes indicate various values of $\lambda'_{\Phi H}$. The gray-shaded region is excluded by the LZ-2025 data, while the neutrino floor for direct DM detection [69] is represented by the gray dashed line. Right: the different contours depict the relic-density-allowed parameter space in the $m_\Phi - \Gamma_\Phi$ plane, considering the genesis of DM during the reheating era. The right y axis, shown in magenta, indicates the variation of $T_{\max}$ with Γ_Φ (left y axis), for $\mathcal{H}_I = 10^{-3}$ GeV. The black star points do not satisfy the LZ-2025 bound on $\sigma_{\Phi N}^{\text{SI}}$. The dashed lines correspond to $\Lambda_{\text{NP}} = T_{\max}$. For both panels, the remaining parameters are fixed to $\lambda_\Phi = 1$ and $\mu_3 = 2m_\Phi$. The left (right) figure shows the WIMP relic-density-allowed parameter space in the radiation-dominated (reheating) epoch.

by DM annihilation or decay. Excesses of these particles in regions of high DM density, such as the Galactic Center, dwarf spheroidal galaxies, and galaxy clusters, could provide indirect evidence for DM. Experiments targeting these signals include Fermi-LAT [36,70–73], MAGIC [74,75], H.E.S.S. [76], and CTA [77,78] for γ rays; IceCube [79] and ANTARES [80–82] for neutrinos; and AMS-02 [83–85] and CALET [86] for charged cosmic rays. Although no confirmed detections have been reported, these searches impose stringent upper limits on DM annihilation and decay rates into SM particles. In particular, the γ -ray observations in the direction of dwarf spheroidal galaxies (Fermi-LAT) and the Galactic Center (H.E.S.S.), together with Planck measurements of the CMB, place upper limits on DM semiannihilation [87], with additional sensitivity expected from CTA.

In this work, we show the constraint on the self-annihilation of the CSDM via the Higgs-portal interaction into SM final states ($b\bar{b}$ and W^+W^-) using the stringent limits from the Fermi-LAT experimental data. In addition, we also show the constraint on the semiannihilation channel $\Phi\Phi \rightarrow \Phi^*h$. To obtain these limits, we use the results of [87] and appropriately rescale them for the CSDM scenario. The self-annihilation cross section into $b\bar{b}$ and W^+W^- decreases with increasing m_Φ and decreasing $\lambda_{\Phi H}$. Consequently, choosing $\lambda_{\Phi H} = 10^{-3}$ allows a significant portion of the parameter space to remain consistent with Fermi-LAT bounds, as illustrated in Fig. 6, where the blue and yellow points correspond to DM annihilation into $b\bar{b}$ and W^+W^- , respectively. In contrast, the semiannihilation cross section depends not only on $\lambda_{\Phi H}$ but also on μ_3 , $\lambda'_{\Phi H}$, and Λ_{NP} . While larger values of μ_3 and Λ_{NP} aid in achieving the correct relic density for a given $\lambda_{\Phi H}$, they are more tightly constrained by the Fermi-LAT limits on the semiannihilation cross

section. As shown in Fig. 6, parts of the parameter space is excluded by the Fermi-LAT bounds (thick blue) and/or falls within the CTA projections (dashed cyan), even though they remain consistent with direct-detection and relic-density constraints. Importantly, the condition $\Lambda_{\text{NP}} > T_{\max}$ is relevant only for the relic-density analysis during the reheating era, whereas $\Lambda_{\text{NP}} > 2m_\Phi$ is required to estimate the semiannihilation rate dominated by the NP coupling. In contrast, self-annihilation into SM particles is unaffected by either condition during the radiation-dominated epoch. Accordingly, in the right (left) panel of Fig. 6, we show only the semiannihilation into Φ^*h (self-annihilation into $b\bar{b}$ and W^+W^- [73]). Some magenta points below the dashed gray exclusion line remain viable because, for smaller Λ_{NP} coupling values, semiannihilation dominates and yields the observed relic abundance during the radiation-dominated epoch.

Finally, after incorporating all relevant bounds, we present in Fig. 7 a summary of the parameter space scan in the $\lambda_{\Phi H} - \Lambda_{\text{NP}}$ plane. The pastel color bar illustrates the variation of m_Φ for two distinct scenarios. In the left panel, freeze-out is assumed to occur during the radiation-dominated era ($T_{\text{rh}} \gg T_{\text{FO}}$), while in the right panel it occurs during the reheating epoch ($T_{\text{FO}} \gg T_{\text{rh}}$), with $T_{\text{rh}} = 5.70$ GeV and $T_{\max} = 5.13$ TeV for the chosen values of $\Gamma_\Phi = 10^{-16}$ and $\mathcal{H}_I = 10^{-4}$ GeV.

In both plots of Fig. 7, the pastel-colored points satisfy relic-density constraints and also respect the criteria $\Lambda_{\text{NP}} > 2m_\Phi$. Additionally, in the right plot, we also show the line $\Lambda_{\text{NP}} = T_{\max}$. The black points superimposed on the pastel regions are excluded by the LZ-2025 direct-detection bound, while the cyan star points overlaid on either pastel or black points fail to meet the Fermi-LAT limits on DM self- and semiannihilation cross sections. In the left panel, after imposing all relevant CSDM constraints, the allowed

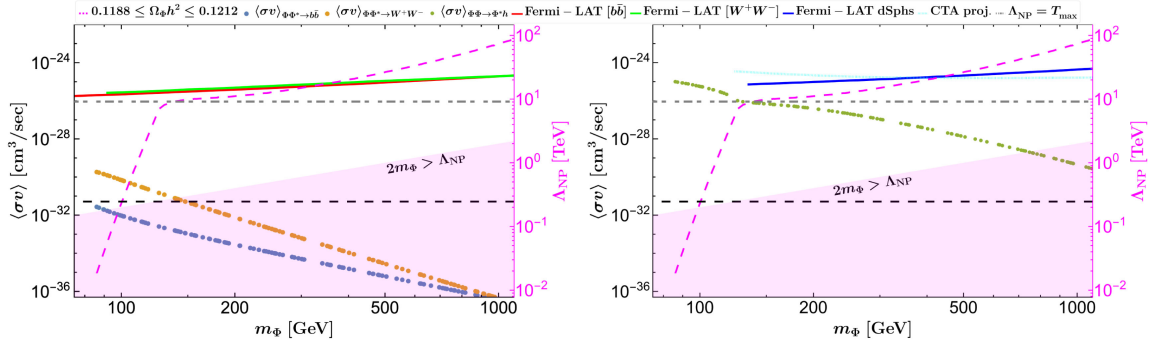


FIG. 6. In the left and right panels of this figure, we present the variation of DM self- and semiannihilation into SM particles (depicted as dotted lines) as a function of m_Φ , respectively. The model parameters are fixed to $\lambda_\Phi = 1$, $\mu_3 = 2m_\Phi$, $\lambda_{\Phi H} = 10^{-3}$, $\lambda'_{\Phi H} = 1$, $\Gamma_\phi = 10^{-16}$ GeV, and $\mathcal{H}_I = 10^{-3}$ GeV. The magenta color-coded right y axes represent the Λ_{NP} scale. The blue and yellow points in the left panel correspond to the self-annihilation of CSDM into $b\bar{b}$ and W^+W^- , respectively, while the green points in the right panel correspond to the semiannihilation of CSDM into Φ^*h . The cyan dotted (CTA) and blue thick (Fermi-LAT) lines, along with the red and green thick Fermi-LAT lines, represent the projected and observed upper limits on the DM self- and semiannihilation cross sections, respectively, as indicated in the figure legend. The magenta long-dashed lines show the relic-allowed parameter space in the $m_\Phi - \Lambda_{\text{NP}}$ plane, with the variation of Λ_{NP} indicated along the right y axis. All magenta points below the gray dot-dashed line are excluded by the EFT validity condition $\Lambda_{\text{NP}} > T_{\text{max}}$, relevant for semiannihilation, while the relic-density-allowed magenta points within the magenta-shaded region are disfavored by $2m_\Phi < \Lambda_{\text{NP}}$. The olive points must satisfy $2m_\Phi < \Lambda_{\text{NP}}$ due to the Λ_{NP} dependence of the semiannihilation cross section. The black dashed line represents $\Lambda_{\text{NP}} = v_{\text{SM}}$ [88–90], where v_{SM} denotes the EWSB VEV.

points are not confined to the Higgs boson resonance region but extend to higher masses while satisfying the EFT validity condition, $\Lambda_{\text{NP}} > 2m_\Phi$, albeit at the expense of larger Higgs-portal coupling $\lambda_{\Phi H}$. The top-left white region corresponds to points that fail to reproduce the observed relic abundance and instead lead to an overabundant DM relic density. This region can, however, be reconciled with the observed abundance if the freeze-out occurs during reheating, as we show in the right panel of Fig. 7. As is evident from the figure, a wide range of DM mass is allowed from the relic density as well as direct and indirect experimental constraints. For our chosen parameters, listed

in the caption of Fig. 7, we have $T_{\text{max}} = 5.13$ TeV, which allows a new-physics scale $\Lambda_{\text{NP}} > 5.13$ TeV. Although we obtain $T_{\text{rh}} = 5.70$ GeV for $\Gamma_\phi = 10^{-16}$ and $\mathcal{H}_I = 10^{-4}$ GeV in this setup, the allowed parameter space may shift for different choices of these inputs. Notably, as we discussed previously just before Sec. III A, this constraint $\Lambda_{\text{NP}} > T_{\text{max}}$ is not strictly required; rather, this is overconstraining. The limit $\Lambda_{\text{NP}} > T_{\text{FO}}$ is sufficient for practical purposes. Therefore, the parameter space in the regime of $\Lambda_{\text{NP}} > T_{\text{max}}$ would not be strictly forbidden.

In conclusion, once relic-density, direct, and indirect bounds are imposed, the allowed points correspond to

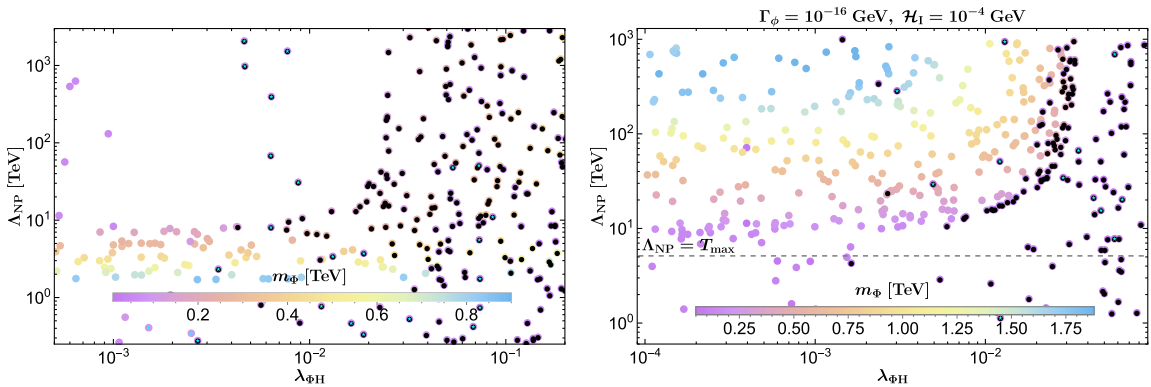


FIG. 7. The relic-density-allowed parameter space is shown in the $\lambda_{\Phi H} - \Lambda_{\text{NP}}$ plane, with the pastel color bar indicating the variation of m_Φ . The remaining parameters in both plots are fixed at $\lambda_\Phi = 1.0$, $\mu_3 = m_\Phi$, and $\lambda'_{\Phi H} = 1.0$ throughout the scan. The left and right plots correspond to the genesis of DM during radiation domination and reheating era (with $T_{\text{rh}} = 5.70$ GeV and $T_{\text{max}} = 5.13$ TeV), respectively. The black points overlaid on the pastel-colored points are excluded by the stringent limit on $\sigma_{\Phi N}^{\text{SI}}$ using LZ-2025 data. All cyan star points overlaid on the black points are excluded by the Fermi-LAT upper limits on dark-matter self-annihilation, $\Phi\Phi^* \rightarrow b\bar{b}(W^+W^-)$, and semiannihilation, $\Phi\Phi \rightarrow \Phi^*h$, cross sections. The gray dashed line corresponds to $\Lambda_{\text{NP}} = T_{\text{max}}$.

TABLE I. All benchmark points satisfy the relic-density constraint as well as bounds from direct, indirect, and collider searches for DM. We fix $\lambda'_{\Phi H} = 1.0$, and for BP4 the corresponding value of T_{max} is 5.13 TeV.

Benchmark points	m_Φ (GeV)	μ_3 (GeV)	$\lambda_{\Phi H}$	Λ_{NP} (GeV)	$\sigma_{\Phi N}^{\text{SI}}$ (cm ²)	T_{rh} (GeV)
BP1	112	112	0.002	1081	2.64×10^{-48}	...
BP2	59.7	59.7	0.001	263	2.74×10^{-48}	...
BP3	106.9	106.9	0.001	556	9.99×10^{-49}	...
BP4	112.72	112.72	0.001	1403	3.46×10^{-49}	5.70

$\lambda_{\Phi H} \lesssim 10^{-2}$, spanning a wide range of CSDM masses, with a new-physics scale $\Lambda_{\text{NP}} \gtrsim 1$ TeV, while lower values may be accessible under appropriate reheating dynamics.

C. Collider search

In this section, we show the production cross sections of the DM signal at future collider experiments [38,44]. We begin by summarizing in Table I a set of benchmark points (BPs) that simultaneously satisfy the relic-density requirement and are consistent with current constraints from direct and indirect DM searches. Benchmark points BP1, BP2, and BP3 correspond to radiation-dominated scenarios, while BP4 corresponds to a scenario in the reheating epoch. In the subsequent analysis, we focus on parameter choices similar to BP1.

In Fig. 8, we present the cross sections for various Φ production channels at the HL-LHC and FCC-ee,

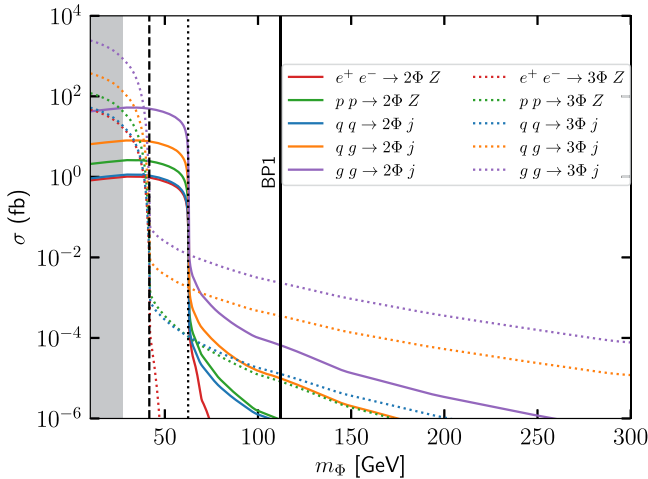


FIG. 8. The CSDM production cross section (σ) as a function of m_Φ at the FCC-ee with $\sqrt{s} = 240$ GeV and at HL-LHC with $\sqrt{s} = 14$ TeV, shown for both 2Φ and 3Φ . In this analysis, we adopt the BP1 parameter values ($\mu_3 = m_\Phi$, $\lambda_{\Phi H} = 0.002$, $\Lambda_{\text{NP}} = 1081$ GeV) to evade current bounds from direct detection (LZ-2025 limit on the spin-independent DM-nucleon cross section) and indirect detection (Fermi-LAT limits on DM annihilation into W^+W^- and $b\bar{b}$). The vertical solid black line indicates $m_\Phi = 112$ GeV, as listed in Table I, while the vertical dashed and dotted black lines mark $m_\Phi = m_h/2$ and $m_\Phi = m_h/3$, respectively. The gray-shaded region ($m_\Phi < 27.3$ GeV) is disfavored by the LHC Higgs boson invisible decay width constraint.

corresponding to center-of-mass energies of $\sqrt{s} = 14$ TeV and $\sqrt{s} = 240$ GeV, respectively, as functions of the dark scalar mass m_Φ . The cross sections are computed using MadGraph5_aMC@NLO v3.5.3 [91], with a transverse momentum cut $p_T > 20$ GeV and a pseudorapidity requirement $|\eta| < 2.5$ imposed on jets produced in association with Φ . The solid and dotted curves correspond to the production of 2Φ and 3Φ , respectively, in association with different particles at the hadron and lepton colliders, as indicated by different colors. The red and green curves represent the associated production of Φ with a Z boson at FCC-ee and HL-LHC, respectively. The blue, orange, and violet curves correspond to associated production with a single jet at the HL-LHC, originating from different partonic initial states, namely, quark-quark (qq), quark-gluon (qg), and gluon-gluon (gg) initial states, respectively. The black dashed and dotted vertical lines indicate the CSDM mass values $m_\Phi = m_h/3$ and $m_\Phi = m_h/2$, respectively, while the thick black line marks the CSDM mass corresponding to BP1 listed in Table I.

For the red solid and dotted curves, corresponding to FCC-ee processes, the cross sections drop to nearly zero at approximately $m_\Phi \simeq (\sqrt{s} - m_Z)/2 \approx 75$ and $m_\Phi \simeq (\sqrt{s} - m_Z)/3 \approx 50$ GeV, respectively. This sharp suppression is purely kinematic in origin and arises from the limited phase space for the on-shell production of Φ beyond these thresholds. In contrast, for the pp collider case, represented by the green, blue, orange, and violet curves, a sudden drop in the cross section is observed at $m_\Phi = m_h/2$ (indicated by the black dotted vertical line). This feature is a direct consequence of the fact that the on-shell decay $h \rightarrow 2\Phi$ becomes kinematically forbidden beyond this mass threshold. A similar drop in the dotted curves occurs at $m_\Phi = m_h/3$ (black dashed vertical line), which can be attributed to the same kinematic constraint affecting the corresponding 3Φ production modes.

As observed, for both the $2\Phi j$ and $3\Phi j$ final states, the gluon-gluon, quark-gluon, and quark-quark initiated subprocesses follow a clear hierarchy in their production cross sections. This ordering directly reflects the corresponding hierarchy of the parton distribution functions (PDFs), with the gluon PDF dominating over the quark PDFs in the relevant Bjorken- x range [92–94]. Consequently, the inclusive contributions to the processes $pp \rightarrow 2/3\Phi j$ are significantly larger than those of $pp \rightarrow 2/3\Phi Z$, primarily

due to the reduced phase-space suppression in the Z -associated production channels.

As can be seen, for BP1, the expected number of signal events expected at the HL-LHC is well below 1 for all considered channels, except for the process $gg \rightarrow 3\Phi j$. Even in this case, the expected signal yield is at most of order $\mathcal{O}(10)$ events, rendering a signal discovery with a conventional cut-based analysis highly challenging. More sophisticated approaches based on machine-learning techniques may significantly enhance the sensitivity of such searches; however, a detailed investigation of these methods lies beyond the scope of the present work and is deferred to future studies.

IV. SUMMARY AND CONCLUSION

The minimal renormalizable Higgs-portal real (complex) scalar DM model, stabilized by a $Z_2(Z_3)$ symmetry, is now strongly constrained by direct, indirect, and collider searches. Among these probes, direct-detection experiments impose the most stringent bounds on the Higgs-portal coupling. This tension can be mitigated by extending the minimal renormalizable setup with additional dark-sector states. Alternatively, an EFT-based extension involving DM and SM fields, and/or a modification of the standard cosmological history, also provides a viable framework for evading this constraint.

In this work, we examined the phenomenology of a complex scalar DM candidate Φ stabilized by a Z_3 symmetry, where we extend the renormalizable CSDM model with a dim-5 operator $\mathcal{O}_5$ involving DM and SM Higgs fields (while noting that the analogous extension for real scalar DM first appears at dim-6). In addition to this, the model also includes a self-interaction term involving the Φ field, which, however, does not play a role in our analysis. Throughout this work, we assumed that the other higher-dimensional operators remain subdominant relative to this leading contribution. After electroweak symmetry breaking, the $\mathcal{O}_5$ operator generates an additional Higgs-portal interaction $\Phi^3 h$. Although this interaction is not constrained by DM direct-detection search, however, this plays a dominant role in determining the relic abundance through semiannihilation. The free parameters associated with $\mathcal{O}_5$, namely, the new-physics scale and its coupling, are further constrained by Fermi-LAT data on the semiannihilation cross section. Additionally, we investigated the freeze-out of the complex scalar DM during the reheating epoch, where regions that are associated with DM

overabundance in the radiation-dominated era may instead provide the correct relic density. Notably, the requirement of EFT validity imposes stringent conditions on the DM mass, the new-physics scale of the theory, and the maximum temperature of the thermal bath.

After imposing the aforementioned constraints, the remaining viable parameter space lies predominantly near the Higgs boson resonance, with the CSDM mass close to the Higgs boson mass and extending above (1 TeV) in the standard radiation-dominated scenario. This restriction can, however, be alleviated if the CSDM freeze-out occurs during the reheating epoch. The corresponding allowed region may be probed by future direct- and indirect-detection experiments. In contrast, searches for CSDM at hadron or lepton colliders remain challenging due to fewer number of events. Consequently, a substantially refined search strategy will be required to achieve observable signal significance, which will be explored in forthcoming work.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: HIGGS INVISIBLE DECAY

In Fig. 9, we present the possible two- and three-body invisible decay channels of the Higgs boson. The corresponding expressions for the decay widths are provided as follows [95]:

$$\Gamma_{h \rightarrow \Phi\Phi^*} = \frac{\lambda_{\Phi H}^2 v^2}{16\pi m_h} \sqrt{1 - 4 \frac{m_\Phi^2}{m_h^2}}, \quad (\text{A1})$$

$$\Gamma_{h \rightarrow \Phi\Phi\Phi} = \frac{1}{3!} \frac{1}{(2\pi)^3} \frac{1}{32\pi m_h^3} \int dm_{12}^2 \int dm_{23}^2 |\mathcal{M}_{h \rightarrow \Phi\Phi\Phi}|^2, \quad (\text{A2})$$

where

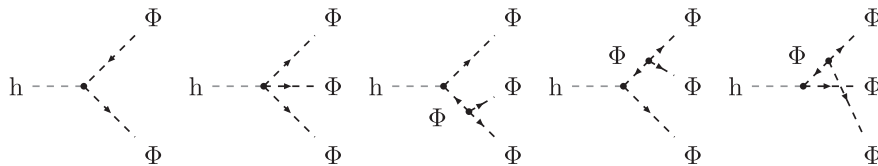


FIG. 9. The Feynman diagrams correspond to the Higgs (h) decays to $\Phi\Phi^*$ and $\Phi\Phi\Phi$.

$$|\overline{\mathcal{M}_{h \rightarrow \Phi\Phi\Phi}}|^2 = \left[\frac{6\lambda'_{\Phi H} v}{\Lambda_{\text{NP}}} + 3 \left(\mu_3 + \frac{\lambda'_{\Phi H} v^2}{\Lambda_{\text{NP}}} \right) \left(\frac{\lambda_{\Phi H} v}{m_{23}^2 - m_{\Phi}^2} + \frac{\lambda_{\Phi H} v}{m_{31}^2 - m_{\Phi}^2} + \frac{\lambda_{\Phi H} v}{m_{12}^2 - m_{\Phi}^2} \right) \right]^2, \quad (\text{A3})$$

$$m_{12}^2 + m_{23}^2 + m_{31}^2 = m_h^2 + 3m_{\Phi}^2, \quad (\text{A4})$$

$$m_{23}^2|_{\text{max}} = (E_2^* + E_3^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} - \sqrt{E_3^{*2} - m_3^2} \right)^2, \quad (\text{A5})$$

$$m_{23}^2|_{\text{min}} = (E_2^* + E_3^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} + \sqrt{E_3^{*2} - m_3^2} \right)^2, \quad (\text{A6})$$

$$E_2^* = (m_{12}^2 - m_1^2 + m_2^2)/2m_{12} \quad \text{and} \quad E_3^* = (m_0^2 - m_{12}^2 - m_3^2)/2m_{12}, \quad (\text{A7})$$

$$m_{12}^2|_{\text{max}} = (m_0 - m_3)^2 \quad \text{and} \quad m_{12}^2|_{\text{min}} = (m_1 + m_2)^2. \quad (\text{A8})$$

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