

Exploring quantum statistics for massive Dirac and Majorana neutrinos using spinor-helicity techniques

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Recently, there has been interest in the applicability of quantum statistics to distinguish Dirac from Majorana neutrinos in multineutrino final states. In particular, debate has arisen over the validity of the Dirac-Majorana confusion theorem in these processes, i.e., that any distinction between the Dirac and Majorana processes goes to zero as the neutrino mass goes to zero. Here we approach this problem equipped with spinor-helicity methods generalized for massive Dirac and Majorana fermions. We explicitly calculate all helicity amplitudes, and their squares, for the decay of a light scalar particle to two neutrinos and two oppositely charged leptons. This allows us to pinpoint the crucial steps which could lead to claims of a violation of the confusion theorem. We show that, if the correct antisymmetrization of Dirac to Majorana amplitudes is used, identification of which is clear in this framework, and all relevant contributions are appropriately summed, a scalar decay into two charged leptons and two neutrinos satisfies the Dirac-Majorana confusion theorem.

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I. INTRODUCTION

The nature of the neutrino as Dirac or Majorana is one of the most important open questions in neutrino physics. Are the neutrinos Dirac fermions with conservation of lepton number, or Majorana fermions with lepton number violated? Answering this question is extremely challenging since any known difference between Dirac and Majorana neutrinos in a physical observable, within the Standard Model (SM) framework, is suppressed by the neutrino mass scale—which is smaller than one-millionth of the mass of the electron. This is the Dirac-Majorana confusion theorem (DMCT), first presented by Kayser and Shrock in Refs. [1,2], respectively. For this theorem, there is not presently a general, model-independent and process-independent proof.

The quest to shed light on the nature of the neutrino motivates searching for processes that could violate the DMCT, and make distinguishing Dirac from Majorana

more experimentally accessible. For processes which violate lepton number, and are zero otherwise, consistency with the DMCT follows from the neutrino mass as a spurion of symmetry breaking—for example, neutrinoless double beta decay [3–7]. However, in processes involving multiple final-state neutrinos as well as additional observable particles, it has been proposed that differences in the quantum statistics of Dirac and Majorana neutrinos could give rise to correlations of the observable particles that are not subject to this suppression. For example, in a recent sequence of papers [8–10], the authors claim to have discovered a violation of the DMCT in B_0 decays to two charged leptons and two neutrinos. This claim has been disputed in Refs. [11,12], where the later emphasizes general arguments that the above violation of the DMCT is not physical. In this work, we revisit the impact of quantum statistics on decays to two-neutrino final states together with additional charged leptons using spinor helicity techniques.

Notably, the DMCT applies to interactions within the framework of the SM, where left-chiral neutrinos interact only via the electroweak force, and right-chiral neutrinos are sterile. The chiral nature of the SM electroweak interactions make spinor-helicity techniques particularly suited to this application. In this work, we use massive spinor helicity formalism, in which massive spinors can be constructed as a sum of massless spinors with shifted momenta—a framework introduced by Mahlon and Parke

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in Ref. [13] for calculating top-quark angular correlations. For massless spinors, helicity and chirality are equivalent, and thus tree-level calculations of processes involving chiral interactions are notably simplified. While the greatest gain in computational efficiency arises for processes involving chiral interactions, the methodology introduced here for neutrino final-state calculations is not limited to such cases, and can be readily extended to frameworks beyond the Standard Model (BSM).

In this work, we primarily highlight a demonstrative example of decays into multineutrino final states. This is the decay of a neutral scalar particle, with a mass roughly that of the B_0 meson, into two charged leptons and two neutrinos via off-shell W bosons. We consider the cases of Dirac and Majorana neutrinos, with particular emphasis on the phases of the spinors so as to calculate the Majorana amplitudes from Dirac amplitudes with correct, physical, antisymmetrization. We explicitly give the Dirac and Majorana amplitudes for each spin configuration, and their square. We show that mathematically there are two ways to construct this antisymmetrization: one that leads to a set of amplitudes that make consistent physics sense, and the other leads to a set of amplitudes with pathologies that make them unphysical. Elucidating the physical distinction between these two choices is particularly clear using the spinor helicity formalism evaluated in the helicity basis.

The outline of this paper is as follows. In Sec. II, we give a brief summary of the spinor-helicity method for massive fermions, before proceeding to Sec. III where we discuss light neutral scalar decay. In Sec. III A we give the four independent amplitudes for our scalar particle decay assuming the neutrinos are Dirac fermions, their square and sum over the spins, assuming the charged leptons are massless for simplicity. In Sec. III B, antisymmetrization possibilities of the Dirac amplitudes to form Majorana amplitudes are discussed, and the physical antisymmetrization is used in Sec. III C to construct a set of three independent amplitudes. The phenomenology and the difference between Dirac and Majorana neutrinos is discussed in Sec. III D, concluding that the DMCT is not violated in these decays. In Sec. III E, we show that considering charged leptons with nonzero mass does not change the conclusions above. In Sec. IV, we explicitly apply the results above to the case of B_0 leptonic decays, and in Sec. V briefly discuss how BSM physics can change this picture. We conclude in Sec. VI, and also include a selection of Appendixes that highlight more detailed aspects of our methods, arguments, and confirmation of techniques.

II. SPINOR-HELICITY TECHNIQUES FOR MASSIVE DIRAC OR MAJORANA FERMIONS

We follow the spinor-helicity technique summarized by Mangano and Parke in Ref. [14], which we briefly review

here. Let $u(q)$ and $v(q)$ represent spinors for a massless fermion and antifermion, respectively, each with four-momentum q ($q^2 = 0$). The complete set of massless spinors is defined as

$$|q\pm\rangle \equiv \gamma_{R/L} u(q) = \gamma_{R/L} v(q),$$

$$\text{and } \langle q \mp | \equiv \bar{u}(q) \gamma_{R/L} = \bar{v}(q) \gamma_{R/L}, \quad (1)$$

where $\gamma_{R/L} \equiv \frac{1}{2}(1 \pm \gamma_5)$, i.e., the $\pm$ notation denotes the chirality of the state, which is equivalent to helicity only in the massless limit. Note that, for example, $|q-\rangle$ represents both a negative helicity massless fermion and a positive helicity massless antifermion. From these definitions, it follows that

$$\gamma_R |q-\rangle = 0, \quad \gamma_L |q+\rangle = 0,$$

$$\langle q- | \gamma_L = 0 \quad \langle q+ | \gamma_R = 0 \quad \text{and} \quad \langle q- | q+\rangle = 0,$$

$$\text{and } q = |q+\rangle \langle q+| + |q-\rangle \langle q-|. \quad (2)$$

Considering also an additional massless momentum k , where $k^2 = 0$, then

$$\langle k \pm | q \pm \rangle = 0,$$

$$\langle k+ | q- \rangle = -\langle q+ | k- \rangle,$$

$$\langle k- | q+ \rangle = (\langle q+ | k- \rangle)^*$$

$$\text{and } \langle k- | q+ \rangle \langle q+ | k- \rangle = 2(k \cdot q), \quad (3)$$

thus $\langle k- | q+ \rangle$ and $\langle q+ | k- \rangle$ are complex conjugate square roots of $2(k \cdot q)$. Many more identities can be derived for these spinors, see, for example, Ref. [14], where a more compact notation was used with $\langle k- | q+ \rangle = \langle kq \rangle$ and $\langle k+ | q- \rangle = [kq]$. Another alternative notation sometimes used is that $\langle k- | q+ \rangle = k_\alpha q^\alpha$ and $\langle k+ | q- \rangle = \tilde{k}^\alpha \tilde{q}_\alpha$, see, for example, Ref. [15]. Here we use a more explicit notation to emphasize the simplicity and understandability of our results to a wide readership.

For a massive fermion, the spinors are constructed by decomposing a massive momentum four-vector, p , with $p^2 = m^2$, into two massless momentum four-vectors. One of these momenta could be a rescaled momentum of a massless particle for the process, as was done by Kleiss and Stirling in Ref. [16]. Here we adopt a different choice: constructing the two massless momentum using the spin four-vector for the massive fermion, s , as was done by Mahlon and Parke in Ref. [13]. We use $\hat{p}$ and $\tilde{p}$ to denote these massless momenta and define them as

$$\hat{p} \equiv \frac{1}{2}(p + ms), \quad \tilde{p} \equiv \frac{1}{2}(p - ms) \quad (4)$$

which are massless momenta, $\hat{p}^2 = 0$, $\tilde{p}^2 = 0$, as $p \cdot s = 0$ and $s^2 = -1$. Also, $2\tilde{p} \cdot \hat{p} = m^2$ and

$$p = \hat{p} + \check{p}, \quad \text{and} \quad s = (\hat{p} - \check{p})/m. \quad (5)$$

Thus, we have traded the two non-null four vectors, (p, s) , for two massless four vectors, $(\hat{p}, \check{p})$; this allows us to use the full powerful machinery of the spinor-helicity method.¹

With these two momenta, one can decompose all massive spinors into two chiral components with each being a spinor for one or the other of these massless momenta, $(\hat{p}, \check{p})$. There is an important phase multiplying each of the massless spinors as follows:

$$\begin{aligned} \bar{U}_\uparrow(p, s) &= e^{-i\psi} \langle \hat{p} + | + e^{i\psi} \langle \check{p} - |, \\ V_\uparrow(p, s) &= e^{-i\psi} |\hat{p} - \rangle + e^{i\psi} |\check{p} + \rangle, \\ \bar{U}_\downarrow(p, s) &= -e^{-i\psi} \langle \check{p} + | + e^{i\psi} \langle \hat{p} - |, \\ V_\downarrow(p, s) &= -e^{-i\psi} |\check{p} - \rangle + e^{i\psi} |\hat{p} + \rangle, \end{aligned} \quad (6)$$

where the phase ψ , which is momentum and spin axis dependent, is given by

$$\psi \equiv \frac{1}{2} \text{Arg}[\langle \hat{p} + | \check{p} - \rangle / m].$$

This way of writing the spinors is particularly powerful if the interactions are of a chiral nature as one of the two terms in the spinors is annihilated.

Further discussion on the spinor conventions and the square root choices in the definition of ψ are given in Appendix A 1. For calculational purposes, it is convenient to factor out an $\exp[-i\psi]$ for all spinors since they will become an overall phase, $\exp[-i(\psi_1 + \psi_2)]$, for all Dirac and Majorana amplitudes.

The above applies for a general spin axis. In the main text of this paper, we will use the language of the helicity basis because it reveals a smooth limit for when the mass goes to zero. However, our results are valid for a general basis. For the helicity basis, the spin axis in the rest frame of the particle is in the boost direction, $\tilde{n}_p$, such that $s = (0, \tilde{n}_p)$, with $s^2 = -1$. Therefore after applying this boost we have

$$\begin{aligned} p &= \gamma m(1, \beta \tilde{n}_p), \quad s = \gamma(\beta, \tilde{n}_p) \text{ giving} \\ \hat{p} &\equiv \frac{1}{2} \gamma(1 + \beta)m(1, +\tilde{n}_p) \xrightarrow{\gamma \rightarrow \infty} \approx \gamma m(1, \tilde{n}_p) = E(1, \tilde{n}_p), \\ \check{p} &\equiv \frac{1}{2} \gamma(1 - \beta)m(1, -\tilde{n}_p) \xrightarrow{\gamma \rightarrow \infty} \approx \frac{1}{4\gamma} m(1, -\tilde{n}_p) \\ &= \left(\frac{m}{2E}\right)^2 E(1, -\tilde{n}_p), \end{aligned} \quad (7)$$

with $p = \hat{p} + \check{p}$ as required. For the helicity basis, spin up ($\uparrow$) will be denoted by an R , and spin down ($\downarrow$) will be

denoted by an L . The expressions on the right give the leading terms in the ultrarelativistic limit, $\gamma \rightarrow \infty$. In this limit, the momentum $\hat{p}$ points in the same direction with very nearly the same energy as the massive particle, whereas $\check{p}$ points in the opposite direction with an energy suppressed by a factor of $(m/2E)^2$. By ignoring the $\check{p}$ terms, one recovers the massless limit.

III. LIGHT SCALAR DECAY AS AN INSTRUCTIVE EXAMPLE

Consider a light scalar particle, Φ , which decays into two off-shell W bosons and then each W decays into a charged lepton and a neutrino, so that the final state is two charged leptons and two neutrinos, as follows:

$$\begin{aligned} \Phi(Q) \rightarrow \{W^+\}^* \{W^-\}^* \rightarrow l^+(\bar{k}) + \nu_1(p_1) \\ + l^-(k) + \nu_2(p_2) \end{aligned} \quad (8)$$

with $M_\Phi \ll M_W$. The four-momenta of the two massless charged leptons are k and $\bar{k}$, for the neutrinos p_1 and p_2 and Q for the scalar particle. The two neutrinos, ν_1 and ν_2 , have the same flavor and mass m_ν . For Dirac neutrinos, we might call ν_1 or ν_2 a neutrino and the other an antineutrino, whereas for Majorana neutrinos this particle-antiparticle designation is not available, that is why we use ν_1 and ν_2 . The Φ particle can be considered a light Higgs-like scalar and the coupling of the W 's to the lepton-neutrino pair is left handed as in the Standard Model. The mass of the Φ particle is chosen well below the W -boson mass so that one cannot kinematically pair either of the neutrinos with one of the charged leptons as both possibilities are kinematically equally favored. For simplicity, we initially set the charge lepton masses to zero, so that we can purely concentrate on the effects of the neutrino mass and the distinction between Dirac and Majorana neutrinos. Later we will address how the charge lepton masses effect this process and also discuss allowing for the possibility of the effects of adding a right-handed W' or Z' boson.

A. Massive Dirac neutrinos

For Dirac neutrinos, we have designated ν_1 as the neutrino and ν_2 as the antineutrino, therefore the amplitude for this decay, $\mathcal{D}[(p_1, s_2), \overline{(p_2, s_2)}]$ [the bar over the (p, s) indicates it is an antifermion] is given by

$$\begin{aligned} \mathcal{D}[(p_1, s_2), \overline{(p_2, s_2)}] &\sim \bar{u}(k) \gamma_L^\mu V(p_2, s_2) \cdot \bar{U}(p_1, s_1) \gamma_L^\mu v(\bar{k}) \\ &= \langle k - | \gamma_L^\mu V(p_2, s_2) \cdot \bar{U}(p_1, s_1) \gamma_L^\mu | \bar{k} - \rangle \end{aligned} \quad (9)$$

$$= -2\langle \bar{k} + | V(p_2, s_2) \cdot \bar{U}(p_1, s_1) | k + \rangle, \quad (10)$$

¹All three polarization vectors for a massive vector particle can be constructed in a similar way using chiral spinors of $\hat{p}$ and $\check{p}$, see the Appendix of Ref. [17].

where $\gamma_L^\mu = \gamma^\mu \gamma_L$, and a Fierz identity is used in moving from Eq. (9) to Eq. (10). We will use the following abbreviation for this amplitude: $\mathcal{D}[S_1, \bar{S}_2]$, where S_i is either L_i or R_i and the momentum is given by p_i . Since the weak interactions are left chiral, only the left-chiral parts of the spinors in Eq. (6) survive. Notice, these components only contain massless spinors, a situation for which the spinor-helicity techniques developed for massless quarks and gluons is ideally suited.

Using the spinors from the previous section, we then have that the Dirac amplitudes in the helicity basis as in Fig. 1 are given by

$$\begin{aligned}\mathcal{D}[L_1, \bar{R}_2] &= 2\langle \bar{k} + |\hat{p}_2\rangle \langle \hat{p}_1 - |k\rangle \{ \langle \hat{p}_1 + |\check{p}_1\rangle / m_\nu \}, \\ \mathcal{D}[R_1, \bar{R}_2] &= 2\langle \bar{k} + |\hat{p}_2\rangle \langle \check{p}_1 - |k\rangle \{ \langle \hat{p}_1 + |\check{p}_1\rangle / m_\nu \}, \\ \mathcal{D}[L_1, \bar{L}_2] &= -2\langle \bar{k} + |\check{p}_2\rangle \langle \hat{p}_1 - |k\rangle \{ \langle \hat{p}_1 + |\check{p}_1\rangle / m_\nu \}, \\ \mathcal{D}[R_1, \bar{L}_2] &= -2\langle \bar{k} + |\check{p}_2\rangle \langle \check{p}_1 - |k\rangle \{ \langle \hat{p}_1 + |\check{p}_1\rangle / m_\nu \}.\end{aligned}\quad (11)$$

A common phase to all of these amplitudes ($-e^{-i(\psi_1+\psi_2)}$) has been removed here. The factors in the $\{\dots\}$ are just phase factors which are unimportant for Dirac neutrinos but will be important when we consider Majorana neutrinos.

Squaring these amplitudes gives

$$\begin{aligned}|\mathcal{D}[L_1, \bar{R}_2]|^2 &\propto (k \cdot \hat{p}_1)(\bar{k} \cdot \hat{p}_2) \\ |\mathcal{D}[R_1, \bar{R}_2]|^2 &\propto (k \cdot \check{p}_1)(\bar{k} \cdot \hat{p}_2) \\ |\mathcal{D}[L_1, \bar{L}_2]|^2 &\propto (k \cdot \hat{p}_1)(\bar{k} \cdot \check{p}_2) \\ |\mathcal{D}[R_1, \bar{L}_2]|^2 &\propto (k \cdot \check{p}_1)(\bar{k} \cdot \check{p}_2),\end{aligned}\quad (12)$$

where they share a common prefactor. Using the expressions for $\hat{p}$ and $\check{p}$ in the helicity basis given in Eq. (7), the relative magnitude of these matrix elements in the

ultrarelativistic limit are as follows:

$$\begin{aligned}|\mathcal{D}[L_1, \bar{R}_2]| : |\mathcal{D}[R_1, \bar{R}_2]| : |\mathcal{D}[L_1, \bar{L}_2]| : |\mathcal{D}[R_1, \bar{L}_2]| \\ \sim \mathcal{O}(1) : \mathcal{O}(1/\gamma_{p_1}) : \mathcal{O}(1/\gamma_{p_2}) : \mathcal{O}(1/(\gamma_{p_1}\gamma_{p_2})),\end{aligned}\quad (13)$$

where γ_{p_i} is the Lorentz factor for the particle with momentum p_i . This demonstrates that there is no helicity suppression in $L\bar{R}$, single suppression in the $R\bar{R}$ and $L\bar{L}$, and double suppression in the $R\bar{L}$ amplitudes.

Summing over all helicity states of the neutrinos, we have

$$\sum_{\text{spins}} |\mathcal{D}[1, \bar{2}]|^2 \propto (k \cdot p_1)(\bar{k} \cdot p_2). \quad (14)$$

If we have not determined experimentally that the neutrino with momentum p_2 is an antineutrino, we also need to add the possibility that it is a neutrino, with matrix element squared given by

$$\sum_{\text{spins}} |\mathcal{D}[2, \bar{1}]|^2 \propto (k \cdot p_2)(\bar{k} \cdot p_1). \quad (15)$$

So that the full amplitude squared is given by

$$\sum_{\text{spins}} |\mathcal{D}[1, \bar{2}]|^2 + |\mathcal{D}[2, \bar{1}]|^2 \propto (k \cdot p_1)(\bar{k} \cdot p_2) + (k \cdot p_2)(\bar{k} \cdot p_1). \quad (16)$$

This is the full Dirac amplitude squared for the decay of the Φ assuming that the neutrinos are Dirac particles; that is, lepton number is conserved. Note that if we integrate this amplitude squared over the full phase space for p_1 and p_2 , we must divide by 2 so as to not double count.

B. Antisymmetrization to form Majorana amplitudes

We now consider the case that the two neutrinos are Majorana. From the Dirac amplitudes, there are mathematically two ways to antisymmetrize with the goal to arrive at

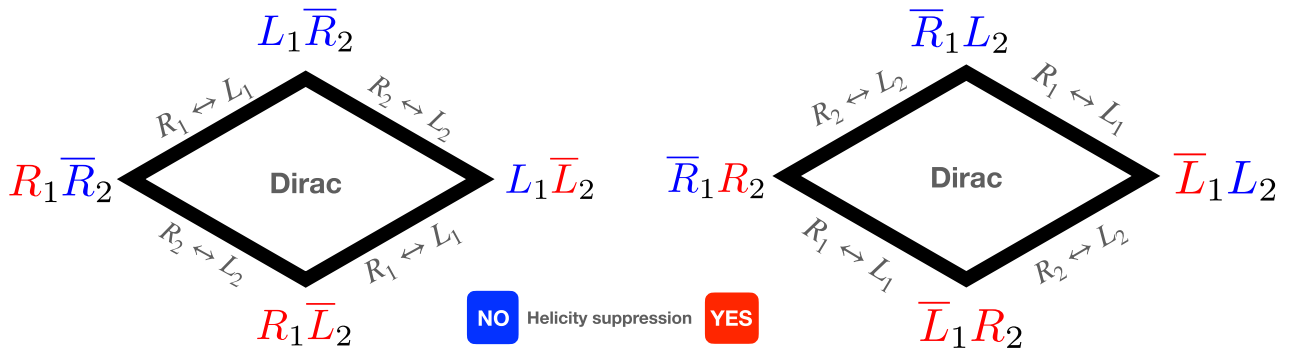


FIG. 1. The eight Dirac amplitudes, obtained from four independent functions. Left is when ν_1 is the fermion and ν_2 is the antifermion and right is vice versa. The letter represents the helicity (L or R), a bar is added if it is an antifermion. The color coding red (blue) denotes when there is (no) helicity suppression. One moves from one vertex to another by $L \leftrightarrow R$ for one or the other of the neutrinos, as indicated. Note, each set of four amplitudes is closed and complete and the right set can be obtained from the left set by interchanging ν_1 and ν_2 . For a given phase space point, (p_1, p_2) , both must be included unless you have determined which is a neutrino and which is an antineutrino.

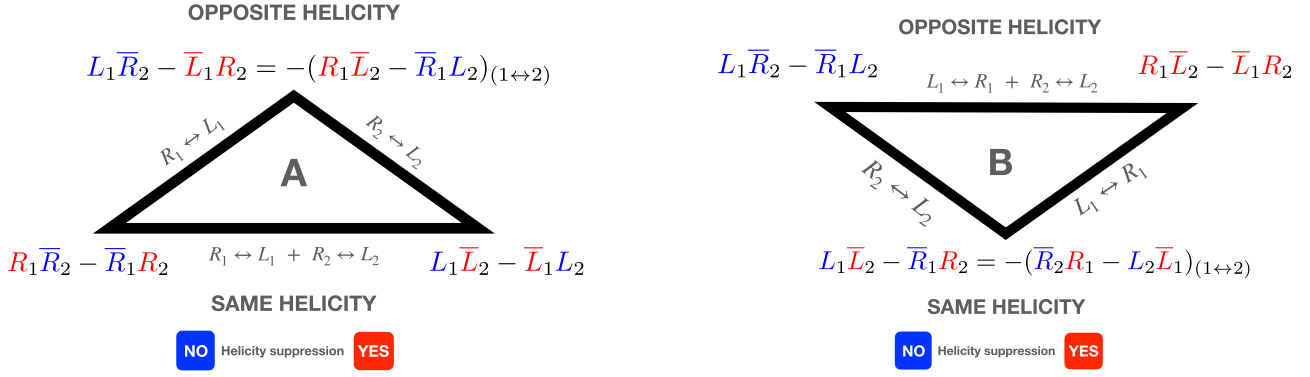


FIG. 2. The two sets of antisymmetrized functions of the Dirac amplitudes with $L_1 \bar{R}_2 - \bar{L}_1 R_2$ (option A, left) and $L_1 \bar{R}_2 - \bar{R}_1 L_2$ (option B, right). From any vertex, one can move to another by $L \leftrightarrow R$ for one or both of 1 and 2. Both sets of functions are closed and complete and all eight Dirac amplitudes are included in each set. As argued in the text, option A is the correct antisymmetrization for Majorana neutrinos.

expressions for the Majorana amplitudes. Beginning with $L_1 \bar{R}_2$, one can antisymmetrize this with either $\bar{L}_1 R_2$ or $\bar{R}_1 L_2$. One can then generate the other helicity combinations by exchanging $L \leftrightarrow R$ on one or both of the neutrinos. We illustrate the results of this exercise in Fig. 2: two sets of three independent functions of the Dirac amplitudes that are both closed and complete.

We then ask which one of these sets of functions represents the amplitudes for a process with two Majorana neutrinos. For such processes there are three *independent* amplitudes: one where the two neutrinos have opposite helicity, and two where the neutrinos have the same helicity, i.e., $\mathcal{M}[L, R]$, $\mathcal{M}[R, R]$, and $\mathcal{M}[L, L]$. Both $\mathcal{M}[R, R]$ and $\mathcal{M}[L, L]$ must vanish when the momenta of the two neutrinos are identical due to the Pauli exclusion principle, whereas for $\mathcal{M}[L, R]$ this is not required since the two neutrinos remain distinguishable by their helicity. The amplitudes $\mathcal{M}[L_1, R_2]$ and $\mathcal{M}[R_1, L_2]$ are not functionally independent (differing only by momentum interchange) but they represent distinct helicity states at a given p_1 and p_2 phase-space point.

The antisymmetrization given in option A satisfies each of these requirements. In this set of amplitudes, the two Dirac amplitudes that are antisymmetrized are identical except for which is fermion and which is an antifermion. The two same-helicity amplitudes vanish when the two neutrino momenta are identical, and there is only one independent opposite-helicity amplitude as required.

Option B has a number of physics pathologies that make this antisymmetrization unacceptable to represent the Majorana amplitudes. First, there are two independent opposite-helicity amplitudes and not one. These two amplitudes are very different in magnitude: one has no helicity suppression, whereas the other is doubly helicity suppressed. There is only one same-helicity amplitude, and this amplitude does not vanish when the two neutrino momenta are identical—violating the Pauli exclusion principle.

Our conclusion here is that the antisymmetrization given by option A, on the left side of Fig. 2, is the only acceptable

way to form Majorana neutrinos and that the $\mathcal{M}[L, R]$ amplitude is given by $L_1 \bar{R}_2 - \bar{L}_1 R_2$, the $\mathcal{M}[R, R]$ amplitude by $R_1 \bar{R}_2 - \bar{R}_1 R_2$, and $\mathcal{M}[L, L]$ amplitude by $L_1 \bar{L}_2 - \bar{L}_1 L_2$. In the next section we explicitly calculate these Majorana amplitudes for Φ decay.

C. Massive Majorana neutrinos

To obtain the amplitudes for the case where the neutrinos are Majorana, we have to antisymmetrize the Dirac amplitudes, so that there can be no distinction between which neutrino is a fermion and which neutrino is an antifermion, leaving everything else unchanged. The correct way to do this follows from the left side of Fig. 2, option A. Below we list these amplitudes, after removing the same overall phase ($-e^{-i(\psi_1 + \psi_2)}$) as in the Dirac case from each.

The *same helicity* Majorana amplitudes are given by

$$\begin{aligned} \mathcal{M}[R_1, R_2] &\equiv \mathcal{D}[R_1, \bar{R}_2] - \mathcal{D}[R_2, \bar{R}_1] = -\mathcal{M}[R_2, R_1] \\ &= +2\langle \bar{k} + |\hat{p}_2 - \rangle \langle \hat{p}_1 - |k + \rangle \{ \langle \hat{p}_1 + |\tilde{p}_1 - \rangle / m_\nu \} \\ &\quad - 2\langle \bar{k} + |\hat{p}_1 - \rangle \langle \hat{p}_2 - |k + \rangle \{ \langle \hat{p}_2 + |\tilde{p}_2 - \rangle / m_\nu \}, \end{aligned} \quad (17a)$$

$$\begin{aligned} \mathcal{M}[L_1, L_2] &\equiv \mathcal{D}[L_1, \bar{L}_2] - \mathcal{D}[L_2, \bar{L}_1] = -\mathcal{M}[L_2, L_1] \\ &= +2\langle \bar{k} + |\tilde{p}_2 - \rangle \langle \hat{p}_1 - |k + \rangle \{ \langle \tilde{p}_1 + |\hat{p}_1 - \rangle / m_\nu \} \\ &\quad - 2\langle \bar{k} + |\tilde{p}_1 - \rangle \langle \hat{p}_2 - |k + \rangle \{ \langle \tilde{p}_2 + |\hat{p}_2 - \rangle / m_\nu \}. \end{aligned} \quad (17b)$$

Note that these two amplitudes are odd functions of 1 and 2 (i.e., $\hat{p}_1 \leftrightarrow \hat{p}_2$ and $\tilde{p}_1 \leftrightarrow \tilde{p}_2$), and therefore vanish when $\hat{p}_1 = \hat{p}_2$ and $\tilde{p}_1 = \tilde{p}_2$. In agreement with the Pauli exclusion principle, the two fermions cannot be in identical momentum and spin states.

For the *opposite helicity* Majorana amplitudes,

$$\begin{aligned}\mathcal{M}[L_1, R_2] &\equiv \mathcal{D}[L_1, \overline{R_2}] - \mathcal{D}[R_2, \overline{L_1}] \\ &= +2\langle \vec{k} + |\hat{p}_2\rangle \langle \hat{p}_1 - |k\rangle \{ \langle \hat{p}_1 + |\tilde{p}_1\rangle / m_\nu \} \\ &\quad + 2\langle \vec{k} + |\tilde{p}_1\rangle \langle \hat{p}_2 - |k\rangle \{ \langle \hat{p}_2 + |\tilde{p}_2\rangle / m_\nu \},\end{aligned}\quad (18a)$$

$$\begin{aligned}\mathcal{M}[R_1, L_2] &\equiv \mathcal{D}[R_1, \overline{L_2}] - \mathcal{D}[L_2, \overline{R_1}] = -\mathcal{M}[L_2, R_1] \\ &= -2\langle \vec{k} + |\tilde{p}_2\rangle \langle \hat{p}_1 - |k\rangle \{ \langle \hat{p}_1 + |\tilde{p}_1\rangle / m_\nu \} \\ &\quad - 2\langle \vec{k} + |\hat{p}_1\rangle \langle \hat{p}_2 - |k\rangle \{ \langle \hat{p}_2 + |\tilde{p}_2\rangle / m_\nu \}.\end{aligned}\quad (18b)$$

Neither of these opposite helicity amplitudes are odd functions in p_1 and p_2 , and so do not vanish when $p_1 = p_2$. This is allowed as the two neutrinos have opposite helicity, so are not in identical states. However, the two amplitudes in Eq. (18) are related to one another as $\mathcal{M}[R_1, L_2] = -\mathcal{M}[L_1, R_2](p_1 \leftrightarrow p_2)$, as expected since they are not independent amplitudes. There are only three independent amplitudes here: one for when both neutrinos are left handed, one when both are right handed, and a

single amplitude for when one is left handed and the other right handed, as required. In addition, these are related to one another by appropriate $\hat{p}_i \leftrightarrow \tilde{p}_i$ interchanges, as this is equivalent to flipping the spin/helicity.

The amplitudes in Eqs. (11), (17), and (18) illustrate the power of the spinor formalism to study the neutrinos nature. Specifically, it becomes trivial to see that at the massless limit both Dirac and Majorana amplitudes are the same. No sum over spin nor integration over neutrino momentum variables are needed for such result to hold. From Eq. (11) and in the helicity basis of Eq. (7), the presence of $\tilde{p}_i$ momentum, excluding the phases inside $\{\dots\}$, in $\mathcal{D}[R_1, \overline{R_2}]$, $\mathcal{D}[L_1, \overline{L_2}]$, and $\mathcal{D}[R_1, \overline{L_2}]$ shows only the Dirac amplitude $\mathcal{D}[L_1, \overline{R_2}]$ is nonzero for $m_\nu \rightarrow 0$. Combining this result with Eqs. (17) and (18), the only amplitudes *not* containing a $\tilde{p}_i$ momentum are the first term of $\mathcal{M}[L_1, R_2]$ and the second term of $\mathcal{M}[R_1, L_2]$. In the limit $m_\nu \rightarrow 0$, these two Majorana amplitudes go to the corresponding Dirac amplitude as follows: $\mathcal{M}[L_1, R_2] \rightarrow \mathcal{D}[L_1, \overline{R_2}]$ and $\mathcal{M}[R_1, L_2] \rightarrow -\mathcal{D}[L_2, \overline{R_1}]$. This result directly follows from the left-chiral nature of neutrino interactions in the SM.

The squares of these Majorana amplitudes are given by

$$\begin{aligned}|\mathcal{M}[R_1, R_2]|^2 &\propto (k \cdot \tilde{p}_1)(\vec{k} \cdot \hat{p}_2) + (k \cdot \tilde{p}_2)(\vec{k} \cdot \hat{p}_1) - \text{Tr}[k\tilde{p}_1\hat{p}_1\vec{k}\hat{p}_2\tilde{p}_2]/(4m_\nu^2), \\ |\mathcal{M}[L_1, L_2]|^2 &\propto (k \cdot \hat{p}_1)(\vec{k} \cdot \tilde{p}_2) + (k \cdot \hat{p}_2)(\vec{k} \cdot \tilde{p}_1) - \text{Tr}[k\hat{p}_1\tilde{p}_1\vec{k}\tilde{p}_2\hat{p}_2]/(4m_\nu^2), \\ |\mathcal{M}[L_1, R_2]|^2 &\propto (k \cdot \hat{p}_1)(\vec{k} \cdot \hat{p}_2) + (k \cdot \tilde{p}_2)(\vec{k} \cdot \tilde{p}_1) - \text{Tr}[k\hat{p}_1\tilde{p}_1\vec{k}\hat{p}_2\tilde{p}_2]/(4m_\nu^2), \\ |\mathcal{M}[R_1, L_2]|^2 &\propto (k \cdot \hat{p}_2)(\vec{k} \cdot \hat{p}_1) + (k \cdot \tilde{p}_1)(\vec{k} \cdot \tilde{p}_2) - \text{Tr}[k\tilde{p}_1\hat{p}_1\vec{k}\tilde{p}_2\hat{p}_2]/(4m_\nu^2).\end{aligned}\quad (19)$$

The “slashes” on the momenta in the traces are omitted for readability. This set of four amplitudes squared has to be summed to obtain the full amplitude squared at each phase space point. All of these amplitudes squared are consistent with the left-chiral contribution to Eq. (1) of Ma and Pantaleone [18], where they calculate the process $e^+e^- \rightarrow NN$, where N is a massive Majorana fermion. An explicit discussion of this may be found in Appendix B.

The $1/m_\nu^2$ in the interference term may be a concern to the reader in the limit that $m_\nu \rightarrow 0$. However, in the helicity basis for which this limit is smooth and using $\hat{p}$ and $\tilde{p}$ given in Eq. (7), it is clear that each of these interference terms are proportional to the neutrino mass squared, such as

$$\begin{aligned}\text{Tr}[k\tilde{p}_1\hat{p}_1\vec{k}\tilde{p}_2\hat{p}_2]/m_\nu^2 &\sim \mathcal{O}(|k||\tilde{p}_1||\hat{p}_1||\vec{k}||\tilde{p}_2||\hat{p}_2|/m_\nu^2) \\ &= \mathcal{O}(m_\nu^2|k||\vec{k}|),\end{aligned}\quad (20)$$

where $|k|$ is the energy of the massless momentum k . As there is no Lorentz $\gamma(=E/m)$ factor for the neutrinos in

the magnitude of this trace, these trace factors go to zero as $m_\nu \rightarrow 0$. This is exactly the DMCT for each helicity configuration.

Summing over all of the spin combinations for the square of the Majorana amplitudes, we have

$$\begin{aligned}\sum_{\text{spins}} |\mathcal{M}[1, 2]|^2 &\propto (k \cdot p_1)(\vec{k} \cdot p_2) \\ &\quad + (k \cdot p_2)(\vec{k} \cdot p_1) - m_\nu^2(k \cdot \vec{k}).\end{aligned}\quad (21)$$

This is the spin summed full amplitude squared for the decay of the Φ assuming that the neutrinos are Majorana particles and that lepton number is not conserved. Note that, if we integrate this amplitude squared over the full phase space for p_1 and p_2 , we further divide by 2 so as to not double count. Except for the last term, $m_\nu^2(k \cdot \vec{k})$, which goes to zero when $m_\nu \rightarrow 0$, this is identical to the expression obtained for the Dirac neutrinos,² Eq. (16).

²The omitted constant factors are identical for the Dirac and Majorana cases, so comparing Eqs. (16) and (21) is exact.

This confirms the DMCT for this process and is in agreement with the general arguments given by Akhmedov and Trautner in Ref. [12].

The only phase space point where this summed Majorana amplitude squared is zero is when the neutrinos are at rest in the Φ rest frame as the two massless charged leptons must have opposite momenta, $(k \cdot \bar{k}) = 2|k||\bar{k}|$, and the spin of the two Majorana neutrinos must be in the direction of the negative charged lepton by angular momentum conservation and therefore the two neutrinos are in the same momentum and spin state. When $p_1 = p_2$ and the two neutrinos are relativistic in the Φ rest frame, the fractional change to squared amplitude from the interference term is of $\mathcal{O}(1/\gamma^2)$, where γm_ν is the energy of the neutrinos in this frame. We will extensively compare the spin summed Dirac and Majorana amplitudes squared in the next section.

D. Comparison of the Dirac and Majorana cases

Angular correlations for the decay of Φ are determined by the matrix element squared, as given for the Dirac and Majorana cases in Eqs. (16) and (21). To summarize, these are given by

$$(k \cdot p_1)(\bar{k} \cdot p_2) + (k \cdot p_2)(\bar{k} \cdot p_1) - \begin{cases} 0 & [\text{Dirac}] \\ m_\nu^2(k \cdot \bar{k}) & [\text{Majorana}] \end{cases} \quad (22)$$

where the same common factor has been suppressed for both cases. Note that here we consider massless charged leptons, but address the impact of their masses in Sec. III E.

Following from Eq. (22), the difference between the Dirac and Majorana amplitudes squared, divided by the Dirac amplitude squared, is given by

$$\frac{\text{D-M}}{\text{D}} \equiv \frac{m_\nu^2(k \cdot \bar{k})}{(k \cdot p_1)(\bar{k} \cdot p_2) + (k \cdot p_2)(\bar{k} \cdot p_1)} = \left[\frac{1}{\gamma_{p_1} \gamma_{p_2}} \right] \left(\frac{(1 - c_{k\bar{k}})}{(1 - \beta_{p_1} c_{p_1 k})(1 - \beta_{p_2} c_{p_2 \bar{k}}) + (1 - \beta_{p_1} c_{p_2 k})(1 - \beta_{p_2} c_{p_1 \bar{k}})} \right), \quad (23)$$

where $c_{qq'} = \cos \theta_{qq'}$ is the cosine of the angle between momenta q and q' , and β_q, γ_q are the relativistic momentum and Lorentz factors for the massive neutrino with momenta q .

In the rest frame of the Φ , the factor $(\dots)$ is of order 1. The factor in $[\dots]$ is *only* of order 1 when both neutrinos are

nonrelativistic, i.e., both $\gamma_{p_1}, \gamma_{p_2}$ of order 1. One may be tempted to consider this in a Lorentz-invariant way by requiring the invariant mass of the two neutrinos, S_{12} , to be close to threshold:

$$S_{12} \equiv (p_1 + p_2)^2 \approx 4m_\nu^2. \quad (24)$$

Accordingly, the difference between Dirac and Majorana neutrinos is only significant in the region of phase space where one can boost to a frame where both neutrinos are nonrelativistic. In Fig. 3, we show both the Dirac and Majorana differential decay rates, and their relative difference, as a function of the square root of S_{12} . Note that the difference between the two is *always* suppressed by $\sim \mathcal{O}(m_\nu^2/M_\Phi^2)$. However, Eq. (24) is only a necessary but not a sufficient condition. To see this, consider Eq. (23) when $p_1 = p_2$. This satisfies the S_{12} requirement, but does not guarantee that there is a significant difference between Dirac and Majorana neutrinos because, at this point, $\gamma_1 \gamma_2$ can still be $\gg 1$.

The correct Lorentz-invariant condition is that

$$(Q \cdot p_1)(Q \cdot p_2) \sim \mathcal{O}(m_\nu^2 M_\Phi^2), \quad (25)$$

recalling that Q is the four-momentum of the decaying Φ . That is, in the rest frame of Φ , both neutrinos must be nonrelativistic. The fraction of phase space that satisfies these conditions is tiny and of $\mathcal{O}(4m_\nu^2/M_\Phi^2)$.³

³For a scattering process rather than the decay of a particle, the same condition will apply with Q replaced by the sum of the momenta of the incoming particles, i.e., in the zero three-momentum frame the two neutrinos must be nonrelativistic.

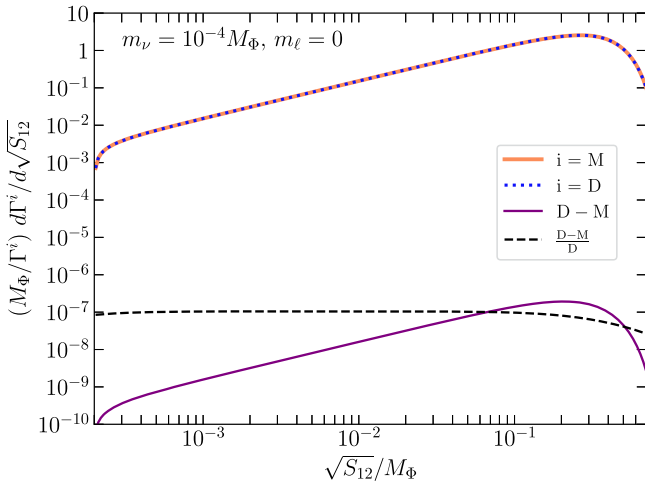


FIG. 3. The differential decay rate for Φ versus the square root of the invariant mass of the two neutrinos, $\sqrt{S_{12}}$, for a neutrino mass of $m_\nu = 10^{-4}M_\Phi$ and massless charged leptons. The Dirac distribution is the red line and the Majorana distribution is the cyan-dashed line. The purple line is the difference between Dirac and Majorana distribution and the black dotted line this difference divided by the Dirac distribution. Note, the roughly flat relative difference between the Dirac and Majorana distributions for all values of $\sqrt{S_{12}}$. The choice $m_\nu = 10^{-4}M_\Phi$ is just representative and shows the relevant physics.

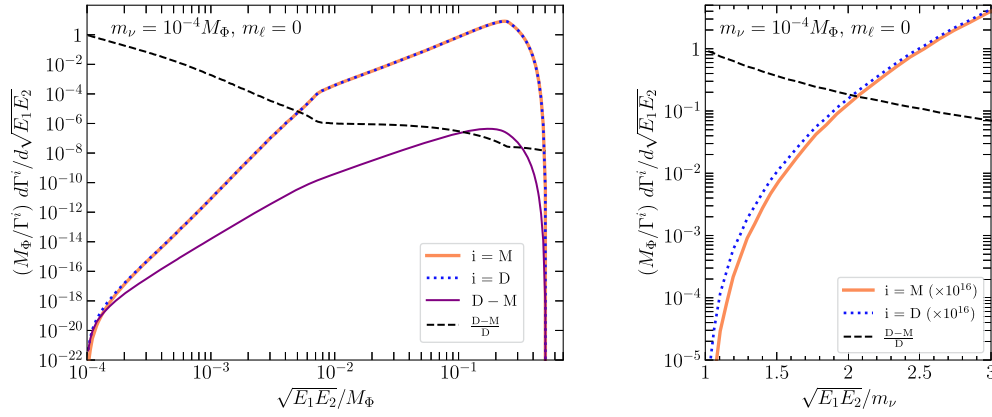


FIG. 4. Left panel: the differential decay rates of Φ versus the square root of the product of the energy of the two neutrinos, $\sqrt{E_1 E_2}$, in the rest frame of Φ , for a neutrino mass of $m_\nu = 10^{-4} M_\Phi$ and massless charged leptons. The Dirac distribution is the red line and the Majorana distribution is the cyan-dashed line. The purple line is the difference between Dirac and Majorana distribution and the black dotted line this difference divided by the Dirac distribution. Note, the relative difference between the Dirac and Majorana distributions only becomes significant for small values of $\sqrt{E_1 E_2}/M_\Phi$ as this is when both neutrinos are nonrelativistic in the Φ rest frame. This region of phase space is suppressed by powers of (m_ν/M_Φ) . Right panel: we enlarge the $\sqrt{E_1 E_2} \sim m_\nu$ region. When the neutrinos are produced at rest, the Majorana process is zero by angular momentum conservation and the Pauli exclusion principle, whereas the Dirac process is nonzero. Therefore $(D - M)/D = 1$ when $\sqrt{E_1 E_2} = m_\nu$.

In Fig. 4 we show both the Dirac and Majorana differential decay rates, and their relative difference, as a function of the square root of $\sqrt{E_1 E_2} \equiv \sqrt{(Q \cdot p_1)(Q \cdot p_2)}/M_\Phi$ in the rest frame of the scalar Φ . As expected, the *relative* difference grows with decreasing $\sqrt{E_1 E_2}$ and reaches a maximum at threshold for both neutrino's production at rest, when $\sqrt{E_1 E_2} = m_\nu$, where the Majorana decay rate is zero.

E. Adding masses to the charged leptons

Here we consider the impact of assigning the charged leptons nonzero masses on our result in Sec. III D. The Dirac amplitudes for massive charged leptons may be obtained by making the following replacements in Eq. (11):

$$\begin{aligned} |k+\rangle &\mapsto \begin{cases} +|\tilde{k}+\rangle & \text{R lepton} \\ +|\hat{k}+\rangle & \text{L lepton,} \end{cases} \\ \langle \bar{k}+| &\mapsto \begin{cases} +\langle \tilde{k}+| & \text{R antilepton} \\ -\langle \hat{k}+| & \text{L antilepton,} \end{cases} \end{aligned} \quad (26)$$

which follows from using the spinors from Eqs. (6) and (9). The common phase introduced by the massive charged fermions features as an overall factor for both Dirac and Majorana neutrinos, and therefore can be consistently factored out and has no physical impact. The all-spin summed Dirac amplitude squared is then given by

$$\sum_{\text{all spins}} |\mathcal{D}[1, \bar{2}]|^2 + |\mathcal{D}[2, \bar{1}]|^2 \propto (k \cdot p_1)(\bar{k} \cdot p_2) + (k \cdot p_2)(\bar{k} \cdot p_1), \quad (27)$$

now with $k^2 = \bar{k}^2 = m_l^2$ instead of zero.

Similarly for the Majorana case, one has to make the replacements from Eq. (26) in Eqs. (17) and (18). The square of the helicity amplitude with $(L\bar{R})_{l-l^+}$ for the charged leptons and $(LR)_{\nu\nu}$ for the neutrinos is given by

$$\begin{aligned} &|\mathcal{M}[L_k \bar{R}_{\bar{k}} | L_1 R_2]|^2 \\ &\propto (\hat{k} \cdot \hat{p}_1)(\hat{\bar{k}} \cdot \hat{p}_2) \\ &+ (\hat{k} \cdot \hat{p}_2)(\hat{\bar{k}} \cdot \hat{p}_1) - \text{Tr}[\hat{k} \hat{p}_1 \hat{\bar{p}}_1 \hat{\bar{k}} \hat{p}_2 \hat{p}_2] / (4m_\nu^2), \end{aligned} \quad (28)$$

and similarly for the other helicity amplitudes. If one sums over the charge lepton configurations, this recovers the expression in Eq. (19), since each term is linear in both k and $\bar{k}$.

The all-spin summed Majorana amplitude squared is again given by

$$\sum_{\text{all spins}} |\mathcal{M}|^2 \propto (k \cdot p_1)(\bar{k} \cdot p_2) + (k \cdot p_2)(\bar{k} \cdot p_1) - m_\nu^2(k \cdot \bar{k}), \quad (29)$$

again with $k^2 = \bar{k}^2 = m_l^2$ rather than zero. So for both the Dirac and Majorana amplitudes squared, the charged lepton mass does not appear *explicitly* but only *implicitly* in the relativistic momentum of the two charged leptons.

Therefore, the difference between Dirac and Majorana amplitudes in Eq. (23) becomes

$$\frac{D - M}{D} = \left[\frac{1}{\gamma_{p_1} \gamma_{p_2}} \right] \left(\frac{(1 - \beta_k \beta_{\bar{k}} c_{k\bar{k}})}{(1 - \beta_{p_1} \beta_k c_{p_1 k})(1 - \beta_{p_2} \beta_{\bar{k}} c_{p_2 \bar{k}}) + (1 - \beta_{p_1} \beta_k c_{p_2 k})(1 - \beta_{p_2} \beta_{\bar{k}} c_{p_1 \bar{k}})} \right). \quad (30)$$

The only difference between this and Eq. (23) is that β_k and $\beta_{\bar{k}}$ have been introduced. As both $\beta_k \approx 1$ and $\beta_{\bar{k}} \approx 1$ in most regions of phase space in the rest frame of the Φ , the effect of adding masses to the charged leptons is minor and does not change the physics conclusion that the DMCT is valid.

Note that, unlike when the charged leptons were treated as massless, the Majorana amplitude squared does *not* exactly vanish when the two neutrinos are at rest in the Φ rest frame. This is because there is a small contribution from when the helicity configuration of the two charged leptons is $L\bar{L}$ and $R\bar{R}$. In both of these configurations, the two neutrinos must have opposite spin and are therefore not in identical states. Thus, the amplitude squared is not forced to be zero by the Pauli exclusion principle, as was the case for massless charged leptons. Nevertheless, the magnitude of the amplitude squared is tiny here due to the additional helicity suppression for the $L\bar{L}$ and $R\bar{R}$ charged lepton configuration. At threshold, $(D - M)/D = 1 - 2m_\ell^2/(M_\Phi - 2m_\nu)^2$ which equals 1 for massless charged leptons, and goes to 1/2 as the mass of the charged leptons approaches its kinematic upper limit, $m_\ell = M_\Phi/2 - m_\nu$.

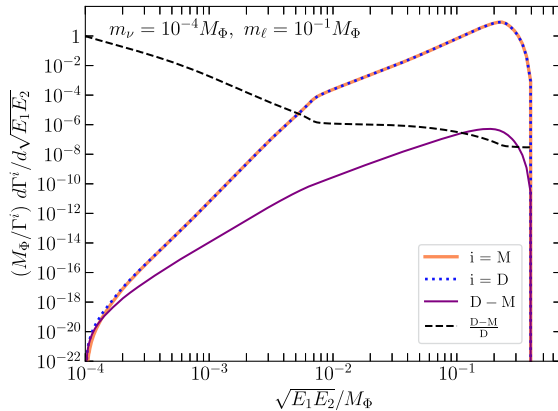


FIG. 5. The differential decay rates of Φ versus the square root of the product of the energy of the two neutrinos, $\sqrt{E_1 E_2}$, in the rest frame of Φ , for a neutrino mass of $m_\nu = 10^{-4} M_\Phi$ and charged lepton mass $m_\ell = 10^{-1} M_\Phi$. The Dirac distribution is the red line and the Majorana distribution is the cyan-dashed line. The purple line is the difference between Dirac and Majorana distribution and the black dotted line this difference divided by the Dirac distribution. Note, the relative difference between the Dirac and Majorana distributions only becomes significant for small values of $\sqrt{E_1 E_2}/M_\Phi$ as this is when both neutrinos are nonrelativistic in the Φ rest frame. The most significant difference between the charged lepton massless, Fig. 4, and massive is the position of the kinematic end point at large $\sqrt{E_1 E_2}/M_\Phi$ and when $\sqrt{E_1 E_2} = m_\nu$ the ratio $(D - M)/D \approx 1 - 2(m_\ell/M_\Phi)^2 = 0.98$, as expected from Eq. (30).

In Fig. 5 we show the decay distribution obtained by integrating the spin-sum matrix elements squared for Dirac and Majorana neutrinos considering a nonzero charged lepton mass. The decay distribution shape is roughly the same as the ones for the massless charged fermion. The inclusion of small charged fermion masses mostly changes the near-threshold curves and makes the relative difference between Dirac and Majorana curves to be slightly smaller than in the $m_\ell = 0$ case, since for massive charge leptons the Majorana decay rate is nonzero at the threshold point.

IV. APPLICATION TO B_0 DECAY

Here we turn our attention to the application of our results to leptonic neutral-meson decays. In Ref. [8], the authors discuss the possibility of using particle correlations in the decays of B_0 to two charged leptons and two neutrinos,

$$B_0 \rightarrow \mu^- \nu_1 \mu^+ \nu_2, \quad (31)$$

to distinguish between Majorana and Dirac neutrinos. The weak interaction part for this decay is identical to the decay of Φ discussed in Sec. III. The B_0 decay has additional complications due to strong interactions, however these do not affect the correlations of the leptons. Therefore, the Φ decay discussed in Sec. III is a good, simple, model to understand the leptonic structure of B_0 decay, in particular to understand the antisymmetrization of the Dirac amplitudes to obtain the Majorana amplitudes.

Unfortunately, in Ref. [8] the authors are not very explicit about how they construct the Dirac amplitudes, and hence the Majorana amplitudes, especially as it applies to the spin/helicity configurations of the two neutrino states. The most direct reference is in Fig. 1 of Ref. [8], where

$$\mathcal{M}^D(p_1, p_2) \sim \mathcal{D}[L_2, \bar{R}_1], \quad (32)$$

and the Dirac amplitude squared is given by

$$|\mathcal{M}^D(p_1, p_2)|^2 = |\mathcal{D}[L_2, \bar{R}_1]|^2 \sim (k \cdot p_2)(\bar{k} \cdot p_1). \quad (33)$$

Note that this assumes that p_1 is the antineutrino momentum. The authors have not included the possibility that p_2 is the antineutrino, which would add the term

$$|\mathcal{D}[L_1, \bar{R}_2]|^2 \sim (k \cdot p_1)(\bar{k} \cdot p_2) \quad (34)$$

to the Dirac matrix-element squared. To make a fair comparison with the Majorana case, as the neutrinos are

not observed, it is imperative that *both* momentum assignments be the antineutrino are summed to construct the Dirac matrix-element squared as

$$|\mathcal{M}^D(p_1, p_2)|_{\text{FULL}}^2 \sim (k \cdot p_1)(\bar{k} \cdot p_2) + (k \cdot p_2)(\bar{k} \cdot p_1), \quad (35)$$

consistent with our Eq. (16).

The construction of the Majorana amplitude given in Ref. [8] is

$$\begin{aligned} \mathcal{M}^M(p_1, p_2) &\sim \mathcal{M}^D(p_1, p_2) - \mathcal{M}^D(p_2, p_1), \\ &\sim \mathcal{D}[L_2, \bar{R}_1] - \mathcal{D}[L_1, \bar{R}_2]. \end{aligned} \quad (36)$$

Note that, if $p_1 = p_2$, this amplitude vanishes. Furthermore, if one squares this Majorana amplitude they arrive at

$$\begin{aligned} |\mathcal{M}^M(p_1, p_2)|^2 &\sim |\mathcal{M}^D(p_1, p_2) - \mathcal{M}^D(p_2, p_1)|^2 \\ &\sim (\bar{k} \cdot p_1)(k \cdot p_2) + (\bar{k} \cdot p_2)(k \cdot p_1) \\ &\quad + (\text{interference terms}). \end{aligned} \quad (37)$$

These interference terms must be sizable in order to make the amplitude squared vanish when $p_1 = p_2$, and therefore they cannot be proportional to powers of the neutrino mass. Except for the interference terms, this matrix element squared is the same as the full Dirac matrix element squared in Eq. (35), and thus the interference terms violate the DMCT. However, this construction of the Majorana amplitude reflects the $\bar{R}_1 L_2 - L_1 \bar{R}_2$ of option B discussed in Sec. III B, which has numerous physics pathologies.

In Ref. [9], a more explicit definition of the Dirac amplitude is given [Eq. (28) of Ref. [9]] as the coherent helicity sum of the Dirac helicity amplitudes,

$$\mathcal{M}^D(p_1, p_2) \sim \sum_{\lambda_1 \lambda_2} \mathcal{D}[\lambda_2, \bar{\lambda}_1], \quad (39)$$

where λ_i is the helicity of the i th neutrino. This is a very unusual prescription for forming the Dirac amplitude as the helicity sum is always performed after squaring the individual amplitudes; see, for example, Ref. [19].

Conceptually, this definition in Eq. (39) from Ref. [9] is very different from Eq. (32), the definition given in Ref. [8]. However, from a numerical point of view the difference is small, because one term in the helicity sum dominates while the other three are helicity suppressed—following from the left-chiral nature of the weak interactions, and the smallness of the neutrino mass.

According to Ref. [9], the Dirac matrix-element squared is given by

$$|\mathcal{M}^D(p_1, p_2)|^2 \sim \left| \sum_{\lambda_1 \lambda_2} \mathcal{D}[\lambda_2, \bar{\lambda}_1] \right|^2. \quad (40)$$

Note, in particular, that in Eq. (40) the helicity sum is performed before squaring. The canonical way to do this sum is as

$$\sum_{\lambda_1 \lambda_2} |\mathcal{D}[\lambda_2, \bar{\lambda}_1]|^2. \quad (41)$$

The latter guarantees that the matrix-element squared yields the same result independent of which spin axis is used for the neutrinos. This follows from the completeness relations given in Appendix A 1. Therefore, using the Dirac amplitude in Eq. (39) would give different results depending on the neutrinos spin axis, and thus is not physical.⁴ Also note again that this does not include the case where p_2 is the momentum of the antineutrino.

The Majorana amplitude in Ref. [9] is constructed as

$$\begin{aligned} \mathcal{M}^M(p_1, p_2) &\sim \mathcal{M}^D(p_1, p_2) - \mathcal{M}^D(p_2, p_1), \\ &\sim \sum_{\lambda_1 \lambda_2} \mathcal{D}[\lambda_2, \bar{\lambda}_1] - \sum_{\lambda_1 \lambda_2} \mathcal{D}[\lambda_1, \bar{\lambda}_2]. \end{aligned} \quad (42)$$

Note that the Majorana matrix element in Eq. (42) exactly vanishes when $p_1 = p_2$: for each term in the first sum there is an identical term in the second. The matrix-element squared is given by

$$|\mathcal{M}^M(p_1, p_2)|^2 \sim \left| \sum_{\lambda_1 \lambda_2} \left(\mathcal{D}[\lambda_2, \bar{\lambda}_1] - \mathcal{D}[\lambda_1, \bar{\lambda}_2] \right) \right|^2. \quad (43)$$

This suffers from the same malady as the Dirac amplitude, in that the answer will depend on the spin projections, whereas a physical answer should be independent of this choice. Numerically, note that this amplitude squared will again be dominated by the $\bar{R}_1 L_2 - L_1 \bar{R}_2$ from option B disputed in Sec. III B.

We conclude that, in Refs. [8,9], either they have incorrectly used the coherent sum of the spin Dirac amplitudes, or they have used the unphysical option B for the antisymmetrizing the Dirac amplitudes to construct the Majorana amplitudes. Following instead from the results of Sec. III, the difference between Dirac and Majorana neutrinos is only significant when the two neutrinos are nearly at rest in the B_0 rest frame. This represents a fraction of phase space suppressed by powers of (m_ν/M_{B_0}) , thereby satisfying the DMCT and in agreement with the general arguments given by Akhmedov and Trautner in Ref. [12].

⁴A further argument against this coherent, rather than incoherent, sum is given in Appendix A 2.

V. BEYOND STANDARD MODEL INTERACTIONS

The spinor-helicity techniques introduced for application to Dirac and Majorana neutrinos may be extended also to beyond Standard Model (BSM) scenarios. We describe an example of this here, in models with an additional right-handed W boson or Z' boson. As was emphasized in Ref. [12], the DMCT is only valid when the interactions of the neutrinos are left chiral, as in the SM. For example, in Ref. [20] the authors consider observables for discriminating between Dirac and Majorana neutrinos with general interactions.

If we add a right-handed W boson, then the Dirac amplitude $(\bar{L}_1 R_2)_{\nu\nu}$ is no longer helicity suppressed by the mass of the neutrinos, but is suppressed by the ratio of the W -boson masses squared. Therefore this amplitude, when added with $(L_1 \bar{R}_2)_{\nu\nu}$ from the left-handed W boson to form the Majorana amplitude, could give a sizable interference when squared. However, at least for the Φ decay discussed in Sec. III, there is a further suppression of the interference from the charged leptons. For the right-handed W boson, the charged leptons are dominantly produced in $(R\bar{L})_{l-l^+}$, whereas for left-handed W bosons the charged leptons are in the $(L\bar{R})_{l-l^+}$ state. Therefore, if the charged leptons are massless there is no interference between these Dirac states. For charged leptons with nonzero mass, this will lead to a suppression which is charged lepton mass dependent. For e^+e^- this suppression will be significant, whereas for $\tau^+\tau^-$ it is not so significant. The techniques introduced here could easily be used to calculate the full matrix amplitude square including a right-handed W boson if needed; however, the discussion here highlights the important features.

For a process for which the neutrino pair is produced by a SM Z boson in the $(L_1 \bar{R}_2)_{\nu\nu}$, and Z' in the $(\bar{L}_1 R_2)_{\nu\nu}$, without a corresponding charged lepton pair, it may be possible to avoid this charged lepton mass suppression of the inference term. However, there will still be a suppression associated with the couplings and mass of the Z' boson compared with those of the Z boson.

VI. CONCLUSIONS

In this work, we introduce the use of massive-spinor helicity methods to processes with massive neutrinos, in the Standard Model interaction framework. We explore the decay of a light scalar to two oppositely charged leptons and two neutrinos to assess whether quantum statistics can be used in this system to distinguish the nature of neutrinos as Dirac or Majorana. A measurable distinction between Dirac and Majorana neutrinos in this process would constitute a violation of the Dirac-Majorana confusion theorem (DMCT).

Using these techniques, we calculated the helicity amplitudes for when the neutrinos are a Dirac particle and antiparticle pair, and also when they are a pair of Majorana neutrinos and thus there can be no particle-antiparticle distinction. The antisymmetrization of the Dirac helicity

amplitudes to form the Majorana amplitudes is explored in detail. A key result is the identification of the correct physical method for this antisymmetrization, which when applied to the two-neutrino final state involves the following:

- (1) For a Dirac amplitude which is *not helicity suppressed*, such as $L_1 \bar{R}_2$, the antisymmetrized is with a Dirac amplitude that is *doubly helicity suppressed*, $\bar{L}_1 R_2$, and similarly vice versa;
- (2) For a Dirac amplitude that is *singly helicity suppressed*, such as $L_1 \bar{L}_2$, the antisymmetrized is with another *singly suppressed* Dirac amplitude, $\bar{L}_1 L_2$.

This antisymmetrization corresponds to the simple interchange between fermion and antifermion for the neutrinos, without any additional spin flip. The helicity suppression is generated by the left-chiral nature of neutrino interactions in the Standard Model. Both above configurations imply that, when the Majorana amplitudes are squared, the interference terms are also helicity suppressed and go to zero as the neutrino mass goes to zero, consistent with the DMCT. This argument may be straightforwardly generalized to final states with more than two neutrinos. Our results are in agreement with the general arguments presented in Ref. [12], and counter to the claims in Refs. [8–10].

Consistency with the DMCT is tied to the left-chiral interactions of neutrinos in the Standard Model framework. The techniques introduced in this work may be applied to BSM scenarios for which quantum statistics may yield observable distinction between Dirac and Majorana neutrinos. Furthermore, our results highlight the relative simplicity and transparency to underlying physics in calculations with spinor-helicity techniques when contrasted with traditional trace techniques.

Note added. Recently, the authors of Ref. [8] presented a tricolored rebuttal of our work, appearing on the e-print archive as Ref. [21]. Their critique may be dissected into two salient points. First, they disagree with our calculation of the square of the Dirac amplitude at a given phase space point, further claiming that our method introduces a violation of lepton number by two units for Dirac neutrinos. Second, they critique our Majorana amplitude calculation's validity on the basis of constructing physical identical-particle states. We disagree on both points. On the first point, for a given phase space point, if one has not determined which of the neutrinos is a fermion or antifermion, both of the neutrino momentum configurations needed to be combined in calculating the total Dirac amplitude squared for that phase space point. Explicitly, the Dirac amplitude squared for both momentum configurations,

$$\Phi(Q) \rightarrow \{l^+(\bar{k}) + \nu(p_1)\} + \{l^-(k) + \bar{\nu}(p_2)\}$$

$$\text{and } \Phi(Q) \rightarrow \{l^+(\bar{k}) + \nu(p_2)\} + \{l^-(k) + \bar{\nu}(p_1)\},$$

need to be added together, since one has not determined that the “neutrino” with momentum p_1 (or p_2) is a neutrino or

antineutrino. There is no two-unit lepton number violation, as we can associate either neutrino momentum with either charged-lepton momentum as indicated by the $\{l^+\nu\}$ and $\{l^-\bar{\nu}\}$ in the previous equation.

With respect to the Majorana neutrino calculation, the authors reiterate their earlier claim that the Majorana amplitude must vanish when the two neutrinos have the same momentum, see Eq. (4) of Ref. [21]. We disagree on this point: as the two Majorana neutrinos can have opposite helicity and thus are not in identical states, therefore the Pauli exclusion principle does not apply.⁵ In our calculation, if the neutrinos have the same momentum *and* helicity, the amplitude vanishes as can be seen in Eq. (17), but not if they have opposite helicity, see Eq. (18). Forcing the amplitude to be zero for all helicity combinations if the two momenta are the same yields their unphysical constructions argued against in our Sec. IV and Appendix A 2.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

The completeness relationships are

$$\begin{aligned} U_\uparrow(p, s)\bar{U}_\uparrow(p, s) &= \frac{1}{2}(1 + \gamma_5 \not{s})(\not{p} + m), & V_\uparrow(p, s)\bar{V}_\uparrow(p, s) &= \frac{1}{2}(1 + \gamma_5 \not{s})(\not{p} - m), \\ U_\downarrow(p, s)\bar{U}_\downarrow(p, s) &= \frac{1}{2}(1 - \gamma_5 \not{s})(\not{p} + m), & V_\downarrow(p, s)\bar{V}_\downarrow(p, s) &= \frac{1}{2}(1 - \gamma_5 \not{s})(\not{p} - m), \end{aligned}$$

$$\text{when summed give } \sum_\lambda U_\lambda(p, s)\bar{U}_\lambda(p, s) = \not{p} + m, \quad \sum_\lambda V_\lambda(p, s)\bar{V}_\lambda(p, s) = \not{p} - m. \quad (\text{A6})$$

We have also explicitly confirmed that the spinors in Table I satisfy all of the identities G.4.34-37 of [15].

APPENDIX A: TECHNICAL APPENDICES

1. A full set of spinors

The full set of spinors for a massive fermion used in this work is given in Table 1 and relationships for the phase ψ are given in Table II. All of these spinors satisfy the appropriate Dirac equation, and are eigenstates of the spin matrix, $\gamma_5 \not{s}$. They are normalized and orthogonal as follows:

$$\begin{aligned} \bar{U}_\lambda(p, s)U_{\lambda'}(p, s) &= 2m\delta_{\lambda\lambda'}, & \bar{V}_\lambda(p, s)V_{\lambda'}(p, s) &= -2m\delta_{\lambda\lambda'}, \\ \bar{U}_\lambda(p, s)V_{\lambda'}(p, s) &= 0, & \bar{V}_\lambda(p, s)U_{\lambda'}(p, s) &= 0, \end{aligned} \quad (\text{A1})$$

for all combinations of λ and λ' equal to $\uparrow$ or $\downarrow$. They are connected by

$$U_\uparrow(p, s) = \gamma_5 V_\downarrow(p, s) \quad U_\downarrow(p, s) = -\gamma_5 V_\uparrow(p, s), \quad (\text{A2})$$

$$\bar{V}_\uparrow(p, s) = \bar{U}_\downarrow(p, s)\gamma_5, \quad \bar{V}_\downarrow(p, s) = -\bar{U}_\uparrow(p, s)\gamma_5. \quad (\text{A3})$$

This multiplication by γ_5 simply changes the sign of one or the other of the terms, since $\gamma_5 = \gamma_R - \gamma_L$. Using the charge conjugation matrix, $\mathcal{C}$, we have

$$V_\uparrow(p, s) = \mathcal{C}(\bar{U}_\uparrow(p, s))^T. \quad (\text{A4})$$

With all of these identities, this set of spinors is fixed except for an overall phase on $U_\uparrow, V_\downarrow, \bar{U}_\downarrow, \bar{V}_\uparrow$ and its complex conjugate on $U_\downarrow, V_\uparrow, \bar{U}_\uparrow, \bar{V}_\downarrow$.

For convenience in this paper, we use this phase freedom to connect the following:

$$\begin{aligned} \bar{U}_\downarrow(p, s) &= -i\bar{U}_\uparrow(p, -s), \\ V_\downarrow(p, s) &= -iV_\uparrow(p, -s), \end{aligned} \quad (\text{A5})$$

so that going from $\uparrow$ to $\downarrow$ is simply achieved by interchanging $\hat{p} \leftrightarrow \check{p}$. We emphasize that this is just a convenient choice, it is not required by any quantum field theory (QFT) property, and the phase freedom remains, but if used, will break these two conditions. The phase choices in Ref. [13] are obtained by multiplying all of $U_\downarrow, V_\uparrow, \bar{U}_\uparrow, \bar{V}_\downarrow$ by $\exp(+i\psi)$ and all of $U_\uparrow, V_\downarrow, \bar{U}_\downarrow, \bar{V}_\uparrow$ by $\exp(-i\psi)$. After this rephasing, the conditions in Eq. (A5) do not hold.

⁵For an equivalent example, consider the ground state of the helium atom.

TABLE I. Dirac spinors for a massive fermion with four-momentum p , mass m ($p^2 = m^2$), and spin four-vector s , ($s^2 = -1$ and $p \cdot s = 0$), we define two massless four-momenta as follows: $\hat{p} \equiv (p + ms)/2$ and $\check{p} \equiv (p - ms)/2$, then $\hat{p}^2 = 0 = \check{p}^2$. The phase $\psi(p, s)$ is momentum and spin axis dependent is given by $\psi \equiv \frac{1}{2} \text{Arg}(\langle \hat{p} + |\check{p}- \rangle / m)$. More on this phase is given in Table II.

Spinors	Outgoing	Incoming
Fermion	$\bar{U}_\uparrow(p, s) = e^{-i\psi} \langle \hat{p} + + e^{i\psi} \langle \check{p} - $ $\bar{U}_\downarrow(p, s) = -e^{-i\psi} \langle \check{p} + + e^{i\psi} \langle \hat{p} - $	$U_\uparrow(p, s) = e^{i\psi} \hat{p} + \rangle + e^{-i\psi} \check{p} - \rangle$ $U_\downarrow(p, s) = -e^{i\psi} \check{p} + \rangle + e^{-i\psi} \hat{p} - \rangle$
Antifermion	$V_\uparrow(p, s) = e^{i\psi} \check{p} + \rangle + e^{-i\psi} \hat{p} - \rangle$ $V_\downarrow(p, s) = e^{i\psi} \hat{p} + \rangle - e^{-i\psi} \check{p} - \rangle$	$\bar{V}_\uparrow(p, s) = e^{-i\psi} \langle \check{p} + + e^{i\psi} \langle \hat{p} - $ $\bar{V}_\downarrow(p, s) = e^{-i\psi} \langle \hat{p} + - e^{i\psi} \langle \check{p} - $

With the identities above, we can derive relations that all the spinors must satisfy the Dirac equation and also be eigenvectors of the spin matrix, $\gamma_5 \not{s}$. For $U_\uparrow(p, s)$, these are

$$\not{p} U_\uparrow(p, s) = m U_\uparrow(p, s), \quad \gamma_5 \not{s} U_\uparrow(p, s) = + U_\uparrow(p, s). \quad (\text{A7})$$

By adding and subtracting these two identities, after multiplying the second by m , we obtain

$$\begin{aligned} [\gamma_R \hat{p} + \gamma_L \check{p}] U_\uparrow(p, s) &= [|\hat{p} + \rangle \langle \hat{p} + | + |\check{p} - \rangle \langle \check{p} - |] U_\uparrow(p, s) \\ &= m U_\uparrow(p, s), \\ [\gamma_R \check{p} + \gamma_L \hat{p}] U_\uparrow(p, s) &= [|\check{p} + \rangle \langle \check{p} + | + |\hat{p} - \rangle \langle \hat{p} - |] U_\uparrow(p, s) \\ &= 0. \end{aligned} \quad (\text{A8})$$

All of the spinors are right or left eigenvectors of the two matrices,

$$|\hat{p} + \rangle \langle \hat{p} + | + |\check{p} - \rangle \langle \check{p} - |, \quad |\check{p} + \rangle \langle \check{p} + | + |\hat{p} - \rangle \langle \hat{p} - |, \quad (\text{A9})$$

with eigenvalues, $\pm m$ or 0, as required, see Table III. All of this is simple to check using spinor-helicity techniques.

2. A technical argument against antisymmetrization option B

An extra argument can be made against the antisymmetrization choice of option B. In fact, we can show the interference term for the antisymmetrization of type B has effectively no physical meaning. To see this, consider the most general phase change in the spinors given in Table I that is also consistent with QFT,

$$\begin{aligned} [U_\downarrow, V_\uparrow, \bar{U}_\uparrow, \bar{V}_\downarrow](p, s) &\rightarrow e^{+i\rho(p, s)} [U_\downarrow, V_\uparrow, \bar{U}_\uparrow, \bar{V}_\downarrow](p, s), \\ [U_\uparrow, V_\downarrow, \bar{U}_\downarrow, \bar{V}_\uparrow](p, s) &\rightarrow e^{-i\rho(p, s)} [U_\uparrow, V_\downarrow, \bar{U}_\downarrow, \bar{V}_\uparrow](p, s). \end{aligned} \quad (\text{A10})$$

The function ρ can be quite general, but has some constraints. Here we only need this phase to be a different arbitrary constant for ν_1 and ν_2 . Using $\Delta\rho \equiv \rho_1 - \rho_2$ and $\Sigma\rho \equiv \rho_1 + \rho_2$, the change above transforms the Dirac amplitudes by

$$\begin{aligned} \mathcal{D}[L_1, \bar{R}_2] &\rightarrow e^{-i\Delta\rho} \mathcal{D}[L_1, \bar{R}_2], & \mathcal{D}[R_2, \bar{L}_1] &\rightarrow e^{-i\Delta\rho} \mathcal{D}[R_2, \bar{L}_1], \\ \mathcal{D}[L_1, \bar{L}_2] &\rightarrow e^{-i\Sigma\rho} \mathcal{D}[L_1, \bar{L}_2], & \mathcal{D}[L_2, \bar{L}_1] &\rightarrow e^{-i\Sigma\rho} \mathcal{D}[L_2, \bar{L}_1], \\ \mathcal{D}[R_1, \bar{R}_2] &\rightarrow e^{+i\Sigma\rho} \mathcal{D}[R_1, \bar{R}_2], & \mathcal{D}[R_2, \bar{R}_1] &\rightarrow e^{+i\Sigma\rho} \mathcal{D}[R_2, \bar{R}_1], \\ \mathcal{D}[R_1, \bar{L}_2] &\rightarrow e^{+i\Delta\rho} \mathcal{D}[R_1, \bar{L}_2], & \mathcal{D}[L_2, \bar{R}_1] &\rightarrow e^{+i\Delta\rho} \mathcal{D}[L_2, \bar{R}_1]. \end{aligned} \quad (\text{A11})$$

Only pairs of amplitudes that have the same phase transformation can be antisymmetrized without introducing an arbitrary phase between them. These are

$$\begin{aligned} (L_1 \bar{L}_2, \bar{L}_1 L_2), & \quad (R_1 \bar{R}_2, \bar{R}_1 R_2), \\ (L_1 \bar{R}_2, \bar{L}_1 R_2), & \quad (R_1 \bar{L}_2, \bar{R}_1 L_2). \end{aligned} \quad (\text{A12})$$

These are exactly the pairs that appear in option A of Fig. 2. As an example, consider the $L_1 \bar{R}_2$ helicity configuration, then the antisymmetrized amplitude transform as

$$\begin{aligned} \text{Option A: } \mathcal{M}[L_1, R_2] &\equiv \mathcal{D}[L_1, \bar{R}_2] - \mathcal{D}[R_2, \bar{L}_1] \\ &\rightarrow e^{-i\Delta\rho} \{ \mathcal{D}[L_1, \bar{R}_2] - \mathcal{D}[R_2, \bar{L}_1] \}, \\ \text{Option B: } \mathcal{K}[L_1, R_2] &\equiv \mathcal{D}[L_1, \bar{R}_2] - \mathcal{D}[L_2, \bar{R}_1] \\ &\rightarrow e^{-i\Delta\rho} \mathcal{D}[L_1, \bar{R}_2] - e^{+i\Delta\rho} \mathcal{D}[L_2, \bar{R}_1]. \end{aligned} \quad (\text{A13})$$

TABLE II. Relationships for the phase factor ψ . Note with this set of square root choices, the interchanging $\hat{p} \leftrightarrow \check{p}$ adds $\pi/2$ to ψ in all of the above and the product of any of the phases with its complex conjugate of the phase, (phase)*, is 1.

Phase	$\langle \hat{p} + \check{p} - \rangle / m \equiv e^{+i2\psi}$	$\langle \check{p} + \hat{p} - \rangle / m = -e^{+i2\psi}$	$(\langle \hat{p} + \check{p} - \rangle / m)^{1/2} \equiv e^{+i\psi}$	$(\langle \check{p} + \hat{p} - \rangle / m)^{1/2} = +ie^{+i\psi}$
(Phase)*	$\langle \check{p} - \hat{p} + \rangle / m = e^{-i2\psi}$	$\langle \hat{p} - \check{p} + \rangle / m = -e^{-i2\psi}$	$(\langle \check{p} - \hat{p} + \rangle / m)^{1/2} = e^{-i\psi}$	$(\langle \hat{p} - \check{p} + \rangle / m)^{1/2} = -ie^{-i\psi}$

TABLE III. The eigenvalues for the two projection matrices given in Eq. (A9). The sum of the two matrices is just $\not{p}$, whereas the difference, line 1 minus line 2, is $m(\gamma_5 \not{s})$, demonstrating that each spinor satisfies the appropriate Dirac equation and has the appropriate spin projection.

Eigenvalues	$\bar{U}_\uparrow, U_\uparrow$	$\bar{U}_\downarrow, U_\downarrow$	$\bar{V}_\uparrow, V_\uparrow$	$\bar{V}_\downarrow, V_\downarrow$
$ \hat{p}+\rangle\langle\hat{p}+ + \check{p}-\rangle\langle\check{p}- $	m	0	0	$-m$
$ \check{p}+\rangle\langle\check{p}+ + \hat{p}-\rangle\langle\hat{p}- $	0	m	$-m$	0

Clearly, option A remains invariant, $|\mathcal{M}[L_1, R_2]|^2 \rightarrow |\mathcal{M}[L_1, R_2]|^2$, while the interference term of option B depends on the choice of $\Delta\rho$,

$$|\mathcal{K}[L_1, R_2]|^2 \rightarrow |\mathcal{D}[L_1, \bar{R}_2]|^2 + |\mathcal{D}[L_2, \bar{R}_1]|^2 - 2\text{Re}\{e^{-2i\Delta\rho}\mathcal{D}[L_1, \bar{R}_2]\mathcal{D}^*[L_2, \bar{R}_1]\}. \quad (\text{A14})$$

Therefore $\mathcal{K}[L_1, R_2]$ cannot be a physical amplitude, further supporting that option A is the only physics choice.

Applying this rephasing to the sum of the Dirac amplitudes, given by Eq. (39),

$$L_1\bar{R}_2 + L_1\bar{L}_2 + R_1\bar{R}_2 + R_1\bar{L}_2 \Rightarrow e^{-i\Delta\rho}L_1\bar{R}_2 + e^{-i\Sigma\rho}L_1\bar{L}_2 + e^{+i\Sigma\rho}R_1\bar{R}_2 + e^{+i\Delta\rho}R_1\bar{L}_2, \quad (\text{A15})$$

we see that the relative phase between each amplitude can be varied and therefore this sum cannot be a physical amplitude.

3. Different bases

Charge-lepton basis. In the neutrino rest frame, choose the neutrino spin to be aligned with the direction of one of the charged leptons in that frame and since they are both massless momenta they remain parallel under any boost. For example, choose $\hat{p}_1 \parallel \vec{k}$ and $\hat{p}_2 \parallel \vec{k}$, then all but one of the Majorana amplitudes in Eqs. (17) and (18) are reduced to one term with no interference term. However, one of the amplitudes consists of two terms, in this case $\mathcal{M}(R, L)$, which leads to the interference term in the amplitude squared summed over spins.

Transverse basis. In the rest frame of the massive particle the spin axis, say x axis, is perpendicular to the boost direction, the z axis, of the particle. So that before the boost $p = (m, 0, 0, 0)$ and $s = (0, 1, 0, 0)$, then after the boost

$$p = \gamma m(1, 0, 0, \beta), \quad s = (0, 1, 0, 0), \quad \text{then} \\ \hat{p} = \frac{1}{2}\gamma m(1, \gamma^{-1}, 0, \beta), \quad \check{p} = \frac{1}{2}\gamma m(1, -\gamma^{-1}, 0, \beta), \quad \text{and} \quad p = \hat{p} + \check{p} \quad \text{with} \quad \hat{p}^2 = 0 = \check{p}^2. \quad (\text{A16})$$

This is easy to generalize to any boost direction and any spin axis perpendicular to the boost.

In this basis, the interference terms for the individual Majorana helicity amplitudes, Eq. (19), is

$$\text{Tr}[k\check{p}_1\hat{p}_1\bar{k}\check{p}_2\hat{p}_2]/m^2 \leq 4\sqrt{(k \cdot \check{p}_1)(\hat{p}_1 \cdot \bar{k})(\bar{k} \cdot \check{p}_2)(\hat{p}_2 \cdot k)} \sim \mathcal{O}(E_1 E_2 |k| |\bar{k}|) \quad (\text{A17})$$

and therefore is not suppressed by the neutrino mass. However, due to cancellations, the sum of the interference terms is still suppressed by the neutrino mass, as this sum is $m_\nu^2(k \cdot \bar{k})$ as given in Eq. (21).

APPENDIX B: APPLICATION TO $e^+e^- \rightarrow NN$

The fermionic component of the leptonic Φ decay in Sec. III may be related to the process $e^+e^- \rightarrow NN$ by a crossing symmetry, where the N are heavy Majorana fermions with mass m_N . Therefore, the spinorial structure of the result for this process by Ma and Pantaleone [18] can be compared with that calculated here as an independent check of our result. Expanding, for example, from our RR Majorana amplitude squared, we obtain

$$|\mathcal{M}[R_1, R_2]|^2 = 16(k \cdot \tilde{p}_1)(\bar{k} \cdot \hat{p}_2) + 16(k \cdot \tilde{p}_2)(\bar{k} \cdot \hat{p}_1) - 4 \text{Tr}[k\tilde{p}_1\hat{p}_1\bar{k}\hat{p}_2\tilde{p}_2]/m_N^2 \quad (\text{B1})$$

$$\begin{aligned} &= 4m_N[\bar{k} \cdot (s_2 - s_1)k \cdot (p_1 - p_2) - k \cdot (s_2 - s_1)\bar{k} \cdot (p_1 - p_2)] \\ &\quad + 4(\bar{k} \cdot p_2)(k \cdot p_1) + 4(\bar{k} \cdot p_1)(k \cdot p_2) - 4m_N^2(k \cdot \bar{k}) + 4(p_1 \cdot p_2 - m_N^2)[(s_2 \cdot \bar{k})(s_1 \cdot k) + (s_2 \cdot k)(s_1 \cdot \bar{k})] \\ &\quad - 4(s_2 \cdot p_1)[(p_2 \cdot \bar{k})(s_1 \cdot k) + (p_2 \cdot k)(s_1 \cdot \bar{k})] + 4(s_2 \cdot p_1)(\bar{k} \cdot k)(p_2 \cdot s_1) - 4(s_2 \cdot s_1)(\bar{k} \cdot k)(p_2 \cdot p_1) \\ &\quad + 4(s_2 \cdot s_1)[(\bar{k} \cdot p_2)(k \cdot p_1) + (k \cdot p_2)(\bar{k} \cdot p_1)] - 4(p_2 \cdot s_1)[(s_2 \cdot \bar{k})(p_1 \cdot k) + (s_2 \cdot k)(p_1 \cdot \bar{k})]. \end{aligned} \quad (\text{B2})$$

This may be used to calculate the left-chiral current contribution to $e^+e^- \rightarrow NN$ by performing the substitution $\bar{k} \rightarrow -k$ and $k \rightarrow -\bar{k}$, where we have insisted on continuity that the “barred” momentum represents the charged anti-fermion momentum. Then it may be contrasted with Eq. (1) of Ref. [18], with $x = \sin^2 \theta_W \mapsto 0$, with which we find perfect agreement for each of the spin contributions.⁶ This demonstrates the power of the spinor-helicity formalism as applied to these processes: the result in Eq. (B1) is more

⁶The terms are organized above for easy contrast with Eq. (1) of Ma and Pantaleone [18]. After performing the $\bar{k} \rightarrow -k$ and $k \rightarrow -\bar{k}$ substitution, matching onto the notation of [18] defined in their Fig. 1 involves the following further notation changes: $k \mapsto p$, $\bar{k} \mapsto \bar{p}$, $s_1 \mapsto s$, $s_2 \mapsto \bar{s}$, $p_1 \mapsto q$, and $p_2 \mapsto \bar{q}$.

compact at illuminating the underlying physics than that from traditional trace techniques.

The result in Eq. (B2) contains interference terms that scale with m_N^2 , and others which represent transverse degrees of freedom, as noted in Ref. [18], and whose revised treatment is presented in Ref. [12]. Summing over spins for the result in Eq. (1) of Ref. [18] is equivalent to summing over helicity configurations for the massless spinors constructed with $\hat{p}$, $\tilde{p}$. Doing so reveals that these transverse terms will cancel, and we recover interference behavior in the total squared matrix-element consistent with that described in Eq. (21). To obtain the additional contributions to $e^+e^- \rightarrow NN$ in Ref. [18] (i.e., where $x \neq 0$), one needs to consider different chiralities of interactions which may be generated by Z exchange, which may straightforwardly be derived using the same spinor-helicity methods as performed in this work.

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