

Axion strings from string axions

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A favored scenario for axions to be dark matter is for them to form a cosmic string network that subsequently decays, allowing for a tight link between the axion mass and relic abundance. We discuss an example in which the axion is protected from quantum gravity effects that would spoil its ability to solve the strong CP problem: namely, a string theoretic axion arising from gauge symmetry in warped extra dimensions. Axion strings arise following the first-order Randall-Sundrum compactification phase transition forming at the junctions of three bubbles during percolation. Their tensions are at the low scale associated with the warp factor, and are parametrically smaller than the usual field-theory axion strings relative to the scale of their decay constant. Simulations of string network formation by this mechanism must be carried out to see whether the axion mass-relic density relation depends on the new parameters in the theory.

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I. INTRODUCTION

The axion solution to the strong CP problem [1–5] provides a highly motivated dark matter candidate [6,7], potentially resolving two of the major mysteries of particle physics through one hypothesis. Conventional axion models involve a complex scalar field $\Phi = \varphi e^{ia/f_a}$, the Peccei-Quinn (PQ) field, whose angular component a is the axion. It is initially a massless Goldstone boson when the global PQ symmetry is spontaneously broken by the vacuum expectation value $\langle \varphi \rangle = f_a$ (known as the axion decay constant). This occurs at high temperatures in the early Universe. At lower temperatures, below the QCD phase transition, the axion gets a small mass because the PQ symmetry is anomalous, and the axion couples to the gluon field strength via $(a/f_a)G\tilde{G}$.

In terms of dark matter predictions, a favored scenario is for the PQ phase transition temperature to be below the reheating temperature after inflation. In this case, a cosmic string network forms, in which a/f_a winds around each string by a multiple of 2π . These strings eventually decay into axion particles and produce a relic density that depends on the single parameter m_a , the axion mass. Then, in principle, experimentalists searching for relic axions could concentrate their efforts on the value of m_a that leads to the observed dark matter abundance. In practice, the string network computations needed to make a precise prediction of m_a are numerically quite demanding, and there is not yet consensus on the range of likely values, although much progress has been made.

Quantum gravity poses a generic problem for field-theory axions: It is believed to break global symmetries, including the PQ symmetry, which would spoil the axion solution to the strong CP problem unless one artificially forbids PQ-breaking operators up to some very high dimension [8–10]. On the other hand, string theories naturally predict light axions that can be protected from such corrections, for example, by having the global symmetry descend from an underlying gauge symmetry. In this case, the symmetry-breaking contributions from quantum gravity are expected to be exponentially suppressed [11]. Recently, Ref. [12] studied string theoretic

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axions and concluded that their cosmic strings have tensions near the Planck scale. They therefore correspond to conventional theories with PQ breaking above the scale of inflation, which never form string networks. These models rely upon the misalignment mechanism [13,14] for getting the axion relic density, which provides no direct link between m_a and the dark matter abundance.

In this work, we discuss a simple counterexample to the above statement, starting from a five-dimensional Randall-Sundrum [15] setup with a warped extra dimension and a U(1) gauge field living in the bulk. The axion is the zero mode of A_5 , the fifth component of the gauge field; hence, it is protected from quantum gravity corrections by the gauge symmetry [16]. Since the starting point does not involve any explicit PQ scalar field Φ , it is not *a priori* obvious what plays the role of the radial component φ that determines the cosmic string profile. The answer turns out to be that φ is the radion, the light degree of freedom that corresponds to fluctuations in the size of the extra dimension. Deep in the core of the string, the radion vanishes, corresponding to a phase in which the field of the low-energy description has dissolved into more elementary constituents, analogous to the quark-hadron phase transition of QCD.

An important difference relative to conventional cosmic strings is the formation mechanism, which is usually a second-order phase transition from a high-temperature $T^2|\phi|^2$ potential to a low-temperature Higgs-like potential $\lambda(|\phi|^2 - v^2)^2$. In the 5D setup, the extra dimension is effectively decompactified at high T and becomes compact during a strongly supercooled first-order transition where bubbles of the broken phase are nucleated. Each bubble contains a region where $\langle a \rangle / f_a$ takes a random value in the interval $[0, 2\pi]$. As the phase transition completes, bubbles coalesce, and at a junction of three bubbles in which a approximately winds around the circle enclosing the junction, a cosmic axion string can form. The dynamics of such a string formation is very different from that in the second-order transition and has not yet been studied in detail. We do not attempt such a simulation here, but rather lay the groundwork for future studies.

II. FRAMEWORK

The geometry of the Randall-Sundrum model can be described by a warped 5D anti-de Sitter (AdS) metric

$$ds^2 = \left(\frac{\varphi}{F}\right)^{2u} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{\ln^2(\varphi/F)}{k^2} du^2 \quad (1)$$

having an ultraviolet (UV) brane at $u = 0$ and an infrared (IR) brane at $u = 1$. The field φ is the radion, which describes dynamical fluctuations of the size of the extra dimension, and $F = \sqrt{24M_5^3/k} \cong \sqrt{24}M_p$, with M_5 being the 5D gravity scale, k the AdS curvature scale, and M_p the

4D Planck scale. The effective 4D description of gravity is general relativity plus small higher derivative corrections if M_5/k is sufficiently large. The radion is naturally stabilized by a potential $V(\varphi)$ at a vacuum expectation value $\langle \varphi \rangle = v \ll k$, which achieves a large hierarchy of scales between the IR and UV branes without the need for fine-tuning. Originally, v was taken to be the TeV scale to explain the standard model weak scale, but here we are interested intermediate scales such as $v \sim 10^{10}$ GeV corresponding to the scale of axion couplings (even though there is no explicit global PQ symmetry in the theory).

The radion plays an important role in the Randall-Sundrum (RS) phase transition at high temperatures, in which the theory is effectively decompactified. It starts near $\varphi = 0$, and at the critical temperature it makes a first-order transition to its vacuum expectation value v . Above the transition temperature T_c , the radion is no longer a good effective description; rather, there is a strongly coupled conformal SU(N) theory, and it becomes confining at T_c .

Generically, the transition can be so supercooled that it never completes [17], especially in regions of parameter space where the 4D effective description is under control. However, it is possible to overcome this obstacle through careful design of the radion potential [18], which comes from the Goldberger-Wise mechanism [19], i.e., integrating out a bulk scalar field whose couplings to φ induce its potential:

$$V(\varphi) = \lambda[\varphi^4 f(\varphi/v) - v^4 f(1)], \quad (2)$$

where $x^4 f(x)$ is a function with a minimum at $x = 1$, whose form depends upon details of the Goldberger-Wise (GW) bulk scalar field Lagrangian. We consider two typical examples denoted by Chacko, Mishra, and Stolarski (CMS) [20] and GW [19], respectively,

$$f(x) = \frac{x^\epsilon}{(1 + \epsilon/4)} - 1, \quad f(x) = (1 - x^\epsilon)^2. \quad (3)$$

The small exponent ϵ relates the large hierarchy of scales between M_p and f_a to a moderately small ratio of scales in the GW field potential being raised to a large power $1/\epsilon$; we take $\epsilon = 0.1$ for definiteness in numerical examples. The coupling λ is expected to be of order $(k/M_5)^6$, which must be small for the effective theory to be trustworthy. The radion's squared mass is given by $m_\varphi^2 = V''(v) = 4\lambda\epsilon v^2$, $2\lambda\epsilon^2 v^2$ for the two respective potentials. Its kinetic term is simply $\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi$, apart from small corrections of order $(\varphi/F)^2$ that we neglect.

To incorporate an axion in the theory, we follow Refs. [16,21] by introducing a U(1) gauge field A_M in the bulk, with A_5 playing the role of the massless axion. We impose the Dirichlet boundary conditions for A_μ on the UV and IR branes, so that the massless A_μ is eliminated. The massive components of A_μ are integrated out by solving

the equation of motion. This gives the relation $F_{\mu 5} = \ln(F/\varphi)(F/\varphi)^{2(u-1)}\partial_\mu\vartheta(x)$, where $\vartheta(x) \equiv 2 \int_0^1 du A_5$ (see Appendix A for details). In the string solution with a winding number of 1, $\vartheta(x)$ also represents the azimuthal angle in the plane transverse to the string. From this, we derive the effective Lagrangian for $\vartheta(x)$ [22],

$$\mathcal{L}_a = \frac{k}{2g_5^2 F^2} \varphi^2 \partial_\mu \vartheta \partial^\mu \vartheta \equiv \frac{1}{2} \left(\frac{\varphi}{v} \right)^2 \partial_\mu a \partial^\mu a, \quad (4)$$

where the 5D gauge coupling g_5 has dimensions of $(\text{mass})^{-1/2}$ and $a \equiv f_a \vartheta$. The decay constant f_a is determined by the periodicity of a in field space, which depends on that of A_5 . It follows from the Wilson loop $\exp(2i \int_0^1 du A_5)$, which is equal to unity for $A_5 = \pi n$, implying that

$$f_a = \frac{\sqrt{k}v}{g_5 F}, \quad (5)$$

in agreement with Ref. [12]: f_a is at the warped scale of the IR brane.

As will be seen, this result for f_a is the one that determines the cosmic string tension, but it does not automatically determine the coupling strength of axions to matter. Reference [21] shows that it depends on whether QCD resides on the branes or in the bulk. Only in the latter case, the axion coupling to gluons is of order $1/f_a$ (5), otherwise the coupling is suppressed by the Planck scale, spoiling the axion solution to the strong CP problem. Having a heavy quark with quantized g_5 charge in the bulk and integrating it out generates a Chern-Simons interaction between A_M and the QCD gluons, resulting in the axion-gluon coupling of order $g^2/(32\pi^2 f_a) a G \tilde{G}$.

Finally, it is natural to wonder whether the identification in (4) can be spoiled by higher-order couplings, such as $\Phi_{GW} F_{MN} F^{MN}$. As we show in Appendix B, these are highly suppressed provided that the mass term of the Goldberger-Wise field is non-negative: $m^2 \geq 0$.

III. STRING SOLUTIONS

For configurations with axion winding by 2π , the energy density of the string is

$$\mathcal{E} = \frac{1}{2} \left(\varphi'(r)^2 + b^2 \frac{\varphi^2}{r^2} \right) + V(\varphi), \quad (6)$$

leading to the equation of motion

$$\varphi'' + \frac{\varphi'}{r} = b^2 \frac{\varphi}{r^2} + V'(\varphi), \quad (7)$$

where $b^2 = (f_a/v)^2 = [24(g_5^2 k)(M_5/k)^3]^{-1}$, which is expected to be small, $b^2 \ll 1$. The exact solution is very

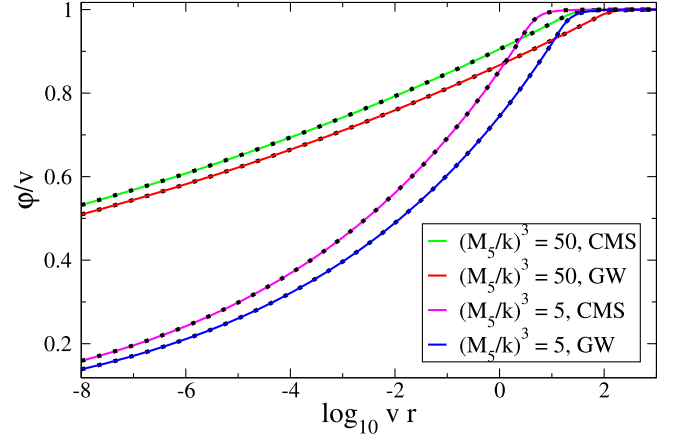


FIG. 1. Left: example cosmic string profiles $\varphi(r)$ for two choices of $(M_5/k)^3$ and the radion potential (CMS [20] and GW [19]) in units of the radion vacuum expectation value v . Solid curves are numerical solutions, dotted are the analytic approximation (8). Other parameters are taken to be $g_5^2 k = 1$, $\lambda = 1/(M_5/k)^6$, $\epsilon = 0.1$.

well approximated by joining its small- and large- r limits at a matching radius $r_m = \sqrt{2b + b^2}/m_\varphi$:

$$\varphi(r) \cong \begin{cases} v(r/r_m)^b/(1 + b/2), & r < r_m, \\ v[1 - b^2/(m_\varphi r)^2], & r > r_m. \end{cases} \quad (8)$$

As examples, we show solutions for $(M_5/k)^3 = 5, 50$, $g_5^2 k = 1$, $\lambda = (M_5/k)^{-6}$, $\epsilon = 0.1$ in Fig. 1. The profiles for the GW and CMS radion potentials differ only because their respective radion masses are not the same.

The tension of the cosmic string is found by integrating the energy density (6), $T = 2\pi \int_0^R dr r \mathcal{E}$. R is the usual infrared cutoff on global cosmic strings, because of the logarithmic growth of the winding term; this is typically cut off by the separation between neighboring strings. The tension can be divided into that coming from the core region $r < r_m$ and that from the exterior region. The exterior contribution is

$$T_{>} = \pi(bv)^2 \left(\frac{b}{4(1 + b/2)} + \ln \frac{R}{r_m} \right). \quad (9)$$

The first term comes from the radion potential by approximating it as $V \cong \frac{1}{2} m_\varphi^2 (v - \varphi)^2$ in the vicinity of its minimum, while the second comes from $b^2 v^2/r^2$. The core contribution to the tension is

$$T_c = \frac{\pi b v^2}{(1 + b/2)^2} + O(b^3 v^2), \quad (10)$$

where $O(b^3 v^2)$ is the part coming from the radion potential that turns out to be independent of λ and ϵ for small b , with coefficient $5/8$ for CMS and $5/168$ for GW. At finite

temperature T , the core string tension receives subleading corrections provided that T lies at a value that is smaller than v (see, for example, [18], where a similar situation with an AdS-Schwarzschild metric was considered).

It is interesting to contrast the relation between the tension and decay constant in this model with that of a generic field-theory PQ string, whose Lagrangian is $|\partial\Phi|^2 - \lambda(|\Phi|^2 - f_a^2/2)^2$, with $\Phi = (\varphi/\sqrt{2})e^{ia/f_a}$. This corresponds to the warped axion model when $b = 1$ and $m_\varphi^2 = 2\lambda v^2$. The cosmic string tension is smaller than its field-theory counterpart since $b^2 \ll 1$ and m_φ is suppressed by ϵ .

IV. STRING PRODUCTION

Unlike the field theoretic PQ phase transition, the RS transition is first order, proceeding by bubble nucleation. Inside each bubble, the axion field will have an approximately uniform phase $\theta_i = a_i/f_a$. If three bubbles meet such that $\theta_1 < \theta_2 < \theta_3$, or vice versa, with the angles approximately covering the circle, then the axion is in a winding configuration, and a cosmic string will start to form in the region between the intersections of the bubbles; see Fig. 2. As the bubbles expand, the interstitial region is topologically restricted to maintain $\varphi = 0$, and the string grows in length until it crosses the horizon or meets another string that it can join with, possibly forming a loop. One might wonder whether the energy release from the bubble collision renders the axion formation unstable. However, given that it is mainly in the form of gravitational waves and particle production, it will not destabilize the string formation, as the winding is topological in nature. Further confirmation can be provided via numerical simulations [23].

This formation mechanism is qualitatively similar to, but different in detail from the Kibble mechanism, which relies upon the tiling of horizon-sized regions, in each of which θ is approximately constant, rather than the collisions of bubbles. Generically, we expect the probability of string formation to be lower in the bubble collision picture.

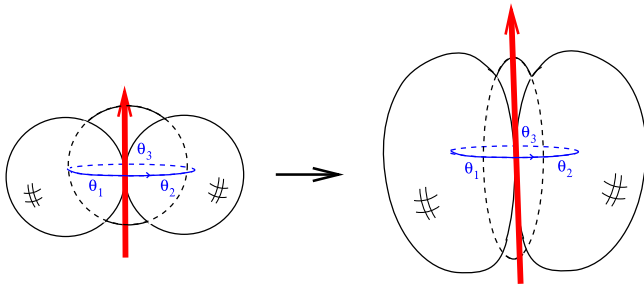


FIG. 2. Intersection of three phase transition bubbles, which can initiate cosmic string formation (red arrow) if their axion phases wind around the circle. After the percolation, the bubbles merge into a nearly spherical one, and the string elongates.

Whereas it is generic for three Hubble patches to be adjoining, since the second-order phase transition occurs at the same time everywhere, in the first-order transition it is more likely for two bubbles to collide than three. The phases may have enough time to equilibrate between two bubbles before they meet a third, thereby avoiding winding. Depending upon how much supercooling occurs, bubble nucleation could be sufficiently rare that three-bubble collisions are suppressed. Once there is such a collision, the probability of having a winding configuration is naively $3!/3^3 = 2/9$.

It is possible that the details of the initial production become unimportant once the string network enters a scaling regime. But even if this occurs, there may still be interesting differences relative to the field-theory axion when it comes to the decays of the strings into axion particles. This motivates numerical simulations of the formation and decay of such networks to supplement their PQ field-theory counterparts, to find out whether the relationship between the ultimate axion mass (acquired during the QCD phase transition) and its relic density becomes dependent on new parameters such as b or the degree of supercooling. Thus far, the production of global strings in a first-order phase transition has only been studied in an M.Sc. thesis [24], but further work in this direction is in progress [25].

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: KINETIC TERM OF THE 5D GAUGE FIELD

In this appendix, we integrate out the five-dimensional $U(1)$ gauge field to obtain the kinetic term for the axion field. The relevant action is

$$S_{U(1)} = -2 \cdot \frac{1}{4g_5^2} \int d^5x \sqrt{-g} g^{MN} g^{PQ} F_{MP} F_{NQ}, \quad (\text{A1})$$

where the extra factor of 2 accounts for the $u \leftrightarrow -u$ orbifold identification. In what follows, we assume that

A_μ depends only on the extra-dimensional coordinate u and satisfies Dirichlet boundary conditions on the boundaries, $A_\mu(u=0) = A_\mu(u=1) = 0$. This means that the 4D field is not dynamical and can be integrated out since its kinetic term vanishes, $F_{\mu\nu} = 0$.

The action in (A1) thus reduces to

$$S_{U(1)} = -\frac{1}{g_5^2} \int d^5x \sqrt{-g} g^{\mu\nu} g^{55} F_{\mu 5} F_{\nu 5}. \quad (\text{A2})$$

Varying with respect to A_μ and A_5 yields the equations of motion (EOMs)

$$\partial_5 \left[\left(\frac{\varphi}{F} \right)^{2u} \frac{\ln(\varphi/F)}{k} F_{\nu 5} \eta^{\nu\mu} \right] = 0, \quad (\text{A3})$$

$$\partial_\mu \left[\left(\frac{\varphi}{F} \right)^{2u} \frac{\ln(\varphi/F)}{k} F_{\nu 5} \eta^{\nu\mu} \right] = 0, \quad (\text{A4})$$

respectively.

We first consider Eq. (A3). The general solution can be written as

$$F_{\mu 5} = \left(\frac{\varphi}{F} \right)^{-2u} \frac{\ln(\varphi/F)}{k} f_\mu(x). \quad (\text{A5})$$

Integrating this with respect to u and taking the $\varphi \ll F$ limit yields the solution for f_μ :

$$f_\mu = -2k \left(\frac{\varphi}{F} \right)^2 \partial_\mu \left[\int_0^1 du A_5 \right]. \quad (\text{A6})$$

The dimensionless axion field ϑ , whose periodicity is 2π , is defined through the gauge-invariant Wilson loop [12,26],

$$\vartheta(x) = 2 \int_0^1 du A_5. \quad (\text{A7})$$

The periodicity comes from the fact that $e^{i\vartheta}$ is a large gauge transformation when ϑ is a multiple of 2π . Substituting (A7) and (A6) into (A5) allows us to write $F_{\mu 5}$ as a function of the dimensionless axion field,

$$F_{\mu 5} = \ln(F/\varphi) \left(\frac{F}{\varphi} \right)^{2(u-1)} \partial_\mu \vartheta(x). \quad (\text{A8})$$

Having obtained an expression for $F_{\mu 5}$ that satisfies (A3), all that remains is to check whether there are any further conditions imposed by (A4). Substituting into (A4) yields

$$\partial_\mu \left[\left(\frac{\varphi}{F} \right)^2 \partial^\mu \vartheta(x) \right] = 0, \quad (\text{A9})$$

which is satisfied, provided we identify $\vartheta(x)$ with the azimuthal angle in the cosmic string solution, while assuming a radial dependence for the radion φ .

APPENDIX B: HIGHER-ORDER OPERATORS

In Sec. II, we identified the cosmic string profile with the radion by comparing the effective couplings for each theory in Eq. (4). Given that we are working within an effective field theory, it is natural to wonder whether higher-dimensional operators can ruin this identification. In this appendix, we shall examine this in detail. In particular, we will examine whether the coupling $\Phi_{GW} F_{MN} F^{MN}$ can spoil our identification.

1. Derivation of leading correction

To that end, consider the action for the massive Goldberger-Wise field,

$$S = -\frac{1}{2} \int d^5x \sqrt{-G} [G^{MN} \partial_M \Phi_{GW} \partial_N \Phi_{GW} + m^2 \Phi_{GW}^2]. \quad (\text{B1})$$

Varying with respect to Φ_{GW} yields the following EOM:

$$\square \Phi_{GW} - m^2 \Phi_{GW} = 0. \quad (\text{B2})$$

Using the metric (1) and allowing only a u dependence for Φ_{GW} simplified the previous equation to

$$\frac{\partial^2 \Phi_{GW}}{\partial [u \ln(\varphi/F)]^2} + 4 \frac{\partial \Phi_{GW}}{\partial [u \ln(\varphi/F)]} - \frac{m^2}{k^2} = 0. \quad (\text{B3})$$

Defining $z \equiv -u \ln(\varphi/F)$ reduces the previous equation to

$$\frac{d^2 \Phi_{GW}}{dz^2} - 4 \frac{d \Phi_{GW}}{dz} - \frac{m^2}{k^2} = 0. \quad (\text{B4})$$

The general solution of the previous equation is

$$\Phi_{GW} = \left(\frac{\varphi}{F} \right)^{-2u} \left[c_+ \left(\frac{\varphi}{F} \right)^{2\nu u} + c_- \left(\frac{\varphi}{F} \right)^{-2\nu u} \right] \equiv \Phi_+ + \Phi_-. \quad (\text{B5})$$

Now it is time to evaluate the integration constants $c_\pm$. We demand that the GW field has some fixed value v_i (with $i = +$ at $u = 0$ and $i = -$ at $u = 1$), which is (following Goldberger and Wise) much smaller than $M_5^{3/2}$. The explicit form of $c_\pm$ (assuming that $\varphi \ll F$) is

$$c_+ \approx v_+, \quad c_- \approx v_- \left(\frac{\varphi}{F} \right)^{4\nu}. \quad (\text{B6})$$

Moving on, we now consider the coupling between Φ_{GW} and the U(1) gauge field F_{MN} . The relevant action

$$S_{coup} = -\frac{c_1}{4\sqrt{M_5}} \int d^5x \sqrt{-G} \Phi_{GW} F_{MP} F_{NQ} G^{MN} G^{PQ}, \quad (\text{B7})$$

where c_1 is an $\mathcal{O}(1)$ dimensionless coefficient. We are interested in the $\mu 5$ component, for which the field strength has the form

$$F_{\mu 5} = \ln(F/\varphi) \left(\frac{\varphi}{F} \right)^{-2(u-1)} \partial_\mu \vartheta. \quad (\text{B8})$$

Plugging everything in, we have the following for each of the components of Φ_{GW} :

$$\begin{aligned} S_{coup}^\pm &= -\frac{c_1 c_\pm}{2\sqrt{M_5}} \int d^5x \frac{\ln(\varphi/F)}{k} \left(\frac{\varphi}{F} \right)^{4u} \ln^2(F/\varphi) \left(\frac{\varphi}{F} \right)^{-4(u-1)} \left(\frac{\varphi}{F} \right)^{-2u \pm 2\nu u} \frac{k^2}{\ln^2(\varphi/F)} \left(\frac{\varphi}{F} \right)^{-2u} \partial_\mu \vartheta \partial^\mu \vartheta \\ &= -\frac{c_1 c_\pm k}{2\sqrt{M_5}} \int d^4x \partial_\mu \vartheta \partial^\mu \vartheta \int_0^1 du \ln(\varphi/F) \left(\frac{\varphi}{F} \right)^{4-4u \pm 2\nu u} \\ &= -\frac{c_1 c_\pm k}{4\sqrt{M_5}(2 \mp \nu)} \int d^4x \left[\left(\frac{\varphi}{F} \right)^{\pm 2\nu} - \left(\frac{\varphi}{F} \right)^4 \right] \partial_\mu \vartheta \partial^\mu \vartheta. \end{aligned} \quad (\text{B9})$$

The leading-order result (after substituting the values for the integration constants $c_\pm$) is, after some massaging, the following:

$$\mathcal{L}_{coup}^\pm = -\frac{c_1}{2 \mp \nu} \cdot \left[\frac{v_\pm}{M_5^{3/2}} \left(\frac{k}{bM} \right)^2 \right] \cdot \frac{k}{2g_5^2} \left(\frac{\varphi}{F} \right)^{2\nu} \partial_\mu \vartheta \partial^\mu \vartheta. \quad (\text{B10})$$

2. String solutions

In this section, we will study the EOM of the string in the presence of the previous correction.

The EOM for the cosmic string is

$$\varphi'' + \frac{\varphi'}{r} = b^2 \frac{\varphi}{r^2} + \alpha \frac{\varphi^{2\nu-1}}{r^2} + V'(\varphi), \quad (\text{B11})$$

where

$$\alpha \equiv -\frac{c_1}{2 \mp \nu} \cdot \left[\frac{v_\pm}{M_5^{3/2}} \left(\frac{k}{bM} \right)^2 \right] \cdot \frac{k}{g_5^2} F^{2\nu-1} f_a^2. \quad (\text{B12})$$

For $\nu \geq 1$, the equation of motion admits a solution that vanishes as a power law at the origin and approaches v at infinity. For $\nu < 1$, the EOM still admits a solution that approaches v at infinity, but the behavior at the origin

changes drastically. To see that, we approximate φ with a power law behavior at the origin:

$$\begin{aligned} \varphi &\approx A \cdot r^\beta \Rightarrow A\beta^2 r^{\beta-2} \\ &\approx Ab^2 r^{\beta-2} + \alpha A^{2\nu-1} r^{(2\nu-1)\beta-2} + Cr^{3\beta}, \end{aligned} \quad (\text{B13})$$

where we made use of the fact that for small r the potentials behave as $\sim r^{3\beta}$ when $\varphi \sim r^\beta$ and $r \ll v^{-1}$. From the previous equation, we have the following (possible) matching conditions:

- (1) $\beta - 2 = (2\nu - 1)\beta - 2 \Rightarrow \beta = 0$. This means that the cosmic string profile approaches a constant at the origin, which is not permissible. Note that this can be avoided if $\nu \geq 1$, since in this case the terms are either of the same order or the correction is subdominant.
- (2) $(2\nu - 1)\beta - 2 = 3\beta \Rightarrow \beta = \frac{1}{\nu-2}$. This means that for $\nu < 1$ we are going to get a diverging φ at the origin, which is not permissible.
- (3) $\beta - 2 = 3\beta \Rightarrow \beta = -1$. This again yields a diverging φ at the origin, and is thus discarded.

From the previous analysis, we deduce the following: Provided that $m^2 \geq 0$, the equation of motion admits finite string solutions and corrections do not spoil our results. On the other hand, if $-4k^2 \leq m^2 < 0$, then the solutions get modified drastically near the origin.

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