

Dynamical gluon effects in twist-3 generalized parton distributions of the proton

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Within the basis light-front quantization framework, we systematically investigate the subleading-twist (twist-3) generalized parton distributions (GPDs) of the proton's valence quarks beyond the Wandzura-Wilczek (WW) approximation. The twist-3 GPDs are not independent; through the equations of motion they decompose into a nongenuine contribution and a genuine twist-3 term. The latter encodes quark–gluon correlations and involves interference between the light-front Fock sectors $|qqq\rangle$ and $|qqqg\rangle$, which are typically neglected in the WW approximation. Using light-front wave functions obtained from diagonalizing the proton light-front Hamiltonian for its $|qqq\rangle$ and $|qqqg\rangle$ Fock components, we compute these GPDs via their overlap representations. To further explore their physical implications, we also evaluate several twist-3–related quantities, including the quark orbital angular momentum, the total quark spin contribution, and the quark spin-orbit correlation. Our results provide new nonperturbative input on higher-twist dynamics particularly multiparton interference effects relevant for future measurements at the EicC and the EIC.

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I. INTRODUCTION

One of the most intriguing problems in modern physics is the exploration of the proton's internal structure and its spin puzzle, both experimentally and theoretically (see Refs. [1,2] and references therein). A powerful tool for probing the proton structure is deeply virtual Compton scattering (DVCS), in which a quark absorbs a virtual photon emitted by an electron, promptly emits a real photon, and then returns to the proton. This process has been measured in numerous experiments, including JLab

CLAS [3–6], JLab Hall A [7,8], HERA H1 [9–11], HERA ZEUS [9,12], and HERA HERMES [13,14]. Further measurements are anticipated at the future Electron–Ion Collider (EIC) [15] and the Electron–Ion Collider in China (EicC) [16].

A variety of distribution functions describe the proton's internal structure at the partonic level. For hard exclusive processes (e.g., DVCS), generalized parton distributions (GPDs) are employed, providing access to the proton's three-dimensional structure. GPDs have attracted significant interest because they are related to conventional observables while encoding much richer information. In the forward limit ($\Delta \rightarrow 0$), one recovers the ordinary parton distribution functions (PDFs), which describe the longitudinal momentum distributions of quarks and gluons in the proton. Conversely, integrating GPDs over the longitudinal momentum fraction yields the electromagnetic form factors (FFs), which characterize the proton's charge and magnetization distributions.

Several theoretical studies of proton GPDs have been carried out using a wide range of QCD-inspired models, including the bag model [17,18], soliton models [19–21],

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constituent quark models (CQMs) [22–26], light-front (LF) quark–diquark models [27–31], meson-cloud models [32,33], and approaches based on AdS/QCD [34–39]. Additional progress has been made within LF quantization frameworks [40–45]. At the same time, Euclidean lattice QCD has emerged as a promising first-principles framework for computing GPDs [46–57].

However, most of the studies mentioned above focus on the leading-twist (twist-2) level. Twist-3 contributions are often considered subleading because they are suppressed by a factor of $1/Q$ in physical observables, where Q denotes the four-momentum transfer of the process. In reality, twist-3 effects are non-negligible and play an important role in revealing the proton’s internal structure [45,58–60]. First, twist-3 distributions are directly connected to quark orbital angular momentum (OAM) and quark spin-orbit correlations [61–63]. Second, they are related to the average transverse color Lorentz force experienced by quarks [64,65]. Third, twist-3 GPDs at nonzero skewness enter the twist-3 DVCS amplitude through the corresponding Compton form factors (CFFs) [66].

In this work, we employ the theoretical framework of basis light-front quantization (BLFQ) [67] and use light-front wave functions (LFWFs), obtained by diagonalizing the LFQCD Hamiltonian, to calculate all twist-3 GPDs in the zero-skewness limit. In contrast to previous studies [45], our approach extends the Fock-space truncation to include the first two sectors, $|qqq\rangle$ and $|qqqg\rangle$, thereby incorporating contributions from quark-gluon-quark correlations. This goes beyond the Wandzura-Wilczek (WW) approximation [68–71]. We further explore the connections between twist-3 GPDs, quark OAM, and quark spin-orbit correlations.

The structure of this work is as follows: Sec. II provides a brief introduction to the BLFQ framework; Sec. III outlines GPDs and presents the twist decomposition of GPDs; Sec. IV reports numerical results for twist-3 zero-skewness GPDs within the first two Fock sectors, along with their properties in the forward limit; Sec. V summarizes the work and discusses future prospects.

II. BASIS LIGHT-FRONT QUANTIZATION

As a nonperturbative approach, BLFQ is grounded in the Hamiltonian formalism and leverages the advantages of LF dynamics. Its core idea is to simultaneously obtain the mass spectrum and bound state wave functions by solving the eigenvalue problem

$$P^- P^+ |\psi\rangle = M^2 |\psi\rangle, \quad (1)$$

where P^+ and P^- denote the longitudinal momentum and the LF Hamiltonian of the system, respectively, with the ‘ $\pm$ ’ components being defined as $P^\pm = P^0 \pm P^3$. Here, M^2 is the eigenvalue corresponding to the LF energy of the system, and the eigenvector $|\psi\rangle$, representing the proton

state, is expanded as

$$|\psi_{\text{proton}}\rangle = \Phi^{qqq}|qqq\rangle + \Phi^{qqqg}|qqqg\rangle + \dots, \quad (2)$$

where Φ are the corresponding LFWFs, and the $\dots$ represents all other possible partonic combinations that can be found inside the proton.

At the initial scale, where the proton state is described by the $|qqq\rangle$ and $|qqqg\rangle$ Fock components, we employ the LF Hamiltonian $P^- = P_{\text{QCD}}^- + P_{\text{C}}^-$, where P_{QCD}^- incorporates the relevant QCD interactions and P_{C}^- models confinement [72]. In the LF gauge $A^+ = 0$, the LF QCD Hamiltonian with one dynamical gluon is given by [72]

$$\begin{aligned} P_{\text{QCD}}^- = \int d^2x^\perp dx^- \bigg\{ & \frac{1}{2} \bar{\psi} \gamma^+ \frac{m_0^2 + (i\partial^\perp)^2}{i\partial^+} \psi \\ & + \frac{1}{2} A_a^i [m_g^2 + (i\partial^\perp)^2] A_a^i + g_s \bar{\psi} \gamma_\mu T^a A_a^\mu \psi \\ & + \frac{1}{2} g_s^2 \bar{\psi} \gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ T^a \psi \bigg\}, \end{aligned} \quad (3)$$

where the first and second terms¹ account for the kinetic energies of the quark (with bare mass m_0) and gluon (with bare mass m_g), respectively, while the last two terms represent the vertex and instantaneous interactions, characterized by the strong coupling g_s . A detailed description and physical interpretation of these terms can be found in Refs. [72–74].

Confinement in the leading Fock sector is implemented following Ref. [75] as

$$P_{\text{C}}^- P^+ = \frac{\kappa^4}{2} \sum_{i \neq j} \left[\vec{r}_{ij\perp}^2 - \frac{\partial_{x_i} (x_i x_j \partial_{x_j})}{(m_i + m_j)^2} \right], \quad (4)$$

where $\vec{r}_{ij\perp} = \sqrt{x_i x_j} (\vec{r}_{i\perp} - \vec{r}_{j\perp})$ denotes the transverse separation between quarks i and j , while κ is the confinement strength. In the $|qqqg\rangle$ sector, we omit an explicit confinement term, assuming that the restricted transverse basis together with the inclusion of a massive gluon effectively captures the essential confinement behavior.

The many-particle basis includes the transverse center-of-mass (CM) motion that is entangled with the intrinsic motion. Consequently, it is essential to introduce a constraint term [67] into the effective Hamiltonian

$$H' = \lambda_L (H_{\text{cm}} - 2b^2 I), \quad (5)$$

where λ_L denotes the Lagrange multiplier, $2b^2$ represents the zero-point energy, and I is the identity operator. By choosing a sufficiently large λ_L , one can shift the excited states of the CM motion to higher energies and ensure that

¹The sign of the second term of Eq. (2) in Ref. [72] should be +.

all low-lying states correspond to the ground state of the CM motion.

We solve the LF Hamiltonian eigenvalue problem within the BLFQ framework [67], expanding the proton in a basis of longitudinal plane waves (in a box of length $2L$ with antiperiodic/periodic boundary conditions for quarks/gluons), transverse two-dimensional harmonic oscillator (2D-HO) functions $\Phi_{nm}(\vec{p}_\perp; b)$ [76], and light-cone helicity spinors. Single-particle states are labeled by $\vec{\alpha} = \{k, n, m, \lambda\}$, with k being the longitudinal quantum number (half-integer for quarks, integer for gluons, excluding zero mode); (n, m) are the 2D-HO quantum numbers, and λ the helicity; states in sectors with multiple color-singlets such as $|qqqg\rangle$ require an additional label to distinguish their color singlet configuration.

The basis is truncated by $N_{\max}$ and $K = \sum_i k_i$ with the sum running over all i -partons from 1 to N , $N_{\max}$ imposes $\sum_i (2n_i + |m_i| + 1) \leq N_{\max}$, while K fixes the longitudinal resolution, with $x_i = k_i/K$. These parameters set the IR/UV scales [76]. Diagonalization yields the proton LFWFs with helicity Λ ,

$$\Psi_{\{x_i, \vec{p}_{\perp i}, \lambda_i\}}^{N, \Lambda} = \sum_{\{n_i, m_i\}} \psi^N(\{\alpha_i\}) \prod_{i=1}^N \Phi_{n_i m_i}(\vec{p}_{\perp i}, b), \quad (6)$$

where $\psi^{N=3}$ and $\psi^{N=4}$ are the eigenvector components for the $|uud\rangle$ and $|uudg\rangle$ sectors.

The Hamiltonian parameters, summarized in Table I with $\{N_{\max}, K\} = \{9, 16.5\}$, are set to reproduce the proton mass and its electromagnetic properties [72]. Note that the vertex quark mass m_f is independent of the kinetic quark mass m_q and encodes nonperturbative effects associated with chiral symmetry breaking in light-front dynamics [77]. With the inclusion of higher Fock sectors in future BLFQ studies, m_f is expected to approach m_q , the effective gluon mass m_g to tend to zero, and b_{inst} sets the UV cutoff for the transverse momentum of the instantaneous gluon. At the model scale, the proton contains approximately 44% probability in the $|qqq\rangle$ sector and 56% in the $|qqqg\rangle$ sector. The resulting LFWFs, appropriate for a low-resolution scale of $\mu_0^2 \sim 0.24 \pm 0.01 \text{ GeV}^2$ [72], have been successfully used to describe a broad range of proton observables, including electromagnetic form factors, radii, PDFs, GPDs, TMDs, and spin and orbital angular momentum [43, 44, 72–74, 78, 79].

TABLE I. Summary of the model parameters [72]. All parameters have units of GeV except for g_s .

m_u	m_d	m_g	κ	m_f	g_s	b	b_{inst}
0.31	0.25	0.50	0.54	1.80	2.40	0.70	3.00

III. GENERALIZED PARTON DISTRIBUTION

In this section, we briefly introduce the definition of twist-3 GPDs. One of the most common definitions is formulated via a bilocal quark-quark correlator, which depends on three variables: x , ξ , and $-t$. Here, x denotes the longitudinal momentum fraction, ξ represents the skewness, and $-t$ stands for the squared momentum transfer. The correlator takes the form

$$F_{\Lambda' \Lambda}^{[\Gamma]}(x, \xi, -t) = \frac{1}{2} \int \frac{dy^-}{4\pi} e^{ixP \cdot y} \langle P', \Lambda' | \bar{\Psi} \left(-\frac{y}{2} \right) \times \mathcal{W} \left(-\frac{y}{2}, \frac{y}{2} \right) \Gamma \Psi \left(\frac{y}{2} \right) | P, \Lambda \rangle_{|y^+=0, \vec{y}^\perp=0}, \quad (7)$$

where $|P, \Lambda\rangle$ describes the momentum and helicity of the initial proton state, while the primed bra vector ($\langle P', \Lambda' |$) corresponds to the final proton state. The Wilson line is defined as $\mathcal{W}(y_2, y_1) \equiv P \exp(ig_s \int_{y_1}^{y_2} dz^\mu A_\mu(z))$; under the light-cone gauge ($A^+ = 0$), this Wilson line reduces to unity. Γ denotes one element of a complete set of Dirac matrices, and $\Psi(x)$ represents the quark field. As we will see, not all components of $\Psi(x)$ are independent dynamical degrees of freedom.

By applying a projection operator $\mathcal{P}^\pm$, the quark field can be decomposed into dynamical and nondynamical components [80], i.e., $\Psi = \Psi^+ + \Psi^-$. Here, $\Psi^\pm = \mathcal{P}^\pm \Psi$ with $\mathcal{P}^\pm = \gamma^\mp \gamma^\pm / 4$. The dynamical component Ψ^+ is expanded in the form of a free fermion field, while the nondynamical component Ψ^- is constrained by the Dirac equation,

$$\Psi^- = \frac{\gamma^+}{2i\partial^+} (m_q - i\gamma^\perp \partial_\perp + g_s \gamma^\perp A_\perp) \Psi^+, \quad (8)$$

where m_q is the quark mass, g_s is the coupling constant, and A_μ is the gluon field. Herein, summation over all repeated indices is implied by default.

For the DVCS process ($ep \rightarrow e'p'\gamma$) associated with GPDs, we adopt a symmetric frame for the initial and final proton momenta. In this frame, the four-momenta of the incoming and outgoing protons are given by

$$P = \left((1 + \xi) \bar{P}^+, -\frac{\vec{\Delta}^\perp}{2}, \frac{M^2 + (\vec{\Delta}^\perp)^2/4}{(1 + \xi) \bar{P}^+} \right), \quad (9)$$

$$P' = \left((1 - \xi) \bar{P}^+, \frac{\vec{\Delta}^\perp}{2}, \frac{M^2 + (\vec{\Delta}^\perp)^2/4}{(1 - \xi) \bar{P}^+} \right), \quad (10)$$

$$\Delta = P' - P = \left(-2\xi \bar{P}^+, \vec{\Delta}^\perp, \frac{t - (\vec{\Delta}^\perp)^2}{2\xi \bar{P}^+} \right), \quad (11)$$

where $\bar{P} = (P + P')/2$ is the average momentum of the proton, Δ is the momentum transfer, $\xi = -\Delta^+/(2\bar{P}^+)$ is

the skewness, M is the proton mass, and $t \equiv -\Delta^2$ is the squared momentum transfer.

Based on the definitions of twist and Dirac matrices, the physical observables for the twist-3 case correspond to the interference terms $\bar{\Psi}^+\Gamma\Psi^-$ and $\bar{\Psi}^-\Gamma\Psi^+$. By contrast, the twist-2 case is characterized by $\bar{\Psi}^+\Gamma\Psi^+$, and the twist-4 case by $\bar{\Psi}^-\Gamma\Psi^-$.

Using this general expression, the GPDs can be computed through the parametrization of Ref. [81]. Several parametrization schemes for GPDs have been developed [82–87], among which the scheme of Ref. [88] is the most extensively studied and is directly related to the present work [82] (see Appendix. B). The full overlap expressions for all twist-3 GPDs are provided in Appendix A. For the corresponding overlap formulas truncated to the lowest Fock sector, we refer the reader to our earlier study [45].

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we present numerical results for zero-skewness twist-3 GPDs calculated via the BLFQ method. The key distinction between this work and previous studies [45] lies in the inclusion of two Fock sectors, $|qqq\rangle + |qqqg\rangle$, which incorporates gluon contributions—specifically, the q - g - q interaction. We further analyze the properties of these twist-3 GPDs, including sum rules, spin-orbit correlations, and OAM. Note that properties such as time-reversal symmetry have been discussed in prior work and are not revisited herein.

The inclusion of two Fock sectors substantially increases the complexity of the overlap representation, primarily due to the emergence of the q - g - q correlator. Each twist-3 GPD consists of three distinct components: the qqq component, which arises from the overlap between the states $\langle qqg|$ and $|qqg\rangle$; the $qqqg$ component, which originates from the overlap between $\langle qqgq|$ and $|qqqg\rangle$; and the “genuine” (abbreviated as “*gen*”) component, which accounts for cross-overlap contributions, i.e., between $\langle qqg|$ and $|qqqg\rangle$, and between $\langle qqgq|$ and $|qqg\rangle$.

A. Twist-3 GPDs

For all results presented here, the rotational symmetry between γ_1 and γ_2 is preserved—not only for each individual component but also for their sum. For simplicity, γ_1 is used for calculations. Given that $\xi = 0$ is fixed in this work, the GPDs are presented as functions of x and $-t$. In the zero-skewness limit, eight of the twist-3 GPDs— H_{2T} , E_{2T} , $\tilde{H}_{2T}$, $\tilde{E}_{2T}$, $\tilde{H}_2$, H'_2 , E'_2 , and $\tilde{E}'_2$ —are found to be consistent with zero within our numerical uncertainty.

Using the overlap representation and the LFWFs obtained by solving the BLFQ framework, we calculate all the twist-3 GPDs. For brevity, we present only the four most extensively studied chiral-even GPDs: these are parametrized from the vector current ($\gamma^\perp$) and pseudovector

current ($\gamma^\perp\gamma_5$), namely $\tilde{E}_{2T}$ (from vector current) and $H'_{2T}, E'_{2T}, \tilde{H}'_{2T}$ (from pseudovector current). These GPDs have been shown to relate to G_i and G'_i (where $i = 1, 4$) [82]. Each two-dimensional subplot distinguishes contributions from the qqq sector (blue solid circles), $qqqg$ sector (red solid squares), genuine (*gen*) contribution (purple solid diamonds), and total GPD (black solid triangles); since the magnitudes of these curves are excessively large, we multiply by x to suppress their values.

In Fig. 1, we present the three-dimensional structure of the twist-3 GPDs in the zero-skewness limit. The results are plotted up to $-t = 1.0 \text{ GeV}^2$, which focuses on the region with meaningful structural features. Beyond the upper limit of the depicted range, the magnitude maintains the depicted trends. That is, for $-t > 1.0 \text{ GeV}^2$ (not shown in the figures), the relevant quantities exhibit a featureless falloff to zero.

For $x\tilde{E}_{2T}$, the region of small x and small $-t$ shows a modest peak with a sign opposite to that in the rest of the kinematic domain; moreover, the u and d quark contributions carry opposite signs throughout the full region. In the case of xH'_{2T} , the distribution is smooth across all kinematics. This behavior arises because its overlap representation does not contain Δ in the denominator, a feature that also makes it the only GPD that remains finite in the $\Delta \rightarrow 0$ limit [corresponding to the twist-3 parton distribution function $g_T(x)$].

For $x\tilde{E}'_{2T}$ and $x\tilde{H}'_{2T}$, the two distributions exhibit similar shapes but opposite signs—both between u and d quarks and between the two GPDs for a given quark flavor. Both GPDs also display fluctuations in the small x and small $-t$ region, with signs consistent with their behavior in the rest of the domain. The peak observed at small x and small $-t$ for all of the above GPDs originate from the inclusion of the nonvalence Fock sector with a dynamical gluon. Furthermore, owing to discretization and truncation effects in the longitudinal direction, we are not able to obtain reliable distributions at even smaller values of x .

In Fig. 2, we fix $-t = 0.25 \text{ GeV}^2$ and plot the four GPDs as functions of x . All GPDs show the same trend, with one prominent peak. The *gen* contribution is much smaller than the other components but remains non-negligible. Most GPDs fluctuate in the small x region: the $qqqg$ sector contribution is particularly notable here, dominating in the small x region and nearly equal to the total GPD (except for H'_{2T}). This behavior can be attributed to their overlap expressions, where division by Δ amplifies contributions at small $-t$. For the remaining x range, the qqq and $qqqg$ sector contributions are nearly equal, while the *gen* contribution is minimal. The peaks of all GPDs cluster in the range $0.2 < x < 0.4$, and all tend to zero in the large x regime.

In Fig. 3, we fix $x = 2.5/16.5$ and plot the four GPDs as functions of $-t$. In subplots (a) and (b), the qqq sector contribution is slightly larger than that of the $qqqg$ sector,

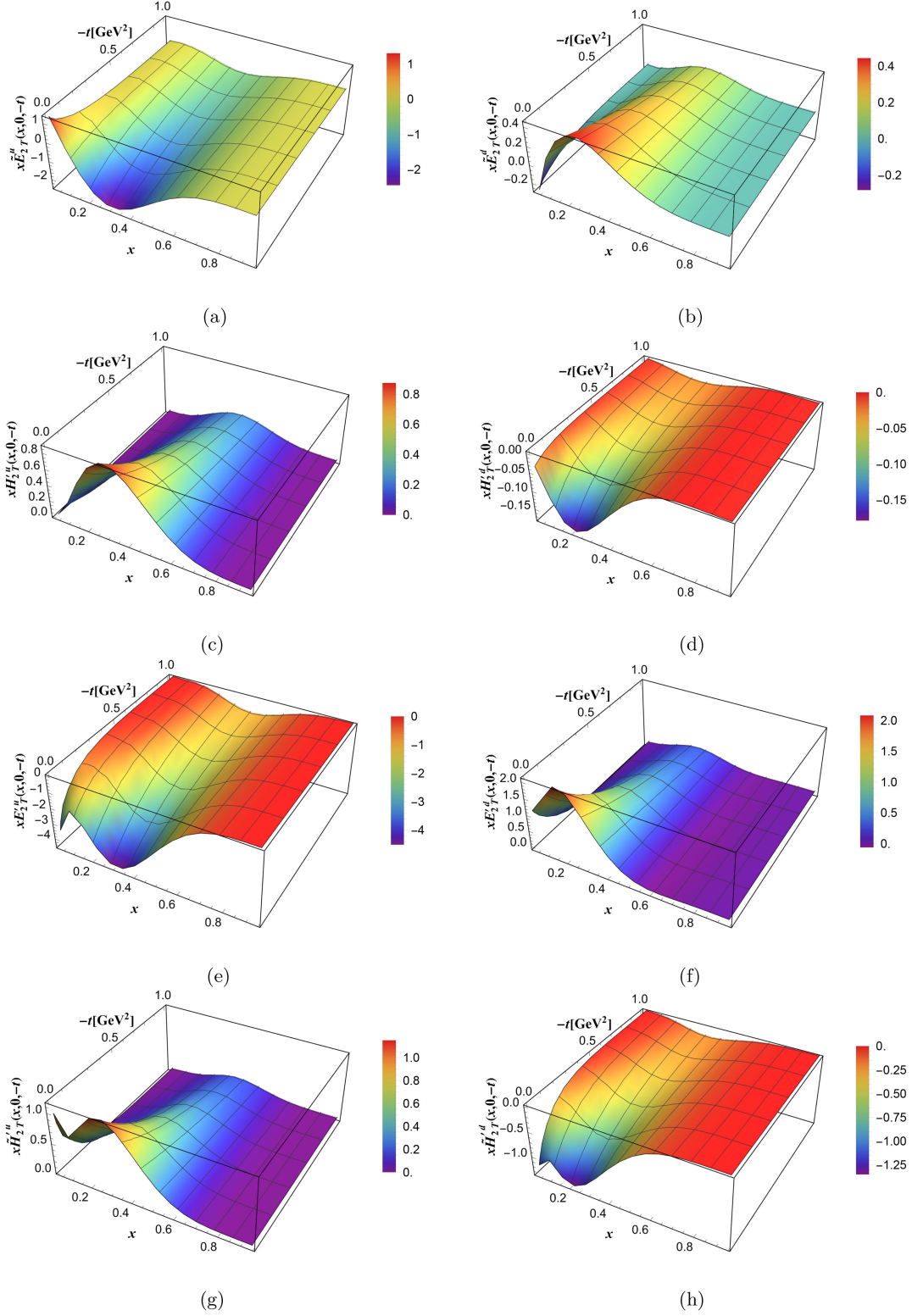


FIG. 1. 3D Plots of the twist-3 quark GPDs $x\tilde{E}_{2T}$, xH'_{2T} , xE'_{2T} , and $x\tilde{H}'_{2T}$ in the proton, evaluated with $N_{\max} = 9$ and $K = 16.5$. The left column panels $\{(a), (c), (e), (g)\}$ correspond to the u quark and the right column panels $\{(b), (d), (f), (h)\}$ correspond to the d quark on the flavor level. The flavor level distributions are given by $X^u_{\text{flavor}} = 2X^u_{\text{quark}}$ and $X^d_{\text{flavor}} = X^d_{\text{quark}}$, where X stands for all the GPDs.

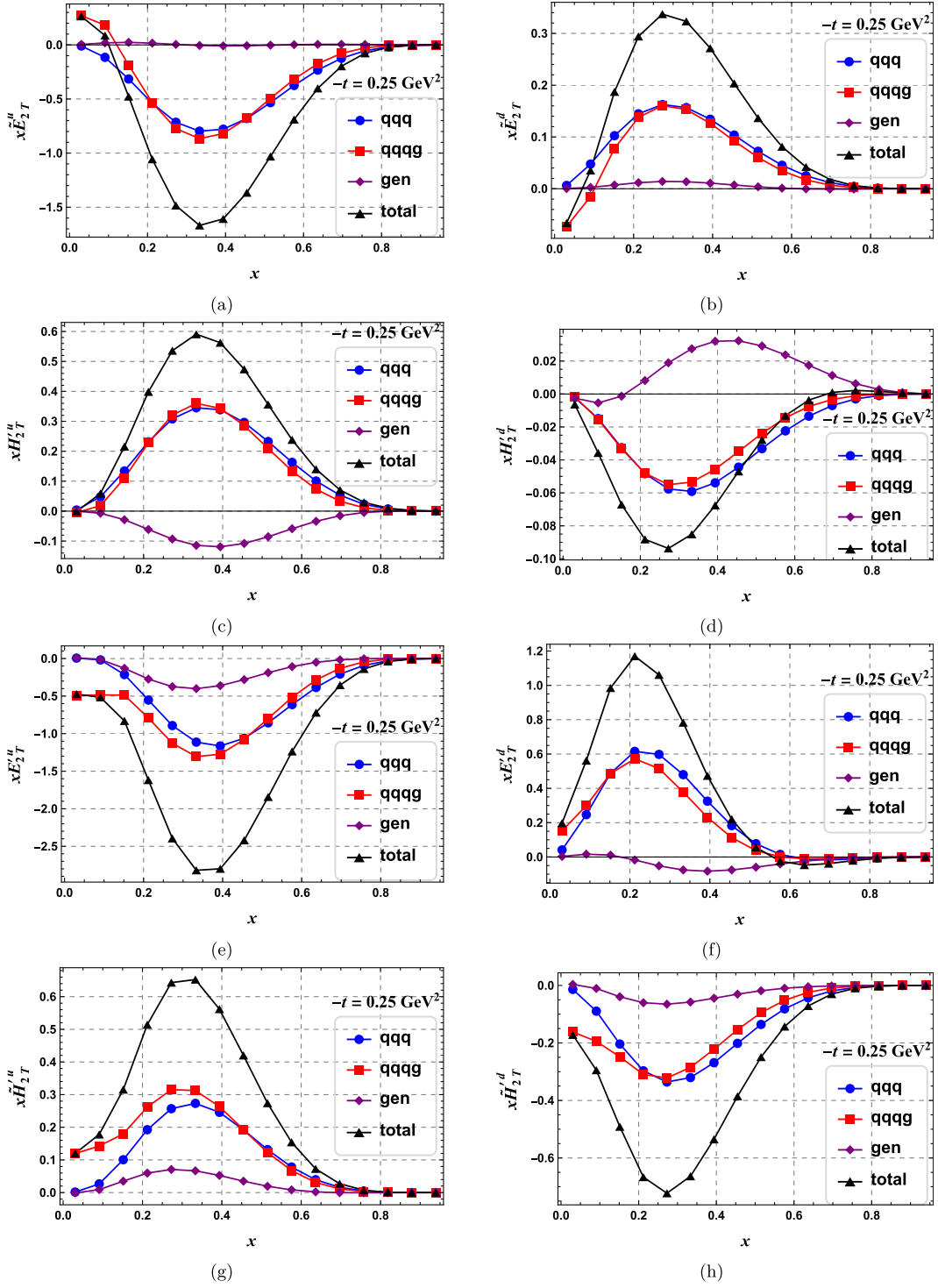


FIG. 2. 2D Plots of the twist-3 quark GPDs $x\tilde{E}_{2T}$, xH'_{2T} , xE'_{2T} , and $x\tilde{H}'_{2T}$ in the proton at fixed $-t = 0.25 \text{ GeV}^2$, evaluated with $N_{\text{max}} = 9$ and $K = 16.5$. The left column panels {(a), (c), (e), (g)} correspond to the u quark and the right column panels {(b), (d), (f), (h)} correspond to the d quark on the flavor level. The flavor level distributions are given by $X_{\text{flavor}}^u = 2X_{\text{quark}}^u$ and $X_{\text{flavor}}^d = X_{\text{quark}}^d$, where X stands for all the GPDs.

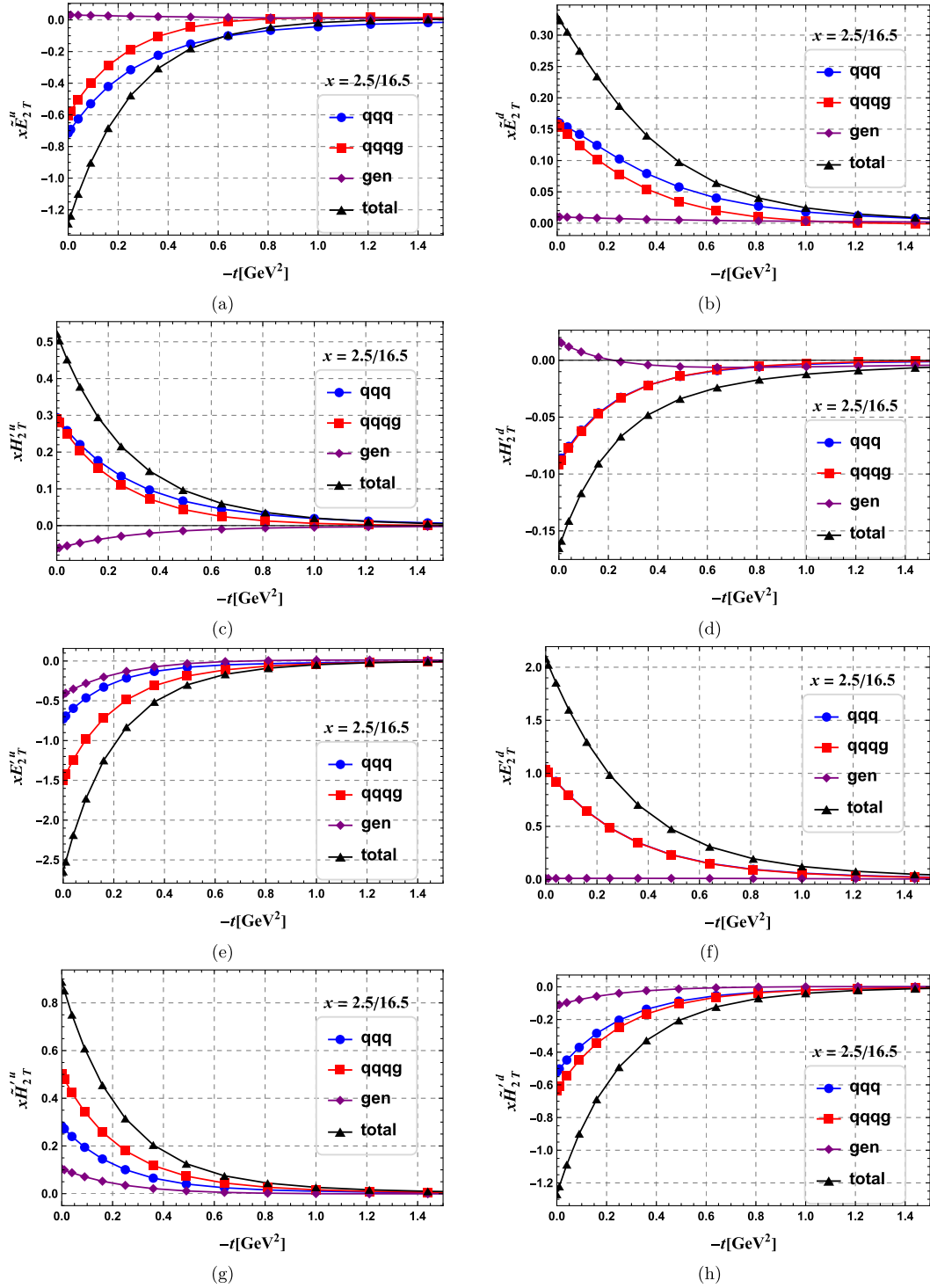


FIG. 3. 2D Plots of the twist-3 quark GPDs $x\tilde{E}_{2T}^u$, xH_{2T}^u , xE_{2T}^d , and $x\tilde{H}_{2T}^d$ in the proton at fixed $x = 2.5/16.5$, evaluated with $N_{\max} = 9$ and $K = 16.5$. The left column panels $\{(a), (c), (e), (g)\}$ correspond to the u quark and the right column panels $\{(b), (d), (f), (h)\}$ correspond to the d quark on the flavor level. The flavor level distributions are given by $X_{\text{flavor}}^u = 2X_{\text{quark}}^u$ and $X_{\text{flavor}}^d = X_{\text{quark}}^d$, where X stands for all the GPDs.

while the *gen* contribution remains small. In subplots (c) and (d), the *qqq* and *qqqg* sector contributions are comparable. In subplot (e), the *qqqg* sector contribution is nearly twice that of the *qqq* sector, while in subplot (f), the two are nearly equal. In subplots (g) and (h), the *qqqg* sector contribution is larger than that of the *qqq* sector. All curves show a decreasing trend as $-t$ increases.

Note that the sign reversal between the u and d distributions naturally reflects the spin-flavor structure of the nucleon in our model, analogous to the expectations from SU(6) symmetry and our previous light-front studies of quark helicities [45,72,89]. Twist-3 GPDs, however, lack a simple parton interpretation, and the origin of their sign reversal is not yet fully understood. It can nevertheless be interpreted as arising from the interplay between quark OAM and spin correlations.

B. Spin-orbit correlation

The parity-odd partner of the quark energy-momentum tensor operator, $\hat{T}_{q5}^{\mu\nu}$, is closely related to quark spin-orbit correlations and has been investigated in Ref. [90]. This operator is defined as

$$\hat{T}_{q5}^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^\mu \gamma_5 i \overleftrightarrow{D}^\nu \psi, \quad (12)$$

where $\overleftrightarrow{D}^\nu = \vec{D}^\nu - \vec{D}^\nu$ denotes the symmetric covariant derivative (with $D^\nu = \partial^\nu - igA^\nu$ as the covariant derivative). To characterize the matrix elements of $\hat{T}_{q5}^{\mu\nu}$ in nucleon states, it is parametrized in terms of five FFs

$$\langle p', s' | \hat{T}_{q5}^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \Gamma_{q5}^{\mu\nu} u(p, s), \quad (13)$$

with

$$\begin{aligned} \Gamma_{q5}^{\mu\nu} = & \frac{\bar{P}^{\{\mu} \gamma^{\nu\}} \gamma_5}{2} \tilde{A}_q(-t) + \frac{\bar{P}^{\{\mu} \Delta^{\nu\}} \gamma_5}{4M} \tilde{B}_q(-t) \\ & + \frac{\bar{P}^{\{\mu} \gamma^{\nu\}} \gamma_5}{2} \tilde{C}_q(-t) + \frac{\bar{P}^{\{\mu} \Delta^{\nu\}} \gamma_5}{4M} \tilde{D}_q(-t) \\ & + Mi \sigma^{\mu\nu} \gamma_5 \tilde{F}_q(-t). \end{aligned} \quad (14)$$

The FFs $\tilde{A}_q(-t)$ and $\tilde{B}_q(-t)$ are associated with the twist-2 contribution obtained by selecting the $\mu\nu = ++$ component of the energy-momentum tensor. In this case, one finds

$$\int dx x \tilde{H}_q(x, \xi, -t) = \tilde{A}_q(-t), \quad (15)$$

$$\int dx x \tilde{E}_q(x, \xi, -t) = \tilde{B}_q(-t). \quad (16)$$

Taking the twist-3 component $\mu\nu = j+$ with $j = 1, 2$, the following relations are obtained [90]

$$\int dx x \tilde{G}_1^q(x, \xi, -t) = -\frac{1}{2} (\tilde{B}_q(-t) + \tilde{D}_q(-t)), \quad (17)$$

$$\begin{aligned} \int dx x \tilde{G}_2^q(x, \xi, -t) = & -\frac{1}{2} (\tilde{A}_q(-t) + \tilde{C}_q(-t)) \\ & + (1 - \xi^2) \tilde{F}_q(-t), \end{aligned} \quad (18)$$

$$\int dx x \tilde{G}_3^q(x, \xi, -t) = -\frac{\xi}{2} \tilde{F}_q(-t), \quad (19)$$

$$\int dx x \tilde{G}_4^q(x, \xi, -t) = -\frac{1}{2} \tilde{F}_q(-t), \quad (20)$$

where $\tilde{G}_i$ are related to H'_{2T} , E'_{2T} , $\tilde{H}'_{2T}$, and $\tilde{E}'_{2T}$ [45,82] (see Appendix B).

We present the FFs $\tilde{A}_q(-t)$, $\tilde{C}_q(-t)$, and $\tilde{F}_q(-t)$ in Fig. 4. We observe that $\tilde{A}_q(-t)$ and $\tilde{C}_q(-t)$ exhibit similar shapes but have opposite signs—both between u - and d -quarks and between the two form factors for a given quark flavor. Moreover, the u -quark contribution is significantly larger than that of the d -quark. In contrast, $\tilde{F}_q(-t)$ is negative for both flavors and displays a similar overall behavior.

Since we focus on extracting the GPDs at $\xi = 0$, the distributions $\tilde{E}$ and $\tilde{G}_1$ lie beyond the scope of this work. Consequently, the corresponding form factors $\tilde{B}_q(-t)$ and $\tilde{D}_q(-t)$ are not determined in the present analysis. For clarity, these form factors could, in principle, be obtained by performing calculations at nonzero ξ , where their contributions can be isolated through the full GPD parametrization.

Several well-established relations involving twist-3 GPDs in the forward limit are worth highlighting. The quark OAM in the nucleon is defined, in the limit $\Delta \rightarrow 0$, as [88]

$$L_z^q = -\lim_{\Delta \rightarrow 0} \int dx x G_2^q(x, 0, \Delta_\perp^2), \quad (21)$$

where the explicit expression for G_2^q is provided in Appendix B. This physical quantity can also be calculated directly from twist-2 GPDs, as defined in Refs. [61–63]. Within our BLFQ approach at the model scale, we obtain $L_z^u = 0.107$ ($L_z^d = 0.0752$) from twist-3 distributions and $L_z^u = 0.0817$ ($L_z^d = 0.0264$) from twist-2 GPDs. This difference may arise from the violation of the Burkhardt-Cottingham (BC) sum rule [91] [$\int dx g_1(x) = \int dx g_T(x)$] in our results. A similar deviation has also been reported in Ref. [58]. In our calculation, the BC sum rule may appear violated due to the Fock sector truncation and the use of separate confining potentials in the longitudinal and transverse directions, which explicitly break rotational invariance. The BC sum rule reflects the equivalence of different spin projections and, more generally, the rotational

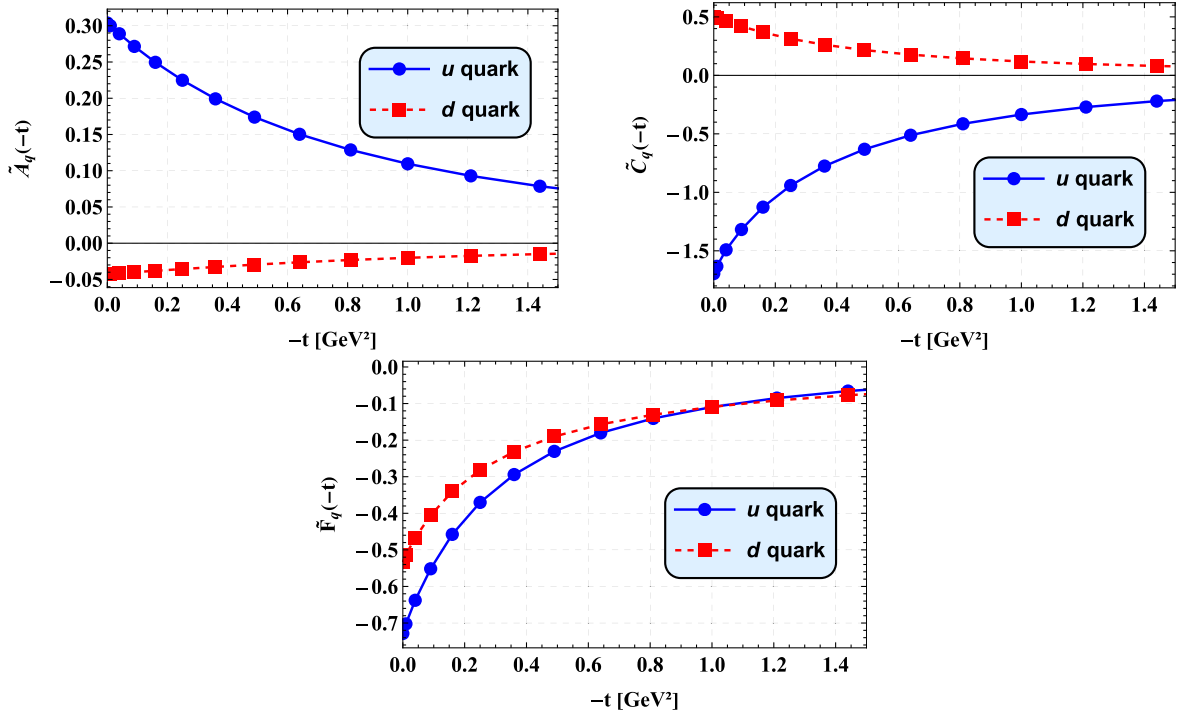


FIG. 4. Plots of the twist-3 FFs $\tilde{A}_q$, $\tilde{C}_q$ and $\tilde{F}_q$ in the proton, evaluated with $N_{\max} = 9$ and $K = 16.5$. The blue circle solid line corresponds to the u quark and the red square dotted line corresponds to the d quark on the quark level.

symmetry of the nucleon spin. As higher Fock sectors are included and the essential QCD interactions are systematically incorporated, rotational invariance is expected to be restored, and the BC sum rule should hold more accurately, fully capturing the genuine twist-3 contribution.

The total quark spin contribution J_z^q can also be related to twist-3 GPDs via [83]

$$J_z^q = \int dx \left(x G_{q,3}(x, 0, 0) - \frac{1}{2} g_1^q(x) \right), \quad (22)$$

where g_1^q is the twist-2 helicity PDF, and $G_{q,3}$ (defined in Ref. [83]) can be converted to $G_{q,3} = -\tilde{E}_{2T}^q$. Using this relation, at the model scale, we find $J_z^u = 0.443$ ($J_z^d = -0.0801$) from twist-3 distributions and $J_z^u = 0.468$ ($J_z^d = -0.0312$) from twist-2 distributions.

For the quark spin-orbit correlation, it is expressed as [90]

TABLE II. Comparison of C_z^u and C_z^d values from different theoretical approaches.

Theoretical approach	C_z^u	C_z^d
BLFQ (model scale, this work)	-0.761	-0.515
NQM, LFCQM, LF χ QSM [92]	≈ -0.8	≈ -0.55
LSS ($\mu^2 = 1 \text{ GeV}^2$) [93]	≈ -0.9	≈ -0.53
Spectator Diquark Model [58]	-0.775	-0.586

$$C_z^q = - \int dx x (\tilde{G}_2^q(x, 0, 0) + 2\tilde{G}_4^q(x, 0, 0)). \quad (23)$$

We list our results in Table II for comparison with other theoretical approaches, including the naive quark model (NQM), the light-front constituent quark model (LFCQM), and the light-front chiral quark-soliton model (LF χ QSM) [92], the Leader-Sidorov-Stamenov (LSS) analysis [93], and the spectator diquark model [58]. The negative signs of the results indicate that the quark spin and Ji OAM are anticorrelated on average [90].

V. CONCLUSION

In this work, we have computed all twist-3 GPDs of the proton in the zero-skewness limit within the BLFQ framework, presenting the four nonvanishing chiral-even distributions for both u and d quarks. Building upon our previous study, we have extended the Fock-space truncation to include a dynamical gluon, thereby incorporating contributions from the $q-g-q$ correlator—i.e., the genuine twist-3 component. The light-front wave functions used in the overlap representation are obtained by diagonalizing the proton light-front Hamiltonian constructed for the $|qqq\rangle$ and $|qqqg\rangle$ Fock sectors, supplemented with a three-dimensional confinement potential.

Our calculations indicate that the genuine twist-3 contribution, while relatively small, is nevertheless

non-negligible; in certain cases, it even induces a sign opposite to that of the nongenuine component. Qualitatively, our results show good overall consistency with those of Ref. [59], except for the $x\tilde{E}_{2T}^d$ distribution and in the region of small x and small $-t$. In addition, we have examined several key twist-3-related quantities—including the quark OAM, the total quark contribution to the nucleon spin, and the quark spin-orbit correlation—and many of these are in reasonable agreement with predictions from other model studies [58,90,93,94].

In future work, we plan to extend our analysis to twist-3 GPDs at nonzero skewness and, more broadly, to the twist-3 distributions of gluons and sea quarks. The nonzero-skewness sector is particularly important, as these twist-3 GPDs enter the twist-3 DVCS amplitudes [66] and will be probed in forthcoming measurements at the EIC and EicC [95]. With continued development of the BLFQ framework, we expect to improve the precision of our calculations, address remaining discrepancies in sum-rule relations, and provide more reliable predictions relevant to understanding the proton spin puzzle.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: OVERLAP REPRESENTATIONS FOR TWIST-3 GPDs

We will present the overlap representations of all zero-skewness twist-3 GPDs in the following. For convenience, the following notations are adopted,

$$[dx]_n = \prod_{i=1}^n \frac{dx_i d^2 \vec{k}_i}{(16\pi^3)^n} 16\pi^3 \delta\left(1 - \sum x_i\right) \times \delta^2\left(\sum \vec{k}_i\right) \delta(x - x_1), \quad (\text{A1})$$

where the subscript 1 denotes the struck quark, and $[\Gamma] = \bar{u}(p', \lambda') \Gamma u(p, \lambda)$ encapsulates the helicity combinations of the struck quark. Additionally, ψ_n^Λ signifies the LFWF $\psi_{i=1,\dots,n}^\Lambda(x_i, p_i, \lambda_i)$, with Λ representing the proton helicity and λ_i represents the quark helicity. The constraint for spectators $\delta_{\lambda_2}^{\lambda'_2} \delta_{\lambda_3}^{\lambda'_3} \delta_{\lambda_4}^{\lambda'_4}$ is implicitly assumed. In the overlap expressions, the symbol Δ refers to the two-dimensional complex representation, i.e., $\Delta = \Delta_1 + i\Delta_2$.

Each overlap representation is divided into three parts. For the $|qqq\rangle$ sector, we have

$$\mathcal{F}_{\Gamma, \Lambda' \Lambda}^{qqq} = \int [dx]_3 \sum_{\lambda'_1, \lambda_1} \left(\frac{p_{1\perp}}{x_1} [\Gamma \gamma^+ \gamma^\perp] + \frac{p'_{1\perp}}{x_1} [\gamma^\perp \gamma^+ \Gamma] + \frac{m_q}{x_1} [\Gamma \gamma^+] + \frac{m_q}{x_1} [\gamma^+ \Gamma] \right) \psi_3^{\Lambda'} \star \psi_3^\Lambda. \quad (\text{A2})$$

A similar expression applies to the $|qqqq\rangle$ sector,

$$\mathcal{F}_{\Gamma, \Lambda' \Lambda}^{qqqq} = \int [dx]_4 \sum_{\lambda'_1, \lambda_1} \left(\frac{p_{1\perp}}{x_1} [\Gamma \gamma^+ \gamma^\perp] + \frac{p'_{1\perp}}{x_1} [\gamma^\perp \gamma^+ \Gamma] + \frac{m_q}{x_1} [\Gamma \gamma^+] + \frac{m_q}{x_1} [\gamma^+ \Gamma] \right) \psi_4^{\Lambda'} \star \psi_4^\Lambda. \quad (\text{A3})$$

Regarding the interference term *gen*, which incorporates the $q - g - q$ interaction, we have

$$\mathcal{F}_{\Gamma, \Lambda' \Lambda}^{gen} = \int [dx]_4 \sum_{\lambda'_1, \lambda_1, \lambda_g} \frac{g_s}{\sqrt{x_g}} \left\{ \left(\frac{\epsilon^j}{x'_1} [\Gamma \gamma^+ \gamma^j] + \frac{\epsilon^j}{x_1} [\gamma^j \gamma^+ \Gamma] \right) \psi_3^{\Lambda'} \star \psi_4^\Lambda + \left(\frac{\epsilon^{j\star}}{x_1} [\gamma^j \gamma^+ \Gamma] + \frac{\epsilon^{j\star}}{x'_1} [\Gamma \gamma^+ \gamma^j] \right) \psi_4^{\Lambda'} \star \psi_3^\Lambda \right\}, \quad (\text{A4})$$

where ϵ^μ is the gluon polarization vector.

The total contribution is the sum of these three components,

$$\mathcal{F}_{\Gamma, \Lambda' \Lambda}^{\text{total}} = \mathcal{F}_{\Gamma, \Lambda' \Lambda}^{qqq} + \mathcal{F}_{\Gamma, \Lambda' \Lambda}^{qqqg} + \mathcal{F}_{\Gamma, \Lambda' \Lambda}^{\text{gen}}. \quad (\text{A5})$$

Using the expressions derived above, we can explicitly formulate the overlap representations for all twist-3 GPDs

$$H_2 = \frac{P^+}{M} \mathcal{F}_{1, \uparrow \uparrow}^{\text{total}}, \quad (\text{A6})$$

$$E_2 = \frac{2P^+}{\Delta^\star} \mathcal{F}_{1, \uparrow \downarrow}^{\text{total}}, \quad (\text{A7})$$

$$\tilde{H}_2 = \frac{P^+}{M} \mathcal{F}_{\gamma_5, \uparrow \uparrow}^{\text{total}}, \quad (\text{A8})$$

$$\tilde{E}_2 = -\frac{2P^+}{\Delta^\star} \mathcal{F}_{\gamma_5, \uparrow \downarrow}^{\text{total}}, \quad (\text{A9})$$

$$H_{2T}^j = \frac{(-i)^{(j-1)} P^+}{M i (\Delta + (-)^j \Delta^\star)} \left(\Delta \mathcal{F}_{\gamma^j, \uparrow \downarrow}^{\text{total}} + \Delta^\star \mathcal{F}_{\gamma^j, \downarrow \uparrow}^{\text{total}} \right), \quad (\text{A10})$$

$$E_{2T}^j = \frac{P^+}{\Delta_j} \left(\mathcal{F}_{\gamma^j, \uparrow \uparrow}^{\text{total}} + \mathcal{F}_{\gamma^j, \downarrow \downarrow}^{\text{total}} \right) + \frac{2MP^+}{i^j \Delta_1 \Delta_2} \left(\mathcal{F}_{\gamma^j, \uparrow \downarrow}^{\text{total}} - (-)^j \mathcal{F}_{\gamma^j, \downarrow \uparrow}^{\text{total}} \right), \quad (\text{A11})$$

$$\tilde{H}_{2T}^j = -\frac{MP^+}{i^j \Delta_1 \Delta_2} \left(\mathcal{F}_{\gamma^j, \uparrow \downarrow}^{\text{total}} - (-)^j \mathcal{F}_{\gamma^j, \downarrow \uparrow}^{\text{total}} \right), \quad (\text{A12})$$

$$\tilde{E}_{2T}^j = \frac{2i^{(j-1)} P^+}{\Delta + (-)^j \Delta^\star} \left(\mathcal{F}_{\gamma^j, \uparrow \uparrow}^{\text{total}} - \mathcal{F}_{\gamma^j, \downarrow \downarrow}^{\text{total}} \right), \quad (\text{A13})$$

$$H_{2T}^j = \frac{P^+}{2M \Delta_j} \left(\Delta \mathcal{F}_{\gamma^j \gamma_5, \uparrow \downarrow}^{\text{total}} + \Delta^\star \mathcal{F}_{\gamma^j \gamma_5, \downarrow \uparrow}^{\text{total}} \right), \quad (\text{A14})$$

$$E_{2T}^j = \frac{2i^{(j-1)} P^+}{\Delta + (-)^j \Delta^\star} \left(\mathcal{F}_{\gamma^j \gamma_5, \uparrow \uparrow}^{\text{total}} + \mathcal{F}_{\gamma^j \gamma_5, \downarrow \downarrow}^{\text{total}} \right) + \frac{2MP^+}{i^{(2-j)} \Delta_1 \Delta_2} \left((-)^j \mathcal{F}_{\gamma^j \gamma_5, \uparrow \downarrow}^{\text{total}} + \mathcal{F}_{\gamma^j \gamma_5, \downarrow \uparrow}^{\text{total}} \right), \quad (\text{A15})$$

$$\tilde{H}_{2T}^j = \frac{(-)^{(j-1)} MP^+}{i^{(2-j)} \Delta_1 \Delta_2} \left(\mathcal{F}_{\gamma^j \gamma_5, \uparrow \downarrow}^{\text{total}} - (-)^j \mathcal{F}_{\gamma^j \gamma_5, \downarrow \uparrow}^{\text{total}} \right), \quad (\text{A16})$$

$$\tilde{E}_{2T}^j = \frac{P^+}{\Delta_j} \left(\mathcal{F}_{\gamma^j \gamma_5, \uparrow \uparrow}^{\text{total}} - \mathcal{F}_{\gamma^j \gamma_5, \downarrow \downarrow}^{\text{total}} \right), \quad (\text{A17})$$

$$H'_2 = \frac{iP^+}{M} \mathcal{F}_{i\sigma^{12} \gamma_5, \uparrow \uparrow}^{\text{total}}, \quad (\text{A18})$$

$$E'_2 = \frac{2iP^+}{\Delta^\star} \mathcal{F}_{i\sigma^{12} \gamma_5, \uparrow \downarrow}^{\text{total}}, \quad (\text{A19})$$

$$\tilde{H}'_2 = \frac{P^+}{M} \mathcal{F}_{i\sigma^{+-} \gamma_5, \uparrow \uparrow}^{\text{total}}, \quad (\text{A20})$$

$$\tilde{E}'_2 = -\frac{2P^+}{\Delta^\star} \mathcal{F}_{i\sigma^{+-} \gamma_5, \uparrow \downarrow}^{\text{total}}, \quad (\text{A21})$$

where M denotes the proton mass. All the numerical results presented in this work are calculated using these overlap representations, and the LFWF obtained from the BLFQ framework.

APPENDIX B: RELATIONS BETWEEN DIFFERENT GPDS

The relations between GPDs defined in Ref. [82] and in [81] are

$$H_{2T} = 2\xi G_4, \quad (\text{B1})$$

$$E_{2T} = 2(G_3 - \xi G_4), \quad (\text{B2})$$

$$\tilde{H}_{2T} = \frac{1}{2} G_1, \quad (\text{B3})$$

$$\tilde{E}_{2T} = -(H + E + G_2) + 2(\xi G_3 - G_4), \quad (\text{B4})$$

$$H'_{2T} = \frac{-t}{4M^2} (\tilde{E} + \tilde{G}_1) + (\tilde{H} + \tilde{G}_2) - 2\xi \tilde{G}_3, \quad (\text{B5})$$

$$E'_{2T} = -(\tilde{E} + \tilde{G}_1) - (\tilde{H} + \tilde{G}_2) + 2(\xi \tilde{G}_3 - \tilde{G}_4), \quad (\text{B6})$$

$$\tilde{H}'_{2T} = \frac{1}{2} (\tilde{E} + \tilde{G}_1), \quad (\text{B7})$$

$$\tilde{E}'_{2T} = 2(\tilde{G}_3 - \xi \tilde{G}_4). \quad (\text{B8})$$

The inversion of above equations is

$$G_1 = 2\tilde{H}_{2T}, \quad (\text{B9})$$

$$G_2 = -(H + E) - \frac{1}{\xi} (1 - \xi^2) H_{2T} + \xi E_{2T} - \tilde{E}_{2T}, \quad (\text{B10})$$

$$G_3 = \frac{1}{2} (H_{2T} + E_{2T}), \quad (\text{B11})$$

$$G_4 = \frac{1}{2\xi} H_{2T}, \quad (\text{B12})$$

$$\tilde{G}_1 = -\tilde{E} + 2\tilde{H}'_{2T}, \quad (\text{B13})$$

$$\tilde{G}_2 = -\tilde{H} + (1 - \xi^2) H'_{2T} - \xi^2 E'_{2T} - \frac{\Delta_\perp^2}{2M^2} \tilde{H}'_{2T} + \xi \tilde{E}'_{2T}, \quad (\text{B14})$$

$$\tilde{G}_3 = -\frac{\xi}{2} (H'_{2T} + E'_{2T}) - \frac{\xi \bar{M}^2}{M^2} \tilde{H}'_{2T} + \frac{1}{2} \tilde{E}'_{2T}, \quad (\text{B15})$$

$$\tilde{G}_4 = -\frac{1}{2}(H'_{2T} + E'_{2T}) - \frac{\bar{M}^2}{M^2}\tilde{H}'_{2T}, \quad (\text{B16})$$

where $\bar{M}^2 = M^2 + t/4$.

The relations between GPDs defined in Ref. [83] and this work are

$$E_{2T}(x, \xi, -t) = 2G_{q,2}(x, \xi, -t), \quad (\text{B17})$$

$$H_{2T}(x, \xi, -t) = G_{q,4}(x, \xi, -t), \quad (\text{B18})$$

$$\tilde{E}_{2T}(x, \xi, -t) = 2\xi G_{q,2}(x, \xi, -t) - G_{q,3}(x, \xi, -t), \quad (\text{B19})$$

$$\tilde{H}_{2T}(x, \xi, -t) = G_{q,1}(x, \xi, -t). \quad (\text{B20})$$

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