

Fate of θ_{12} under $\mu - \tau$ reflection symmetry in light of the first JUNO results

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The recent Jiangmen Underground Neutrino Observatory (JUNO) measurements of θ_{12} and Δm_{21}^2 open a new avenue for probing flavor symmetric structures in the lepton sector. Motivated by this, we study a model in which $\mu - \tau$ reflection symmetry naturally emerges from an underlying A_4 flavor symmetry within a type-II seesaw framework. Beyond its standard predictions of $\theta_{23} = 45^\circ$ and $\delta_{CP} = \pm\pi/2$, the framework yields testable predictions for θ_{12} that can be probed by JUNO. Two viable scenarios arise, one predicting $\sin^2\theta_{12} \gtrsim 0.335$, which is strongly disfavored by the latest JUNO results. Correlations between θ_{12} and model parameters further enhance the model's predictivity. Future measurements at DUNE and T2HK will provide complementary tests of this scenario.

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I. INTRODUCTION

The precision era of neutrino oscillation physics has been marked by substantial progress in probing lepton mixing parameters [1–8]. Recently, the next-generation Jiangmen Underground Neutrino Observatory (JUNO) has released its first measurements based on reactor antineutrino data [9–11]. With only 59.1 days of exposure, JUNO has already achieved unprecedented sensitivity to the solar oscillation parameters, $\sin^2\theta_{12}$ and Δm_{21}^2 [12]. The reported best-fit values with 1σ uncertainties for the normal mass ordering scenario are

$$\begin{aligned}\sin^2\theta_{12} &= 0.3092 \pm 0.0087, \\ \Delta m_{21}^2 &= (7.50 \pm 0.12) \times 10^{-5} \text{ eV}^2.\end{aligned}\quad (1)$$

Even with its initial dataset, JUNO provides tighter constraints than current global fit analyses [13], further supported by independent reactor measurements from the SNO+ Collaboration [14]. In turn, the solar sector is now strongly constrained, enabling more incisive tests of lepton mixing frameworks and placing tighter limits on viable flavor models [15–21]. Nonetheless, several fundamental questions remain to be addressed.

At present, the key unknowns in the neutrino oscillation sector are the neutrino mass ordering i.e., the sign of Δm_{31}^2 ($\Delta m_{31}^2 > 0$ corresponds to normal ordering and $\Delta m_{31}^2 < 0$ to inverted ordering), the octant of the atmospheric angle θ_{23}

(lower octant if $\theta_{23} < 45^\circ$ and upper octant if $\theta_{23} > 45^\circ$), and the Dirac CP -violating phase δ_{CP} . The first JUNO results [12] already establish the foundation for its long-term program to determine the neutrino mass ordering, while the remaining unknowns (θ_{23} and δ_{CP}) will be probed with high sensitivity by upcoming long-baseline experiments such as DUNE [22–24] and T2HK [25,26]. The release of the first JUNO data have motivated a wide range of theoretical studies [27–38], including frameworks that predict nontrivial correlations among lepton mixing parameters [39–43]. Consequently, such scenarios are now being subject to increasingly stringent constraints from precision measurements in the solar sector.

In this context, we explore a flavor symmetric framework in which the precise JUNO measurements [12] play a decisive role, with further scrutiny expected from future long-baseline experiments [22–26]. We focus on a realization of $\mu - \tau$ reflection symmetry, which conventionally predicts a maximal atmospheric mixing angle, $\theta_{23} = 45^\circ$ and a maximal Dirac CP -violating phase, $\delta_{CP} = \pm\pi/2$ [44]. In its canonical formulation [45–56], the symmetry strictly fixes θ_{23} and δ_{CP} , while leaving the reactor mixing angle θ_{13} and solar mixing angle θ_{12} unconstrained, allowing them to take values consistent with experimental data. In the realization considered here, the framework further imposes a nontrivial constraint on θ_{12} , thereby enabling a direct confrontation with JUNO's precision results [12].

In this work, we present a framework in which $\mu - \tau$ reflection symmetry emerges naturally from an underlying A_4 flavor symmetry.¹ Thanks to the A_4 structure [75], the framework establishes a predictive relation among the

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¹A number of selected studies based on A_4 in the literature can be found in Refs. [57–74].

parameters, resulting in a constrained value of the solar mixing angle θ_{12} . The model is realized within a type-II seesaw mechanism [76–79] incorporating two types of $SU(2)_L$ triplet scalars, Δ and Δ' . The specific choice of the vacuum expectation value (VEV) of these scalars gives rise to two different cases of $\mu - \tau$ reflection symmetry, each leading to distinctive constraints on the lepton mixing parameters. In light of the recent JUNO results [12], we focus here on the exact $\mu - \tau$ reflection symmetry scenario, which, in addition to the canonical predictions for θ_{23} and δ_{CP} , also constrains the solar mixing angle θ_{12} . This notable prediction is a direct consequence of the underlying correlations among the model parameters governing the neutrino mass matrix. Possible deviations from this exact scenario would result in a nonmaximal θ_{23} and determine its octant. Such scenarios could be probed by upcoming experiments such as DUNE [22–24] and T2HK [25,26], but they lie beyond the scope of the present study.

The structure of this paper is as follows. In Sec. II, we outline the theoretical framework of the model based on A_4 flavor symmetry and discuss the emergence of a $\mu - \tau$ reflection symmetric neutrino mass matrix. In Sec. III, we present the numerical results of the model, including possible correlations involving the mixing angle θ_{12} and examine their consistency with the recent JUNO results. Finally, we provide concluding remarks in Sec. IV. In Appendix A, we provide a brief discussion of the corrections to the oblique parameters. The A_4 algebra and its tensor product rules are given in Appendix B. In Appendix C, we discuss the determination of lepton mixing parameters.

II. MODEL FRAMEWORK

We consider a type-II seesaw model embedded within an A_4 flavor symmetry, in which $\mu - \tau$ reflection symmetry arises naturally. Neutrino masses are generated via the type-II seesaw mechanism, and the A_4 symmetry governs the leptonic mixing pattern. We introduce two types of $SU(2)_L$ triplet scalars, Δ_i and Δ' , transforming as A_4 singlets $(1, 1'', 1')$ and a triplet (3) , respectively.² The leptons L and l_{R_i} ($i = 1, 2, 3$) are assigned as 3 and $(1, 1'', 1')$ under A_4 , respectively. The Higgs-like doublets ϕ_α ($\alpha = e, \mu, \tau$), responsible for generating charged lepton masses, are assigned as triplets under A_4 . In addition, a discrete Z_3 symmetry is imposed to ensure that the charged lepton mass matrix remains diagonal, so that the leptonic mixing pattern is solely governed by the neutrino sector. Under the Z_3 symmetry, nontrivial charges are assigned exclusively to l_{R_i} and ϕ_α , whereas all other particles transform trivially. The particle content and their transformation properties under the various symmetries are summarized in Table I.

²The corrections to the oblique parameters [80], induced by the presence of multiple $SU(2)_L$ triplet scalars, are briefly discussed in Appendix A.

TABLE I. Particle content and their transformation properties under different symmetries, where $i = 1, 2, 3$, and $\alpha = e, \mu, \tau$. To avoid confusion, the ω charge associated with the Z_3 symmetry is indicated in **boldface**, distinguishing it from the ω that appears in the A_4 tensor product rules (see Appendix B) and VEV alignment.

Fields	$SU(2)_L$	$U(1)_Y$	A_4	Z_3
L	2	$-1/2$	3	1
l_{R_i}	1	-1	$(1, 1'', 1')$	$(1, \omega^2, \omega)$
ϕ_α	2	$1/2$	$(3, 3, 3)$	$(1, \omega, \omega^2)$
Δ_i	3	1	$(1, 1'', 1')$	1
Δ'	3	1	3	1

The transformations of Δ_i and Δ' are chosen such that their VEV alignments, $\langle \Delta_i \rangle = u_i$ and $\langle \Delta' \rangle = \frac{u'}{\sqrt{3}}(\pm 1, \omega, \omega^2)$, naturally realize the $\mu - \tau$ reflection symmetry. Depending on the sign choice in the VEV alignment of Δ' , two distinct scenarios arise. Consequently, the model predicts the salient features of $\mu - \tau$ reflection symmetry, namely, $\theta_{23} = 45^\circ$ and $\delta_{\text{CP}} = \pm\pi/2$. Beyond the standard $\mu - \tau$ reflection predictions, the structure imposed by the underlying A_4 symmetry tightly correlates the model parameters, resulting in robust and testable predictions for the solar mixing angle θ_{12} . Imposing neutrino oscillation constraints yields a lower bound on $\sin^2 \theta_{12}$ in one scenario, thereby enabling the latest JUNO results to probe this realization of the model. Having discussed the model framework, we next examine the generation of neutrino masses and the resulting mixing pattern.

A. Lagrangian and neutrino mass matrix

Following the charge assignments of the particles under the various symmetries given in Table I, the invariant Yukawa Lagrangian that governs the leptonic sector can be formulated as follows:

$$\mathcal{L} = \mathcal{L}_\ell + \mathcal{L}_\nu, \quad (2)$$

where $\mathcal{L}_\ell$ is responsible for generating the charged lepton masses, and the neutrino masses can be extracted from $\mathcal{L}_\nu$, given by

$$\begin{aligned} -\mathcal{L}_\ell &= y_e(\bar{L} \otimes \phi_e)_1 \otimes (l_{R_1})_1 + y_\mu(\bar{L} \otimes \phi_\mu)_1 \\ &\quad \otimes (l_{R_2})_{1''} + y_\tau(\bar{L} \otimes \phi_\tau)_{1''} \otimes (l_{R_3})_{1'} + \text{H.c.}, \\ -\mathcal{L}_\nu &= \alpha_1(\bar{L}^c \otimes L)_1 \otimes (i\tau_2 \Delta_1)_1 + \alpha_2(\bar{L}^c \otimes L)_1 \\ &\quad \otimes (i\tau_2 \Delta_2)_{1''} + \alpha_3(\bar{L}^c \otimes L)_{1''} \otimes (i\tau_2 \Delta_3)_{1'} \\ &\quad + \beta(\bar{L}^c \otimes L)_{3S} \otimes (i\tau_2 \Delta')_3 + \text{H.c.} \end{aligned} \quad (3)$$

The lower indices in parentheses $[\dots]_p$; $p = 1, 1', 1'', 3_S, 3$ denote the A_4 transformation of enclosed fields. The VEV alignments for ϕ_α are chosen as

$$\begin{aligned} \phi_e &= v_1(1, 0, 0)^T, & \phi_\mu &= v_2(0, 1, 0)^T, \\ \phi_\tau &= v_3(0, 0, 1)^T. \end{aligned} \quad (4)$$

This leads to a diagonal charged lepton mass matrix, implying that the leptonic mixing arises solely from the neutrino sector. We now rewrite $\mathcal{L}_\nu$ in its expanded form using the A_4 tensor product rules,

$$\begin{aligned} -\mathcal{L}_\nu = & \alpha_1(\bar{L}_1^c L_1 + \bar{L}_2^c L_2 + \bar{L}_3^c L_3)i\tau_2 \Delta_1 \\ & + \alpha_2(\bar{L}_1^c L_1 + \omega \bar{L}_2^c L_2 + \omega^2 \bar{L}_3^c L_3)i\tau_2 \Delta_2 \\ & + \alpha_3(\bar{L}_1^c L_1 + \omega^2 \bar{L}_2^c L_2 + \omega \bar{L}_3^c L_3)i\tau_2 \Delta_3 \\ & + \beta[(\bar{L}_2^c L_3 + \bar{L}_3^c L_2)i\tau_2 \Delta'_a + (\bar{L}_3^c L_1 + \bar{L}_1^c L_3)i\tau_2 \Delta'_b \\ & + (\bar{L}_1^c L_2 + \bar{L}_2^c L_1)i\tau_2 \Delta'_c] + \text{H.c.} \end{aligned} \quad (5)$$

Here, the scalar Δ' , transforming as a triplet under A_4 , is defined as $\Delta' \equiv (\Delta'_a, \Delta'_b, \Delta'_c)^T$. Once the scalar fields Δ_i and Δ' acquire VEVs, neutrino masses are generated. We adopt a specific VEV alignment consistent with the minimization conditions of the scalar potential [81–83], given by³

$$\langle \Delta_i \rangle = u_i, \quad \langle \Delta' \rangle = \frac{u'}{\sqrt{3}}(\pm 1, \omega, \omega^2)^T, \quad (6)$$

where $\omega^3 = 1$. We choose the couplings α_i, β and the VEVs u_i, u' to be real, so that the CP -violation in the leptonic sector originates solely from the ω appearing in the A_4 tensor product rules. Thus, using Eqs. (5) and (6), the neutrino mass matrix can be extracted, which exhibits the $\mu - \tau$ reflection symmetric structure, given by

$$\mathcal{M}_\nu = \begin{pmatrix} A & C & C^* \\ C & B & D \\ C^* & D & B^* \end{pmatrix}, \quad (7)$$

$$\begin{aligned} \text{where, } A &= \sum_{i=1}^3 \alpha_i u_i, & B &= \sum_{i=1}^3 \omega^{(2+i)} \alpha_i u_i, \\ D &= \pm \frac{\beta u'}{\sqrt{3}}, & C &= \omega^2 \frac{\beta u'}{\sqrt{3}} \equiv (\pm) D \omega^2. \end{aligned} \quad (8)$$

The parameters A and D are real, whereas B and C are complex. As indicated in Eq. (8), the parameters C and D are not independent, but are related through $C = (\pm) \omega^2 D$. Moreover, the parameters A and B are also not entirely arbitrary; rather, they arise from similar combinations of the underlying couplings and VEVs, reflecting a common structural origin. These relations are a direct consequence of the underlying A_4 symmetry together with the specific VEV alignments of Δ_i, Δ' . As a result, the neutrino mass matrix in Eq. (7) is left with only four effective free parameters. This reduces the parameter freedom of the

mass matrix compared to a completely general realization of $\mu - \tau$ reflection symmetry [44].

This reduced number of independent parameters has important implications for the flavor structure of leptonic mixing, specifically for the mixing angle θ_{12} . In the conventional $\mu - \tau$ reflection symmetry, θ_{12} remains unconstrained, as the neutrino mass matrix contains sufficient freedom among its elements. However, in our framework, the additional structure imposed by the A_4 symmetry and VEV alignments induces nontrivial correlations among the matrix elements. Since the solar mixing angle is particularly sensitive to the relative structure of the (1–2) sector of the mass matrix, these relations translate into correlations involving θ_{12} . As we will discuss in the next section, these correlations manifest differently in two distinct scenarios once experimental constraints are imposed. While both scenarios exhibit correlated behavior of θ_{12} with the model parameters, case-II leads to a more restricted parameter space, resulting in a lower bound on $\sin^2 \theta_{12}$, whereas case-I allows a broader range consistent with current data. Therefore, the predictivity of θ_{12} in this framework arises from the interplay between the A_4 imposed structure, the specific vacuum alignments, and the experimental constraints.

III. NUMERICAL ANALYSIS AND MODEL PREDICTIONS

We now discuss a detailed numerical analysis of the model based on the neutrino mass matrix obtained in Eq. (7), which exhibits an exact $\mu - \tau$ reflection symmetry. Since the charged lepton mass matrix is diagonal, the leptonic mixing arises solely from the neutrino sector. As a consequence of the $\mu - \tau$ reflection symmetry, the atmospheric mixing angle and the Dirac CP -violating phase are fixed to $\theta_{23} = 45^\circ$ and $\delta_{CP} = \pm \pi/2$, respectively, independent of the specific values of the model parameters. In light of the recent JUNO results [12], we focus on observables that play a decisive role in testing the framework. In particular, the correlated behavior of the solar parameters $\sin^2 \theta_{12}$ and Δm_{21}^2 imposes strong constraints on the model.

We present two viable scenarios corresponding to the two possible VEV alignments of the scalar Δ' . The associated VEV alignments are $\langle \Delta' \rangle \propto (1, \omega, \omega^2)^T$ and $\langle \Delta' \rangle \propto (-1, \omega, \omega^2)^T$, which lead to $D = +\beta \frac{u'}{\sqrt{3}}$ (case-I) and $D = -\beta \frac{u'}{\sqrt{3}}$ (case-II), respectively, in Eq. (7). The model involves four free parameters: two real parameters A and D , and one complex parameter B , parametrized as $B = r e^{i\theta}$. We carry out a parameter scan over the following ranges:

$$\begin{aligned} A &= [10^{-4}, 10^{-1}] \text{ eV}, & |D| &= [10^{-4}, 10^{-1}] \text{ eV}, \\ r &= [10^{-4}, 10^{-1}] \text{ eV}, & \theta &= [0, 2\pi]. \end{aligned} \quad (9)$$

³Another possible choice of VEV alignment is $\langle \Delta' \rangle \propto (\pm 1, \eta, \eta^{-1})^T$, where $\eta^2 = \omega$, which leads to identical predictions and is phenomenologically equivalent.

For the numerical analysis, we impose the 3σ constraints on neutrino oscillation parameters from the Astroparticle and High Energy Physics (AHEP) global fit data [84,85], given by

$$\begin{aligned}\Delta m_{21}^2 &= [6.94, 8.14] \times 10^{-5} \text{ eV}^2, \\ \Delta m_{31}^2 &= [2.47, 2.63] \times 10^{-3} \text{ eV}^2, \\ \sin^2 \theta_{12} &= [0.271, 0.369], \\ \sin^2 \theta_{13} &= [0.02000, 0.02405].\end{aligned}\quad (10)$$

As noted above, the mixing angle θ_{23} is predicted to be 45° , which lies within the 3σ allowed range of AHEP global fit data.

We next present our model predictions for both scenarios mentioned above. The diagonalization procedure for the mass matrix in Eq. (7), together with the determination of the associated mixing angles, is outlined in Appendix C. We find that inverted ordering is disfavored in case-II, while case-I remains consistent with the current constraints. In the following, we focus on the normal ordering case, as it is the scenario probed by the recent JUNO measurements. The solar mixing angle θ_{12} exhibits a robust correlation with the model parameters in both scenarios. For case-I, the predicted values of θ_{12} remain consistent with current data. In contrast, case-II predicts a lower bound on the solar mixing angle, $\sin^2 \theta_{12} \gtrsim 0.335$, which is nearly excluded at the 3σ level considering the JUNO results [12], although it remains compatible with the AHEP global fit data [84].

A. Constraints on model parameters from the solar mixing angle θ_{12}

We begin by examining the correlation between the solar mixing angle θ_{12} and the model parameters. We find that $\sin^2 \theta_{12}$ exhibits a strong dependence on the ratios of model parameters, defined as $r_1 \equiv |D/A|$, $r_2 \equiv |D/B|$, and $r_3 \equiv |A/B| \equiv r_2/r_1$. The allowed parameter spaces of the model are shown as blue and green scatter points

corresponding to case-I and case-II, respectively. In Fig. 1, we show the correlation between $\sin^2 \theta_{12}$ and the ratio r_1 . The left and right panels correspond to case-I and case-II, respectively. The allowed 3σ ranges of $\sin^2 \theta_{12}$ from JUNO and AHEP global fit are indicated by the brown and light red bands, respectively.

Similarly, we present the correlations for the ratios r_2 and r_3 in Figs. 2 and 3, respectively. The color code remains the same as in Fig. 1. The correlations observed in Figs. 1, 2, and 3 highlight the interplay between $\sin^2 \theta_{12}$ and the model parameter ratios. In view of the recent JUNO measurement, these relations translate into stringent tests of the model. Once the mass-squared difference (Δm_{21}^2 and Δm_{31}^2) constraints are imposed, the free parameters of the model become correlated, leading to fixed ratios. The additional requirement of compatibility with $\sin^2 \theta_{12}$ further confines these ratios to a narrow region.

In case-I, imposing the 3σ AHEP constraints confines the parameter ratios to the ranges: $r_1 \sim [1.3, 2.4]$, $r_2 \sim [0.57, 0.70]$, and $r_3 \sim [0.28, 0.42]$. While, for case-II the corresponding allowed ranges are $r_1 \sim [2.4, 3.0]$, $r_2 \sim [1.5, 1.8]$, and $r_3 \sim [0.50, 0.78]$. In addition, case-II predicts a lower bound on the solar mixing angle, $\sin^2 \theta_{12} \gtrsim 0.335$. As a result, this scenario survives only within a very narrow region of the AHEP global fit parameter space and is almost excluded by the JUNO measurements. Notably, the origin of this constraint lies in the intrinsic correlation among the solar parameters, which we discuss next.

B. Solar parameter correlations in the light of JUNO results

We now turn to the correlation between the solar parameters $\sin^2 \theta_{12}$ and Δm_{21}^2 . The resulting predictions can be directly compared with the AHEP global fit data [84] and the JUNO measurements [12]. Figure 4 illustrates the model correlations in the $(\sin^2 \theta_{12} - \Delta m_{21}^2)$ plane. The left panel represents the model predictions for case-I in blue,

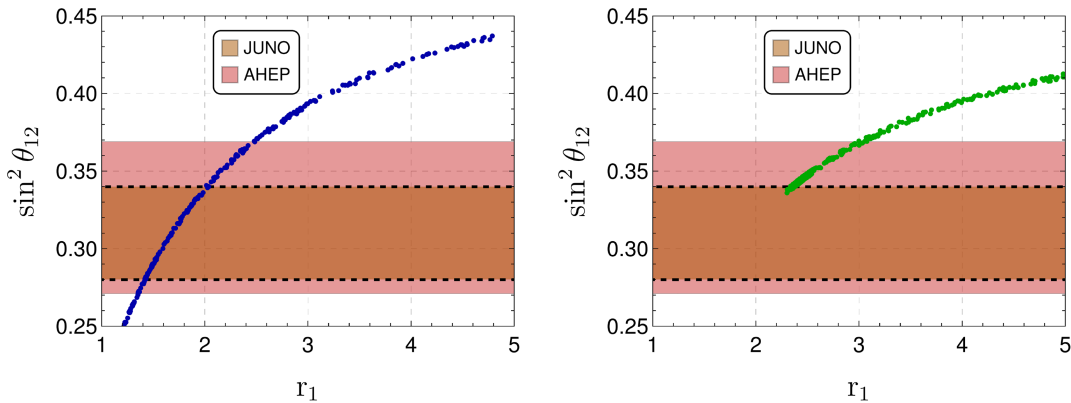


FIG. 1. Correlation between $\sin^2 \theta_{12}$ and the model parameter ratio r_1 . The predictions for case-I and case-II are shown in blue and green, respectively, in the left and right panels. The brown and light red bands represent the allowed 3σ ranges from JUNO and AHEP, respectively.

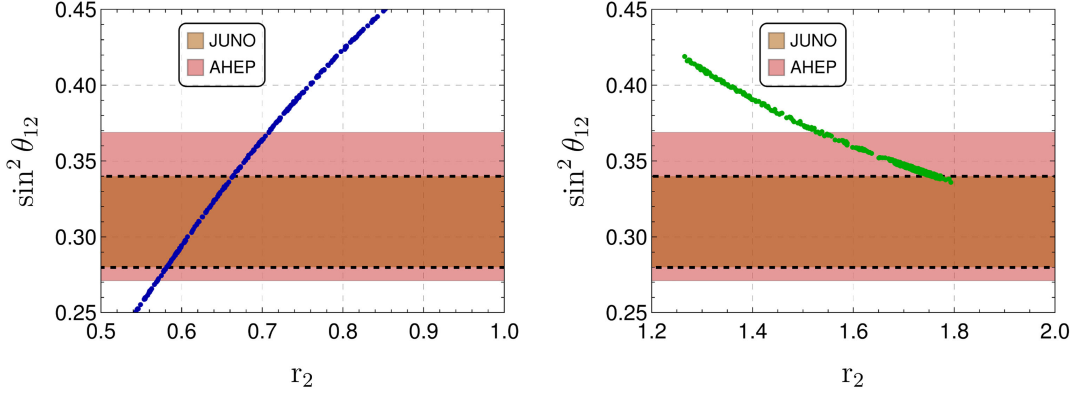


FIG. 2. Correlation between $\sin^2 \theta_{12}$ and the model parameter ratio r_2 . The color code remains the same as in Fig. 1.

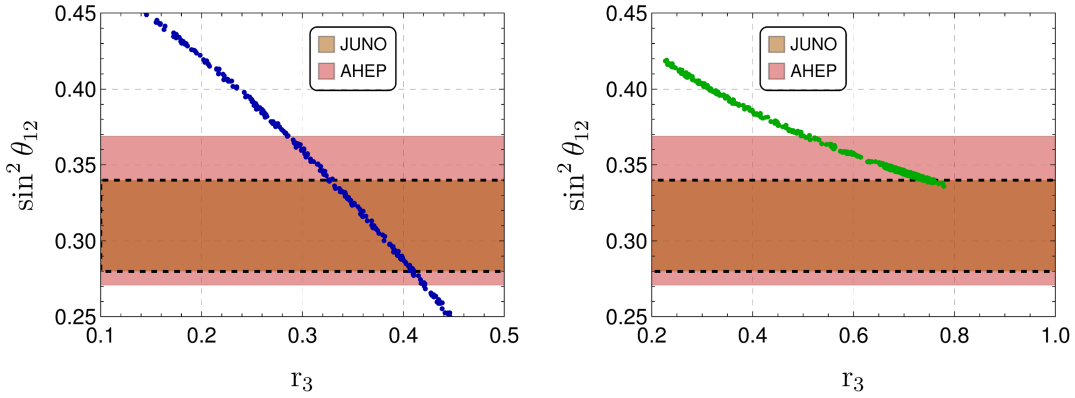


FIG. 3. Correlation between $\sin^2 \theta_{12}$ and the model parameter ratio r_3 . The color code remains the same as in Fig. 1.

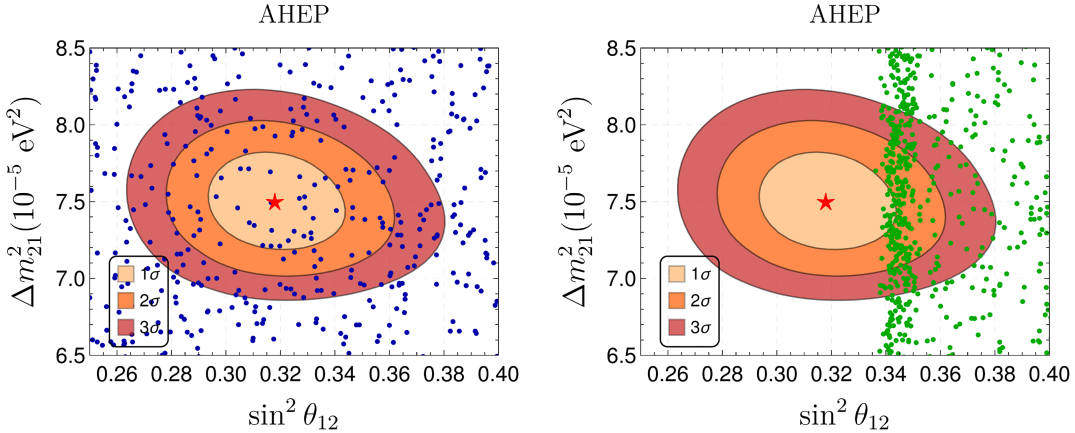


FIG. 4. Correlation between the solar parameters $\sin^2 \theta_{12}$ and Δm_{21}^2 predicted by the model, compared with the AHEP global fit data [84]. Case-I and case-II are shown in blue and green in the left and right panels, respectively.

whereas the right panel shows the corresponding predictions for case-II in green. For the comparison, we consider the $1, 2, 3\sigma$ contours from the AHEP global fit in the $(\sin^2 \theta_{12} - \Delta m_{21}^2)$ plane, with the corresponding best-fit point indicated by a red star. After imposing the constraints from the reactor mixing angle θ_{13} and the atmospheric

mass-squared difference Δm_{31}^2 , we find that the allowed parameter space in case-I, shown by blue points, spans the entire allowed region. In contrast, the parameter space in case-II, shown by green points, exhibits a lower bound on the solar mixing angle, $\sin^2 \theta_{12} \gtrsim 0.335$. Nevertheless, both scenarios remain compatible with the AHEP global fit data.

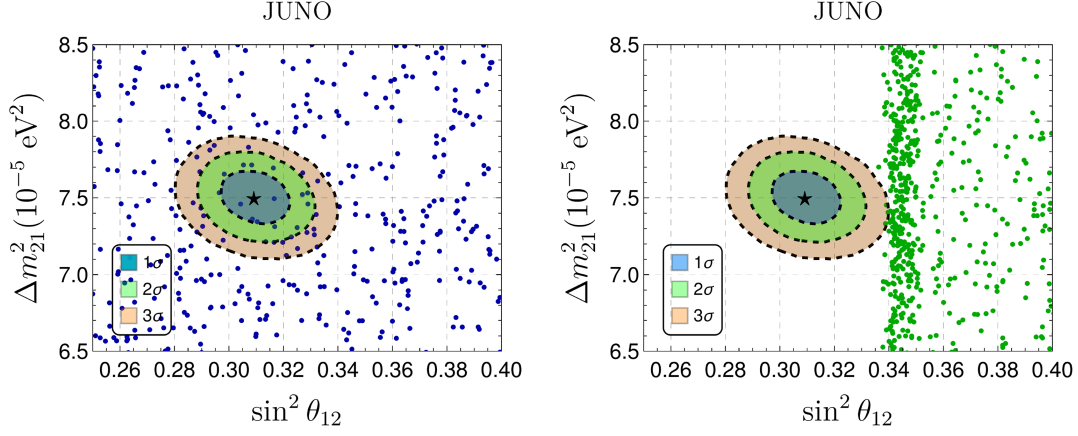


FIG. 5. Correlation between the solar parameters $\sin^2 \theta_{12}$ and Δm_{21}^2 predicted by the model, compared with the JUNO results [84]. Case-I and case-II are shown in blue and green in the left and right panels, respectively.

However, the presence of this lower bound in case-II suggests a restricted compatibility with the experimental data. This observation becomes particularly compelling considering the recent JUNO measurement of $\sin^2 \theta_{12}$. We therefore proceed to confront the allowed parameter space of the model with the JUNO data, which provides a more stringent and decisive test of the framework. The allowed parameter space of the model and their comparison with the 1, 2, 3 σ contours of JUNO are shown in Fig. 5. The corresponding best-fit value is indicated by a black star. As in Fig. 4, the left (right) panel corresponds to case-I (case-II). Since the blue points span the entire allowed region in the $(\Delta m_{21}^2 - \sin^2 \theta_{12})$ plane, case-I remains a viable scenario. In contrast, for case-II, the model predicts a lower bound $\sin^2 \theta_{12} \gtrsim 0.335$, causing all the green points to lie outside the 3 σ JUNO contours. Consequently, in light of the JUNO results, this scenario of the model is nearly excluded.

Therefore, by comparing Figs. 4 and 5, we observe that prior to the JUNO measurements, case-II was also consistent and remained well within the 2 σ range of the AHEP global fit data. However, once the JUNO results are taken into account, only the first scenario, case-I ($D = +\beta \frac{u'}{\sqrt{3}}$), remains compatible, while the second one ($D = -\beta \frac{u'}{\sqrt{3}}$) is almost ruled out.

IV. CONCLUSIONS

We have developed a framework in which $\mu - \tau$ reflection symmetry is naturally realized as a consequence of an underlying A_4 flavor symmetry. Neutrino masses are generated via the type-II seesaw mechanism through the introduction of two $SU(2)_L$ triplet scalars: Δ_i and Δ' , transforming as singlets and a triplet under A_4 , respectively. The specific VEV alignments of these scalars lead to an exact $\mu - \tau$ reflection symmetry in the neutrino sector. Depending on the choice of VEV alignment for Δ' , two distinct viable scenarios emerge. An additional Z_3 symmetry

ensures a diagonal charged lepton mass matrix, leaving the leptonic mixing entirely governed by the neutrino sector. The resulting $\mu - \tau$ reflection symmetry then fixes the atmospheric mixing angle to $\theta_{23} = 45^\circ$ and the Dirac CP -violating phase to $\delta_{CP} = \pm\pi/2$.

Beyond the well-known predictions arising from $\mu - \tau$ reflection symmetry, our framework yields nontrivial and testable implications for the solar mixing angle θ_{12} . In particular, when combined with the solar mass-squared difference Δm_{21}^2 , the allowed parameter space of the model can be directly confronted with the latest JUNO measurements. Among the two distinct scenarios, one predicts $\sin^2 \theta_{12} \gtrsim 0.335$, which is strongly disfavored by the JUNO results, although it remains compatible with the current AHEP global fit data. In both cases, $\sin^2 \theta_{12}$ exhibits a robust correlation with the ratios of the underlying model parameters. Imposing the neutrino oscillation constraints significantly narrows the allowed range of these ratios, thereby reducing the parametric freedom of the model and enhancing its phenomenological implications.

Furthermore, future measurements of the θ_{23} octant at DUNE and T2HK will play an important role in probing extensions of this framework that introduce controlled deviations from exact $\mu - \tau$ reflection symmetry. The observation of a nonmaximal θ_{23} with a specific octant preference would provide compelling evidence in favor of such scenarios.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: ELECTROWEAK PRECISION OBSERVABLES: S , T , AND U PARAMETERS

In this section, we briefly discuss the corrections to the oblique parameters, namely S , T , and U parameters [80], induced by the presence of $SU(2)_L$ triplet scalars (Δ_i, Δ'). Within the type-II seesaw framework, these triplet scalars carry electroweak charges and therefore interact directly with the gauge bosons W and Z . As a consequence, they contribute to the vacuum polarization amplitudes of W and Z bosons, leading to modifications in the oblique parameters. The contributions of a scalar multiplet with general weak isospin and hypercharge to the oblique parameters have been computed in Ref. [86], while their explicit evaluation in the type-II seesaw framework, involving a scalar triplet, can be found in Refs. [87,88]. In our framework, the scalar sector is extended to include the multiple triplet scalars, each acquiring a VEV as given in Eq. (6). The presence of such multiple triplet scalars modifies the corresponding expressions [87,88], primarily through mixing among the triplet scalars. Nevertheless, their mixing with the Higgs doublet remains negligible due to the smallness of the triplet VEVs, which are required for neutrino mass generation [see Eq. (9)].

These $SU(2)_L$ triplet scalars Δ_i and Δ' are assigned as singlets and triplet of A_4 , respectively. In the 2×2 matrix notation, they can be expressed as

$$\Delta_q \equiv \begin{pmatrix} \Delta_q^+/\sqrt{2} & \Delta_q^{++} \\ \Delta_q^0 & -\Delta_q^+/\sqrt{2} \end{pmatrix}, \quad (\text{A1})$$

where Δ_q represents $\Delta_1, \Delta_2, \Delta_3, \Delta'_a, \Delta'_b, \Delta'_c$. After spontaneous symmetry breaking, these triplet scalars mix among themselves, neutral part with neutral and singly (doubly) charged with singly (doubly) charged scalars, respectively. We denote the resulting mass eigenstates as H_i^0, H_j^+, H_k^{++} , with corresponding masses M_i^0, M_j^+, M_k^{++} , respectively. The relation between gauge eigenstates and mass eigenstates is given by

$$\Delta_q^0 = U_{qi}^0 H_i^0, \quad \Delta_q^+ = U_{qj}^+ H_j^+, \quad \Delta_q^{++} = U_{qk}^{++} H_k^{++}. \quad (\text{A2})$$

Taking into account the mixing relations defined in Eq. (A2), and utilizing the general expressions for the oblique parameters of scalar multiplets derived in Ref. [86], we obtain the following expressions:

$$S = -\frac{1}{3\pi} \sum_{p=-1}^1 \sum_{q,i} |U_{qi}^{s_p}|^2 \left[p \ln M_{p,i}^2 + 6(p - s_W^2 Q_p)^2 \xi \left(\frac{M_{p,i}^2}{M_Z^2}, \frac{M_{p,i}^2}{M_Z^2} \right) \right], \quad (\text{A3})$$

$$T = \frac{1}{16\pi s_W^2} \sum_{p=-1}^1 \sum_{q,i,j} U_{qi}^{s_p} U_{qj}^{*s_{p-1}} \left[(2 + p - p^2) \eta \left(\frac{M_{p,i}^2}{M_W^2}, \frac{M_{p-1,j}^2}{M_W^2} \right) \right], \quad (\text{A4})$$

$$U = \frac{1}{6\pi} \left[\sum_j \ln(M_j^+)^4 - \sum_i \ln(M_i^0)^2 - \sum_k \ln(M_k^{++})^2 \right] + \frac{1}{\pi} \sum_{p=-1}^1 \left[\sum_{q,i,j} U_{qi}^{s_p} U_{qj}^{*s_{p-1}} (-2 - p + p^2) \xi \left(\frac{M_{p,i}^2}{M_W^2}, \frac{M_{p-1,j}^2}{M_W^2} \right) + \sum_{q,i} |U_{qi}^{s_p}|^2 2(p - s_W^2 Q_p)^2 \xi \left(\frac{M_{p,i}^2}{M_Z^2}, \frac{M_{p,i}^2}{M_Z^2} \right) \right], \quad (\text{A5})$$

where $M_{p,i} = \{M_i^0, M_i^+, M_i^{++}\}$, $s_p = \{0, +, ++\}$ and $Q_p = \{0, 1, 2\}$ for $p = \{-1, 0, 1\}$, and $s_W^2 \equiv \sin^2 \theta_W$, θ_W is the weak mixing angle.

$$\begin{aligned} \xi(x, y) &= \frac{4}{9} - \frac{5}{12}(x+y) + \frac{1}{6}(x-y)^2 \\ &\quad + \frac{1}{4} \left[x^2 - y^2 - \frac{1}{3}(x-y)^3 - \frac{x^2 + y^2}{x-y} \right] \ln \frac{x}{y} - \frac{1}{12} d(x, y) f(x, y), \\ d(x, y) &= -1 + 2(x+y) - (x-y)^2, \\ f(x, y) &= \begin{cases} -2\sqrt{d(x, y)} \left[\arctan \left(\frac{x-y+1}{\sqrt{d(x, y)}} \right) - \arctan \left(\frac{x-y-1}{\sqrt{d(x, y)}} \right) \right], & \text{if } d(x, y) > 0 \\ 0 & \text{if } d(x, y) = 0 \\ \sqrt{-d(x, y)} \ln \left[\frac{x+y-1+\sqrt{-d(x, y)}}{x+y-1-\sqrt{-d(x, y)}} \right], & \text{if } d(x, y) < 0 \end{cases} \end{aligned} \quad (\text{A6})$$

$$\eta(x, y) = x + y - \frac{2xy}{x-y} \ln \frac{x}{y}. \quad (\text{A7})$$

In the above expressions, $M_{p-1,i}$ is formally undefined for $p = -1$, however, the corresponding terms vanish identically for $p = -1$ and therefore do not contribute.

In the limit of small mass splittings among the triplet components, the contributions to the S , T , U parameters admit simplified expressions [88]. While a detailed discussion of scenarios involving multiple triplet scalars is certainly of interest, it is beyond the scope of the current focus of the work. Nevertheless, it is important to note that such scalar sectors generically induce corrections to the ρ parameter, thereby placing constraints on the VEVs of the triplet scalars. For instance, in the minimal type-II seesaw scenario with a single triplet Δ , one typically obtains the bound $v_\Delta \leq \mathcal{O}(1)$ GeV, which is expected to be even more restrictive in scenarios involving multiple triplets. However, in our framework, the triplet VEVs required for neutrino mass generation are naturally small. As evident from Eqs. (8) and (9), they remain sufficiently small even for $\mathcal{O}(1)$ Yukawa couplings. As a result, the induced corrections to the ρ parameter and other electroweak observables are negligibly small, ensuring consistency with current experimental limits [13].

APPENDIX B: TENSOR PRODUCT RULES OF A_4

The A_4 symmetry is a non-Abelian discrete flavor group. It corresponds to the group of even permutations of four objects and is isomorphic to the symmetry group of a regular tetrahedron. The group contains 12 elements and can be generated by two generators S and T obeying the relations

$$S^2 = T^3 = (ST)^3 = I. \quad (\text{B1})$$

In the basis where S and T are real matrices, the generators are given by

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \quad (\text{B2})$$

The A_4 group has four irreducible representations, three of them are one dimensional (i.e., three singlets) 1, $1'$, and $1''$, and one of them is three dimensional (triplet) 3. The multiplication rules for these representations are given as follows:

$$\begin{aligned} 1 \times 1 &= 1 = 1' \times 1'', & 1' \times 1' &= 1'', & 1'' \times 1'' &= 1', \\ 1 \times 3 &= 3, & 3 \times 3 &= 1 + 1' + 1'' + 3_S + 3_A, \end{aligned} \quad (\text{B3})$$

where 3_S and 3_A denote the two independent triplet contractions arising from the product of two triplets.

If $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$ are two triplets, then their multiplication rules are constructed as follows [75]:

$$\begin{aligned} (ab)_1 &= a_1 b_1 + a_2 b_2 + a_3 b_3, \\ (ab)_{1'} &= a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3, \\ (ab)_{1''} &= a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3, \\ (ab)_{3_S} &= (a_2 b_3 + a_3 b_2, a_3 b_1 + a_1 b_3, a_1 b_2 + a_2 b_1)^T, \\ (ab)_{3_A} &= (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1)^T, \end{aligned} \quad (\text{B4})$$

where ω is the cube root of unity defined by $\omega = e^{2\pi i/3}$. The triplets 3_S and 3_A denote the symmetric and antisymmetric contractions, respectively.

APPENDIX C: DETERMINATION OF LEPTON MIXING PARAMETERS

The neutrino mass matrix given in Eq. (7) can be diagonalized by a unitary transformation as follows:

$$U_\nu^T \mathcal{M}_\nu U_\nu = \text{diag}(m_1, m_2, m_3), \quad (\text{C1})$$

where U_ν is the unitary matrix, and $m_{1,2,3}$ denote the physical neutrino mass eigenvalues. In the present framework, the charged lepton mass matrix is diagonal. Consequently, U_ν directly corresponds to the leptonic mixing Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix,

$$U_{\text{PMNS}} \equiv U_\nu. \quad (\text{C2})$$

We choose the symmetric parametrization of the lepton mixing matrix [76,85] given by

$$\begin{aligned} U_{\text{PMNS}} &= P(\delta_1, \delta_2, \delta_3) U_{23}(\theta_{23}, \phi_{23}) \\ &\times U_{13}(\theta_{13}, \phi_{13}) U_{12}(\theta_{12}, \phi_{12}), \end{aligned} \quad (\text{C3})$$

where $P(\delta_1, \delta_2, \delta_3)$ is a diagonal matrix of unphysical phases, and the U_{ij} are complex rotations in the ij plane, as for example,

$$U_{23}(\theta_{23}, \phi_{23}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} e^{-i\phi_{23}} \\ 0 & -\sin \theta_{23} e^{i\phi_{23}} & \cos \theta_{23} \end{pmatrix}. \quad (\text{C4})$$

The phases ϕ_{12} and ϕ_{13} are relevant for neutrinoless double beta decay, while the combination $\delta_{CP} = \phi_{13} - \phi_{12} - \phi_{23}$ is the usual Dirac CP -violating phase measured in neutrino oscillations. The leptonic mixing angles can be extracted from the elements of the PMNS matrix through the following relations:

$$s_{13} = |U_{\text{PMNS}}|_{13}, \quad s_{12} = \frac{|U_{\text{PMNS}}|_{12}}{c_{13}}, \quad s_{23} = \frac{|U_{\text{PMNS}}|_{23}}{c_{13}}, \quad (\text{C5})$$

where $s_{13,12,23}$ and c_{13} denote the shorthand notations for $\sin \theta_{13,12,23}$ and $\cos \theta_{13}$, respectively.

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