

Direct detection of mechanism-agnostic fast-moving dark matter

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We present a comprehensive framework for interpreting electron recoil signals induced by fast-moving dark matter (DM), applicable across a wide range of theoretically motivated models. Amid both null results in conventional weakly interacting massive particle searches and growing interest in alternative DM scenarios, we focus on (semi-)relativistic DM components that can arise from mechanisms such as DM annihilation, decay, or cosmic-ray acceleration. These boosted DM candidates produce distinct experimental signatures that differ qualitatively from nonrelativistic DM, necessitating a dedicated treatment. Our framework incorporates relativistic kinematics and atomic effects through ionization form factors, enabling accurate predictions of differential cross sections in both low- and high-energy regimes. We demonstrate how atomic effects become negligible at high recoil energies, validating the free-electron approximation in specific parameter regions. Furthermore, we highlight the complementarity between low-threshold direct detection experiments and high-threshold neutrino observatories in probing fast-moving DM across broad kinematic domains. This formalism provides a robust and model-independent foundation for interpreting current and future searches for relativistic DM.

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I. INTRODUCTION

The existence of dark matter (DM) in the universe provides compelling observational evidence—all gravity-based—for physics beyond the Standard Model (SM), as no SM particle has yet been shown to consistently account for DM-related observations. One of the earliest and most well-motivated candidate frameworks involves weakly interacting massive particles (WIMPs), which are thermally produced in the early Universe and remain consistent with the observed cosmological evolution. The WIMP framework predicts weakly interacting DM candidates with weak-scale masses to be consistent with the observed DM relic abundance, $\Omega_C h^2 \simeq 0.12$ [1]. Interestingly, other fundamental issues, such as the gauge hierarchy problem, also motivate new physics at the weak scale (often referred to as the “WIMP miracle”), further energizing the community and driving experimental efforts, including DM direct and

indirect detection as well as collider searches. However, despite these extensive efforts over the past few decades, no conclusive evidence of DM has been observed through nongravitational interactions (see, e.g., Refs. [2,3]), gradually motivating the community to reconsider and broaden the theoretical framework.

While many alternative scenarios continue to envision nonrelativistic DM with different mass scales, a series of ideas predicting fast-moving or even relativistic DM in the present Universe have emerged over the past decade. Typical scenarios assume particular mechanisms through which a certain fraction of ambient DM acquires nontrivial boost factors in the present Universe, hence boosted, while remaining consistent with the overall cosmological history. Moreover, many of these models offer new realizations of the WIMP miracle or alternative ways for preserving the thermal DM framework, while largely evading the strong experimental constraints placed on conventional WIMP DM [4–7]. Earlier ideas include two-component boosted DM [4] with the assisted freeze-out mechanism [5], semi-annihilation [8], self-annihilation [9,10], DM-induced nucleon decay [11,12], and decaying DM [13–16]. More recent proposals include charged cosmic-ray acceleration [17–21], cosmic-ray neutrino acceleration [22–24], various astrophysical processes [25–31], and inelastic collision of cosmic rays with the atmosphere [32,33].

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Unlike the conventional DM candidates, the expected phenomenology of fast-moving DM is qualitatively different. As mentioned above, since only a certain fraction of the relic DM becomes boosted, large-volume detectors—including ton-scale DM direct detection experiments [29,33–51] and large-volume neutrino detectors such as DUNE [52–61], Super-/Hyper-Kamiokande [4,6,7,47,49,52–54,57–63], Ice-Cube [4,7,13,14,57,62,64], prototype detectors for DUNE [65,66], and detectors in the Short Baseline Neutrino Program [66]—are preferred to achieve signal sensitivity in regions of parameter space beyond existing constraints. Moreover, by construction, the incoming DM particles carry sufficient energy to produce a variety of relativistic experimental signatures, ranging from simple electron or nucleon scattering [4,7,33,35,37–46,51–54,57,58,65–67] to deep inelastic scattering [56,57], nuclear inelastic scattering [59], and even DM upscattering followed by cascade decays [35,54,55,57,65,67–69]. Motivated by these intriguing proposals, searches for fast-moving or boosted DM have been conducted in various experiments—including Super-Kamiokande (SK) [70,71], COSINE-100 [72,73], PandaX [74,75], CDEX [76,77], NEWSdm [78], and ICARUS-Gran Sasso [79]—across both elastic and inelastic scattering channels. Beyond these experimental search efforts, fast-moving DM may also have intriguing astrophysical and cosmological implications [80,81] such as the core-cusp problem [82] and DM self-heating [83].

Independent of theoretical developments and experimental efforts along this line, the phenomenology of fast-moving DM received significant attention in connection with the now-subsided XENON1T electron recoil excess [84]. Since the typical energy scale of the reported electron signals is $\mathcal{O}(\text{keV})$, the electron binding energy is no longer negligible, and proposals to explain the excess (e.g., Refs. [33,38–42,44–46,85]) incorporated the effects from ionization form factors (also often referred to as atomic excitation form factors) into their calculations. Indeed, the importance of the ionization form factors was already highlighted in Refs. [86–92] in the context of electron scattering induced by nonrelativistic DM, as they affect the electron-DM scattering rates. Several of these studies further emphasized how different ionization form factor formalisms influence the recoil momentum spectrum, especially at high momentum transfers. Many studies implementing the ionization form factors have employed the plane-wave approximation [86,93], even in the analysis of fast-moving DM. However, this method has limitations compared to more sophisticated approaches that compute outgoing electron wave functions by solving the Schrödinger equation [87,90–92] or using the relativistic Dirac-Hartree-Fock method [88,89]. These differences become particularly significant at high momentum transfers, where the effects of fast-moving DM are most pronounced. A recent study of ours [51] revisited and extended the atomic ionization formalism, systematically comparing different methods for formulating

the ionization form factor, specifically in the context of fast-moving DM interactions mediated by either a vector or a pseudoscalar mediator.

Building on the context described above, in this work, we fully generalize the program initiated in Ref. [51] by undertaking the following tasks:

- (i) Establishing the formalism for electron scattering with any fast-moving DM across a broad range of well-motivated DM scenarios.
- (ii) Identifying the effective regime in which atomic effects can be neglected, validating the use of the free-electron approximation.
- (iii) Demonstrating the complementarity between low- and high-energy-threshold experiments in the search for (fast-moving) DM.

First, in our formalism, the incoming (fast-moving) DM is treated as relativistic, rather than assuming the nonrelativistic limit (see, e.g., Ref. [89]), and the wave functions for both the bound electrons (initial state) and free electrons (final state) are taken as solutions to the Dirac equation instead of the Schrödinger equation. The resulting ionization form factors depend on both the momentum transfer and the recoil electron energy. This relativistic treatment provides the flexibility to accommodate various Lorentz interaction structures through the ionization form factors. We will consider scenarios where interactions between the SM particles and scalar or fermionic DM are mediated by scalar, pseudoscalar, vector, and axial vector mediators through renormalizable operators.

Obviously, the formalism should reduce to that of free electrons in the limit of large recoil energy, which also serves as a consistency check of the formalism. For a given recoil energy, the ionization form factor determines the range of momentum transfers capable of producing that recoil. In the low recoil energy regime, the form factor is spread over a broad range of allowed momenta. However, we will demonstrate that in the high recoil energy limit, the ionization form factor becomes sharply peaked around a single momentum transfer value—effectively approaching a Dirac δ function—thereby recovering the result for free electrons. The boundary between the low and high recoil energy regimes—and thus the boundary for the effective regime where the free-electron approximation is valid—depends on the details of the underlying model. We identify the parameter regions where incorporating the ionization form factor is essential for accurate signal rate estimates and quantify the resulting differences across all DM scenarios considered. We further find that, contrary to the common expectation that the ionization form factor suppresses event rates, its inclusion can enhance the signal, depending on the model specifics—particularly in the case of a pseudoscalar mediator.

Finally, we emphasize that the search for fast-moving DM benefits significantly from the complementarity between low- and high-energy-threshold experiments. Low-threshold

detectors, such as DM direct detection experiments, are particularly sensitive to low-energy electron recoils and can probe lighter fast-moving DM candidates or heavier DM with a mild boost. In contrast, high-threshold detectors, including large-volume neutrino observatories, excel at identifying higher-energy interactions associated with more massive or strongly boosted DM particles. Together, these experimental approaches cover a broad range of recoil energies and DM parameter space, enabling a more comprehensive and robust exploration of fast-moving DM models than would be possible with either class of detector alone. For illustration, we consider the XENONnT [94] and Super-Kamiokande [95] experiments, which respectively employ the ionization form factor and free-electron models to estimate signal sensitivity, in the contexts of two-component boosted DM and cosmic-ray-induced boosted DM scenarios.

This paper is organized as follows: in Sec. II, we provide the notation and the general formulation of the DM interaction with the bound electrons by providing the necessary formulas for the differential cross section and the elementary interactions of interest. Section III presents a quantitative numerical analysis that identifies the kinematic regions where the ionization form factor becomes significant and, consequently, where atomic physics effects materially impact the limits. Section IV presents sensitivity estimates both with the ionization form factor included and under the free-electron approximation, and compares the two, assuming a fixed reference DM flux. We then apply these results in Sec. V to two benchmark fast-moving DM scenarios with realistic fluxes: two-component boosted DM and cosmic-ray-induced boosted DM. Section VI provides the summary and outlook. Results for various DM-mediator spin combinations are presented in Appendix A.

II. SETUP

A. Electron recoils

1. Kinematics

In this section, we discuss the scattering between an electron and a DM particle—denoted by χ of mass m —incoming with a speed v and an energy E . We assume that the DM particle χ is either a fermion or a scalar that interacts elastically with an electron e , as follows:

$$\chi(p^\mu)e(k^\mu) \rightarrow \chi(p'^\mu)e(k'^\mu), \quad (1)$$

where p^μ and k^μ denote the four-momenta of the DM particle and the electron, respectively, with unprimed and primed quantities referring to the initial and final states. As shown in the Feynman diagram in Fig. 1, the interaction is mediated by a t -channel exchange of a particle X with mass m_X , which couples to χ and e via the couplings g_χ and g_e , respectively. As mentioned earlier, the mediator can be a vector, scalar, axial vector, or pseudoscalar, provided the interaction

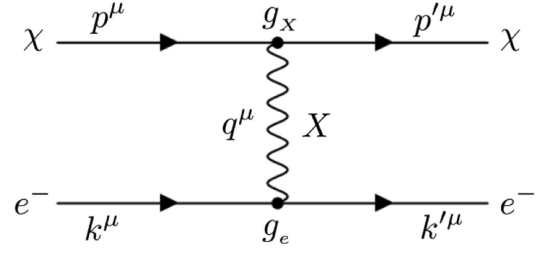


FIG. 1. Feynman diagram depicting the interaction between a DM particle and a target electron.

remains renormalizable. Further details on the interaction structures will be presented later.

For clarity and future reference, we adopt the following labeling for the four-momenta:

$$\begin{aligned} p^\mu &= (E, \mathbf{p}), & p'^\mu &= (E', \mathbf{p}'), \\ k^\mu &= (E_e, \mathbf{k}), & k'^\mu &= (E'_e, \mathbf{k}'). \end{aligned} \quad (2)$$

The deposited energy is defined as the energy difference between the outgoing electron and the incoming electron, $\Delta E \equiv E'_e - E_e$, while the momentum transfer is given by $\mathbf{q} \equiv \mathbf{p} - \mathbf{p}'$. Applying energy conservation and the on-shell condition, the deposited energy is given by

$$\Delta E = E - \sqrt{E^2 + q^2 - 2pq \cos \theta_q}, \quad (3)$$

where θ_q is the angle between $\mathbf{p}$ and $\mathbf{q}$. Note that we use the notation $p = |\mathbf{p}|$ and $q = |\mathbf{q}|$ to denote the magnitudes of the momenta—a convention we will maintain throughout the paper. Solving the above relation for q , we find

$$q = p \cos \theta_q \pm \sqrt{p^2 \cos^2 \theta_q + \Delta E(\Delta E - 2E)}. \quad (4)$$

Let us analyze the above equation in the limiting cases $\cos \theta_q = \pm 1$. First, for nonzero deposited energy, energy conservation implies that the quantity $\Delta E(\Delta E - 2E)$ is strictly negative, as it equals $E'^2 - E^2$. With this in mind, the expression under the square root becomes $E'^2 - m^2 \geq 0$, ensuring that the square root is real. Consequently, the deposited energy ΔE must be chosen carefully to guarantee that the momentum transfer q remains physical. The condition ensuring physical values of q , consistent with energy conservation, is given by¹

$$\Delta E \leq E - m. \quad (5)$$

Second, we note that the case when $\cos \theta_q = -1$ is physically not allowed since it leads to negative values as $q = -p \pm p'$, where we have used $p'^2 = E'^2 - m^2 \geq 0$.

¹ $\Delta E \geq E + m$ is also allowed, but it violates energy conservation.

The reason that q is negative here is that $p > p'$ for nonzero ΔE . We are now left with the condition $\cos \theta_q = 1$, which corresponds to the maximum and minimum values of q . We denote these values by $q^\pm$, which are determined by

$$q^\pm = p \pm \sqrt{p^2 + \Delta E(\Delta E - 2E)}. \quad (6)$$

Although the condition defined in Eq. (5) is required to ensure real values of q , energy conservation imposes an upper kinematic bound on ΔE . For a free, stationary electron, the maximum scattered electron energy occurs at $\cos \theta_q = 1$ and is given by

$$E_e'^{\max} = m_e \frac{(E - m_e)^2 + E^2 - m^2}{(E - m_e)^2 - E^2 + m^2}. \quad (7)$$

This leads to the maximum energy deposition

$$\Delta E^{\max} = E_e'^{\max} - E_e \leq E - m, \quad (8)$$

where the equality holds when $m = E_e$.

Finally, we comment on the nonrelativistic limit of the above equation. Using the approximation $E \approx m + \frac{1}{2}mv^2$, where v is the velocity of the DM particle, and assuming $m \gg \Delta E$, we obtain

$$q_{\text{non-rel}}^\pm \approx p \pm \sqrt{p^2 - 2m\Delta E}. \quad (9)$$

Here, we have neglected the $\frac{1}{2}m\Delta Ev^2$ term, as it is subleading compared to the p^2 term. Equation (9) agrees with the result in Ref. [89]. For the nonrelativistic case, one uses Eq. (9) with $p \approx mv$, while in the relativistic case, one uses Eq. (6) with $p = \gamma mv$ and $E = \gamma m$, where γ is the Lorentz factor.

2. Generalized differential cross section

We now discuss the full formulation of the cross section for the χ - e scattering, taking into account that electrons are bound to atoms. For concreteness, we use the fully relativistic, spin-averaged matrix element squared, denoted by $|\mathcal{M}|^2$, throughout this paper. In the DM literature—particularly in the context of WIMP physics—a nonrelativistic treatment of the matrix element squared is often preferred. This is typically done using the Born approximation, where the cross section is parametrized as

$$\sigma_B = \frac{\mu^2}{16\pi m^2 m_e^2} \overline{|\mathcal{M}|_B^2}, \quad (10)$$

where μ is the χ - e reduced mass, and $\overline{|\mathcal{M}|_B^2}$ is given by

$$\overline{|\mathcal{M}|_B^2} = \frac{16g_e^2 g_X^2 m_e^2 m^2}{(q^2 + m_X^2)^2}. \quad (11)$$

For later uses, we define the fully relativistic dark matter form factor $|F_{\text{DM}}|^2$ and its Born approximation counterpart $|F_{\text{DM}}|_B^2$ as follows:

$$|F_{\text{DM}}|^2 \equiv \frac{\overline{|\mathcal{M}|^2}}{|\mathcal{M}|_B^2(q = \alpha_{\text{em}} m_e)}, \quad (12)$$

$$|F_{\text{DM}}|_B^2 \equiv \frac{\overline{|\mathcal{M}|_B^2}}{|\mathcal{M}|_B^2(q = \alpha_{\text{em}} m_e)}, \quad (13)$$

where we have chosen a reference momentum transfer of $\alpha_{\text{em}} m_e$, with α_{em} denoting the fine-structure constant. Defining the parameter $\bar{\sigma} \equiv \sigma_B(q = \alpha_{\text{em}} m_e)$, which serves as a reference cross section and encapsulates model-dependent parameters such as couplings, we can express the squared matrix element as

$$\overline{|\mathcal{M}|^2} = \frac{16\pi m^2 m_e^2 \bar{\sigma}}{\mu^2} \times |F_{\text{DM}}|^2. \quad (14)$$

Hence, the differential cross section can be generally written as [40]

$$d\sigma = \frac{m^2 m_e^2 \bar{\sigma}}{4\pi E E' E_e E_e' \mu^2 v} |F_{\text{DM}}|^2 \times \delta(E' + E_e' - E - E_e) \frac{|f_{e_i \rightarrow e_f}|^2}{(2\pi)^3} d^3 q d^3 k', \quad (15)$$

where $|f_{e_i \rightarrow e_f}|$ is the transition factor from a bound state electron e_i to a free electron e_f and the volume element $d^3 p'$ is replaced with $d^3 q$. The Dirac δ function here can be re-expressed in terms of the angle, θ_q , as

$$\delta(E' - (E + E_e - E_e')) = \frac{E'}{pq \sin \theta_q} \delta(\theta_q). \quad (16)$$

Since the transition factor does not depend on the angular component of the vector, $\mathbf{q}$, we can integrate over the solid angle element $d\hat{\Omega}_q$, simplifying the differential cross section to

$$d\sigma = \frac{m^2 m_e^2 \bar{\sigma}}{2\mu^2} \frac{q dq}{p E E_e E_e' v} |F_{\text{DM}}|^2 \frac{|f_{e_i \rightarrow e_f}|^2}{(2\pi)^3} d^3 k'. \quad (17)$$

We now express the differential cross section in terms of the recoil energy E_r , defined via $E_e' = m_e + E_r$. Using the relation $d^3 k' = E_e' k' d\Omega_{k'} dE_r$, we obtain

$$\frac{d\sigma}{dE_r} = \frac{m^2 m_e^2 \bar{\sigma}}{4\mu^2 p k'^2 E E_e v} \int_{q^-}^{q^+} q dq |F_{\text{DM}}|^2 |f_{\text{ion}}|^2, \quad (18)$$

where $|f_{\text{ion}}|^2$ is the dimensionless ionization form factor, defined as

$$|f_{\text{ion}}|^2 \equiv \frac{2k'^3}{(2\pi)^3} \int d\Omega_{k'} |f_{e_i \rightarrow e_f}|^2. \quad (19)$$

For a spherically symmetric transition factor squared, we have

$$|f_{\text{ion}}|^2 = \frac{4k'^3}{(2\pi)^2} |f_{e_i \rightarrow e_f}|^2. \quad (20)$$

Finally, the differential cross section above becomes

$$\frac{d\sigma}{dE_r} = \frac{m^2 m_e^2 \bar{\sigma}}{4\mu^2 p k'^2 E E_e v} \int_{q^-}^{q^+} q dq |F_{\text{DM}}|^2 |f_{\text{ion}}|^2. \quad (21)$$

It is straightforward to see that the fully relativistic treatment of the differential cross section smoothly reduces to the standard WIMP case in the nonrelativistic limit, under the following substitutions:

$$p \sim mv, \quad E E_e \sim m m_e, \quad \text{and} \quad k' \sim \sqrt{2m_e E_r}. \quad (22)$$

This yields the standard WIMP formula

$$\frac{d\sigma^{\text{WIMP}}}{dE_r} = \frac{\bar{\sigma}_B}{8\mu^2 E_r v^2} \int_{q_{\text{NR}}}^{q_{\text{NR}}^+} q dq |F_{\text{DM}}|_B^2 |f_{\text{ion}}|^2. \quad (23)$$

Note that we have used the approximate form factor $|F_{\text{DM}}|_B^2$ in place of the full relativistic form $|F_{\text{DM}}|^2$, and the nonrelativistic limits $q_{\text{NR}}^\pm$ instead of $q^\pm$. In this context, the form factor typically takes one of two limiting forms:

$$|F_{\text{DM}}|_B^2 = \begin{cases} 1 & \text{if } m_X \gg q, \\ \frac{\alpha_{\text{em}}^2 m_e^2}{q^2} & \text{if } m_X \ll q. \end{cases} \quad (24)$$

Equation (21) provides the full treatment of the χ - e interaction, incorporating atomic physics effects through the ionization form factor. It is instructive to show that the simpler case of a free, stationary electron—neglecting atomic effects—can be easily recovered. This is accomplished by making the following replacement in Eq. (15):

$$|f_{e_i \rightarrow e_f}|^2 \rightarrow (2\pi)^3 \delta^3(\mathbf{q} - \mathbf{k}'), \quad (25)$$

which corresponds to treating the electron as free and at rest. For a spherically symmetric ionization form factor, this replacement implies

$$|f_{\text{ion}}|^2 \rightarrow 2 \frac{k'^3}{q^2} \delta(q - k'). \quad (26)$$

Substituting this into the cross section expression yields the simplified differential cross section for a free electron:

$$\frac{d\sigma^{\text{free}}}{dE_r} = \frac{m^2 m_e \bar{\sigma}}{2\mu^2 p^2} |F_{\text{DM}}|_{q=k'}^2, \quad (27)$$

where we have used $E_e = m_e$ for an electron at rest. In the following section, we examine the effects of atomic physics by analyzing the impact of the ionization form factor $|f_{\text{ion}}|^2$ on the kinematically allowed region of the parameter space. Before doing so, however, we first describe the class of interactions under consideration.

B. Dark matter models

Here, we outline the types of interactions we aim to study, classifying them based on the spin of the DM particle and the structure of the Lorentz interaction. In this work, we consider a generic DM particle χ in two forms: a spin-1/2 fermion denoted by ψ and a spin-0 scalar denoted by ϕ . The generic mediator particle X responsible for the interaction is classified as one of the following types: vector (V^μ), scalar (S), axial vector (A^μ), or pseudoscalar (P). Given the set of dark matter particles

$$\chi = \{\psi, \phi\}, \quad (28)$$

and the set of the mediators

$$X = \{V^\mu, S, A^\mu, P\}, \quad (29)$$

there are seven distinct combinations that can be described by renormalizable interaction Lagrangians. To clarify our notation, each interaction type is abbreviated using two characters: the first denotes the DM particle type, and the second denotes the mediator type. For example, FV refers to fermionic dark matter with a vector mediator, while SP represents scalar dark matter with a pseudoscalar mediator. The coupling between a particle and a mediator is denoted by superscripting the particle symbol and subscripting the mediator symbol. For example, the coupling between fermionic dark matter and an axial vector mediator is labeled as g_A^ψ , while the coupling of the electron to a scalar mediator is labeled as g_S^e .

In all cases, the spin-averaged squared matrix element shares a common denominator of the form $(m_X^2 - t)^2$, where t is the Mandelstam variable associated with the t -channel exchange diagram, as shown in Fig. 1. Furthermore, as evident from Eqs. (21) and (27), the cross section appears as an overall factor in the reference cross section $\bar{\sigma}$. Consequently, all interaction scenarios include this common prefactor, defined by Eqs. (10) and (11). For clarity, we denote it as

$$\overline{\sigma}_{\chi X} = \frac{\mu^2}{\pi} \frac{(g_X^e g_X^\chi)^2}{(m_X^2 + \alpha_{\text{em}}^2 m_e^2)^2}. \quad (30)$$

This notation is important, as we will present exclusion plots based on this cross section for various interaction types. For example, in the case of fermionic DM with a vector mediator, limits on the cross section are expressed as bounds

TABLE I. Dimensionless parameters used in the shape analysis of the coupling-normalized matrix element squared, $|\overline{\mathcal{A}}|^2$. Except for the boost factor, all quantities are normalized by the electron mass.

Parameter	Definition
Boost factor γ	E/m
Dark matter mass α	m/m_e
Mediator mass β	m_X/m_e
Electron recoil energy x	E_r/m_e

on $\overline{\sigma}_{\text{FV}}$, from which constraints on the couplings and other model parameters can be derived accordingly.

We note that throughout this work, we use the full expression for the differential cross section, as given in Ref. [41], for the different interaction types. Nevertheless, it is instructive to examine the analytic behavior of the differential cross section in two limiting cases, as will be discussed shortly. For practical purposes, and in terms of the squared matrix element, the differential cross section for scattering off free electrons can be derived from Eq. (27) as

$$\frac{d\sigma_{\chi X}^{\text{free}}}{dE_r} = \frac{(g_X^e g_X^\gamma)^2}{32\pi m_e m_\chi^2 \gamma^2} |\mathcal{A}|_{\chi X}^2, \quad (31)$$

where the condition $\gamma \gg 1$ is assumed, and the coupling-normalized matrix element squared is defined as

$$|\overline{\mathcal{A}}|_{\chi X}^2 \equiv \frac{|\mathcal{M}|_{\chi X}^2}{(g_X^e g_X^\gamma)^2}. \quad (32)$$

For simplicity, we express the cross section parameters in terms of the four dimensionless quantities defined in Table I. In this case, the differential cross section becomes

$$\frac{d\sigma_{\chi X}^{\text{free}}}{dx} = \frac{(g_X^e g_X^\gamma)^2}{32\pi m_e^2} \frac{|\overline{\mathcal{A}}|_{\chi X}^2(x, \alpha, \gamma, \beta)}{\alpha^2 \gamma^2}, \quad (33)$$

where we explicitly express that $|\overline{\mathcal{A}}|_{\chi X}^2$ is a function of the four dimensionless parameters.

After expressing $|\overline{\mathcal{A}}|^2$ in terms of the dimensionless parameters, we analyze its behavior in two limiting cases: (i) the heavy mediator limit ($\beta \gg 1$), where the expressions are expanded and truncated at terms of order $\mathcal{O}(1/\beta^6)$, and (ii) the light mediator limit ($\beta \ll 1$), where the expressions are expanded and truncated at terms of order $\mathcal{O}(\beta^2)$. Table II summarizes the leading-order behavior of the cross section in the two limiting cases.

Instead of describing each limiting case for every interaction individually—and to avoid repetition—we highlight the generic features that emerge across interaction types as follows:

- (i) *Heavy mediators*: in the heavy mediator regime ($\beta \gg 1$), the differential cross sections generally do not exhibit a decreasing trend with recoil energy E_r , denoted here as x . Most distributions start out flat in x , although some cases show a rising behavior depending on specific parameter choices. A notable exception occurs in pseudoscalar-mediated interactions: the FP case displays a quadratic rise, while the SP case exhibits a linear increase with x . The flat behavior is particularly pronounced when the squared matrix element includes constant terms (independent of x) proportional to γ^2 , which is especially evident in the FV, FA, and SV cases. For example, in the FV case, the distribution transitions from flat to rising when $x \gg \sqrt{2}\alpha\gamma$, a condition satisfied only within a restricted region of parameter space where $\sqrt{2}\alpha\gamma < x^{\text{max}}$. In contrast, scalar-mediated interactions (FS and SS) exhibit weaker flatness. The FS case transitions from flat to rising when $x \gg 2\alpha$, while for the SS case, this transition occurs at $x > 2$.
- (ii) *Light mediators*: in the light mediator regime ($\beta \ll 1$), the differential cross section distributions are generally falling or flat. With the exception of the FP case, all interactions exhibit a decreasing trend at small values of x , and in some cases, this trend continues up to x^{max} . This falling behavior is particularly characteristic of the three scalar dark matter interactions: SV, SS, and SP. In other cases, the extent

TABLE II. Summary table of the coupling-normalized matrix element squared, $|\overline{\mathcal{A}}|^2$ in terms of the dimensionless parameters defined in Table I at two mediator mass limits: heavy and light mediators.

χX	Heavy mediators: $ \overline{\mathcal{A}} _{\chi X}^2(x, \alpha, \gamma, \beta \gg 1)$	Light mediators: $ \overline{\mathcal{A}} _{\chi X}^2(x, \alpha, \gamma, \beta \ll 1)$
FV	$8 \frac{2\alpha^2 \gamma^2 - x(\alpha^2 + 2\alpha\gamma + 1) + x^2}{\beta^4}$	$2 - \frac{2(\alpha^2 + 2\alpha\gamma + 1)}{x} + \frac{4\alpha^2 \gamma^2}{x^2}$
FS	$4 \frac{4\alpha^2 + 2x(1 + \alpha^2) + x^2}{\beta^4}$	$1 + \frac{2(\alpha^2 + 1)}{x} + \frac{4\alpha^2}{x^2}$
FA	$8 \frac{2\alpha^2(2 + \gamma^2) + x(1 - 2\alpha\gamma + \alpha^2) + x^2}{\beta^4}$	$2 + \frac{16\alpha^2}{\beta^4} + \frac{2(1 - 2\alpha\gamma + \alpha^2)}{x} + \frac{4\alpha^2(1 + \gamma^2)}{x^2}$
FP	$4 \frac{x^2}{\beta^4}$	$1 - \frac{\beta^2}{x}$
SV	$8\alpha \frac{2\alpha\gamma^2 - x(\alpha + 2\gamma)}{\beta^4}$	$2\alpha \frac{2\alpha\gamma^2 - x(\alpha + 2\gamma)}{x^2}$
SS	$8\alpha^2 \frac{2 + x}{\beta^4}$	$2\alpha^2 \frac{2 + x}{x^2}$
SP	$\frac{8x\alpha^2}{\beta^4}$	$\frac{2\alpha^2}{x}$

of the decrease depends on the value of the γ parameter. For instance, in the FV case, the distribution transitions from falling to flat when $x \gg \sqrt{2}\alpha\gamma$, and a similar transition occurs in the FS case when $x \gg 2\alpha$.

- (iii) *General observations:* the analysis reveals several distinctive features associated with specific interaction types. Scalar and pseudoscalar mediators show no dependence on the γ parameter, a property that holds beyond leading-order approximations. Additionally, the FP interaction is entirely independent of α , whereas all scalar dark matter interactions (SV, SS, and SP) exhibit at least a linear dependence on α .

Overall, these observations illustrate how the mediator type, kinematic variables, and interaction structure collectively shape the behavior of differential cross sections, offering valuable insights into expected DM detection rates and exclusion limits, which will be elaborated upon shortly.

C. Ionization form factor

Here, we aim to provide a quick overview of the ionization form factor. The ionization form factor $|f_{\text{ion}}|^2$ relies on the estimation of the transition factor squared $|f_{e_i \rightarrow e_f}|^2$, which is defined as the overlap of the initial bound electron state $|e_i\rangle$ and the final free-electron state $|e_f\rangle$. The general transition factor is given by

$$f_{e_i \rightarrow e_f} = \langle e_f | \mathcal{O} e^{i\mathbf{q}\cdot\mathbf{r}} | e_i \rangle = \int d^3\mathbf{r} \psi_{e_f}^*(\mathbf{r}) \mathcal{O} e^{i\mathbf{q}\cdot\mathbf{r}} \psi_{e_i}(\mathbf{r}), \quad (34)$$

where $\mathcal{O}$ is the operator associated with the mediator type, ψ_{e_i} is the initial state electron wave function representing the original bound electron, and ψ_{e_f} is the final state electron wave function representing the outgoing electron. For concreteness, we consider the vector mediator case for which $\mathcal{O} = \mathbf{1}$. In this work, we focus our attention on the relativistic ionization form factor, while different treatments have been considered in various studies; see, for example, Refs. [40,86–91,93,96–100].

The use of relativistic treatment allows for simple natural generalization for the various Lorentz interactions as has been pointed out in Ref. [89]. The transition factor in this case is defined through wave functions that are solutions to the Dirac equation as

$$f_{e_i \rightarrow e_f}^{\text{DCE}} = \int d^3\mathbf{r} \psi_{E,\kappa',m'}^\dagger(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} \psi_{n\kappa m}(\mathbf{r}), \quad (35)$$

where the superscript DCE explicitly indicates the fact that the ionization form factor is related to the Dirac continuous energy case. Here $\psi_{n\kappa m}$ is the general bound-state solution to the eigenvalue problem $\hat{H}\psi_{n\kappa m} = E_{n\kappa}\psi_{n\kappa m}$, where $\hat{H}$ is the relativistic Dirac Hamiltonian

$$\hat{H} = \vec{\alpha} \cdot \hat{\mathbf{p}} + m_e(\beta - 1) + V_{\text{eff}}(r), \quad (36)$$

where $\vec{\alpha}$ and β denote the usual Dirac matrices in the standard representation. The effective potential V_{eff} depends solely on the radial coordinate, assuming that the electrons are subject to a spherically symmetric potential. We employ the Dirac-Hartree-Fock-Slater form for V_{eff} that implements the electron-nucleus and electron-electron interactions [98].² Therefore, the solution $\psi_{n\kappa m}$ depends on radius $r = |\mathbf{r}|$ only, as follows:

$$\psi_{n\kappa m}(\mathbf{r}) = \frac{1}{r} \begin{pmatrix} P_{n\kappa}(r) \Omega_{\kappa m}(\hat{\mathbf{r}}) \\ i Q_{n\kappa}(r) \Omega_{-\kappa m}(\hat{\mathbf{r}}) \end{pmatrix}, \quad (37)$$

where $\Omega_{\kappa m}$ is the two-component spherical spinor. Note that the above full Dirac wave function is a four-component spinor, with the upper (lower) two components corresponding to the large (small) component radial wave functions, denoted by P (Q). For the free-state solution $\psi_{E,\kappa m}$, the principal quantum number n is replaced with the recoil energy E_r . Finally, the Dirac quantum number κ is determined by the orbital and total angular momentum quantum numbers, ℓ and j , respectively, as follows:

$$\kappa = (\ell - j)(2j + 1). \quad (38)$$

The operator between the two wave functions in Eq. (35) is governed by the form of the interaction. In other words, this is where the model dependence appears in the ionization form factor. For example, as specified earlier, Eq. (35) assumes that the dark matter mediator is a vector boson that interacts with electrons through a vector coupling. In this case, the operator is simply $e^{i\mathbf{q}\cdot\mathbf{r}}$ times a 4×4 identity matrix I , which leaves the large and small components unmixed when performing the dot product. We can readily consider the situation where the mediator interacts with electrons through scalar (γ^0), axial vector (γ_5), or pseudo-scalar ($\gamma^0\gamma_5$) with the identity matrix in Eq. (35) replaced by the γ matrices in the respective brackets as per Ref. [89]. Denoting the vector, scalar, axial vector, and pseudoscalar mediators by V, S, A, and P, respectively, we express the squared transition factor for each mediator as follows:

$$\begin{aligned} |f_{e_i \rightarrow e_f}^{\text{DCE}}|_V^2 &= \sum_{\kappa'} \sum_L C_{\kappa\kappa'}^L (R_{P,P} + R_{Q,Q})^2, \\ |f_{e_i \rightarrow e_f}^{\text{DCE}}|_S^2 &= \sum_{\kappa'} \sum_L C_{\kappa\kappa'}^L (R_{P,P} - R_{Q,Q})^2, \\ |f_{e_i \rightarrow e_f}^{\text{DCE}}|_A^2 &= \sum_{\kappa'} \sum_L D_{\kappa\kappa'}^L (R_{P,Q} - R_{Q,P})^2, \\ |f_{e_i \rightarrow e_f}^{\text{DCE}}|_P^2 &= \sum_{\kappa'} \sum_L D_{\kappa\kappa'}^L (R_{P,Q} + R_{Q,P})^2, \end{aligned} \quad (39)$$

where $L = |\ell - \ell'|$ with ℓ and ℓ' encapsulated in κ . Here, R is the radial integral, and the two subscripts, either P or Q ,

²We assume that the interactions potentially arising via new mediator exchanges are negligible.

indicate which components contribute to the integral, as follows:

$$R_{X,Y} = \int_0^\infty X_{E,\kappa'} Y_{n\kappa} j_L(qr) dr, \quad (40)$$

where $j_L(x)$ is the usual spherical Bessel function. Finally, the overall coefficient $C_{\kappa\kappa'}^L$ is written in terms of the Wigner 3- j symbol as

$$C_{\kappa\kappa'}^L = \frac{1}{4} (-1)^{j+j'-\ell-\ell'} (2L+1) \begin{pmatrix} \ell' & \ell & L \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{pmatrix} j' & L & j \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}^{-2} \left[(-1)^{j+j'-\ell-\ell'} (2j+1)(2j'+1) \begin{pmatrix} \ell' & \ell & L \\ 0 & 0 & 0 \end{pmatrix}^2 \right. \\ \left. + 8\sqrt{\ell'(\ell'+1)\ell(\ell+1)} \begin{pmatrix} \ell' & \ell & L \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell' & \ell & L \\ -1 & 1 & 0 \end{pmatrix} - 4(\kappa'+1)(\kappa+1) \begin{pmatrix} \ell' & \ell & L \\ -1 & 1 & 0 \end{pmatrix}^2 \right]. \quad (41)$$

The coefficient $D_{\kappa\kappa'}^L$ has a similar form structure and can be obtained from the expression for $C_{\kappa\kappa'}^L$ by making simple index substitutions: specifically, replacing κ with $-\kappa$ and ℓ with $|1/2 - \kappa| - 1/2$. For a detailed discussion of the numerical stability of the above integrals and the methods used to compute the relativistic wave functions, see Refs. [51,88,89,98]. Reference [101] highlighted the importance of accurately calculating the ionization form factor, which requires precise wave-function modeling to preserve the orthogonality of free-electron states. Our results show excellent agreement with Ref. [101], as well as with Ref. [89].

III. EFFECTIVE LIMIT OF IONIZATION FORM FACTOR

Equipped with the full relativistic treatment, we return to one of our original questions: “How does the ionization form factor deform the cross section?” In particular, we are interested in whether the full treatment of the cross section incorporating the ionization form factor in Eq. (21) can be approximated to the simpler free-electron formalism in Eq. (27). To this end, we investigate two aspects: we study (i) how the ionization form factor behaves at increasing electron recoil energies and whether it converges to a desired asymptotic form, and (ii) the impact of the ionization form factor on the cross section for a wide range of the kinematic parameter space, focusing on low electron recoil energies only.

A. Asymptotic behavior at high energies

As we mentioned earlier in Eqs. (25)–(26), at some limit, the ionization form factor should be able to retrieve the free-electron case. To this end, we take the ionization form factor, varying the electron recoil energy as in Fig. 2. For illustration purposes, we chose two example energy shells of the xenon atom: 2 p (top panel) and 3 s (bottom panel). We study the shape of the ionization form factor as the electron recoil energy increases: 100 keV (black), 1 MeV (red), 10 MeV (green), and 100 MeV (blue) for all of the mediator types. At each energy, we show the ionization form

factor curves: vector (solid), axial vector (dashed), scalar (dot-dashed), and pseudoscalar (dotted). We observe that, at each energy E_r , the curves are centralized at momentum transfer values of $q = k'$ as required by momentum conservation. At low electron recoil, the curves are broadly distributed, showing clear model dependence between various mediator types. However, as the recoil energy increases, the curves exhibit a sharper peak centered at $q = k'$. We clearly see that as the energy increases, the shape of the ionization form factor behaves like a Dirac δ function, conforming to the free-electron treatment. In other words, if the electron recoil energy is high ($E_r \gtrsim 10$ MeV), the ionization form factor is unlikely to deform the cross section, validating the free initial electron assumption.

B. Effective behavior at low energies

Our results in Fig. 2 clearly suggest that the effects due to the ionization form factor are significant at low electron recoil energies. In fact, many DM detectors operate at electron recoil energy near the keV scale, making them sensitive to the ionization form factor. We now aim to quantitatively determine the validity of the free-electron approximation by numerically analyzing the relevant regions of parameter space. A straightforward approach to assess this is by comparing the total cross sections—for example, through the quantity $|1 - \sigma/\sigma^{\text{free}}|$, where σ denotes the total cross section computed from Eq. (21) and σ^{free} corresponds to that from Eq. (27). While this method can provide a useful estimate of the approximation’s validity in many cases, it may be less reliable in others. This limitation arises because integrating to obtain the total cross section inherently discards valuable information about the shape of the differential cross section. To retain the shape information, we define the following measure³:

³This is an example measure. We find that similar conclusions can be drawn using alternative measures, such as mean absolute difference, maximum absolute difference, and maximum square difference.

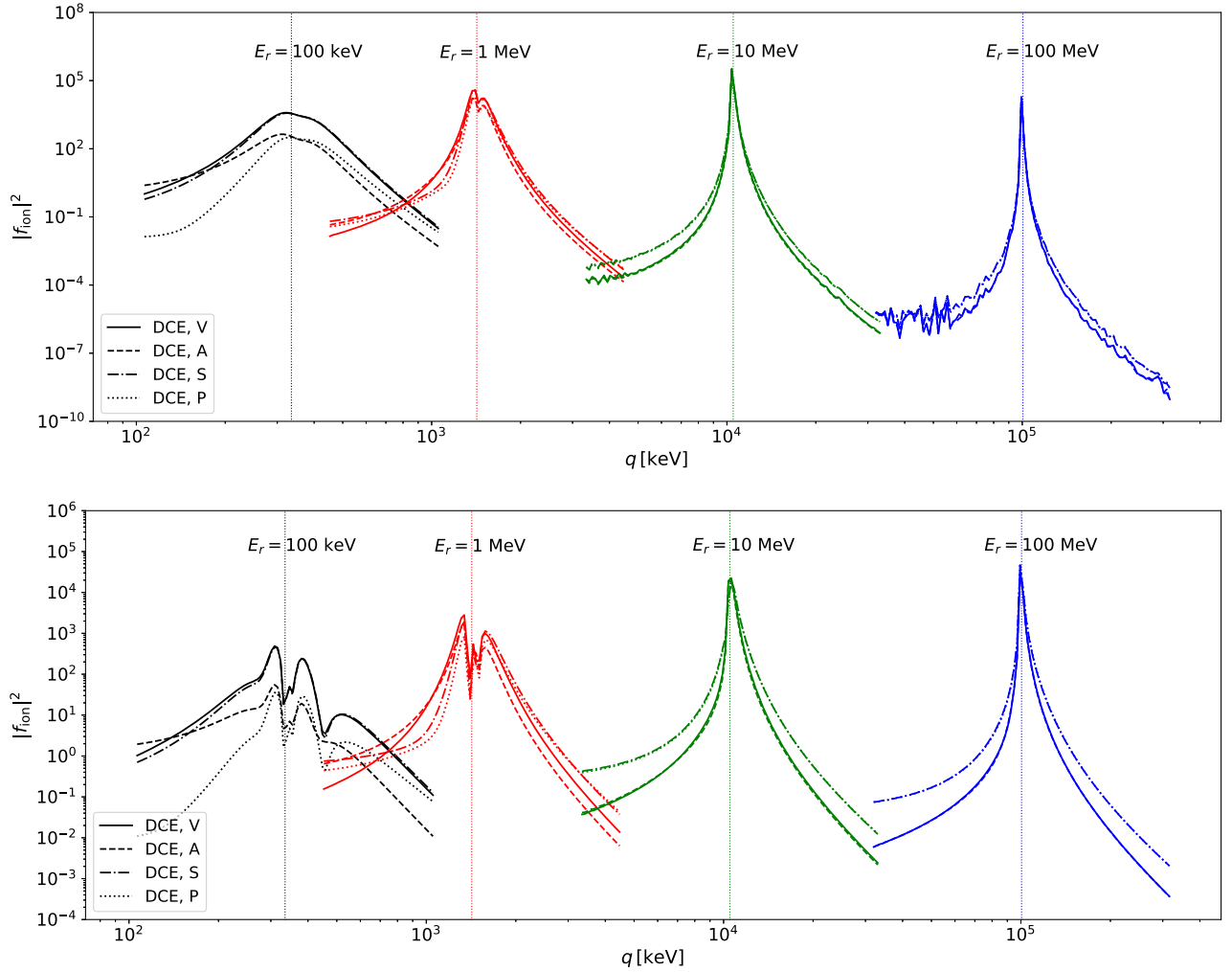


FIG. 2. Asymptotic behavior of the ionization form factor of the xenon atom for the 2 p (top) and 3 s (bottom) shells. All ionization form factor curves are estimated at various electron recoil energies: 100 keV (black), 1 MeV (red), 10 MeV (green), and 100 MeV (blue). The four mediator cases are represented by solid (vector), dashed (axial vector), dot-dashed (scalar), and dotted (pseudoscalar) curves. The vertical dotted lines represent the corresponding q' value for each of the given E_r values.

$$R_\sigma \equiv \sqrt{\frac{\sum_{i=1}^N \left(\frac{d\sigma^{\text{free}}}{dE_r^i} - \frac{d\sigma}{dE_r^i} \right)^2}{\sum_{i=1}^N \left(\frac{d\sigma}{dE_r^i} \right)^2}}. \quad (42)$$

One may argue that the free-electron assumption is valid in regions where R_σ approaches zero, but becomes unreliable when R_σ deviates significantly from zero. For illustration, we consider the recoil energy range from 0 to 200 keV with a fixed interval of 0.5 keV. Consequently, the index i runs from 1 to $N = 400$. Although this choice is primarily illustrative, the selected energy range is relevant to certain DM detectors, such as the XENONnT experiment. We choose energy levels from 3s to 5p, which account for effectively 44 electrons of the xenon atom, as we find that the contributions from the remaining innermost electrons are negligible.

In Fig. 3, we show color maps of R_σ in the kinematic parameter space (m_χ, γ) for the vector mediator, shown at

three representative mediator masses: $m_\chi = 50$ keV (left), $m_\chi = 100$ MeV (center), and $m_\chi = 3m_\chi$ (right). We henceforth adopt these mass choices for the rest of the paper. Several noteworthy observations follow. Notice that the range of the color map is different based on the case. First and foremost, the validity of the free-electron assumption—and consequently, the necessity of incorporating the ionization form factor—depends critically on the specifics of the underlying model. For example, when the mediator mass is $m_\chi = 100$ MeV, R_σ ranges from 0.003 to 0.8, whereas for $m_\chi = 3m_\chi$, it spans from 0.003 to 2. Nevertheless, a common feature in all panels is the hatched region in the bottom left corner, corresponding to portions of the parameter space that cannot yield electron recoil energies above 1 keV.

At a fixed mediator mass, as illustrated in the left and center panels, we observe a gradual decrease in the value of

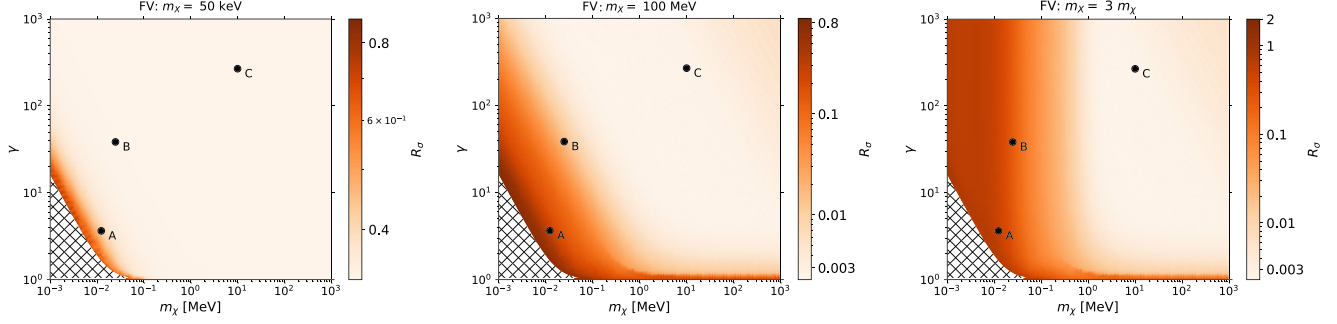


FIG. 3. R_σ for the FV case representing different mediator mass regimes of $m_\chi = 50$ keV (left), $m_\chi = 100$ MeV (center), and $m_\chi = 3m_\chi$ (right). The color scale in each panel is normalized to the maximum R_σ value within that panel. The hatched region corresponds to the portion of the parameter space that does not yield an electron recoil above 1 keV.

R_σ from the bottom left to the top right corners. This trend indicates that atomic effects on the cross section diminish with increasing energy γm_χ . This observation is consistent with the common expectation that electrons can be approximated as free particles when the incoming DM particle carries sufficiently high energy. While this holds true in many cases, the precise behavior depends on the specific form of the squared matrix element and the dynamics of its convolution with the ionization form factor. For example, the descent in the R_σ value to the top right corner is more pronounced for a mediator mass of 100 MeV than for 50 keV, as heavier mediators are more likely to facilitate larger momentum transfer to the target electron. In contrast, the value of R_σ for the dynamic choice $m_\chi = 3m_\chi$ (right panel) appears more sensitive to the DM mass than to the boost factor.

These observations indicate that the impact of the ionization form factor is highly dependent on the mediator mass choice, making it too difficult to draw a generalized conclusion. To develop our intuition on the atomic effects in the FV case, we choose three representative benchmark points A, B, and C (marked in Fig. 3), as follows:

$$\begin{aligned} \text{Point A} &= \{m_\chi = 15 \text{ keV}, \gamma = 4\}, \\ \text{Point B} &= \{m_\chi = 25 \text{ keV}, \gamma = 40\}, \\ \text{Point C} &= \{m_\chi = 10 \text{ MeV}, \gamma = 300\}. \end{aligned} \quad (43)$$

For these benchmark points, the energy γm_χ ranges from well below the electron mass (A), to near the electron mass (B), and to significantly above the electron mass (C). The full differential cross section given by Eq. (21) (dashed red) and that of the free-electron assumption in Eq. (27) (solid blue) at the three points are contrasted in Fig. 4. Here, the points A, B, and C are arranged from top to bottom, while the mediator mass scenarios $m_\chi = 50$ keV, $m_\chi = 100$ MeV, and $m_\chi = 3m_\chi$ are arranged from left to right.

In the left panel, the mediator mass is chosen to be much smaller than that of the electron mass, corresponding to the FV case in the $\beta \ll 1$ limit as defined in Table II. For small

values of x , i.e., electron recoil energies much smaller than the electron mass, the dominant contribution in the matrix element squared is due to the $\alpha^2 \gamma^2 / x^2$ term. This consequently makes the differential cross section in Eq. (33) dominated by a $1/E_r^2$ dependence, leading to the falling behavior observed in all distributions in the left panel. Moreover, the distributions incorporating atomic effects closely resemble those based on the free-electron treatment, with only mild deviations. This is reflected in the estimated R_σ measure, which decreases from 0.43 to 0.33 as we move from point A to point C. Neglecting atomic physics effects in this regime can lead to errors in the cross section at the level of several tens of percent.

In the center panel, the mediator mass lies in the $\beta \gg 1$ regime (see the left column of Table II for the FV case). In this limit, the matrix element squared is dominated by the constant term, making the leading contribution independent of the electron recoil energy. As a result, the differential cross section becomes approximately flat over a broad range of electron recoil energy values. We observe that the cross section obtained using the atomic effects treatment closely matches that of the free-electron approximation, exhibiting minimal deviations. Indeed, the associated error remains well below the percent level across most of the parameter space, as illustrated in the center panel of Fig. 3.

In the right panel, the mediator mass is chosen as $m_\chi = 3m_\chi$, making it dependent on the DM mass. Consequently, the parameter β spans a wide range, varying from values much smaller than 1 to those much greater than 1. As the DM mass increases, the form of the matrix element squared transitions from that corresponding to the right column to the left column of Table II. First, this explains why the right panel of Fig. 3 exhibits growing sensitivity to the DM mass. Second, it accounts for the gradual shift in the distributions shown in the right panels of Fig. 4—from steeply falling to nearly flat—as one moves from the top to the bottom panel, i.e., as the DM mass increases. In effect, this choice of mediator mass encapsulates and reinforces the observations discussed above. The discrepancy between the atomic effects treatment and

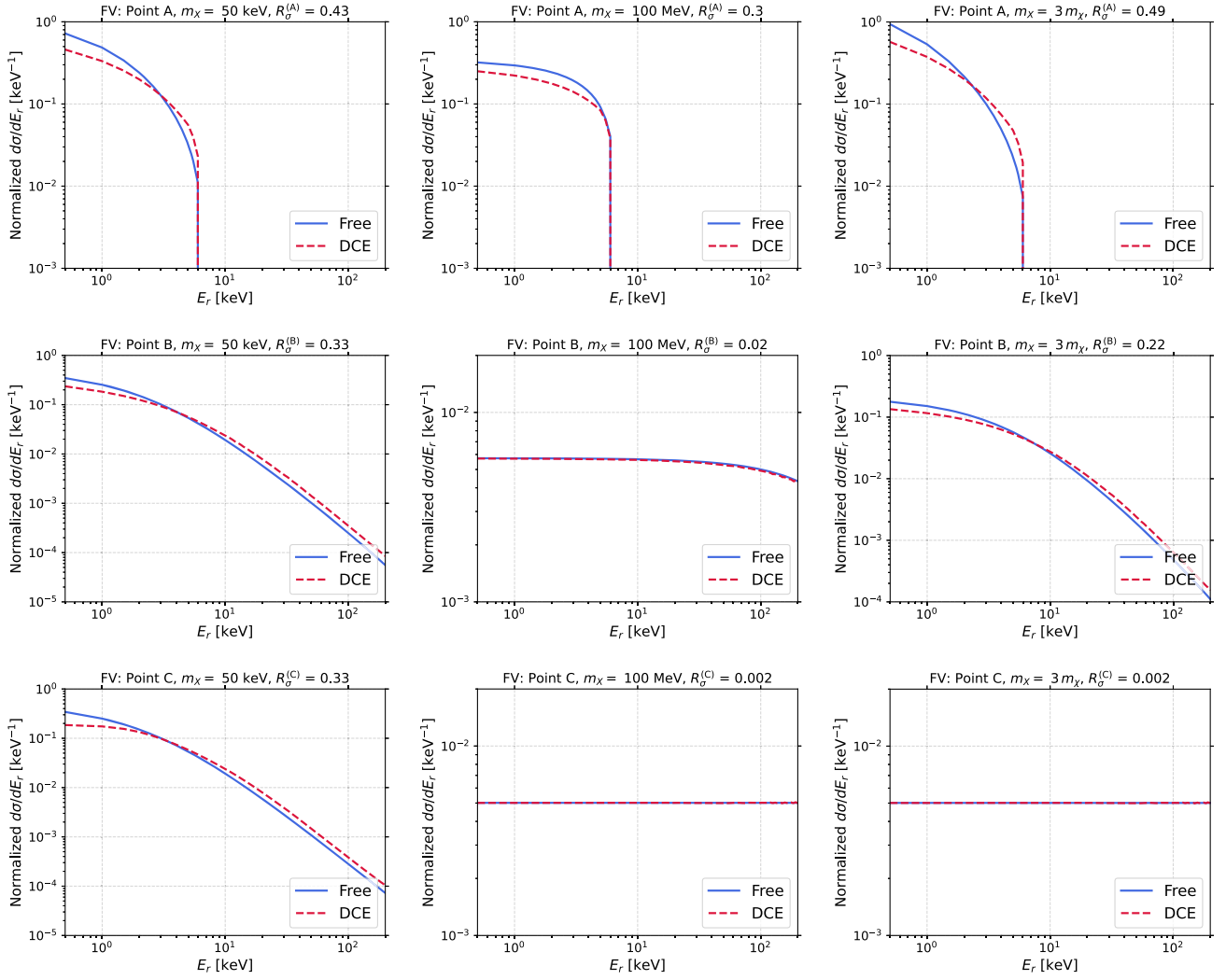


FIG. 4. Differential cross sections for the FV case in terms of the electron recoil energy for the free-electron assumption in solid blue and for the relativistic ionization form factor in dashed red. Left, center, and right panels, respectively, correspond to mediator masses $m_\chi = 50$ keV, $m_\chi = 100$ MeV, and $m_\chi = 3m_\chi$. Top, middle, and bottom panels, respectively, correspond to points A, B, and C as indicated in Fig. 3.

the free-electron approximation ranges from substantial at low dark matter masses to negligible at high masses. This variation reflects the underlying transition from a light mediator regime (on the order of a few keV) to a heavy mediator regime (on the order of a GeV).

A key observation that encapsulates the FV case is that the differential cross section exhibits either a flat or falling behavior, depending on the mediator mass scenario and the specific kinematic parameters. Notably, the value of the measure R_σ becomes negligible when the distribution is flat. In such cases, the free-electron approximation can be reliably employed without significantly underestimating or overestimating the cross section.

A question one may pose at this point is whether or not the observations above go through irrespective of the choice of the mediator type. To address this question, we consider a contrasting scenario that is diametrically different from the

FV case. Figure 5 presents color maps of R_σ for the FP case, using the same mediator mass choices as in the previous analysis. Note that the matrix element squared in both the $\beta \gg 1$ and $\beta \ll 1$ limits shows a rising behavior with the electron recoil energy. In the left panel, we observe a gradual descent in the deviation between the atomic effects treatment and the free-electron approximation with increasing γm_χ . In comparing the center panel of the FP case to that of the FV case, we noticed a flipped behavior. The value of the measure R_σ ranges from small to extremely high as the energy γm_χ increases. The right panel shows a mixed behavior of the first two panels, featuring an intermediate region where the variation remains at or below the percent level. Additionally, the variation is driven by the DM mass, owing to the choice scheme of the mediator mass.

The distributions at representative benchmark points A, B, and C, corresponding to those used earlier, are shown in

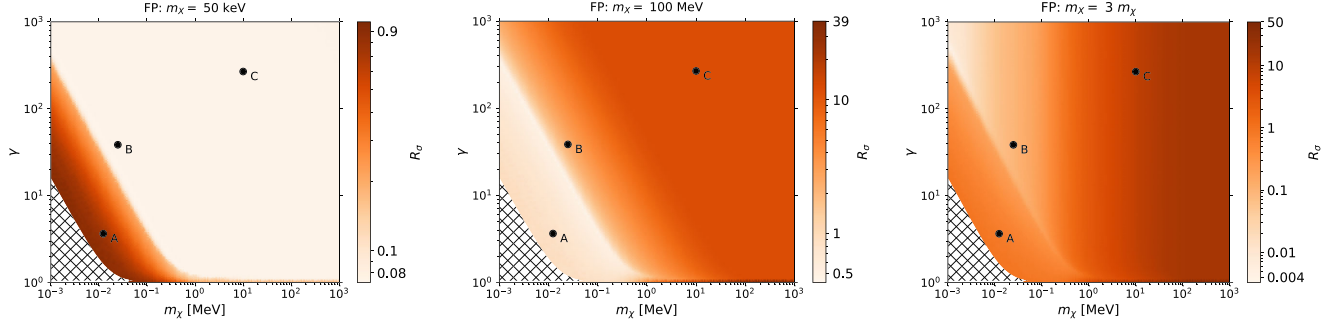


FIG. 5. R_σ for the FP case representing different mediator mass regimes of $m_X = 50$ keV, $m_X = 100$ MeV, and $m_X = 3m_X$ in the left, center, and right panels, respectively. The color scale in each panel is normalized to the maximum R_σ value within that panel. The hatched region corresponds to the portion of the parameter space that does not yield an electron recoil above 1 keV.

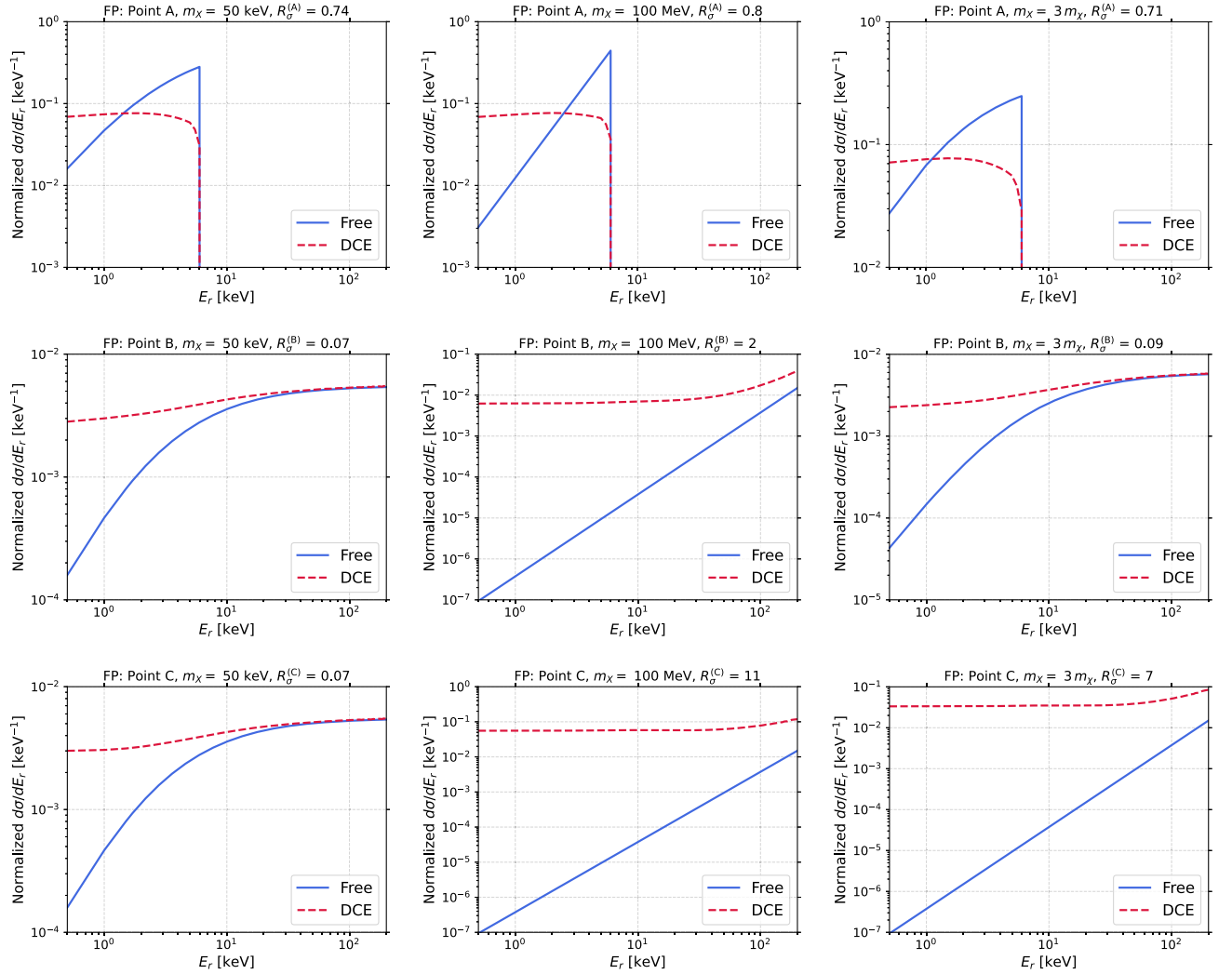


FIG. 6. Differential cross section for the FP case in terms of the electron recoil energy for the free-electron treatment in solid blue and for the relativistic ionization form factor in dashed red. Left, center, and right panels, respectively, correspond to mediator masses $m_X = 50$ keV, $m_X = 100$ MeV, and $m_X = 3m_X$. Top, middle, and bottom panels, respectively, correspond to the kinematic points A, B, and C as indicated in Fig. 5.

Fig. 6. When the mediator mass is small, as in the left panels, the distributions based on the free-electron treatment exhibit a rising trend and become noticeably flat with increasing electron recoil energy. The center panels reveal a different trend and offer new insight. Here, the distributions are originally monotonically increasing, and atomic effects tend to flatten this behavior, leading to extremely large variations. In the FP case, we find that R_σ for a light mediator, whereas it remains non-negligible for a heavy mediator—in clear contrast to the FV case. This indicates that the deviation is minimized when the distribution is flat, but becomes significantly larger when the distribution is falling (as in the FV case) or rising (as here). Finally, the right panels exhibit mixed behavior and allow a small region of the kinematic parameter space (near point B) to display a flat trend, for which atomic effects are negligible.

IV. DETECTION PROSPECTS AT DETECTORS

Building on the formalism discussed thus far, we now examine the detection prospects at experimental facilities. For the time being, we defer the discussion of the origin of the DM and its properties to the next section. For a DM signal to be observable at a detector, the maximum electron recoil energy must be greater than the experimental threshold

$$E_r^{\max} > E_r^{\text{thr}}. \quad (44)$$

In other words, there exists a nonzero probability of detecting DM with the given mass and energy as long as the above condition holds. This energy threshold varies across experiments; for instance, certain direct detection experiments are sensitive to energy deposits below 1 keV, while large-volume neutrino experiments typically require energies above several tens of MeV. Figure 7 illustrates the relevant parameter space in terms of DM speed and mass. The contour lines represent

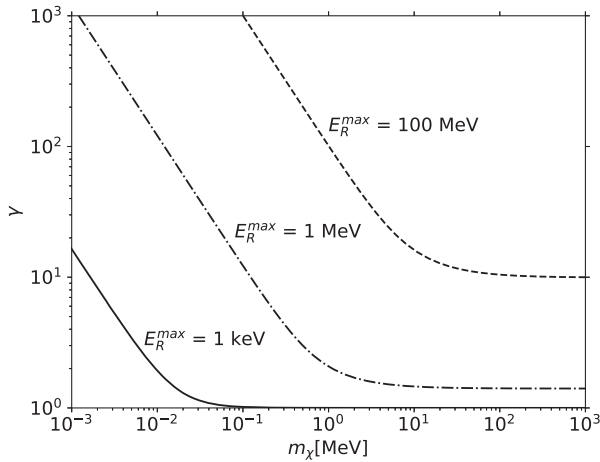


FIG. 7. Maximum recoil energy based on the kinematics of the DM particle, where solid, dash-dotted, and dashed contours represent 1 keV, 1 MeV, and 100 MeV, respectively.

the maximum attainable electron recoil energy, with illustrative thresholds set at 1 keV, 1 MeV, and 100 MeV. For example, a DM particle sitting below the solid curve cannot produce electron recoils above 1 keV by any means, rendering this region of parameter space inaccessible to experiments even with low-energy detection thresholds of ~ 1 keV. In contrast, for large-volume neutrino experiments with MeV-scale energy thresholds, the region below the dot-dashed line becomes inaccessible.

By focusing on scenarios where DM moves at speeds closer to the speed of light, we observe that both low- and high-threshold experiments are theoretically capable of detecting fast-moving DM. In this scenario, we can take advantage of the fact that atomic electrons can be effectively treated as free particles at rest. Therefore, the total number of observable signal events is given by

$$N^{\text{free}} = \Phi \Delta T N_T \int_{E_r^-}^{E_r^+} \frac{d\sigma^{\text{free}}}{dE_r} dE_r, \quad (45)$$

where $E_r^\pm$ are defined as

$$E_r^+ = \min[E_r^{\max}, E_r^{\text{cut}}], \quad (46)$$

$$E_r^- = \max[E_r^{\min}, E_r^{\text{thr}}], \quad (47)$$

and E_r^{cut} represents an upper limit imposed by detector saturation or other experimental constraints. In addition to Eq. (45), and for validation in low-threshold experiments, we also consider the full event rate including atomic effects. In this case, the total number of observable signal events is given by

$$N = \Phi \Delta T N_T \int_{E_r^-}^{E_r^+} \frac{d\sigma}{dE_r} dE_r, \quad (48)$$

where the integrand is given by Eq. (21). The first factor in Eqs. (45) and (48) represents the flux of the incoming DM, which depends on the source of the DM and its production mechanism. For the purposes of isolating the atomic effects, we fix the flux value to a reference value:

$$\Phi^{\text{ref}} = 10^{-7} \text{ cm}^{-2} \text{ s}^{-2}. \quad (49)$$

Of course, as mentioned above, the flux is not constant in reality but varies with different model parameters. In the next section, we show how results can be readily obtained for realistic flux models through rescaling. The observation time and the number of target electrons are denoted by ΔT and N_T , respectively. Our general approach involves four parameters:

$$\text{free parameters} = \{m_\chi, \gamma, m_X, \overline{\sigma_{\chi\chi}}\}, \quad (50)$$

where the coupling combination $g_\chi g_e$ can always be inferred from the reference cross section $\overline{\sigma_{\chi\chi}}$. In the following,

we illustrate the experimental sensitivity at two representative experiments:

- (i) *Super-Kamiokande*: SK is a water-based large-volume neutrino experiment with a volume of $22.4 \times 10^3 \text{ m}^3$. The energy threshold of the experiment is chosen to be 100 MeV, and on average, the experiment detects an order of 726 sub-GeV and 197 multi-GeV electron recoil events annually [102]. SK has been collecting data for about 20 years, which we set as an exposure time.
- (ii) *XENONnT experiment*: the primary objective of the multiton xenon-based experiment is to search for WIMP interactions with target nuclei; however, it also offers valuable sensitivity to electron recoil events. To facilitate a direct comparison between the predicted signal rate and the observed rate reported by the XENONnT Collaboration [103], based on an exposure of 1.16 ton-years, we compute the signal rate while accounting for efficiency corrections, selection criteria, and binning. Our analysis focuses on the energy range ($1 \text{ keV} < E_R < 140 \text{ keV}$) [103].

We perform a signal-to-background log-likelihood analysis, with the exclusion significance defined as

$$\sigma_{\text{exc}} = \sqrt{2 \sum_{i=1}^N \left[B_i \ln \left(\frac{B_i}{B_i + S_i} \right) + S_i \right]}, \quad (51)$$

where i runs over energy bins. Here, S denotes the signal predicted either by Eqs. (45) or (48), depending on the case, while B represents the background reported by the corresponding experiment.

For illustrations, Fig. 8 presents the 2σ exclusion limits for the FV case (top) and the FP case (bottom). The mediator mass choices are again $m_X = 50 \text{ keV}$ (left), $m_X = 100 \text{ MeV}$ (center), and $m_X = 3m_\chi$ (right). Before delving into the model-specific results, we first highlight several general conventions that are consistently applied across all exclusion plots and analyses in this work. These key conventions are summarized as follows:

- (i) Exclusion limits from the XENONnT experiment are uniformly represented by blue-shaded regions, while those from the SK experiment are represented in red. For XENONnT, the solid contours correspond to the limits derived from the signal defined by Eq. (48), while the dashed contours correspond to those derived from the signal defined by Eq. (45). For SK, all contours represent limits derived from the signal defined by Eq. (45).
- (ii) The intensity of the shading indicates the strength of the exclusion: lighter shades correspond to weaker constraints, whereas darker shades indicate stronger exclusions.
- (iii) The number labeled on each contour line indicates the reference cross section, in cm^2 , as defined in Eq. (30), at which a 2σ exclusion is achieved.

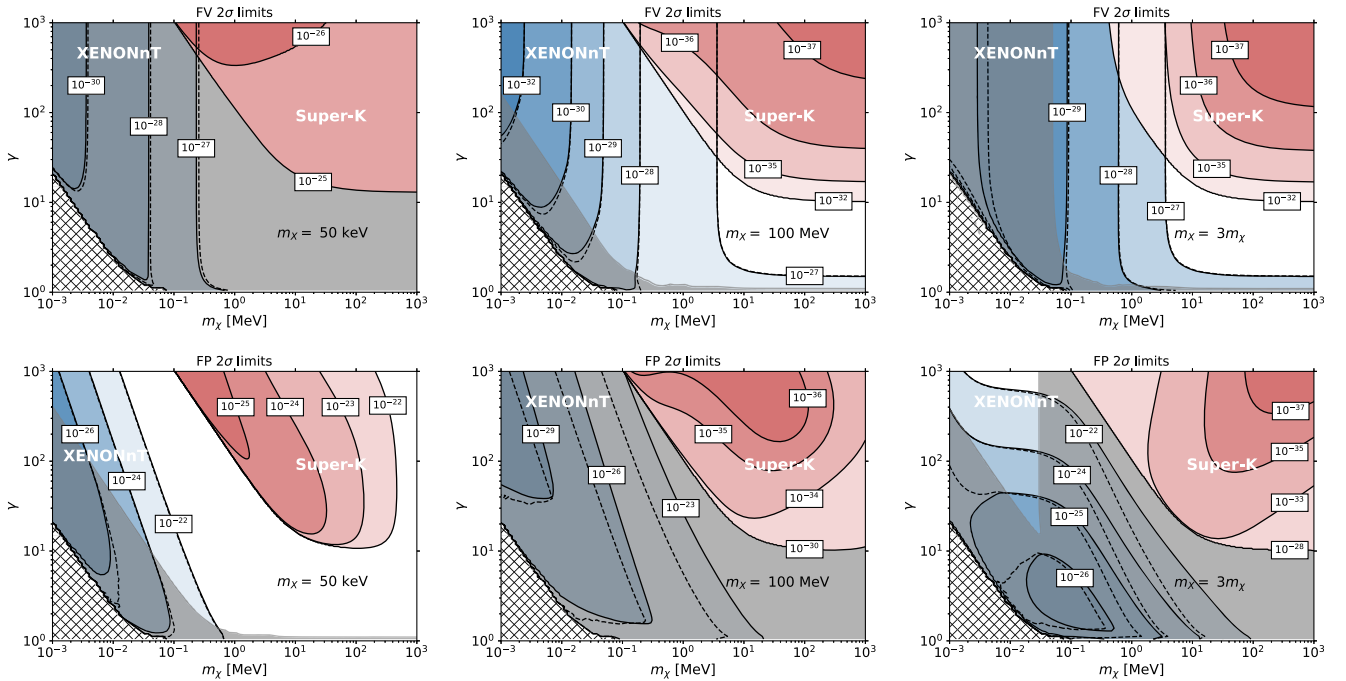


FIG. 8. 2σ exclusion limits for the fixed-flux scenario: FV (top) and FP (bottom). Left, center, and right panels respectively correspond to mediator masses $m_X = 50 \text{ keV}$, $m_X = 100 \text{ MeV}$, and $m_X = 3m_\chi$. The contour lines are indicated by the cross section in cm^2 where the blue (red) shaded regions correspond to the XENONnT (SK) limits. The gray shaded regions represent areas of the parameter space where $R_\sigma > 0.1$. The dashed contours in the XENONnT region indicate the corresponding limits derived from the signal defined in Eq. (45).

- (iv) The gray-shaded region corresponds to $R_\sigma > 0.1$. This roughly translates to a theoretical uncertainty on the differential cross section greater than 10%. In other words, regions without gray shading correspond to areas in the (m_χ, γ) parameter space where the free-electron approximation remains approximately valid. This uncertainty applies only to the results associated with the XENONnT experiment, as XENONnT is a low-threshold experiment potentially sensitive to atomic effects, whereas SK operates at the MeV-scale threshold.
- (v) Owing to its lower energy threshold, XENONnT generally probes a wider region of the kinematic parameter space, spanning both low and high values of m_χ and γ , depending on the specific model. In contrast, SK is primarily sensitive to the upper right region of the parameter space, where fast-moving DM particles are sufficiently energetic to induce detectable recoils. The hatched region in the bottom left corner denotes the portion of parameter space where DM-induced electron recoil energies above 1 keV cannot be produced.

We now turn to model-specific observations. In the FV case shown in the top panel, the exclusion limits are stronger for the heavy mediator scenario compared to the light mediator, as expected. Notably, for XENONnT, the shape of the exclusion contour lines remains approximately consistent across mediator masses, with a gradual decrease in exclusion strength observed from left to right. This observation admits a straightforward interpretation. At XENONnT, the electron recoil energies lie well below the electron mass, making the dimensionless parameter x small. Consequently, the dominant term in the squared matrix element is proportional to $\alpha^2\gamma^2$ for both light and heavy mediator scenarios, as shown in Table II. In contrast to XENONnT, the exclusion limits at SK exhibit a different trend. For instance, in the light mediator case, the limits are stronger in the left region, whereas for a heavy mediator, the strongest limits appear in the upper right region. This behavior can again be understood by examining the shape of the differential cross section, as given in Eq. (33), and informed by the structure of the squared matrix element summarized in Table II. SK operates at high electron recoil energies, corresponding to the regime $x \gg 1$. In this limit, the differential cross section for the light mediator case becomes a decreasing function of the recoil energy. Moreover, not only does the shape exhibit a falling trend, but the total cross section also decreases with increasing $\alpha\gamma$, resulting in a higher event rate—and thus stronger limits—in the left region of the parameter space. In contrast, for the heavy mediator case, the differential cross section remains relatively flat over a broad range of recoil energies, while the total cross section increases with $\alpha\gamma$. Consequently, the event rate—and hence the exclusion strength—is enhanced in the right region. These trends in the differential cross

section for the FV case, across light, heavy, and varying mediator masses, are illustrated in Fig. 4.

In the FP case shown in the bottom panel, the differential cross section exhibits an overall rising trend, as seen in Fig. 6. This rise is steeper for the heavy mediator scenario compared to the light mediator case, which tends to stabilize as the electron recoil energy increases. At XENONnT, a comparison between the FP and FV cases reveals that the exclusion limits gradually weaken from left to right, although the contour lines exhibit a slight tilt. This behavior arises from the fact that, unlike the FV case, the squared matrix element in the FP scenario is independent of both α and γ , as shown in Table II. At SK, where $x \gg 1$, the matrix element squared for light mediator is always flat, and the differential cross section is inversely proportional to $\alpha^2\gamma^2$. As a result, the exclusion limit is stronger toward the left side of the SK region of interest. In the heavy mediator case, the differential cross section scales as $(x/\alpha\gamma)^2$, leading to a stronger exclusion limit near the optimal region where the event rate is maximized, i.e., where $(x/\alpha\gamma)^2$ is maximized.

Across all panels of Fig. 8, for XENONnT we show, in addition to the limits derived from the full treatment including atomic effects (solid curves), the corresponding limits from the free-electron approximation (dashed curves). It is evident that the differences between the two treatments are large within the gray region and negligible outside it. This comparison delineates the parameter space in which the free-electron approximation is valid.

A summary of the limits for the various spin cases considered in this work is presented in Appendix A.

V. APPLICATION TO BENCHMARK DM MODELS

In this section, we consider two representative mechanisms that can accelerate DM particles to high velocities. Specifically, we explore annihilation and cosmic-ray interactions as potential sources of fast-moving DM. Our focus is on characterizing the resulting DM flux, understanding the physical processes involved in its production, and discussing the implications for detection. Here, we briefly summarize the key ingredients needed to estimate the flux and refer readers to the sources cited in the Introduction and throughout this section for further details. Our focus is on interpretability and practical utility: specifically, we show how the limits derived for a fixed reference flux in the previous section can be straightforwardly reinterpreted for realistic model fluxes.

A. Two-component boosted dark matter

To illustrate the flux of fast-moving or boosted DM (BDM) originating from the Galactic Center (GC), we consider a two-component dark matter model; one species is heavier than the other. In this scenario, stable, relativistic particles in the dark sector are produced through the annihilation of heavier DM in the GC. This BDM produced

in the present Universe is subdominant, i.e., the expected flux is much smaller than that of the ambient (nonrelativistic) DM. However, its large Lorentz boost factor allows it to overcome relatively high energy thresholds and be detected in large-volume terrestrial detectors through interactions with electrons, nucleons, or nuclei. This mechanism combines aspects of both direct and indirect detection: BDM is generated through halo DM annihilation (as in indirect searches) but detected through scattering in detectors (similar to direct detection experiments).

The example model of interest consists of two DM species, ψ_A and ψ_χ , with masses m_A and m_χ , respectively. For illustration, we assume that both ψ_A and ψ_χ are Dirac fermions. The dominant component, ψ_A , does not interact directly with the SM, while ψ_χ interacts through a vector portal, e.g., dark photon. The primary production mechanism of BDM is a pair annihilation, $\psi_A \bar{\psi}_A \rightarrow \psi_\chi \bar{\psi}_\chi$. Since ψ_χ particles are created with a large boost factor, they can be detected in neutrino experiments such as SK or in ton-scale direct detection experiments such as XENONnT. For discussions of production mechanisms, the thermal evolution of the DM species, and constraints, see, for example, Refs. [4,5,7].

The flux of these BDM particles reaching Earth, Φ_{GC} , depends on the ψ_A density, annihilation rate, and spatial distribution of ψ_A in the GC. The differential flux per unit solid angle Ω per BDM energy E_χ is given by

$$\frac{d\Phi_{\text{GC}}}{d\Omega dE_\chi} = \frac{1}{4} \frac{r_{\text{Sun}}}{4\pi} \left(\frac{\rho_{\text{local}}}{m_A} \right)^2 J \langle \sigma v \rangle_{AA \rightarrow \chi\chi} \frac{dN_\chi}{dE_\chi}, \quad (52)$$

where r_{Sun} is the distance from the GC to the Sun, ρ_{local} is the local ψ_A density, taken as 0.3 GeV/cm^3 [104], and J is the integral along the line of sight. The term $\langle \sigma v \rangle_{AA \rightarrow \chi\chi}$ represents the velocity-averaged annihilation cross section, and the BDM energy distribution is simply given by

$$\frac{dN_\chi}{dE_\chi} = 2\delta(E_\chi - m_A), \quad (53)$$

where the prefactor 2 describes the fact that two BDM particles are created per annihilation. For a practical estimate, the total flux over the entire sky is parametrized in Ref. [4] as

$$\Phi_{\text{GC}}^{4\pi} = 4.0 \times 10^{-7} \text{ cm}^{-2} \text{ s}^{-1} \times \left(\frac{\langle \sigma v \rangle_{AA \rightarrow \chi\chi}}{5 \times 10^{-26} \text{ cm}^3/\text{s}} \right) \left(\frac{20 \text{ GeV}}{\gamma m_\chi} \right)^2. \quad (54)$$

The top left panel of Fig. 9 shows the BDM flux Φ^{BDM} as a function of m_χ and γ . The flux increases as γm_χ decreases, primarily due to the $(\gamma m_\chi)^{-2}$ dependence in Eq. (54). The contour lines (bottom left) maintain a similar shape across different masses and speeds, suggesting that lower mass

and speed particles are more detectable compared to higher mass and speed particles.

B. DM boosted by cosmic rays

DM can also be accelerated to high energies through interactions with cosmic-ray (CR) electrons. While (a dominant fraction of) DM is nonrelativistic, scattering with energetic CR electrons can impart significant energy in the present Universe, potentially making (a certain fraction of) DM detectable in direct detection experiments.

The underlying process involves a CR electron transferring a portion of its energy to a DM particle during a collision. The resulting DM kinetic energy depends on both the initial energy of the electron and the scattering angle. The maximum kinetic energy transferable to the DM particle is determined by the electron's energy T_e and mass. Given that CR electrons span a broad energy spectrum—from a few MeV to several GeV—they are capable of boosting DM particles across a wide range of kinetic energies.

The flux of CR-boosted DM depends on the local DM density ρ_{local} , the CR electron flux $d\Phi_e/dT_e$, and the interaction cross section between DM and electrons. The differential flux is given by [40,105]

$$\frac{d\Phi^{\text{CR}}}{dT_\chi} = D_{\text{eff}} \rho_{\text{local}} \int_{T_e^{\text{min}}}^{\infty} dT_e \frac{1}{m_\chi} \frac{d\Phi_e}{dT_e} \frac{d\sigma_{e\chi}}{dT_\chi}. \quad (55)$$

Here, $D_{\text{eff}} = 10 \text{ kpc}$ represents the effective distance over which DM is considered. The CR electron flux $d\Phi_e/dT_e$ is parametrized in Ref. [106]. If the original DM is nonrelativistic, the differential cross section $d\sigma_{e\chi}/dT_\chi$ takes a simple form of $\bar{\sigma}/T_\chi^{\text{max}}$, where we use a benchmark cross section of 10^{-29} cm^2 as in Ref. [105]. An independent analysis of the benchmark choice of the cross section is illustrated in Appendix B. The maximum kinetic energy is

$$T_\chi^{\text{max}} = \frac{T_e^2 + 2m_e T_e}{T_e + (m_\chi + m_e)^2/2m_\chi}. \quad (56)$$

The lower integration limit is

$$T_e^{\text{min}} = \left(\frac{T_\chi}{2} - m_e \right) \left(1 \pm \sqrt{1 + \frac{2T_\chi (m_e + m)^2}{m (2m_e - T_\chi)^2}} \right). \quad (57)$$

In terms of γ , we express the flux as

$$\frac{d\Phi^{\text{CR}}}{d\gamma} = m_\chi \frac{d\Phi^{\text{CR}}}{dT_\chi}. \quad (58)$$

The top right panel of Fig. 9 illustrates the expected CR-boosted DM flux. Compared to BDM, Φ^{CR} decreases more rapidly at high γm_χ , due to the rapid fall of the CR electron flux with energy.

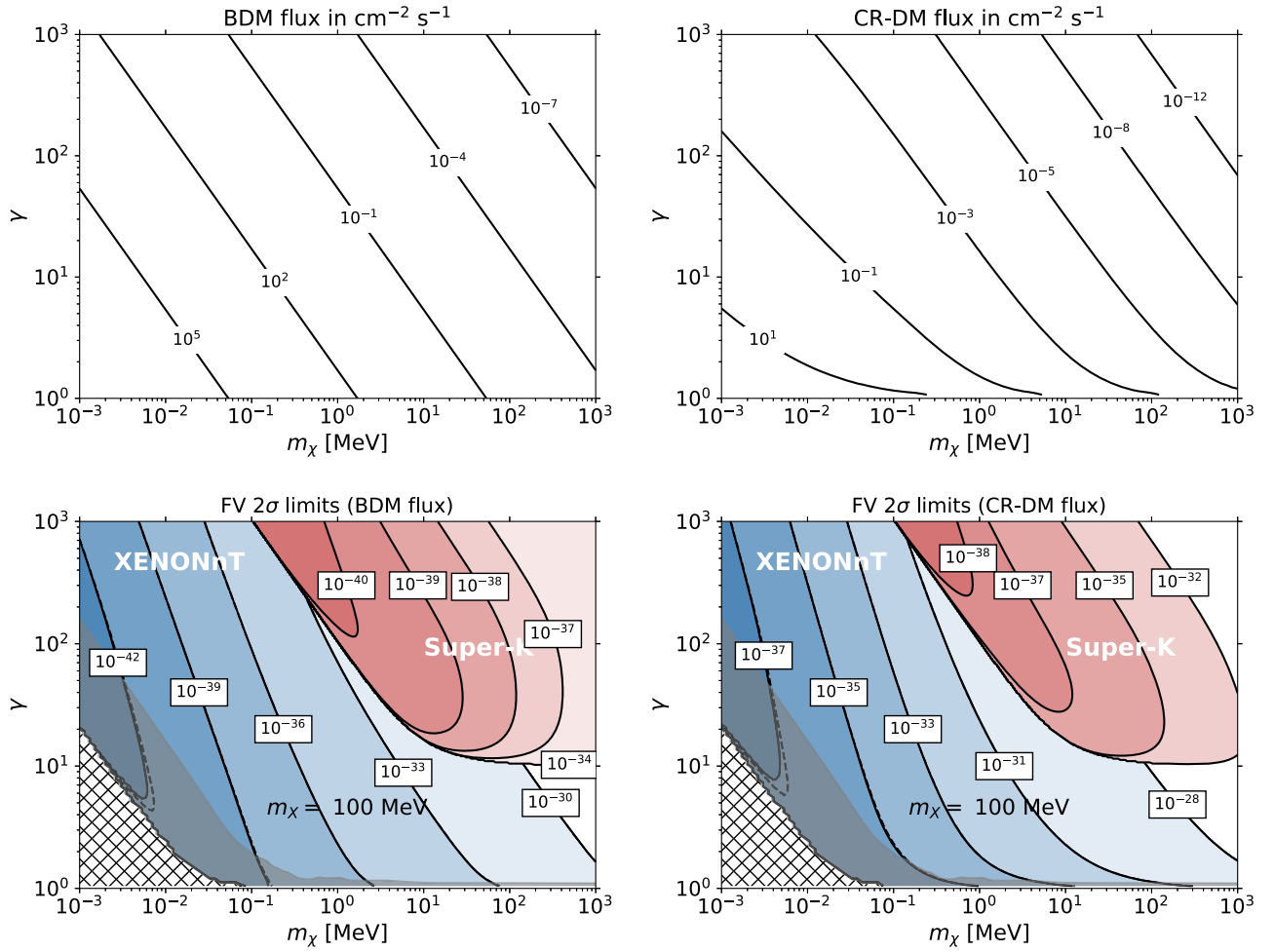


FIG. 9. BDM and CR-DM fluxes (limits) in the top (bottom) panel shown in the two kinematic parameter space (m_χ, γ) .

We now illustrate how the limits can be obtained immediately for realistic fluxes. In this example, we choose the FV case with a mediator mass $m_\chi = 100 \text{ MeV}$. Earlier, we performed the analysis for a fixed reference flux (top center panel of Fig. 8). Given the BDM and CR-DM fluxes at each DM mass and boost factor (top panels of Fig. 9), one can rescale the flux, which in turn rescales the derived cross section limits, yielding the bottom panels of Fig. 9.

Focusing first on the BDM case (bottom left panel), the shape and orientation of the flux contours reorient the resulting limits. For instance, at fixed flux, the XENONnT contours were primarily vertical, whereas with the BDM flux, they became diagonal. The sensitivity still weakens gradually with increasing DM mass. We find that XENONnT covers a broad region of parameter space, with cross section limits ranging from 10^{-42} to 10^{-30} cm^2 .

The SK case similarly demonstrates how flux reorients the contours. With a constant flux, the limits were strongest in the upper right portion of the SK region and weakened toward the lower left. By contrast, the BDM flux increases toward the lower left, so the SK limits become stronger on

the left side of the region. Similar conclusions hold for the CR-DM flux case.

For completeness and to help orient readers unfamiliar with the sensitivity presentation in this work, Appendix B shows example projections of our limits onto the traditional DM mass-cross section parameter space.

VI. SUMMARY AND OUTLOOK

In this work, we have developed a comprehensive and mechanism-agnostic framework for interpreting electron-recoil signals induced by fast-moving dark matter. Our analysis systematically incorporates relativistic kinematics, mediator-dependent matrix elements, and atomic physics effects through a fully relativistic ionization form factor. This formalism provides a unified description of scattering processes for both fermionic and scalar DM across vector, scalar, pseudoscalar, and axial vector mediators.

We quantitatively demonstrated the asymptotic behavior of the relativistic ionization form factor and identified the energy regimes in which atomic effects are relevant. At high

recoil energies, the form factor approaches the free-electron limit, validating the use of the free-electron approximation in those regions. Conversely, at lower energies, atomic effects can significantly modify the predicted event rates—sometimes enhancing rather than suppressing the signal—depending on the Lorentz structure of the interaction.

By comparing XENONnT and Super-Kamiokande, we highlighted the complementarity between low- and high-threshold experiments in probing fast-moving DM across wide kinematic ranges. We applied our formalism to two benchmark scenarios—two-component boosted DM and cosmic-ray-accelerated DM—and demonstrated how exclusion limits can be rescaled straightforwardly for realistic flux models. These results illustrate how the shape and orientation of the exclusion contours depend not only on detector thresholds but also on the underlying DM production mechanisms.

Overall, our framework bridges the gap between atomic-scale and relativistic treatments, enabling consistent interpretation of results from current and future direct-detection and neutrino-based experiments. It establishes a foundation for model-independent searches for relativistic DM, providing a quantitative map between flux models, mediator dynamics, and detector responses.

Looking ahead, several directions merit further exploration. A natural extension is to incorporate directional and time-dependent signatures, which may help discriminate between cosmic-ray-induced and annihilation-driven boosted DM. Future work may also include nuclear-recoil channels and composite or inelastic mediator models, as well as the integration of machine-learning-based analyses for nonstandard recoil spectra. The formalism developed here can also inform studies of neutrino-DM interplay and quantum-sensor-based detection in emerging experiments [107]. Through these avenues, the mechanism-agnostic approach introduced in this paper paves the way for a deeper and more unified understanding of fast-moving dark matter across the next generation of terrestrial and astrophysical searches.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: ADDITIONAL LIMIT PLOTS

In this appendix, we provide a list of limit plots corresponding to all renormalizable interactions studied in Table II. These plots follow the same conventions as those of Fig. 8. We group interaction cases by the mediator types, where vector mediator cases FV and SV are compared against each other first. Then we move to the scalar mediator, followed by the pseudoscalar mediator and finally the axial vector mediator.

In Fig. 10, we present the 2σ exclusion limits for the vector mediator scenario. Beyond the general features i–v outlined in Sec. IV, we highlight several model-specific observations. Notably, there are strong similarities between the exclusion limits for fermionic and scalar DM, both in the light and heavy mediator regimes, across the two experiments. To better understand these similarities, we examine the structure of the differential cross sections for the two cases. In the context of the XENONnT experiment, where the electron recoil energy is relatively low, the dominant contributions to the differential cross section arise from the $x \ll 1$ regime (see Table II). Conversely, for the SK experiment, which probes much higher electron recoil energies, the relevant terms come from the $x \gg 1$ limit. For instance, in the light mediator case at XENONnT, the dominant contribution to the squared matrix element for both fermionic and scalar DM is given by the $4\alpha^2\gamma^2/x^2$ term, as shown in Table II. Similarly, for the heavy mediator case at XENONnT, the leading term in both scenarios is $16\alpha^2\gamma^2/\beta^2$. Overall, exclusion limits for the heavy mediator tend to be stronger than those for the light mediator. This is because the differential cross section is relatively flat in the heavy mediator regime, whereas it decreases with energy in the light mediator case, as illustrated in Fig. 4. For the vector mediator, and especially in the case of the XENONnT experiment, the exclusion limits across all scenarios are primarily driven by terms proportional to $\alpha^2\gamma^2$. This common dependence leads to the observed uniformity in the exclusion contours across the panels in Fig. 10. The contour lines exhibit similar shapes, with the strongest exclusions occurring on the left side of the blue-shaded region, gradually weakening toward the right. Analogous features are observed in the SK experiment as well.

In Fig. 11, we examine the exclusion limits in the case where the interaction is mediated by a scalar particle. The top panels correspond to fermionic DM, while the bottom panels show results for scalar DM. The left, center, and right panels depict scenarios with light, heavy, and dynamic mediator mass choices, respectively. The general considerations listed as items i–v in Sec. IV remain applicable here as well. There is a noticeable degree of similarity in the exclusion contours across the different panels of the figure, particularly in the case of XENONnT. In the $x \ll 1$ regime, relevant for low-energy recoils, the squared matrix element is dominated by $16\alpha^2/\beta^4$ for heavy mediators and by

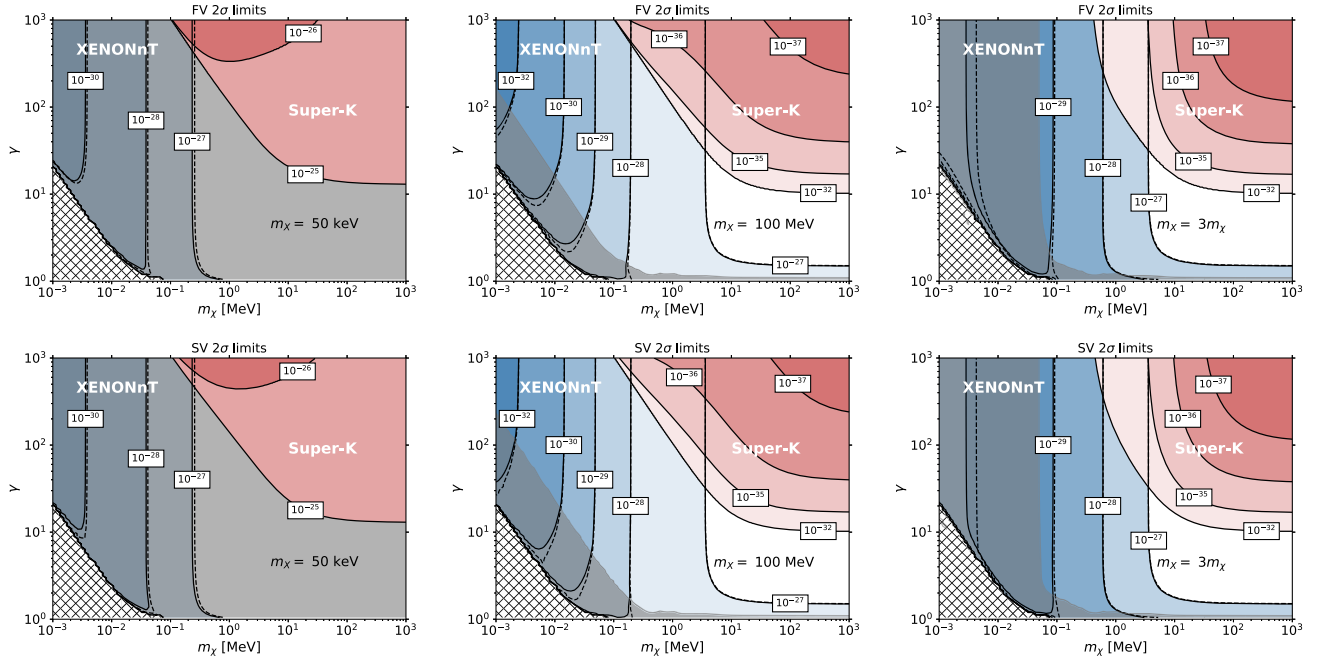


FIG. 10. Vector mediator 2σ exclusion limits for the FV case (top) and the SV case (bottom). Left, center, and right panels, respectively, correspond to mediator masses $m_X = 50$ keV, $m_X = 100$ MeV, and $m_X = 3m_\chi$. The contour lines are indicated by the cross section in cm^2 where the blue (red) shaded regions correspond to the XENONnT (SK) limits. The gray-shaded regions represent areas of the parameter space where $R_\sigma > 0.1$. The dashed contours in the XENONnT region indicate the corresponding limits derived from the signal defined in Eq. (45).

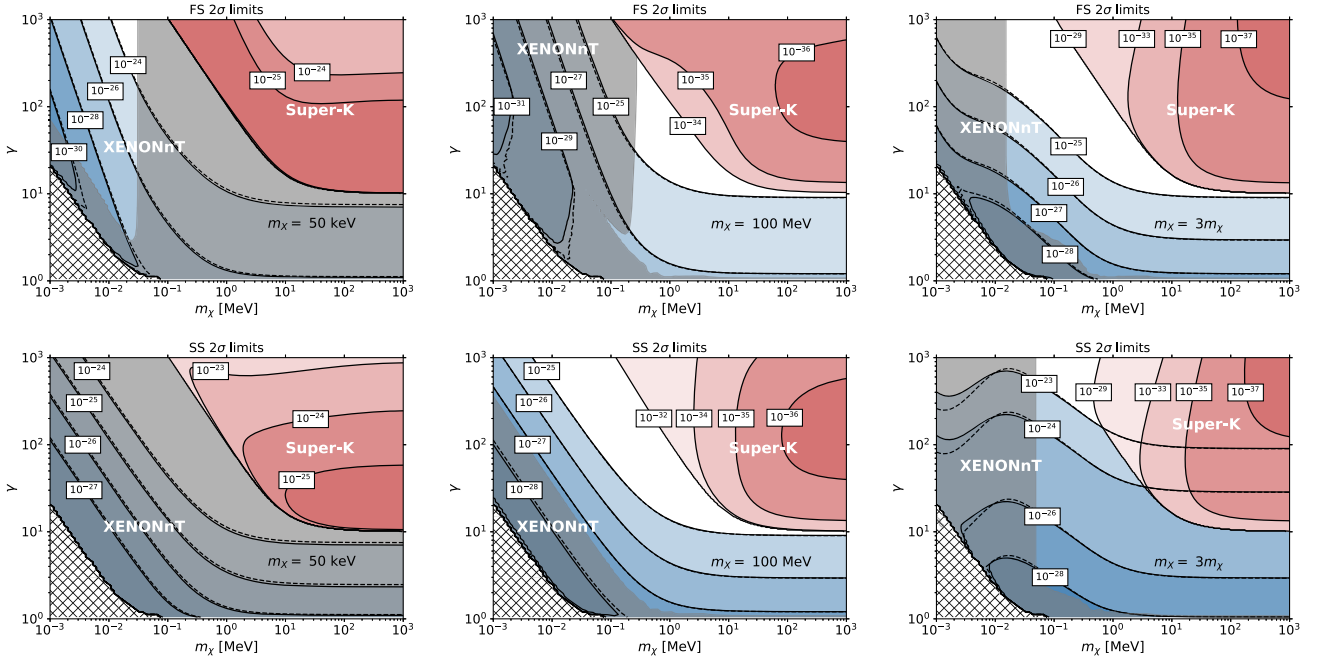


FIG. 11. Scalar mediator 2σ exclusion limits for the FS case (top) and the SS case (bottom). Left, center, and right panels, respectively, correspond to mediator masses $m_X = 50$ keV, $m_X = 100$ MeV and $m_X = 3m_\chi$. The contour lines are indicated by the cross section in cm^2 where the blue (red) shaded regions correspond to the XENONnT (SK) limits. The gray-shaded regions represent areas of the parameter space where $R_\sigma > 0.1$. The dashed contours in the XENONnT region show the limits derived from the signal defined in Eq. (45).

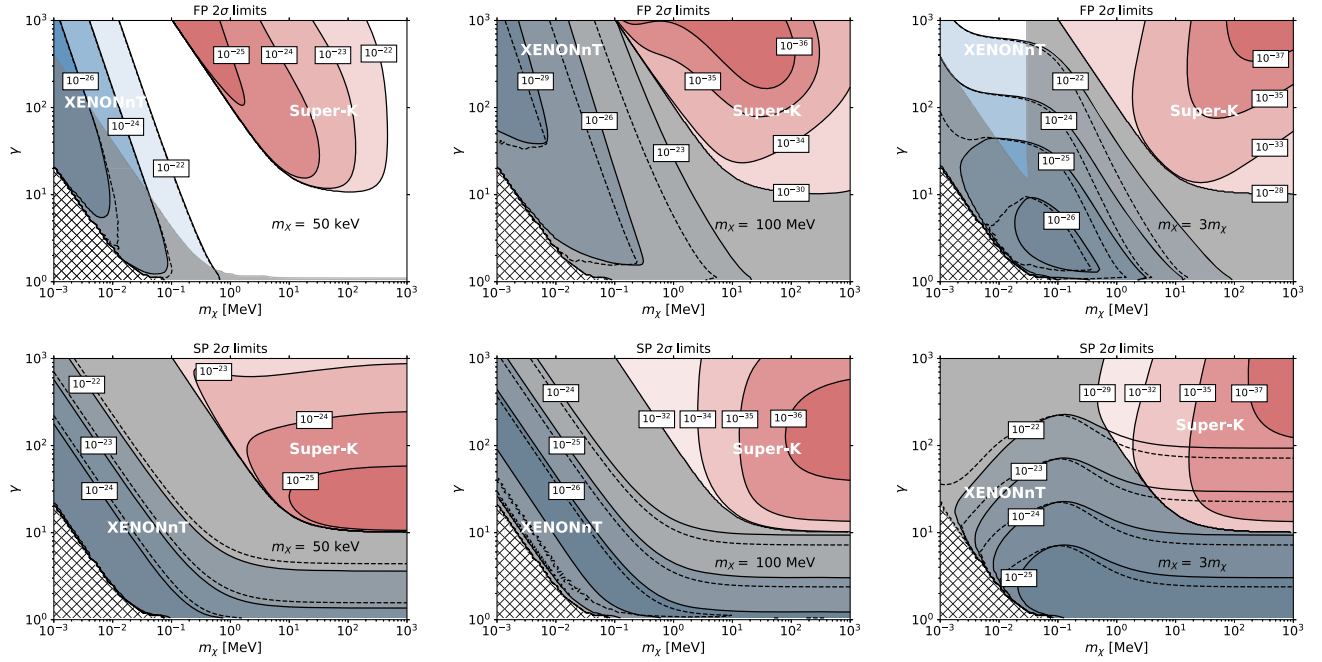


FIG. 12. Pseudoscalar mediator 2σ exclusion limits for the FP case (top) and the SP case (bottom). Left, center, and right panels, respectively, correspond to mediator masses $m_X = 50$ keV, $m_X = 100$ MeV, and $m_X = 3m_X$. The contour lines are indicated by the cross section in cm^2 where the blue (red) shaded regions correspond to the XENONnT (SK) limits. The gray-shaded regions represent areas of the parameter space where $R_\sigma > 0.1$. The dashed contours in the XENONnT region denote the corresponding limits derived using the signal defined in Eq. (45).

$4\alpha^2/x^2$ for light mediators, in both FS and SS cases. For the SK experiment, where $x \gg 1$ dominates due to higher recoil energies, we observe some differences in the leading terms of the matrix elements. These distinctions can be seen in Table II. For instance, in the light mediator scenario, both FS and SS matrix elements fall as $1/x^2$, but with different behaviors: the SS case exhibits a continuous decline, while the FS case is stabilized by a constant contribution. Similarly, for the heavy mediator, the FS matrix element grows quadratically with x , whereas the SS case exhibits linear growth. A notable feature in the scalar mediator case is the absence of dependence on the parameter γ (see Table II) in the squared matrix element. This omission significantly impacts the exclusion limits, especially when the matrix element is either constant or scales inversely with x , as is often the case for XENONnT limits in all panels and for the SK limits in the light mediator scenario. In such cases, regions of parameter space with lower γ values are more strongly constrained due to an enhanced cross section. This can be illustrated with a simple example from the XENONnT exclusion in the bottom center panel of Fig. 11. According to Eq. (33), the differential cross section behaves as

$$\frac{d\sigma}{dx} \propto |\mathcal{A}|^2/(\alpha^2\gamma^2). \quad (\text{A1})$$

Taking the SS case with a heavy mediator, we have $|\mathcal{A}|^2 = 16\alpha^2/\beta^4$, leading to a differential cross section that

scales as $16/(\beta^4\gamma^2)$ and hence making the limit stronger towards lower γ values. The right panel corresponds to the dynamical choice of the mediator mass ($m_X = 3m_X$), where the mediator mass ranges from an order of a keV in the left part of the parameter space to an order of a GeV in the right part of the parameter space. This transition allows an admixture effect of both light and heavy mediators.

In Fig. 12, we present the exclusion limits in the case of a pseudoscalar mediator. The top panels correspond to fermionic DM, while the bottom panels show the scalar DM case. The left, center, and right panels illustrate the scenarios with $m_X = 50$ keV, $m_X = 100$ MeV and $m_X = 3m_X$. As before, the general considerations listed in items i–v in Sec. IV also apply here. To avoid redundancy, we note that the bottom panels of this figure closely resemble the corresponding panels in Fig. 11. This is expected due to the similar structure of the squared matrix element in both cases, as shown in Table II. In contrast, the top panels exhibit a different behavior. In the FP case with a light mediator, the squared matrix element is approximately unity.⁴ Consequently, the differential cross section is simply proportional to $1/(\alpha^2\gamma^2)$, indicating that regions with smaller values of α and γ are more strongly constrained. This trend is clearly visible in both the XENONnT and

⁴This approximation is true since the apparent β^2/x term in Table II has a subleading contribution in the limit $\beta^2 \ll 1$.

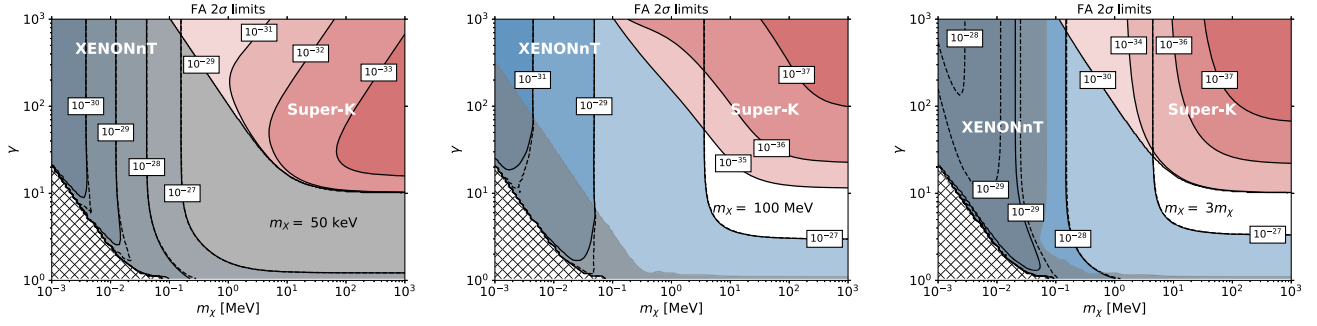


FIG. 13. Axial vector mediator 2σ exclusion limits. Left, center, and right panels, respectively, correspond to mediator masses $m_\chi = 50$ keV, $m_\chi = 100$ MeV, and $m_\chi = 3m_\chi$. The contour lines are indicated by the cross section in cm^2 where the blue (red) shaded regions correspond to the XENONnT (SK) limits. The gray-shaded regions represent areas of the parameter space where $R_\sigma > 0.1$. In the XENONnT region, the dashed contours indicate limits derived from the signal defined in Eq. (45).

SK limits. This particular case, where $|A|^2 = 1$, provides a valuable reference point for isolating the effects of detector kinematics and specifications without complications from model-dependent dynamics. As such, it serves as a baseline to understand the general shape and orientation of the exclusion contours. Once this baseline is established, deviations observed in other scenarios can be interpreted in terms of the additional contributions arising from the matrix element structure specific to each model.

We now focus on a specific interaction scenario in which the mediator is an axial vector and the DM particle is fermionic. Figure 13 presents the 2σ exclusion limits, with the mediator mass choices following the same as the previous cases. The general considerations outlined in items i–v for the vector mediator continue to apply here as well. Here, we observe that the differential cross sections in the FV and FA cases become very similar in the heavy mediator regime. This similarity carries over to the exclusion limits, explaining the nearly identical constraints seen in both the XENONnT and SK experiments. For the light mediator case, the FA scenario shares qualitative features with the FS case. In both, the cross section decreases with increasing x and tends to flatten out at large x . However, the rate of flattening differs: the FA case flattens more rapidly due to the additional $16\alpha^2/\beta^4$ term in the squared matrix element. As a result, in the FA case, the limits tend to strengthen with increasing α , particularly noticeable in the SK experiment as shown in the left panel of Fig. 13. This effect is less pronounced in the XENONnT experiment, as it primarily probes much smaller x values where the influence of this term is comparatively weaker.

APPENDIX B: TRADITIONAL LIMIT PLOTS—CASE STUDY

In this appendix, we present a case study illustrating how traditional limit plots appear for a specific choice of DM spin and mediator type. We consider fermionic dark matter with a vector mediator (FV), taking the mediator mass to be

$m_\chi = 100$ MeV. We then adopt the two benchmark models discussed in Sec. V.

For the BDM case, a realistic choice depends on the details of the mechanism by which the fast-moving DM particle is generated. In the two-component BDM scenarios, the fast-moving DM χ is generated through an annihilation of a heavier species ψ_A through the process $\psi_A \bar{\psi}_A \rightarrow \psi_\chi \bar{\psi}_\chi$. Typically, the heavier species are assumed to annihilate at rest, given their nonrelativistic nature, leading to the mass ratio $m_A/m_\chi = \gamma$. In other words, the speed of χ is set by the ratio of the masses of the two species. Therefore, a choice of the heavy mass leads to a choice of the boost Lorentz factor. Two examples are depicted in the left ($\gamma = 50$) and center ($\gamma = 200$) panels of Fig. 14.

For the CR-boosted DM case, a realistic choice involves performing an integration over the allowed DM speeds. A single DM particle obtains a range of kinetic energy T_χ and hence a range of a Lorentz boost factor γ when it is boosted by CR. The γ range is set by a lower value of 1 and a maximum value obtained by the kinetic energy of the original CR T_e . The allowed kinetic energy ranges for the CR and the DM are, respectively, as follows:

$$\text{CR: } T_e^{\min}(\gamma) \leq T_e \leq T_e^{\max}, \quad (\text{B1})$$

and

$$\text{DM: } 1 \leq \gamma. \quad (\text{B2})$$

Here $T_e^{\min}(\gamma)$ is given by Eq. (57), a function in T_χ and hence a function in γ and $T_e^{\max}$ is set to be 90 GeV according to Ref. [105]. For each value of γ , the CR kinetic energy is set by the range in Eq. (B1). The integration over γ has an upper limit γ^{top} at which $T_e^{\min}(\gamma = \gamma^{\text{top}}) = T_e^{\max}$. An example of DM boosted by CR is depicted in the right panel of Fig. 14.

In all panels of Fig. 14, and in particular for the XENONnT experiment, two types of limits are displayed. Solid lines correspond to limits derived from the full event

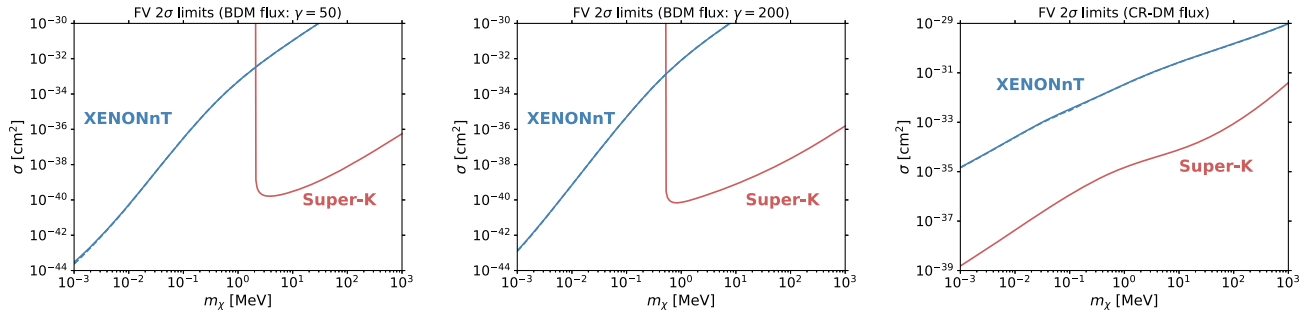


FIG. 14. Vector mediator 2σ exclusion limits for the FV case at mediator mass $m_\chi = 100$ MeV. The blue lines correspond to the limits at XENONnT, while the red lines correspond to the limits at SK. Left and center panels represent the two-component BDM case at boost factors $\gamma = 50$ and $\gamma = 200$, respectively, while the right panel corresponds to the CR-DM.

rate defined in Eq. (48), while dashed lines—appearing nearly identical to the solid ones—are obtained using the rate defined in Eq. (45). As expected, the free-electron approximation of the cross section produces limits that are significantly similar to those of the full treatment, including the atomic effects. This, of course, is due to the choice of mediator and the boost factors as detailed in Secs. III and IV.

It is notable to point out that the derived limits of the two-component BDM case at SK have a clear lower mass cutoff,

depending on the available boost factor. A larger boost factor allows SK to probe a wider range of DM masses. This is evident in the left and center panels of Fig. 14. In contrast, for the CR-DM case (right panel), this sharp lower mass edge is absent. Because CR scattering produces a continuous range of boost factors, low mass DM particles can receive sufficiently large boosts to become detectable at SK. As a result, the mass reach does not show a sharp cutoff as in the fixed-boost BDM case.

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