

Black hole singularity is a surface not a point

Andrew J. S. Hamilton ^{*}

JILA and Department of Astrophysical and Planetary Sciences, Box 440, University of Colorado Boulder, Colorado 80309, USA

Tyler McMaken [†]

Department of Mathematics and Physics, University of Mary, 7500 University Drive, Bismarck, North Dakota 58504, USA



(Received 3 January 2026; accepted 15 June 2026; published 30 July 2026)

It is widely repeated in the popular literature and elsewhere that the singularity at the center of a black hole is a point. It is not true. Two observers who free fall into a spherical black hole along two different angular trajectories at the same time t do not encounter each other at the central singularity; rather, they lose causal contact with each other already well away from the singularity. Counter-intuitively, in general relativity two points can be spatially close yet causally distant. The singularity is a surface not a point. The story for rotating black holes is more complicated, but the same conclusion holds. For a rotating black hole, the singular surface almost certainly resides at its inner horizon, where even the tiniest classical or quantum perturbations ignite the exponential mass inflation instability, precipitating collapse to a spacelike singular surface. There are implications for quantum gravity. We argue that, whatever the ultimate theory of quantum gravity may be, the quantum states of a black hole probably reside at its effectively two-dimensional singular surface, which coevolves unitarily with, and in thermodynamic equilibrium with, the hot atmosphere of trapped Hawking radiation that the black hole generates within its event horizon.

DOI: [10.1103/1pvt-3pn5](https://doi.org/10.1103/1pvt-3pn5)

I. INTRODUCTION

The most energetic places in the known Universe occur at the big bang and at the singularities of black holes. Unsurprisingly, not only seasoned experts, but also students and the general public are fascinated by these places, where the known laws of physics break down.

The purpose of this paper is to push back against the widespread misconception that the singularity at the center of a black hole is a point. Sections [IA](#) and [IB](#) of this Introduction offer context reviewing mainstream treatments of singularities within spherical (Schwarzschild) and rotating (Kerr) black holes, respectively.

Section [II](#) demonstrates a central result of this paper, that the singularity at the center of a spherical, Schwarzschild black hole is best characterized as a surface not a point. The key argument is that different angular locations on the

singularity are causally disconnected, in spite of being metrically close. Section [II A](#) calculates the boundary of the region of visibility seen by an infaller approaching the singularity, who loses all causal contact with neighboring infallers approaching the same singularity. The loss of causal contact between neighboring regions is a known feature of gravitational collapse in both black holes and cosmology [\[1–3\]](#).

Geroch, Kronheimer, and Penrose [\[4\]](#) proposed extending a general relativistic spacetime by including on its boundary “ideal points,” defined to be distinct if they do not share the same past or future, that is, if they are causally disconnected. Ideal points include not only those on a singularity, but also points at infinity. Reference [\[4\]](#) emphasize that the definition of ideal points depends on the causal structure of a spacetime, not on its metric. The review by [\[5\]](#) offers an entry to the literature on causal boundaries.

Section [III](#) generalizes to a rotating, Kerr black hole. Unlike the Schwarzschild singularity, the singularity within a mathematical Kerr black hole is often described as a timelike, one-dimensional ring. However, as argued in Sec. [III A](#), in a real rotating black hole, the geometry of a Kerr black hole is almost certainly truncated at its inner horizon by classical or quantum perturbations that trigger the mass inflation instability, precipitating collapse to a spacelike singularity. If so, the singularity of a Kerr black

^{*}Contact author: Andrew.Hamilton@colorado.edu; <http://jila.colorado.edu/~ajsh/>

[†]Contact author: tcmmaken@umary.edu

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](#) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

hole resides at its inner horizon, and its structure mirrors the spherical analysis of Sec. II.

Section IV discusses some implications for quantum gravity.

For reference, Appendices A and B review the well-known solutions of the geodesic equations in, respectively, Schwarzschild and Kerr black holes.

A. Spherical black hole review

Most textbooks on general relativity tend to be guarded in their treatment of singularities of black holes, even in the simplest case, that of a spherical uncharged black hole, a Schwarzschild black hole. Commonly the textbooks point out that the tidal force diverges at the singularity of a Schwarzschild black hole at zero radius $r = 0$ and that the usual Schwarzschild time and radial coordinates t and r , which are, respectively, timelike and spacelike outside the horizon, switch to become, respectively, spacelike and timelike inside the horizon. An infaller who falls inside the horizon is inexorably dragged inward toward the singularity. Absent a consensus theory of quantum gravity, what happens at the singularity remains enigmatic.

One classic text that does address the Schwarzschild singularity is that of Rindler [6], who rightfully disputes the notion of a singular point but nonetheless mischaracterizes the singularity as a one-dimensional line. He writes on page 159, “we tend to think of the Schwarzschild locus $R = 0$ [...] as a single spatial point, the permanent center of the horizon sphere [...] And this is false!” Rindler then clarifies on page 161 while describing the Kruskal [7,8] diagram of a Schwarzschild black hole: “the universe suddenly appears out of nowhere as an infinite line” (a white hole singularity), which “immediately flares into a long drawn out double trumpet” before collapsing back to a line (a black hole singularity) “momentarily, and then again nothing.” The singular lines are spacelike, consistent with the conceptual picture that proper time comes into existence at the white hole singularity and comes to an end at the black hole singularity, after the throat of the so-called Einstein-Rosen bridge becomes infinitely long and vanishingly thin.

A Kruskal diagram is a spacetime diagram constructed from radial and time coordinates arranged such that radially moving light always moves at 45° . Its virtue is that it clarifies the causal structure of the geometry, since timelike worldlines must evolve generally upward, at less than 45° from vertical. A Penrose diagram [9] (Fig. 1) shares with a Kruskal diagram the property that radially moving light always moves at 45° but, in addition, maps spatial and temporal infinity to a finite position. Like the Kruskal diagram, the Penrose diagram shows that the past (white hole) and future (black hole) singularities are extended in the spacelike t direction.

Rindler’s [6] characterization of the singularities as lines stems from the fact that they occur at zero radius $r = 0$,

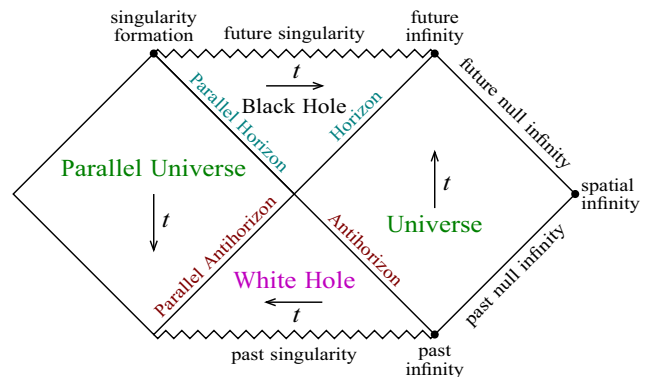


FIG. 1. Penrose diagram of the analytically extended Schwarzschild geometry. A Penrose diagram [9] is a spacetime diagram constructed from radial and time coordinates arranged such that radially moving light, whether outgoing ($\nearrow$) or ingoing ($\searrow$), always moves at 45° , and spatial and temporal infinity are mapped to a finite position. The analytically extended geometry contains not only a Universe and a Black Hole, but also a Parallel Universe and a White Hole. The Parallel Universe and White Hole are time-reversed versions of the Universe and Black Hole. In a real black hole formed from gravitational collapse, there is no Parallel Universe or White Hole, and the Antihorizon and Parallel Horizon are replaced by the dimming, redshifting surface of whatever collapsed into the black hole long ago. The arrows show the direction of the Schwarzschild time coordinate t in each of the regions. Inside the Black Hole and White Hole, the time coordinate t is spacelike. The endpoints of the future singularity are labeled from the perspective of observers in the Universe, “singularity formation” and “future infinity.”

where the angular dimensions shrink to zero. A primary purpose of the present paper is to show that this seemingly obvious inference is false. The root reason is that, in general relativity, being spatially close does not necessarily imply being causally close.

A sentiment similar to Rindler’s is expressed in the weighty tome by Chandrasekhar [10], who, in discussing the Kruskal diagram of the Schwarzschild geometry, writes on page 92, “the centre emerges as a singular line of the Schwarzschild space-time.”

The popular literature tends to be more pointed. It is unified in its declaration that the singularity of a black hole is a point. Greene in “Fabric of the Cosmos” [11] writes on page 337, “According to general relativity, all that makes up a black hole is crushed together at a single minuscule point at the black hole’s center.”

deGrasse Tyson in “Death by Black Hole” [12] writes on page 284, “Meanwhile the stuff within the event horizon has collapsed to an infinitesimal point at the black hole’s center.”

Bartusiak in the preface of “Black Hole” [13] writes, “The idea of the black hole is actually quite simple. It has a mass, and it has a spin. In some ways, it is as elementary an entity as an electron or a quark. But what confounded physicists for so long was the black hole’s ultimate nature: it is matter squeezed to a point.”

The picture that the singularity is a point is seemingly well founded. It comes from the fact that a spherical black hole is described by the Schwarzschild [14] metric [Eq. (A1) in Appendix A] and that the central singularity, where the curvature diverges, is at zero radius, $r = 0$. The radius of a two-dimensional sphere enclosing the singularity diminishes to zero at the singularity.

A mathematical relativist would be more cautious in their language. They would point out that the central “point” of the Schwarzschild metric is not part of the spacetime manifold, because general relativity fails there: there is no locally inertial frame at the singularity. Rigorously, within general relativity, it is legitimate to make statements only about regions asymptotically close to, but not at, the singularity. The present paper will follow the relativist’s caution.

B. Rotating black hole review

The Schwarzschild geometry describes a spherical, nonrotating black hole. But real black holes rotate [15]. A celebrated example is Cygnus-X1, an x-ray binary consisting of a $19M_{\odot}$ giant star accreting onto a $15M_{\odot}$ black hole [16], whose spin parameter a has been measured to be at least 0.95 of the maximum possible spin [17].

Rotating black holes are described by the Kerr [18,19] geometry. The big difference between a spherical (Schwarzschild) and a rotating (Kerr) black hole is that a rotating black hole has not one but two horizons, an outer and an inner horizon. As in a Schwarzschild black hole, space falls faster than light inside the outer horizon of a Kerr black hole [20]. But in a Kerr black hole, centrifugal repulsion slows the inflow of space inside the outer horizon, back down to the speed of light at the inner horizon.

As first elucidated by Carter in 1968 [21], in the exact, analytically extended, mathematical solution for the Kerr geometry, the inner horizon is a gateway to remarkable and astonishing phenomena, including wormholes, white holes, and a central ring singularity. The ring singularity is timelike and, thanks to the centrifugal force, infinitely gravitationally repulsive. Its ringlike nature can be ascertained by noting that it occurs along the equatorial plane ($\theta = \pi/2$) at Boyer-Lindquist [22] radius $r = 0$, where r is an oblate spheroidal coordinate related to Cartesian coordinates $\{x, y, z\}$ by the transformation $x^2 + y^2 + z^2 = r^2 + a^2 \sin^2 \theta$. Whenever the spin a is nonzero, the singularity spreads out into a ring of radius a .

The divergingly repulsive character of the ring singularity is not its only unphysical feature. Adjacent to the ring singularity is an azimuthal tunnel through which an observer, if moving sufficiently rapidly retrograde, can go into their own past, violating causality [21]. The ring singularity is a source of unpredictability: timelike and lightlike worldlines emerge from it without causal precedent. Naturally one should be skeptical that timelike

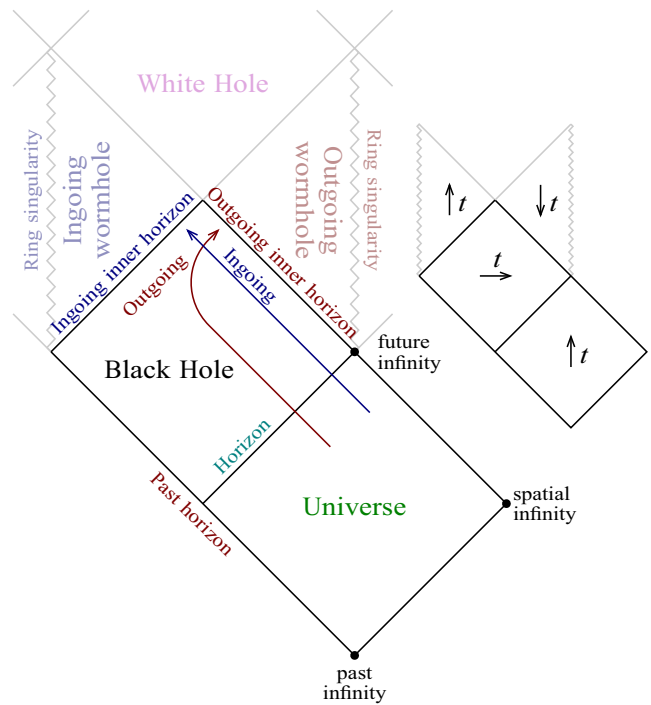


FIG. 2. Penrose diagram of the analytically extended Kerr geometry. After passing through the Horizon into the Black Hole region where the global time coordinate t becomes spacelike, geodesics may continue through either the ingoing or outgoing portions of the inner horizon. Beyond each inner horizon is a wormhole where coordinate time t reverts to being timelike, and an infinitely gravitationally repulsive ring singularity is visible. The wormholes exit to a white hole and more. These parts of the analytic continuation are shown in ghostly lines and labels; they are prevented from coming into existence by the Poisson-Israel [23] mass inflation instability described in Sec. III A. The inset shows the direction of coordinate time t in the various regions. Proper time of course always increases upward in a Penrose diagram.

singularities, violations of causality, wormholes, and other such phantasmagoria could happen in reality.

Mathematicians call the boundary of unpredictability the Cauchy horizon. In a Kerr black hole, the Cauchy horizon is the inner horizon. Section III A argues that, in a realistic rotating black hole, the inner horizon replaces the ring singularity as the boundary of the spacetime.

The analysis of the nature of the Kerr interior in Sec. III will be focused on its overall causal structure and the behavior of ingoing and outgoing geodesics. The Kerr geometry’s inner horizon is in fact composed of two distinct surfaces, here called the outgoing inner horizon (through which outgoing geodesics pass) and the ingoing inner horizon (through which ingoing geodesics pass). Qualitatively, outgoing means going against the superluminal inflow of space between the horizons, while ingoing means going with the flow. Of course both outgoing and ingoing geodesics are compelled to fall inward, to smaller radius r , by the superluminal inflow.

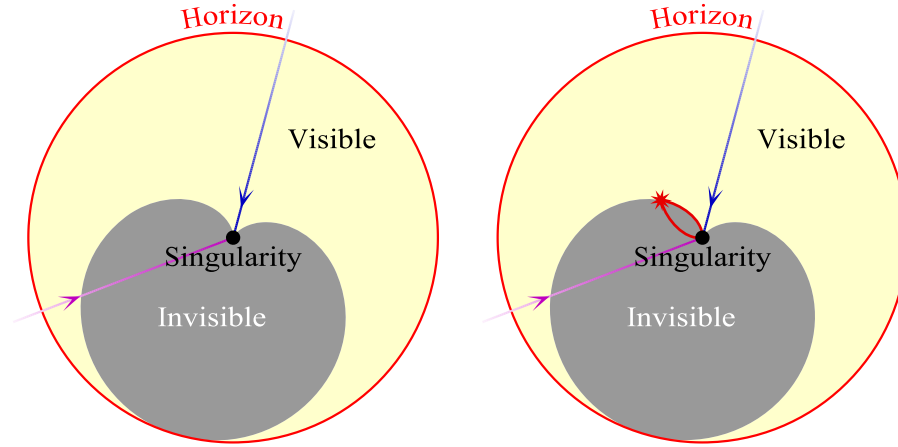


FIG. 3. Left: the light yellow shaded region shows the region visible to a blue infaller who falls radially to the singularity of a Schwarzschild black hole; the dark gray shaded region shows the region that remains invisible to the infaller. The boundary of the invisible region has the shape of a cardioid, Eq. (1). If another, purple, infaller falls along a different radial direction, the two infallers not only fail to meet at the singularity, they lose causal contact with each other already some distance from the singularity. Since the two infallers fall to two causally disconnected places, the singularity cannot be a point. Right: the same, showing in red the shortest causal path joining the two infallers asymptotically near the singularity. The shortest causal path is a pair of light rays that start at the starred point, move in opposite angular directions, and reach the infallers asymptotically near the singularity. The shortest causal path remains nonzero even though the spatial distance between the infallers tends to zero.

The causal structure of the Kerr geometry is illuminated by its Penrose diagram, Fig. 2. As illustrated in the diagram, outgoing geodesics fall through the outgoing inner horizon into a region of the Kerr geometry where the time coordinate t (the coordinate with respect to which the geometry is time-translation symmetric) progresses backward; while ingoing geodesics fall through the ingoing inner horizon to a region where the time coordinate t progresses forward. The two regions beyond the inner horizons can be called outgoing and ingoing wormholes, because the centrifugal force continues to slow the inflow of space, turning it around inside the wormholes to become an outflow of space.

The Kerr geometry can be analytically continued beyond the wormholes to a white hole and other universes. However, that story will not be pursued here, because none of it happens in reality. Instead, as will be elaborated in Sec. III, the inner horizon will itself probably become a spacelike singular surface, rendering the rotating black hole's interior and singularity topologically identical to that of Schwarzschild.

II. SPHERICAL BLACK HOLES

According to the textbook claims of Sec. IA, the central singularity in a Schwarzschild black hole has zero radius and therefore should be viewed as asymptotically approaching a point or line with vanishing angular dimensions. The goal of this section is to show that this view is misleading, not only from the perspective of causal geodesics (Secs. IIA and IIB), but also from ray-traced

visualizations tracking observables within orthonormal frames (Sec. IIC).

A. Two infallers do not meet each other at the singularity

Figure 3 shows the trajectories of two observers, one blue, one purple, who free fall radially into a Schwarzschild black hole along two different angular positions. You can imagine that they do their best to fall in at the same time t , although, because of the finite light travel time between them, they always see the other observer farther out in radius than they themselves are.

The defining feature of a black hole is that space inside the horizon falls faster than light [20] (the special relativistic rule is that nothing can move *through* space faster than light; spacetime itself can do whatever general relativity prescribes). So light that is emitted from inside the horizon is forced to move inward, even if the light is trying to move outward.

Consider what the blue infaller sees as they approach the singularity. As Fig. 3 (which is mathematically accurately drawn) illustrates, the blue infaller near the singularity cannot see all of the spacetime inside the horizon. There is a certain maximum angular motion that a light ray can have inside the black hole's horizon. That maximally sideways-moving light bounds what an infaller can see. The region beyond that boundary is invisible to the infaller.

The calculation of geodesics in the Schwarzschild geometry is outlined in Appendix A 1. The null geodesics with the maximum angular motion inside the horizon turn

out to be those with infinite angular momentum per unit energy. The expression for the radius r versus angle θ along such geodesics [Eq. (A8)], as seen by an observer at angular position θ_{obs} , is pretty:

$$r = r_s \sin^2\left(\frac{\theta - \theta_{\text{obs}}}{2}\right), \quad (1)$$

where r_s is the Schwarzschild radius, the radius of the horizon. Equation (1) is the equation of a cardioid. The cardioid is the heart-shaped boundary of the zone of invisibility depicted in Fig. 3. The cardioid just kisses the horizon at the antipode of the observer's angular position θ_{obs} .

The last that the blue infaller sees of the purple infaller is at the boundary of the invisible gray region, already well away from the singularity. Far from encountering each other at the singularity, the blue and purple infallers lose causal contact with each other already some distance from the singularity. Because both infallers fall at the same time t , both infallers would appear to follow the same worldline on the Penrose diagram 1. The singularity is therefore causally separated not only along the spacelike t direction, but also along angular directions for two infallers with the same coordinate t . Since the two infallers fall to causally disconnected places, the singularity cannot be a point or a line: it must be a surface.

B. Contradiction?

The argument seems to have led to a contradiction. On the one hand, two infallers reaching the singularity are causally separated. On the other hand, the Schwarzschild metric shows that the spatial distance between the infallers goes to zero at the singularity. The resolution of this conundrum is that, in general relativity, two points can be spatially close yet causally distant.

In everyday experience, things that are spatially close are causally close. It can be hard to adjust one's intuition to the fact that this is not true in general relativity. But being close in one sense does not necessarily imply being close in another sense. For example, in special relativity the spacetime distance ds along a null (lightlike) path is zero, even though the null path connects points that are separated in both time and space. Relativity has a tendency to defy intuition.

Where does the intuition that spatially close implies causally close go wrong? In the Schwarzschild geometry, the spatial separation between infallers approaching the singularity is measured along the angular direction. But, as Fig. 3 illustrates, the angular region that an infaller can measure with a ruler diminishes near the singularity to a sliver, shrinking to a cusp at the singularity. Near the singularity, there is no actual observer who can measure anything but a small part of the circumference of a sphere that encloses the singularity.

The shortest causal path between two infallers reaching the singular surface at the same time t but at two different angular positions is a pair of null geodesics each with the maximum possible angular momentum, as illustrated by the red lines in the right panel of Fig. 3. A quantitative measure of the causal distance along null geodesics in general relativity is the affine distance [24]. Appendix A 3 defines affine distance and shows that, up to an arbitrary overall normalization factor, the shortest causal distance between infallers separated by angle θ is given by Eq. (A17).

C. The view near the singularity

The notion that the singularity is a surface becomes more compelling when one considers what an infaller sees as they approach the singularity. Because space falls faster than light inside the horizon, an observer inside the horizon of a Schwarzschild black hole is always looking at light that was emitted above them, even if they are looking downward. For example, if the observer falls feet first through the horizon, they continue to see their feet below them, but they are seeing an image of their feet as they used to be above their head. An infaller looking down never gets to see the singularity below them.

Figure 4 shows eight frames from a general-relativistically ray-traced visualization of what an observer would see free falling into a Schwarzschild black hole, an issue first addressed by [24]. In the visualization, the observer starts from rest at infinity and free falls radially with zero angular momentum.

The black hole in Fig. 4 is painted with a dark red grid, the past horizon of the black hole [26]. In a real black hole, the past horizon is physically the redshifting, dimming surface of the star or whatever else collapsed to the black hole long ago. The infaller does not catch up with the image of the collapsed star; rather, the image of the collapsed star remains ahead of the infaller, still redshifting and dimming away.

When the infaller falls through the horizon, the horizon appears to split into two. In the Penrose diagram 1, an observer passing through the horizon goes from seeing the surface labeled Antihorizon to seeing two surfaces, the Parallel Horizon and the Horizon. The event horizon, or future horizon [26], the whitish grid in Fig. 4, appears out of nowhere. This is the surface of no return, within which space falls faster than light. The event horizon becomes visible only after the infaller has passed through it. The event horizon and the sky above are colored in Fig. 4 with an appropriately red- or blueshifted blackbody color.

As the infaller approaches the singularity, they have the impression of landing on a flat plane. The two horizons, the event and past horizons, appear to converge toward this flat plane. The infaller has the impression that they finally catch up with the past horizon at the singularity. The appearance of flattening can be attributed to the diverging tidal force near the singular surface. The mathematical details that give

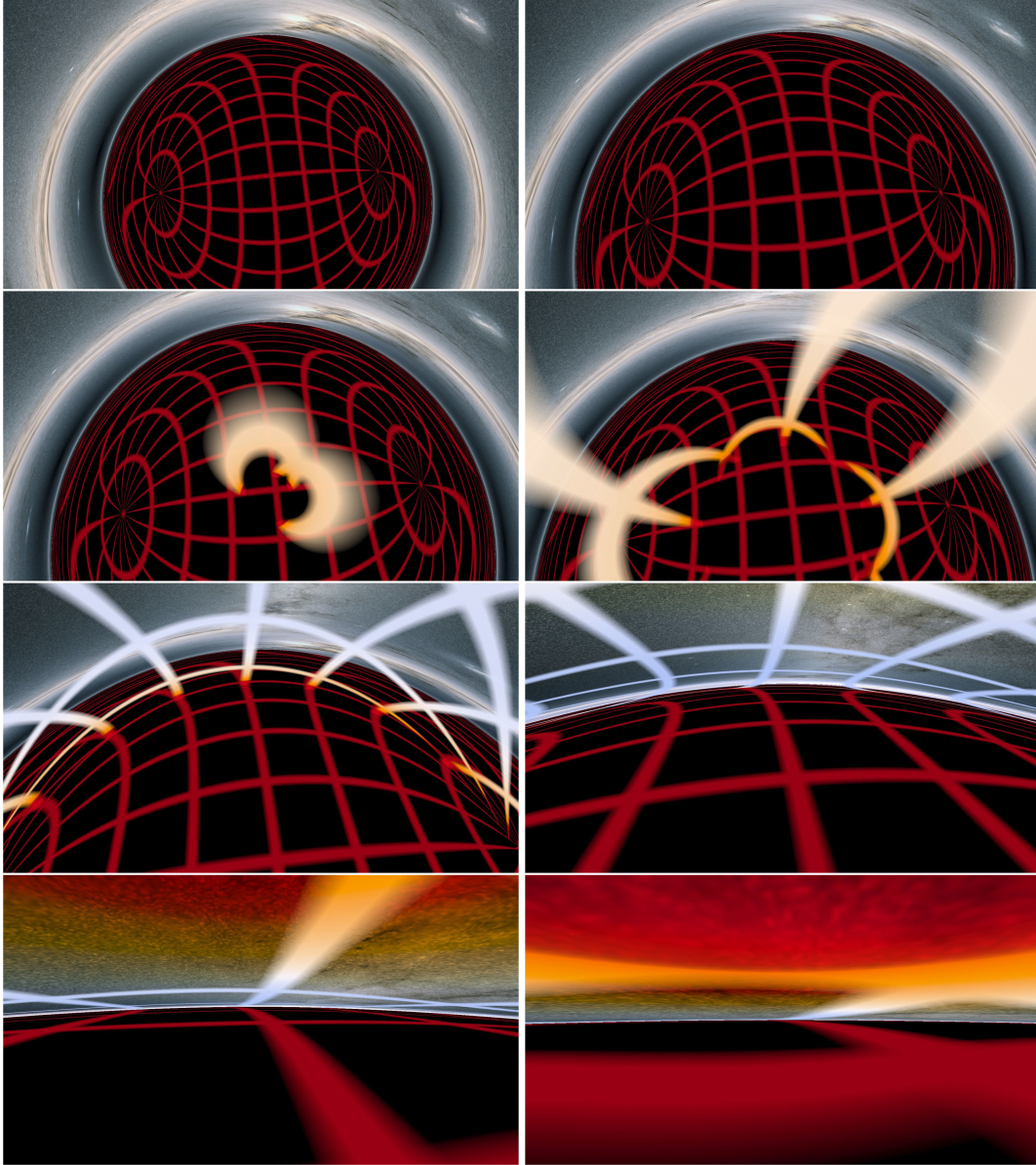


FIG. 4. Eight frames from a visualization of the view seen by an observer who free falls radially through the horizon of a Schwarzschild black hole [24]. The infaller is on a geodesic that starts at rest at infinity and falls radially with zero angular momentum. From left to right and top to bottom, the observer is at radii 1.5, 1.01, 0.99, 0.9, 0.5, 0.1, 0.01, and 0.001 horizon radii. The dark red grid is the black hole’s past horizon, which in a real black hole is replaced by the dimming, redshifting surface of the star or whatever else collapsed to the black hole long ago. The event or future horizon is painted with a grid colored with an appropriately red- or blueshifted blackbody color. The background is an image of the Milky Way from Gaia Data Release 3 [25].

rise to the observed surfacelike behavior are given in Appendix A 2.

III. ROTATING BLACK HOLES

In rotating black holes, the singularity is commonly presented as a ring of zero thickness, hidden in a timelike region behind two horizons. However, as claimed in Sec. IB, in realistic settings the Kerr geometry’s inner horizon is unstable and should collapse into a spacelike singular surface, blocking off any hypothetical wormholes

and ring singularities. Section III A elaborates this argument. Then, Sec. III B proceeds to show how two infallers will not be causally close upon reaching the inner horizon (and therefore the resulting singularity), mirroring the Schwarzschild analysis of Sec. II A.

A. Mass inflation

In the same year (1968) that Carter [21] revealed the Kerr geometry in all its glory, Penrose [27] pointed out that an observer who reaches the outgoing inner horizon of a black

hole will see the outside Universe infinitely blueshifted (Penrose considered a spherical charged, Reissner-Nordström, black hole, but the same holds true for a rotating, Kerr, black hole). In the Penrose diagram 2, the outgoing inner horizon is a null line extending from the future infinity of the Universe. Penrose conjectured that the diverging concentration of energy collecting at the inner horizon would cause it to collapse, preventing any of the bizarre features of the geometry inside the inner horizon from coming into existence.

The reason for the infinite blueshift at the inner horizon has to do with the behavior of geodesics near horizons. As first pointed out by Carter [28], the Hamilton-Jacobi equation in the Kerr geometry is separable, and therefore geodesics are exactly solvable, as outlined in Appendix B. It is convenient to describe Kerr geodesics in terms of a set of Hamilton-Jacobi parameters P_t , P_x , P_y , and P_ϕ , defined by Eqs. (B17). These parameters depend on the conserved energy E and azimuthal angular momentum L of the geodesic, but they themselves are not constants of motion. As the Hamilton-Jacobi parameters change along a geodesic, their signs dictate whether each respective coordinate increases or decreases as proper time progresses forward. In particular, above the outer horizon, the radial Hamilton-Jacobi parameter P_x can either be negative (corresponding to infalling particles) or positive (corresponding to outfalling particles), while the time parameter P_t is always negative (since particles must move forward in time t). But in the Black Hole region of the Kerr geometry, between the outer and inner horizons, the radial Hamilton-Jacobi parameter P_x must always be negative (infalling), signifying that space is falling inward faster than light, while the time parameter P_t can be either positive (outgoing, up-rightward in Penrose diagram 2) or negative (ingoing, up-leftward in Penrose diagram 2). The expression (B16) for the 4-momentum of a particle in the Boyer-Lindquist tetrad frame shows that, at the inner horizon, outgoing ($P_t > 0$) and ingoing ($P_t < 0$) geodesics see each other infinitely blueshifted. The infinite blueshift comes from the diverging denominator $1/\sqrt{|\Delta_x|}$ in the time and radial components p_t and p_x of the tetrad-frame momentum.

As outgoing and ingoing streams approach each other near the inner horizon, they see each other more and more blueshifted, the blueshift diverging as the reciprocal of the horizon function, $1/|\Delta_x|$. At the inner horizon, where the horizon function Δ_x goes to zero, the blueshift becomes infinite, whereupon the streams exceed the speed of light through each other and fall through separate outgoing and ingoing inner horizons into two separate regions of spacetime, the outgoing and ingoing wormholes in the Penrose diagram 2. Time t progresses in opposite directions in the two wormholes, backward in the outgoing wormhole ($P_t > 0$) and forward in the ingoing wormhole ($P_t < 0$).

As long as the spacetime is a complete vacuum, as the Kerr solution assumes, there is no contradiction to outgoing

and ingoing frames exceeding the speed of light through each other. But as soon as there are even the tiniest amounts of outgoing and ingoing light or matter, it is impossible for the outgoing and ingoing streams to exceed the speed of light through each other. In any real astronomical black hole, light and matter accreted from outside the horizon will fuel both outgoing (prograde) and ingoing (retrograde) streams at the inner horizon. Inevitably, the Kerr solution must break down at the inner horizon of any real black hole.

At the outer horizon, where the horizon function Δ_x also reaches zero, one may wonder why a similar breakdown does not occur as it does at the inner horizon thanks to the diverging denominator $1/\sqrt{|\Delta_x|}$ in expression (B16). In the outside Universe, time t progresses forward, equivalent to a negative Hamilton-Jacobi time parameter, $P_t < 0$. Continuity across the outer horizon requires P_t to remain negative there, meaning that all geodesics are necessarily ingoing as they cross the horizon. No outgoing geodesics cross the outer horizon. Another way of thinking about the distinction is that light rays near the event horizon tend to move away from it, while light rays near the inner horizon are driven closer to it. The event horizon is a repeller of light rays, the inner horizon an attractor.

The backreaction on the Kerr geometry caused by counterstreaming of outgoing and ingoing streams near the inner horizon was eventually calculated in a seminal paper by Poisson and Israel in 1990 [23]. Their first paper concerned spherical charged (Reissner-Nordström) black holes, but they soon followed with a paper on rotating black holes [29]. They showed that the backreaction was such as to cause an exponential growth in the interior mass, a phenomenon they dubbed “mass inflation.” The proper energy density in the center-of-mass frame of the counterstreaming streams also exponentiates, as does the Weyl curvature (tidal force).

As reviewed by [30], there has been some controversy as to the final outcome of mass inflation. The original scenario contemplated by Poisson and Israel was that the cross flow of outgoing and ingoing streams would be driven by a “Price tail” [31] of gravitational radiation generated when the black hole first collapsed. As confirmed by many subsequent researchers (see, e.g., [32] and references therein), the outcome in this scenario is a “weak, null singularity.” The singularity is weak in the sense that, although the tidal force diverges to infinity, it diverges in such a short proper time that objects in the spacetime are scarcely distorted.

However, as pointed out by [33], the Price tail of gravitational waves decays too rapidly to be the primary source of backreaction. In any real black hole, accretion of matter or radiation (cosmic microwave background photons if nothing else) soon (in a matter of seconds for a stellar-mass black hole) dominates the outgoing and ingoing streams impinging on the inner horizon. As long as there is ongoing accretion, the outcome is collapse to a spacelike

singularity. Numerical calculations of an accreting, rotating black hole [34] indicate that the collapse is of the chaotic variety discovered by Belinskii, Khalatnikov and Lifshitz in 1970 [1,2].

Recent calculations [35,36] using the apparatus of quantum field theory in curved spacetime show that, even in a pure vacuum in the absence of outside accretion, quantum effects arising from a Kerr black hole's own gravity destabilize the inner horizons. The renormalized expectation value of the quantum energy-momentum tensor remains finite and well behaved at the outer horizon, but it diverges at both outgoing and ingoing inner horizons. Calculations of the Hawking radiation seen by an infaller [37] point to the same conclusion. As the infaller approaches the black hole and passes the outer horizon, they continue to experience a tiny amount of Hawking radiation (we continue to use that term here, even though it most commonly refers to the asymptotically distant radiation predicted by [38]). But once the infaller reaches either inner horizon, they see a diverging amount of non-thermal radiation [37]. An outgoing infaller sees a diverging ingoing Hawking flux, while an ingoing infaller sees a diverging outgoing flux. The results continue to hold in the limit of a spherical (Schwarzschild) black hole, whose inner horizon is at zero radius; although curiously enough, the calculations are most challenging in the limit of small spin, the series expansions requiring more terms to achieve convergence.

The bottom line of these deliberations is that, although the Kerr line element provides an excellent approximation to the geometry of a real rotating black hole down to near its inner horizon, that approximation breaks down dramatically just above the inner horizon. Counterpropagating outgoing and ingoing streams, generated either by accretion from outside or by the black hole's own quantum self-irradiation, become hyperrelativistic as they approach the inner horizon, driving the center-of-mass density and curvature to exponentially huge values. Classically, the likely outcome [30,34] is Belinskii-Khalatnikov-Lifshitz collapse to a spacelike singular surface at the inner horizon. A more complete understanding awaits a consensus theory of quantum gravity.

B. Geodesics

In a rotating black hole, as in a spherical black hole, the boundary between regions visible and invisible to an infaller between the horizons is set by photons that have the largest angular motion. The Carter constant $\mathcal{K}$, which for a spherical black hole equals the total angular momentum squared, is infinite for photons with the largest angular motion:

$$\mathcal{K} = \infty. \quad (2)$$

The ratio J of azimuthal to total angular momentum is

$$J \equiv \frac{L}{\sqrt{\mathcal{K}}} = \sin \theta_J = \pm \sqrt{1 - y_J^2}, \quad (3)$$

where $y_J \equiv -\cos \theta_J$ is the largest latitude reached on the geodesic ($J = 0$ and $y_J = \pm 1$ for geodesics that reach the poles, $J = \pm 1$ and $y_J = 0$ on the equator). In the limit of infinite $\mathcal{K}$, the Hamilton-Jacobi solution (B21) simplifies to

$$\begin{aligned} -\int_{x_0} \frac{dx}{P_x} &= \frac{1}{\sqrt{\mathcal{K}}} \int_{r_0} \frac{dr}{\sqrt{(r - e_-)(e_+ - r)}} \\ &= \frac{2}{\sqrt{\mathcal{K}}} \left[\operatorname{atan} \sqrt{\frac{r - e_-}{e_+ - r}} \right]_{r_0}^r \\ &= \int_{y_0} \frac{dy}{P_y} = \frac{1}{\sqrt{\mathcal{K}}} \int_{y_0} \frac{dy}{\sqrt{(y + y_J)(y_J - y)}} \\ &= \frac{2}{\sqrt{\mathcal{K}}} \left[\operatorname{atan} \sqrt{\frac{y + y_J}{y_J - y}} \right]_{y_0}^y, \end{aligned} \quad (4)$$

where $e_{\pm}$ are the radii of the ergospheres at latitude θ_J ,

$$e_{\pm} = M \pm \sqrt{M^2 - Q^2 - a^2 \cos^2 \theta_J}. \quad (5)$$

Define the angle ψ by

$$\psi \equiv -\sqrt{\mathcal{K}} \int_{x_0} \frac{dx}{P_x} = \sqrt{\mathcal{K}} \int_{y_0} \frac{dy}{P_y}, \quad (6)$$

which starts at $\psi = 0$ at the position x_0, y_0 . A null trajectory passing through an emitter or observer at radius $r_0 \equiv a \cot(ax_0)$ and latitude $y_0 \equiv -\cos \theta_0$ is given parametrically in terms of the angle ψ by

$$\begin{aligned} r &= M - \frac{1}{2}(e_+ - e_-) \cos(\psi + \psi_r) \\ &= e_- + (e_+ - e_-) \sin^2 \left(\frac{\psi + \psi_r}{2} \right), \end{aligned} \quad (7a)$$

$$y = -y_J \cos(\psi + \psi_y), \quad (7b)$$

where

$$\psi_r \equiv \operatorname{acos} \left(-\frac{2(r_0 - M)}{e_+ - e_-} \right), \quad (8a)$$

$$\psi_y \equiv \operatorname{acos} \left(-\frac{y_0}{y_J} \right). \quad (8b)$$

The radius r and polar height y vary periodically with the angle ψ , with period 2π .

Figure 5 illustrates trajectories in radius r and latitude θ (azimuth ϕ being projected to 0) for an infaller who reaches the inner horizon at latitude $\theta_0 = \frac{1}{4}\pi$, equivalent to inclination angle $\frac{1}{2}\pi - \theta_0 = \frac{1}{4}\pi$ above the equator.

Compared to the case of a nonrotating (Schwarzschild) black hole shown in Fig. 3, visibility inside a rotating black hole is complicated by the fact that the rotation drags

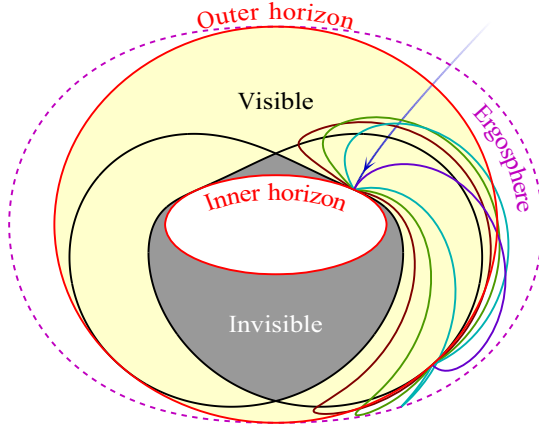


FIG. 5. Regions between the outer and inner horizons visible and invisible to an infaller who reaches the inner horizon of a Kerr black hole. The black hole here has spin parameter $a = 0.8M_*$, and the infaller (blue line) falls with zero angular momentum $L = 0$ along a geodesic at latitude $\theta_0 = 45^\circ$. The boundary of the (in)visible region is set by null trajectories with infinite Carter constant $\mathcal{K}$ and asymptotically small azimuthal angular momenta J ; such trajectories, dragged around by the rotation of the black hole, spiral around the black hole divergently many times, hugging the $J = 0$ surface (black lines) all the way between the outer and inner horizons. Null geodesics with infinite $\mathcal{K}$ and various azimuthal angular momenta J are shown on the radius-latitude $r - \theta$ plane (projected to azimuth $\phi = 0$) as solid lines passing through the observer at the inner horizon. To reach the observer at latitude θ_0 , $|J|$ cannot exceed $J_{\max} = \sin \theta_0$; the trajectories shown have $|J|/J_{\max}$ equal to 0 (black), $\frac{1}{4}$ (brown), $\frac{1}{2}$ (green), $\frac{3}{4}$ (cyan), and 1 (purple). For each $|J| < J_{\max}$ there are two trajectories, which approach the observer from, respectively, lower and higher latitude. Null geodesics with infinite $\mathcal{K}$ start as outgoing at (infinitesimally outside) the outer horizon, switch to ingoing inside the ergosphere, fall back through the outer horizon, down to the inner horizon. All the geodesics start on the outer horizon at the same latitude antipodal, $\theta = \frac{1}{2}\pi - \theta_0$, to the final latitude θ_0 at the inner horizon (but not antipodal in azimuth). Compare to Fig. 3 for a Schwarzschild black hole.

frames around it. The Boyer-Lindquist time and azimuthal coordinates t and ϕ along a geodesic, Eqs. (B22a), are not the best choice at or inside the horizon, because they diverge at the horizon thanks to the $1/\Delta_x$ factor. A better choice of time and azimuthal coordinates are principal null coordinates t_{pn} and ϕ_{pn} that corotate with the black hole along the ingoing or outgoing principal null congruences:

$$t_{\text{pn}} \equiv t \pm \int \frac{dx}{\Delta_x}, \quad \phi_{\text{pn}} = \phi \pm \int \frac{\omega_x dx}{\Delta_x}, \quad (9)$$

where the lower (−) sign is ingoing and the upper (+) sign outgoing. The corotating coordinates t_{pn} and ϕ_{pn} , also known as Kerr-Eddington-Finkelstein coordinates (often symbolized u/v and $\tilde{\phi}$), are constant along the ingoing (−) or outgoing (+) principal null geodesics. The Boyer-

Lindquist time coordinate t , which is spacelike between horizons, decreases along ingoing geodesics inside the outer horizon, per the Penrose diagram 2. The corotating time coordinate t_{pn} similarly decreases along ingoing geodesics; indeed it decreases not just between the horizons, but between the outer and inner ergospheres. Along outgoing geodesics, the ingoing corotating coordinate t_{pn} decreases between horizons, diverges at horizons, and increases outside horizons. The corotating azimuthal coordinate ϕ_{pn} decreases or increases in tandem with t_{pn} .

Between outer and inner horizons, the Hamilton-Jacobi parameter P_t can be either positive (outgoing) or negative (ingoing). The t component $p_t = P_t/(\rho\sqrt{|\Delta_x|})$ of the tetrad-frame momentum, Eq. (B16), diverges at horizons, where $\Delta_x = 0$. At horizons, outer or inner, ingoing and outgoing frames stream through each other at the speed of light; ingoing and outgoing frames see each other infinitely blueshifted. The divergence occurs only at horizons; ingoing and outgoing frames that lie strictly between horizons stream through each other at less than the speed of light. As described in Sec. III A, all objects that fall through the outer horizon are necessarily ingoing, so no divergence occurs there. However, ingoing infallers with sufficiently positive angular momentum L (prograde) can turn around between the horizons and become outgoing at the inner horizon; and ingoing objects can emit outgoing light from below the outer horizon. Light, whether ingoing or outgoing, tends to move away from the outer horizon, so light does not collect there. By contrast, ingoing and outgoing light always converges toward the inner horizon. Even the tiniest streams of ingoing and outgoing light at the inner horizon ignite the mass inflation instability there.

For definiteness, consider an infaller who is ingoing when they reach the inner horizon. The ingoing infaller sees ingoing light to have finite blue- or redshift. Ingoing light with infinite $\mathcal{K}$ necessarily has negative azimuthal angular momentum J (and outgoing light has positive J). Ingoing corotating coordinates t_{pn} and ϕ_{pn} along a geodesic are, from Eqs. (B22a),

$$t_{\text{pn}} = \int_{x_0} \left(\frac{P_t}{P_x} - 1 \right) \frac{dx}{\Delta_x} + \int_{y_0} \frac{P_\phi}{P_y} \frac{\omega_y dy}{\Delta_y}, \quad (10a)$$

$$\phi_{\text{pn}} = \int_{x_0} \left(\frac{P_t}{P_x} - 1 \right) \frac{\omega_x dx}{\Delta_x} + \int_{y_0} \frac{P_\phi}{P_y} \frac{dy}{\Delta_y}. \quad (10b)$$

If outgoing instead of ingoing frames were considered, then outgoing coordinates t_{pn} and ϕ_{pn} should be used, with a + instead of a − sign in Eqs. (10).

Abbreviate

$$r_\diamond \equiv \frac{1}{2}(r_+ - r_-) = \sqrt{M^2 - Q^2 - a^2}, \quad (11a)$$

$$e_\diamond \equiv \frac{1}{2}(e_+ - e_-) = \sqrt{M^2 - Q^2 - a^2 y_J^2}, \quad (11b)$$

and

$$c_r \equiv \cos(\psi + \psi_r), \quad s_r \equiv \sin(\psi + \psi_r), \quad (12a)$$

$$c_y \equiv \cos(\psi + \psi_y), \quad s_y \equiv \sin(\psi + \psi_y). \quad (12b)$$

The polar contribution to t_{pn} , Eq. (10a), can be incorporated into the radial contribution by using $-dx/P_x = dy/P_y$ to recast it as

$$\int_{y_0} \frac{P_\phi \omega_y dy}{\Delta_y P_y} = - \int_{x_0} \frac{P_\phi \omega_y dx}{\Delta_y P_x} = - \int_{x_0} aJ \frac{dx}{P_x / \sqrt{\mathcal{K}}}, \quad (13)$$

valid in the limit of infinite $\mathcal{K}$, when $P_\phi = L = \sqrt{\mathcal{K}}J$. The time and azimuthal angles t_{pn} and ϕ_{pn} along null geodesics with $\mathcal{K} = \infty$ integrate to

$$t_{\text{pn}} = -(\pm\xi - \xi_0 + 2\xi_\diamond i_r), \quad (14a)$$

$$\phi_{\text{pn}} = -(\pm\alpha - \alpha_0 + 2\alpha_\diamond i_r + \beta - \beta_0), \quad (14b)$$

in which overall minus (−) signs have been introduced because, as described following Eqs. (9), t_{pn} and ϕ_{pn} decrease along ingoing geodesics between the outer and inner ergospheres; for outgoing geodesics with respect to outgoing t_{pn} and ϕ_{pn} , the overall signs in Eqs. (14) would be plus (+). The r -dependent functions ξ and α in Eqs. (14) vary periodically with $\psi + \psi_r$, with period 2π . The $\pm$ signs in $\pm\xi$ and $\pm\alpha$ in Eqs. (14) are the sign of $\sin(\psi + \psi_r)$, and the integer $i_r \equiv [(\psi + \psi_r)/\pi]$ is the π phase of $\psi + \psi_r$, which increments by 1 at the same time as the $\pm$ sign of $\pm\xi$ or $\pm\alpha$ flips sign, ensuring that t_{pn} and ϕ_{pn} vary smoothly as ξ and α pass through their radial extrema. The functions ξ , α , and β in Eqs. (14) are

$$\xi \equiv \text{sgn}(c_r) \left(\frac{2M^2 - Q^2}{2r_\diamond} \ln \left(\frac{e_\diamond + a|J||s_r| - r_\diamond|c_r|}{e_\diamond + a|J||s_r| + r_\diamond|c_r|} \right) + e_\diamond|c_r| \right) - 2M \ln \left(1 + \frac{e_\diamond|s_r|}{a|J|} \right), \quad (15a)$$

$$\xi_\diamond \equiv \frac{2M^2 - Q^2}{r_\diamond} \ln \left(\frac{e_\diamond + r_\diamond}{a|J|} \right) - e_\diamond, \quad (15b)$$

$$\alpha \equiv \text{sgn}(c_r) \frac{a}{r_\diamond} \ln \left(\frac{r_\diamond|s_r| + a|J||c_r|}{r_\diamond + e_\diamond|c_r|} \right), \quad (15c)$$

$$\alpha_\diamond \equiv \frac{a}{r_\diamond} \ln \left(\frac{r_\diamond + e_\diamond}{a|J|} \right), \quad (15d)$$

$$\beta \equiv \text{acot}(|J|c_y, s_y). \quad (15e)$$

The r -dependent functions ξ and α have minima, respectively, $\xi = -\xi_\diamond$ and $\alpha = -\alpha_\diamond$ at $\psi + \psi_r = 0$ where $r = e_-$, pass through 0 at $\psi + \psi_r = \frac{1}{2}\pi$ where $r = M$, and

reach maxima $\xi = \xi_\diamond$ and $\alpha = \alpha_\diamond$ at $\psi + \psi_r = \pi$ where $r = e_+$. At outer or inner horizons, $s_r = \pm a|J|/e_\diamond$ and $c_r = \pm \sqrt{1 - s_r^2}$, with signs set by the phase of $\psi + \psi_r$. The sequence as $\psi + \psi_r$ increases from 0 to 2π is inner horizon at $\psi + \psi_r \in [0, \frac{1}{2}\pi]$, outer horizon at $\psi + \psi_r \in [\frac{1}{2}\pi, \pi]$, outer horizon at $\psi + \psi_r \in [\pi, \frac{3}{2}\pi]$, and inner horizon at $\psi + \psi_r \in [\frac{3}{2}\pi, 2\pi]$. The values of ξ and α at outer and inner horizons are

$$\xi_\pm \equiv \pm \left(\frac{2M^2 - Q^2}{r_\diamond} \ln \left(\frac{e_\diamond}{a|J|} \right) - r_\diamond \right) - 2M \ln 2, \quad (16a)$$

$$\alpha_\pm \equiv \pm \frac{a}{r_\diamond} \ln \left(\frac{e_\diamond}{a|J|} \right), \quad (16b)$$

where the upper and lower signs are at the outer and inner horizons, respectively. Note that the parts of the sequence inside the inner horizon do not occur in a real rotating black hole, because the mass inflation instability invalidates the Kerr solution there.

An observer at r_0 watching an emitter at r sees the emitter as they used to be a time t_{pn} ago. During that time the corotating coordinate ϕ_{pn} at r has changed by $\omega_x t_{\text{pn}}$, so the observer sees the emitter at azimuthal coordinate

$$\phi_{\text{pn}} - \omega_x t_{\text{pn}}. \quad (17)$$

In particular, an ingoing observer at the inner horizon r_- watching an ingoing light ray with infinite $\mathcal{K}$ falling through the outer horizon r_+ sees the emitter at azimuthal coordinate

$$\begin{aligned} \phi_{\text{pn}} - \omega_x t_{\text{pn}} &= - \left((\alpha_+ - \alpha_-) + (\beta_+ - \beta_-) - \frac{a}{r_+^2 + a^2} (\xi_+ - \xi_-) \right) \\ &= \frac{4a}{r_+^2 + a^2} \left(M \ln \left(\frac{e_\diamond}{a|J|} \right) - r_\diamond \right) - (\beta_+ - \beta_-) \end{aligned} \quad (18)$$

relative to their own azimuthal coordinate ϕ_{pn} . The relative azimuthal coordinate (18) is not coordinate gauge-invariant, but the difference in the perceived azimuthal coordinates of different emitters at the same outer horizon radius r_+ is coordinate gauge-invariant. A takeaway from Eq. (18) is that the perceived azimuthal coordinate $\phi_{\text{pn}} - \omega_x t_{\text{pn}}$ diverges logarithmically as $|J| \rightarrow 0$, indicating that an observer at the inner horizon sees geodesics with tiny J (and infinite $\mathcal{K}$) circulate the black hole many times, dragged around by the black hole's rotation. Numerical experiment confirms that the boundary of the (in)visible regime is determined by geodesics with the smallest azimuthal angular momentum, $J \sim 0$. The geodesics hug the $J = 0$ surface in radius r and latitude y , spiraling around in azimuth ϕ_{pn} , all the way from

outer to inner horizon. The resulting (in)visible region is illustrated in Fig. 5.

IV. IMPLICATIONS FOR QUANTUM GRAVITY

The conceptual adjustment that the singularity of a black hole is a surface not a point has repercussions for quantum gravity.

Without committing to any particular theory of quantum gravity, this section advances four claims about its desired features:

- (1) The quantum states of a black hole reside at its singular surface not at its horizon.
- (2) Quantum mechanics requires that the singular surface evolve unitarily.
- (3) The singular surface coevolves unitarily with its own Hawking radiation.
- (4) The singular surface probably evolves rapidly toward thermodynamic equilibrium.

We emphasize that we are not advocating that the singular surface should, in a full theory of quantum gravity, be interpreted as literally a two-dimensional geometric surface. In string theory, for example, there is a fundamental length scale, and there are T-dualities that exchange small and large scales. In loop quantum gravity, spacetime resolves into a discrete spin network. Whatever thing the singular surface resolves into, it will possess a Hilbert space of states, along with equations that describe its unitary evolution. The present paper is agnostic as to what those states and equations might be.

A. The quantum states of a black hole reside at its singular surface

The states of a black hole are commonly conceived to reside, or at least to be encoded somehow, on its horizon. The idea traces to Hawking [38], who first showed that a black hole behaves like a thermodynamic object with entropy equal to $1/(4G)$ times the area of its horizon. In statistical mechanics, the entropy counts the logarithm of the number of quantum states occupied by the system. The notion that a black hole's entropy really can be treated in this statistical mechanical way gains support from the fact that leading theories of quantum gravity, such as string theory, e.g., [39], and loop quantum gravity, e.g., [40], are able to reproduce the correct formula for entropy.

However, a prominent takeaway from the present paper is that someone freely falling through the event horizon does not encounter anything untoward there. An observer looking at an astronomical black hole from the outside is not looking at its event horizon. Rather, they are looking at the exponentially dimming, redshifting surface of the star that collapsed long ago, which [26] called the past horizon and [24] dubbed the “illusory horizon.” When an observer falls through the event horizon, they do not catch up with the collapsed star but rather continue to see it ahead of

them, still redshifting (or blueshifting) away, as depicted in Fig. 4.

Since the observer near the singularity can perceive only the part of spacetime lying within their past light cone, their view asymptotically kisses the singular surface at just a single point in the spatial $\{t, \theta, \phi\}$ directions. The rest of the singular surface remains hidden from view, behind the past horizon.

In quantum field theory, the statistical state of a system in which an observer has incomplete information is described by a density matrix acting on the Hilbert space of states. Associated with the density matrix is an entropy, obtained by tracing over the observer's ignorance. This suggests that the entropy of the black hole is a trace over the unseen states at the singular surface, and this remains true not only for observers outside the black hole, but also for those deep inside, down to where they are about to hit the singular surface.

If the states of a black hole reside at its singular surface, what does it mean to say that the states are encoded on its horizon? Carlip [41] has argued that the entropy of any local Killing horizon whose transverse oscillations satisfy a conformal symmetry in the orthogonal $t-r$ direction reproduces the Bekenstein-Hawking entropy formula. He argues that this could explain the simplicity and universality of the entropy formula. He does not require that microscopic states be physically localized to the horizon; a (near-)horizon symmetry, without reference to its source, is sufficient to reproduce the entropy.

Modern approaches to holography usually discard the possibility that information on the horizon is encoded solely by interior degrees of freedom because of the mismatch between the entropy's area scaling and the bulk interior's volume scaling, which can grow arbitrarily large without substantially changing the horizon area. Instead, degrees of freedom are usually treated as being encoded nonlocally (via, e.g., entanglement wedge reconstruction), rendering the question of where states reside meaningless in a gauge-invariant sense. The presence of nonlocal, correlational degrees of freedom is explored further in Sec. IV C. But this paper suggests that the more important source of quantum states lies not throughout the volume of the bulk but rather confined to the lower-dimensional structure of the singularity. This claim is supported by the recent calculations of [35–37], which show that the quantum energy-momentum and Hawking radiation seen by an infaller remain finite and tiny at the outer horizon and throughout the bulk but diverge at the inner horizon, where, as argued in Sec. III A, the Poisson-Israel inflationary instability likely precipitates collapse to a singular surface.

B. Unitary evolution of the singular surface

In general relativity, the singularity of the Schwarzschild geometry is a boundary beyond which the geometry cannot be continued. The singularity is the three-dimensional

boundary of four-dimensional spacetime. The dimension that comes to an end at the singularity is the time (the timelike radial) dimension, so the singular boundary is spacelike.

This cannot be true in quantum gravity. A fundamental tenet of quantum mechanics is that evolution must be unitary. Quantum mechanics deals with probabilities of quantum states at one time evolving into quantum states at another time. Unitary evolution means that amplitudes of states at one time are related by a unitary transformation to amplitudes at another time, which ensures that the sum of probabilities (squared amplitudes) is always one. Presumably quantum gravity replaces the *exitium in nihilum* (“exit into nothing”) predicted by general relativity by “something,” and that something should evolve unitarily.

Suppose that the black hole has collapsed and settled down. Unitarity requires that infallers who subsequently fall in and encounter the singular surface at different times t and angular locations θ, ϕ should count the same set of quantum states. If unitarity holds, the singular surface should be not a three-dimensional classical boundary but rather an effective two-dimensional surface that evolves unitarily.

Although classically the time coordinate t is spacelike at the singular boundary, it does provide a natural time ordering of events at the singularity, insofar as objects that fall to the singularity at later times t probe the singularity at later times than objects that fell in earlier.

We do not offer a proof of unitarity, which is notoriously tricky. Rather, we assert that unitarity is a necessary desideratum for a successful quantum description of a black hole’s singular surface.

C. Hawking radiation as a mediator of unitary evolution

Recent calculations using the apparatus of quantum field theory in curved spacetime [35–37] reveal diverging quantum energy-momentum and Hawking radiation at the inner horizons of black holes (including spherical black holes, where the inner horizon is at zero radius, $r = 0$). This fact suggests that the singular surface should be thought of not as having infinitesimal thickness but rather as being accompanied by a roiling atmosphere of Hawking radiation. The situation is reminiscent of that in quantum field theory, where a point particle such as an electron or photon should be thought of not as an isolated point but rather as accompanied by a roiling cloud of virtual particles polarized out of the vacuum by the presence of the particle.

Hawking radiation is a natural mediator of unitary evolution. A Hawking pair, drawing its energy from the black hole’s curvature, is created in a maximally entangled state. One of the pair falls to the singular surface, while its partner either escapes the black hole or else also falls to the singular surface. The singularity “observes” the first of the pair, but that observation does not destroy the entanglement

but rather shares the entanglement with other states of the singularity. If the partner escapes to the outside where it is observed, then likewise that observation does not destroy the entanglement but rather shares the entanglement with other states of the outside system, so that the singularity and outside world remain entangled with each other. If, instead of escaping, the partner falls to another place on the singular surface, then the partner shares its entanglement with that region of the singular surface, thereby entangling different spacelike-separated parts of the singular surface. When another Hawking pair is created, it repeats the process of further entangling. In this fashion the Hawking radiation that escapes the black hole becomes thoroughly entangled not only with the interior of the black hole, but also with other Hawking radiation that escapes.

D. Evolution to thermodynamic equilibrium

If the singular surface coevolves unitarily with its hot atmosphere of trapped Hawking radiation, then any disturbance from the outside should quickly dissipate, leaving the surface and its Hawking radiation in a condition of mutual thermodynamic equilibrium.

In the AdS-CFT correspondence [42], the reflecting AdS boundary is crucial to reflect a black hole’s Hawking radiation back into the black hole, allowing the Hawking radiation to come into thermodynamic equilibrium and ensuring unitarity.

In the present picture there is no need for a reflecting AdS boundary to confine the Hawking radiation. The great bulk of the Hawking radiation is generated inside the event horizon, and the trapped Hawking radiation is forced back to the singular surface. The Hawking radiation that dribbles out to the world beyond the event horizon is but a stifled remnant of the black hole’s fiery core.

V. CONCLUSION

This paper has argued that the widely repeated statement that the singularity at the center of a black hole is a point is incorrect. Although points near the singularity of a Schwarzschild black hole are spatially close, they are causally distant. As illustrated in Fig. 3, when two observers at two different angular positions fall to the singularity, they do not meet each other at the singularity; rather, they lose causal contact with each other already well away from the singularity. The singularity is a two-dimensional surface not a point.

The same is true for rotating black holes. In an astronomically realistic black hole, the Kerr geometry provides an accurate approximation down to the region just above the inner horizon, but it fails dramatically at the inner horizon. Even the tiniest combination of outgoing and ingoing perturbations, whether instigated by accretion from outside [34] or by the black hole’s own quantum self-irradiation, [35–37], drive the Poisson-Israel [23] exponential instability,

precipitating collapse to a spacelike singular surface. The instability and collapse effectively truncate the Kerr geometry at its inner horizon. Infallers who reach the inner horizon at two different angular positions are, as in a Schwarzschild black hole, causally distant, Fig. 5.

An observer looking at a black hole from the outside is not looking at its event horizon; rather they are looking at the exponentially dimming, redshifting surface of the star that collapsed long ago, which [26] called the past horizon and [24] dubbed the illusory horizon. When an observer falls through the event horizon, they do not catch up with the collapsed star but rather continue to see it ahead of them, still redshifting (or blueshifting) away. Since the infaller can see only within their past light cone, which narrows to a cusp at the singular surface, the singular surface remains hidden from view behind the past or illusory horizon, all the way down to the point where the infaller hits the singular surface. At this point the infaller has the impression that they catch up with the past or illusory horizon, which, thanks to the diverging tidal force, gives the appearance of flattening to a flat surface, Fig. 4.

Section IV discusses the implications for quantum gravity. It argues four key points: (1) regardless of whatever the final theory of quantum gravity might be, the quantum states of a black hole reside at its singular surface; (2) the singular surface should evolve unitarily; (3) it should coevolve with its own trapped Hawking radiation; and (4) it should evolve rapidly to a state of thermodynamic equilibrium. An infaller who has not (quite) reached the singular surface never gets to see the singular surface: it is always hidden behind the past horizon, even for infallers asymptotically close to it. For an infaller, the entropy of the black hole is a trace over the unseen states of the singular surface, which remains true not only for observers outside the event horizon, but also for those deep inside, down to where the observer is just about to hit the singular surface.

Calculations using the apparatus of quantum field theory in curved spacetime [35–37] show that as an infaller finds themselves approaching the inner horizon, the quantum energy-momentum and Hawking radiation they see diverge, indicating that the singular surface is hot. The calculations indicate that the great bulk of Hawking radiation is generated inside the event horizon; only a tiny fraction leaks out to the Universe outside. The trapped Hawking radiation is forced inward back to the singular surface. In AdS-CFT, an AdS boundary is needed to reflect a black hole’s Hawking radiation back to its event horizon, allowing the black hole to come into thermal equilibrium with its own Hawking radiation and enabling the closed system to evolve unitarily. In the present picture there is no need for an AdS boundary, because the profuse Hawking radiation generated inside the event horizon is trapped by the black hole’s own gravity. Any disturbance from outside should be able to dissipate rapidly, leaving the singular

surface and its attendant Hawking radiation in a condition of mutual thermodynamic equilibrium. We conjecture that, as in AdS-CFT, the system evolves unitarily, as quantum mechanics demands. Treating the black hole singularity as a hot, quantum correlated surface provides a path forward to understanding the true quantum gravitational nature of the most energetic places in the known Universe.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: GEODESICS IN THE SCHWARZSCHILD GEOMETRY

1. Geodesics

The Schwarzschild line element for a black hole of mass M is, in terms of Schwarzschild time t and polar coordinates r, θ, ϕ (units $c = 1$),

$$ds^2 = -\Delta dt^2 + \frac{1}{\Delta} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (\text{A1})$$

where Δ is the horizon function

$$\Delta \equiv 1 - \frac{r_s}{r}, \quad r_s \equiv 2GM. \quad (\text{A2})$$

With units of the speed of light c restored, the Schwarzschild radius is $r_s = 2GM/c^2$. Without loss of generality, a geodesic can be taken to follow $\phi = 0$. For a particle of mass m , conservation of energy E , angular momentum L , and rest mass m implies that the 4-momentum $p^\mu = mdx^\mu/d\tau$ along the geodesic satisfies

$$p_t = -E, \quad p_\theta = L, \quad p_\mu p^\mu = -m^2. \quad (\text{A3})$$

Equations (A3) imply that the radial component p^r of the 4-momentum satisfies

$$p^r = \pm(E^2 - U)^{1/2}, \quad (\text{A4})$$

where U is the effective potential

$$U \equiv \left(m^2 + \frac{L^2}{r^2}\right)\Delta. \quad (\text{A5})$$

Equation (A4) solves to give the angle θ as a function of radius r along the geodesic:

$$\theta = \int \frac{L dr}{\sqrt{(E^2 - m^2)r^4 - L^2 r^2 \Delta}}. \quad (\text{A6})$$

For a massless particle, $m = 0$, geodesics depend only on the ratio

$$J \equiv \frac{L}{E} \quad (\text{A7})$$

of the angular momentum to energy. The maximum possible angular momentum inside the horizon is $J \rightarrow \infty$, in which case Eq. (A6) simplifies to

$$\theta = \int \frac{dr}{\sqrt{r(r_s - r)}} = 2 \arcsin \sqrt{\frac{r}{r_s}}, \quad (\text{A8})$$

which gives the cardioid equation (1).

2. Flattening of the singular surface perceived by an infaller

In the locally orthonormal tetrad frame of a timelike observer with the 4-momentum given by Eqs. (A3), the coordinate-frame 4-momentum p^μ of a photon will transform into a tetrad-frame 4-momentum $p^m = e^m_\mu p^\mu$, where $g_{\mu\nu} e_m^\mu e_n^\nu = \eta_{mn}$. The angle χ this photon will be observed to make with respect to the outward radial direction will then satisfy

$$\tan \chi = \frac{p^2}{p^1}. \quad (\text{A9})$$

For an infalling observer free falling with energy E and zero angular momentum, $L = 0$, watching a light ray with angular momentum per unit energy J , the angle χ satisfies

$$\frac{J}{r} = \frac{\sin \chi}{(E/m) + \cos \chi \sqrt{(E/m)^2 - \Delta}}. \quad (\text{A10})$$

In the limit $r \rightarrow 0$, where $\Delta \equiv 1 - r_s/r \rightarrow -\infty$, Eq. (A10) simplifies to

$$\tan \chi \rightarrow \frac{J}{r} \sqrt{\frac{r_s}{r}} \rightarrow \infty, \quad (\text{A11})$$

that is,

$$\chi \rightarrow \frac{\pi}{2}, \quad (\text{A12})$$

independent of the energy E of the infaller. Equation (A10) indicates that null rays observed by an infaller approaching the singularity will become aberrated to the direction perpendicular to the radial axis, corresponding to a great circle on the observer's sky. If the observer has a finite angular momentum L , then the observer's transverse motion further aberrates the view in the transverse direction, but it traces out the same great circle on the sky. The past horizon thus has the appearance of flattening into an infinite plane, as illustrated in Fig. 4, regardless of the state of motion of the infaller.

3. Causal (affine) distance between infallers

The proper spatial distance between two infallers who reach the singularity at the same Schwarzschild time t but along different radial directions goes to zero at the singularity, but the causal distance between the two, the shortest causal path joining them, does not go to zero. The shortest causal path is illustrated by the red lines in the right panel of Fig. 3, a pair of null geodesics each with the maximum possible angular momentum, $J \rightarrow \infty$.

The affine distance λ along a geodesic, massive or massless, is defined by

$$p^\mu = m \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{d\lambda}. \quad (\text{A13})$$

The affine distance remains finite in the massless limit $m \rightarrow 0$ and thus provides a measure of causal distance along a null geodesic. Physically, the affine distance is the proper distance along a light ray measured by rulers parallel-transported along the light ray [24]. The affine distance depends on a frame-dependent normalization factor proportional to the observed frequency ω_{obs} of the light ray. Affine distances measured in different frames are related by a Lorentz boost.

The affine distance λ along a null geodesic is obtained by integrating $d\theta/d\lambda = p^\theta$ or equivalently $dr/d\lambda = p^r$. Normalized to a frame at rest at infinity, the affine distance is

$$\lambda = \frac{1}{J} \int r^2 d\theta = \int \frac{dr}{\sqrt{1 - J^2 \Delta / r^2}}. \quad (\text{A14})$$

Normalized to the frame of an observer, the affine distance λ_{obs} is

$$\lambda_{\text{obs}} = \omega_{\text{obs}} \lambda, \quad (\text{A15})$$

where ω_{obs} is the observed frequency (energy) relative to that at rest at infinity. For an observer with angular momentum L_{obs} at radius r_{obs} observing a null geodesic with $J \rightarrow \infty$, the observed frequency is

$$\omega_{\text{obs}} = \frac{J \sqrt{L_{\text{obs}}^2 + r_{\text{obs}}^2}}{r_{\text{obs}}^2}. \quad (\text{A16})$$

The shortest causal path joining infallers reaching the singularity is realized by a pair of photons emitted in opposite directions with maximum angular momentum, $J \rightarrow \infty$, from a point halfway (in angle) between the infallers, illustrated by the red lines in the right panel of Fig. 3. The causal path has two symmetrically equal parts, each following the path of a cardioid, Eq. (1). If the angular separation between the two infallers near the singularity is 2θ , then the observed affine distance along the shortest causal path is $2\lambda_{\text{obs}}$, twice the affine distance along each

individual null segment:

$$2\lambda_{\text{obs}} = \frac{2\omega_{\text{obs}}}{J} \int_0^\theta r^2 d\theta = \frac{2r_s^2 \sqrt{L_{\text{obs}}^2 + r_{\text{obs}}^2}}{r_{\text{obs}}^2} \int_0^\theta \sin^4 \frac{\theta}{2} d\theta$$

$$= \frac{r_s^2 \sqrt{L_{\text{obs}}^2 + r_{\text{obs}}^2}}{r_{\text{obs}}^2} \left(\frac{3}{4} \theta - \sin \theta + \frac{1}{8} \sin 2\theta \right). \quad (\text{A17})$$

The θ -dependent factor in parentheses on the right-hand side of Eq. (A17) is $\approx \frac{1}{40} \theta^5$ at small separations θ and equal to $\frac{3}{4} \pi$ at $\theta \rightarrow \pi$.

It should be emphasized that the affine distance λ_{obs} is a perceived observer-dependent distance. To fix the normalization would require making a gauge choice of observer. One canonical choice might be an observer radially infalling at the horizon, where $J \rightarrow \infty$ light rays begin their journey, $L_{\text{obs}} = 0$ and $r_{\text{obs}} = r_s$, in which case the observer-dependent factor in Eq. (A17) reduces to r_s . Another possible choice might an observer radially infalling near the singular surface, in which case the observer-dependent factor is r_s^2/r_{obs} , which diverges as $r_{\text{obs}} \rightarrow 0$.

The conclusion is that, although a causal distance between points on the singular surface can be defined, that distance cannot be construed as a Lorentz-invariant measure of separation between points on the singular surface.

APPENDIX B: GEODESICS IN THE KERR-NEWMAN GEOMETRY

1. Kerr-Newman line element

As first pointed out by Carter [28], geodesics in the Kerr-Newman [18,43] geometry are Hamilton-Jacobi separable. Although it is most common to express the Kerr-Newman line element in terms of Boyer-Lindquist [22] ellipsoidal coordinates t , r , θ , and ϕ , separation of variables is most natural with respect to a different set of radial and angular coordinates x and y in place of the Boyer-Lindquist radius r and polar angle θ , Eqs. (B2). In terms of coordinates t , x , y , and ϕ adapted to facilitate separation [44], the line element for Kerr-Newman and related separable geometries is

$$ds^2 = \rho^2 \left[-\Delta_x \left(\frac{dt - \omega_y d\phi}{1 - \omega_x \omega_y} \right)^2 + \frac{dx^2}{\Delta_x} + \frac{dy^2}{\Delta_y} + \Delta_y \left(\frac{d\phi - \omega_x dt}{1 - \omega_x \omega_y} \right)^2 \right]. \quad (\text{B1})$$

Separability requires that ω_x and Δ_x are functions only of the radial coordinate x , while ω_y and Δ_y are functions only of the polar coordinate y . The conformal factor ρ^2 must be a separated sum of functions of x and y . The Einstein equations that follow from the line element (B1) prove to be separable [44]. Solving the Einstein equations with zero source of energy-momentum, or with

energy-momentum sourced by a central charge, yields the Kerr-Newman geometry for a rotating black hole of mass M , charge Q , and spin a . For the Kerr-Newman geometry, the radial and angular coordinates x and y are related to the Boyer-Lindquist radius r and polar angle θ by

$$x = \frac{1}{a} \arctan \frac{a}{r}, \quad y = -\cos \theta, \quad (\text{B2})$$

whose differentials are related by

$$dx = -\frac{dr}{R^2}, \quad dy = \sin \theta d\theta, \quad (\text{B3})$$

with R defined by

$$R \equiv \sqrt{r^2 + a^2}. \quad (\text{B4})$$

For the Kerr-Newman geometry, the conformal factor ρ^2 in the line element (B1) is the separated sum

$$\rho^2 = r^2 + a^2 \cos^2 \theta, \quad (\text{B5})$$

ω_x is the angular velocity of the Boyer-Lindquist tetrad frame through the coordinates, ω_y is the angular momentum of principal null geodesics,

$$\omega_x = \frac{a}{R^2}, \quad \omega_y = a \sin^2 \theta, \quad (\text{B6})$$

Δ_x is the horizon function, and Δ_y the polar function,

$$\Delta_x = \frac{1}{R^2} \left(1 - \frac{2Mr + Q^2}{R^2} \right), \quad \Delta_y = \sin^2 \theta. \quad (\text{B7})$$

The vanishing of the horizon function Δ_x defines the radial location of outer and inner horizons r_+ and r_- , at

$$r_{\pm} = M \pm \sqrt{M^2 - Q^2 - a^2}. \quad (\text{B8})$$

The vanishing of the polar function Δ_y defines the location of north and south poles, at $\theta = 0$ and π . Ergospheres, boundaries of regions within which the black hole's rotation drags around frames faster than light, occur where $g_{tt} = 0$, at

$$e_{\pm} = M \pm \sqrt{M^2 - Q^2 - a^2 \cos^2 \theta}. \quad (\text{B9})$$

2. Geodesics from Hamilton-Jacobi separation

The Hamilton-Jacobi equation governing geodesics of a particle of rest mass m (and zero charge, if the black hole is charged; see [44] for generalization to a charged particle) is

$$g^{\mu\nu} p_\mu p_\nu = -m^2, \quad (\text{B10})$$

where the covariant momenta p_μ are, in the Hamilton-Jacobi approach, defined to be the derivatives of the particle action S with respect to the coordinates:

$$p_\mu \equiv \frac{\partial S}{\partial x^\mu}. \quad (\text{B11})$$

In geometries such as Kerr that are time-translation symmetric and azimuthally symmetric, the covariant time and azimuthal momenta p_t and p_ϕ define the conserved energy E and azimuthal angular momentum L of the particle,

$$p_t = -E, \quad p_\phi = L. \quad (\text{B12})$$

The Hamilton-Jacobi equation (B10) is separable if the action S is a separated sum of functions of the coordinates [28]:

$$S = -Et + L\phi + S_x + S_y, \quad (\text{B13})$$

in which S_x and S_y are, respectively, functions of x and y alone. The line element (B1) defines not only a metric, but also a tetrad, a foursome of locally inertial axes at each point, encoded in a vierbein matrix e^m_μ :

$$ds^2 = \eta_{mn} e^m_\mu e^n_\nu dx^\mu dx^\nu, \quad (\text{B14})$$

where $\eta_{mn} \equiv \text{diag}\{-1, 1, 1, 1\}$ is the Minkowski metric of special relativity. Latin and Greek indices denote, respectively, tetrad and coordinate indices. Recast into the tetrad frame, the Hamilton-Jacobi equation (B10) is

$$\eta^{mn} p_m p_n = -m^2, \quad (\text{B15})$$

in which the covariant tetrad-frame momenta $p_m = e^m_\mu p_\mu$ of the particle are

$$p_m = \frac{1}{\rho} \left\{ \frac{P_t}{\sqrt{|\Delta_x|}}, \frac{P_x}{\sqrt{|\Delta_x|}}, \frac{P_y}{\sqrt{\Delta_y}}, \frac{P_\phi}{\sqrt{\Delta_y}} \right\}, \quad (\text{B16})$$

with Hamilton-Jacobi parameters P_m defined in terms of the coordinate-frame momenta p_μ by

$$P_t = -E + L\omega_x, \quad P_\phi = L - E\omega_y, \quad (\text{B17a})$$

$$P_x = -p_x \Delta_x, \quad P_y = p_y \Delta_y. \quad (\text{B17b})$$

In terms of the Hamilton-Jacobi parameters P_m , the Hamilton-Jacobi equation (B15) is

$$\frac{-P_t^2 + P_x^2}{\Delta_x} + \frac{P_y^2 + P_\phi^2}{\Delta_y} = -m^2 \rho^2. \quad (\text{B18})$$

The Hamilton-Jacobi equation (B18) separates as

$$-\left(\frac{-P_t^2 + P_x^2}{\Delta_x} + m^2 \rho^2\right) = \frac{P_y^2 + P_\phi^2}{\Delta_y} + m^2 a^2 \cos^2 \theta = \mathcal{K}, \quad (\text{B19})$$

where $\mathcal{K}$ is a separation constant, the Carter constant. Physically, the Carter constant $\mathcal{K}$ is a constant of motion associated with conservation of total angular momentum squared. The separated Hamilton-Jacobi equations (B19) imply that

$$P_x = \pm \sqrt{P_t^2 - (\mathcal{K} + m^2 \rho^2) \Delta_x}, \quad (\text{B20a})$$

$$P_y = \pm \sqrt{-P_\phi^2 + (\mathcal{K} - m^2 a^2 \cos^2 \theta) \Delta_y}. \quad (\text{B20b})$$

The expressions (B20) for the Hamilton-Jacobi parameters P_x and P_y can be interpreted as effective radial and polar potentials governing the motion of the particle in the radial x and polar y directions. The coordinates along a particle geodesic follow from integrating $mdx^\mu/d\tau = p^\mu = e^m_\mu p^k$, which in the present case involves integrating $dy/dx = p^y/p^x = -P_y/P_x$, equivalent to the implicit equation

$$-\frac{dx}{P_x} = \frac{dy}{P_y}. \quad (\text{B21})$$

The time and azimuthal coordinates t and ϕ along the trajectory follow from integrating

$$dt = \frac{P_t dx}{P_x \Delta_x} + \frac{\omega_y P_\phi dy}{P_y \Delta_y}, \quad (\text{B22a})$$

$$d\phi = \frac{\omega_x P_t dx}{P_x \Delta_x} + \frac{P_\phi dy}{P_y \Delta_y}. \quad (\text{B22b})$$

3. Conditions on Hamilton-Jacobi parameters

The Hamilton-Jacobi equation (B18) can be rearranged as

$$P_t^2 - P_x^2 = \left(\frac{P_y^2 + P_\phi^2}{\Delta_y} + m^2 \rho^2\right) \Delta_x, \quad (\text{B23})$$

which shows that $P_t^2 - P_x^2$ must change sign across a horizon, where the horizon function Δ_x passes through zero. Equation (B23) shows that the Hamilton-Jacobi parameters P_t and P_x must satisfy

$$\begin{aligned} |P_t| &> |P_x| & \text{if } \Delta_x > 0, \\ |P_t| &= |P_x| & \text{if } \Delta_x = 0, \\ |P_t| &< |P_x| & \text{if } \Delta_x < 0. \end{aligned} \quad (\text{B24})$$

The Hamilton-Jacobi parameters P_m must be continuous, including across horizons. The conditions (B24) imply that P_t must have the same sign everywhere throughout any connected region where the horizon function Δ_x is positive, while P_x must have the same sign everywhere throughout any connected region where Δ_x is negative. In summary, the signs of the Hamilton-Jacobi parameters P_t and P_x must be as follows.

- (1) Outside the outer horizon, in the Universe part of the Kerr geometry in Fig. 2, the time parameter P_t must be negative, reflecting the fact that the time coordinate t must be timelike and increasing with the proper time of any particle. The radial parameter P_x can be either positive (outfalling) or negative (infalling).
- (2) Between the outer and inner horizons, in the Black Hole part of the geometry in Fig. 2, the radial parameter P_x must be negative, reflecting the fact that the radius is timelike and decreasing with the proper time of any particle. The time parameter P_t can be either positive (outgoing) or negative (ingoing).
- (3) Inside the inner horizon, the time parameter P_t is positive in the outgoing wormhole and negative in the ingoing wormhole. The radial parameter P_x can be either positive (outfalling) or negative (infalling).

-
- [1] V. A. Belinskii, I. M. Khalatnikov, and E. M. Lifshitz, Oscillatory approach to a singular point in the relativistic cosmology, *Adv. Phys.* **19**, 525 (1970).
 - [2] V. A. Belinskii, I. M. Khalatnikov, and E. M. Lifshitz, A general solution of the Einstein equations with a time singularity, *Adv. Phys.* **31**, 639 (1982).
 - [3] L. Andersson, H. van Elst, W. C. Lim, and C. Uggla, Asymptotic silence of generic cosmological singularities, *Phys. Rev. Lett.* **94**, 051101 (2005).
 - [4] R. Geroch, E. H. Kronheimer, and R. Penrose, Ideal points in space-time, *Proc. R. Soc. A* **327**, 545 (1972).
 - [5] A. García-Parrado and J. M. M. Senovilla, Causal structures and causal boundaries, *Classical Quantum Gravity* **22**, R1 (2005).
 - [6] W. Rindler, *Essential Relativity: Special, General, and Cosmological* (Springer-Verlag, New York, Heidelberg, Berlin, 1977).
 - [7] M. D. Kruskal, Maximal extension of Schwarzschild metric, *Phys. Rev.* **119**, 1743 (1960).
 - [8] G. Szekeres, On the singularities of a Riemann manifold, *Publ. Math. Debrecen* **7**, 285 (1960).
 - [9] R. Penrose, Conformal treatment of infinity, *Gen. Relativ. Gravit.* **43**, 901 (2011).
 - [10] S. Chandrasekhar, *The Mathematical Theory of Black Holes* (Clarendon Press, Oxford, England, 1983).
 - [11] B. Greene, *The Fabric of the Cosmos* (Alfred A. Knopf, New York, 2004).
 - [12] N. d. Tyson, *Death by Black Hole* (W. W. Norton & Company, New York, 2007).
 - [13] M. Bartusiak, *Black Hole: How an Idea Abandoned by Newtonians, Hated by Einstein, and Gambled On by Hawking Became Loved* (Yale University Press, New Haven, CT, 2016).
 - [14] K. Schwarzschild, Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie, *Sitzungsber. Preuss. Akad. Wiss. Berlin Klasse Math. Phys. Tech.* **1916**, 189 (1916).
 - [15] C. S. Reynolds, Observational constraints on black hole spin, *Annu. Rev. Astron. Astrophys.* **59**, 117 (2021).
 - [16] J. A. Orosz, J. E. McClintock, J. P. Aufdenberg, R. A. Remillard, M. J. Reid, R. Narayan, and L. Gou, The mass of the black hole in Cygnus X-1, *Astrophys. J.* **742**, 84 (2011).
 - [17] L. Gou, J. E. McClintock, M. J. Reid, J. A. Orosz, J. F. Steiner, R. Narayan, J. Xiang, R. A. Remillard, K. A. Arnaud, and S. W. Davis, The extreme spin of the black hole in Cygnus X-1, *Astrophys. J.* **742**, 85 (2011).
 - [18] R. P. Kerr, Gravitational field of a spinning mass as an example of algebraically special metrics, *Phys. Rev. Lett.* **11**, 237 (1963).
 - [19] R. P. Kerr, Discovering the Kerr and Kerr-Schild metrics, in *The Kerr Spacetime: Rotating Black Holes in General Relativity*, edited by D. L. Wiltshire, M. Visser, and S. Scott (Cambridge University Press, Cambridge, England, 2009), pp. 38–72.
 - [20] A. J. S. Hamilton and J. P. Lisle, The river model of black holes, *Am. J. Phys.* **76**, 519 (2008).
 - [21] B. Carter, Global structure of the Kerr family of gravitational fields, *Phys. Rev.* **174**, 1559 (1968).
 - [22] R. H. Boyer and R. W. Lindquist, Maximal analytic extension of the Kerr metric, *J. Math. Phys. (N.Y.)* **8**, 265 (1967).
 - [23] E. Poisson and W. Israel, Internal structure of black holes, *Phys. Rev. D* **41**, 1796 (1990).
 - [24] A. J. S. Hamilton and G. Polhemus, Stereoscopic visualization in curved spacetime: Seeing deep inside a black hole, *New J. Phys.* **12**, 123027 (2010).
 - [25] The color of the sky from Gaia's early data release 3 (2020).
 - [26] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge University Press, Cambridge, England, 1973).
 - [27] R. Penrose, Structure of space-time, in *Battelle Rencontres: 1967 Lectures in Mathematics and Physics*, edited by C. de Witt-Morette and J. A. Wheeler (W. A. Benjamin, New York, 1968), pp. 121–235.

- [28] B. Carter, Hamilton-Jacobi and Schrödinger separable solutions of Einstein's equations, *Commun. Math. Phys.* **10**, 280 (1968).
- [29] C. Barrabès, W. Israel, and E. Poisson, Collision of light-like shells and mass inflation in rotating black holes, *Classical Quantum Gravity* **7**, L273 (1990).
- [30] T. McMaken and A. J. S. Hamilton, Geometry near the inner horizon of a rotating, accreting black hole, *Phys. Rev. D* **103**, 084014 (2021).
- [31] R. H. Price, Nonspherical perturbations of relativistic gravitational collapse. I. Scalar and gravitational perturbations, *Phys. Rev.* **5**, 2419 (1972).
- [32] M. Dafermos and J. Luk, The interior of dynamical vacuum black holes I: The C^0 -stability of the Kerr Cauchy horizon, *Ann. Math.* **202**, 309 (2025).
- [33] A. J. S. Hamilton and P. P. Avelino, The physics of the relativistic counter-streaming instability that drives mass inflation inside black holes, *Phys. Rep.* **495**, 1 (2010).
- [34] A. J. S. Hamilton, Mass inflation followed by Belinskii-Khalatnikov-Lifshitz collapse inside accreting, rotating black holes, *Phys. Rev. D* **96**, 084041 (2017).
- [35] N. Zilberman, M. Casals, A. Ori, and A. C. Ottewill, Two-point function of a quantum scalar field in the interior region of a Kerr black hole, *Phys. Rev. D* **106**, 125011 (2022).
- [36] N. Zilberman, M. Casals, A. Ori, and A. C. Ottewill, Quantum fluxes at the inner horizon of a spinning black hole, *Phys. Rev. Lett.* **129**, 261102 (2022).
- [37] T. McMaken and A. J. S. Hamilton, Hawking radiation inside a rotating black hole, *Phys. Rev. D* **109**, 065023 (2024).
- [38] S. W. Hawking, Particle creation by black holes, *Commun. Math. Phys.* **43**, 199 (1975).
- [39] A. Strominger and C. Vafa, Microscopic origin of the Bekenstein-Hawking entropy, *Phys. Lett. B* **379**, 99 (1996).
- [40] A. Ashtekar, J. Baez, A. Corichi, and K. Krasnov, Quantum geometry and black hole entropy, *Phys. Rev. Lett.* **80**, 904 (1998).
- [41] S. Carlip, Black hole entropy from conformal field theory in any dimension, *Phys. Rev. Lett.* **82**, 2828 (1999).
- [42] J. Maldacena, The large N limit of superconformal field theories and supergravity, *Adv. Theor. Math. Phys.* **2**, 231 (1998).
- [43] E. T. Newman, E. Couch, K. Chinnapared, A. Exton, A. Prakash, and R. Torrence, Metric of a rotating, charged mass, *J. Math. Phys. (N.Y.)* **6**, 918 (1965).
- [44] A. J. S. Hamilton, The interior structure of rotating black holes II. Uncharged black holes, *Phys. Rev. D* **84**, 124056 (2011).