

Color-magnetic correlations in SU(2) and SU(3) lattice QCD

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We study the two-point field-strength correlation $g^2 \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^b(s') \rangle$ in the Landau gauge in SU(2) and SU(3) quenched lattice QCD, as well as the gluon propagator $g^2 \langle A_\mu^a(s) A_\nu^b(s') \rangle$. The Landau-gauge gluon propagator $g^2 \langle A_\mu^a(s) A_\mu^a(s') \rangle$ is well described by the Yukawa-type function e^{-mr}/r with $r \equiv |s - s'|$ for $r = 0.1\text{--}1.0$ fm in both SU(2) and SU(3) QCD. Next, motivated by color-magnetic instabilities in the QCD vacuum, we investigate the perpendicular-type color-magnetic correlation, $C_\perp(r) \equiv g^2 \langle H_z^a(s) H_z^a(s + r\hat{\perp}) \rangle$ ($\hat{\perp}$: unit vector on the xy plane), and the parallel-type one, $C_\parallel(r) \equiv g^2 \langle H_z^a(s) H_z^a(s + r\hat{\parallel}) \rangle$ ($\hat{\parallel}$: unit vector on the tz plane). These two quantities reproduce all the correlation of $g^2 \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^b(s') \rangle$, due to the Lorentz and global SU(N_c) color symmetries in the Landau gauge. Curiously, the perpendicular-type color-magnetic correlation $C_\perp(r)$ is found to be always negative for arbitrary r , except for the same-point correlation at $r = 0$. In contrast, the parallel-type color-magnetic correlation $C_\parallel(r)$ is always positive. In the infrared region of $r \gtrsim 0.4$ fm, $C_\perp(r)$ and $C_\parallel(r)$ strongly cancel each other, which leads to a significant cancellation in the sum of the field-strength correlations as $\sum_{\mu,\nu} g^2 \langle G_{\mu\nu}^a(s) G_{\mu\nu}^a(s') \rangle \propto C_\perp(|s - s'|) + C_\parallel(|s - s'|) \simeq 0$. Finally, we decompose the field-strength correlation into quadratic, cubic, and quartic terms of the gluon field A_μ in the Landau gauge. For the perpendicular-type color-magnetic correlation $C_\perp(r)$, the quadratic term is always negative, which is explained by the Yukawa-type gluon propagator. The quartic term gives a relatively small contribution. In the infrared region, the cubic term is positive and tends to cancel with the quadratic term, resulting in a small value of $C_\perp(r)$.

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I. INTRODUCTION

Quantum chromodynamics (QCD) is established as the fundamental theory of the strong interaction, and it is described as an SU(N_c) non-Abelian gauge theory with the color number $N_c = 3$ [1–3]. Stemming from its non-Abelian nature, QCD shows the asymptotic freedom, that is, its coupling decreases with the renormalization scale. Therefore, perturbative QCD [4] is workable for the analysis of high-energy hadron reactions in the framework of the parton model [5,6].

At low energies, however, the QCD coupling becomes strong, and perturbative QCD is no more applicable. Reflecting the strong-coupling nature in the infrared region, QCD exhibits nonperturbative phenomena such as color confinement [7] and dynamical chiral symmetry breaking [8], and the QCD vacuum structure itself becomes highly nontrivial [9].

For instance, due to the asymptotic freedom, QCD has a color-magnetic instability, which induces spontaneous emergence of color-magnetic fields [10–16]. In other words, the system with zero color-magnetic field is energetically unstable. The nonzero color-magnetic QCD vacuum is called the Savvidy vacuum and/or the Copenhagen vacuum. Actually, the gluon condensate takes a large positive value, i.e., $\langle G_{\mu\nu}^a G_{\mu\nu}^a \rangle = 2(\langle H_a^2 \rangle - \langle E_a^2 \rangle) > 0$, in QCD in the Minkowski space, which means significant excess of color-magnetic fields rather than color-electric fields.

Historically, Savvidy first showed the magnetic instability in the Yang-Mills theory in 1977, using the one-loop effective potential with constant H , that is, the system with a constant color-magnetic field H has lower energy than that of the perturbative vacuum with $H = 0$ [10]. In fact, the energy density $\varepsilon(H)$ of a constant color-magnetic field is given by

$$\varepsilon(H) - \varepsilon(0) = \frac{1}{2} H^2 + \frac{11(gH)^2}{48\pi^2} \left(\ln \frac{gH}{\mu^2} - \frac{1}{2} \right) - i \frac{(gH)^2}{8\pi} \quad (1)$$

with the gauge coupling g at the renormalization point μ in the SU(2) Yang-Mills theory at the one-loop level.

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Here, the logarithmic term includes the β -function coefficient with a minus sign, and this term causes instability of the $H = 0$ system in the case of asymptotic freedom. The energy minimum is achieved at nonzero color-magnetic field as

$$gH = \mu^2 \exp\left(-\frac{24\pi^2}{11g^2}\right). \quad (2)$$

This color-magnetic instability is logarithmic-type and cannot be removed by higher-order corrections.

The solution of the one-loop effective action with $H(x)$ is known to be a multivortex system, which was found by the Copenhagen group around 1980 [15]. The multivortex system is inhomogeneous and clearly breaks translational and rotational symmetry, while the physical vacuum is Poincare invariant.

Then, at an infrared scale, such inhomogeneous systems must appear as randomly directed domains to recover the rotational symmetry in the physical QCD vacuum [12,13,16], like magnetic materials above the Curie temperature. In this way, the multivortex system is conjectured to form a fluctuating stochastic domain structure at a large scale. Considering the fluctuating color fields in the QCD vacuum [12,13], Kirzhnits *et al.* showed that a possible randomization of QCD can produce an effect similar to localization, resulting in a linear interquark potential [17]. Dosch and Simonov proposed the “stochastic vacuum model” for gauge-invariant field-strength correlators and demonstrated that its infrared exponential damping yields an asymptotic linear potential [18–20].

In spite of the importance of the color-magnetic instability, which is directly related to the QCD-vacuum structure, such analytical research has not been well studied beyond the one-loop effective action. One obvious reason is due to theoretical difficulties to analyze nonperturbative properties in infrared QCD.

Nowadays, lattice QCD [21–23] is a standard theoretical method for the analysis of nonperturbative QCD, and it is numerically performed with the Monte Carlo method based on the Euclidean path integral of QCD. In lattice QCD, as an importance sampling, lattice gauge configurations are numerically generated, and their set can be regarded as a typical ensemble of the QCD vacuum. Therefore, one obtains much important information of the nonperturbative-QCD vacuum by analyzing the lattice QCD configurations.

As a pioneering work, based on the stochastic vacuum model, Di Giacomo *et al.* [24–28] and Bali *et al.* [29] found that the gauge-invariant field-strength correlator exhibits infrared exponential damping in lattice QCD.

Motivated by these studies, we study the field-strength correlation and its overall behavior in lattice QCD [30]. In this paper, using lattice QCD, we mainly investigate the color-magnetic correlation in the Landau gauge, which has

many advantages in terms of symmetries and minimal gauge-field fluctuations.

The organization of this paper is as follows. In Sec. II, we present the formalism of color-magnetic correlations in QCD in the Landau gauge. Section III summarizes the setup of lattice QCD calculations in the case of SU(2) and SU(3) color, respectively. In Sec. IV, we show the lattice QCD result of the Landau-gauge gluon propagator. Section V shows the numerical results of the color-magnetic correlations in SU(2) and SU(3) lattice QCD. In Sec. VI, we analyze the color-magnetic correlations in terms of the order of the gluon field. Section VII is devoted for the summary and concluding remarks.

II. COLOR-MAGNETIC CORRELATIONS IN QCD

This section presents the formalism of the color-magnetic correlation $g^2 \langle H_z^a(s) H_z^a(s') \rangle$, which is generalized to the field-strength correlation $g^2 \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^b(s') \rangle$, in the Landau gauge in SU(N_c) QCD.

A. Color-magnetic correlations in the Landau gauge

SU(N_c) QCD is described by the quark field $q(s)$ and the gluon field $A_\mu(s) = A_\mu^a(s) T^a \in \mathfrak{su}(N_c)$ (T^a : SU(N_c) generators), and its Lagrangian is expressed as [4,22]

$$\mathcal{L} = -\frac{1}{2} \text{Tr} G_{\mu\nu} G^{\mu\nu} + \bar{q}(i\not{D} - m)q. \quad (3)$$

The quark field $q_{c,f}^\alpha(s)$ has a color index $c = 1, 2, \dots, N_c$ in the fundamental representation, as well as a Dirac spinor index α and a flavor index f . We have used the covariant derivative $D_\mu \equiv \partial_\mu + igA_\mu$ and the field strength

$$\begin{aligned} G_{\mu\nu} &\equiv G_{\mu\nu}^a T^a \equiv \frac{1}{ig} [D_\mu, D_\nu] \\ &= \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \in \mathfrak{su}(N_c), \end{aligned} \quad (4)$$

with the gauge coupling g . The color-magnetic field $H_i(s)$ ($i = 1, 2, 3$) is given by the spatial field strength,

$$H_i(s) = H_i^a(s) T^a = \frac{1}{2} \epsilon_{ijk} G_{jk}(s) \in \mathfrak{su}(N_c). \quad (5)$$

In this paper, we investigate the field-strength correlation $g^2 \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^b(s') \rangle$, which is generally gauge variant in QCD, and its analysis requires some gauge fixing. (As an alternative, one could consider a gauge-invariant quantity similar to that by linking the two field strengths with suitable parallel transporters [23]. However, this method gives uncertainty regarding how to make the connection.)

To investigate the correlation, we mainly use the Landau gauge because it is widely used due to its many useful symmetries, such as Lorentz, translational, and global color symmetries. In Euclidean QCD, we define the Landau

gauge so as to minimize the global quantity,

$$R[A_\mu^a] \equiv \int d^4s \{A_\mu^a(s)A_\mu^a(s)\}, \quad (6)$$

via a gauge transformation [31–34], which leads to the local gauge fixing condition $\partial_\mu A_\mu = 0$. (Note that such a global definition cannot be applied in the Minkowski metric.) In this global definition, the Landau gauge has a clear physical interpretation that it strongly suppresses in total artificial gauge-field fluctuations associated with the gauge degrees of freedom [32].

Considering the nontrivial color-magnetic picture for the QCD vacuum, we investigate the following two types of color-magnetic correlation in the Landau gauge in lattice QCD [30],

(1) Perpendicular-type color-magnetic correlation

$$C_\perp(r) \equiv g^2 \langle H_z^a(s) H_z^a(s + r\hat{\perp}) \rangle \quad (7)$$

with $\hat{\perp}$ an arbitrary unit vector on the xy plane,

(2) Parallel-type color-magnetic correlation

$$C_\parallel(r) \equiv g^2 \langle H_z^a(s) H_z^a(s + r\hat{\parallel}) \rangle \quad (8)$$

with $\hat{\parallel}$ an arbitrary unit vector on the tz plane.

In Euclidean Landau-gauge QCD, we note the relation

$$\begin{aligned} \langle H_z^a(s) H_z^a(s + r\hat{z}) \rangle &= \langle H_z^a(s) H_z^a(s + r\hat{t}) \rangle, \\ i.e., \\ \langle G_{xy}^a(s) G_{xy}^a(s + r\hat{z}) \rangle &= \langle G_{xy}^a(s) G_{xy}^a(s + r\hat{t}) \rangle, \end{aligned} \quad (9)$$

due to the four-dimensional rotational invariance. Furthermore, as shown in the next subsection, any field-strength correlation $g^2 \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^b(s') \rangle$ can be expressed with the two correlations $C_\perp(r)$ and $C_\parallel(r)$ in the Landau gauge.

In the lattice QCD formalism, $C_\perp(r)$ and $C_\parallel(r)$ correspond to the spatial correlation of two plaquettes, as illustrated in Fig. 1.

B. General field-strength correlation

In Euclidean QCD in the Landau gauge, all two-point field-strength correlations $g^2 \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^b(s') \rangle$ can be expressed with the two correlations $C_\perp(r)$ and $C_\parallel(r)$, due to the Lorentz and global SU(N_c) color symmetries.

Without loss of generality, we set

$$s^R \equiv s - s' = r\hat{t} = (0, 0, 0, r) \text{ (relatively temporal)} \quad (10)$$

by a suitable four-dimensional rotation. Then, because of color and reflection symmetries, the field-strength correlation is written by

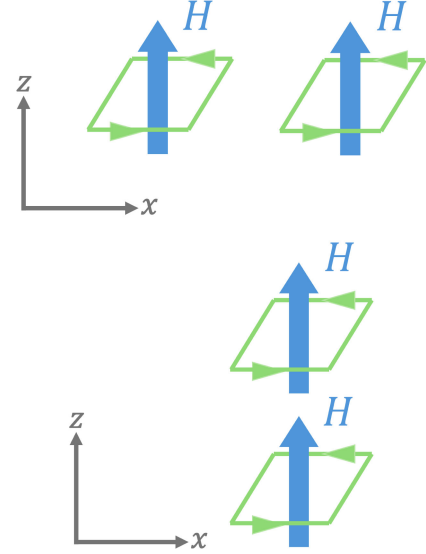


FIG. 1. Plaquette correlations corresponding to the spatial color-magnetic correlations at the same Euclidean time, $C_\perp(r)$ and $C_\parallel(r)$, in lattice QCD. The upper plaquette correlation corresponds to $C_\perp(r) = g^2 \langle H_z^a(s) H_z^a(s + r\hat{x}) \rangle$, and the lower one $C_\parallel(r) = g^2 \langle H_z^a(s) H_z^a(s + r\hat{z}) \rangle$.

$$g^2 \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^b(s') \rangle = \frac{\delta^{ab}}{N_c^2 - 1} (\delta_{\mu\alpha} \delta_{\nu\beta} - \delta_{\mu\beta} \delta_{\nu\alpha}) K_{\mu\nu}(r) \quad (\text{no sum}). \quad (11)$$

This quantity is antisymmetric with respect to the exchange of α and β . Because it is also antisymmetric under the exchange of μ and ν , $K_{\mu\nu}(r)$ has to be symmetric. Since the tensor factor in Eq. (11) becomes zero for $\mu = \nu$, the diagonal component of $K_{\mu\nu}(r)$ can be any value and is set to zero here. Then, $K_{\mu\nu}$ is expressed as

$$K_{\mu\nu}(r) = K_{\nu\mu}(r), \quad K_{\mu\mu}(r) = 0 \quad (\text{no sum}), \quad (12)$$

and one obtains from Eq. (11)

$$K_{\mu\nu}(r) = g^2 \langle G_{\mu\nu}^a(s) G_{\mu\nu}^a(s') \rangle \quad (\text{no sum on } \mu, \nu). \quad (13)$$

Due to the rotational symmetry, $K_{\mu\nu}(r)$ has only two independent components,

$$K_{tx}(r) = K_{ty}(r) = K_{tz}(r) = C_\perp(r) \quad (14)$$

and

$$K_{xy}(r) = K_{yz}(r) = K_{zx}(r) = C_\parallel(r). \quad (15)$$

Here, we have used the following relations based on four-dimensional rotation symmetry in the Euclidean spacetime:

$$\begin{aligned}
K_{ty}(r) &= g^2 \langle G_{ty}^a(r\hat{t}) G_{ty}^a(0) \rangle = g^2 \langle G_{xy}^a(r\hat{x}) G_{xy}^a(0) \rangle \\
&= g^2 \langle H_z^a(r\hat{x}) H_z^a(0) \rangle = C_{\perp}(r)
\end{aligned} \tag{16}$$

by exchanging t and x , and

$$\begin{aligned}
K_{xy}(r) &= g^2 \langle G_{xy}^a(r\hat{t}) G_{xy}^a(0) \rangle = g^2 \langle G_{xy}^a(r\hat{z}) G_{xy}^a(0) \rangle \\
&= g^2 \langle H_z^a(r\hat{z}) H_z^a(0) \rangle = C_{\parallel}(r)
\end{aligned} \tag{17}$$

by exchanging t and z .

In this way, we obtain for $s^R \equiv s - s' = r\hat{t} = (0, 0, 0, r)$ (relatively temporal)

$$\begin{aligned}
g^2 \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^b(s') \rangle &= \frac{\delta^{ab}}{N_c^2 - 1} [\{\delta_{\mu 4}(\delta_{\alpha 4}\delta_{\nu\beta} - \delta_{\beta 4}\delta_{\nu\alpha}) + [(\mu, \alpha) \leftrightarrow (\nu, \beta)]\} C_{\perp}(r) + \epsilon_{\mu\nu k 4} \epsilon_{\alpha\beta k 4} C_{\parallel}(r)] \\
&= \frac{\delta^{ab}}{N_c^2 - 1} [\{\delta_{\mu 4}(\delta_{\alpha 4}\delta_{\nu\beta} - \delta_{\beta 4}\delta_{\nu\alpha}) + \delta_{\nu 4}(\delta_{\beta 4}\delta_{\mu\alpha} - \delta_{\alpha 4}\delta_{\mu\beta})\} C_{\perp}(r) + (\hat{\delta}_{\mu\alpha}\hat{\delta}_{\nu\beta} - \hat{\delta}_{\mu\beta}\hat{\delta}_{\nu\alpha}) C_{\parallel}(r)], \tag{18}
\end{aligned}$$

with $r = |s - s'|$ and $\hat{\delta}_{\mu\nu} \equiv \delta_{\mu\nu} - \hat{s}_{\mu}^R \hat{s}_{\nu}^R = \delta_{\mu\nu} - \hat{t}_{\mu} \hat{t}_{\nu}$. For the derivation, we have used its antisymmetric property under the exchange of $\mu \leftrightarrow \nu$ and $\alpha \leftrightarrow \beta$, respectively. By applying a four-dimensional rotation to Eq. (18), all two-point field-strength correlations $g^2 \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^b(s') \rangle$ are expressed with $C_{\perp}(r)$ and $C_{\parallel}(r)$ in the Landau gauge. In particular, one finds

$$\sum_{\mu, \nu} g^2 \langle G_{\mu\nu}^a(s) G_{\mu\nu}^a(s') \rangle = 6[C_{\perp}(r) + C_{\parallel}(r)], \tag{19}$$

which is a single-valued function of $r = |s - s'|$ in the Landau gauge.

III. GLUON AND FIELD STRENGTH IN SU(2) AND SU(3) LATTICE QCD

For the nonperturbative analysis of the color-magnetic correlation, we use SU(2) and SU(3) lattice QCD Monte Carlo calculations with the standard plaquette action at the quenched level. In lattice QCD, field variables are formulated on a lattice with the spacing a , and their interactions are controlled by the gauge coupling g [22,23].

In lattice QCD, the gluon field $A_{\mu}(s) \in \mathfrak{su}(N_c)$ and the field strength $G_{\mu\nu}(s) \in \mathfrak{su}(N_c)$ are exponentiated to form the link variable $U_{\mu}(s)$ and the plaquette $\square_{\mu\nu}(s)$, respectively,

$$\begin{aligned}
U_{\mu}(s) &= e^{iagA_{\mu}(s)} \in \text{SU}(N_c), \\
\square_{\mu\nu}(s) &= e^{ia^2gG_{\mu\nu}(s)} \in \text{SU}(N_c).
\end{aligned} \tag{20}$$

For the lattice data analysis, we use the translational and rotational invariance in Euclidean Landau-gauge QCD. For the spatial correlation, we take both on-axis and off-axis lattice data. On the statistical error of the lattice data, the jackknife error estimate is adopted.

A. SU(3) lattice QCD setup

For the SU(3) lattice QCD calculations, we adopt the four lattices as shown in Table I.

The lattice spacing a is determined so as to reproduce the string tension $\sigma = 0.89 \text{ GeV/fm}$ at $\beta = 5.7, 5.8$, and 6.0 [32,35]. For the lattice spacing at $\beta = 6.2$, we refer to Ref. [33]. The gauge configurations are picked up with the interval of 1000 sweeps, after the thermalization of 20,000 sweeps. In this study, we use 200 gauge configurations at $\beta = 5.7$ and 5.8 , and 800 gauge configurations at $\beta = 6.0$. For the gluon propagator, we also use 50 gauge configurations at $\beta = 6.2$. For the Landau gauge fixing, we use the ordinary iterative maximization algorithm [31] with an over-relaxation parameter of 1.6.

In the Landau gauge, since the gluon field is globally minimized, we define SU(3) gluon fields with the link variable as

$$\mathcal{A}_{\mu}(s) \equiv \frac{1}{2iag} [U_{\mu}(s) - U_{\mu}^{\dagger}(s)]|_{\text{traceless}} \in \mathfrak{su}(3) \tag{21}$$

in the fundamental representation. This definition is often used in the Landau gauge in lattice QCD. In the following, we simply denote $\mathcal{A}_{\mu}(s)$ by $A_{\mu}(s)$ and mainly use it for the SU(3) lattice calculation.

TABLE I. Lattice parameter $\beta \equiv 2N_c/g^2$ and the lattice size for the SU(3) lattice QCD calculations at the quenched level. The corresponding lattice spacing a and the physical volume $(La)^4$ are also listed.

β	Lattice size	Spacing a	Phys. vol. $(La)^4$	Config.#
5.7	16^4	0.186 fm	$(3.0 \text{ fm})^4$	200
5.8	16^4	0.152 fm	$(2.4 \text{ fm})^4$	200
6.0	24^4	0.104 fm	$(2.5 \text{ fm})^4$	800
6.2	48^4	0.0726 fm	$(3.48 \text{ fm})^4$	50

In SU(3) lattice QCD, we define the field strength

$$G_{\mu\nu}(s) \equiv \partial_\mu A_\nu(s) - \partial_\nu A_\mu(s) + ig[A_\mu(s), A_\nu(s)] \in \text{su}(3) \quad (22)$$

using the Landau-gauge gluon field in Eq. (21) and the forward derivative, satisfying

$$\partial_\mu f(s) = \frac{1}{a} [f(s + a\hat{\mu}) - f(s)]. \quad (23)$$

We mainly use the field strength $G_{\mu\nu}(s)$ for lattice QCD calculations of field-strength correlations.

We also introduce SU(3) “plaquette field strength”

$$\mathcal{G}_{\mu\nu}(s) \equiv \frac{1}{2ia^2g} [\square_\mu(s) - \square_\mu^\dagger(s)]|_{\text{traceless}} \in \text{su}(3) \quad (24)$$

defined with the plaquette variable in the Landau gauge. $\mathcal{G}_{\mu\nu}(s)$ coincides with $G_{\mu\nu}(s)$ in the continuum limit.

B. SU(2) lattice QCD setup

For the SU(2) lattice QCD calculations, we adopt the three lattices as shown in Table II.

The lattice spacing a is determined to reproduce the string tension $\sigma = 0.89$ GeV/fm [36]. The gauge configurations are picked up with the interval of 200 sweeps, after the thermalization of 2000 sweeps. We use 400 gauge configurations at each β . The Landau gauge fixing is achieved by the ordinary iterative maximization algorithm [31] with an over-relaxation parameter of 1.7.

In SU(2) lattice QCD, the gluon field $A_\mu(s) \in \text{su}(2)$ is directly obtained from the link-variable $U_\mu(s)$ in the fundamental representation as

$$U_\mu(s) = e^{ia g A_\mu(s)} = e^{ia g A_\mu^a(s) \tau^a / 2} = e^{i\tau^a \theta^a} = \cos \theta + i\tau^a \hat{\theta}^a \sin \theta \in \text{SU}(2), \quad (25)$$

with $\theta^a \equiv agA_\mu^a(s)/2$, $\theta \equiv (\theta^a \theta^a)^{1/2}$ and $\hat{\theta}^a \equiv \theta^a / \theta$. In SU(2) lattice QCD, we define the field strength

TABLE II. Lattice parameter $\beta \equiv 2N_c/g^2$ and the lattice size for the SU(2) lattice QCD calculations at the quenched level. The corresponding lattice spacing a and the physical volume $(La)^4$ are also listed.

β	Lattice size	Spacing a	Phys. vol. $(La)^4$	Config.#
2.3	16^4	0.18 fm	$(2.9 \text{ fm})^4$	400
2.4	24^4	0.127 fm	$(3.0 \text{ fm})^4$	400
2.5	32^4	0.09 fm	$(2.9 \text{ fm})^4$	400

$$G_{\mu\nu}(s) \equiv \partial_\mu A_\nu(s) - \partial_\nu A_\mu(s) + ig[A_\mu(s), A_\nu(s)] \in \text{su}(2) \quad (26)$$

using the Landau-gauge gluon field $A_\mu(s)$ in Eq. (25) and the forward derivative ∂_μ .

IV. LANDAU-GAUGE GLUON PROPAGATOR

To begin with, we study the gluon propagator, which is closely related to the color-magnetic correlation. (Indeed, in Abelian gauge theories, the gauge-field propagator and the field strength are directly related.) The gluon propagator is one of the most fundamental quantities in QCD [37–39] and has been well studied, particularly in the Landau gauge in lattice QCD [31–34].

In this section, we numerically investigate the gluon propagator, i.e., the gluon two-point function,

$$D_{\mu\nu}^{ab}(s, s') \equiv g^2 \langle A_\mu^a(s) A_\nu^b(s') \rangle = \frac{\delta^{ab}}{N_c^2 - 1} D_{\mu\nu}(s - s'), \quad (27)$$

$$D_{\mu\nu}(s - s') \equiv g^2 \langle A_\mu^a(s) A_\nu^a(s') \rangle, \quad (28)$$

in the Landau gauge in Euclidean SU(N_c) lattice QCD. In the Landau gauge, all components of the gluon propagator can be expressed with the scalar combination of

$$D(r) \equiv g^2 \langle A_\mu^a(s) A_\mu^a(s') \rangle, \quad (29)$$

which is a single-valued function of the four-dimensional Euclidean spacetime distance $r \equiv |s - s'|$ [32]. We briefly summarize the general properties of the Landau-gauge gluon propagator in Appendix A.

Figure 2 shows the gluon propagator $D(r)$ in the Landau gauge in SU(3) and SU(2) quenched lattice QCD. In both SU(2) and SU(3) QCD, the Landau-gauge gluon propagator $D(r)$ is well described with a Yukawa-type function in the wide region of $r = 0.1$ – 1.0 fm, that is,

$$g^2 \langle A_\mu^a(s) A_\mu^a(s') \rangle \simeq Am \frac{e^{-mr}}{r} \equiv D_{\text{Yukawa}}(r), \quad (30)$$

although the gluon propagator might take a different form in the far infrared region of $r \gg 1$ fm [33].

From the fit analysis with the Yukawa-type function as shown in Fig. 2, the gluonic mass parameter m is estimated as $m \simeq 0.66$ GeV for SU(3) QCD [32] and $m \simeq 0.676$ GeV for SU(2) QCD. The best-fit parameters are summarized in Table III. [The definition of the coefficient A is different from that in Ref. [32] by $3(N_c^2 - 1)g^2$.]

Since the Yukawa-type gluon propagation is natural in the three-dimensional spacetime instead of the four-dimensional one [32], this might relate to some dimensional reduction hidden in nonperturbative QCD [40].

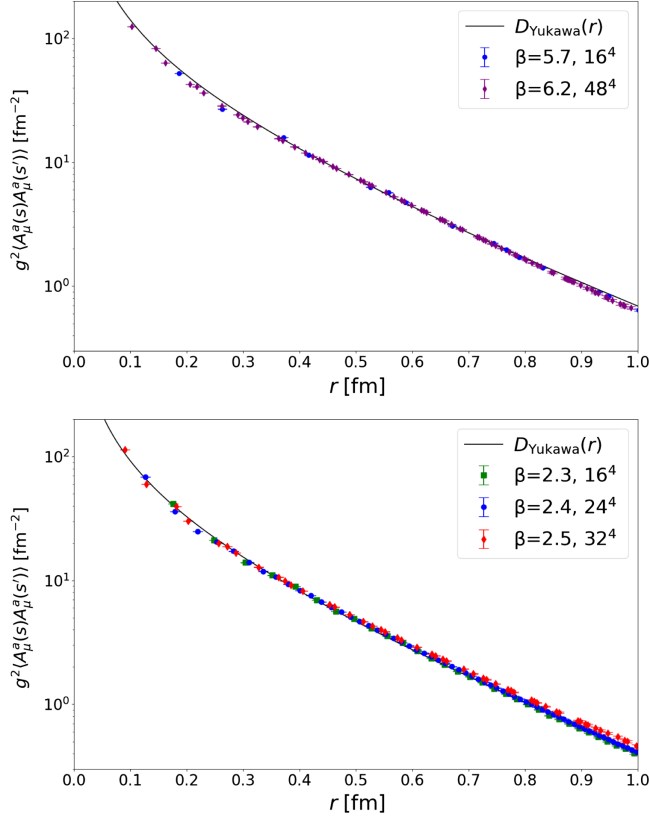


FIG. 2. Landau-gauge gluon propagator $g^2 \langle A_\mu^a(s) A_\mu^a(s') \rangle$ plotted against $r \equiv |s - s'|$ in SU(3) (upper) and SU(2) (lower) lattice QCD. The curve is the best-fit Yukawa-type function $D_{\text{Yukawa}}(r)$ in Eq. (30).

Next, we examine every tensor component of $D_{\mu\nu}(s)$ using its Fourier image,

$$\tilde{D}_{\mu\nu}(p) \equiv \int d^4s e^{-ip \cdot s} D_{\mu\nu}(s). \quad (31)$$

In Euclidean Landau-gauge QCD, due to the $O(4)$ Lorentz symmetry, the tensor structure of $\tilde{D}_{\mu\nu}(p)$ is uniquely determined as

$$\tilde{D}_{\mu\nu}(p) = (\delta_{\mu\nu} - \hat{p}_\mu \hat{p}_\nu) \tilde{D}(p^2) \quad (32)$$

with $p^2 = p^\mu p^\mu$ and $\hat{p}^\mu \equiv p^\mu / \sqrt{p^2}$.

TABLE III. Best-fit parameters on the gluonic mass m and the coefficient A in the Yukawa-type function $D_{\text{Yukawa}}(r)$ in Eq. (30) for the Landau-gauge gluon propagator $g^2 \langle A_\mu^a(s) A_\mu^a(s') \rangle$ in SU(3) and SU(2) lattice QCD.

$\text{SU}(N_c)$	m	A
SU(3)	0.660(4) GeV	5.87(2)
SU(2)	0.676(6) GeV	3.78(2)

For the Yukawa-type propagator, $D(r) = A m e^{-mr}/r$, $\tilde{D}(p^2)$ is written by [32]

$$\tilde{D}(p^2) = \frac{1}{3} \tilde{D}_{\mu\mu}(p) = \frac{4\pi^2 A m}{3(p^2 + m^2)^{3/2}}, \quad (33)$$

and, as shown in Appendix A, we derive

$$D_{\mu\nu}(s - s') = \frac{A m^2}{3} [F(mr) \delta_{\mu\nu} - G(mr) \hat{s}_\mu^R \hat{s}_\nu^R] \quad (34)$$

with $r \equiv |s - s'|$, $\hat{s}^R \equiv (s - s')/r$ and

$$F(w) \equiv \frac{1}{w} \left[e^{-w} \left(1 + \frac{1}{w} + \frac{2}{w^2} + \frac{2}{w^3} \right) - \frac{2}{w^3} \right], \quad (35)$$

$$G(w) \equiv \frac{1}{w} \left[e^{-w} \left(1 + \frac{4}{w} + \frac{8}{w^2} + \frac{8}{w^3} \right) - \frac{8}{w^3} \right]. \quad (36)$$

V. LATTICE QCD RESULT FOR COLOR-MAGNETIC CORRELATIONS

In this section, we study the color-magnetic correlations, $C_\perp(r)$ and $C_\parallel(r)$, in the Landau gauge using SU(2) and SU(3) lattice QCD at the quenched level. Note again that all two-point field-strength correlations $g^2 \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^b(s') \rangle$ are expressed with these two correlations. For the lattice data analysis, we use rotational invariance via relabeling the spacetime axis, as well as translational invariance, in Euclidean Landau-gauge QCD.

In the adopted β region, $C_\perp(r)$ and $C_\parallel(r)$ at different β values are found to be approximately single-valued functions of $r \equiv |s - s'|$ in lattice QCD. [In Appendix B, we present the lattice QCD data on the color-magnetic correlations, $C_\perp$ and $C_\parallel$, for each β in SU(3) and SU(2), respectively, to provide comprehensive information.]

A. Perpendicular-type color-magnetic correlation

First, we investigate the perpendicular-type color-magnetic correlation

$$C_\perp(r) \equiv g^2 \langle H_z^a(s) H_z^a(s + r \hat{\perp}) \rangle \quad (37)$$

in the Landau gauge. Here, $\hat{\perp}$ is an arbitrary unit vector on the xy plane. Figure 3 shows the numerical result of $C_\perp(r)$ in SU(3) and SU(2) lattice QCD.

Curiously, the perpendicular-type color-magnetic correlation is *always negative*, $C_\perp(r) < 0$, for all values of r , except for the same-point correlation at $r = 0$, $C_\perp(0) = g^2 \langle H_z^a(s) H_z^a(s) \rangle$, which is trivially positive. The lattice unit value of the same-point correlation $C_\perp(0)$ is as follows: In SU(3) lattice QCD, $C_\perp(0) = 7.20, 6.71$, and 6.09 at $\beta = 5.7, 5.8$, and 6.0 , respectively. In SU(2)

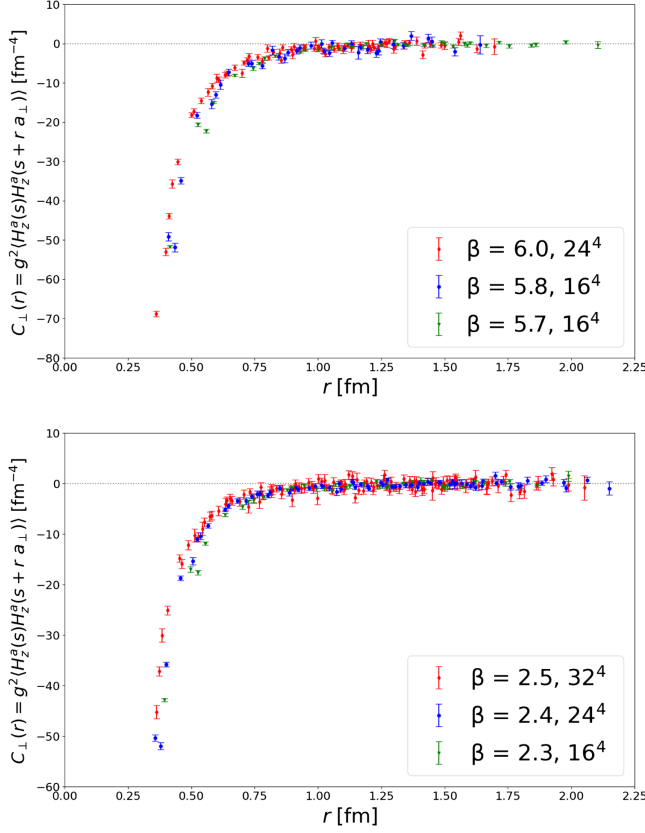


FIG. 3. Perpendicular-type color-magnetic correlation $C_{\perp}(r) \equiv g^2 \langle H_z^a(s) H_z^a(s + r \hat{\perp}) \rangle$ in the Landau gauge in SU(3) (upper) and SU(2) (lower) lattice QCD. $\hat{\perp}$ denotes a unit vector on the xy plane.

lattice QCD, $C_{\perp}(0) = 5.46, 4.63$, and 4.09 at $\beta = 2.3, 2.4$, and 2.5 , respectively.

We note that an “always negative” correlation is rare in physics, whereas “always positive” and “alternating” correlations have been observed in various areas of physics.

One might suspect that the gauge fixing introduces an unphysical effect. However, we examined the gauge-invariant field-strength correlation extracted from the plaquette correlators, as shown in Fig. 4, and obtained a similar result. In fact, the gauge-invariant correlation corresponding to the perpendicular-type color-magnetic correlation $C_{\perp}(r)$ is always negative, except for the same-point correlation $C_{\perp}(0)$, which is positive at $r = 0$.

Now, we explore the physical implications of $C_{\perp}(r)$ always being negative except for $r = 0$. This seems to contradict the picture of continuously varying color-



FIG. 4. Example of the plaquette correlator to extract the gauge-invariant field-strength correlation in lattice QCD.

magnetic fields in the QCD vacuum. In fact, this lattice result suggests that the QCD vacuum differs significantly from homogeneous color-magnetic systems [10] or multi-vortex systems [15], even at small distances of the order of the lattice spacing $a = 0.1\text{--}0.2$ fm.

Instead, our result supports the picture that color-magnetic fields are highly stochastic in the QCD vacuum [18–20, 24–29], similar to the spaghetti vacuum [16]. In other words, even if some color-magnetic domains exist, they would be tiny, and randomness dominates even at small distances of about $a = 0.1\text{--}0.2$ fm.

B. Parallel-type color-magnetic correlation

Next, we investigate the parallel-type color-magnetic correlation

$$C_{\parallel}(r) \equiv g^2 \langle H_z^a(s) H_z^a(s + r \hat{\parallel}) \rangle \quad (38)$$

in the Landau gauge. Here, $\hat{\parallel}$ denotes an arbitrary unit vector on the tz plane.

Figure 5 shows the numerical result of $C_{\parallel}(r)$ in SU(3) and SU(2) lattice QCD. The parallel-type color-magnetic correlation is always positive, $C_{\parallel}(r) > 0$, for all distances r .

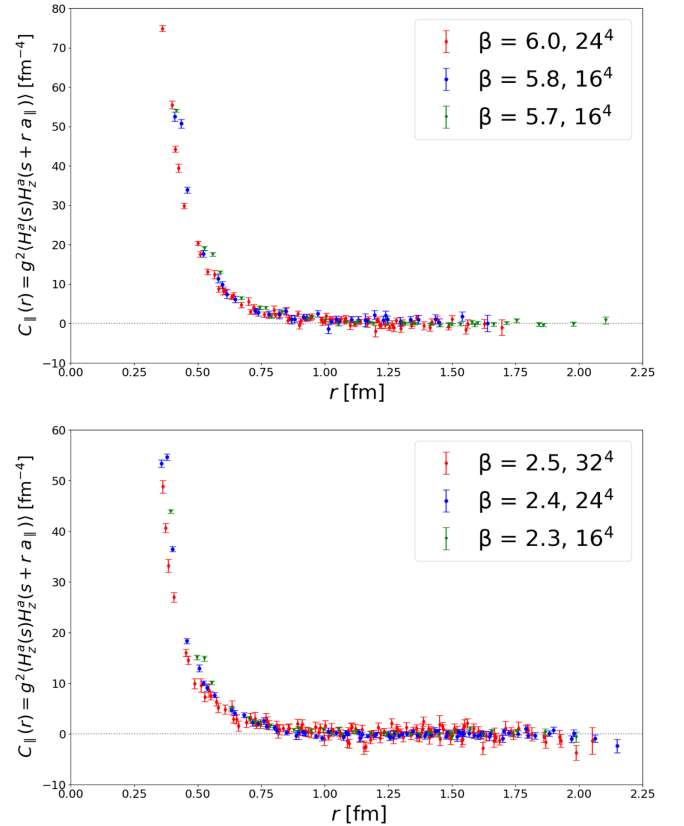


FIG. 5. Parallel-type color-magnetic correlation $C_{\parallel}(r) \equiv g^2 \langle H_z^a(s) H_z^a(s + r \hat{\parallel}) \rangle$ in the Landau gauge in SU(3) (upper) and SU(2) (lower) lattice QCD. $\hat{\parallel}$ denotes a unit vector on the tz plane.

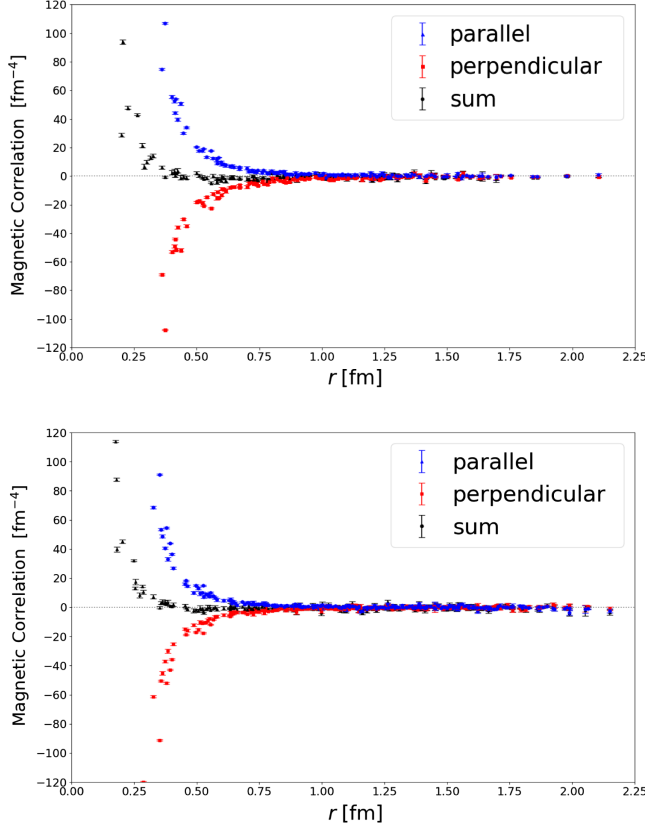


FIG. 6. The color-magnetic correlation of the parallel-type $C_{\parallel}(r)$, the perpendicular-type $C_{\perp}(r)$, and their sum $C_{\perp}(r) + C_{\parallel}(r)$ in the Landau gauge in SU(3) (upper) and SU(2) (lower) lattice QCD.

Unlike the case of $C_{\perp}(r)$, this positive correlation seems natural in continuum field theories. In particular, in regular Abelian gauge theories, such a positive correlation can be explained by magnetic flux conservation, although it is nontrivial in non-Abelian cases.

Figure 6 shows the color-magnetic correlation of the parallel-type $C_{\parallel}(r)$, the perpendicular-type $C_{\perp}(r)$, and their sum $C_{\perp}(r) + C_{\parallel}(r)$ in the Landau gauge in SU(3) and SU(2) lattice QCD.

For all values of r , the sum $C_{\perp}(r) + C_{\parallel}(r)$ decreases monotonically with r and seems to be always positive,

$$C_{\perp}(r) + C_{\parallel}(r) = \frac{1}{6} \sum_{\mu,\nu} g^2 \langle G_{\mu\nu}^a(s) G_{\mu\nu}^a(s') \rangle > 0 \quad (39)$$

$$\begin{aligned} \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^a(s') \rangle_{\text{quad}} &\equiv \langle (\partial_{\mu} A_{\nu}^a - \partial_{\nu} A_{\mu}^a)(s) (\partial_{\alpha} A_{\beta}^a - \partial_{\beta} A_{\alpha}^a)(s') \rangle, \\ \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^a(s') \rangle_{\text{cubic}} &\equiv 2ig \langle \text{Tr} \{ (\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu})(s) [A_{\alpha}, A_{\beta}](s') \} \rangle + 2ig \langle \text{Tr} \{ [A_{\mu}, A_{\nu}](s) (\partial_{\alpha} A_{\beta} - \partial_{\beta} A_{\alpha})(s') \} \rangle, \\ \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^a(s') \rangle_{\text{quartic}} &\equiv -2g^2 \langle \text{Tr} \{ [A_{\mu}, A_{\nu}](s) [A_{\alpha}, A_{\beta}](s') \} \rangle, \end{aligned} \quad (44)$$

where the factor of two comes from $\text{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab}$.

with $r = |s - s'|$. For $r = 0$, this sum leads to a positive value for the gauge-invariant gluon condensate, $g^2 \langle G_{\mu\nu}^a G_{\mu\nu}^a \rangle > 0$.

In the infrared region of $r \gtrsim 0.4$ fm, the parallel-type color-magnetic correlation $C_{\parallel}(r)$ strongly cancels with the perpendicular-type correlation $C_{\perp}(r)$ as

$$C_{\parallel}(r) \simeq -C_{\perp}(r), \quad (40)$$

which physically means

$$\langle H_z^a(s) H_z^a(s + r\hat{z}) \rangle \simeq -\langle H_z^a(s) H_z^a(s + r\hat{x}) \rangle, \quad (41)$$

and such a relation leads to a strong cancellation for the sum of Landau-gauge field-strength correlations,

$$\sum_{\mu,\nu} g^2 \langle G_{\mu\nu}^a(s) G_{\mu\nu}^a(s') \rangle = 6[C_{\perp}(r) + C_{\parallel}(r)] \simeq 0, \quad (42)$$

in the four-dimensional Euclidean spacetime.

In Appendix C, we also consider the correlation of the SU(3) “plaquette field strength” $\mathcal{G}_{\mu\nu}$ defined in Eq. (24). Regarding the lattice result, the field-strength correlation defined by $\mathcal{G}_{\mu\nu}$ is similar to that defined by $G_{\mu\nu}$.

VI. DECOMPOSITION AND DISCUSSION

In this section, we try to analyze the lattice QCD result of the color-magnetic correlation in the Landau gauge, particularly considering the origin of the negative correlation of $C_{\perp}(r) < 0$.

A. Decomposition of the field-strength correlation in terms of the gluon field

In terms of the number of the gluon field A_{μ} , we decompose the field-strength correlation $\langle G_{\mu\nu}^a(s) G_{\alpha\beta}^a(s') \rangle$ into three parts, i.e., quadratic, cubic, and quartic terms,

$$\begin{aligned} \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^a(s') \rangle &= \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^a(s') \rangle_{\text{quad}} \\ &\quad + \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^a(s') \rangle_{\text{cubic}} \\ &\quad + \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^a(s') \rangle_{\text{quartic}}. \end{aligned} \quad (43)$$

The quadratic, cubic, and quartic terms are defined by

Among the three terms, the quadratic term can be directly expressed with the gluon propagator (28), i.e., $D_{\mu\nu}(s-s') \equiv g^2 \langle A_\mu^a(s) A_\nu^a(s') \rangle$, as

$$g^2 \langle G_{\mu\nu}^a(s) G_{\alpha\beta}^a(s') \rangle_{\text{quad}} = \partial_\mu^s \partial_\alpha^{s'} D_{\nu\beta}(s-s') - \partial_\mu^s \partial_\beta^{s'} D_{\nu\alpha}(s-s') - \partial_\nu^s \partial_\alpha^{s'} D_{\mu\beta}(s-s') + \partial_\nu^s \partial_\beta^{s'} D_{\mu\alpha}(s-s'). \quad (45)$$

In the Landau gauge, due to the Lorentz symmetry, this quantity can be expressed using the scalar combination of the gluon propagator $D(r) \equiv g^2 \langle A_\mu^a(s) A_\mu^a(s') \rangle$, which is a single-valued function of the four-dimensional Euclidean distance $r \equiv |s-s'|$.

B. Decomposition of perpendicular-type color-magnetic correlation in the Landau gauge

In terms of the gluon field A_μ , the color-magnetic correlation $\langle H_z^a(s) H_z^a(s') \rangle$ is decomposed as

$$\begin{aligned} \langle H_z^a(s) H_z^a(s') \rangle &= \langle (\partial_x A_y^a - \partial_y A_x^a)(s) (\partial_x A_y^a - \partial_y A_x^a)(s') \rangle + 4ig \langle \text{Tr} \{ (\partial_x A_y - \partial_y A_x)(s) [A_x, A_y](s') \} \rangle \\ &\quad - 2g^2 \langle \text{Tr} \{ [A_x, A_y](s) [A_x, A_y](s') \} \rangle, \end{aligned} \quad (46)$$

and the quadratic term of A_μ can be described by the gluon propagator $D_{\mu\nu}(s-s')$, or more specifically by $D(r)$ in the Landau gauge.

In the case of the Yukawa-type gluon propagator (30), we calculate the quadratic term in the color-magnetic correlation at the same time point of $s-s' = (x, y, z, 0)$ as

$$\begin{aligned} g^2 \langle H_z^a(s) H_z^a(s') \rangle_{\text{quad}} &\equiv g^2 \langle (\partial_1 A_2^a - \partial_2 A_1^a)(s) (\partial_1 A_2^a - \partial_2 A_1^a)(s') \rangle = g^2 \epsilon_{ij3} \epsilon_{kl3} \partial_i^s \partial_k^{s'} \langle A_j^a(s) A_l^a(s') \rangle \\ &= -\epsilon_{ij3} \epsilon_{kl3} \partial_i^s \partial_k^{s'} D_{jl}(s-s') \\ &= -\frac{Am^2}{3} \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} F(mr) + \frac{1}{r} \frac{d}{dr} G(mr) + \frac{z^2}{r^2} \left\{ \frac{2}{r^2} G(mr) - \frac{1}{r} \frac{d}{dr} G(mr) \right\} \right] \end{aligned} \quad (47)$$

with the cylindrical radial coordinate $\rho \equiv (x^2 + y^2)^{1/2}$ and $r \equiv |s-s'| = (\rho^2 + z^2)^{1/2}$. (See Appendix D.)

For the Yukawa-type gluon propagator, the quadratic term in the perpendicular-type color-magnetic correlation $C_\perp(r)$ ($\hat{\perp} \in xy$ -plane) is always negative,

$$\begin{aligned} g^2 \langle H_z^a(s) H_z^a(s+r\hat{\perp}) \rangle_{\text{quad}} &= g^2 \langle (\partial_x A_y^a - \partial_y A_x^a)(s) (\partial_x A_y^a - \partial_y A_x^a)(s+r\hat{\perp}) \rangle \\ &= -\frac{Am^4}{3} \frac{1}{mr} \left(1 + \frac{1}{mr} + \frac{1}{m^2 r^2} \right) e^{-mr} < 0, \end{aligned} \quad (48)$$

and the quadratic term in the parallel-type color-magnetic correlation $C_\parallel(r)$ ($\hat{\parallel} \in zt$ -plane) is always positive,

$$\begin{aligned} g^2 \langle H_z^a(s) H_z^a(s+r\hat{\parallel}) \rangle_{\text{quad}} &= g^2 \langle (\partial_x A_y^a - \partial_y A_x^a)(s) (\partial_x A_y^a - \partial_y A_x^a)(s+r\hat{\parallel}) \rangle \\ &= \frac{2Am^4}{3} \frac{1}{m^2 r^2} \left(1 + \frac{1}{mr} \right) e^{-mr} > 0. \end{aligned} \quad (49)$$

Then, if the quadratic term is dominant, the negative behavior of the perpendicular-type color-magnetic correlation, $C_\perp(r) < 0$, could be explained by the Yukawa-type

gluon propagator. However, the real situation is not so simple.

Figure 7 shows individual contributions of the quadratic, cubic, and quartic terms in the perpendicular-type color-magnetic correlation $C_\perp(r)$ in SU(3) and SU(2) lattice QCD.

The quadratic term is always negative, as was demonstrated with the Yukawa-type gluon propagator in the Landau gauge. The quadratic term seems to be quantitatively reproduced by the Yukawa-type gluon propagator, as shown in Fig. 7.

The cubic term is positive for large r and negative for small r . In the infrared region, the magnitude of the cubic term is similar to that of the quadratic term. Consequently, the positive cubic term tends to cancel with the negative quadratic term, resulting in a small value of $C_\perp(r)$, in the infrared region.

Since the cubic term of the gauge field is unique to non-Abelian gauge theories, its significant contribution in the QCD vacuum indicates the distinction between QCD and Abelian gauge theories.

Compared to the quadratic and cubic terms, the quartic term contributes relatively little as an absolute value. In the infrared region, however, the contribution of the quartic term is similar in magnitude to the sum of the contributions of the quadratic and cubic terms, because they cancel each other.

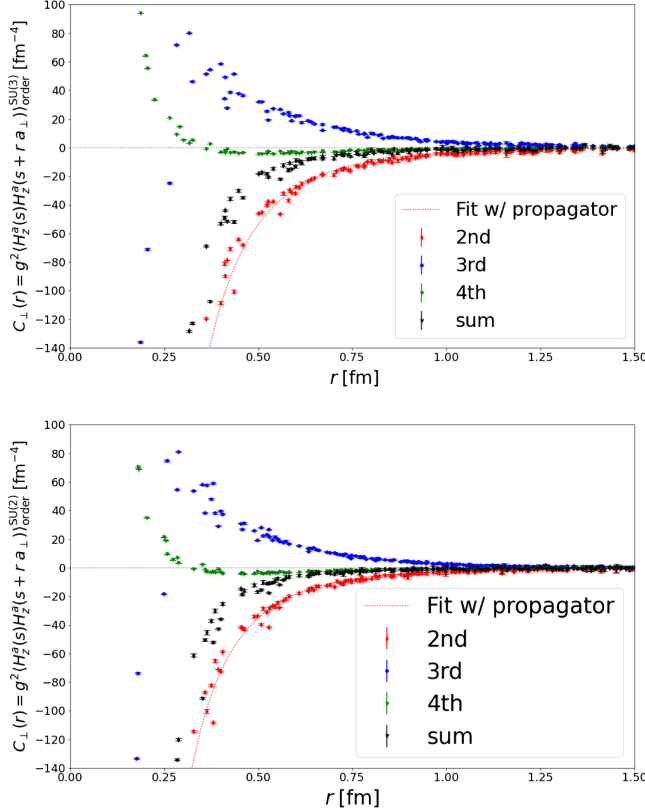


FIG. 7. Each contribution of the quadratic (red), cubic (blue), and quartic (green) terms of the perpendicular-type color-magnetic correlation $C_{\perp}(r)$ (black) in the Landau gauge in SU (3) (upper) and SU(2) (lower) lattice QCD. The red dotted line denotes the curve of Eq. (48), derived from the Yukawa-type propagator (30).

VII. SUMMARY AND CONCLUSION

Using SU(2) and SU(3) quenched lattice QCD, we have studied the field-strength correlation $g^2 \langle G_{\mu\nu}^a(s) G_{\mu\nu}^b(s') \rangle$, as well as the gluon propagator, in the Landau gauge.

To begin with, we have investigated the Landau-gauge gluon propagator $D(r) \equiv g^2 \langle A_{\mu}^a(s) A_{\mu}^a(s') \rangle$ and have found that it is well described by the Yukawa-type function e^{-mr}/r with $r \equiv |s - s'|$ for $r = 0.1\text{--}1.0$ fm in both SU(2) and SU(3) lattice QCD.

We have investigated the perpendicular-type color-magnetic correlation, $C_{\perp}(r) \equiv g^2 \langle H_z^a(s) H_z^a(s + r\hat{\perp}) \rangle$ ($\hat{\perp} \in xy$ plane), and the parallel-type one, $C_{\parallel}(r) \equiv g^2 \langle H_z^a(s) H_z^a(s + r\hat{\parallel}) \rangle$ ($\hat{\parallel} \in zt$ - plane). These two quantities reproduce all the correlation of $g^2 \langle G_{\mu\nu}^a(s) G_{\mu\nu}^b(s') \rangle$, due to the Lorentz and global $SU(N_c)$ color symmetries in the Landau gauge.

Curiously, we have found that the perpendicular-type color-magnetic correlation $C_{\perp}(r)$ is always negative for all values of r , except for the same-point correlation. In contrast, the parallel-type color-magnetic correlation $C_{\parallel}(r)$ is always positive. In the infrared region of $r \gtrsim 0.4$ fm, $C_{\perp}(r)$ and

$C_{\parallel}(r)$ strongly cancel each other, which leads to a significant cancellation in the sum of the field-strength correlations as $\sum_{\mu,\nu} g^2 \langle G_{\mu\nu}^a(s) G_{\mu\nu}^a(s') \rangle \propto C_{\perp}(|s - s'|) + C_{\parallel}(|s - s'|) \simeq 0$.

Finally, we have decomposed the field-strength correlation into quadratic, cubic, and quartic terms of the gluon field A_{μ} in the Landau gauge. For the perpendicular-type color-magnetic correlation $C_{\perp}(r)$, the quadratic term is always negative, which is explained by the Yukawa-type gluon propagator. The quartic term gives a relatively small contribution. In the infrared region, the cubic term is positive and tends to cancel with the quadratic term, resulting in a small value of $C_{\perp}(r)$.

We have considered the physical implications of the negativity of the perpendicular-type color-magnetic correlation, $C_{\perp}(r) < 0$, which seems to contradict the picture of continuously varying color-magnetic fields. This result suggests that the QCD vacuum differs significantly from homogeneous color-magnetic systems [10] or multi-vortex systems [15], even at small distances of the order of the lattice spacing $a = 0.1\text{--}0.2$ fm.

In Abelian gauge theories, the negativity of the perpendicular-type magnetic correlation could be explained by magnetic flux conservation, although the ordinary QED vacuum is trivial. For example, center projected QCD in the maximal center gauge [41] would yield a negative value for the perpendicular-type magnetic correlation. However, in non-Abelian gauge theories, such a simple explanation is not applied, and the negative color-magnetic correlation of $C_{\perp}(r) < 0$ remains a puzzle in QCD.

In this context, it is interesting to study magnetic correlations in Abelian-projected QCD in the maximally Abelian gauge, instead of the Landau gauge. In Abelian-projected QCD, all magnetic correlations can be expressed using the diagonal-gluon propagator [36,42,43]. Despite the Abelian gauge theory, the perpendicular-type magnetic correlation is nontrivial because magnetic flux conservation or the Abelian Bianchi identity is violated by the appearance of magnetic monopoles [44,45].

Finally, we discuss the physical consequences of the fact that $C_{\perp}(r)$ is always negative except for $r = 0$. This indicates that the QCD vacuum differs significantly from homogeneous color-magnetic [10] or multivortex systems [15], even at distances of the order of the lattice spacing $a = 0.1\text{--}0.2$ fm. Instead, our result supports the picture that color-magnetic fields are highly stochastic in the QCD vacuum [15–20,24–29], such as the spaghetti vacuum [16]. In other words, even if some color-magnetic domains exist, they would be tiny, and randomness dominates even at the small distance scales.

In the context of stochastic magnetic fields, there is an interesting physical case of a spin system. In 1979, Parisi and Sourlas demonstrated that a three-dimensional spin system in the presence of Gaussian random magnetic fields can be mathematically described as a one-dimensional spin system in the absence of magnetic fields [46]. In fact,

dimensional reduction occurs for the spin system in a stochastic magnetic field.

As an interesting possibility, the random color-magnetic fields at the infrared scale might lead to some dimensional reduction effects for the QCD vacuum [32,40], like the Parisi-Sourlas mechanism [46]. Indeed, the Yukawa-type gluon propagator could be interpreted as a sign of dimensional reduction of the QCD vacuum [32]. It is interesting to explore the possibility that infrared dimensional reduction occurs through the random color-magnetic fields in the QCD vacuum.

The lattice QCD calculations have been performed by SQUID at RCNP at Osaka University.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: LANDAU-GAUGE GLUON PROPAGATOR AND YUKAWA-TYPE FUNCTION

In Appendix A, we investigate the Landau-gauge gluon propagator $D_{\mu\nu}(s-s') \equiv g^2 \langle A_\mu^a(s) A_\nu^a(s') \rangle$ and the Yukawa-type propagator (30), which gives an approximation of $D_{\mu\mu}(r)$ for $r = 0.1\text{--}1.0$ fm.

1. General formulae on Landau-gauge propagator

First, we present the general formalism of the Landau-gauge propagator of gauge fields $A_\mu(s)$ in most gauge theories, including QCD and quantum electrodynamics (QED), in four-dimensional Euclidean spacetime.

Using the inverse Fourier transform of Eq. (32), the coordinate-space propagator $D_{\mu\nu}(s)$ is generally expressed as

$$\begin{aligned} D_{\mu\nu}(s) &= \int \frac{d^4 p}{(2\pi)^4} \tilde{D}(p^2) (\delta_{\mu\nu} - \hat{p}_\mu \hat{p}_\nu) e^{ip \cdot s} \\ &= f(r) \delta_{\mu\nu} - g(r) \hat{s}_\mu \hat{s}_\nu \end{aligned} \quad (\text{A1})$$

with the four-dimensional distance $r = |s|$ and $\hat{s} \equiv s/r$. From the relations

$$\begin{aligned} D_{\mu\mu}(s) &= 4f(r) - g(r), \\ \hat{s}_\mu \hat{s}_\nu D_{\mu\nu}(s) &= f(r) - g(r), \end{aligned} \quad (\text{A2})$$

one finds

$$f(r) = \frac{1}{3} [D_{\mu\mu}(s) - \hat{s}_\mu \hat{s}_\nu D_{\mu\nu}(s)], \quad (\text{A3})$$

$$g(r) = \frac{1}{3} [D_{\mu\mu}(s) - 4\hat{s}_\mu \hat{s}_\nu D_{\mu\nu}(s)]. \quad (\text{A4})$$

Using the Poisson integral representation,

$$J_n(z) = \frac{(z/2)^n}{\sqrt{\pi} \Gamma(n + \frac{1}{2})} \int_0^\pi d\theta \cos(z \cos \theta) \sin^{2n} \theta, \quad (\text{A5})$$

we obtain

$$\begin{aligned} D_{\mu\mu}(s) &= 3 \int \frac{d^4 p}{(2\pi)^4} \tilde{D}(p^2) e^{ip \cdot s} \\ &= \frac{3}{4\pi^3} \int_0^\infty dp p^3 \tilde{D}(p^2) \int_0^\pi d\theta \sin^2 \theta e^{ipr \cos \theta} \\ &= \frac{3}{4\pi^2 r} \int_0^\infty dp p^2 \tilde{D}(p^2) J_1(pr), \end{aligned} \quad (\text{A6})$$

$$\begin{aligned} \hat{s}_\mu \hat{s}_\nu D_{\mu\nu}(s) &= \int \frac{d^4 p}{(2\pi)^4} [1 - (\hat{p} \cdot \hat{s})^2] \tilde{D}(p^2) e^{ip \cdot s} \\ &= \frac{1}{4\pi^3} \int_0^\infty dp p^3 \tilde{D}(p^2) \int_0^\pi d\theta \sin^4 \theta e^{ipr \cos \theta} \\ &= \frac{3}{4\pi^2 r^2} \int_0^\infty dp p \tilde{D}(p^2) J_2(pr). \end{aligned} \quad (\text{A7})$$

2. Yukawa-type propagator

Next, we investigate the Landau-gauge propagator $D_{\mu\nu}(s)$ corresponding to the Yukawa-type propagator (30). For the Yukawa-type propagator, the expression of $\tilde{D}(p^2)$ is given by Eq. (33) [32], and we obtain

$$\begin{aligned} D_{\mu\mu}(s) &= \frac{3}{4\pi^2 r} \int_0^\infty dp p^2 \tilde{D}(p^2) J_1(pr) \\ &= \frac{Am}{r} \int_0^\infty dp \frac{p^2}{(p^2 + m^2)^{3/2}} J_1(pr) \\ &= \frac{Am}{r} \left(1 + m \frac{d}{dm} \right) \int_0^\infty dp \frac{J_1(pr)}{(p^2 + m^2)^{1/2}} \\ &= \frac{Am}{r} \left(1 + m \frac{d}{dm} \right) \left(\frac{1}{mr} - \frac{e^{-mr}}{mr} \right) \\ &= Am \frac{e^{-mr}}{r}. \end{aligned} \quad (\text{A8})$$

Here, we have used

$$\int_0^\infty dx \frac{J_\nu(2ax)}{(x^2 + y^2)^{1/2}} = I_{\nu/2}(ay) K_{\nu/2}(ay) \quad (\text{A9})$$

and its special case of $\nu = 1$, $x = p$, $y = m$ and $a = r/2$:

$$\begin{aligned} \int_0^\infty dp \frac{J_1(pr)}{(p^2 + m^2)^{1/2}} &= I_{1/2}\left(\frac{mr}{2}\right) K_{1/2}\left(\frac{mr}{2}\right) \\ &= \frac{2}{\sqrt{\pi mr}} \sinh\left(\frac{mr}{2}\right) \cdot \sqrt{\frac{\pi}{mr}} e^{-mr/2} \\ &= \frac{1}{mr} - \frac{e^{-mr}}{mr}. \end{aligned} \quad (\text{A10})$$

Similarly, we obtain

$$\begin{aligned} \hat{s}_\mu \hat{s}_\nu D_{\mu\nu}(s) &= \frac{3}{4\pi^2 r^2} \int_0^\infty dp p \tilde{D}(p^2) J_2(pr) \\ &= \frac{Am}{r^2} \int_0^\infty dp \frac{p}{(p^2 + m^2)^{3/2}} J_2(pr) \\ &= -\frac{Am}{r^2} \int_0^\infty dp \frac{d}{dp} \left[\frac{1}{(p^2 + m^2)^{1/2}} \right] J_2(pr) \\ &= \frac{Am}{r^2} \int_0^\infty dp \frac{1}{(p^2 + m^2)^{1/2}} \frac{d}{dp} J_2(pr) \\ &= \frac{Am}{2r} \int_0^\infty dp \frac{1}{(p^2 + m^2)^{1/2}} [J_1(pr) - J_3(pr)] \\ &= \frac{Am}{r} \left[\frac{2}{m^3 r^3} - \left(\frac{1}{mr} + \frac{2}{m^2 r^2} + \frac{2}{m^3 r^3} \right) e^{-mr} \right] \end{aligned} \quad (\text{A11})$$

using Eq. (A9) with $\nu = 3$, $x = p$, $y = m$ and $a = r/2$,

$$\begin{aligned} \int_0^\infty dp \frac{J_3(pr)}{(p^2 + m^2)^{1/2}} &= I_{3/2}\left(\frac{mr}{2}\right) K_{3/2}\left(\frac{mr}{2}\right) \\ &= \frac{2}{\sqrt{\pi mr}} \left[\cosh\left(\frac{mr}{2}\right) - \frac{\sinh\left(\frac{mr}{2}\right)}{mr/2} \right] \\ &\quad \times \left(1 + \frac{2}{mr} \right) \sqrt{\frac{\pi}{mr}} e^{-mr/2} \\ &= \frac{1}{mr} \left(1 - \frac{4}{m^2 r^2} \right) - \frac{e^{-mr}}{mr} \left(1 + \frac{2}{mr} \right)^2. \end{aligned} \quad (\text{A12})$$

Inserting Eqs. (A8) and (A11) into Eqs. (A3) and (A4), we obtain

$$\begin{aligned} f(r) &= \frac{Am}{3r} \left[e^{-mr} \left(1 + \frac{1}{mr} + \frac{2}{m^2 r^2} + \frac{2}{m^3 r^3} \right) - \frac{2}{m^3 r^3} \right] \\ &= \frac{Am^2}{3} F(mr), \end{aligned} \quad (\text{A13})$$

$$\begin{aligned} g(r) &= \frac{Am}{3r} \left[e^{-mr} \left(1 + \frac{4}{mr} + \frac{8}{m^2 r^2} + \frac{8}{m^3 r^3} \right) - \frac{8}{m^3 r^3} \right] \\ &= \frac{Am^2}{3} G(mr) \end{aligned} \quad (\text{A14})$$

with $F(w)$ and $G(w)$ defined in Eqs. (35) and (36).

APPENDIX B: COLOR-MAGNETIC CORRELATIONS AT EACH β

In Appendix B, we present the color-magnetic correlations for each β to provide comprehensive information on

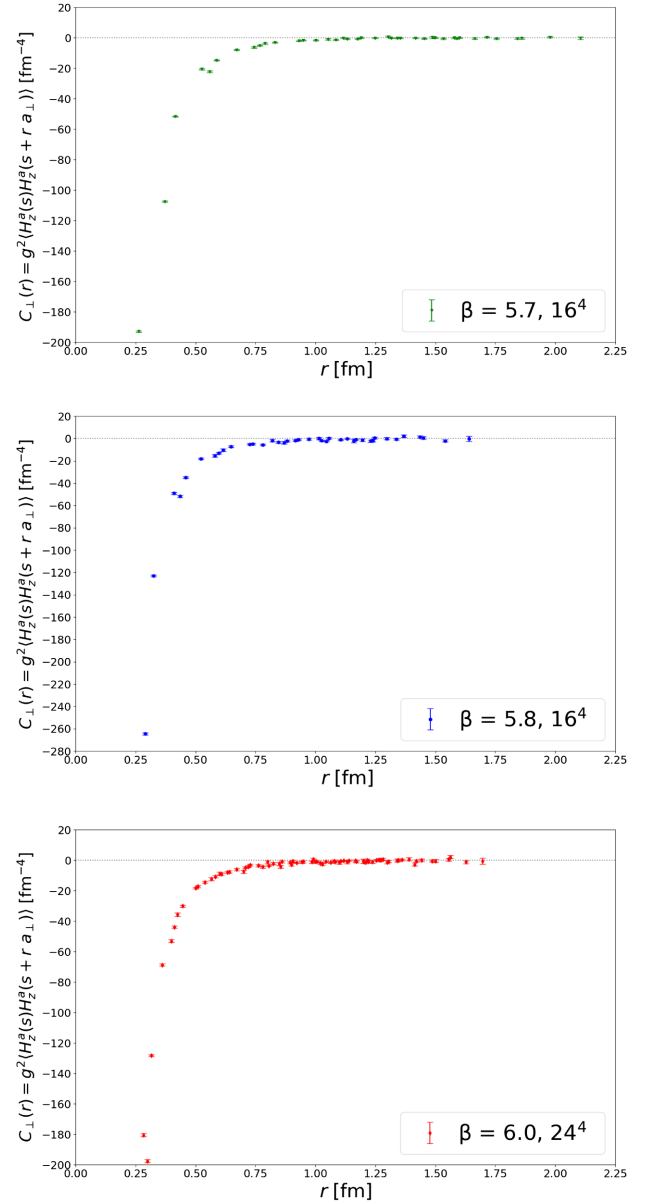


FIG. 8. Perpendicular-type color-magnetic correlations $C_\perp(r)$ in $SU(3)_{\text{color}}$ lattice QCD at $\beta = 5.7, 5.8$, and 6.0 .

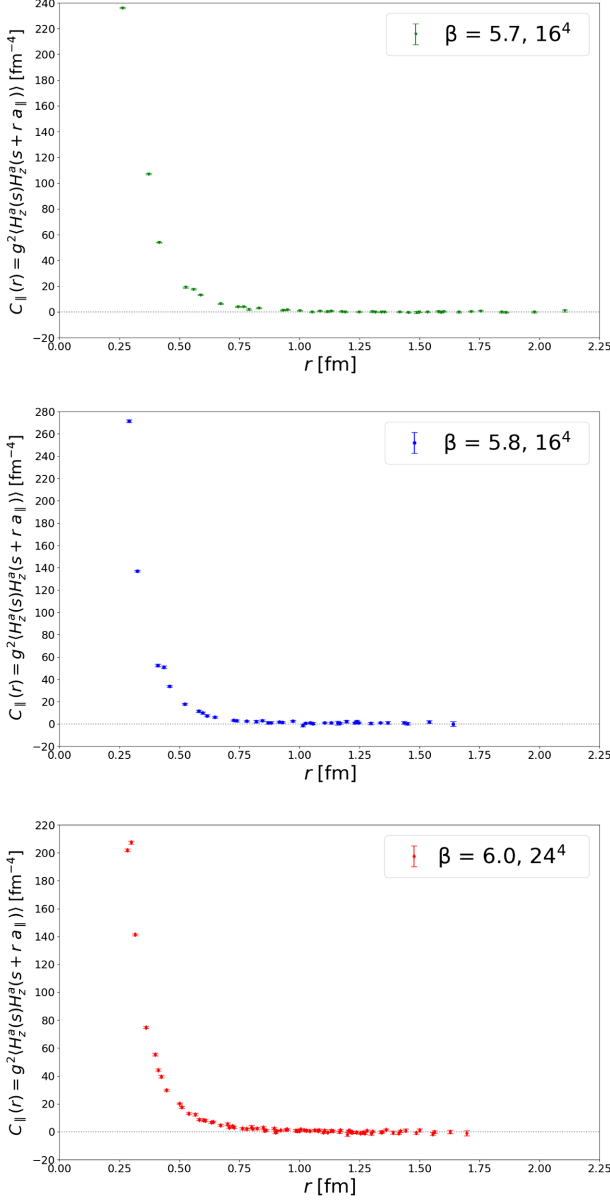


FIG. 9. Parallel-type color-magnetic correlations $C_{\parallel}(r)$ in $SU(3)_{\text{color}}$ lattice QCD at $\beta = 5.7, 5.8$, and 6.0 .

lattice QCD data, since the scaling property is nontrivial for the field-strength correlation.

Figures 8 and 9 show $SU(3)_{\text{color}}$ lattice QCD at each β for the color-magnetic correlations of the perpendicular-type $C_{\perp}(r)$ and of the parallel-type $C_{\parallel}(r)$, respectively.

Figures 10 and 11 show $SU(2)_{\text{color}}$ lattice QCD at each β for the color-magnetic correlations of the perpendicular-type $C_{\perp}(r)$ and of the parallel-type $C_{\parallel}(r)$, respectively.

APPENDIX C: CORRELATION OF PLAQUETTE FIELD STRENGTH

In Appendix C, we examine the correlation of the plaquette field strength $\mathcal{G}_{\mu\nu}(s)$, which is defined in Eq. (24), in the Landau gauge in $SU(3)$ lattice QCD.

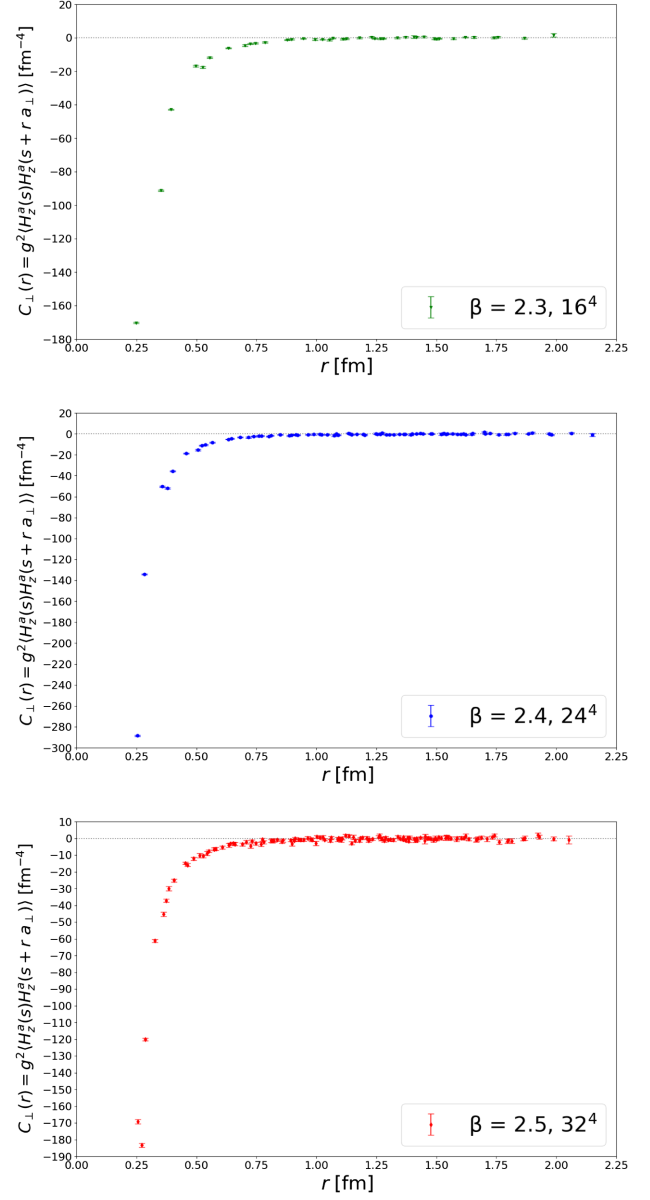


FIG. 10. Perpendicular-type color-magnetic correlations $C_{\perp}(r)$ in $SU(2)_{\text{color}}$ lattice QCD at $\beta = 2.3, 2.4$, and 2.5 .

We denote the corresponding color-magnetic field by $\mathcal{H}_i \equiv \epsilon_{ijk} \mathcal{G}_{jk}(s)/2$ and show the color-magnetic correlation $g^2 \langle \mathcal{H}_i^a(s) \mathcal{H}_i^a(s') \rangle$ in Fig. 12.

The top of Fig. 12 shows the perpendicular-type color-magnetic correlation, $C_{\perp}^P(r) \equiv g^2 \langle \mathcal{H}_z^a(s) \mathcal{H}_z^a(s + r \hat{\perp}) \rangle$ ($\hat{\perp} \in xy$ plane), which is found to be always negative like $C_{\perp}(r)$, except for the same point of $r = 0$. The middle figure shows the parallel-type correlation, $C_{\parallel}^P(r) \equiv g^2 \langle \mathcal{H}_z^a(s) \mathcal{H}_z^a(s + r \hat{\parallel}) \rangle$ ($\hat{\parallel} \in zt$ plane), which is always positive like $C_{\parallel}(r)$. The bottom figure shows the sum of the perpendicular type and parallel type, $C_{\perp}^P(r) + C_{\parallel}^P(r)$. As before, we observe a significant cancelation between the

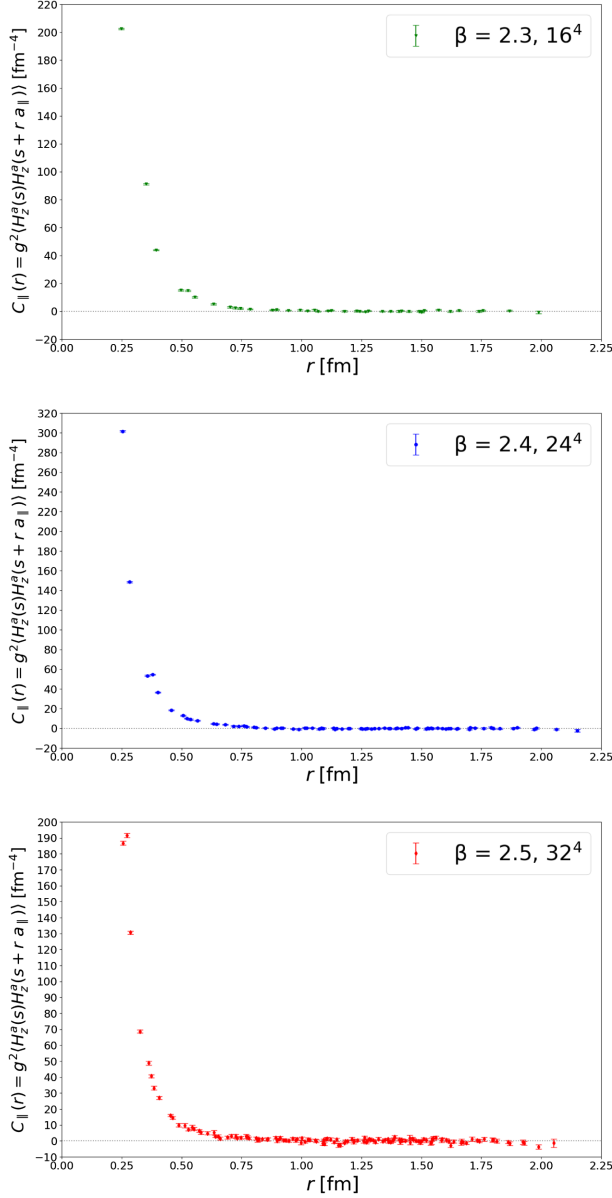


FIG. 11. Parallel-type color-magnetic correlations $C_{\parallel}(r)$ in $SU(2)_{\text{color}}$ lattice QCD at $\beta = 2.3, 2.4$, and 2.5 .

perpendicular-type and parallel-type color-magnetic correlations in the infrared region of $r \gtrsim 0.4$ fm.

APPENDIX D: QUADRATIC TERM IN COLOR-MAGNETIC CORRELATION IN LANDAU GAUGE AND YUKAWA PROPAGATOR

In Appendix D, we derive the quadratic term in the color-magnetic correlation in the Landau gauge, including the case of the Yukawa-type gluon propagator (30).

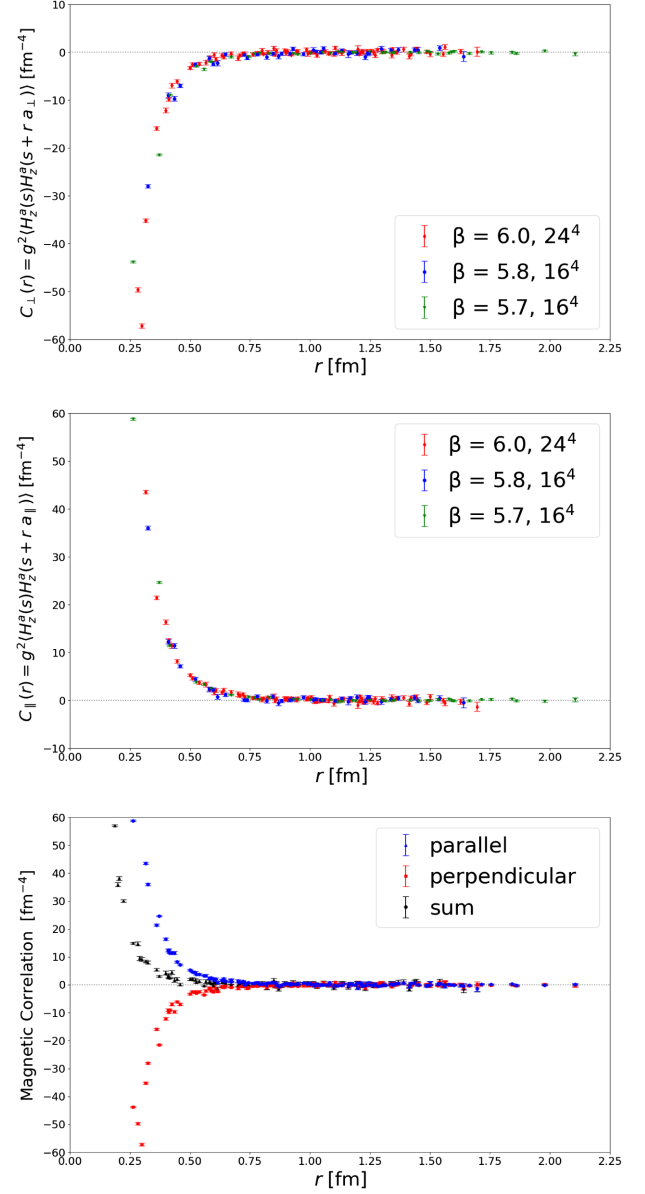


FIG. 12. Color-magnetic correlations using the plaquette field strength $\mathcal{G}_{\mu\nu}$ in the Landau gauge in $SU(3)$ lattice QCD. The top shows the perpendicular type of $C_{\perp}^P(r) \equiv g^2 \langle \mathcal{H}_z^a(s) \mathcal{H}_z^a(s + r \hat{\perp}) \rangle$ ($\hat{\perp} \in xy$ plane). The middle shows the parallel type of $C_{\parallel}^P(r) \equiv g^2 \langle \mathcal{H}_z^a(s) \mathcal{H}_z^a(s + r \hat{\parallel}) \rangle$ ($\hat{\parallel} \in zt$ plane). The bottom shows the sum (black) of $C_{\perp}^P(r)$ (red) and $C_{\parallel}^P(r)$ (blue).

1. General formalism on quadratic terms in color-magnetic correlation in the Landau gauge

First, we present the general formalism on the quadratic term in the color-magnetic correlation in the Landau gauge in four-dimensional Euclidean spacetime.

Due to the Lorentz symmetry, the Landau-gauge propagator $D_{\mu\nu}(s - s') \equiv g^2 \langle A_{\mu}^a(s) A_{\nu}^a(s') \rangle$ generally takes the

form of Eq. (A1), i.e., $D_{\mu\nu}(s) = f(r)\delta_{\mu\nu} - g(r)\hat{s}_\mu\hat{s}_\nu$, with $r \equiv |s|$ and $\hat{s} \equiv s/r$.

The quadratic term in the color-magnetic correlation is generally expressed by the propagator $D_{\mu\nu}(s)$ as

$$\begin{aligned} g^2 \langle H_z^a(s) H_z^a(s') \rangle_{\text{quad}} &\equiv g^2 \langle \epsilon_{ij3} \partial_i A_j^a(s) \epsilon_{kl3} \partial_k A_l^a(s') \rangle \\ &= g^2 \epsilon_{ij3} \epsilon_{kl3} \partial_i^s \partial_k^{s'} \langle A_j^a(s) A_l^a(s') \rangle \\ &= -\epsilon_{ij3} \epsilon_{kl3} \partial_i^s \partial_k^s D_{jl}(s - s'). \end{aligned} \quad (\text{D1})$$

Without loss of generality, we can set $s = (x, y, z, 0)$ and $s' = 0$ using translation and four-dimensional Euclidean rotation on the zt plane to obtain

$$\begin{aligned} g^2 \langle H_z^a(s) H_z^a(0) \rangle_{\text{quad}} &= -\epsilon_{ij3} \epsilon_{kl3} \partial_i^s \partial_k^s D_{jl}(s) \\ &= -\epsilon_{ij3} \epsilon_{kl3} \partial_i \partial_k [f(r)\delta_{jl} - g(r)\hat{s}_j\hat{s}_l] \\ &= -(\delta_{ik} - \delta_{i3}\delta_{k3}) \partial_i \partial_k f(r) + \epsilon_{ij3} \epsilon_{kl3} \partial_k \left\{ g(r)\hat{s}_j \frac{1}{r} \hat{s}_{il} \right\} \\ &= -(\partial_x^2 + \partial_y^2) f(r) + \epsilon_{ij3} \epsilon_{kl3} \partial_k \left\{ \frac{g(r)}{r} \hat{s}_j \right\} \end{aligned} \quad (\text{D2})$$

with $r \equiv |s|$, $\hat{s} \equiv s/r$, $\delta_{ij} \equiv \delta_{ij} - \hat{s}_i\hat{s}_j$, and $\partial_i \hat{s}_j \equiv \frac{1}{r} \hat{\delta}_{ij}$. Using the cylindrical coordinate (ρ, ϕ, z) for s , we get the quadratic term in the color-magnetic correlation in the Landau gauge,

$$\begin{aligned} g^2 \langle H_z^a(s) H_z^a(0) \rangle_{\text{quad}} &= -\frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} f(r) - (\delta_{jk} - \delta_{j3}\delta_{k3}) \left\{ \frac{d}{dr} \left(\frac{g(r)}{r} \right) \hat{s}_k \hat{s}_j + \frac{g(r)}{r^2} \hat{\delta}_{kj} \right\} \\ &= -\frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} f(r) - \left\{ \frac{d}{dr} \left(\frac{g(r)}{r} \right) + \frac{g(r)}{r^2} \right\} + \hat{z}^2 \left\{ \frac{d}{dr} \left(\frac{g(r)}{r} \right) - \frac{g(r)}{r^2} \right\} \\ &= -\left[\frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} f(r) + \frac{1}{r} \frac{d}{dr} g(r) + \frac{z^2}{r^2} \left\{ \frac{2}{r^2} g(r) - \frac{1}{r} \frac{d}{dr} g(r) \right\} \right]. \end{aligned} \quad (\text{D3})$$

2. Color-magnetic correlation for Yukawa-type propagator

For the Yukawa-type propagator (30), $f(r)$ and $g(r)$ are given by Eqs. (A13) and (A14), respectively. Substituting them into Eq. (D3) yields the following.

In the case of $s = r\hat{\perp} = (x, y, 0, 0)$, i.e., $z = 0$ and $\rho = r$, corresponding to $C_\perp(r)$, we get

$$\begin{aligned} g^2 \langle H_z^a(r\hat{\perp}) H_z^a(0) \rangle_{\text{quad}} &= -\left[\frac{1}{r} \frac{d}{dr} r \frac{d}{dr} f(r) + \frac{1}{r} \frac{d}{dr} g(r) \right] \\ &= -\frac{Am^4}{3} \left[\frac{1}{w} \frac{d}{dw} w \frac{d}{dw} F(w) + \frac{1}{w} \frac{d}{dw} G(w) \right] \Big|_{w=mr} \\ &= -\frac{Am^4}{3} \left[\frac{1}{w} \frac{d}{dw} \left(w \frac{d}{dw} F(w) + G(w) \right) \right] \Big|_{w=mr} \\ &= +\frac{Am^4}{3} \left[\frac{1}{w} \frac{d}{dw} \left\{ e^{-w} \left(1 + \frac{1}{w} \right) \right\} \right] \Big|_{w=mr} \\ &= -\frac{Am^4}{3} e^{-w} \frac{1}{w} \left(1 + \frac{1}{w} + \frac{1}{w^2} \right) \Big|_{w=mr} < 0. \end{aligned} \quad (\text{D4})$$

In the case of $s = r\hat{z} = (0, 0, z, 0)$, i.e., $z = r$ and $\rho = 0$, corresponding to $C_{\parallel}(r)$, we get

$$\begin{aligned}
 g^2 \langle H_z^a(r\hat{z}) H_z^a(0) \rangle_{\text{quad}} &= - \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial r}{\partial \rho} f'(r) + \frac{2}{r^2} g(r) \right] \Big|_{\rho \rightarrow 0} \\
 &= -2 \left[\frac{1}{r} \frac{d}{dr} f(r) + \frac{1}{r^2} g(r) \right] \\
 &= -\frac{2Am^4}{3} \left[\frac{1}{w} \frac{d}{dw} F(w) + \frac{1}{w^2} G(w) \right] \Big|_{w=mr} \\
 &= \frac{2Am^4}{3} e^{-w} \frac{1}{w^2} \left(1 + \frac{1}{w} \right) \Big|_{w=mr} > 0.
 \end{aligned} \tag{D5}$$

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