

## All axion dark matter from supersymmetric models

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Supersymmetric models accompanied by certain anomaly-free discrete  $R$  symmetries  $\mathbb{Z}_n^R$  are attractive in that 1. the  $R$  symmetry (which can arise from compactified string theory as a remnant of the broken 10-d Lorentz symmetry) forbids unwanted superpotential terms while allowing for the generation of an accidental, approximate global  $U(1)_{PQ}$  symmetry needed to solve the strong  $CP$  problem and 2. they provide a *raison d'être* for an otherwise *ad-hoc*  $R$ -parity conservation. We augment the minimal supersymmetric Standard Model (MSSM) by two additional  $\mathbb{Z}_n^R$ - and Peccei-Quinn (PQ)-charged fields  $X$  and  $Y$  wherein supersymmetry (SUSY) breaking at an intermediate scale  $m_{\text{hidden}}$  leads to PQ breaking at a scale  $f_a \sim 10^{11}$  GeV leading to a SUSY Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) axion. The same SUSY breaking can trigger  $R$ -parity breaking via higher-dimensional operators leading to tiny  $R$ -violating couplings of order  $(f_a/m_P)^N$  so that *both*  $U(1)_{PQ}$  and  $R$  parity emerge as accidental, approximate symmetries. For  $\mathbb{Z}_4^R$  and  $\mathbb{Z}_8^R$ , we find only an  $N = 1$  suppression. Then the lightest SUSY particle (LSP) of the MSSM becomes unstable with a lifetime of order  $10^{-3} - 10^1$  s so the LSPs all decay away before the present epoch. That leaves a universe with all axion cold dark matter and no weakly interacting massive particles (WIMPs) in accord with recent LZ(2024) WIMP search results.

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**Introduction.** One of the successes of the Standard Model (SM) is that it provides a rationale for why globally conserved quantum numbers like baryon (B) and lepton (L) number are conserved: they are expected to be accidental, approximate symmetries that arise only because SM gauge invariance doesn't allow such terms. They are expected to arise via higher dimension operators [1] such as the Weinberg dimension-5 operator  $\mathcal{L} \ni (\lambda/m_P)LHLH$ , where  $L$  and  $H$  are the  $SU(2)_L$  lepton and Higgs doublets. The Weinberg operator gives rise to massive neutrinos as required by neutrino oscillations. Higher dimensional operators giving rise to for instance proton decay are also expected. In fact, all higher order field operators are expected to occur in the SM effective field theory, suppressed by

appropriate powers of the Planck mass  $m_P$ , unless explicitly forbidden.

Of course, the SM is plagued by the well-known fine-tuning problems known as the gauge hierarchy problem (GHP) and the strong  $CP$  problem. We will assume here the supersymmetric solution to the GHP as exemplified in the minimal supersymmetric Standard Model (MSSM) [2] where all quadratic divergences to the Higgs mass cancel due to the (super)symmetry. The MSSM is also supported by a variety of virtual effects including gauge coupling unification [3], radiative electroweak symmetry breaking (REWSB) for a top-quark mass  $m_t \sim 100\text{--}200$  GeV [4], matching theory expectations to the measured value of the Higgs mass  $m_h \simeq 125$  GeV [5] and precision electroweak measurements [6]. We will also assume the axionic solution to the strong  $CP$  problem as exemplified by the supersymmetry (SUSY) DFSZ axion model [7,8] since both the MSSM and DFSZ require two Higgs doublets and the SUSY DFSZ model provides a Kim-Nilles solution [9] to the SUSY  $\mu$  problem [10].

Even within the SUSY DFSZ-augmented MSSM (referred to here as the PQMSSM), a variety of additional issues can occur. The first of these is the little hierarchy problem (LHP) which has been exacerbated by lack of

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SUSY signal at LHC [11]. We believe that part of the LHP arose due to early overestimates of fine-tuning in supersymmetric models [12]. Using the model-independent measure  $\Delta_{EW}$  [13], then many previously popular models such as constrained minimal supersymmetric Standard Model (CMSSM) [14], minimal gauge mediated SUSY breaking model (mGMSB) [15] and minimal anomaly-mediated SUSY breaking model (mAMSB) [16] and varieties of split SUSY [17] do indeed have a LHP [18] but other models such as non-universal Higgs model (NUHM) [19–22], natural anomaly-mediated SUSY breaking model (nAMSB) [23] and natural generalized mirage mediation [24] have large portions of parameter space that remain natural and so do not suffer from the LHP. These natural SUSY models are typified by light Higgsinos with mass  $\lesssim 350$  GeV and highly mixed top squarks with mass  $m_{\tilde{t}_1} \lesssim 2\text{--}3$  TeV (due to a large trilinear soft  $A$  term, as is generic in gravity mediation, and which also lifts  $m_h \rightarrow 125$  GeV).

A second issue with the MSSM is that it generically allows for large L- and B-violating processes via the superpotential terms

$$W_{RPV} \ni \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c + \mu'_i L_i H_u \quad (1)$$

(where  $i, j$ , and  $k$  are generation indices) which then requires the rather *ad-hoc* imposition of  $R$ -parity conservation (RPC) [25–27]:  $R = (-1)^{3(B-L)-2s}$  (where  $s$  is spin). There are rather severe limits on products of  $R$ -parity violating (RPV) couplings due to proton decay

$$\lambda'_{11k} \lambda''_{11k} \lesssim 10^{-25} \quad (2)$$

for  $m_{\tilde{q}} \sim 1$  TeV and so this motivates the *ad-hoc* assumption of RPC that sets these to zero. Other strong limits from  $n - \bar{n}$  oscillation and double nucleon decay provide bounds of order  $\lambda'' \lesssim 10^{-3} - 10^{-4}$  [28] (depending on sfermion mass). For more RPV coupling bounds from other processes, see e.g. Refs. [25–27].

An implication of RPC is that the lightest SUSY particle is absolutely stable and if neutral (like the lightest neutralino  $\tilde{\chi}_1^0$ ), then it may provide a good candidate for cold dark matter (CDM) in the universe [29,30]. For natural SUSY models with light Higgsinos, the thermally produced (TP) neutralino relic density is underproduced [31] with  $\Omega_{\tilde{\chi}_1^0}^{TP} h^2 \sim 0.01$ . This is not of great concern within the PQMSSM because the axion is expected to make up the bulk of CDM [32]. Also, the reduced abundance of Higgsino-like lightest SUSY particles (LSPs) helped PQMSSM models avoid constraints from WIMP direct detection experiments since the light Higgsinos would make up typically only  $\sim 10\%$  of the CDM [33]. However, under new strong limits from the Lux-Zeplin

TABLE I. Derived MSSM field  $R$  charge assignments for various anomaly-free discrete  $\mathbb{Z}_N^R$  symmetries which are consistent with  $SU(5)$  or  $SO(10)$  unification (from Lee *et al.* Ref. [39]).

| Multiplet | $\mathbb{Z}_4^R$ | $\mathbb{Z}_6^R$ | $\mathbb{Z}_8^R$ | $\mathbb{Z}_{12}^R$ | $\mathbb{Z}_{24}^R$ |
|-----------|------------------|------------------|------------------|---------------------|---------------------|
| $H_u$     | 0                | 4                | 0                | 4                   | 16                  |
| $H_d$     | 0                | 0                | 4                | 0                   | 12                  |
| $Q$       | 1                | 5                | 1                | 5                   | 5                   |
| $U^c$     | 1                | 5                | 1                | 5                   | 5                   |
| $E^c$     | 1                | 5                | 1                | 5                   | 5                   |
| $L$       | 1                | 3                | 5                | 9                   | 9                   |
| $D^c$     | 1                | 3                | 5                | 9                   | 9                   |
| $N^c$     | 1                | 1                | 5                | 1                   | 1                   |

(LZ) experiment in 2024 [34], even natural SUSY with a reduced abundance of TP LSPs seems ruled out [33,35].<sup>1</sup>

A third set of issues pertain to the PQMSSM: 1. from whence does the required global  $U(1)_{PQ}$  arise, 2. how does the (cosmologically favored) PQ scale  $f_a \sim 10^{11}$  GeV arise and 3. does the DFSZ axion have the *axion quality* [37] required to satisfy the strong  $CP$  problem where an axion misalignment angle  $\bar{\theta} \lesssim 10^{-10}$  is required [38]?

A solution to the PQMSSM problems was proposed in Refs. [39,40] where a discrete  $R$ -symmetry  $\mathbb{Z}_n^R$  was invoked. Discrete  $R$  symmetries are expected to arise from various string compactifications as the remnant of the breakdown of 10-d Lorentz symmetry [41,42]. The anomaly-free discrete  $R$  symmetries that forbid the SUSY  $\mu$  term (the first step in solving the SUSY  $\mu$  problem), suppress  $p$  decay and are consistent with grand unified reps were tabulated by Lee *et al.* in Ref. [39] and found to be  $\mathbb{Z}_4^R$ ,  $\mathbb{Z}_6^R$ ,  $\mathbb{Z}_8^R$ ,  $\mathbb{Z}_{12}^R$ , and  $\mathbb{Z}_{24}^R$ . Under  $R$  symmetries, the superspace coordinates  $\theta_a$  transform non-trivially so that the different elements of superfields transform differently. Since the superpotential  $W$  contributes to the Lagrangian as  $\mathcal{L} \sim \int d^2\theta W$ , the superpotential carries an overall  $R$  charge  $+2$ . The various  $R$  charges of MSSM superfields are listed in Table I for the different  $\mathbb{Z}_n^R$  symmetries.

It was emphasized in Ref. [43] that the hypothesized  $\mathbb{Z}_n^R$  could generate the global  $U(1)_{PQ}$  required by axions and also forbid any RPV terms in the superpotential. It also allows for a Majorana singlet neutrino superfield mass term (thus allowing for see-saw neutrinos) and it forbids dangerous dimension-5 operators that could destabilize the proton.

In Ref. [40], in a PQMSSM model which invoked two additional PQ fields  $X$  and  $Y$ , the strongest of the discrete  $R$  symmetries,  $\mathbb{Z}_{24}^R$ , was found to forbid superpotential terms up through  $(1/m_P)^6$ , thus allowing for a solution to the

<sup>1</sup>It may be possible to avoid the LZ constraint in the PQMSSM in regions of model parameter space where large entropy dilution occurs, when the saxion mass  $m_s \lesssim 2m_{LSP}$  and when  $f_a$  is large  $\gtrsim 10^{14}$  GeV [36].

axion quality problem.<sup>2</sup> Under SUSY breaking via a large trilinear soft term, then the  $X$  and  $Y$  fields developed vacuum expectation value (vevs) of order  $\sim 10^{11}$  GeV, thus spontaneously breaking the global PQ symmetry and generating an axion decay constant  $f_a \sim 10^{11}$  GeV, in the cosmologically favored sweet spot where the axion provides the bulk of the CDM relic density. The SUSY  $\mu$  term was also generated with  $\mu = \lambda_\mu f_a^2 / m_P \sim m_{\text{weak}}$ .

We begin with the PQMSSM model of Ref. [40] (base model  $B_{II}$  of [44]) with superpotential

$$W = f_u Q H_u U^c + f_d Q H_d D^c + f_\ell L H_d E^c + f_\nu L H_u N^c \quad (3)$$

$$+ f X^3 Y / m_P + \lambda_\mu X^2 H_u H_d / m_P + M_N N^c N^c / 2 \quad (4)$$

where generation indices are suppressed. We focus on the PQ portion of the scalar potential where intermediate-scale field vevs for  $X$  and  $Y$  dominate over the EW-scale vevs that also develop via REWSB as usual. Then the  $F$ -term scalar potential is given by

$$V_F = |3f\phi_X^2\phi_Y/m_P|^2 + |f\phi_X^3/m_P|^2 \quad (5)$$

and is augmented by

$$V_{\text{soft}} \ni m_X^2 |\phi_X|^2 + m_Y^2 |\phi_Y|^2 + (f A_f \phi_X^3 \phi_Y / m_P + \text{H.c.}) \quad (6)$$

where the  $\phi_{X,Y}$  are the scalar components of the  $X$  and  $Y$  PQ superfields. Minimization of the scalar potential yields vevs  $v_X, v_Y \sim 10^{11}$  GeV thus breaking the  $\mathbb{Z}_n^R$  and global  $U(1)_{PQ}$  symmetries with axion decay constant  $f_a = \sqrt{v_X^2 + 9v_Y^2}$  and  $\mu = \lambda_\mu v_X^2 / m_P$ . For the special case of an assumed  $\mathbb{Z}_{24}^R$  symmetry, then all nonrenormalizable operators suppressed by powers of  $(1/m_P)^6$  or less are forbidden thus solving the axion quality problem.

However, now we concern ourselves with nonrenormalizable operators containing  $R$ -parity violating combinations,<sup>3</sup>

$$W \ni (X/m_P)^p (Y/m_P)^q \times (LLE^c \text{ or } LQD^c \text{ or } U^c D^c D^c) \quad (7)$$

where the  $\mathbb{Z}_n^R$  charges for the  $R$ -parity violating combinations are listed in Table II and the superpotential charge is  $Q(W) = 2 \bmod n = 2, 2+n, 2+2n \dots$ .

*Trilinear  $R$ -parity violation from  $\mathbb{Z}_n^R$  models.* Here, we will adopt the notation that  $QQQ$  stands for a trilinear product of visible sector superfields which are RPV.

<sup>2</sup>Reference [44] emphasizes that for lower values of  $f_a \sim 10^9$  GeV, lower orders of  $n \sim 8$  or 12 for  $\mathbb{Z}_n^R$  may be sufficient to solve the axion quality problem.

<sup>3</sup>Our methods bear some similarities to the analysis presented in Ref. [45].

TABLE II. Derived MSSM field  $R$  charge assignments for  $R$ -parity violating combinations for various anomaly-free discrete  $\mathbb{Z}_n^R$  symmetries which are consistent with  $SU(5)$  or  $SO(10)$  unification.

| Combination   | $\mathbb{Z}_4^R$ | $\mathbb{Z}_6^R$ | $\mathbb{Z}_8^R$ | $\mathbb{Z}_{12}^R$ | $\mathbb{Z}_{24}^R$ |
|---------------|------------------|------------------|------------------|---------------------|---------------------|
| $LLE^c$       | 3                | 11               | 11               | 23                  | 23                  |
| $LQD^c$       | 3                | 11               | 11               | 23                  | 23                  |
| $U^c D^c D^c$ | 3                | 11               | 11               | 23                  | 23                  |
| $LH_u$        | 1                | 7                | 5                | 13                  | 25                  |

- (1)  $\mathbb{Z}_4^R$  case: For the  $\mathbb{Z}_4^R$  case, we can take  $R$  charges as  $Q_X = +1$  and  $Q_Y = -1$ . Then a lowest order operator including a  $QQQ$  combination is  $W \ni \lambda(Y/m_P)QQQ$  and so when  $X$  and  $Y$  obtain vevs, we obtain an  $R$ -parity violating coupling of order  $(v_Y/m_P) \sim 10^{-7}$ .
- (2)  $\mathbb{Z}_6^R$  case: For the  $\mathbb{Z}_6^R$  case, we can take  $R$  charges as  $Q_X = -1$  and  $Q_Y = 11$ . Then a lowest order operator including a  $QQQ$  combination is  $W \ni \lambda(X/m_P)^3 QQQ$  and so when  $X$  and  $Y$  obtain vevs, we obtain an  $R$ -parity violating coupling of order  $(v_X/m_P)^3 \sim 10^{-21}$ .
- (3)  $\mathbb{Z}_8^R$  case: For the  $\mathbb{Z}_8^R$  case, we can take  $R$  charges as  $Q_X = -1$  and  $Q_Y = 5$ . Then a lowest order operator including a  $QQQ$  combination is  $W \ni \lambda(X/m_P)^1 QQQ$  and so when  $X$  and  $Y$  obtain vevs, we obtain an  $R$ -parity violating coupling of order  $(v_X/m_P)^1 \sim 10^{-7}$ .
- (4)  $\mathbb{Z}_{12}^R$  case: For the  $\mathbb{Z}_{12}^R$  case, we can take  $R$  charges as  $Q_X = -1$  and  $Q_Y = 5$ . Then a lowest order operator including a  $QQQ$  combination is  $W \ni \lambda(Y/m_P)^3 QQQ$  and so when  $X$  and  $Y$  obtain vevs, we obtain an  $R$ -parity violating coupling of order  $(v_Y/m_P)^3 \sim 10^{-21}$ .
- (5)  $\mathbb{Z}_{24}^R$  case: For the  $\mathbb{Z}_{24}^R$  case, we can take  $R$  charges as  $Q_X = -1$  and  $Q_Y = 5$ . Then a lowest order operator including a  $QQQ$  combination is  $W \ni \lambda(X/m_P)^2 (Y/m_P) QQQ$  and so when  $X$  and  $Y$  obtain vevs, we obtain an  $R$ -parity violating coupling of order  $(v_X^2 v_Y / m_P^3) \sim 10^{-21}$ .

Thus, to summarize, under the  $\mathbb{Z}_4^R$  and  $\mathbb{Z}_8^R$  discrete  $R$  symmetries, we expect  $R$ -parity violating processes with couplings  $\lambda \sim 10^{-7}$  whilst under the  $\mathbb{Z}_6^R$ ,  $\mathbb{Z}_{12}^R$  and  $\mathbb{Z}_{24}^R$  symmetries we expect RPV with  $\lambda \sim 10^{-21}$ . For the  $n = 4$  and 8 cases, thus some additional RPV coupling suppression of some of the couplings (such as lepton or baryon triality [46,47]) would be required to fulfill the  $p$ -decay constraint Eq. (2). Lepton triality  $L_3$  would also forbid the potentially troublesome bilinear RPV terms from arising.

The expected tiny RPV couplings are small enough that the  $\tilde{\chi}_1^0$  LSP should be stable on the scale of collider experiments, so the usual SUSY  $E_T$  signatures [11] should ensue. The rough scale for visibility of LSP decay in LHC

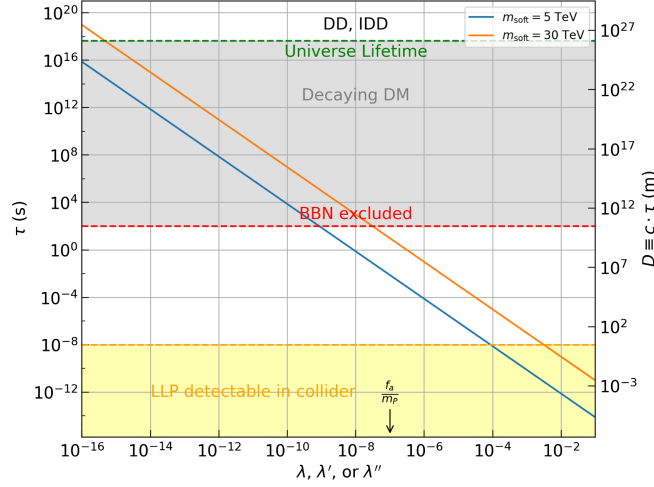


FIG. 1. Lifetime of lightest neutralino (left vertical axis) versus RPV couplings  $\lambda, \lambda'$  or  $\lambda''$  for  $m_{\text{soft}} = 5$  and  $30$  TeV and  $m_{\tilde{\chi}_1^0} = 200$  GeV. On the right vertical axis, we show the approximate decay length  $D$  (meters). The locus of *all axion dark matter* is indicated by the arrow labeled  $f_a/m_P$ . At that point, when the  $X$  and  $Y$  fields obtain intermediate scale vevs  $v_X, v_Y \sim 10^{11}$  GeV, then the axion decay constant  $f_a \sim 10^{11}$  GeV and the generic RPV couplings are of order  $\sim 10^{-7}$ .

collider detectors is that  $d = \beta c \gamma \tau \lesssim 5$  m. Approximating this as  $d \lesssim c\tau$ , then we expect  $\tau \lesssim 2 \times 10^{-8}$  s.

But on longer timescales—such as those affecting cosmology—other constraints become relevant. Under RPV, the (photinolike)  $\tilde{\chi}_1^0$  decay width for a single RPV coupling is given by<sup>4</sup>

$$\Gamma(\tilde{\chi}_1^0) = \frac{3\alpha\lambda_{111}^2}{128\pi^2} \frac{m_{\tilde{\chi}_1^0}^5}{m_{\text{soft}}^4} \quad (8)$$

where  $m_{\text{soft}}$  is the SUSY sfermion mass scale which can be in the 10–40 TeV range for natural SUSY under  $\Delta_{\text{EW}}$  emergent from the string landscape [49]. Including gaugino and Higgsino mixing angle factors can reduce this rate whilst including more decay modes other than just  $\lambda'_{111} \neq 0$  will increase the decay rate—thus, we take this formula to be an order-of-magnitude estimate.

The associated lifetime  $\tau_{\tilde{\chi}_1^0}$  is shown vs coupling  $\lambda$  in Fig. 1 for  $m_{\text{soft}} = 5$  and  $30$  TeV. The lifetime will obviously shorten if more decay modes are included.

The lifetime of late-decaying neutral relics from the big bang is severely constrained by big bang nucleosynthesis (BBN) which requires the relic particle decay debris not to affect the successful predictions of light element abundances as calculated in the standard cosmology. We adopt the results from Fig. 10 of Jedamzik [50] where BBN-allowed regions of late decaying relics are plotted in the *would-be* LSP abundance  $\Omega_\chi h^2$  vs  $\tau_\chi$  plane for relic particle  $\chi$ . For our

natural SUSY case with a Higgsino-like LSP,  $\Omega_{\tilde{\chi}_1^0}^{TP} h^2 \sim 0.01$  and for  $m_{\tilde{\chi}_1^0} \sim 100$  GeV, the lifetime  $\tau$  is constrained by BBN to be  $\lesssim 10^2$  s. This bound is indicated in Fig. 1 by the red-dashed line, and the allowed region is below it. From the plot, we see that  $\lambda$  values  $\lesssim 10^{-8}$ – $10^{-9}$  are then excluded. For  $\tau \gtrsim \tau_u \sim 4.3 \times 10^{17}$  s ( $\tau_u$  is the age of the universe) then such  $\lambda$  values would again be possible, but then perhaps excluded by the LZ(2024) direct WIMP detection constraints. Also, for  $\tau \gtrsim \tau_u$ , WIMPs would be susceptible to indirect detection via WIMP-WIMP annihilation to gamma rays or antimatter or via decaying dark matter searches. In particular, the above  $\mathbb{Z}_n^R$  cases with  $\lambda \sim 10^{-21}$  may be LZ excluded whilst the cases  $\mathbb{Z}_4^R$  and  $\mathbb{Z}_8^R$  with  $\lambda \sim 10^{-7}$  are allowed. For these latter cases, the *would-be* relic Higgsino-like WIMPs all decay away on times scales of order  $10^{-3}$ – $10^1$  s after the big bang. Since our considerations all arise from an assumed SUSY DFSZ axion model, we may infer that the coherent-oscillation (CO) produced axions [51–53] comprise all the remaining dark matter from this class of supersymmetric models. In this case, the axion relic density (in the case where PQ symmetry breaks before the end of inflation, and where additional axion production/dilution effects are neglected [54]) is given by the standard expression [55]

$$\Omega_a^{CO} h^2 \simeq 0.24 \theta_i^2 f(\theta_i) (f_a/10^{12} \text{ GeV})^{7/6} \quad (9)$$

where  $f_a$  is the PQ scale,  $\theta_i$  is the initial misalignment angle and  $f(\theta_i) \simeq [\log(e/(1 - \theta_i^2/\pi^2))]^{7/6}$  is the anharmonicity factor. This seems to be in accord with recent results from the LZ experiment [34] which are strong enough to exclude thermally underproduced natural Higgsino-like WIMPs, assuming no nonthermal processes such as entropy dilution are occurring [see Fig. 8(a) of Ref. [56]]. We also show in Fig. 1 the yellow-shaded region wherein late-decaying LSPs (long-lived particles) produced at the LHC collider might decay within the detector geometry. For the white, unshaded region between yellow and gray, we expect the usual missing  $E_T$  signatures from natural SUSY at LHC to ensue (for a recent review, see Ref. [56]).

**Conclusions.** The MSSM augmented by a discrete  $\mathbb{Z}_n^R$  symmetry is well motivated in that the discrete  $R$ -symmetry forbids the  $\mu$  term and other unwanted superpotential terms while allowing for desired ones. In an example two-extra field model where the MSSM is augmented by  $X$  and  $Y$  fields, the  $\mathbb{Z}_n^R$  symmetry can generate both the global  $U(1)_{PQ}$  (as an accidental, approximate global symmetry) which solves the strong  $CP$  problem with  $f_a \sim 10^{11}$  GeV (cosmological sweet spot for axion dark matter) and generates a Kim-Nilles  $\mu$  term  $\sim 100$ – $350$  GeV while also forbidding RPV superpotential terms as renormalizable operators. The higher  $\mathbb{Z}_n^R$  symmetries can also solve the

<sup>4</sup>Some more complete  $\tilde{\chi}_1^0$  decay formulas are given in Ref. [48].

axion quality problem. RPV couplings can be generated via nonrenormalizable operators with expected magnitude  $(f_a/m_P)^N$  where  $N = 1$  for  $\mathbb{Z}_4^R$  and  $\mathbb{Z}_8^R$  and  $N = 3$  for the remaining  $n = 6, 12$  and  $24$ . [Thus, both  $U(1)_{PQ}$  and  $R$  parity emerge from the underlying  $\mathbb{Z}_n^R$  as approximate, accidental symmetries.] The  $N = 1$  case leads to suppressed RPV couplings expected of order  $\lambda_{ijk}, \lambda'_{ijk}$  and  $\lambda''_{ijk} \sim 10^{-7}$ . In such a case, the LSP of the MSSM, a thermally underproduced Higgsino-like WIMP, would be unstable with lifetime of order  $\tau \sim 10^{-3}$ – $10$  s. This would result in SUSY models with *all* SUSY DFSZ axion dark matter since the WIMPs will have decayed before BBN is finished. These results are in accord with recent WIMP search results from LZ(2024) [34] which seem to rule out thermally produced natural SUSY WIMPs, even with a diminished abundance. The SUSY DFSZ axions have a very diminished  $a\gamma\gamma$  coupling due to the presence of

Higgsinos circulating in the triangle diagram and so are beyond present axion haloscope detection capabilities [57].

The regions of parameter space we find are in accord with the analysis of Higaki *et al.* [58] showing how Affleck-Dine baryogenesis can occur with tiny RPV couplings  $\lambda \sim 10^{-9}$ – $10^{-6}$  and with  $m_{3/2} \gtrsim 10$  TeV. See also Akita and Otsuka [59]. See also Ref. [60] wherein it is argued that axion dark matter is the reason that supermassive black holes form at cosmic dawn (at redshift  $z \sim 10$ ).

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*Data availability.* The data supporting this study's findings are available within the article.

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