

Parton spin correlations and $\mathcal{CP}$ properties in Higgs boson decay at future lepton colliders

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We present a phenomenological study of partonic spin correlations and $\mathcal{CP}$ properties in $H \rightarrow gg$ decay channel at future lepton colliders. We investigate two classes of observables: Lund observable defined based on subjects and four-point energy-energy correlator (E4C) between particles inside two jets. Our results show that the E4C with energy weighted to the power of $n = 4$ achieves the strongest sensitivity to the spin correlations of gluons from Higgs boson decay. Under the assumption of ideal identification of different gluon splitting modes, we estimate that future lepton colliders operating at $\sqrt{s} = 240$ GeV with an integrated luminosities of 5.6 ab^{-1} can successfully probe gluon spin correlations, while 20 ab^{-1} of data can probe the $\mathcal{CP}$ -mixing angle in the Hgg coupling to $\lesssim 0.03\pi$ using E4C. We outline strategies for extending this framework to realistic detector-level analyses, which can provide a new pathway for the precision test of Standard Model and searches for new physics.

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I. INTRODUCTION

Spin correlations are widely investigated at colliders for precision measurements of the Standard Model (SM) and searches for physics beyond the SM (BSM), as well as to test the quantum nature (entanglement and test Bell inequalities) of microscopic interactions at high-energy colliders [1,2]. These studies usually consider measuring spin correlations through the decay products of heavy particles. For example, as the lifetime of the top quark is much shorter than the timescale of hadronization, the spin correlation of top quark pairs can be measured through the kinematics of the decay products, which has been used to determine the $\mathcal{CP}$ properties of top quark Yukawa, searches for top squarks and heavy Higgs bosons [3–7].

Recently, great progress has been made in probing the effects of quantum entanglement using the spin correlations between the top quark pair at the Large Hadron Collider (LHC) [8,9].

For partons, including gluons and light quarks, which evolve into QCD jets through showering and hadronization on a timescale of 10^{-24} s, extracting their spin correlations requires a detailed understanding of how such correlations are preserved or modified during jet formation. This is a longstanding challenge in collider physics that demands sustained theoretical and computational efforts. To this end, parton showers including spin correlations up to next-to-leading logarithmic (NLL) accuracy are recently being developed [10–16]. Complementarily, quantum approaches to simulating QCD branching in real time have been proposed [17–19], aiming to perform coherent summations in parton showers beyond the perturbative approximation.

Simultaneously, it is crucial to identify and optimize the level of quantum correlations that current and future experiments can realistically probe and utilize these information to probe BSM physics. In this work, we initiate a study of parton spin correlations in hadronic Higgs decays, focusing primarily on the decay channel $H \rightarrow gg$. To probe these correlations, we investigate two

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spin-sensitive observables: a Lund-plane-based variable defined at the jet level [13], and the four-point energy-energy correlator (E4C) constructed from particles inside jets. We note that energy correlators have been used to study QCD spin physics, see, e.g., Refs. [20–28]. Our work represents the first application of E4C to probe parton spin correlations in Higgs boson decay and beyond Standard Model physics. For a review of other applications of energy correlators, we refer to [29]. We consider the $ZH \rightarrow \nu\bar{\nu}H$ production channel at future lepton colliders [30,31], which provides a clean environment that avoids the need for jet reclustering when constructing the E4C observable.

This work is organized as follows. We first discuss, in Sec. II, the theoretical framework of $H \rightarrow gg$ with a $\mathcal{CP}$ -mixing phase incorporated, and introduce the spin-sensitive observables. In Sec. III, we present the strategies employed to preserve spin correlations between partons in the parton shower at leading order, and compare the correlation strength to that predicted by matrix element calculations. Building on this, in Sec. IV we present the sensitivity to probe spin correlations and test the $\mathcal{CP}$ properties of the Hgg coupling at future lepton colliders. Finally, in Sec. V we summarize our findings and outline strategies for future improvements.

II. THEORETICAL FRAMEWORK

We consider the spin correlations of gluons in $H \rightarrow gg$ decay. As the spin correlations can be sensitive to the $\mathcal{CP}$ properties of Hgg coupling, we parametrize the Higgs effective interaction by introducing a $\mathcal{CP}$ -mixing phase Δ :

$$\mathcal{L}_{\text{eff}} = \frac{1}{4} \frac{\alpha_s}{3\pi} \frac{h}{v} (\cos \Delta G_{\mu\nu}^a G^{a,\mu\nu} + \sin \Delta G_{\mu\nu}^a \tilde{G}^{a,\mu\nu}) H. \quad (1)$$

Here, $G_{\mu\nu}^a$ is the reduced gluon field strength tensor, which is invariant under $\mathcal{CP}$ transformations, and $\tilde{G}_{\mu\nu}^a$ is its dual, which is $\mathcal{CP}$ -odd. The matrix element of $H \rightarrow gg$ is given by

$$\begin{aligned} \mathcal{M}_{H \rightarrow gg}^{h_1 h_2} = & -\frac{1}{4} \frac{\alpha_s}{3\pi v} \left(\epsilon_{\mu\nu\alpha\beta} k_1^\mu k_2^\nu \epsilon_{h_1}^{*\alpha} \epsilon_{h_2}^{*\beta} \sin \Delta \right. \\ & \left. - [(k_1 \cdot k_2)(\epsilon_{h_1}^* \cdot \epsilon_{h_2}^*) - (k_1 \cdot \epsilon_{h_2}^*)(k_2 \cdot \epsilon_{h_1}^*)] \cos \Delta \right), \end{aligned} \quad (2)$$

for gluons with momenta p_1 and p_2 , helicities h_1 and h_2 , and corresponding polarization vectors ϵ_{h_1} and ϵ_{h_2} , respectively. In the Higgs boson rest frame, Eq. (3) takes the form

$$\mathcal{M}_{++} = -\frac{1}{4} \frac{\alpha_s}{3\pi v} \frac{m_H^2}{2} e^{i\Delta}, \quad \mathcal{M}_{--} = -\frac{1}{4} \frac{\alpha_s}{3\pi v} \frac{m_H^2}{2} e^{-i\Delta}, \quad (3)$$

where $\mathcal{M}_{++}$ ($\mathcal{M}_{--}$) represents the matrix element for the two gluons produced with helicity $++$ ($--$).

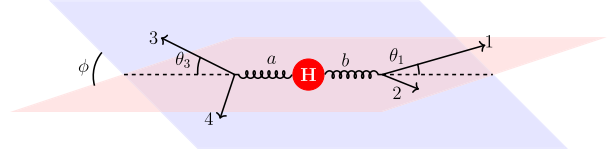


FIG. 1. The definition of azimuthal difference ϕ in Higgs rest frame for $H \rightarrow gg$ at the parton level.

The amplitude where gluons are of different helicity vanishes due to angular momentum conservation. $\mathcal{CP}$ transformation flips the helicity of gluon, we have $\mathcal{M}_{++} \rightarrow \mathcal{M}_{--}$. Thus the amplitude $\mathcal{M}_{++} + \mathcal{M}_{--} \propto \cos \Delta$ is $\mathcal{CP}$ even, while $\mathcal{M}_{++} - \mathcal{M}_{--} \propto \sin \Delta$ is $\mathcal{CP}$ odd.

A. Observables

Spin correlations of partons have been studied through jet substructures by constructing the azimuthal difference ϕ between the planes of splittings in two separate jets, known as the Lund observable [13] (see Fig. 1):

$$\phi \equiv \arccos \frac{(\vec{p}_1 \times \vec{p}_2) \cdot (\vec{p}_3 \times \vec{p}_4)}{(|\vec{p}_1 \times \vec{p}_2|)(|\vec{p}_3 \times \vec{p}_4|)}, \quad (4)$$

where $\vec{p}_1$ and $\vec{p}_2$ are the momentum for the two subjects within one jet with $|\vec{p}_1| > |\vec{p}_2|$, while $\vec{p}_3$ and $\vec{p}_4$ for the two subjects within the other jet with $|\vec{p}_3| > |\vec{p}_4|$.

An azimuthal difference $\phi_{(ij,kl)}$ can also be defined at the level of individual particles within two separated jets, where (ij) and (kl) denote pairs of particles originating from separated jets. Building on this, one can construct an infrared-safe observable sensitive to spin correlations: the four-point energy-energy correlator (E4C), corresponding to the azimuthal difference weighted by the product of the energies of the four particles raised to a power $n \geq 1$ [32]:

$$\text{E4C}(\phi, n) = \sum_{ij,kl} \int \frac{d\sigma}{\sigma} \frac{E_i^n E_j^n}{Q_1^{2n}} \frac{E_k^n E_l^n}{Q_2^{2n}} \delta(\phi - \phi_{(ij,kl)}). \quad (5)$$

Here $\{E_i, E_j\}$ correspond to energy of particle $\{i, j\}$ in one jet with jet energy Q_1 , $\{E_k, E_l\}$ for particle $\{k, l\}$ in another jet with jet energy Q_2 . The E4C is computed by summing over all such pairs $\{i, j\}$ and $\{k, l\}$ that define the same ϕ . Note that the calculation of the energy correlator uses the differential cross section defined in the lab frame, whereas the azimuthal angle is measured in the Higgs boson rest frame.

III. PARTON SHOWER AT LEADING ORDER

We do not fully understand how the correlation information propagation in showering and jet fragmentation occurs. As a leading-order approximation, we estimate gluon spin correlations in Higgs decay at the parton level by incorporating a one-step gluon splitting. In this approach, the two partons from the splitting are used to approximate

subjects for the Lund observable and to approximate the jet-level energy correlator. This framework allows for an analytical calculation using spin-dependent splitting function, see, e.g., Ref. [33].

A. Splitting function

Consider the splitting $g_1 \rightarrow a_1 b_1$ and $g_2 \rightarrow a_2 b_2$, where $a_i b_i$ can be a quark antiquark pair or a pair of gluons. Equation (6) gives the factorization for the primary splitting process of the two gluons coming from $H \rightarrow g_1 g_2$.

$$|\mathcal{M}|^2 \simeq \frac{z_1(1-z_1)}{k_{1\perp}^2} \frac{z_2(1-z_2)}{k_{2\perp}^2} \mathcal{M}_{H \rightarrow gg}^{h_1 h_2} \left(\mathcal{M}_{H \rightarrow gg}^{h'_1 h'_2} \right)^* \times D_{g_1 \rightarrow a_1 b_1}^{h_1 h'_1}(z_1, k_{1\perp}) D_{g_2 \rightarrow a_2 b_2}^{h_2 h'_2}(z_2, k_{2\perp}). \quad (6)$$

Here $\mathcal{M}_{H \rightarrow gg}^{h_1 h_2}$ is defined in Eq. (3). z_i is the energy fraction carried by parton a_i from gluon g_i with helicity h_i , and $k_{i\perp}$ is the small transverse momenta. $D_{g_i \rightarrow a_i b_i}^{h_i h'_i}$ is the decay matrix of gluon g_i in the collinear limit:

$$D_{g_i \rightarrow a_i b_i}^{h_i h'_i}(z_i, k_{i\perp}) = \epsilon_{h_i, \mu} \epsilon_{h'_i, \nu}^* P_{a_i b_i \leftarrow g_i}^{\mu\nu}(z_i, k_{i\perp}), \quad (7)$$

and gluon splitting kernel $P_{ab \leftarrow g}^{\mu\nu}$ is given by the Altarelli-Parisi functions [34–36]:

$$\begin{aligned} P_{q\bar{q} \leftarrow g}^{\mu\nu}(z, k_\perp) &= P_{q\bar{q} \leftarrow g}^{\mu\nu}(z, k_\perp) \\ &= T_F \left[-g^{\mu\nu} + 4z(1-z) \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \right], \\ P_{gg \leftarrow g}^{\mu\nu}(z, k_\perp) &= 2C_A \left[-g^{\mu\nu} \left(\frac{z}{1-z} + \frac{1-z}{z} \right) \right. \\ &\quad \left. - 2z(1-z) \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \right]. \end{aligned} \quad (8)$$

For quark splitting, there is no spin dependence for a collinear splitting, leaving a $\delta_{ss'}$ factor where $s(s')$ denotes the spin of the fermion, as shown below:

$$\begin{aligned} P_{qg \leftarrow q}^{ss'}(z, k_\perp) &= P_{q\bar{q} \leftarrow q}^{ss'}(z, k_\perp) = \delta_{ss'} C_F \frac{1+z^2}{1-z}, \\ P_{gq \leftarrow q}^{ss'}(z, k_\perp) &= P_{g\bar{q} \leftarrow q}^{ss'}(z, k_\perp) = \delta_{ss'} C_F \frac{1+(1-z)^2}{z}. \end{aligned} \quad (9)$$

Using Eqs. (6)–(8), we can compute the Lund observables and the E4C for events of the process $H \rightarrow gg$. At leading order, the predictions obtained using the splitting functions should agree in the collinear limit with those from the matrix element calculation performed by MadGraph5_aMC@NLO (MadGraph), where the tree-level amplitude for $H \rightarrow gg \rightarrow a_1 b_1 a_2 b_2$ can be evaluated. To carry out this comparison, we simulate the process $e^+ e^- \rightarrow ZH$ with $H \rightarrow gg \rightarrow a_1 b_1 a_2 b_2$ and $Z \rightarrow \nu \bar{\nu}$ in MadGraph, with

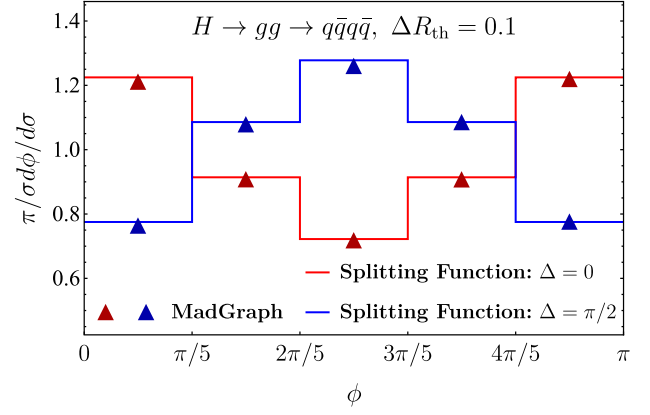


FIG. 2. The normalized distribution of the observable ϕ for $H \rightarrow gg \rightarrow q\bar{q}q\bar{q}$ using splitting function (solid line), full matrix element calculations using MadGraph.

the understanding that the intermediate gluons (gg) are treated as off shell. The transverse momentum p_T of final state quarks are required to satisfy $p_T \geq 0.1$ GeV. Using the truth-level information of the four final-state partons, we reconstruct the Higgs boson kinematics and boost the system to the Higgs rest frame. In the Higgs boson rest frame, to identify the two parton pairs originating from successive gluon splittings, we consider all possible pairings of the four partons and select the configuration in which the opening angle ΔR between the two partons inside each pair is smaller than a chosen threshold ΔR_{th} .

For the specific case of $H \rightarrow gg \rightarrow q\bar{q}q\bar{q}$ and the choices of $\Delta R_{\text{th}} = 0.1$, we present in Fig. 2 the normalized distribution $\pi/\sigma d\sigma/d\phi$, which corresponds to the Lund observable as we will discuss later. The distribution obtained from MadGraph is shown as triangle markers. The corresponding distribution computed using the splitting functions is shown as solid lines, where good agreement with that from the matrix element calculation is observed. Furthermore, the distribution does not change much for choices of larger ΔR_{th} , as the dominant contribution is from the soft region.

B. E4C vs Lund observables

We come to investigate the effectiveness of Lund observables and E4C in revealing the spin correlations of gluon pair from Higgs decay. The Lund observable is defined in the region with $z > z_{\text{cut}}$ to ensure infrared safety, where z is the energy fraction carried by the softer parton from the splitting. Using the splitting-function approach, we obtain for $H \rightarrow gg$ the distribution of the Lund observable ϕ ,

$$\text{Obs}(\phi, x) \equiv \frac{\pi d\sigma}{\sigma d\phi} = 1 + A_{\text{Lund}}(x) \cos(2\phi - 2\Delta), \quad (10)$$

with the parameter choice $x = z_{\text{cut}}$. Similarly, the E4C(ϕ, x) distribution can be written in the normalized form

TABLE I. Correlation strength $A_{\text{Lund}}(x)$ for two benchmark values of $z_{\text{cut}} = 0.1, 0.4$ and $A_{\text{E4C}}(x)$ for two benchmark values of $n = 1, 4$.

Correlation strength	$q\bar{q}q\bar{q}$	$gggg$	$q\bar{q}gg$	$jjjj$
$A_{\text{Lund}}(z_{\text{cut}} = 0.1)$	0.4204	0.0028	-0.0346	0.0003
$A_{\text{Lund}}(z_{\text{cut}} = 0.4)$	0.9481	0.0115	-0.1044	0.0011
$A_{\text{E4C}}(n = 1)$	0.4516	0.0027	-0.0349	0.0003
$A_{\text{E4C}}(n = 4)$	0.6944	0.0070	-0.0695	0.0007

$$\text{Obs}(\phi, x) \equiv \frac{\text{E4C}(\phi, x)}{\mathcal{N}(x)} = 1 + A_{\text{E4C}}(x) \cos(2\phi - 2\Delta), \quad (11)$$

with $x = n$ and $\mathcal{N}(x) = \int_0^\pi d\phi \text{E4C}(\phi, x)$. In both cases, the term $\cos(2\phi - 2\Delta)$ encodes the expected spin-correlation structure, while $A_{\text{Lund/E4C}}(x)$ quantifies the strength of the correlation for the parameter choice x .

We present the correlation strengths $A_{\text{Lund}}(x)$ for two benchmark values of z_{cut} and $A_{\text{E4C}}(x)$ for two benchmark values of n in Table I. Results are shown for the different splitting modes $X \in \{gggg, ggq\bar{q}, q\bar{q}q\bar{q}, jjjj\}$, where q denotes the set of light quarks u, c, d, s , and j denotes the inclusive mode combining gluons and the light quarks.

We observe that $A_{\text{Lund}}(x)$ increases with z_{cut} , where the energy of the softest particle becomes larger. This behavior

is consistent with the expectation from Eq. (8): the term governing the spin correlation between the two gluons from Higgs decay is proportional to $z(1-z)$, which is maximized at $z = 0.5$ for both the $g \rightarrow gg$ and $g \rightarrow q\bar{q}$ splittings. Since the E4C is an energy-weighted observable that receives enhanced contributions from regions in which both particles resulting from a splitting are hard, it is natural that $A_{\text{E4C}}(x)$ increases from $n = 1$ to $n = 4$, as observed Table I. This increase is observed to saturate for larger n , and we therefore restrict our discussion to $n \leq 4$. For the $g \rightarrow gg$ splitting mode, this trend is further supported by expectations from the soft theorem [37]: soft radiation of massless gauge bosons obeys universal soft limits in which spin information is retained by the hard particles. Consequently, the E4C suppresses contributions from events where spin correlations fail to propagate through splittings dominated by soft emissions. The same reasoning applies to the case of Lund observable: increasing z_{cut} removes an even larger fraction of soft configurations, thereby enhancing the contribution from harder splittings and leading to a larger value of $A_{\text{Lund}}(x)$.

By convoluting the correlation strength of each splitting mode with its corresponding splitting probability, where approximately 93% are $gggg$ modes, 7% are $q\bar{q}gg$ modes, and 0.1% are $q\bar{q}q\bar{q}$ modes, we obtain the correlation strength $A_{\text{Lund/E4C}}(x)$ for the inclusive channel $jjjj$, as listed in Table I. The resulting $A_{\text{Lund/E4C}}(x)$ is significantly

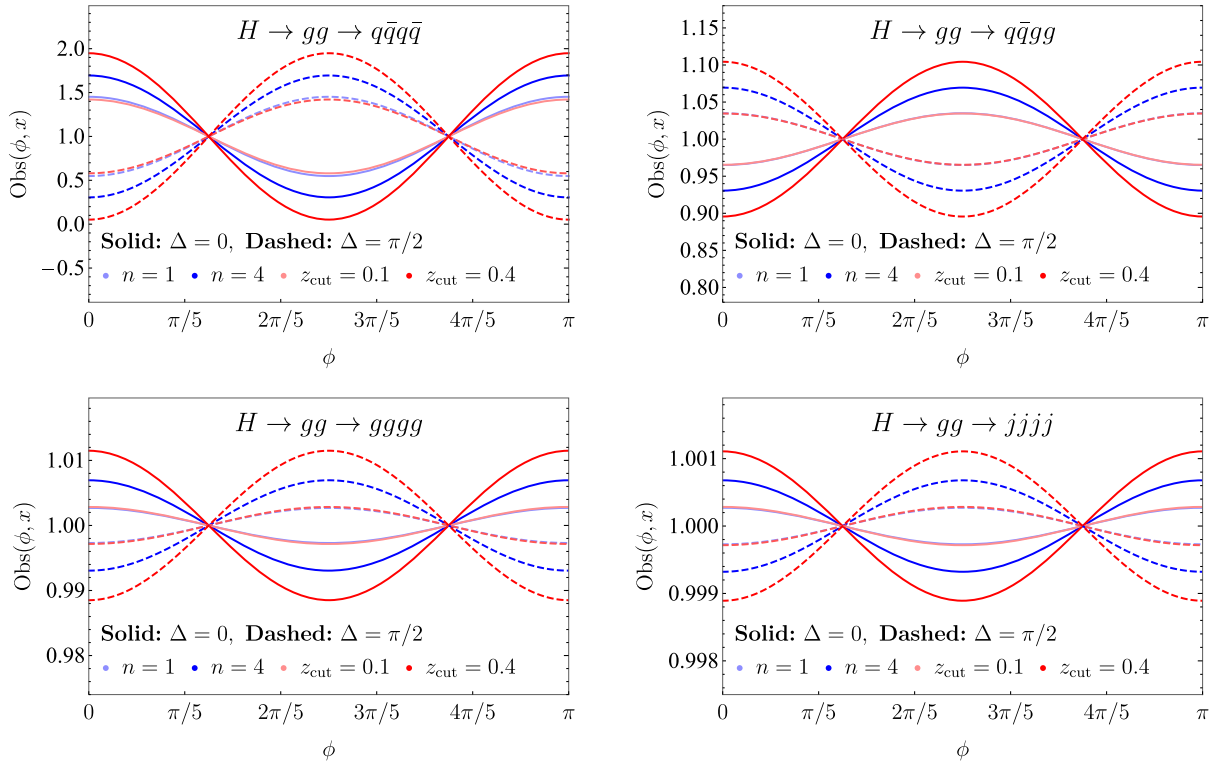


FIG. 3. $\text{Obs}(\phi, x)$ for the Lund observable with $z_{\text{cut}} = 0.1$ and $z_{\text{cut}} = 0.4$, as well as for the E4C observable with $n = 1$ and $n = 4$. Results for both $\Delta = 0$ ($\mathcal{CP}$ -even) and $\Delta = \pi/2$ ($\mathcal{CP}$ -odd) in a different splitting mode are presented. Since $A_{\text{E4C}}(n = 1)$ and $A_{\text{Lund}}(z_{\text{cut}} = 0.1)$ are very close, the corresponding curves overlap.

TABLE II. Cut efficiency ϵ_z for $z_{\text{cut}} = 0.1$ and $z_{\text{cut}} = 0.4$ when considering the Lund observable.

ϵ_z	$qqqq$	$gggg$	$q\bar{q}gg$	$jjjj$
$z_{\text{cut}} = 0.1$	0.64	0.32	0.45	0.33
$z_{\text{cut}} = 0.4$	0.07	0.02	0.01	0.02

smaller than that of the individual splitting modes, due to partial cancelations between the two dominant contributing channels: $gggg$ and $q\bar{q}gg$, which have correlations of opposite sign. The distribution $\text{Obs}(\phi, x)$ for the Lund observable and the E4C are further shown in Fig. 3, considering both cases $\Delta = 0$ and π , for different splitting modes.

Among all splitting modes, the case $z_{\text{cut}} = 0.4$ shows a relatively large $A_{\text{Lund}}(x)$. However, this enhancement comes at the cost of reduced statistics due to the $z > z_{\text{cut}}$ requirement. Table II highlights the efficiency ϵ_z of imposing this requirement for $z_{\text{cut}} = 0.4$ and $z_{\text{cut}} = 0.1$, which underscores the trade-off between maximizing correlation strength and retaining sufficient event rates.

IV. PHENOMENOLOGY AT FUTURE LEPTON COLLIDERS

We investigate the spin correlation between the two gluons from Higgs decay. For a future lepton collider operating at a center-of-mass energy of $\sqrt{s} = 250$ GeV, we consider the dominant Higgs production channel $e^+e^- \rightarrow ZH$ with a total cross section of $\sigma_{ZH} = 204.7 \text{ fb}^{-1}$, and the leptonic decay channels of $Z \rightarrow l\bar{l}$ with $l\bar{l}$ including e^+e^- , $\mu^+\mu^-$ and the invisible $\nu\bar{\nu}$ channels. For these decay modes, the hadronic activities are purely from the Higgs decay. We neglect non-Higgs backgrounds such as ZZ , assuming they can be efficiently suppressed by reconstructing the invariant mass of the Higgs boson from the hadronic activities. In the $Z \rightarrow \nu\bar{\nu}$ channel, $H \rightarrow gg$ can be separated from $H \rightarrow b\bar{b}$ and $H \rightarrow VV^* \rightarrow jjjj$ with an efficiency of $\epsilon_{gg} = 90\%$, and a mistag rate of approximately 3%, using event-level analysis techniques [38]. As a result, backgrounds from such decay modes of the Higgs boson are subdominant and will be neglected after applying the cutting efficiency ϵ_{gg} . A similar level of discrimination can be extended to the charged lepton decay channels of Z . The kinematics of the Higgs boson can be reconstructed from the gluon jets which allows boosting to the Higgs rest frame for the construction of Lund observables and E4C.

The number of signal events after the above selection for each splitting mode is given by

$$N_X = \sigma_{ZH} \mathcal{L} \text{Br}(Z \rightarrow l\bar{l}) \text{Br}(H \rightarrow gg) P_{gg \rightarrow X} \epsilon_z \epsilon_{\text{cut}} \epsilon_{gg}, \quad (12)$$

with $\text{Br}(Z \rightarrow l\bar{l}) \simeq 0.27$, $\text{Br}(H \rightarrow gg) = 0.0857$, and $P_{gg \rightarrow X}$ corresponding to the probability of gg splitting to

TABLE III. Significance to exclude the uncorrelated hypothesis using the choices of E4C($\phi, n = 1$) and E4C($\phi, n = 4$), and the Lund observable with $z_{\text{cut}} = 0.1$ and $z_{\text{cut}} = 0.4$, at $\mathcal{L} = 5.6 \text{ ab}^{-1}$.

q	$q\bar{q}q\bar{q}$	$gggg$	$q\bar{q}gg$	$jjjj$
$z_{\text{cut}} = 0.1$	1.3608	0.1661	0.6688	0.0175
$z_{\text{cut}} = 0.4$	0.6847	0.1047	0.3646	0.0108
$n = 1$	1.836	0.2795	1.007	0.0292
$n = 4$	2.8812	0.7204	2.005	0.0731

X . ϵ_z is only applied for the Lund observable with z_{cut} applied. We use ϵ_{cut} to account for the minimal kinematic requirements for a jet (lepton) to be reconstructed at the detector, which are realized by requiring the corresponding parton (lepton) to have $p_T \geq 20(10) \text{ GeV}$, $\eta < 5(2.5)$, and $\Delta R > 0.4$ between these particles. Such requirements implemented with MadGraph lead to $\epsilon_{\text{cut}} \simeq 0.63$.

A. Spin correlation measurement

With $\Delta = 0$, we first examine the hypothesis that the spins are uncorrelated. Under this assumption, $\text{Obs}(\phi, x)$ is expected to be uniformly distributed in ϕ . Using $M = 5$ bins uniformly distributed in ϕ from 0 to π and assuming each splitting modes at jet level can be fully separated, the significance for excluding the uncorrelated hypothesis is given by

$$q = \sqrt{-2 \sum_{i=1}^M N_i^{\text{obs}} - N_i + N_i^{\text{obs}} \log \frac{N_i}{N_i^{\text{obs}}}}, \quad (13)$$

where N_i^{obs} corresponds to the SM prediction with spin correlations and N_i is the signal event number predicted by the hypothesis to be tested at i th bin. For the uncorrelated hypothesis, we have $N_i = N_{\text{tot}}/5$, where N_{tot} is the total event number integrating over all bins. The significance obtained at various choices of x for two benchmark values of the luminosity $\mathcal{L} = 5.6$ and $\mathcal{L} = 20 \text{ ab}^{-1}$ [39] is shown in Tables III and IV, respectively.

We observe that the significance of excluding the uncorrelated hypothesis is generally higher when using E4C compared to the Lund observable, due to the reduced statistics from applying z_{cut} . Furthermore, in the limit of

TABLE IV. Same as Table III, with the luminosity changed to $\mathcal{L} = 20 \text{ ab}^{-1}$.

q	$q\bar{q}q\bar{q}$	$gggg$	$q\bar{q}gg$	$jjjj$
$z_{\text{cut}} = 0.1$	2.517	0.3138	1.2639	0.0331
$z_{\text{cut}} = 0.4$	1.2939	0.1979	0.6891	0.0248
$n = 1$	3.4697	0.5283	1.9031	0.0551
$n = 4$	5.4449	1.3614	3.7891	0.1381

perfect separation between distinct splitting modes at jet level, future lepton colliders can exclude the uncorrelated hypothesis in either $q\bar{q}q\bar{q}$ or $q\bar{q}gg$ splitting mode at 95% CL. The inclusive mode $jjjj$, while offering the largest statistical sample, shows negligible power to distinguish the correlated from the uncorrelated hypothesis, given its tiny correlation strength from the cancellations between different modes.

As a ballpark estimate of the potential improvement from sub-jet tagging in evading the cancellation of the correlation strength in the inclusive $jjjj$ mode, we consider applying a cut on the classification score $\text{score}(g \rightarrow q\bar{q})$ introduced in [40], which was designed to distinguish the splitting channels $g \rightarrow gg$ and $g \rightarrow q\bar{q}$. Using the efficiencies reported in Fig. 1 of Ref. [40], we estimate the modified significance by rescaling the inclusive $jjjj$ sensitivity according to the surviving event fraction and the enhanced purity of the $q\bar{q}gg$ component after the cut. We find that applying a cut $\text{score}(g \rightarrow q\bar{q}) > 0.35$, which increases the purity of the $q\bar{q}gg$ component from 0.07 to 0.24 and thus is assumed to significantly alleviates the cancellation effect, improves the estimated significance from 0.1381 to 1.3422, corresponding to roughly an order-of-magnitude enhancement in sensitivity. Since the tagging strategy in [40] targets secondary splittings, while our setup focuses on subjets originating from the first splitting, further improvements may be achievable with dedicated sub-jet-tagging techniques optimized for our analysis.

B. $\mathcal{CP}$ measurement

As Lund observable and E4C are $\mathcal{CP}$ -sensitive observables, we further explore their potential in probing the $\mathcal{CP}$ properties of the Hgg coupling at future lepton colliders. While indirect constraints on the $\mathcal{CP}$ -mixing angle in the Hgg coupling can be derived from neutron electric dipole moment (nEDM) measurements [41–43], colliders enable direct probes of the $\mathcal{CP}$ property of Hgg coupling. At the LHC, such probes rely on the azimuthal angle difference between the two associated jets in the $H + 2j$ production channel [44–46]. In contrast, the Lund observable/E4C constructed from Higgs decay products provides a complementary and potentially first direct probe of the $\mathcal{CP}$ property of Hgg coupling at lepton colliders. For a given nonzero $\mathcal{CP}$ -mixing angle Δ , we evaluate the exclusion sensitivity by computing the statistical significance using Eq. (13), where N_i denotes the expected number of events in the i th bin for the corresponding Δ hypothesis. The sensitivities are shown in Fig. 4 for all four splitting modes, where the horizontal dashed line indicates the exclusion limit with $q = 2$. We observe that for the best scenario of using E4C($\phi, n = 4$) in the $q\bar{q}q\bar{q}$ channel, a $\mathcal{CP}$ -mixing angle as small as $\Delta \approx 0.03\pi$ can be excluded at future lepton colliders with $\sqrt{s} = 240$ GeV and $\mathcal{L} = 20 \text{ ab}^{-1}$ data. In comparison, the $\mathcal{CP}$ -sensitive observable in the $H + 2j$ production channel has not yet demonstrated sensitivity to the $\mathcal{CP}$ -mixing angle using the current dataset with $\mathcal{L} = 139 \text{ fb}^{-1}$ LHC [44]. Nevertheless, the $\mathcal{CP}$ -mixing

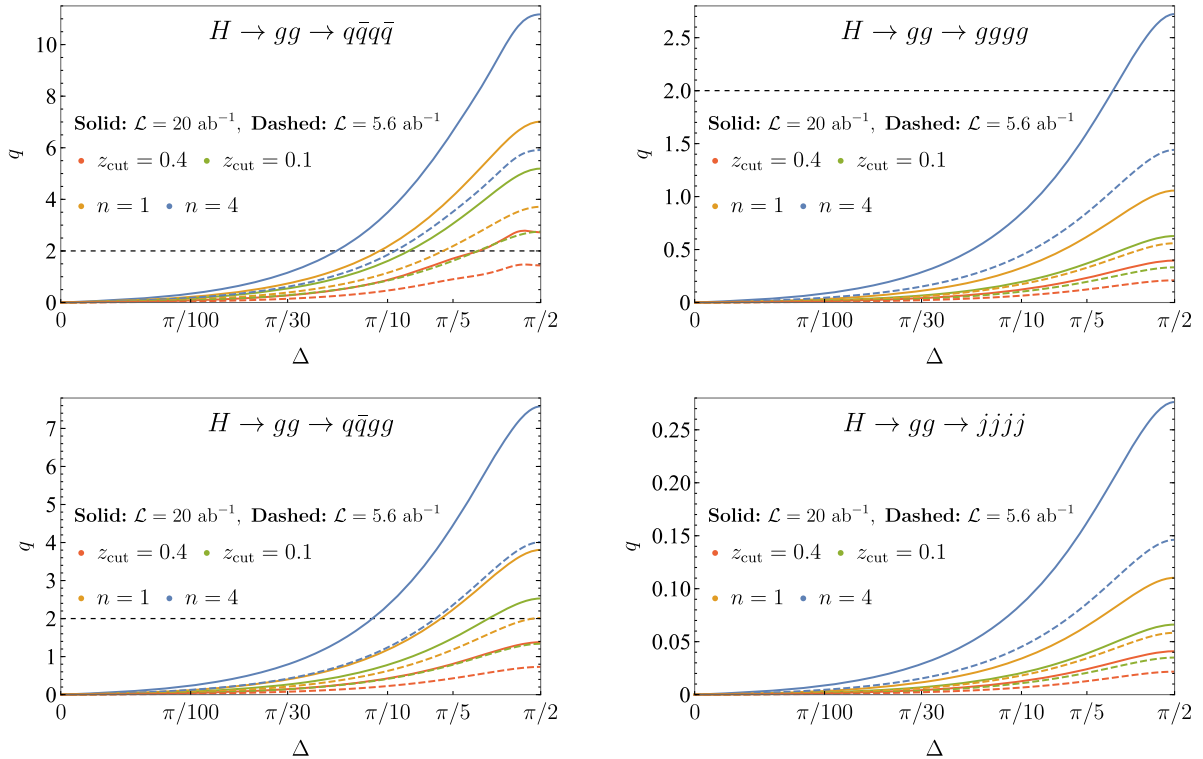


FIG. 4. The significance q to exclude a $\mathcal{CP}$ mixing hypothesis with mixing angle Δ , at $\mathcal{L} = 20$ and 5.6 ab^{-1} .

angle can be constrained to $\Delta \lesssim 0.23\pi$ through a global fit including both $HG_{\mu\nu}^a G^{a,\mu\nu}$ and $HG_{\mu\nu}^a \tilde{G}^{a,\mu\nu}$ operators [44]. For HL-LHC with $\mathcal{L} = 3000 \text{ fb}^{-1}$, the same global fit is expected to improve the constraint to about $\Delta \lesssim 0.11\pi$ [44].

V. CONCLUSIONS

In this work, we studied observables sensitive to gluon spin correlations in Higgs decays by looking at jet substructure observables. Two classes of observables were investigated: the four-point energy correlator (E4C) with different energy-weighting power and a Lund-plane-based variable. While the Lund observable requires a cut on subjet energy fraction to ensure IR safety, as well as to enhance the sensitivity to gluon spin correlations, it simultaneously reduces the available event statistics. Taking these competing effects into account, we find that the E4C observable with an energy-weighting power of $n = 4$ provides the highest sensitivity to gluon spin correlations at the parton level.

Assuming ideal discrimination between different gluon splitting modes at the jet level, gluon spin correlations can be observed at future lepton colliders operating at $\sqrt{s} = 240 \text{ GeV}$ with an integrated luminosity of $\mathcal{L} = 5.6 \text{ ab}^{-1}$, particularly through the splitting channel $gg \rightarrow q\bar{q}q\bar{q}$. Although the inclusive gluon splitting mode provides the largest event yield, its sensitivity is strongly reduced due to cancellations between contributions from $g \rightarrow q\bar{q}$ and $g \rightarrow gg$ splittings, resulting in a much weaker net correlation. Extending this framework to direct measurements of the $\mathcal{CP}$ structure of the Hgg coupling, we find that the $gg \rightarrow q\bar{q}q\bar{q}$ channel can probe the $\mathcal{CP}$ -mixing angle to $\Delta\phi \simeq 0.03\pi$ at $\sqrt{s} = 240 \text{ GeV}$ with an integrated luminosity of $\mathcal{L} = 20 \text{ ab}^{-1}$.

The strategy of using energy-energy correlation can also be extended to potentially improve the intrajet spin correlation measurement at LHC [40]. Yet, this strategy extending to probe the $b\bar{b}$ spin correlations and the $\mathcal{CP}$ structure of the $Hb\bar{b}$ coupling in $H \rightarrow b\bar{b}$ decay is strongly limited, as the quark splittings into vector bosons do not preserve the initial quark spin correlations, in agreement with expectations from soft theorems.

Future work will focus on the development of dedicated subjet-tagging techniques to enable separations of different splitting modes at the jet level. This will allow a full exploitation of the potential of the E4C observable at future lepton colliders for probing partonic spin correlations and for searching for new physics beyond the Standard Model.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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