

Multipole structure of the $N \rightarrow \Delta$ transition generalized parton distributions

June-Young Kim^{*} and Hyun-Chul Kim[†]

Department of Physics and Institute of Quantum Science, Inha University, Incheon 22212, Republic of Korea



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We establish the multipole structure of the $N \rightarrow \Delta$ transition at the level of the generalized parton distributions (GPDs). We decompose the four transition GPDs into one monopole, two dipole, and one quadrupole components in the transverse plane by a multipole expansion of the covariant transition matrix element in terms of the three-dimensional spin-transition tensors and the transverse momentum transfer. These multipole components are in one-to-one correspondence with the light-front helicity amplitudes. In the zero-skewness limit, the multipole GPDs define impact-parameter transition densities, which generalize the transverse transition charge densities to the x -dependent level. These transition densities arise from nondiagonal matrix elements between two distinct hadronic states and must therefore be distinguished from the diagonal densities of the nucleon and the Δ . The multipole transition densities visualize the monopole, dipole, and quadrupole structures of the partonic $N \rightarrow \Delta$ transition in the transverse plane.

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I. INTRODUCTION

Understanding the internal structure of the $\Delta(1232)$, the first excited state of baryons, is a fundamental problem in hadron physics [1]. Since the ephemeral Δ decays strongly into πN [2] and cannot be used as a target, its structure is usually accessed through the electromagnetic (EM) $N \rightarrow \Delta$ transition, characterized by the magnetic dipole ($M1$), electric quadrupole ($E2$), and Coulomb quadrupole ($C2$) form factors [3]. Experiments [4–14] based on pion photo- and electroproduction off the nucleon have been carried out over decades, and established that the $M1$ transition dominates, whereas the measured quadrupole ratios R_{EM} and R_{SM} are only a few percent in magnitude. These small but nonvanishing quadrupole amplitudes indicate the deformation of the Δ [15], which arises from the pion cloud in dynamical models [16,17] or from the polarization of the vacuum (the Dirac continuum), as shown in the chiral quark-soliton model (χ QSM) [18]. In the transverse charge densities constructed from the transition form factors, these amplitudes appear as dipole and quadrupole patterns in the transverse plane [19].

The form factors, however, provide information only on the densities in the transverse plane. A more complete

description of the transition is furnished by the generalized parton distributions (GPDs), which correlate the longitudinal momentum of the quark with its transverse position inside the baryon [20–25]. At zero skewness, the GPDs provide a probabilistic interpretation in impact-parameter space or in the transverse plane [26,27]. The concept of the GPDs can be extended to the nucleon-to-resonance transition matrix elements, which makes it possible to image the partonic structure of baryon resonances. Recently, hard exclusive $\pi^- \Delta^{++}$ electroproduction was measured at the Thomas Jefferson National Accelerator Facility (JLab) [28], exclusive $\pi \Delta(1232)$ production and the deeply virtual Compton process $eN \rightarrow e\gamma\pi N$ were analyzed theoretically [29,30], and dedicated research programs on transition GPDs will soon be available at the JLab upgrades and the Electron-Ion Collider (EIC) [31,32].

On the theoretical side, the vector $N \rightarrow \Delta$ transition GPDs were first parametrized in Ref. [23] in terms of three functions. A fourth Lorentz structure was subsequently proposed to match the number of independent light-front helicity amplitudes [25], but it turned out not to be independent of the original three. A complete and independent fourth structure was identified only recently in Ref. [33], and its kinematic and dynamical implications were explored in Refs. [33–35].

The aim of the present work is to establish the multipole structure of the $N \rightarrow \Delta$ transition at the level of the GPDs. To this end, we perform a multipole expansion of the covariant matrix element in terms of the three-dimensional spin-transition tensors and the transverse momentum transfer. The four transition GPDs are then decomposed into one monopole, two dipole, and one quadrupole components in

^{*}Contact author: jun-young.kim@inha.ac.kr

[†]Contact author: hchkim@inha.ac.kr

the transverse plane, each of which corresponds to a definite light-front helicity amplitude. The multipole GPDs extend the multipole structure of the transition form factors [3] to the x -dependent level. In the zero-skewness limit, they define impact-parameter transition densities, which generalize the transverse charge densities of Ref. [19] and visualize the partonic structure of the $N \rightarrow \Delta$ transition.

The present paper is organized as follows: Sec. II reviews the covariant parametrization of the $N \rightarrow \Delta$ transition GPDs and their relation to the EM transition form factors. In Sec. III we introduce the spin-transition tensors in the symmetric light-front frame, with which we derive the multipole expansion of the matrix element and the corresponding light-front helicity amplitudes. In Sec. IV we take the zero-skewness limit and visualize the impact-parameter transition densities in the monopole, dipole, and quadrupole channels. Section V is devoted to the summary and conclusions. Technical details are compiled in the Appendixes.

II. GENERAL FORMALISM

In this section, we recapitulate the general formalism for the $N \rightarrow \Delta$ transition GPDs. We define the nonlocal light-cone matrix element, introduce the two covariant bases of the transition GPDs, and relate their first Mellin moments to the EM transition form factors, which serve as the starting point of the multipole analysis in Sec. III.

A. Generalized parton distributions

The transition matrix element of the nonlocal vector operator between the proton N^+ and the Δ^+ baryon states is defined as

$$\mathcal{M}_{\text{GPDs}}^{[\not{n}]} = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle \Delta^+ | \bar{\psi}(-\lambda n/2) \not{n} \tau^3 \psi(\lambda n/2) | N^+ \rangle, \quad (1)$$

where ψ denotes the quark field operator, λ is the light-cone separation between the two fields, and n is a lightlike vector satisfying $n^2 = 0$. We use the shorthand $|N^+\rangle \equiv |N^+(p, \sigma)\rangle$ and $\langle \Delta^+ | \equiv \langle \Delta^+(p', \sigma') |$. The four-momenta and spin polarizations of the initial and final states are denoted by (p, σ) and (p', σ') , respectively. These arguments are suppressed hereafter. The matrix τ^3 denotes the SU(2) flavor matrix for the $N \rightarrow \Delta$ transition.

This matrix element is parametrized in terms of the transition GPDs. Three vector transition GPDs were first introduced in Ref. [23], in analogy with the EM $N \rightarrow \Delta$ transition form factors, to which their first Mellin moments are related. As discussed in Sec. I, however, four independent Lorentz structures are required to match the number of light-front helicity amplitudes [25], and the complete parametrization was established in Ref. [33].

The four $N^+ \rightarrow \Delta^+$ transition GPDs are defined through the light cone vector-current matrix element,

$$\mathcal{M}_{\text{GPDs}}^{[\not{n}]} = \sqrt{\frac{2}{3}} \bar{u}_\alpha \left[\sum_{I=1,2,3,X} G_I(x, \xi, t) \mathcal{K}_I^{\alpha\mu} n_\mu \right] u. \quad (2)$$

Here $G_I(x, \xi, t)$, with $I = 1, 2, 3, X$, denote the four $N^+ \rightarrow \Delta^+$ transition GPDs, and $\sqrt{2/3}$ is the isospin Clebsch-Gordan coefficient for this transition. We use the symmetric kinematics

$$P = \frac{p + p'}{2}, \quad \Delta = p' - p, \quad (3)$$

and define

$$t = \Delta^2, \quad \xi = -\frac{\Delta \cdot n}{2P \cdot n}. \quad (4)$$

The variable x denotes the average light-cone momentum fraction carried by the quark with respect to $P \cdot n$, while ξ is the skewness parameter measuring the longitudinal momentum transfer and t is the invariant momentum transfer squared. The spinors $\bar{u}_\alpha \equiv \bar{u}_\alpha(p', \sigma')$ and $u \equiv u(p, \sigma)$ denote the Rarita-Schwinger spinor for the outgoing Δ baryon and the Dirac spinor for the incoming nucleon, respectively. They satisfy the on-shell Dirac equations

$$\bar{u}_\alpha \not{p}' = m_\Delta \bar{u}_\alpha, \quad \not{p} u = m_N u, \quad (5)$$

with $p'^2 = m_\Delta^2$ and $p^2 = m_N^2$. These on-shell conditions yield

$$\Delta \cdot P = \frac{m_\Delta^2 - m_N^2}{2}, \quad (6a)$$

$$P^2 + \frac{\Delta^2}{4} = \frac{m_N^2 + m_\Delta^2}{2}. \quad (6b)$$

The Rarita-Schwinger spinor also satisfies the subsidiary condition

$$\bar{u}_\alpha \gamma^\alpha = 0. \quad (7)$$

The tensors $\mathcal{K}_I^{\alpha\mu}$ encode the independent Lorentz structures constructed from the available four-vectors p and p' , the metric tensor $g^{\mu\nu}$, the Levi-Civita tensor $\epsilon^{\mu\nu\rho\sigma}$, and the Dirac matrices. Their explicit expressions are given by [33]

$$\mathcal{K}_1^{\alpha\mu} = \frac{1}{m_N} (\Delta^\alpha \gamma^\mu - \not{\Delta} g^{\alpha\mu}) \gamma_5, \quad (8a)$$

$$\mathcal{K}_2^{\alpha\mu} = \frac{2}{m_N^2} (\Delta^\alpha P^\mu - (\Delta \cdot P) g^{\alpha\mu}) \gamma_5, \quad (8b)$$

$$\mathcal{K}_3^{\alpha\mu} = \frac{1}{m_N^2} (\Delta^\alpha \Delta^\mu - \Delta^2 g^{\alpha\mu}) \gamma_5, \quad (8c)$$

$$\mathcal{K}_X^{\alpha\mu} = m_N n^\alpha \gamma^\mu \gamma_5. \quad (8d)$$

The first three structures correspond to the Belitsky-Radyushkin (BR) parametrization, whereas the last one was introduced in Ref. [33]. The choice of basis, however, is not unique. Another commonly used basis is that of the Goeke-Polyakov-Vanderhaeghen (GPV) GPDs [23], motivated by the Jones-Scadron multipole form factors [3]. The GPV GPDs are related to the BR GPDs by the linear transformation

$$3m_\Delta m_N (m_\Delta + m_N) \begin{pmatrix} H_M \\ H_E \\ H_C \end{pmatrix} = \begin{pmatrix} (3m_\Delta + m_N)(m_\Delta + m_N) - t & 2(m_\Delta^2 - m_N^2) & 2t \\ m_\Delta^2 - m_N^2 + t & 2(m_\Delta^2 - m_N^2) & 2t \\ 4m_\Delta^2 & 2(3m_\Delta^2 + m_N^2 - t) & 2(m_\Delta^2 - m_N^2 + t) \end{pmatrix} \begin{pmatrix} m_N G_1 \\ m_\Delta G_2 \\ m_\Delta G_3 \end{pmatrix}. \quad (9)$$

We use Eq. (9) as the working definition for converting between the GPV and BR bases. The full covariant decomposition of the GPV parametrization will not be reproduced here. Its explicit form is given in Refs. [23,33].

B. Electromagnetic form factors

The first moment of the nonlocal operator in Eq. (1) reduces to the local EM current and thus relates the transition GPDs in Eq. (2) to the corresponding EM transition form factors. Note that only the isovector component of the EM current contributes to the $N \rightarrow \Delta$ transition. In the Jones-Scadron covariant parametrization [3], written with the same Lorentz structures as in Eq. (8), the EM current matrix element is expressed as

$$\begin{aligned} \mathcal{M}_{\text{FF}}^{[\gamma\mu]} &= \langle \Delta^+ | \bar{\psi}(0) \gamma^\mu \frac{\tau^3}{2} \psi(0) | N^+ \rangle \\ &= \sqrt{\frac{2}{3}} \bar{u}_\alpha \left[\sum_{I=1,2,3} G_I^*(t) \mathcal{K}_I^{\alpha\mu} \right] u. \end{aligned} \quad (10)$$

Here the Lorentz structures are identical to those in Eq. (8), but the dynamical quantities are form factors, depending only on the invariant momentum transfer squared t . The asterisk is used to distinguish the EM form factors from the GPDs. The first moments of the transition GPDs are then given by

$$\int dx G_{1,2,3}(x, \xi, t) = 2G_{1,2,3}^*(t), \quad (11a)$$

$$\int dx G_X(x, \xi, t) = 0. \quad (11b)$$

Similarly, the first moments of the GPV GPDs yield the multipole form factors

$$\int dx H_{M,E,C}(x, \xi, t) = 2G_{M,E,C}^*(t), \quad (12)$$

where $G_M^*(t)$, $G_E^*(t)$, and $G_C^*(t)$ denote the magnetic dipole, electric quadrupole, and Coulomb quadrupole form factors, respectively. They are related to $G_{1,2,3}^*(t)$ by the same linear transformation as in Eq. (9).

III. MULTIPOLE EXPANSION

The Lorentz covariant structure of the matrix element in Eqs. (2) and (8) can be analyzed by performing a multipole expansion in terms of the three-dimensional polarization tensors and the angular dependence of the transverse momentum transfer. This expansion provides a natural way to relate the angular structure of the matrix element to the underlying spatial distributions. We first introduce the polarization tensors, specify the symmetric-frame kinematics, and then construct the corresponding multipole basis and helicity amplitudes.

A. Polarization tensors in spherical basis

The spin structure of the diagonal matrix element can be expressed in terms of the polarization tensor, which is well understood (see Ref. [36]). We extend this construction to the nondiagonal matrix element in Eq. (1) by choosing a normalization consistent with the diagonal case:

$$T_{LM}(\sigma', \sigma) \equiv \frac{\sqrt{2L+1}}{\sqrt{(2S+1)}} C_{S\sigma LM}^{S'\sigma'}, \quad (13)$$

where $C_{S\sigma LM}^{S'\sigma'}$ denotes Clebsch-Gordan (CG) coefficients. For the $N \rightarrow \Delta$ transition, the spin quantum numbers of the initial and final states are fixed to $S = 1/2$ and $S' = 3/2$, respectively. The allowed polarization tensors $T_{LM} \equiv T_{LM}(\sigma', \sigma)$ are then restricted by the angular-momentum selection rule to

$$T_{1M}, \quad M = 0, \pm 1, \quad (14a)$$

$$T_{2M}, \quad M = 0, \pm 1, \pm 2. \quad (14b)$$

Thus, only the dipole ($L = 1$) and quadrupole ($L = 2$) transitions contribute, while the monopole ($L = 0$) and higher multipoles ($L \geq 3$) are excluded by this constraint.

B. Polarization tensors in Cartesian basis

The spherical polarization tensors in Eq. (14) can be represented in terms of Cartesian spin-transition tensors.

We introduce the corresponding Cartesian vector and rank-two tensor components as

$$V^i \equiv V^i(\sigma', \sigma), \quad Q^{ij} \equiv Q^{ij}(\sigma', \sigma), \quad (15)$$

with $i, j = 1, 2, 3$. Using the standard spherical-Cartesian convention [36], the vector components are related by

$$T_{10} = V^3, \quad T_{1\pm 1} = \mp \frac{1}{\sqrt{2}}(V^1 \pm iV^2), \quad (16)$$

and the rank-two components by

$$T_{20} = \sqrt{\frac{3}{2}}Q^{33}, \quad (17a)$$

$$T_{2\pm 1} = \mp (Q^{13} \pm iQ^{23}), \quad (17b)$$

$$T_{2\pm 2} = \frac{1}{2}(Q^{11} - Q^{22} \pm 2iQ^{12}). \quad (17c)$$

Here Q^{ij} is symmetric and traceless,

$$Q^{ij} = Q^{ji}, \quad Q^{ii} = 0. \quad (18)$$

These spin-transition tensors can be represented by coupling the spin-1 polarization vector to the Pauli spin operator. We define

$$V^i = \sqrt{\frac{3}{2}} \sum_{\lambda'} C_{1\lambda' \frac{1}{2}\sigma}^{\frac{3}{2}\sigma'} \hat{e}_{\lambda'}^{*i}, \quad (19a)$$

$$Q^{ij} = \sum_{\lambda' s'} C_{1\lambda' \frac{1}{2}s'}^{\frac{3}{2}\sigma'} \frac{1}{2} \sigma_{s' \sigma}^{\{i} \hat{e}_{\lambda'}^{*j\}}. \quad (19b)$$

where $a^{\{i} b^{j\}} = a^i b^j + a^j b^i$. The vector $\hat{e}_{\lambda'}^{*i}$ is the complex conjugate of the spherical polarization vector, with

$$\hat{e}_0^i = (0, 0, 1), \quad \hat{e}_{\pm 1}^i = \mp \frac{1}{\sqrt{2}}(1, \pm i, 0). \quad (20)$$

It is related to the Cartesian-to-spherical transformation matrix U^{CS} through

$$\hat{e}_{\lambda'}^{*i} = (-1)^{-\lambda'} U_{i, -\lambda'}^{\text{CS}}. \quad (21)$$

The quantity $(\sigma')_{s' \sigma}$ denotes the Pauli matrix element.

Equation (19a) is therefore directly related to T_{1M} in Eq. (14a) by the spherical-Cartesian transformation (21). In the rank-two case, Eq. (19b), the Pauli matrix element is first rewritten as a rank-one spin tensor, which introduces the CG coefficient for coupling the spin-1/2 state to a rank-one spin operator:

$$(\sigma')_{s' \sigma} = \sqrt{3} \sum_{\mu=-1,0,1} C_{\frac{1}{2}1\mu}^{\frac{3}{2}\sigma'} \hat{e}_{\mu}^{*i}. \quad (22)$$

The symmetric-traceless projection of the two spin-1 vectors in Eq. (19b) then selects the $L = 2$ component and introduces the CG coefficient for $1 \otimes 1 \rightarrow 2$. Together with the CG coefficient already present in Eq. (19b), these coefficients are recoupled into the single coefficient that defines T_{2M} in Eq. (14b). In contrast, the antisymmetric part of the same product projects onto the $L = 1$ component. Using Eq. (22), one readily obtains

$$i\epsilon^{ijk} V^k = \sqrt{\frac{3}{2}} \sum_{\lambda' s'} C_{1\lambda' \frac{1}{2}s'}^{\frac{3}{2}\sigma'} (\sigma_{s' \sigma}^i \hat{e}_{\lambda'}^{*j} - \sigma_{s' \sigma}^j \hat{e}_{\lambda'}^{*i}). \quad (23)$$

This relation reflects the fact that the antisymmetric product of two spin-1 vector operators transforms as a spin-1 tensor.

Equations (19)–(23) provide the Cartesian working representation of the spin-transition tensors. This representation will serve as the practical working expression for the explicit evaluation of the Rarita-Schwinger and Dirac spinors in the multipole expansion (see Appendixes A–C).

C. Symmetric frame in light-front quantization

We specify the light-front (LF) kinematics for the non-diagonal $N \rightarrow \Delta$ matrix element entering the GPD parametrization in Eqs. (2) and (8). The LF components of a four-vector are defined as $a = (a^+, a^-, \mathbf{a}_\perp)$, with $a^\pm = (a^0 \pm a^3)/\sqrt{2}$. The scalar product is then given by

$$a \cdot b = a^+ b^- + a^- b^+ - \mathbf{a}_\perp \cdot \mathbf{b}_\perp. \quad (24)$$

We work in the standard symmetric frame commonly used for GPDs,

$$P = (P^+, P^-, \mathbf{0}_\perp), \quad (25a)$$

$$\Delta = (\Delta^+, \Delta^-, \mathbf{\Delta}_\perp). \quad (25b)$$

The plus component of the momentum transfer in Eq. (25b) is determined to be $\Delta^+ = -2\xi P^+$ from Eq. (4) by choosing the light cone vector as $n^\mu = (1, 0, 0, -1)/(\sqrt{2}P^+)$ in Minkowski coordinates. The on-shell conditions in Eq. (6), together with the symmetric-frame condition in Eq. (25), fix the minus components of Eq. (25) as

$$P^- = \frac{2\xi(m_\Delta^2 - m_N^2) + 2m_\Delta^2 + 2m_N^2 + \mathbf{\Delta}_\perp^2}{8P^+(1 - \xi^2)}, \quad (26a)$$

$$\Delta^- = \frac{2(m_\Delta^2 - m_N^2) + \xi(2m_\Delta^2 + 2m_N^2 + \mathbf{\Delta}_\perp^2)}{4P^+(1 - \xi^2)}. \quad (26b)$$

In this frame, the remaining angular dependence enters only through the direction of the transverse momentum transfer,

$$\Delta_{\perp} = |\Delta_{\perp}|(\cos \theta, \sin \theta). \quad (27)$$

It can be organized in terms of the irreducible two-dimensional tensors

$$X_0 = 1, \quad (M = 0), \quad (28a)$$

$$X_1^i(\theta) = \frac{\Delta_{\perp}^i}{|\Delta_{\perp}|}, \quad (M = \pm 1), \quad (28b)$$

$$X_2^{ij}(\theta) = \frac{\Delta_{\perp}^i \Delta_{\perp}^j}{|\Delta_{\perp}|^2} - \frac{1}{2} \delta^{ij}, \quad (M = \pm 2), \quad (28c)$$

which form a convenient tensor basis for the multipole expansion.

In the forward limit $\xi, t \rightarrow 0$, the four-momenta (25) reduce to

$$P = \left(P^+, \frac{m_{\Delta}^2 + m_N^2}{4P^+}, \mathbf{0}_{\perp} \right), \quad (29a)$$

$$\Delta = \left(0, \frac{m_{\Delta}^2 - m_N^2}{2P^+}, \mathbf{0}_{\perp} \right). \quad (29b)$$

For $m_{\Delta} \neq m_N$, the minus component of the momentum transfer, Δ^- , remains nonzero, so that $\Delta^{\mu} \neq 0$. The vanishing momentum transfer is recovered only in the diagonal case, $m_{\Delta} = m_N$.

D. Matrix element in multipole basis

In the symmetric frame specified in Sec. III C, evaluating the Dirac and Rarita-Schwinger spinors allows the covariant parametrizations in Eqs. (2) and (8) to be mapped onto the four independent multipole structures defined in the bases of Eqs. (15) and (28),

$$\begin{aligned} \mathcal{M}_{\text{GPDs}}^{[\gamma]} &= Q^{33} X_0 M_0 - i \epsilon^{3kl} V^l X_1^k(\theta) \frac{|\Delta_{\perp}|}{m_N} M_1^V \\ &+ Q^{3k} X_1^k(\theta) \frac{|\Delta_{\perp}|}{m_N} M_1^Q + Q^{kl} X_2^{kl}(\theta) \frac{|\Delta_{\perp}|^2}{m_N^2} M_2. \end{aligned} \quad (30)$$

The coefficients M_0 , M_1^V , M_1^Q , and M_2 are scalar functions of (x, ξ, t) , which define the corresponding multipole GPDs. Their subscripts indicate the rank of the transverse multipole tensor: M_0 is associated with the scalar component X_0 , M_1^V and M_1^Q with the two independent vector components proportional to X_1^k , and M_2 with the rank-two tensor X_2^{kl} . These functions are related to the covariant GPDs $G_{1,2,3,X}$ through linear combinations, given explicitly in Appendix D.

At leading twist, QCD evolution is determined by the partonic operator. The operator in Eq. (1) is flavor

nonsinglet and does not mix with gluon operators. The GPDs nevertheless depend on scale. We do not calculate this scale dependence here, since the ansatz below is used only to illustrate the multipole patterns.

E. Light-front helicity amplitudes

The multipole decomposition can also be expressed in terms of light-front helicity amplitudes. This representation is useful because the transverse multipole rank is reflected in the corresponding helicity-flip structure. We define the helicity amplitudes by factoring out the azimuthal phase,

$$\mathcal{M}_{\text{GPDs}}^{[\gamma]} = 2e^{i(\sigma-\sigma')\theta} A_{\sigma'\sigma}, \quad (31)$$

where θ is the azimuthal angle of $\Delta_{\perp}$, as defined in Eq. (27). The light-front helicity amplitudes $A_{\sigma'\sigma} \equiv A_{\sigma'\sigma}(x, \xi, t)$ then depend only on the GPD variables.

Using the multipole representation in Eq. (30), one obtains

$$A_{\frac{11}{22}} = -A_{-\frac{1}{2}-\frac{1}{2}} = \frac{M_0}{\sqrt{6}}, \quad (32a)$$

$$A_{\frac{31}{22}} = A_{-\frac{3}{2}-\frac{1}{2}} = \frac{|\Delta_{\perp}|}{8m_N} (2\sqrt{3}M_1^V - \sqrt{2}M_1^Q), \quad (32b)$$

$$A_{\frac{1}{2}-\frac{1}{2}} = A_{-\frac{11}{22}} = \frac{|\Delta_{\perp}|}{8m_N} (2M_1^V + \sqrt{6}M_1^Q), \quad (32c)$$

$$A_{\frac{3}{2}-\frac{1}{2}} = -A_{-\frac{31}{22}} = -\frac{|\Delta_{\perp}|^2}{2\sqrt{2}m_N^2} M_2. \quad (32d)$$

The power of $|\Delta_{\perp}|$ in each term matches the helicity difference $|\sigma' - \sigma|$. Accordingly, the helicity-conserving amplitude is governed by the monopole GPD M_0 , the amplitudes with $|\sigma' - \sigma| = 1$ by the two independent dipole GPDs M_1^V and M_1^Q , and the amplitude with $|\sigma' - \sigma| = 2$ by the quadrupole GPD M_2 . The light-front helicity amplitudes therefore provide a direct physical interpretation of the multipole GPDs introduced above.

IV. MECHANICAL INTERPRETATION OF THE TRANSITION MATRIX ELEMENTS

In this section, we turn to the spatial interpretation of the multipole GPDs derived in Sec. III. In the zero-skewness limit, their two-dimensional Fourier transforms define the impact-parameter transition densities, which visualize the monopole, dipole, and quadrupole structures of the $N \rightarrow \Delta$ transition in the transverse plane. We also discuss how these transition densities should be interpreted in comparison with the diagonal densities of the nucleon and the Δ .

A. Impact-parameter distributions

We now consider the zero-skewness limit, $\xi \rightarrow 0$. In this limit, $\Delta^+ = 0$, so that the initial and final hadron states carry the same plus momentum. The Fourier transform with respect to $\Delta_\perp$ can therefore be interpreted as resolving the transverse structure of the matrix element at fixed longitudinal momentum, rather than as an overlap between states with different plus momenta [26,27]. In this limit,

$$|\Delta_\perp|^2 = -t. \quad (33)$$

We define the corresponding impact-parameter-space distribution for the $N \rightarrow \Delta$ transition by

$$\frac{1}{2} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b} \mathcal{M}_{\text{GPDs}}^{\text{[f]}} \bigg|_{\xi \rightarrow 0} = \rho(x, \mathbf{b}; \sigma', \sigma), \quad (34)$$

where $\mathbf{b}$ denotes the transverse impact parameter

$$\mathbf{b} = |\mathbf{b}|(\cos \phi, \sin \phi). \quad (35)$$

Since the matrix element is nondiagonal in the hadronic states, $\rho(x, \mathbf{b}; \sigma', \sigma)$ is not a probability density of quarks inside either the nucleon or the Δ . Rather, it is an impact-parameter-space transition density associated with the light-front quark operator in Eq. (1). At fixed longitudinal momentum fraction x , it gives the dependence of the $N \rightarrow \Delta$ transition amplitude on the transverse impact parameter $\mathbf{b}$, defined with respect to the light-front transverse center of momentum [27]. The baryon mass difference does not affect the definition of the transverse center of momentum as long as $\Delta^+ = 0$, so that the impact-parameter representation is as well defined as in the equal-mass case [34].

This construction may be regarded as an x -dependent extension of transverse $N \rightarrow \Delta$ transition densities obtained at the form-factor level [19]. A three-dimensional representation in (x, b_x, b_y) should therefore be read as showing the x and transverse impact-parameter dependence of the transition density, not as an ordinary probability density.

Applying the Fourier transform in Eq. (34) to the momentum-space multipole decomposition in Eq. (30) gives the corresponding impact-parameter-space representation. The transition density can then be written in the same spin-multipole basis as

$$\begin{aligned} \rho(x, \mathbf{b}; \sigma', \sigma) = & Q^{33} \rho_0(b) + \epsilon^{3kl} V^l X_1^k(\phi) \rho_1^V(b) \\ & + i Q^{3k} X_1^k(\phi) \rho_1^Q(b) + Q^{kl} X_2^{kl}(\phi) \rho_2(b), \end{aligned} \quad (36)$$

where $b = |\mathbf{b}|$. The radial functions ρ_0 , ρ_1^V , ρ_1^Q , and ρ_2 are the Fourier-Bessel transforms of the corresponding multipole GPDs M_0 , M_1^V , M_1^Q , and M_2 . They represent the monopole, the two independent dipole, and the quadrupole components of the $N \rightarrow \Delta$ transition density in the

transverse plane. The radial functions in Eq. (36) are defined as

$$\rho_0(x, b) = \tilde{M}_0(x, 0, b), \quad (37a)$$

$$\rho_1^{V,Q}(x, b) = \frac{1}{m_N} \frac{d}{db} \tilde{M}_1^{V,Q}(x, 0, b), \quad (37b)$$

$$\rho_2(x, b) = -\frac{1}{m_N^2} b \frac{d}{db} \frac{1}{b} \frac{d}{db} \tilde{M}_2(x, 0, b), \quad (37c)$$

where the functions $\tilde{M}$ are defined as the Fourier transforms of the corresponding multipole GPDs.

$$\tilde{M}(x, 0, b) = \frac{1}{2} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b} M(x, 0, t). \quad (38)$$

B. Visualization of the partonic structure

We now evaluate the transition density for specific polarization states of the initial nucleon and final Δ baryon. For the unpolarized quark operator (1), the transition density between longitudinally polarized states ($\sigma' = \sigma = 1/2$) is given by

$$\rho(x, \mathbf{b}; \sigma' = 1/2, \sigma = 1/2) = \sqrt{\frac{2}{3}} \rho_0(x, b). \quad (39)$$

Thus, only the monopole component contributes in this case.

For transversely polarized states, the transition density acquires additional angular structures in the transverse plane. We choose the transverse spin along the x direction and define

$$|N, s_x = 1/2\rangle = \frac{1}{\sqrt{2}} (|\sigma = 1/2\rangle + |\sigma = -1/2\rangle), \quad (40a)$$

$$\begin{aligned} |\Delta, s_x = 1/2\rangle = & \frac{1}{\sqrt{8}} (\sqrt{3}|\sigma = 3/2\rangle + |\sigma = 1/2\rangle \\ & - |\sigma = -1/2\rangle - \sqrt{3}|\sigma = -3/2\rangle), \end{aligned} \quad (40b)$$

For these polarization states, one obtains

$$\begin{aligned} \rho(x, \mathbf{b}; s'_x = 1/2, s_x = 1/2) = & \sqrt{\frac{1}{6}} \rho_0(x, b) + \rho_1^V(x, b) \sin \phi \\ & - \frac{\sqrt{6}}{4} \rho_2(x, b) \cos 2\phi. \end{aligned} \quad (41)$$

The first term is the monopole contribution, while the terms proportional to $\sin \phi$ and $\cos 2\phi$ represent the dipole and quadrupole distortions of the transverse transition density, respectively.

Using a multipole ansatz, we visualize Eqs. (39) and (41) in Fig. 1. This ansatz is intended to highlight the multipole patterns of the distributions rather than their absolute

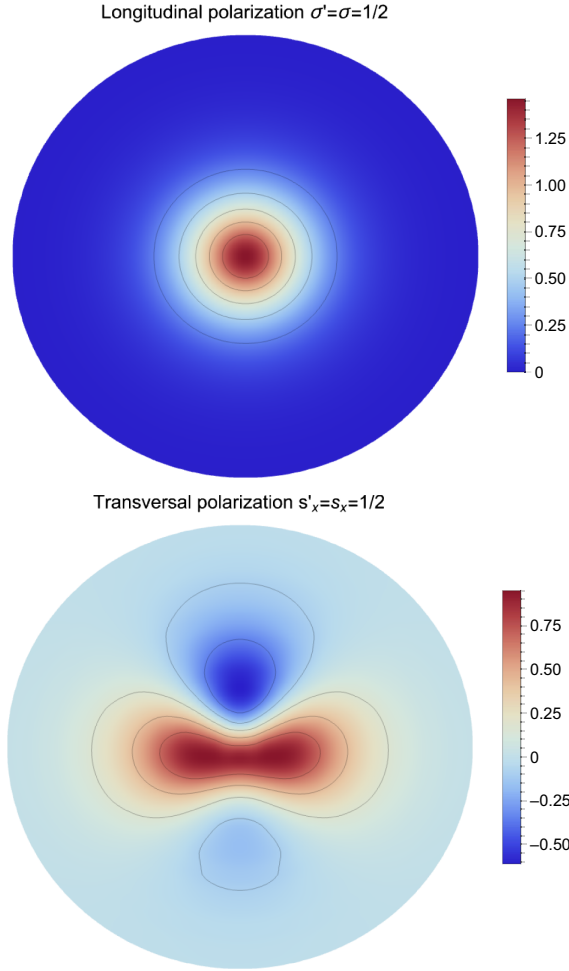


FIG. 1. Visualization of the transition densities when the nucleon and the Δ baryon are longitudinally (upper panel) and transversely (lower panel) polarized. The angular patterns are shown using a simple multipole ansatz.

strengths, which depend on the underlying dynamics and are beyond the scope of the present study. The allowed multipoles follow from spin and symmetry. Their relative strengths and spatial profiles depend on the assumed GPDs. The individual multipole contributions are shown separately in Fig. 2.

C. Interpretation and discussion

The physical meaning of the transition density must be carefully distinguished from that of the diagonal densities of the nucleon and the Δ . The diagonal density, obtained from the $N \rightarrow N$ or $\Delta \rightarrow \Delta$ matrix element at zero skewness, is positive definite in its helicity-conserving component and answers the question of where the quarks are located inside the given baryon. In contrast, the transition density arises from the overlap of two distinct hadronic states, so that it obeys no positivity condition and no number sum rule. Its moments are instead given by the transition form factors, and the structure that survives in the forward limit obeys the

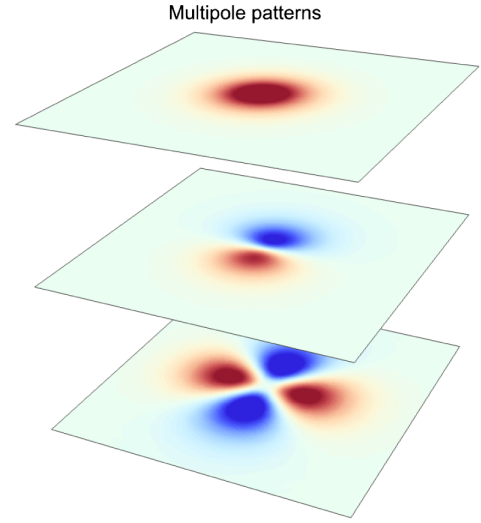


FIG. 2. Impact-parameter distributions of unpolarized quarks for the proton and Δ baryon transversely polarized along the x axis. The distributions exhibit richer multipole patterns, consisting of monopole (upper layer), dipole (middle layer), and quadrupole (lower layer) structures. The relative strengths of the multipoles are set by the ansatz.

zero sum rule in Eq. (11b) imposed by current conservation [34]. It answers a different question, namely, at which transverse position the quark operator converts the nucleon into the Δ . For the same reason, the quadrupole pattern of the transition density should not be identified with the intrinsic deformation of either the nucleon or the Δ alone. It characterizes the spin-angular structure of the $N \rightarrow \Delta$ overlap, whereas the intrinsic deformation of each baryon is encoded in the corresponding diagonal densities. The density interpretation of the forward-limit ($\Delta_\perp = 0$) transition matrix element was clarified in Ref. [34], where the partonic operator acts as a number density in the partonic states while the matrix element connects different hadron states. In the forward limit, the monopole GPD M_0 becomes proportional to G_X and thus to the tensor-polarized transition parton density, whose magnitude was estimated in the mean-field approach and found to be parametrically small [34,35]. The present work extends this distinction to the impact-parameter densities at nonzero transverse momentum transfer, where the dipole and quadrupole structures appear.

A region with a large magnitude corresponds to a transverse position where the quark operator gives a strong contribution to the transition amplitude. Being an amplitude rather than a probability density, the transition density carries a sign that reflects the relative phase of the overlap. Consequently, different regions in the transverse plane may interfere constructively or destructively in the integrated form factor.

The multipole structures encode the spin-angular content of the transition, as visualized in Figs. 1 and 2. When both baryons are longitudinally polarized, only the monopole component contributes, as shown in Eq. (39), so that the

transition density is axially symmetric in the transverse plane (upper panel of Fig. 1). For transverse polarization along the x axis, the azimuthal symmetry is broken according to Eq. (41). The dipole term, proportional to $\sin \phi$, displaces the density perpendicular to the transverse spin direction, whereas the quadrupole term, proportional to $\cos 2\phi$, generates the characteristic elongation of the density, which in the present ansatz is aligned with the spin direction (lower panel of Fig. 1). The separate monopole, dipole, and quadrupole layers are displayed in Fig. 2. These patterns originate from the interference among different helicity amplitudes and are the x -resolved counterparts of the angular structures observed in the transition charge densities at the form-factor level [19].

Finally, the visualization presented in this section is based on a simple multipole ansatz and illustrates the angular patterns of the transition densities rather than their absolute magnitudes. A quantitative description of the multipole GPDs requires dynamical input. First estimates exist in the forward limit, where the tensor-polarized density was computed in the mean-field approach [34,35]. Extending these calculations to nonzero transverse momentum transfer within the χ QSM gives access to the dipole and quadrupole multipole GPDs and is under way.

V. SUMMARY AND CONCLUSIONS

In the present work, we have established the multipole structure of the $N \rightarrow \Delta$ transition at the level of the generalized parton distributions. Evaluating the light-front Rarita-Schwinger and Dirac spinors in the symmetric frame, we performed a multipole expansion of the covariant matrix element in terms of the three-dimensional spin-transition tensors and the transverse momentum transfer. The four covariant transition GPDs were decomposed into one monopole (M_0), two dipole (M_1^V and M_1^Q), and one quadrupole (M_2) components in the transverse plane. Each multipole GPD corresponds to a definite light-front helicity amplitude, the helicity flip of which matches the rank of the transverse multipole tensor. The explicit relations between the multipole and covariant GPDs were also derived.

In the zero-skewness limit, the multipole GPDs define the impact-parameter transition densities, which extend the transverse transition charge densities from the form-factor level to the x -dependent level. For longitudinally polarized states, only the monopole component contributes, whereas for transversely polarized states the dipole and quadrupole components distort the density in the transverse plane. We emphasized that the transition density must be distinguished from the diagonal densities of the nucleon and the Δ . It arises from the overlap of two distinct hadronic states and obeys neither a positivity condition nor a number sum rule. The quadrupole pattern of the transition density thus characterizes the spin-angular structure of the $N \rightarrow \Delta$ overlap rather than the intrinsic deformation of either baryon alone.

To connect these multipoles to experiment, one must calculate the scattering amplitude for a process such as $eN \rightarrow e\gamma\Delta$. This is needed to determine which combinations of GPDs, convoluted with the hard scattering kernels, enter the cross section and asymmetries. We have not carried out this calculation.

The multipole GPDs introduced in the present work provide a model-independent framework for analyzing the partonic structure of the $N \rightarrow \Delta$ transition, which will be probed by hard exclusive reactions at the JLab upgrades and the EIC. A quantitative evaluation of the multipole GPDs within the χ QSM, which naturally realizes the large- N_c spin-flavor symmetry, is under way and will be reported elsewhere.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: MATRIX ELEMENTS OF SPIN-MULTIPOLE OPERATORS

The explicit matrix elements of the spin-multipole structures appearing in the multipole expansion of the nonlocal vector-current matrix element in Eq. (30) are listed below. The rows correspond to $\sigma' = 3/2, 1/2, -1/2, -3/2$ from top to bottom, and the columns to $\sigma = 1/2, -1/2$ from left to right:

$$Q^{33} = \begin{pmatrix} 0 & 0 \\ \sqrt{\frac{2}{3}} & 0 \\ 0 & -\sqrt{\frac{2}{3}} \\ 0 & 0 \end{pmatrix}, \quad (\text{A1a})$$

$$Q^{3i}X_1^i = \begin{pmatrix} -\frac{1}{2\sqrt{2}}e^{-i\theta} & 0 \\ 0 & \frac{1}{2}\sqrt{\frac{3}{2}}e^{-i\theta} \\ \frac{1}{2}\sqrt{\frac{3}{2}}e^{i\theta} & 0 \\ 0 & -\frac{1}{2\sqrt{2}}e^{i\theta} \end{pmatrix}, \quad (\text{A1b})$$

$$i\epsilon^{3kj}V^jX_1^k = \begin{pmatrix} -\frac{\sqrt{3}}{2}e^{-i\theta} & 0 \\ 0 & -\frac{1}{2}e^{-i\theta} \\ -\frac{1}{2}e^{i\theta} & 0 \\ 0 & -\frac{\sqrt{3}}{2}e^{i\theta} \end{pmatrix}, \quad (\text{A1c})$$

$$Q^{ij}X_2^{ij} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}}e^{-2i\theta} \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\sqrt{2}}e^{2i\theta} & 0 \end{pmatrix}. \quad (\text{A1d})$$

APPENDIX B: LIGHT-FRONT RARITA-SCHWINGER AND DIRAC SPINORS

In this Appendix, we collect the light-front (LF) expressions for the Rarita-Schwinger and Dirac spinors. The LF Rarita-Schwinger spinor is obtained by coupling a spin-1 polarization vector to a spin-1/2 Dirac spinor through the Clebsch-Gordan coefficients [37]:

$$u^\alpha(p, \sigma) = \sum_{\lambda s} C_{1\frac{1}{2}s}^{\frac{3}{2}\sigma} \epsilon^\alpha(p, \lambda) u(p, s). \quad (\text{B1})$$

In LF quantization, the Dirac spinor is given by [38–41]

$$u(p, s) = \frac{1}{\sqrt{\sqrt{2}p^+}} (\not{p} + m) \not{\chi}_s, \quad (\text{B2})$$

where $\chi_s = (\phi_s, 0)$ denotes the rest-frame Dirac spinor, with ϕ_s being the two-component spinor. The dimensionless LF vector is defined as $\bar{n} = (1, 0, 0, -1)/\sqrt{2}$, and m denotes the on-shell mass. The explicit representation of Eq. (B2) reads [42,43]

$$u\left(p, +\frac{1}{2}\right) = \frac{1}{\sqrt{2\sqrt{2}p^+}} \begin{pmatrix} \sqrt{2}p^+ + m \\ p_R \\ \sqrt{2}p^+ - m \\ p_R \end{pmatrix}, \quad (\text{B3})$$

$$u\left(p, -\frac{1}{2}\right) = \frac{1}{\sqrt{2\sqrt{2}p^+}} \begin{pmatrix} -p_L \\ \sqrt{2}p^+ + m \\ p_L \\ -\sqrt{2}p^+ + m \end{pmatrix}, \quad (\text{B4})$$

where $p_{R,L} = p^x \pm ip^y$. The spin-1 polarization vector is given by

$$\epsilon^\alpha(p, \lambda = \pm 1) = \left(0, \mp \frac{1}{\sqrt{2}} \frac{p_{R,L}}{p^+}, \hat{e}_{\perp \pm 1}\right), \quad (\text{B5a})$$

$$\epsilon^\alpha(p, \lambda = 0) = \left(\frac{p^+}{m}, \frac{p_\perp^2 - m^2}{2mp^+}, \frac{p_\perp}{m}\right). \quad (\text{B5b})$$

The transverse components of the rest-frame polarization vectors $\hat{e}_\lambda$ and momentum $\mathbf{p}$ are given by

$$\hat{e}_{\perp \pm 1} = \mp \sqrt{\frac{1}{2}}(1, \pm i), \quad p_\perp^i = (p^x, p^y). \quad (\text{B6})$$

With these conventions, Eq. (B1) gives the explicit LF Rarita-Schwinger spinor.

APPENDIX C: LIGHT-FRONT SPINOR BILINEARS

In this Appendix, we present the calculation of the bilinear matrix elements constructed from the LF Rarita-Schwinger and Dirac spinors for each spin projection, using the conventions summarized in Appendix B. In the symmetric frame discussed in Sec. III C, the Dirac bilinears can be expressed as

$$\bar{u}(p', s') \not{\epsilon}_5 u(p, s) = 2\sqrt{1 - \xi^2} \sigma^3, \quad (\text{C1a})$$

$$\bar{u}(p', s') \gamma_5 u(p, s) = \frac{1}{\sqrt{1 - \xi^2}} (\kappa \sigma^3 - \Delta_\perp^j \sigma^j), \quad (\text{C1b})$$

where the spin indices are suppressed, with

$$\sigma^i \equiv \phi_s^\dagger \sigma^i \phi_s. \quad (\text{C2})$$

The kinematic factor κ in Eq. (C1b) is defined as

$$\kappa = m_\Delta - m_N + \xi(m_\Delta + m_N). \quad (\text{C3})$$

For the spin-1 polarization vector appearing in the final-state Rarita-Schwinger spinor, it is useful to record the following relations in the symmetric frame at nonzero ξ . We have

$$\epsilon^{+*}(p', \lambda') = \frac{P^+(1 - \xi)}{m_\Delta} \delta_{\lambda'0}, \quad (\text{C4a})$$

$$\epsilon^*(p', \lambda') \cdot \Delta = a_1 \delta_{\lambda'0} + a_2 \hat{\epsilon}_{\perp \lambda'}^* \cdot \Delta_{\perp}, \quad (\text{C4b})$$

where

$$a_1 = \frac{P^+(1-\xi)}{m_{\Delta}} \Delta^- - \frac{\xi(\Delta_{\perp}^2 - 4m_{\Delta}^2)}{4(1-\xi)m_{\Delta}} - \frac{\Delta_{\perp}^2}{2m_{\Delta}}, \quad (\text{C5a})$$

$$a_2 = -\frac{1}{1-\xi}. \quad (\text{C5b})$$

Using the notation in Eqs. (C1)–(C5), the matrix elements of the spinor bilinears are derived as

$$\bar{u}^{\alpha}(p', \sigma') \Delta_{\alpha} \not{n} \gamma_5 u(p, \sigma) = 2\sqrt{1-\xi^2} \left[a_1 Q^{33} + a_2 Q^{3k} \Delta_{\perp}^k + \frac{a_2}{2} \sqrt{\frac{2}{3}} i \epsilon^{3kj} V^j \Delta_{\perp}^k \right], \quad (\text{C6a})$$

$$\bar{u}^{\alpha}(p', \sigma') \Delta_{\alpha} \gamma_5 u(p, \sigma) = \frac{1}{\sqrt{1-\xi^2}} \left[\kappa a_1 Q^{33} + (\kappa a_2 - a_1) Q^{3k} \Delta_{\perp}^k + \sqrt{\frac{2}{3}} \frac{\kappa a_2 + a_1}{2} i \epsilon^{3kj} V^j \Delta_{\perp}^k - a_2 Q^{lk} \Delta_{\perp}^l \Delta_{\perp}^k \right], \quad (\text{C6b})$$

$$\bar{u}^{\alpha}(p', \sigma') n_{\alpha} \gamma_5 u(p, \sigma) = \frac{1-\xi}{m_{\Delta} \sqrt{1-\xi^2}} \left[\kappa Q^{33} - Q^{3k} \Delta_{\perp}^k + \frac{1}{2} \sqrt{\frac{2}{3}} i \epsilon^{3kj} V^j \Delta_{\perp}^k \right], \quad (\text{C6c})$$

$$\bar{u}^{\alpha}(p', \sigma') n_{\alpha} \not{n} \gamma_5 u(p, \sigma) = \frac{2(1-\xi)}{m_{\Delta}} \sqrt{1-\xi^2} Q^{33}. \quad (\text{C6d})$$

APPENDIX D: RELATIONS BETWEEN COVARIANT AND MULTIPOLE GPDS

Matching the covariant GPD parametrization to the multipole expansion of the nonlocal vector-current matrix element, we obtain the following relations between the multipole GPDs and the covariant GPDs:

$$M_0(x, \xi, t) = \frac{2\sqrt{2}}{\sqrt{3}\sqrt{1-\xi^2}} \left[\left\{ \frac{(1-\xi^2)}{m_N} a_1 - \frac{(1-\xi)\bar{m}}{m_N m_{\Delta}} \kappa \right\} G_1 + \left\{ \frac{\kappa a_1}{m_N^2} - \frac{\delta_{N\Delta} \bar{m}(1-\xi)}{m_N^2 m_{\Delta}} \kappa + \frac{|\Delta_{\perp}|^2}{2m_N^2} a_2 \right\} G_2 \right. \\ \left. - \left\{ \frac{\xi \kappa a_1}{m_N^2} + \frac{(1-\xi)t}{2m_N^2 m_{\Delta}} \kappa + \frac{\xi |\Delta_{\perp}|^2}{2m_N^2} a_2 \right\} G_3 + \left\{ \frac{m_N}{m_{\Delta}} (1-\xi)(1-\xi^2) \right\} G_X \right], \quad (\text{D1a})$$

$$M_1^Q(x, \xi, t) = \frac{2\sqrt{2}}{\sqrt{3}\sqrt{1-\xi^2}} \left[\left\{ (1-\xi^2) a_2 + \frac{(1-\xi)\bar{m}}{m_{\Delta}} \right\} G_1 + \left\{ \frac{\kappa a_2 - a_1}{m_N} + \frac{\delta_{N\Delta} \bar{m}(1-\xi)}{m_N m_{\Delta}} \right\} G_2 \right. \\ \left. - \left\{ \xi \frac{\kappa a_2 - a_1}{m_N} - \frac{(1-\xi)t}{2m_N m_{\Delta}} \right\} G_3 \right], \quad (\text{D1b})$$

$$M_1^V(x, \xi, t) = -\frac{2}{3\sqrt{1-\xi^2}} \left[\left\{ (1-\xi^2) a_2 - \frac{(1-\xi)\bar{m}}{m_{\Delta}} \right\} G_1 + \left\{ \frac{\kappa a_2 + a_1}{m_N} - \frac{\delta_{N\Delta} \bar{m}(1-\xi)}{m_N m_{\Delta}} \right\} G_2 \right. \\ \left. - \left\{ \xi \frac{\kappa a_2 + a_1}{m_N} + \frac{(1-\xi)t}{2m_N m_{\Delta}} \right\} G_3 \right], \quad (\text{D1c})$$

$$M_2(x, \xi, t) = -\frac{2\sqrt{2}}{\sqrt{3}\sqrt{1-\xi^2}} [a_2 G_2 - \xi a_2 G_3], \quad (\text{D1d})$$

where $\bar{m} = (m_{\Delta} + m_N)/2$ and $\delta_{N\Delta} = m_{\Delta} - m_N$.

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