

Charmonium-like exotic hadron productions in e^+e^- collisions at the BESIII energy with the PACIAE model

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Inspired by the BESIII observation of exotic hadron $G(3900)$ in the $e^+e^- \rightarrow D\bar{D}$ process [M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. Lett.* **133**, 081901 (2024)], we use the parton and hadron cascade model PACIAE to simulate the productions of charmonium-like exotic hadrons, including $X(3872)$, $Z_c(3900)^0$, and $G(3900)$ in the e^+e^- annihilations at $\sqrt{s} = 4.95$ GeV. The charmonium-like candidates are recombined by the dynamically constrained phase-space coalescence model using component mesons $D\bar{D}$ [for $G(3900)$] or $D\bar{D}^*/\bar{D}D^*$ [for $X(3872)$, $Z_c(3900)^0$, and $G(3900)$] in the PACIAE simulated final hadronic state. We then calculate, for the first time, the charmonium-like candidate's orbital angular momentum quantum number in its rest frame and perform the spectral classification for each of the above candidates. For $G(3900)$, as its J^{PC} is 1^{--} , it is identified as the P -wave $D\bar{D}$ or $D\bar{D}^*/\bar{D}D^*$ state. Meanwhile, for $X(3872)$ and $Z_c(3900)^0$, as their J^{PC} s are, respectively, 1^{++} and 1^{+-} , they are identified as the S -wave $D\bar{D}^*/\bar{D}D^*$ candidates. It is observed that the yield of the P -wave $G(3900)$ composed of $D\bar{D}^*/\bar{D}D^*$ is significantly larger than that composed of $D\bar{D}$. Moreover, in the $D\bar{D}^*/\bar{D}D^*$ channel, the yield of the P -wave $G(3900)$ is around three times as large as the yield of the S -wave $Z_c(3900)^0$, and it is around two orders of magnitude as large as the yield of the S -wave $X(3872)$. Finally, significant discrepancies are observed in the transverse momentum spectra and the rapidity distributions among the $X(3872)$, $Z_c(3900)^0$, and $G(3900)$ states. These discrepancies are proposed as valuable criteria for identifying the different charmonium-like exotic hadrons from each other.

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I. INTRODUCTION

The exotic hadrons (unusual hadrons) containing more than three quarks/antiquarks are allowed and expected by the quark model and the quantum chromodynamics (QCD) [1,2]. The first exotic hadron $X(3872)$ was observed in e^+e^- collisions by the Belle Collaboration in 2003 [3]. Since then, more and more experimental measurements and theoretical explanations about the exotic hadrons have been published. There are experimental studies from the

BABAR [4], the Belle [5–7], the BESIII [8–11], and the LHCb [12] Collaborations. The theoretical methods include the lattice QCD (LQCD) [13–17], the QCD sum rule [18–20], the Schrödinger equation Gaussian expansion method [21], the quasi three-body unitary model [22], and our phenomenological model, the Monte Carlo simulation method [23]. We refer to the review papers in Refs. [24–29].

Our phenomenological model Monte Carlo simulation method is composed of the parton and hadron cascade model PACIAE [30,31] and the quantum statistical mechanics based on the dynamically constrained phase-space coalescence model (DCPC) [32], i.e., PACIAE + DCPC. It has been successfully employed in describing the formations of exotic hadrons in the e^+e^- annihilations at BESIII energies and in the proton-proton (pp) collisions at LHC energies [23,33–36]. Among our method, the model of the LQCD [13–17], the QCD sum rule [18–20], and the Schrödinger equation Gaussian expansion

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method [21], etc., each has its advantages and disadvantages. They are mutually nonexclusive and even indispensable mutually. All of them should be studied together with the experiments to further reveal the nature of the exotic hadrons.

After the *BABAR* and Belle Collaborations observed a peak around 3.9 GeV in $e^+e^- \rightarrow D\bar{D}/D\bar{D}^*$ and interpreted it as a $G(3900)$ candidate [4,6], the BESIII Collaboration has also recently reported its observation of the $G(3900)$ peak in the $e^+e^- \rightarrow D^0\bar{D}^0/D^+D^-$ processes [11]. Meanwhile, its structure has been identified as the P -wave $D\bar{D}^*/\bar{D}D^*$ resonance in Ref. [37]. The Belle Collaboration had measured the mass and the decay width of $X(3872)$ in the decay channel of $X(3872) \rightarrow D^{*0}\bar{D}^0$ [5]. The quantum numbers J^{PC} of $X(3872)$ was determined as 1^{++} by the LHCb Collaboration [38]. The $Z_c(3900)^0$ was first observed by the BESIII Collaboration in the decay channel of $Z_c(3900)^0 \rightarrow \pi^0 J/\psi$ [9]. Later, its J^{PC} was determined as 1^{+-} by this collaboration [37,39].

In this paper, we use the PACIAE model to simulate the productions of the charmonium-like exotic hadrons, including $X(3872)$, $Z_c(3900)^0$, and $G(3900)$ in the e^+e^- annihilations at the BESIII upper energy of $\sqrt{s} = 4.95$ GeV. The charmonium-like candidate is then recombined by the DCPC model using constituent mesons of $D\bar{D}$ or $D\bar{D}^*/\bar{D}D^*$ in the PACIAE model's simulated final hadronic state. Here, $D\bar{D}$ refers to $D^0\bar{D}^0$ or D^+D^- , while $D\bar{D}^*/\bar{D}D^*$ refers to $D^0\bar{D}^{*0}/\bar{D}^0D^{*0}$ or D^+D^{*-}/D^-D^{*+} . For $G(3900)$, its component mesons could be $D\bar{D}$ or $D\bar{D}^*/\bar{D}D^*$. However, for $X(3872)$ and $Z_c(3900)^0$, their component mesons could be only the $D\bar{D}^*/\bar{D}D^*$, as they are not observed experimentally in the $D\bar{D}$ final state [3,5,9,10,39]. We then calculate the charmonium-like candidate's orbital angular momentum quantum number in its rest frame for the first time. The spectral classification is performed for each of the above charmonium-like candidates according to the spectroscopic notation of $^{2S+1}L_J$ [40], where S , L , and J are, respectively, the spin, the orbital angular momentum, and the total angular momentum (physical spin) quantum numbers of the charmonium-like candidates. The wave shapes of $S, P, D, \dots$ are just corresponding to the $L = 0, 1, 2, \dots$, respectively.

For $G(3900)$, as its J^{PC} is 1^{--} [37], it is identified as the P -wave $D\bar{D}$ or $D\bar{D}^*/\bar{D}D^*$ state. Moreover, for $X(3872)$ and $Z_c(3900)^0$, as their J^{PC} s are, respectively, 1^{++} and 1^{+-} [37–39], they are identified as the S -wave $D\bar{D}^*/\bar{D}D^*$ state. We will compare the yields, the transverse momentum spectra, and the rapidity distributions among the P -wave $D\bar{D}$ or $D\bar{D}^*/\bar{D}D^*$ state $G(3900)$ and the S -wave $D\bar{D}^*/\bar{D}D^*$ states $X(3872)$ and $Z_c(3900)^0$. These discrepancies will provide valuable criteria for identifying the different charmonium-like exotic hadrons from each other.

II. MODEL

In the PACIAE model, the simulation of e^+e^- annihilation is first executed by PYTHIA6/PYTHIA8 [41,42] with presetting of temporarily turning off string fragmentation and with postsetting of breaking up diquark/antidiquark randomly. The partonic initial state is available after partons are randomly assigned on the surface of the sphere with radius of 1 fm. Then, this partonic initial state proceeds $2 \rightarrow 2$ partonic rescattering. Here, the lowest order perturbative quantum chromodynamics (pQCD) cross section is introduced for the partonic rescattering processes. For the cases involving heavy quarks, such as $qc \rightarrow qc$ and $gc \rightarrow gc$, the effect of the heavy quark mass has been considered in the cross section calculation [43]. The final partonic state is reached after the partonic rescattering. Third, the final partonic state is hadronized by the Lund string fragmentation regime and/or the Monte Carlo coalescence model. It results an intermediately final hadronic state. Finally, this intermediate final hadronic state goes through the hadronic rescattering and the kinetic freeze-out happens, resulting in a final hadronic state (FHS). Here, the considered $2 \rightarrow 2$ hadronic rescattering processes are taken from Ref. [44] with an empirical cross section [30].

The DCPC model is based on the quantum statistical mechanics [45,46]. In this model, the yield of the N -particle cluster reads

$$Y_N = \int \cdots \int_{E_\alpha \leq E \leq E_\beta} \delta_{12 \cdots N} \frac{d\mathbf{x}_1 d\mathbf{p}_1 \cdots d\mathbf{x}_N d\mathbf{p}_N}{h^{3N}}, \quad (1)$$

where the E_α and E_β are the cluster's lower and upper energy thresholds. The $\mathbf{x}_i$ and $\mathbf{p}_i$ are, respectively, the i th particle three-coordinate and three-momentum in the center-of-mass system (cms) of the e^+e^- collision. It is assumed that if the cluster exists naturally, certain dynamical constraints (the component constraint, the coordinate constraint, and the momentum constraint) should be satisfied.

Taking a charmonium-like candidate (i.e., the cluster mentioned above) composed of D^0 and $\bar{D}^{*0}$ (i.e., the component hadrons mentioned above) as an example, the dynamical constraint is then

$$\delta_{12} = \begin{cases} 1, & \text{if } 1 \equiv D^0, 2 \equiv \bar{D}^{*0}, R_i \leq R_0, \text{ and} \\ & m_0 - \Delta m \leq m_{\text{inv}} \leq m_0 + \Delta m \\ 0, & \text{otherwise} \end{cases}, \quad (2)$$

where m_0 refers to the mass of the charmonium-like candidate, Δm is the mass uncertainty (a free parameter), which is estimated as the half decay width of the charmonium-like candidate. Table I lists m_0 and Δm for the charmonium-like exotic hadrons $X(3872)$, $Z_c(3900)^0$, and $G(3900)$ [5,11,39]. R_0 and R_i ($i = 1, 2$) are, respectively, the radius of the charmonium-like cluster (a free

TABLE I. m_0 and Δm for the charmonium-like exotic hadrons $X(3872)$, $Z_c(3900)^0$, and $G(3900)$.

	$X(3872)$	$Z_c(3900)^0$	$G(3900)$
m_0 (MeV/c ²)	3872.9	3893.0	3872.5
Δm (MeV/c ²)	1.95	22.1	89.85

parameter) and the position vector length of the component meson i in the rest frame of the cluster. In this frame, the three-coordinate of the constituent meson (D^0 and $\bar{D}^{*0}$) is denoted as $\mathbf{x}_i^*$. It is obtained by first Lorentz transforming $\mathbf{x}_i$ to the cms of the constituent mesons and then propagating the earlier freeze-out component meson freely to the freeze-out time of the later component meson [47,48]. R_0 is assumed to be in the range of $1 \text{ fm} < R_0 < 10 \text{ fm}$ as the relative distances between two component mesons should be larger than the sum of the radius of two component mesons and less than the interaction range of 20 fm [49,50]. The invariant mass (m_{inv}) reads

$$m_{\text{inv}} = \sqrt{\left(\sum_{i=1}^2 E_i\right)^2 - \left(\sum_{i=1}^2 \mathbf{p}_i\right)^2}, \quad (3)$$

where E_i and $\mathbf{p}_i$ ($i = 1, 2$) are the component meson's (D^0 and $\bar{D}^{*0}$) energy and three-momentum in the cms of the e^+e^- collisions, respectively.

This PACIAE + DCPC model has been successfully applied to calculate the yield of the exotic states, such as the $X(3872)$ [23,33,34], the $Z_c^\pm(3900)$ [51], the $P_c(4312)$, the $P_c(4440)$, the $P_c(4457)$ [35], the Ω_c^0 [52], and the $X(2370)$ [36] based on the PACIAE simulated partonic or hadronic final states.

To generate the charmonium-like candidate composed of $D\bar{D}$ for instance, a component meson list consists of D^+ , D^- , D^0 , and $\bar{D}^0$ taken from the PACIAE model simulated FHS should be first constructed. Then, a two-layer cycle over the component mesons in the list is then implemented. Each combination in the cycle, if it is D^+D^- or $D^0\bar{D}^0$ and satisfies the corresponding constraints in Eq. (2), counts as a charmonium-like candidate. The meson list is then updated by removing the used meson. A new two-layer cycle is executed on the updated list. Repeat these steps until the list is empty or the rest in the list is unable to generate a charmonium-like candidate. The production of the charmonium-like candidate composed of $D\bar{D}^*/\bar{D}D^*$ is done in a similar way.

In the charmonium-like candidate rest frame, its orbital angular momentum (OAM) $\mathbf{l}^*$ is the sum over the OAMs of its constituent mesons. This is expressed as

$$\mathbf{l}^* = \mathbf{x}_1^* \times \mathbf{p}_1^* + \mathbf{x}_2^* \times \mathbf{p}_2^*, \quad (4)$$

where $\mathbf{p}_i^*$ ($i = 1, 2$) is the three-momentum of the constituent meson. It is obtained by Lorentz transforming $\mathbf{p}_i$ to

the cms of the constituent mesons (i.e., the charmonium-like candidate rest frame). According to quantum mechanics, the OAM should be quantized by

$$l^{*2} = L(L+1)\hbar^2, \quad (5)$$

where $\hbar$ is the reduced Planck constant, and L is the orbital angular momentum quantum number of the charmonium-like candidate. As L should be an integer, the solution of Eq. (5) reads

$$L = \text{round}\left(\frac{-1 + \sqrt{1 + 4l^{*2}/\hbar^2}}{2}\right), \quad (6)$$

where the function $\text{round}(X)$ returns X 's nearest integer. Such a quantization rule is denoted as a quantum axiom in Ref. [53].

For the charmonium-like candidate composed of $D\bar{D}$, its parity is $P = P_D * P_{\bar{D}} * (-1)^L = (-1)^L$, and its C -parity is $C = (-1)^{L+S}$, where S is the total spin of D and $\bar{D}$ [40]. For the charmonium-like candidate composed of $D\bar{D}^*/\bar{D}D^*$, its parity is $P = P_{D(\bar{D})} * P_{\bar{D}^*(D^*)}(-1)^L = (-1)^L$. Since $D\bar{D}^*$ is not an eigenstate of the charge conjugate operator C , the C -parity is defined for the symmetric/antisymmetric combination $(D\bar{D}^* \pm \bar{D}D^*)/\sqrt{2}$. For the symmetric (antisymmetric) combination, its C -parity is $-1(+1)$ [37]. Having the spin S and the orbital angular momentum quantum number L , we are able to determine the charmonium-like state's quantum number J^{PC} , where $J = |L - S|, \dots, L + S$. For the general exotic hadron, its quantum number J^{PC} has to be known by the five-dimensional angular correlation and/or the cross section angular distribution analysis [54,55].

III. RESULTS AND DISCUSSIONS

The PACIAE 4.0 model [31] is used to simulate the e^+e^- collisions at $\sqrt{s} = 4.95 \text{ GeV}$. A total of 500 million e^+e^- collision events are generated with the default model parameters. Then, the charmonium-like candidates are recombined by the DCPC model with component mesons of $D\bar{D}$ or $D\bar{D}^*/\bar{D}D^*$ in the PACIAE simulated FHS. According to the method in the end of the "Model" section, the J^{PC} s for the charmonium-like S -, P -, and D -wave candidates composed of $D\bar{D}$ are, respectively, 0^{++} , 1^{--} , and 2^{++} . Moreover, the J^{PC} s for the S -, P -, and D -wave charmonium-like candidates composed of $D\bar{D}^*/\bar{D}D^*$ are, respectively, $1^{+\pm}$, $(0, 1, 2)^{-\pm}$, and $(1, 2, 3)^{+\pm}$. For $G(3900)$, as its J^{PC} is 1^{--} [37], it is identified as the P -wave $D\bar{D}$ or $D\bar{D}^*/\bar{D}D^*$ state. Meanwhile, for $X(3872)$ and $Z_c(3900)^0$, as their J^{PC} s are, respectively, 1^{++} and 1^{+-} [37–39], they are identified as the S -wave $D\bar{D}^*/\bar{D}D^*$ state. The event averaged yields of the P -wave $D\bar{D}$ and $D\bar{D}^*/\bar{D}D^*$ state $G(3900)$, as well as the S -wave $D\bar{D}^*/\bar{D}D^*$ states $X(3872)$ and $Z_c(3900)^0$, are tabulated in Table II. It is observed that the yield of the P -wave

TABLE II. The PACIAE + DCPC model simulated event averaged yield of the P -wave $D\bar{D}$ and $D\bar{D}^*/\bar{D}D^*$ state $G(3900)$, as well as the S -wave $D\bar{D}^*/\bar{D}D^*$ states $X(3872)$ and $Z_c(3900)^0$ in e^+e^- collisions at $\sqrt{s} = 4.95$ GeV.

	P -wave $D\bar{D}$	P -wave $D\bar{D}^*/\bar{D}D^*$	S -wave $D\bar{D}^*/\bar{D}D^*$	S -wave $D\bar{D}^*/\bar{D}D^*$
	$G(3900)$	$G(3900)$	$X(3872)$	$Z_c(3900)^0$
Yield	3.87×10^{-4}	6.08×10^{-4}	5.81×10^{-6}	1.97×10^{-4}

$G(3900)$ composed of $D\bar{D}^*/\bar{D}D^*$ is significantly larger than that composed of $D\bar{D}$. Moreover, in the $D\bar{D}^*/\bar{D}D^*$ channel, the yield of the P -wave $G(3900)$ is around three times as large as the yield of the S -wave $Z_c(3900)^0$, and it is around two orders of magnitude as large as the yield of the S -wave $X(3872)$.

We have also investigated the effect of varying the coalescence radius R_0 on the yield of the charmonium-like candidates. With increasing R_0 from 1 to 10 fm, the yields of the charmonium-like candidates first increase and then saturate at $R_0 \approx 2.5$ fm. Moreover, in order to study the impact of different methods for extracting the value of L for the charmonium-like candidates, another function “trunc(X)” is utilized in Eq. (6). It truncates the number X to an integer by removing the fractional part of X . With the change of the method from “round(X)” to “trunc(X)”, the yields of the P -wave $D\bar{D}$ or $D\bar{D}^*/\bar{D}D^*$ state $G(3900)$ and the S -wave $D\bar{D}^*/\bar{D}D^*$ state $Z_c(3900)^0$ will change. However, the yield of the S -wave $D\bar{D}^*/\bar{D}D^*$ state $X(3872)$ remains the same. Meanwhile, the hierarchy for the yields of the charmonium-like exotic hadrons composed of

$D\bar{D}^*/\bar{D}D^*$ does not change, i.e., $\text{yield}(G(3900)) > \text{yield}(Z_c(3900)^0) > \text{yield}(X(3872))$.

The upper panels in Figs. 1 and 2 show, respectively, the simulated y and p_T single-differential distributions for the P -wave $D\bar{D}$ (solid triangles up) and $D\bar{D}^*/\bar{D}D^*$ (empty triangles up) state $G(3900)$, as well as the S -wave $D\bar{D}^*/\bar{D}D^*$ states $X(3872)$ (empty circles) and $Z_c(3900)^0$ (empty triangles down) in e^+e^- collisions at $\sqrt{s} = 4.95$ GeV. In the upper panels of Fig. 1, there is a peak in both the first bin of $[-0.25, -0.2]$ and the last bin of $[0.2, 0.25]$. Moreover, in the upper panels of Fig. 2, there is a peak in the first p_T bin of $0-0.1$ GeV/c. This is because among our simulated final state events of e^+e^- annihilations, aimed on the $D\bar{D}$ production for instance, there are not only two-hadron events of $D + \bar{D}$ but also more than two-hadron events of $D + \bar{D} + \pi^+ + \pi^-$. In the former events, the transverse momentum of D is just opposite to that of $\bar{D}$ leading to the zero transverse momentum of the charmonium-like candidate if the final state photons are neglected, which is responsible for the peaks mentioned in the above rapidity and transverse momentum distributions. The latter events are responsible for the rest of the p_T distribution. So far, there is no experimental results of the $X(3872)$, $Z_c(3900)^0$, and $G(3900)$ productions in the $e^+ + e^- \rightarrow \pi^+ + \pi^- + D + \bar{D}$ and $e^+ + e^- \rightarrow \pi^+ + \pi^- + D + \bar{D}^*$ [56]. Thus, our results could serve as a prediction of these exotic hadron productions in the e^+e^- annihilation processes with four-hadron final states containing a pair of charmed mesons.

In both y and p_T single-differential distributions, obvious discrepancies are observed among the P -wave

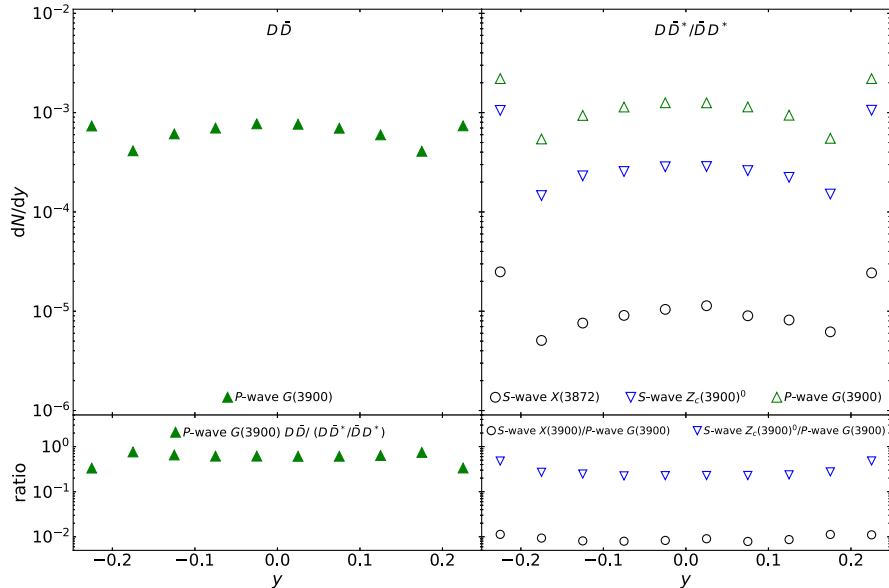


FIG. 1. Upper panels: the simulated y single-differential distributions of the P -wave $D\bar{D}$ (solid triangles up) and $D\bar{D}^*/\bar{D}D^*$ (empty triangles up) state $G(3900)$, as well as the S -wave $D\bar{D}^*/\bar{D}D^*$ states $X(3872)$ (empty circles) and $Z_c(3900)^0$ (empty triangles down) in e^+e^- collisions at $\sqrt{s} = 4.95$ GeV. Lower panels: the ratio between two distributions denoted by the legend.

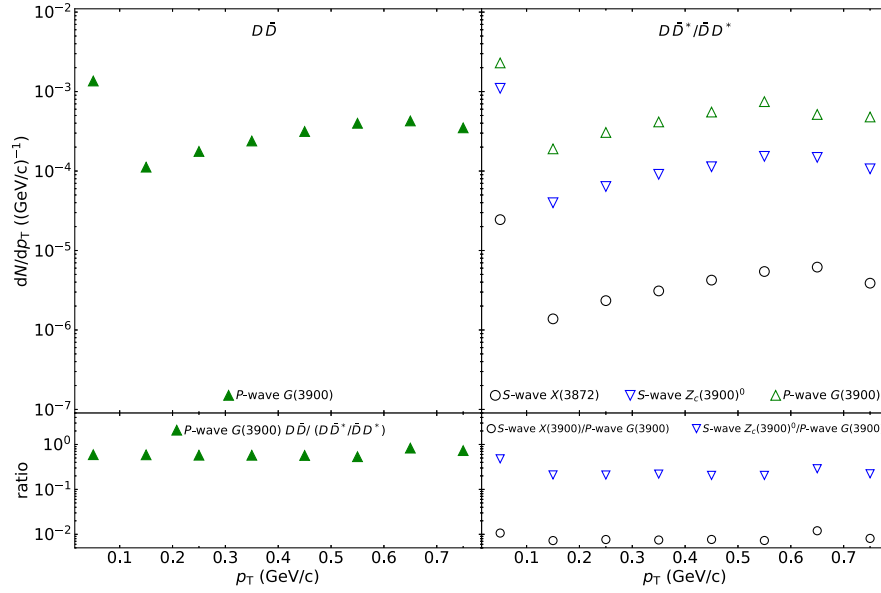


FIG. 2. Upper panels: the simulated p_T single-differential distributions of the P -wave $D\bar{D}$ (solid triangles up) and $D\bar{D}^*/\bar{D}D^*$ (empty triangles up) state $G(3900)$, as well as the S -wave $D\bar{D}^*/\bar{D}D^*$ states $X(3872)$ (empty circles) and $Z_c(3900)^0$ (empty triangles down) in e^+e^- collisions at $\sqrt{s} = 4.95$ GeV. Lower panels: the ratio between the two distributions denoted by the legend.

$D\bar{D}$ or $D\bar{D}^*/\bar{D}D^*$ state $G(3900)$ and the S -wave $D\bar{D}^*/\bar{D}D^*$ states $X(3872)$ and $Z_c(3900)^0$. Since the mass of $G(3900)$, $X(3872)$, and $Z_c(3900)^0$ are very close, it is not easy to distinguish them experimentally from the $D\bar{D}^*/\bar{D}D^*$ channel. These discrepancies could serve as effective criteria to identify the different charmonium-like exotic hadrons from each other. We strongly suggest the experimental measurement of the γ and p_T single-differential distributions of the charmonium-like candidates in the e^+e^- annihilations at the BESIII energies and comparing them with our simulated results.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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