

Dilaton effective field theory across the conformal edge

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Dilaton effective field theory can be employed to analyze lattice data in gauge theories that lie in close proximity of the lower edge of the conformal window. Under special conditions, we show that it can be used as a diagnostic tool to distinguish near-conformal, yet confining, theories from infrared conformal ones. We demonstrate this efficacy by analyzing two sets of lattice measurements taken from the literature. For the $SU(3)$ theory coupled to $N_f = 8$ Dirac fermions transforming in the fundamental representation, our analysis favors confinement. For the $SU(2)$ theory with $N_f = 1$ adjoint fermion, our fits favor infrared conformal behavior. We discuss future lattice measurements, and analysis refinements, that can further test this framework.

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I. INTRODUCTION

In four dimensions, asymptotically free gauge theories are realized in different zero-temperature phases, depending on the type and number N_f of light matter fields included. In the absence of matter fields, Yang-Mills theories confine, and the presence of few fermions does not alter qualitatively this feature, except for introducing continuous global symmetries and their spontaneous breaking. Conversely, for a large enough number of fermions, long distance physics is governed by the appearance of an infrared fixed point, with its associated conformal invariance. This can be shown perturbatively, when N_f is close to its upper bound insuring asymptotic freedom [1]. The conformal behavior is expected to persist over a finite range of $N_f > N_{fc}$, with the lower bound defining the edge of the conformal window. The determination of N_{fc} , and the characterization of the theory near it, is an interesting strong-coupling problem.

Lattice field theory is the natural instrument to study the problem, and in recent years it has led to significant

advances—see the review in Ref. [2] and references therein, as well as the more recent Refs. [3–7]. These studies explore a finite range of fermion masses that suppress infrared sensitivity to the lattice volume, and yield a discrete spectrum of bound states, as expected in confining theories. Some of these studies have reported evidence for the existence of a relatively light, flavor-singlet particle, together with a set of similarly light pseudoscalar particles, separated in mass from heavier bound states [7–28].

In a gauge theory outside of the conformal window, the pseudoscalars are naturally interpreted as pseudo-Nambu-Goldstone Bosons (pNGBs), associated with the spontaneous breaking of an approximate internal global symmetry acting on the matter fields. With the light flavor-singlet interpreted as a dilaton associated with the spontaneous breaking of approximate dilatation symmetry, the behavior of a gauge theory taken to be just below the edge of the window has been successfully described employing dilaton effective field theory (dEFT) [29–41]. See also the review in Ref. [42] and references therein, including the precursors in Refs. [43,44], the early study in the context of electroweak symmetry breaking [45], as well as recent phenomenological applications in Refs. [46–52], and theoretical developments in Refs. [53–58].

In this paper, we expand on this idea, noting that the same dEFT can also be used just inside the conformal window, in the presence of a finite fermion mass. The dilaton and pNGBs continue to exist there as relatively light states. Therefore, dEFT can be employed to explore near-edge gauge theories without an inside-versus-outside bias,

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and hence used as a diagnostic tool to discriminate between confining and conformal behavior. We test this framework by applying it to two gauge theories in proximity to the lower edge of the conformal window. We start with the well studied $SU(3)$ gauge theory with $N_f = 8$ Dirac fermions in the fundamental representation [17,18], finding, as assumed in Refs. [29–41], that it is outside the conformal window. We then analyze the $SU(2)$ gauge theory coupled to $N_f = 1$ Dirac fermion transforming in the adjoint representation [7,28], finding evidence that it is inside.

In Sec. II, we discuss the general framework. We then implement it to fit the lattice data for the above two gauge theories in Sec. III. In Sec. IV, we summarize our results and discuss future directions. We relegate to the Appendix some technical details about our fitting procedure, and tables reproducing the data we take from Refs. [17,59] and [28,60,61].

II. DILATON EFFECTIVE FIELD THEORY

In the dEFT Lagrangian density, the canonically normalized dilaton field χ has a potential $V(\chi)$ and couplings to the pNGBs:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - V(\chi) + \frac{P}{4} \chi^2 \text{Tr}[\partial_\mu \Sigma (\partial^\mu \Sigma)^\dagger] + \frac{R}{4} \chi^y \text{Tr}[\Sigma X^\dagger + X \Sigma^\dagger]. \quad (1)$$

The matrix field Σ describes the pNGBs. The factor χ^2 multiplying their kinetic term acts as a conformal compensator, and P is a positive dimensionless parameter to be fit to lattice data. The final term describes explicit breaking of the internal global symmetry arising from the finite mass m assigned to the fermions in the underlying gauge theory. The χ dependence of the terms in Eq. (1) follows from a spurion analysis, explained in Refs. [39,42], where higher-order EFT corrections, including both loop effects and additional operators in the Lagrangian, are also discussed. Here we work at leading order in the EFT expansion, using the Lagrangian in Eq. (1) at the tree level.

The form of Σ and the parameter matrix X are dictated by the internal symmetry and the appropriate breaking pattern. In the case of gauge theories with matter in complex representations (as with the $SU(3)$, $N_f = 8$ theory), Σ transforms on the left and right according to the $SU(N_f)_L \times SU(N_f)_R$ symmetry, and satisfies the non-linear constraint $\Sigma \Sigma^\dagger = \mathbb{1}_{N_f \times N_f}$, while $X = \mathbb{1}_{N_f \times N_f}$.

For theories with matter in real representations, the global symmetry is enhanced to $SU(2N_f)$, and the pNGBs span the $SU(2N_f)/SO(2N_f)$ coset. Hence, $\Sigma \equiv SX = XS^T$ transforms as the symmetric representation of $SU(2N_f)$, with S satisfying $SS^\dagger = \mathbb{1}_{2N_f \times 2N_f}$, and with X

symmetric and traceless. For the $SU(2)$ theory with adjoint matter ($N_f = 1$), we choose $X = \tau^1$, the first Pauli matrix.¹

The factor χ^y in the final term of $\mathcal{L}$ is also a conformal compensator, with the scaling dimension y determined by fits to lattice data. Near the edge of the conformal window, the anomalous dimension of the fermion bilinear $\bar{\psi}\psi$ is expected to approach 1 [62], implying $y \approx 2$. This value is also motivated in earlier dilaton EFT studies [29]. The non-negative constant R , of dimension $4 - y$, is induced by the presence of the underlying fermion mass m , defined at the lattice cutoff. It vanishes in the limit $m \rightarrow 0$, and we take its m -dependence to be linear:

$$R = B_f m, \quad (2)$$

where B_f is a positive constant.

For the dilaton potential $V(\chi)$, we adopt the form

$$V(\chi) = A\chi^4 + B\chi^\Delta, \quad (3)$$

where $\Delta (> 0)$, A , and B are real parameters, and where the second term describes the expected deformation of conformality present even in the limit $m = 0$. The full scalar potential is given by

$$W(\chi) = V(\chi) - \frac{1}{2} x N_f R \chi^y, \quad (4)$$

with $x = 1$ for complex representations, and $x = 2$ for real and pseudoreal ones. For the case $y < 4$, Δ , and with $R > 0$, the second term dominates for small χ , driving the system away from the origin and toward a stable, symmetry breaking minimum of $W(\chi)$.

We note in passing that our previous studies [39,42] of the $SU(3)$ gauge theory with $N_f = 8$ fermions in the fundamental representation were conducted with a constraint on the parameters of the potential $V(\chi)$ such that it has a symmetry breaking minimum, insuring that the spontaneous breaking of approximate conformal symmetry persists in the limit $R = 0$. Here we proceed more generally, allowing lattice fits to determine whether or not $V(\chi)$ has a symmetry-breaking minimum, that is whether the theory is just outside or just inside the conformal window. Our approach allows for the possibility that $V(\chi)$ could approach a much-employed logarithmic form [39–44] in the marginal-deformation limit $\Delta \rightarrow 4$.

III. FITS TO LATTICE DATA

The parameters of the dEFT in Eq. (1) derive from the underlying gauge theory that UV completes it. They are directly related to the decay constants (F_d , F_π) and masses (M_d , M_π) of the dilaton and pNGBs. Lattice studies of the

¹For pseudoreal representations, X and Σ are antisymmetric matrices, and the pNGBs describe the $SU(2N_f)/Sp(2N_f)$ coset.

underlying gauge theory typically present values for these quantities as functions of a common mass m of the fermions, defined at the UV cutoff.

The quantity F_d is the vacuum expectation value (VEV) of χ , determined in terms of the dEFT parameters by extremizing $W(\chi)$:

$$4AF_d^{4-y} + \Delta BF_d^{\Delta-y} - \frac{1}{2}xN_f R y = 0. \quad (5)$$

The dilaton mass is given by

$$M_d^2 = 12AF_d^2 + \Delta(\Delta-1)BF_d^{\Delta-2} - \frac{1}{2}xN_f R y(y-1)F_d^{y-2}. \quad (6)$$

With $\Sigma \equiv \exp[2i\Pi/F_\pi]$, canonical normalization of the pNGB kinetic term gives

$$F_\pi^2 = PF_d^2, \quad (7)$$

and

$$M_\pi^2 = RF_d^y/F_\pi^2. \quad (8)$$

Lattice measurements exist for only the quantities F_π^2 , M_π^2 , and M_d^2 , so we reorganize the above equations to obtain three useful fit equations. The first, independent of $V_\Delta(\chi)$, is

$$M_\pi^2 F_\pi^{2-y} - Cm = 0, \quad (9)$$

where $C \equiv B_f/P^{y/2}$ is treated as a fit parameter. Two additional equations can be obtained in which the

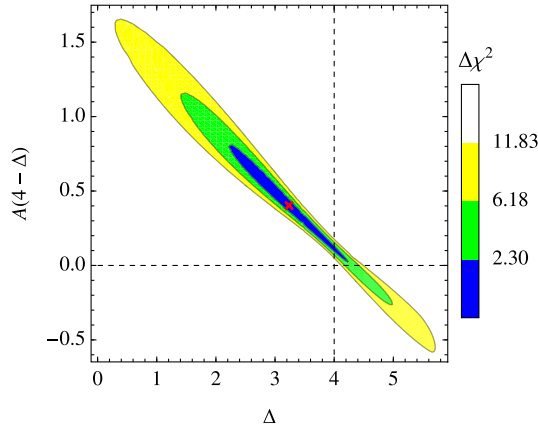


FIG. 1. Contours of $\Delta\chi^2$ indicating confidence intervals (equivalent to 1σ , 2σ , and 3σ , respectively), for A and Δ in the $SU(3)$ gauge theory with $N_f = 8$ Dirac fermions in the fundamental representation. The central value denoted by the red dot is reported in Table I. For each point of the plot, we minimize the χ^2 in respect to the other four fit parameters. The region $A > 0$ and $\Delta < 4$ is favored.

exponential dependence on the fit parameter y is removed. They take the form

$$\frac{M_\pi^2}{F_\pi^2} - \frac{2}{yxN_f P^2} \left[4A + \Delta B \left(\frac{F_\pi^2}{P} \right)^{\frac{\Delta-4}{2}} \right] = 0, \quad (10)$$

and

$$\frac{M_d^2}{F_\pi^2} - \frac{4(4-y)A}{P} - \frac{\Delta(\Delta-y)B}{P} \left(\frac{F_\pi^2}{P} \right)^{\frac{\Delta-4}{2}} = 0. \quad (11)$$

As we employ Eqs. (9)–(11) to fit the lattice data, we will find it helpful to depict the form of $V'(\chi)$ in a way that incorporates the uncertainty in the fit parameters. Noting that $V'(F_d) = [\frac{1}{2}xyN_f P^{1/2}] M_\pi^2 F_\pi$, and employing Eq. (10), we can display the form of $V'(F_d)$, simply by plotting $M_\pi^2 F_\pi$ versus F_π ($= P^{-1/2} F_d$). This plot can be extended to show the form of $V'(\chi)$ even at values of χ below a potentially nonvanishing VEV, where it is negative, by continuation of F_π to small values.

A. $SU(3)$ gauge theory with $N_f = 8$ fermions in the fundamental representation

The lattice data obtained from Refs. [17,59] are summarized in Table III of the Appendix. The masses and decay constants of the 63 pNGBs and the masses of the dilaton are listed there for five values of the underlying fermion mass m . As noted in the Appendix, correlations are neglected. The choice of lattice coupling β and the range of fermion mass m employed in the lattice study lead to masses M_π and M_d well below the inverse lattice spacing,

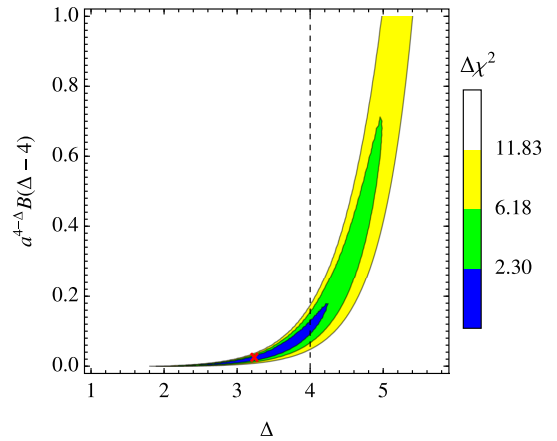


FIG. 2. Contours of $\Delta\chi^2$ indicating confidence intervals (equivalent to 1σ , 2σ , and 3σ , respectively), for B and Δ , in the $SU(3)$ gauge theory with $N_f = 8$ Dirac fermions in the fundamental representation. The central value denoted by the red dot is reported in Table I. For each point of the plot, we minimize the χ^2 in respect to the other four fit parameters. The region with $B < 0$ and $\Delta < 4$ is favored, consistent with being outside the conformal window.

TABLE I. Central values in the dEFT fit for the $SU(3)$ gauge theory with $N_f = 8$ Dirac fermions in the fundamental representation. Lattice data for M_π , M_d , and F_π are incorporated into this fit, for five different values of the underlying fermion mass m , corresponding to 15 data points. The lattice measurements are taken from Refs. [17,59], and reproduced in Table III. Dimensional quantities are presented in units of the lattice spacing, a . We do not quote individual uncertainties for all of the fit parameters here, as the likelihood in parameter space is non-Gaussian and exhibits strong correlations between parameters. Instead, joint confidence regions for A , B and Δ are shown in Figs. 1 and 2. For y , we conservatively estimate the uncertainty to be 5%, reflecting the spread of acceptable fits.

Parameter	Central value
y	2.081
$a^{3-y}C$	7.16
Δ	3.237
P	0.1050
A	0.527
$a^{4-\Delta}B$	-0.0328
$\chi^2/d.o.f.$	2.70/9

and also below the masses of heavier states, which are not included in the dEFT analysis.

Results for A , B , and Δ are shown in Figs. 1 and 2 in the form of $\Delta\chi^2$ contours. The favored upper left quadrants together correspond to values $\Delta < 4$, $A > 0$, and $B < 0$. The central values of all six fit parameters are shown in Table I. Those for y and P are consistent with earlier studies. The central values of Δ (< 4), A (positive), and B (negative) imply that the theory lies just outside the

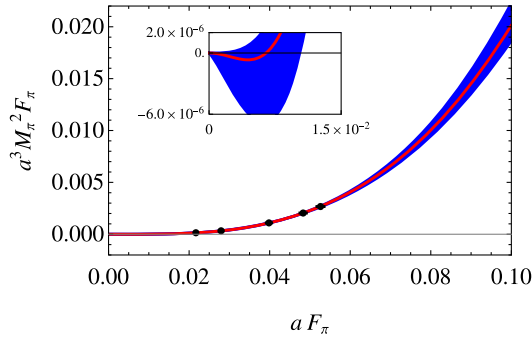


FIG. 3. Plot of $a^3 M_\pi^2 F_\pi$ versus $a F_\pi$, in the $SU(3)$ theory with $N_f = 8$ fermions in the fundamental representation [17,59]. The red curve is based on the central values of Table I, with the five data points denoted by the black discs. The blue shaded region represents the uncertainty in the fitting curve, equivalent to the 1σ confidence level. The red curve reaches the horizontal axis at a nonzero value of $a F_\pi$, corresponding to the limit $m \rightarrow 0$ in a confining gauge theory. For larger values of $F_\pi \propto F_d$, we have $M_\pi^2 F_\pi \propto V'(F_d)$. The curve is extrapolated downward, where it becomes negative, by continuing Eq. (10) to unphysical values of F_π , demonstrating that the region near the origin is unstable, as shown in the inset.

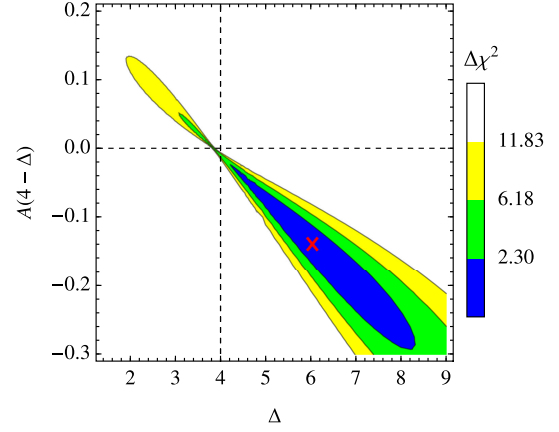


FIG. 4. Contours of $\Delta\chi^2$ indicating confidence intervals (equivalent to 1σ , 2σ , and 3σ , respectively), for A and Δ in the $SU(2)$ gauge theory with $N_f = 1$ Dirac fermion in the adjoint representation. The fit includes only data taken at the three lightest fermion mass points. The central value is reported in Table II. For each point of the plot, we minimize the χ^2 in respect to the other four fit parameters. The region with $\Delta > 4$ and $A > 0$ is favored, consistently with infrared conformal behavior.

conformal window, with $V(\chi)$ having a symmetry breaking minimum, as assumed in our earlier studies [42]. This is compatible with the expectation from the renormalization scheme invariant, multiloop analysis of Ref. [63], as well as the analysis of Ref. [3], which combines lattice and perturbative inputs.

A graphical depiction of this conclusion is shown in Fig. 3. We plot $M_\pi^2 F_\pi$ as a function of $F_\pi \propto F_d$. The red curve is based on the central values of Table I. It shows $M_\pi^2 F_\pi$ vanishing at a finite value of F_π (corresponding to

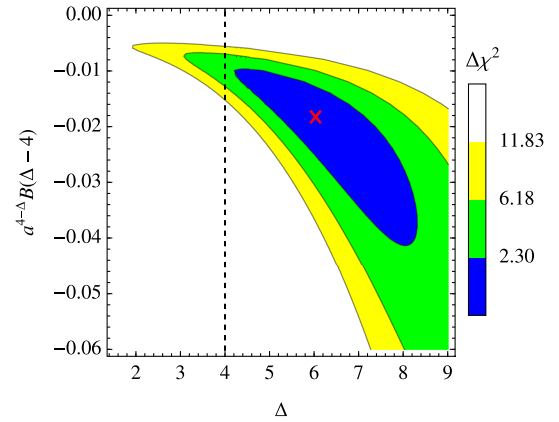


FIG. 5. Contours of $\Delta\chi^2$ indicating confidence intervals (equivalent to 1σ , 2σ , and 3σ , respectively), for B and Δ in the $SU(2)$ gauge theory with $N_f = 1$ Dirac fermion in the adjoint representation. The fit includes only data taken at the three lightest fermion mass points. The central value is reported in Table II. For each point of the plot, we minimize the χ^2 in respect to the other four fit parameters. The region with $\Delta > 4$ and $B < 0$ is favored, consistently with infrared conformal behavior.

TABLE II. Central values for the dEFT fit for the $SU(2)$ gauge theory with $N_f = 1$ Dirac fermion in the adjoint representation. The lattice data are taken from Refs. [28,60,61], and reproduced in Table IV, and have been restricted to the three ensembles with smallest fermion mass and $\beta = 2.1$. We take the fermion mass used in our fits to be the quantity (m_{PCAC}) from the lattice study, which is defined using the partially conserved axial current (PCAC) relation (see Ref. [64]). Lattice data for M_π , M_d , F_π , and m_{PCAC} have been incorporated into this fit, corresponding to nine data points. Dimensional quantities are presented in units of the lattice spacing, a . We do not quote individual uncertainties for all the fit parameters due to the non-Gaussian, strongly correlated likelihood. Instead, joint confidence regions for A , B and Δ are shown in Figs. 4 and 5, and we estimate an uncertainty of 10% for y .

Parameter	Central value
y	2.19
$a^{3-y}C$	7.69
Δ	6.03
P	0.131
A	0.0683
$a^{4-\Delta}B$	-0.0090
$\chi^2/d.o.f.$	0.616/3

$m \rightarrow 0$), and increasing continuously with increasing F_π . In this range, $M_\pi^2 F_\pi \propto V'(F_d)$. We extend the red curve to lower values of F_π , where $V' < 0$, by continuing the fit equations (9)–(10) into this regime. The blue shaded region represents uncertainty in the fitting curve equivalent to the 1σ level. The five data points, depicted by the black discs, lie well above the value of F_π where $V'(F_d)$ vanishes, and well within a region where the uncertainty in the fit is small.

B. $SU(2)$ gauge theory with $N_f = 1$ fermion in the adjoint representation

We again employ Eqs. (9)–(11) to fit the lattice data [7,28,60,61]. In this case, with the spontaneous breaking pattern $SU(2) \rightarrow SO(2)$, there are two pNGBs. We list the measurements performed with Wilson fermions and coupling $\beta = 2.1$, for the largest available volumes, in Table IV of the Appendix. We identify the fermion mass m_{PCAC} , defined following Ref. [64], with the mass parameter m in our fits. We include in our analysis only the three ensembles with the lightest mass, $aM_{\pi,d} < 0.7$. We tested the stability of the fits by including one or more heavier ensembles.

Results for A , B , and Δ are shown in Figs. 4 and 5 in the form of $\Delta\chi^2$ contours, with central values listed in Table II. Among the central values, those for y and P are similar to

²As discussed in Ref. [7], ensembles with $\beta > 2.1$ yield a systematic decrease in the anomalous dimension of the fermion condensate, indicating to us that in those ensembles the Wilson fermion mass does not reach low enough values to assess the existence of an IR fixed point.

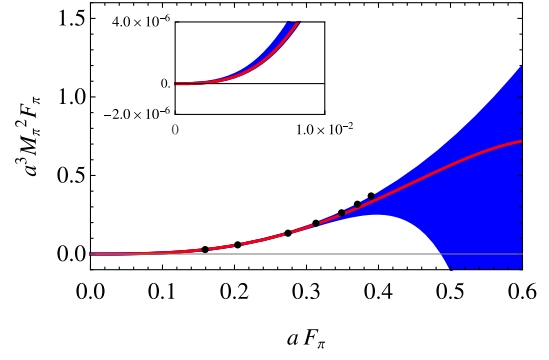


FIG. 6. Plot of $a^3 M_\pi^2 F_\pi$ versus $a F_\pi \propto a F_d$, in the $SU(2)$ theory with $N_f = 1$ fermion in the adjoint representation [28,60,61], restricting attention to ensembles with $\beta = 2.1$. The red curve is based on the central values of Table II, using only the three lightest ensembles. The data points are depicted by the black discs. The red curve reaches the horizontal line at $F_\pi = 0$, as shown in the inset, corresponding to the limit $m \rightarrow 0$ in an infrared conformal theory. Throughout the range, $a^3 M_\pi^2 F_\pi \propto V'(F_d)$. The blue shaded region represents the uncertainty in the fitting curve, equivalent to the 1σ confidence level.

those of Table I. In particular, $y \approx 2$, as expected near the edge of the conformal window [62]. We again have $A > 0$ and $B < 0$, but now $\Delta > 4$. For these values, $B\chi^\Delta$ is an irrelevant operator that becomes less important at low energies. Although it explicitly breaks scale invariance, its effect on the full dilaton potential in Eq. (4) diminishes in the massless limit, consistent with the claim that the gauge theory lies inside the conformal window. It is also the case that $V(\chi)$ turns down at large χ , but the turn down sets in well beyond the range of applicability of the dEFT.

We depict this in Fig. 6, by plotting $M_\pi^2 F_\pi \propto V'(F_d)$ as a function of $F_\pi \propto F_d$. The red curve, based on the central values of Table II, shows $V'(F_d)$ vanishing at $F_\pi = 0$ (corresponding to the limit $m \rightarrow 0$ in an infrared conformal gauge theory), and increasing with F_π through values well beyond the location of the data points, depicted by the black discs. The data points lie low enough so that the 1σ -fit uncertainty, represented by the shaded region, is relatively small.

We conclude that available lattice measurements indicate that this theory is inside the conformal window. It is governed in the infrared by the extremum of $V(\chi)$ at $\chi = 0$, and all the masses and decay constants vanish in the limit $m = 0$. Additional lattice ensembles with small fermion mass and large β will be helpful to confirm this preliminary conclusion.

IV. SUMMARY AND DISCUSSION

We employed dEFT as a diagnostic tool to discriminate between infrared-conformal and near-conformal, confining gauge theories. Our approach draws on spectroscopic lattice measurements performed with finite mass for the

elementary fermions, in a range of parameter space in which both the pNGBs and the dilaton are clearly lighter than other states. This feature may be expected to emerge naturally in theories near the lower edge of the conformal window, for small enough fermion masses. We took the dEFT to apply smoothly through the transition between confinement and conformality.

We tested this approach with two lattice studies meeting these requirements. For the $SU(3)$ gauge theory coupled to $N_f = 8$ Dirac fermions in the fundamental representation, we used measurements obtained with staggered fermions, restricted to the largest available lattice couplings and volumes [17,18], to infer that the theory is outside the conformal window (near conformal but confining in the $m \rightarrow 0$ limit). For the $SU(2)$ theory with $N_f = 1$ fermions in the adjoint representation, we used data obtained with Wilson fermions [7,28], by restricting attention to ensembles in which the relevant masses are significantly smaller than the lattice cutoff, to minimize lattice artifacts. Our analysis indicates that this theory is inside the conformal window (infrared conformal in the $m \rightarrow 0$ limit).

We made several simplifying assumptions. We ignored systematic effects in the lattice measurements, such as correlations between individual measurements, and contamination from finite-volume and finite size effects. We included only leading-order terms in the dEFT Lagrangian. We verified explicitly that the lattice measurements employed small enough fermion masses to explore only small-field dEFT excursions.

Our results should be taken as preliminary, as indicated by the large uncertainty in some of the outputs—in particular the scaling dimension Δ . Future precision lattice measurements, particularly for the $SU(2)$ theory, could overcome these limitations, by exploring lattice parameters with smaller fermion masses and less contamination from lattice artifacts.

The framework developed here nevertheless provides a systematic EFT description that can accommodate both confining an infrared-conformal possibilities on equal footing, and can be extended to incorporate additional observables as further input. It can also be applied to other gauge theories. It would be interesting, for example, to apply our analysis to the $SU(3)$ theory coupled to $N_f = 10$ Dirac fermions in the fundamental representation [14], as well as to lattice data for the theory with $N_f = 2$ sextet fermions [20–25], and to theories with other gauge groups, such as $Sp(4)$ [4,6].

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DATA AVAILABILITY

The data that support the findings of this article are openly available [65].

APPENDIX: MAXIMUM LIKELIHOOD ANALYSIS AND LATTICE DATA

We here collect some technical details needed to reproduce our main results. First, we summarize in Tables III and IV the spectroscopic measurements for pNGBs and dilaton, taken from Refs. [17,28]. These measurements are combined with Eqs. (9)–(11), to define the χ^2 , in the following way.

Given the N fitting functions, $\vec{f} = \{f_i\}_{i=1}^N$, which depend on the K parameters of the model, $\vec{a} = \{a_k\}_{k=1}^K$, and on M experimentally measurable quantities, $\vec{x} = \{x_j\}_{j=1}^M$, we define the $N \times N$ covariance matrix $\vec{C}$ as

$$\vec{C} \equiv O^T C O, \quad (A1)$$

where C is the $M \times M$ covariance matrix of the observables, and $O_j^i = \frac{\partial f_i}{\partial x_j}$ is the $M \times N$ matrix of partial derivatives. The χ^2 function is the following:

$$\chi^2(\vec{a}) \equiv \vec{f}^T(\vec{a}, \vec{x}) \vec{C}^{-1} \vec{f}(\vec{a}, \vec{x}). \quad (A2)$$

The best fit value for $\vec{a}$ is obtained by minimizing numerically the function $\chi^2(\vec{a})$.

As we do not have access to the covariance matrix for the measurements listed in Tables III and IV, we take C to be diagonal, with nonvanishing elements given by the variance due to the statistical errors of the measurements, σ_j^2 . We do not include systematic effects, assuming that they are smaller than the statistical ones, for those

TABLE III. Numerical lattice measurements taken from Table IX of Ref. [17] with $\beta = 4.8$ (see also the data release [59]), for the $SU(3)$ theory coupled to $N_f = 8$ fermions transforming in the fundamental representation. The errors in parentheses include statistical and systematics, but correlations are neglected. The fermion mass, am , appears as a parameter in the lattice staggered action, and hence there is no associated uncertainty.

am	aM_π	aF_π	aM_d
0.00889	0.2253(37)	0.05262(88)	0.301(12)
0.0075	0.2057(34)	0.04831(80)	0.2744(85)
0.005	0.1657(27)	0.03982(66)	0.2408(87)
0.00222	0.1087(18)	0.02794(47)	0.1545(83)
0.00125	0.0811(13)	0.02168(36)	0.1174(48)

TABLE IV. Numerical lattice measurements for the $SU(2)$ theory coupled to $N_f = 1$ fermion transforming in the adjoint representation [28,60,61], for ensembles with $\beta = 2.1$, and the largest available volumes. The errors in parentheses are statistical only, and correlations are neglected. These numerical results are obtained with Wilson fermions; hence a measure of symmetry breaking effects due to finite fermion mass is provided by the PCAC mass, am_{PCAC} , extracted according to the process described in Ref. [64].

am_{PCAC}	aM_π	aF_π	aM_d
0.15326(21)	0.97363(51)	0.38982(94)	0.847(34)
0.13820(17)	0.92461(45)	0.37075(71)	0.716(57)
0.12200(23)	0.86745(64)	0.34875(92)	0.690(38)
0.10251(16)	0.79101(51)	0.31313(97)	0.565(36)
0.08011(19)	0.69504(63)	0.27437(70)	0.469(33)
0.04969(12)	0.53319(48)	0.20450(67)	0.373(29)
0.03246(14)	0.42018(32)	0.15925(36)	0.295(13)

measurements we retain in the analysis. The functions f_i are defined by the left-hand side of Eqs. (9)–(11), respectively. For the $SU(3)$ theory, we treat the five measurements of aM_π , aF_π , and aM_d as the observables. The five choices of am are treated as numerical values of a parameter that is known without uncertainty—the mass of

the staggered fermions in the lattice action. For the $SU(2)$ theory, our χ^2 analysis makes use of the measurements of am_{PCAC} , aM_π , aF_π , and aM_d , restricting attention to the three ensembles with the lightest masses, among those with $\beta = 2.1$.

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