

Incoherent particle production in ultraperipheral heavy ion collisions

L. A. Harland-Lang 

Department of Physics and Astronomy, University College London, London, WC1E 6BT, United Kingdom



(Received 18 November 2025; accepted 8 May 2026; published 21 July 2026)

In this paper, we present the first complete treatment of incoherent photon-initiated production in ultraperipheral heavy ion collisions, focusing on the dilepton and diphoton final states. In the former case we compare to the A Toroidal LHC Apparatus ATLAS measurement of dielectron production and find that our predictions match the data very well. In the latter case we show that this contribution is too small to explain the observed broad in diphoton acoplanarity background to the purely coherent light-by-light scattering signal, after the inclusion of appropriate fiducial cuts. We in addition consider a new “quark emission” topology, which while naïvely might be expected to dominate the incoherent diphoton production channel, is in fact found to be kinematically suppressed. Finally, we revisit QCD-initiated diphoton production and issue of theoretical uncertainties in this case. We find that this again cannot explain the observed size of the higher acoplanarity background. The production mechanism leading to this background to light-by-light production, and the reason for its observed enhancement in comparison to the dilepton case, therefore remains unclear.

DOI: [10.1103/1d4d-1tx6](https://doi.org/10.1103/1d4d-1tx6)

I. INTRODUCTION

The ultraperipheral collision (UPC) of heavy ions occurs when the impact parameter separation of the ions is much larger than the range of QCD, and hence no color flow occurs between the colliding ions. In this environment, photon-initiated (PI) particle production is particularly enhanced, and this has enabled a rich set of experimental measurements at the LHC [1–8] and relativistic heavy ion collider [9].

The dominant PI production mechanism leading to UPC processes is due to purely coherent production, that is where both ions act coherently as a source of photons, and hence the cross section is enhanced by a factor of Z^4 , where Z is the ion electric charge. However, it is equally possible for either (or both) ions to interact incoherently, due to photon emission from the individual nucleons within the ion, such that the ion necessarily breaks up after the interaction. This contribution, in the dominant case where only one ion interacts incoherently, is expected to be suppressed by a factor of $\sim 1/Z$ and hence enter at the percent level with respect to the purely coherent case for typical LHC ion beams. While small, this may therefore not be negligible at the level of precision being aimed for at the LHC. Moreover the kinematics are in general rather different, with the incoherent mechanism dominantly leading to larger

momentum transfers to the final-state system, as typically measured via the final-state acoplanarity, a proxy for the transverse momentum of the centrally produced system.

This incoherent production mechanism is particularly topical in light of the A Toroidal LHC Apparatus (ATLAS) measurement [3] of dielectron production in UPCs, where the contribution from the higher acoplanarity tail is extracted via a template fitting technique. However, until now no model for incoherent production differential in the dielectron acoplanarity has been available, in order to compare these data with theoretical expectations. In this paper we therefore resolve this unsatisfactory state of affairs, by providing the first full account of incoherent PI production in UPCs, including a complete treatment of the initial-state photon flux, the final-state kinematics, and the survival factor probability of no additional ion-ion inelastic interactions. This will be made available in the SuperChic Monte Carlo (MC) generator. We will compare against the ATLAS data and find rather good agreement in general, validating the overall approach.

Beyond dilepton production, a further strong motivation for considering this issue is the case of light-by-light (LbyL) scattering, as measured by both the ATLAS [2] and compact muon solenoid (CMS) [7] collaborations. Here, the contribution to the fiducial region from larger diphoton acoplanarities (that is, larger diphoton transverse momenta), which is where the incoherent contribution is expected to dominate, is found to be rather large and of order the size of the coherent signal. In these analyses, this is assumed to be due to QCD-initiated production, namely via the $gg \rightarrow \gamma\gamma$ subprocess as part of an overall color-singlet exchange between

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

the beams. However, in a recent study [10] it was suggested that incoherent PI production may instead be relevant here, with their estimates suggesting it may enter at the $\sim 20\%$ level with respect to the coherent signal.

We will therefore apply the same approach developed in this paper to model incoherent dilepton production to the LbyL scattering case. We find that after accounting for all relevant effects omitted in the initial study of Ref. [10], due in particular to the kinematic cuts on the final-state photons, the incoherent PI contribution is expected to be small, at the subpercent level. As we will discuss, this is significantly lower than the size of background observed by both ATLAS and CMS.

With this in mind, we will also consider for the first time the possible contribution from photon emission directly from the quark line in the nucleon of the incoherently interacting ion. While naïvely enhanced by a factor of $\sim 1/\alpha^2$ in comparison to the “signal-like” incoherent contribution, namely that which proceeds via loop-induced LbyL scattering, we find that accounting for all relevant factors this is somewhat suppressed relative to it. This therefore cannot explain the sizeable background observed by ATLAS and CMS. Related to this, we briefly comment on the possibility of purely exclusive two-photon photoproduction off the nucleon. On general grounds this is expected to again be suppressed, being proportional to the valence generalized quark Parton Distribution Functions (PDFs), although we raise the possibility of a related inelastic channel, a full analysis of which is left to future work.

Finally, we revisit the case of QCD-initiated production, for which the central predictions in Ref. [11] were found to lie below the ATLAS and CMS extractions of the higher acoplanarity background by an order of magnitude or more. We will consider all possible sources of theoretical uncertainty but do not find any convincing possibility that these may explain the size of the background. The production mechanism leading to this background, and the reason for its enhancement in the LbyL case in comparison to the dilepton channel, therefore remains unclear, although in the concluding section we discuss some possible steps to shed light on this issue.

The outline of this paper is as follows: in Sec. II we present the theoretical framework behind our approach, discussing in Sec. II A the basic overview, in Sec. II B the modeling of the nucleon-ion cross section, and in Sec. II C the treatment of the ion-ion survival factor. In Sec. III we present results. We start in Sec. III A with an approximate analysis within the photon PDF framework, and then in Sec. III B present a comparison of the predictions within the full theoretical treatment to the ATLAS data on dielectron production. In Sec. IV we next consider the case of LbyL scattering, starting in Sec. IV A with the signal-like contribution, before considering the quark emission case in Sec. IV B, commenting on the possibility of purely exclusive two-photon photoproduction off the nucleon in Sec. IV C,

and finally revisiting the QCD-initiated case in Sec. IV D. In Sec. V we summarize our results, with particular focus on the implications for the LbyL measurement.

II. THEORY

A. Basic framework

For the case of incoherent PI production, the strongly dominant contribution is due to the mixed case where one ion continues to interact coherently; as we will see this contributes at the percent level to the cross section and hence production due to incoherent emission from both ions will enter at the sub-per-mille level, and can be safely ignored. To first approximation, as in, e.g., Ref [10] we have

$$\sigma_{AA}^{\text{incoh}} = A\sigma_{pA} + (Z - A)\sigma_{nA}, \quad (1)$$

that is, the incoherent AA cross section is given in terms of the individual proton (p) and neutron (n) – A cross sections, suitably summed over the nucleon content in the ion. A representative Feynman diagram for this pA/nA cross section is shown in Fig. 1(b), while the corresponding purely coherent case is shown in Fig. 1(a) for comparison. For incoherent production, the photon emission from the nucleon can either be elastic or inelastic, exactly as in PI production in pp collisions (see, e.g., Ref. [12]), and hence the upper particle line in Fig. 1(b) can either be interpreted as either a nucleon or quark/antiquark line depending on whether elastic or inelastic emission is considered.

B. Nucleon-ion cross section

As we will discuss further below, there are various reasons why Eq. (1) requires modification, but nonetheless this provides the basic framework for the calculation. We are therefore interested in calculating the nucleon-ion cross section for the PI production of some system X . This can be

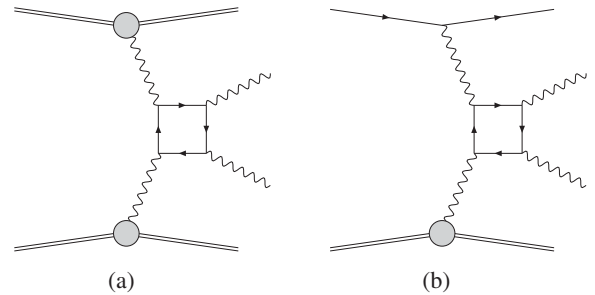


FIG. 1. Representative Feynman diagrams for light-by-light scattering in AA ultraperipheral collisions in the case of (a) coherent and (b) incoherent production. The lower double line corresponds to the ion beam that undergoes coherent photon emission, while the upper line corresponds to the incoherent interaction. The latter can be interpreted as either a nucleon or quark/antiquark line depending on whether elastic or inelastic photon emission from the individual nucleons is considered.

done following the approach outlined in Refs. [12,13], with the cross section given by

$$\sigma_{pA} = \frac{1}{2s} \int dx_1 dx_2 d^2 q_{1\perp} d^2 q_{2\perp} d\Gamma \alpha(Q_1^2) \alpha(Q_2^2) \times \frac{1}{\tilde{\beta}} \frac{\rho_1^{\mu\mu'} \rho_2^{\nu\nu'} M_{\mu'\nu'}^* M_{\mu\nu}}{Q_1^2 Q_2^2} \delta^{(4)}(q_1 + q_2 - p_X), \quad (2)$$

where the momentum fractions $x_{1,2}$ and $\tilde{\beta}$ are defined in Ref. [14], while $M^{\mu\nu}$ corresponds to the $\gamma\gamma \rightarrow X(k)$ production amplitude and $d\Gamma = \prod_{j=1}^N d^3 k_j / 2E_j (2\pi)^3$ is the standard phase space volume. We have focused on the pA cross section for brevity, but the nA cross section is given in the same way. Here, ρ is the density matrix of the virtual photon, which takes the general form

$$\rho_i^{\alpha\beta} = 2 \int \frac{dM_i^2}{Q_i^2} \left[- \left(g^{\alpha\beta} + \frac{q_i^\alpha q_i^\beta}{Q_i^2} \right) F_1(x_{B,i}, Q_i^2) + \frac{(2p_i^\alpha - \frac{q_i^\alpha}{x_{B,i}})(2p_i^\beta - \frac{q_i^\beta}{x_{B,i}}) x_{B,i}}{Q_i^2} F_2(x_{B,i}, Q_i^2) \right], \quad (3)$$

where $x_{B,i} = Q_i^2 / (Q_i^2 + M_i^2 - m_p^2)$ for a hadronic system of mass M_i and we note that the definition of the photon momentum q_i as outgoing from the hadronic vertex is opposite to the usual deep inelastic scattering convention. Here, the integral over M_i^2 is understood as being performed simultaneously with the phase space integral over the photon momenta, i.e., is not fully factorized from it.

The inputs for the elastic and inelastic structure functions are described in the Appendix. However, here we discuss a modification that must be applied to the case of elastic proton emission (as discussed in the Appendix we can safely drop the elastic neutron emission case) to account for the fact that the protons are bound inside the ions. It in particular should be the case that in order for incoherent emission from the ion to occur, the proton cannot scatter into an already occupied state in the ion, due to the Pauli exclusion principle. This Pauli blocking effect is typically modeled by requiring that the momentum transfer to the proton should be roughly larger than the typical Fermi momentum, $p_F \sim 250$ MeV, of the nucleons in the ion (see Ref. [15] for a recent summary). More precisely, in the Fermi gas model this is accounted for by modifying the elastic proton form factors such that [16]

$$F_i^{\text{el,p/A}}(x_{B,i}, Q_i^2) = C_{\text{Pauli}}(Q_i^2) F_i^{\text{el,p}}(x_{B,i}, Q_i^2), \quad (4)$$

where $F_i^{\text{el,p}}$ are the usual free proton elastic form factors and the modification factor $C(Q_i^2)$ is given by

$$C_{\text{Pauli}}(Q^2) = \frac{3Q_{\text{rec}}}{4p_F} \left[1 - \frac{1}{12} \left(\frac{Q_{\text{rec}}}{p_F} \right)^2 \right], \quad (5)$$

if $Q_{\text{rec}} > 2p_F$ and $C(Q_i^2) = 1$ otherwise. Here

$$Q_{\text{rec}}^2 = \left(\frac{Q^2}{2m_p} \right)^2 + Q^2. \quad (6)$$

This factor therefore reduces the corresponding elastic cross section by suppressing the region of lower photon Q^2 . As the proton form factor is rather peaked towards low values of Q^2 this reduces the corresponding cross section by a factor of ~ 5 , as we will discuss in the following sections. We note that in principle we could account directly for resonant excitation of the ions through photon emission, by suitably accounting for the inelastic ion form factors. However, as shown in Ref. [17], using a simple model for the dominant giant dipole resonant excitation of the ion, this contribution is expected to be at the per mille level with respect to the purely coherent cross section, and so we neglect it in what follows.

We note that, while Eq. (2) is the general expression that we use, it is useful to make connect with simpler formulations when calculating the impact of the survival factor as well for the case where only the on-shell $\gamma\gamma \rightarrow X$ amplitudes are available. To do this, taking the high-energy limit, and using the gauge invariance of the PI amplitude to drop the terms $\sim q_i$ in Eq. (3) we have

$$\rho_i^{\alpha\beta} = 2 \int \frac{dM_i^2}{Q_i^2} \left[-g^{\alpha\beta} F_1(x_{B,i}, Q_i^2) + 2 \frac{q_{i\perp}^\alpha q_{i\perp}^\beta x_{B,i}}{Q_i^2 x_i^2} F_2(x_{B,i}, Q_i^2) \right]. \quad (7)$$

For elastic photon emission, the F_2 term dominates and we can write

$$\sigma = \frac{1}{2s} \int dx_1 dx_2 d^2 q_{1\perp} d^2 q_{2\perp} d\Gamma \times \frac{1}{\tilde{\beta}} |T(q_{1\perp}, q_{2\perp})|^2 \delta^4(q_1 + q_2 - k), \quad (8)$$

where T is the process amplitude, given by

$$T(q_{1\perp}, q_{2\perp}) = \mathcal{N}_1^p(x_1, Q_1^2) \mathcal{N}_2^A(x_2, Q_2^2) q_{1\perp}^\mu q_{2\perp}^\nu \mathcal{M}_{\mu\nu}, \quad (9)$$

with the normalization factors

$$\mathcal{N}_1^p(x_i, Q_i^2) = \frac{2\alpha(Q_i^2)^{1/2} F_E(Q_i^2)^{1/2}}{x_i Q_i^2} C_{\text{Pauli}}^{1/2}(Q_i^2),$$

$$\mathcal{N}_2^A(x_i, Q_i^2) = \frac{2\alpha(Q_i^2)^{1/2} F_p(Q_i^2) G_E(Q_i^2)}{x_i Q_i^2}, \quad (10)$$

for the case that the incoherent (proton) beam is labeled $i = 1$ and the ion is labeled $i = 2$. The various form factors in the above expression are defined in the Appendix. In the full calculation we of course include both configurations,

and note that when applying any cuts on the final-state system care must be taken to ensure that the appropriate boost is performed from the pA to the AA frame.

Finally, we note that, in the equivalent photon limit it was shown in Ref. [18] that Eq. (9) can be recast via

$$q_{1\perp}^\mu q_{2\perp}^\nu \mathcal{M}_{\mu\nu} = \begin{cases} -\frac{1}{2}(\mathbf{q}_{1\perp} \cdot \mathbf{q}_{2\perp})(\mathcal{M}_{++} + \mathcal{M}_{--})(J_z^P = 0^+) \\ -\frac{i}{2}[(\mathbf{q}_{1\perp} \times \mathbf{q}_{2\perp})](\mathcal{M}_{++} - \mathcal{M}_{--})(J_z^P = 0^-) \\ +\frac{1}{2}((q_{1\perp}^x q_{2\perp}^x - q_{1\perp}^y q_{2\perp}^y) + i(q_{1\perp}^x q_{2\perp}^y + q_{1\perp}^y q_{2\perp}^x))\mathcal{M}_{-+}(J_z^P = +2^+) \\ +\frac{1}{2}((q_{1\perp}^x q_{2\perp}^x - q_{1\perp}^y q_{2\perp}^y) - i(q_{1\perp}^x q_{2\perp}^y + q_{1\perp}^y q_{2\perp}^x))\mathcal{M}_{+-}(J_z^P = -2^+) \end{cases} \quad (11)$$

where $\mathcal{M}_{\pm\pm}$ corresponds to the $\gamma(\pm)\gamma(\pm) \rightarrow X$ helicity amplitudes and J_z^P the spin parity of the corresponding $\gamma\gamma$ configuration. We will make use of this expression when calculating cross sections for processes where only the on-shell $\gamma\gamma \rightarrow X$ amplitudes are readily available, as is the case for LbyL scattering.

C. Survival factor

When calculating the cross section for incoherent UPC production we must account for the “no-overlap” probability that the colliding ions do not interact inelastically, as we do in the purely coherent case. This is as usual most clearly discussed in impact parameter space. We therefore start with the cross section, written in impact parameter space, for the PI production of a system X in pA collisions (as before focusing on the proton case for brevity, but with it understood that the neutron case follows in a similar fashion). It can be shown that this is given by

$$\sigma_{pA \rightarrow pAX} = \int d^2b_{1\perp} d^2b_{2\perp} \times \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} |\tilde{\mathbf{N}}_p(x_1, b_{1\perp})|^2 |\tilde{\mathbf{N}}_A(x_2, b_{2\perp})|^2 \sigma_{\gamma\gamma \rightarrow X}. \quad (12)$$

Here

$$\tilde{\mathbf{N}}(x, b_\perp) \equiv \frac{1}{(2\pi)} \int d^2q_\perp \mathbf{N}(x, q_\perp) e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp}, \quad (13)$$

and

$$\mathbf{N}_{p,A}(x_i, q_{i\perp}) = \frac{x_i \mathcal{N}_1^{p,A}(x_i, Q_i^2)}{2\pi} \mathbf{q}_{i\perp}. \quad (14)$$

To calculate the contribution from inelastic photon emission from a single ion, due to incoherent emission from the protons in the ion, we then simply write

$$\sigma_{AA \rightarrow pAX}^{\text{inel,el}} = \int d^2b_{1\perp} d^2b_{2\perp} \int d^2\tilde{b}_\perp T_{A,p}(\tilde{b}_\perp) \Gamma_{AA}(s, b'_\perp) \times \frac{d^2\sigma_{pA \rightarrow pAX}}{d^2b_{1\perp} d^2b_{2\perp}}, \quad (15)$$

where $\mathbf{b}'_\perp = \mathbf{b}_\perp - \tilde{\mathbf{b}}_\perp$, with $\mathbf{b}_\perp = \mathbf{b}_{1\perp} + \mathbf{b}_{2\perp}$. Here T_A is the transverse proton density, given by

$$T_{A,p}(b_\perp) = \int dz \rho_p(r), \quad (16)$$

where the proton density $\rho_p(r)$ is given in terms of the standard Woods-Saxon parametrization. This is therefore normalized such that

$$\int d^2b_\perp T_{A,p}(b_\perp) = Z, \quad (17)$$

where Z is the ion atomic number. In Eq. (15), $\Gamma_{A_1A_2}$ represents the usual probability that no inelastic scattering occurs at impact parameter $b'_\perp$. It is typically written in terms of the ion-ion opacity $\Omega_{A_1A_2}$ via

$$\Gamma_{A_1A_2}(s, b'_\perp) \equiv \exp(-\Omega_{A_1A_2}(s, b'_\perp)). \quad (18)$$

This is given in terms of the opacity due to nucleon-nucleon interactions, Ω_{nn} , which is in turn given by a convolution of the nucleon-nucleon scattering amplitude A_{nn} and the transverse nucleon densities T_n ; see Ref. [11] for a more detailed discussion.

The physical origin of the additional integrals in Eq. (15) is demonstrated in Fig. 2. In particular, the impact parameter between the proton that emits the photon and the center of the other ion (which interacts coherently) is given by $\mathbf{b}_\perp$, while the impact parameter between the center of two ions is given by $\mathbf{b}'_\perp$, which is itself given in terms of $\mathbf{b}_\perp$ and the transverse position $\tilde{\mathbf{b}}_\perp$ of the proton in the inelastically interacting ion.

If we simply ignore the survival factor then the integral over T_A simply drops out of Eq. (15) and we have

$$\sigma_{AA \rightarrow pAX}^{\text{inel,el}} = Z \sigma_{pA \rightarrow pAX}, \quad (19)$$

using the normalization (17). That is, the total inelastic cross section simply becomes a sum over the individual Z proton-ion interactions; for the neutron contribution we simply replace $T_{A,p}$ with the transverse neutron density, $T_{A,n}$, for which the normalization matches the $\sim(A - Z)$ prefactor as

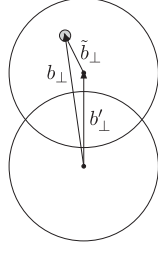


FIG. 2. Diagram indicating the definitions of the impact parameter vectors defined in Eq. (15).

we would expect. This is the approach taken in Ref. [10]. However, we can see that this will certainly be an approximation that will overestimate the cross section. In particular, for certain impact parameter configurations it will not be the case that the entirety of the nucleons in the ion can take part in the interaction, due to the requirement that the ions themselves do not overlap, or more precisely interact inelastically.

To account for this effect, we simply define

$$\Gamma_{A_1 A_2}^{\text{inel,el}}(s, b_\perp) = \int d^2 \tilde{b}_\perp T_A(\tilde{b}_\perp) \Gamma_{AA}(s, b'_\perp), \quad (20)$$

in which case the cross section (15) becomes

$$\sigma_{AA \rightarrow pAX}^{\text{inel,el}} = \int d^2 b_{1\perp} d^2 b_{2\perp} \Gamma_{AA}^{\text{inel,el}}(s, b_\perp) \frac{d^2 \sigma_{pA \rightarrow pAX}}{d^2 b_{1\perp} d^2 b_{2\perp}}, \quad (21)$$

i.e., it has the form as a standard proton-ion UPC cross section, but with a modified survival factor, and form factor on the inelastic side. This can therefore be readily translated to momentum space, and evaluated following the approach most recently outlined in Refs. [19,20]. In particular, defining

$$\mathcal{P}^{\text{inel,el}}(s, k_\perp) \equiv \frac{1}{(2\pi)^2} \int d^2 b_\perp e^{i\mathbf{k}_\perp \cdot \mathbf{b}_\perp} \Gamma_{A_1 A_2}^{\text{inel,el}}(s, b_\perp)^{1/2}, \quad (22)$$

in terms of Eq. (20), then to account for the survival factor we simply replace Eq. (9) with

$$T_{S^2}(q_{1\perp}, q_{2\perp}) = \int d^2 k_\perp T(q'_{1\perp}, q'_{2\perp}) \mathcal{P}^{\text{inel,el}}(s, k_\perp), \quad (23)$$

where $q'_{1\perp} = q_{1\perp} - k_\perp$ and $q'_{2\perp} = q_{2\perp} + k_\perp$. For the F_1 contribution, we consistently include the F_1 term in Eq. (7), but add this incoherently with the dominant electric dipole contribution described above; see Ref. [21] for further details. For inelastic photon emission from a nucleon we apply the approach of the elastic case, but with the elastic form factors suitably replaced by the inelastic structure functions. On physical grounds, we can expect a relatively smooth transition between the elastic and inelastic cases in the low Q_i^2 , and by doing so we account for the physically

relevant process and $q_{i\perp}$ dependence of the survival factor. In principle for higher Q^2 values a more complete procedure should be pursued, as outlined in Ref. [22]; however, in reality this makes a small difference to the cross section predictions of interest, that is within other uncertainties, and hence we do not do this here.

III. RESULTS

A. First look: Photon PDFs and survival factors

As a first look, we can simplify the full expression (2) in the high-energy and on-shell approximation, and neglecting the survival factor, to give

$$\sigma_{pA} \approx \int dx_1 dx_2 f_{\gamma/p}(x_1, \mu^2) f_{\gamma/A}(x_2, \mu^2) \hat{\sigma}(\gamma\gamma \rightarrow X), \quad (24)$$

for the case that beam 1 interacts incoherently, and similarly for the case that the incoherent and coherent beams are swapped. Here the $f_{\gamma/p,A}$ is the photon PDF in a proton (ion), which is defined as in, e.g., Ref. [23], as follows:

$$\begin{aligned} x f_{\gamma/p,A}(x, \mu^2) &= \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{dz}{z} \int_{\frac{x^2 m_i^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \cdot \left[\left(z p_{\gamma q}(z) + \frac{2x^2 m_i^2}{Q^2} \right) \right. \\ &\quad \left. \times F_2^{p,A}(x/z, Q^2) - z^2 F_L^{p,A}\left(\frac{x}{z}, Q^2\right) \right], \end{aligned} \quad (25)$$

where m_i is the nucleon or ion mass, depending on the PDF under consideration. By considering the relative sizes of these photon PDFs, or more precisely the luminosity,

$$\frac{d\mathcal{L}}{dm_{\gamma\gamma}^2} = \int dx_1 dx_2 f_{\gamma/p}(x_1, \mu^2) f_{\gamma/A}(x_2, \mu^2) \delta(sx_1 x_2 - m_{\gamma\gamma}^2), \quad (26)$$

with $\mu^2 = m_{\gamma\gamma}^2$, we can estimate the relative size of the incoherent to coherent cross sections.

For the incoherent case, we will scale the proton (neutron) PDF contributions appropriately by Z ($A - Z$), and we consistently account for the fact that either ion can interact incoherently. We will also consider the impact of applying further cuts on the final state. In particular, for UPC processes it is standard to apply a cut on the transverse momentum of the centrally produced system, p_T^X , to be below 1–2 GeV, in order to reduce the impact of various nonexclusive backgrounds. The incoherent case due to inelastic photon emission from the nucleons in the ion will be dominantly affected by such a cut, as here the inelastic nucleon form factors are significantly broader in Q^2 than the elastic and purely coherent cases. To estimate the impact of this, we can follow the approach of Ref. [22]

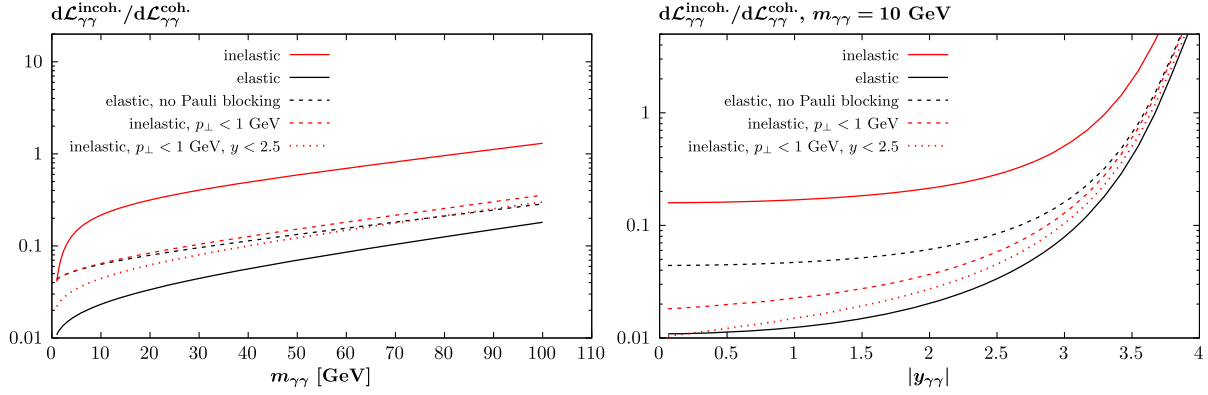


FIG. 3. Ratio of incoherent to coherent luminosities (26) as a function of (left) the diphoton invariant mass, and (right) the absolute diphoton rapidity, for $m_{\gamma\gamma} = 10$ GeV. Results are also shown in the inelastic case with additional cuts placed on the final-state quark in $q \rightarrow q\gamma$ emission generating the corresponding photon PDF, assuming LO kinematics.

and consider modifications to the photon PDF (25) for the incoherent, inelastic case. We can estimate this by considering leading order (LO) kinematics for the $q \rightarrow q\gamma$ splitting that generates the inelastic photon PDF, and then applying a cut of $p_{\perp} < 1$ GeV on the final-state quark in such an emission. We in particular require that

$$p_{\perp}^q = [(1-z)Q^2]^{1/2} < 1 \text{ GeV}. \quad (27)$$

As the central system transverse momentum is dominantly given by the recoil against the inelastic side this will provide a good matching to a $p_T^X < 1$ GeV cut. Further to this, exclusivity vetos are applied in experimental analyses, by requiring that no additional activity is present within the veto region. Without going into the specific experimental configurations of any given analysis, we can provide some demonstration of this by simply requiring that the rapidity of the final-state quark, again applying LO kinematics, lies above 2.5 units for the case that $\sqrt{s_{nn}} = 5.02$ TeV; see Ref. [22] for a definition of the kinematics in this case.

Results are shown in Fig. 3 for the ratio of the incoherent to coherent luminosities, as a function of (left) the diphoton invariant mass, and (right) the absolute diphoton rapidity, for $m_{\gamma\gamma} = 10$ GeV. Focusing first on the inelastic case, we can see that absent additional cuts this constitutes $\sim 20\% - 100\%$ of the coherent signal, depending on the invariant mass region considered. As $m_{\gamma\gamma}$ increases, this in particular increases significantly in size, due to the larger phase space available for inelastic emission (although we note that for larger invariant masses the absolute cross section is strongly suppressed).

This result is in line with the recent study of Ref. [10] [see Fig. 5 (left), published version], where based on this it is speculated that such incoherent emission may form a sizeable component of the coherent signal for existing experimental data. However, we can see from our results that once a cut as in Eq. (27) is applied, to match the signal selection in such analyses, the incoherent inelastic

contribution is significantly reduced, as we would expect from the fact that a large fraction of the photon PDF will come from the higher Q^2 region that is removed by such a cut. The result is now at the $\sim 2\% - 10\%$ level. For the event selection of most existing data, for which lower values of $m_{\gamma\gamma}$ dominantly contribute, the leads us to expect a percent level contribution. Applying the rapidity veto cut on top of this further reduces this, as we can see from the figure.

Turning to the elastic case, before Pauli blocking effects (4) are applied this provides a comparable contribution to the inelastic case, after imposing suitable cuts. However, Pauli blocking reduces this by a factor of $\sim 3 - 5$. The final contribution is again at the percent level. For the luminosity ratios vs diphoton rapidity, similar conclusions hold. We can see that at very forward rapidity the incoherent contribution begins to dominate, due to the sharply falling coherent flux with x , but these are in a region where the absolute cross section is strongly suppressed, and are also outside current experimental selections.

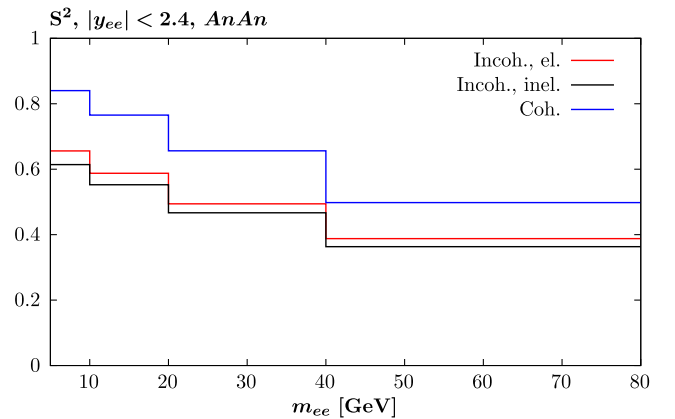


FIG. 4. Predicted survival factors for coherent and incoherent electron pair production, within the ATLAS [3] event selection, for $|y_{ee}| < 2.4$ and as a function of the dielectron invariant mass. Here, the survival factor corresponds to the cross section ratio using Eq. (23) to that using Eq. (9).

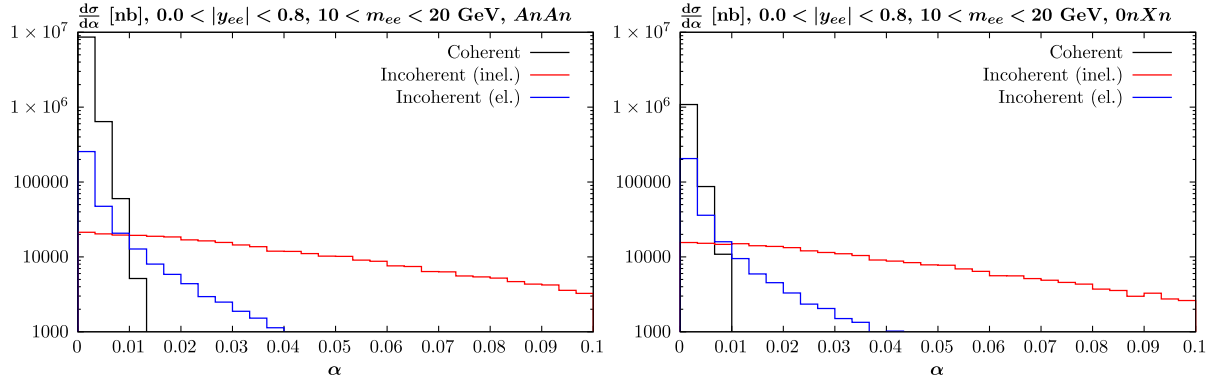


FIG. 5. Predicted coherent and incoherent electron pair production cross sections, differential with respect to the dielectron acoplanarity $\alpha = 1 - \Delta\phi_{ee}/\pi$, within the ATLAS [3] event selection, for $|y_{ee}| < 0.8$. Results are given for the (left) $AnAn$ and (right) $0nXn$ zero degree calorimeter (ZDC) selections, where 0, A, X correspond to no, any and greater than zero neutron emissions on the corresponding ion side, respectively.

Finally, we note that in addition to the above effects, the survival factor probability of no additional ion-ion interactions must be accounted for, as discussed in the previous section. This is shown in Fig. 4 for the event selection of the ATLAS measurement [3] of electron pair production in PbPb UPCs, which we will consider again in the following section. We can see that this provides some further suppression in the incoherent cross sections, that is somewhat more significant than in the purely coherent case. This is as we may expect given that even for nucleon-ion impact parameters that are themselves larger than the ion radius, for some ion-ion geometries there may nonetheless be ion overlap (see Fig. 2).

B. Comparison to ATLAS data on dielectron production

In this section we present predictions for electron pair production, and comparisons to the ATLAS measurement [3] of this process. The full event selection is as described in this paper, but of particular relevance is the cut, $p_{\perp}^{ee} < 2$ GeV, on the dielectron transverse momentum, which as discussed above tends to reduce the incoherent cross section rather significantly. In this analysis, a template fit is performed to the dielectron acoplanarity, $\alpha = 1 - \Delta\phi_{ee}/\pi$, with the coherent signal distribution modelled by the Starlight MC [24] and incoherent distribution modeled using single-dissociative events in pp collisions from SuperChic [12] (the only available possibility at the time), in both cases interfaced to Pythia [25] to account for final state radiation effects. In the latter case, as it is the same inelastic proton structure functions that enter the calculation of incoherent inelastic production presented here, we expect the acoplanarity distribution calculated in this way to closely match the results here; in particular, while the elastic beam is a proton in the ATLAS sample, rather than an ion, and hence has a broader Q^2 distribution, the dominant contribution to the dielectron acoplanarity in the tail region is due to the inelastic beam.

Predictions for these acoplanarity distributions are shown in Fig. 5, with the dielectron rapidity chosen to match the ATLAS results (Fig. 3 of Ref. [3]). As these are not provided corrected to the cross section level in the ATLAS analysis we do not present a direct comparison, but broadly speaking the trend observed in the data, and expected on general grounds, is present. In particular, the data are dominantly composed of a coherent signal that is sharply peaked at low acoplanarity, and a much broader tail due to the incoherent inelastic contribution. The incoherent elastic contribution has a broader acoplanarity distribution than the coherent signal, but this nonetheless falls significantly faster than the inelastic case, as we would expect. Due to the difference between this and the inelastic case, this contribution may therefore not be in general accounted for in the two component template fit performed by ATLAS, although as we will see below it is expected to be subdominant.

Results are also shown for the $0nXn$ ZDC selection. This is accounted for following the approach of Ref. [19], although some care is needed in the interpretation. In particular, for coherent production we simply include the probability for the $0nXn$ (which we define to include $Xn0n$ as well) final state, i.e., with additional ion excitation of one beam and without it for the other. However, for incoherent production (both elastic and inelastic) the incoherent ion will break up and will therefore automatically lie in the Xn category. Therefore, for the $0nXn$ category, we should multiply the incoherent cross sections by a $0nAn$ excitation probability, if beam 2 interacts inelastically, where in this case we do not include the corresponding $An0n$ possibility (which would correspond in part to the $XnXn$ final state), and vice versa if beam 1 interacts inelastically. For the $XnXn$ case, this is calculated in the usual way for coherent production, while for incoherent production it is by definition given by the remaining cross section that is not $Xn0n$. Broadly, the result of this is that the $0nXn$ requirement tends to reduce the coherent cross section more significantly than the incoherent, as is observed in Fig. 5 (right).

TABLE I. Predicted and measured incoherent, inelastic electron pair production fractional contributions to the purely coherent case, within the ATLAS [3] event selection, for $|y_{ee}| < 0.8$, $10 < m_{ee} < 20$ GeV. Results are shown for different ZDC selections.

	$AnAn$	$0nXn$	$XnXn$
ATLAS [3], $ y_{ee} < 0.8$, $10 < m_{ee} < 20$ GeV	0.043 ± 0.0026	0.099 ± 0.006	0.13 ± 0.01
Central prediction	0.039	0.12	0.17

Turning now to a more direct comparison to the data, incoherent to coherent cross section fractions are presented in Ref. [3], extracted using the template fitting approach described above, and for certain requirements on the dielectron rapidity and invariant mass. In the main published analysis, these are in particular given for the case that an additional $|y_{ee}| < 0.8$, $10 < m_{ee} < 20$ GeV cut is placed in the dielectron system. The results are given in Table I for the three relevant ZDC selections considered in the analysis. While the $0n0n$ case is also considered by ATLAS, this is predicted to have no incoherent contribution, and indeed the ATLAS data are consistent with this; hence we do not consider this case further in what follows. We can see that broadly the predicted fractions match the data rather well. We will discuss the theoretical

uncertainties further below but note that a $\sim 20\%$ uncertainty (at least) is certainly expected. Hence within these (and the experimental) uncertainties the agreement is very good.

Further comparisons to the unpublished results presented in Ref. [26] are shown in Figs. 6 and 7, as a function of the dielectron invariant mass and rapidity. Broadly speaking, the agreement is rather good, although in more detail there are regions, e.g., in the lowest invariant mass bin and for the $0nXn$ ZDC selection, where the agreement is somewhat less good. Nonetheless, this can give us some broad confidence in the overall results. In particular, and looking ahead to Sec. IV, this level of agreement validates the predictions for the incoherent background in the case of LbyL scattering to a reasonable level of precision.

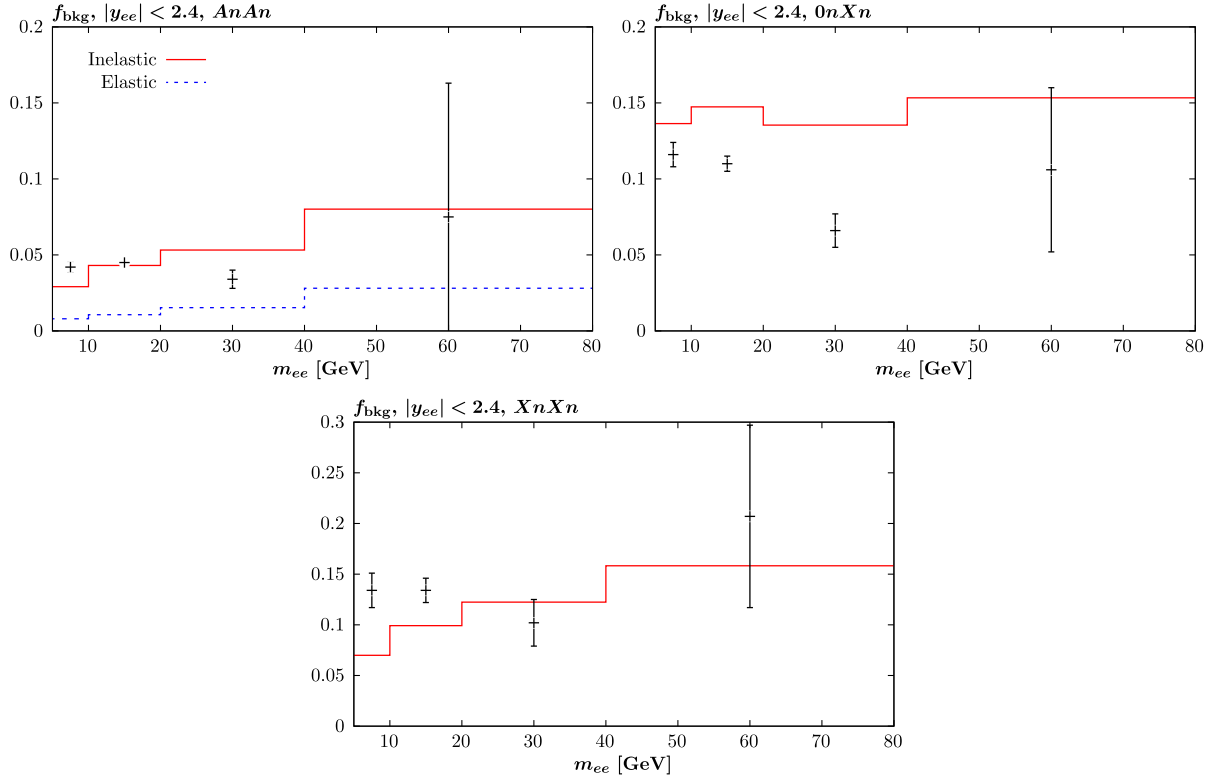


FIG. 6. Predicted incoherent, inelastic electron pair production fractional contributions to the purely coherent case, within the ATLAS [3] event selection, for $|y_{ee}| < 2.4$ and as a function of the dielectron invariant mass. The measured (unpublished) ATLAS values taken from Ref. [26], and results are given for the (top left) $AnAn$, (top right) $0nXn$ and (bottom) $XnXn$ ZDC selections, as defined in the caption of Fig. 5. For the $AnAn$ case the elastic incoherent contribution is also shown for comparison, but we emphasise that the ATLAS data correspond dominantly to the purely inelastic contribution, and hence this should not be included when comparing to these data.

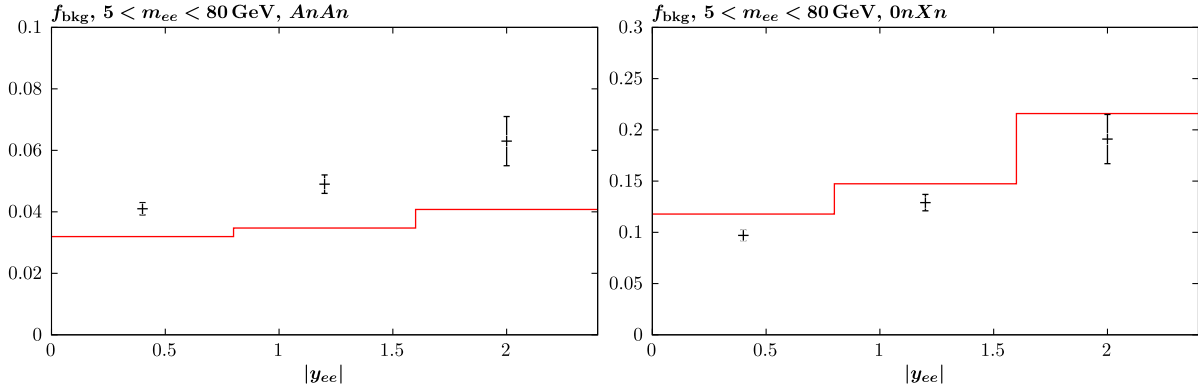


FIG. 7. Predicted incoherent, inelastic electron pair production fractional contributions to the purely coherent case, within the ATLAS [3] event selection, for $5 < m_{ee} < 80$ GeV and as a function of the magnitude of the dielectron rapidity. The measured ATLAS values, taken from Ref. [26], are also shown, and results are given for the (left) *AnAn* and (right) *0nXn* ZDC selections, as defined in the caption of Fig. 5.

In terms of the uncertainties on the predictions, a straightforward source of uncertainty on the inelastic contribution comes from the PDF uncertainty on the parton-level process. The PDF uncertainty due to the initial-state quark/antiquarks can be calculated using the standard MSHT20qed_nnlo PDFs [27] eigenvectors, and is found to be rather small, $\sim 1\%$. There is in addition an uncertainty due to the modeling of the lower Q^2 , W^2 region, but this is lower still. In broad terms, we expect this source of uncertainty to follow that of the photon PDF itself in Ref. [27], i.e., indeed be at the percent level. The above PDF set is, however, that of a free proton, and hence no account of nuclear modifications to the proton PDF has been accounted for here. To evaluate this, we have investigated the impact of changing from the p to Pb PDFs using the nNNPDF3.0 next-to-leading order PDF sets [28]. Using the Pb set tends to increase the predicted cross section somewhat, by $\sim 2\%$ – 5% , depending on the kinematic region considered. Further to this, there is some uncertainty due to the modeling of the survival factor, although as discussed in Ref. [29] this is expected to be very small, and moreover it will to some extent cancel in the cross section ratios considered here.

The naïve uncertainty is therefore at the $\sim 5\%$ level, and is hence rather small. However, this does not account for what is most likely to be the dominant uncertainty source, namely the impact of the experimental exclusivity requirement on the data. We do not apply any such veto in the above comparisons; however, we have seen from Fig. 3 that imposing a simple cut on the final-state quark assuming LO $q \rightarrow q\gamma$ kinematic in the inelastic process leads to a $\sim 20\%$ reduction in the expected cross section. A more precise calculation would provide a full account of the particle multiplicity distribution of the ion dissociation system, which is beyond the scope of the current study. Nonetheless, conservatively we can take an uncertainty of that order here, although we note that this effect will only act to reduce the predicted cross sections, which can therefore be viewed as upper bounds from this point of view.

In the elastic case, the dominant source of uncertainty is due to the modeling of the Pauli blocking effect as in Eq. (4). We have seen from Fig. 3 that this reduces the predicted contribution by a significant factor of ~ 3 – 5 . The physical origin of this effect is well understood, and accounting for it is a key feature of modeling nuclear scattering experiments, e.g., in the case of electron [30] and neutrino scattering off nuclear targets (see Ref. [31] for a recent summary). However, there is certainly some uncertainty in the precise implementation of this effect. To assess this, we investigate the effect of reducing the Fermi momentum used in Eq. (5) from 250 to 200 MeV. While this represents a rather aggressive level of reduction that is not necessarily favoured by data or models of nuclear structure, it will provide some indication of the level of model dependence. We find that this tends to increase the elastic cross section by $\sim 10\%$ – 20% , and so we can take this as a representative estimate for the corresponding uncertainty.

IV. LIGHT-BY-LIGHT SCATTERING

A. Signal-like contribution

We now consider the case of LbyL scattering, in particular having in mind the large acoplanarity backgrounds that are observed (and subtracted, using a template fit) in the ATLAS [2] and CMS [7] measurements of this process. We begin by considering the signal-like incoherent contribution to this process, i.e., as shown in Fig. 1 (right), while in the following section we will discuss additional contributions that should in general be accounted for here.

The predicted acoplanarity distributions for coherent and incoherent production are shown in Fig. 8 for the CMS event selection. For the ATLAS selection, the corresponding distributions are rather similar, and so are omitted for brevity. The incoherent contributions, which as we will see explicitly below enter at the percent level, are scaled by a factor of 10 (100) in the elastic (inelastic) cases for ease of

TABLE II. Predicted percentage signal-like contribution from incoherent production to the coherent light-by-light scattering, within the ATLAS [2] and CMS [7] event selections. Results with and without the acoplanarity cut $\alpha < 0.01$ (where $\alpha = 1 - \Delta\phi_{\gamma\gamma}/\pi$), are shown for comparison, although for both measurements this cut is applied.

	Incoherent, elastic	Incoherent, inelastic
ATLAS [2] selection	0.9	2.1
ATLAS [2] selection, $\alpha < 0.01$	0.5	0.4
CMS [7] selection	0.9	2.3
CMS [7] selection, $\alpha < 0.01$	0.4	0.4

comparison. This result is to be compared with Fig. 5 of Ref. [7], where the corresponding measurement is shown. We can see that the shape of the coherent contribution is clearly peaked towards low acoplanarity values, as expected, while the incoherent inelastic leads to precisely the sort of tail that is observed in the data. However, it is only after scaling this inelastic contribution by a factor of 100 that the size of this is seen to roughly match that observed in the data. The shape of the elastic contribution is rather more similar to the signal case, but is again significantly smaller than it. Finally, the contribution from the quark emission process that we will discuss further in Sec. IV B is shown for comparison.

In more detail, in Table II we show the predicted percentage contribution from incoherent production to coherent LbyL scattering, within the ATLAS [2] and CMS [7] event selections. We can see that for both elastic and inelastic production these are at the $\sim 0.5\%$ level after the acoplanarity cut $\alpha < 0.01$ is imposed, as it is for both measurements. We note that if the acoplanarity cut is not imposed the inelastic contribution is instead $\sim 2\%$, and if we in addition increase the upper cut on the diphoton transverse momentum from 1 to 2 GeV to match that imposed in the ATLAS dielectron analysis [3] then this increases further to $\sim 4\%$, i.e., in line with the prediction in Table I, as we would expect given the similar kinematic regions and the fact that the relative incoherent contribution from this type of diagram should be relatively process independent.

Once the tighter $p_\perp$ and acoplanarity cuts are imposed, on the other hand, incoherent production is expected to provide a very minor contribution in the signal region, and therefore cannot explain the size of the larger acoplanarity background observed in both analysis. In particular, although in Refs. [2,7] precise cross section extractions are not given, the number of background events is quoted, and gives a good indication of the measured backgrounds. In both of these analyses a template fit is performed, with the shape of the high acoplanarity background tail modeled using the SuperChic prediction [11] for QCD-initiated diphoton production. The normalization is then fitted to the data,

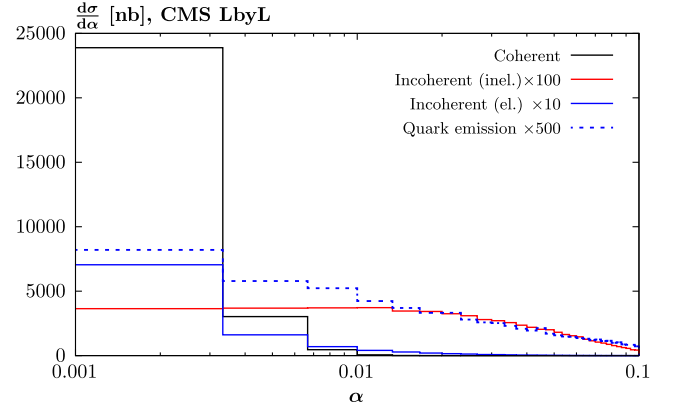


FIG. 8. Predicted coherent and incoherent light-by-light scattering cross sections, differential with respect to the diphoton acoplanarity $\alpha = 1 - \Delta\phi_{\gamma\gamma}/\pi$, within the CMS [7] event selection. The incoherent contributions are scaled as indicated in the legend for ease of comparison. The incoherent quark emission case, with a $\eta_q > \pm 3$ cut imposed, depending on the relevant incoherent beam, is also shown; this is discussed in Sec. IV B.

along with the coherent signal. As we will discuss further in Sec. IV D, while this production mechanism is different, this results in a similar (broad) shape to the acoplanarity distribution to the PI incoherent inelastic contribution, and hence such extractions can be treated relatively agnostically with respect to the precise production mechanism.

It is found in the ATLAS case that the ratio of background to signal events is $12/45 \sim 0.27$, while in the CMS case it is $10/13 \sim 0.77$. Given the significant statistical uncertainty on the latter result these are roughly consistent with each other. Therefore, broadly speaking these suggest a ratio of background to signal of ~ 0.5 , which is clearly significantly higher, by roughly 2 orders of magnitude, than the predictions in Table II. In other words, these cannot be the dominant background contribution. We recall in particular that the theoretical uncertainty on these predictions is expected to be $O(10\%)$, while the good agreement with the ATLAS data on dielectron production lends further evidence to the fact that this background is accurately modeled.

In the elastic case, we can see again from Fig. 8 that the predicted acoplanarity distribution is significantly narrower than the inelastic component, and more in line with the coherent signal. Therefore such a contribution will not contribute significantly to the observed backgrounds. On the other hand, it will also not be effectively subtracted by the template fitting procedure, and hence should be explicitly accounted for in future analyses, even if as we can see the expected contribution is small.

B. Quark emission diagrams

Considering again the Feynman diagram in Fig. 1 in the inelastic case, namely where the upper line is interpreted as quark/antiquark parton within a constituent nucleon in the

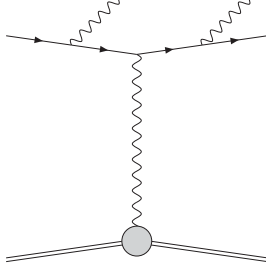


FIG. 9. Representative quark emission Feynman diagram for incoherent contribution to light-by-light scattering signal in AA ultraperipheral collisions.

ion, it is clear that this is not the only contributing diagram. There are in particular various diagrams where either one or both photons are emitted directly by the upper quark line. This is shown in Fig. 9, where the other diagrams with the photons emitted in different configurations by the upper quark lines are implicit. We note that there are in general various possible sources of collinear singularities in these diagrams, but these are not present in the phase space region relevant here. In particular, the final-state collinear quark-photon singularity is removed by the requirement that the photons are produced centrally and with sufficient $p_\perp$, while the outgoing quark line must lie outside the central region, or more precisely the exclusivity veto region, which can extend further again. The initial-state singularity, due to a collinear splitting from the coherent photon, is similarly removed by the exclusivity requirement that the final-state quark lies in the opposite hemisphere to the direction of this photon.

This class of diagram is particularly relevant as we can see by comparison of the two cases that it actually enters at a lower order in α . In particular, while the signal-like incoherent production cross section is $O(\alpha^5)$, the quark emission cross section is $O(\alpha^3)$, and hence naïvely should be enhanced by a significant factor of $1/\alpha^2 \sim 10^4$, which given that the signal-like incoherent contribution enters at the percent level of the coherent, might suggest that this is even larger (by a factor of $\sim 10^2$) than the purely coherent signal. We note that this difference in the order in α also clearly demonstrates that there is no question of gauge invariance from treating the diagrams individually, although in principle they should be added at the amplitude level, with interference included; we will comment further on this below.

However, there are various factors that will counteract this enhancement. First, it is known from basic maximally helicity violating considerations that the $\gamma(p_a)q(p_b) \rightarrow \gamma(p_1)\gamma(p_2)q(p'_b)$ amplitude [32] is proportional to the spinor production of the incoming and outgoing quark momenta, as follows:

$$\mathcal{M}(\gamma(p_a)q(p_b) \rightarrow \gamma(p_1)\gamma(p_2)q(p'_b)) \propto \langle p_b p'_b \rangle. \quad (28)$$

This implies (combined with the fact that the rest of the amplitude is perfectly regular in this limit) that the

scattering amplitude in fact vanishes in the limit of forward quark scattering, with the outgoing quark momentum $p'_b \propto p_b$. As this is precisely the limiting case of the phase space region of interest, namely where the outgoing quark travels in the forward direction and does not enter the rapidity veto region, we can expect this to suppress the corresponding cross section. Indeed, an explicit check of the, e.g., outgoing quark pseudorapidity distribution demonstrates that the contribution from the forward region is strongly suppressed relative to the standard signal-like case, with a preference for the phase space region towards the edge of the veto region. We can see this in Fig. 10, which shows the normalized distribution with respect to the outgoing quark pseudorapidity, η_q , for both of these contributions. This is defined in the lab frame, which is distinct from the nucleon-ion rest frame in which the incoherent cross section is typically calculated. The difference in the two cases is clear, although we note that this is also driven by the t -channel enhancement from the photon propagator in the signal-like case that is not present for the quark emission process.

In addition to this, we can see that the signal-like diagram in general receives a t -channel $\sim 1/Q^2$ enhancement from the upper photon propagator,¹ which in the quark emission diagrams is replaced by a factor of order $\sim 1/M_{\gamma q}^2$, where $M_{\gamma q}$ is the invariant mass of the initial-state quark-photon system. For the inelastic incoherent case we are considering, the average photon Q^2 is $O(\text{GeV}^2)$ and is significantly larger than in the case of elastic incoherent, and certainly purely coherent, production. Nonetheless this will clearly lead to a relative suppression in comparison to the signal-like incoherent case, which increases with $M_{\gamma q}$ and hence the diphoton invariant mass $M_{\gamma\gamma}$. Very roughly speaking, if we assume that $\langle Q^2 \rangle = 1 \text{ GeV}^2$ and $M_{\gamma q} \sim M_{\gamma\gamma}$ then for LbyL scattering in the ATLAS and CMS event selection this will lead to a suppression of

$$\frac{1 \text{ GeV}^2}{M_{\gamma\gamma}^2} \sim \frac{1}{25},$$

i.e., 1–2 orders of magnitude, and in reality given $M_{\gamma q} > M_{\gamma\gamma}$, closer to 2, although clearly the precise level of suppression will depend on the specifics of the processes and their kinematics. Nonetheless, we can broadly expect this to reduce the overall enhancement of the quark emission process with respect to the signal-like from $\sim 10^4$ to $\sim 10^2$, and further at higher masses.

Moreover, we note that the LbyL scattering, $\gamma\gamma \rightarrow \gamma\gamma$, cross section itself receives some further numerical enhancement from the summation over the different particles in the intermediate loop. From Ref. [33] the (dominant) lepton loop

¹We note that as usual one factor of $1/Q^2$ is effectively canceled by the numerator of the upper $qq\gamma$ vertex.

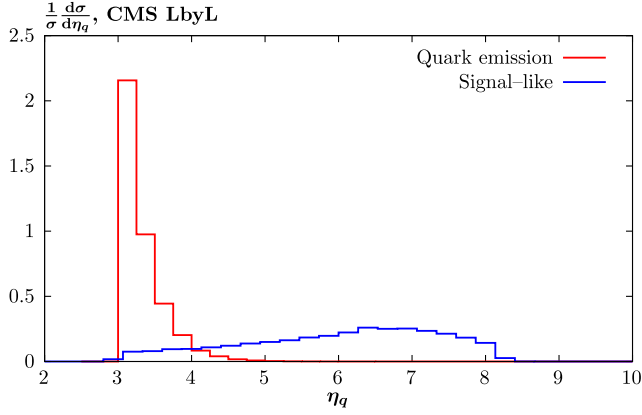


FIG. 10. Normalized distribution with respect to the pseudorapidity, η_q , of the outgoing quark line, in the lab frame, for quark emission and signal-like incoherent contributions to the light-by-light scattering signal in AA ultraperipheral collisions. The case where the incoherently interacting ion is in the positive z direction is considered for concreteness, and the final-state photons lie within the CMS [7] LbyL scattering fiducial region, while a cut of $\eta_q > 3$ is imposed.

contribution to the LO amplitude for, e.g., all positive photon helicities is given by

$$\mathcal{M}_{++++} = 8\alpha^2, \quad (29)$$

in the massless lepton limit. Hence summing over the three lepton species we have an enhancement of $\sim (8 \times 3)^2 \sim 600$. Clearly the precise level of enhancement will depend on the precise structure of the corresponding quark emission diagrams, but we note that given the photons are in this case purely emitted from the quark line, this will lead to a further $\sim 1/e_q^4$ suppression with respect to the signal-like case, i.e., by roughly an order of magnitude.

Finally, we recall that there are additional cuts placed on the diphoton system, namely to restrict the transverse momentum to be $p_{\perp}^{\gamma\gamma} < 1$ GeV as well as on the acoplanarity. Given that the individual transverse momenta are peaked at 2.5 GeV, i.e., not significantly higher than this cut, this will not necessarily reduce the cross section dramatically, in particular having in mind that we are considering the suppression relative to the incoherent signal-like contribution, which is also suppressed by such a cut. Nonetheless this will lead to further suppression, which will certainly become more relevant for higher photon transverse momenta.

Putting the above effects together, we may expect this contribution to enter at something like the same level as the purely signal-like incoherent contribution, but not to be significantly enhanced with respect to it. To verify this, we have implemented this process following the approach of Ref. [13]. Namely, this type of diagram can be evaluated using the general expression (2) but making the substitution

$$\rho_1^{\mu\mu'} \rho_2^{\nu\nu'} M_{\mu'\nu'}^* M_{\mu\nu} \rightarrow \frac{Q_1^2}{4\pi\alpha(Q_1^2)} \int \frac{dM_1^2}{Q_1^2} \rho_2^{\mu\mu'} \sigma_{\mu\mu'}^1, \quad (30)$$

where the integration is as usual performed simultaneously with the other phase space integrals, and similarly if beam 2 instead interacts incoherently. At LO we have

$$\sigma_{\mu\mu'}^i = \sum_{j=q,\bar{q}} f_j(x_{B,i}, \mu_F^2) \langle A_\mu^i A_{\mu'}^{i*} \rangle, \quad (31)$$

where A_μ^i is the corresponding $\gamma^* + q \rightarrow \gamma\gamma + q$ amplitude as in Fig. 9, with a collinear initial-state quark/antiquark from beam i , carrying proton momentum fraction $x_{B,i}$. Further details of the precise implementation of this, and the relation to the standard collinear result are given in Appendix A of Ref. [13]. These amplitudes are calculated from the standalone output of MadGraph5_aMC@NLO [34,35].

As discussed above, the singularity structure of this process requires that we impose an exclusivity veto on the final-state quark, in order to remove the collinear quark-photon regions. We do this by simply imposing that the final-state quark pseudorapidity lies above some cut that is outside the central region where the photons are present. We in particular consider $\eta_q > 3-4.5$ for the case that beam 1 (defined as traveling in the positive z direction) interacts incoherently, and similarly in the negative rapidity region if it is beam 2. The lower limit is almost certainly too loose to match the exclusivity vetoes imposed experimentally, but nonetheless gives some estimate. For simplicity, we neglect the impact of the ion-ion survival factor, though this may in general further suppress the relative contribution from the quark emission diagrams. We find that the predicted cross section is rather strongly sensitive to the precise cut, but even for the looser $\eta_q > 3$ case, the predicted cross section is a factor of ~ 3 lower than the signal-like contribution within the ATLAS or CMS fiducial regions. For more stringent cuts, this reduces even further.

Therefore, we can safely ignore this type of contribution, and indeed any interference it has with the signal-like case. More broadly, there are of course various one-loop corrections to the quark emission diagrams that would then enter at the some order in α as the signal-like one, but these will clearly be further suppressed, and can be neglected. We note that in the case of lepton pair production, there is a similar topology of diagram, where the lepton pair is emitted from the quark line directly. However, as discussed in Sec. 3 of Ref. [13] these enter at the percent level in general in comparison to the signal-like incoherent case and hence for current purposes we can safely neglect this.

C. Exclusive two-photon photoproduction: Brief remarks

Having considered the leading-order inelastic contribution to the case that the two photons are emitted directly

from the interacting nucleon, as per Fig. 9, it is worth commenting on the fact that such a process can also proceed purely elastically, that is with the outgoing nucleon (but not ion) remaining intact. Indeed, this has been discussed in detail in Refs. [36–38] in the context of lepton-nucleon scattering. The key observation here is that the production amplitude is purely imaginary, and proportional to the valence generalized quark distributions taken at the border values of $\xi = \pm x$ (i.e., precisely in the skewed regime relevant to gluon-initiated central exclusive production processes [39], in that case for the generalized gluon PDF).

As an aside, we note that the cancellation of the real amplitude in fact follows from the same observation made above, see Eq. (28), given that the amplitude precisely corresponds to such forward kinematics. However, from the point of view of phenomenology the relevant factor is that the C-parity of the overall proceeds leads to only the generalized valence quark distributions contributing, which will therefore strongly suppress the contribution from this process, in particular for the intermediate x values of relevance here. Therefore it is not expected to be a relevant source of background to the LbyL scattering signal. It remains possible, however, that this t -channel quark exchange topology may nonetheless play a role if we move away from the purely exclusive region, although in general this will be suppressed by the cut on the diphoton $p_\perp$; a detailed analysis of these issues is left to future work.

D. QCD-initiated production revisited

Having considered the various possible purely quantum electrodynamics (QED) contributions for the higher acoplanarity background to LbyL scattering, we have seen that these are not expected to provide anything beyond a percent-level correction in the signal (low acoplanarity) region. However, as discussed above, this is in contrast to the data-driven determinations by both ATLAS [2] and CMS [7], which find contributions that are at least an order of magnitude higher than this. In both of these analyses, the shape of the high acoplanarity background tail is modeled using the SuperChic prediction [11] for QCD-initiated diphoton production. However, the normalization is allowed to be free in the fit, and is also found to be significantly higher than this QCD-initiated contribution predicted in Ref. [11], by roughly an order of magnitude or more.

With this in mind, we briefly revisit the case of QCD-initiated production here, and discuss the extent to which such a larger normalization could be consistent with expectations. We recall that this process proceeds according to the “Durham” perturbative QCD model for exclusive QCD-initiated production (see, e.g., Ref. [39] for a review), namely the underlying process is due $gg \rightarrow \gamma\gamma$ scattering, i.e., exactly as in Fig. 1 (left) but with the initial-state photons replaced by gluons, while an additional soft “screening” gluon must then be exchanged between the interacting nucleons to ensure a color-singlet exchange.

As discussed in detail in Ref. [11], when generalizing the proton-proton case to that of heavy ion collisions, we can consider two possibilities, namely coherent and incoherent production. In the former case, the ion acts as a coherent source of gluons and remains intact afterwards, while in the latter we simply consider the case that the individual nucleons can act incoherently as individual sources of gluons, with the ion breaking up, in both cases in the appropriate color-singlet configuration; these mechanisms are fully analogous to the cases of coherent and incoherent PI production discussed above.

In Ref. [11] the cases where both ions interacted either coherently or incoherently were considered, and so for completeness we extend this here to consider the “mixed” case where one ion interacts coherently and one interacts incoherently. We have also improved the modeling of the incoherent case, making use of the approach discussed previously for the case of PI production. In particular, in Ref. [11] the ion-ion survival factor was accounted for by multiplying the nucleon-nucleon cross section by

$$\langle S_{\text{incoh}}^2 \rangle = \frac{\int d^2 b_{1\perp} d^2 b_{2\perp} T_n(b_{1\perp}) T_n(b_{2\perp}) e^{-\Omega_{A_1 A_2}(b_{1\perp} - b_{2\perp})}}{\int d^2 b_{1\perp} d^2 b_{2\perp} T_n(b_{1\perp}) T_n(b_{2\perp})}, \quad (32)$$

where $b_{i\perp}$ are as usual the impact parameters between the ion centers and the interacting nucleons. In other words, this approximates the result by neglecting the range of the underlying nucleon-nucleon interaction. We can, however, readily account for this by generalizing the approach described in Sec. II C. Namely, Eq. (20) becomes

$$\begin{aligned} \Gamma_{A_1 A_2}^{\text{QCD, incoh}}(s, b_\perp) \\ = \int d^2 \tilde{b}_{1\perp} d^2 \tilde{b}_{2\perp} T_A(\tilde{b}_{1\perp}) T_A(\tilde{b}_{2\perp}) \Gamma_{AA}(s, b'_\perp), \end{aligned} \quad (33)$$

where $\mathbf{b}'_\perp = \mathbf{b}_\perp - \tilde{\mathbf{b}}_{1\perp} - \tilde{\mathbf{b}}_{2\perp}$ and then the cross section can be calculated in the same manner. The previous approach will tend to underestimate the cross section somewhat, although the effect is rather mild, at the $\sim 10\%$ level. The mixed case is calculated in a similar fashion.

Given the sizeable large acoplanarity background observed in the ATLAS and CMS data, we also, in order to be as aggressive as is reasonably possible in terms of the size of the predicted incoherent cross section, now omit the additional pp survival factor in the calculation. That is, the above expression is assumed to capture the totality of the soft survival effects. As discussed in Ref. [11], there are reasons to believe that this will overestimate the expected cross section. In particular, as the exclusive production process occurs at the ion periphery, the nucleon density is low and hence the average number of nucleon-nucleon interactions contained in the above expression can be below 1. Nonetheless, this can serve to provide an upper bound on

TABLE III. Predicted percentage gluon-initiated (QCD) contributions the coherent light-by-light scattering final state, within the ATLAS [2] and CMS [7] event selections. Results with and without the acoplanarity cut $\alpha < 0.01$ (where $\alpha = 1 - \Delta\phi_{\gamma\gamma}/\pi$), are shown for comparison, although for both measurements this cut is applied. The MSHT20qed_LO PDFs [40] are used.

	QCD, incoherent	QCD, mixed	QCD, coherent
ATLAS [2] selection	7.6	1.4	0.7
ATLAS [2] selection, $\alpha < 0.01$	1.2	0.4	0.6
CMS [7] selection	9.4	1.7	0.8
CMS [7] selection, $\alpha < 0.01$	1.7	0.5	0.5

the cross section, with the question of accounting for the data kept in mind. We have also corrected the implementation of the coherent production cross section used in Ref. [11], where the Pomeron-nucleon form factor was not implemented correctly. This results in a somewhat larger prediction cross section, but a factor of up to ~ 2 , although as we will see the predicted rate remains negligible. Finally, we now make use of an LO PDF set, namely the MSHT20qed_LO PDFs [40], for which the gluon in the relevant x region tends to be rather larger than at next-to-next-to-leading order (NNLO); a PDF set at the latter order was in particular used in Ref. [11]. This results in a significantly larger predicted cross section, by up to an order of magnitude, which as discussed further below is in better agreement with the collider detector at fermilab (CDF) measurement of exclusive diphoton production [41].

In Table III we show the predicted QCD-initiated cross section for the LbyL diphoton final state. We note that *incoherent* corresponds here to the case that both ions interact incoherently, which is in contrast to the nomenclature used above in the PI case, where this referred to the case that one ion interacted incoherently; here, we use the label *mixed* to refer to this case. We can see that the fractional contributions within the ATLAS and CMS event selection are at the subpercent level, and indeed the per-mille level once the acoplanarity and diphoton transverse momentum cuts are imposed. The fully incoherent case is predicted to give the largest contribution, as we might expect given the short-range nature of the QCD interaction and consequent suppression from requiring the ion to interact as a fully coherent QCD source. The mixed and coherent are somewhat comparable after the full event selection is applied. In comparison to Ref. [11] the predicted coherent and incoherent cases are significantly larger, as expected from the discussion above. However, even in this most aggressive scenario with respect to the size of the cross sections, these remain at the percent level or less once we have accounted for all fiducial cuts.

The predicted acoplanarity distributions are shown in Fig. 11. As expected, and already observed in Ref. [11], the QCD-initiated contribution has a rather broader acoplanarity tail than the LbyL scattering signal. This is, however, less

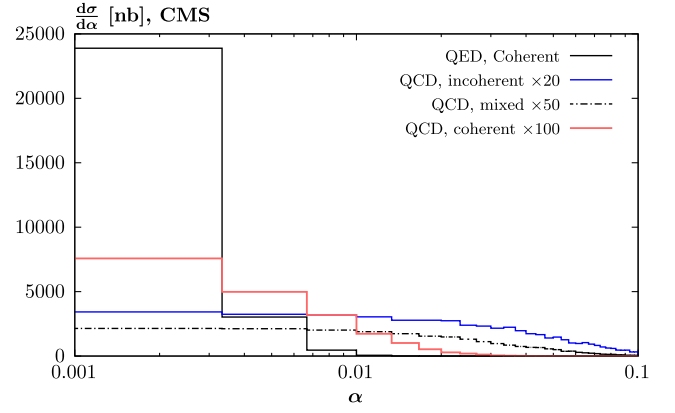


FIG. 11. Predicted coherent photon-initiated (QED) and gluon-initiated (QCD) light-by-light scattering cross sections, differential with respect to the diphoton acoplanarity $\alpha = 1 - \Delta\phi_{\gamma\gamma}/\pi$, within the CMS [7] event selection. The QCD contributions are scaled as indicated in the legend for ease of comparison.

evident in the purely coherent case, due to the coherence requirement restricting the initial-state transverse momentum transfer to be relatively low. Therefore, it is only the incoherent and mixed contributions that can, it appears, potentially explain the large acoplanarity backgrounds observed by ATLAS and CMS. Moreover, we can see that it is only after scaling these contributions up by significant factors that this is possible; in other words, these can potentially describe the shape of this background in acoplanarity, but not the size of it.

However, some care is possibly needed here. In particular, there are various important sources of uncertainty in the modeling of this process. Indeed it is explicitly commented on in the CMS analysis [7] that the “large uncertainties” in the modeling of this process justify the overall normalization being left free. Now, while it is certainly true that there are important sources of uncertainty here, as discussed in, e.g., Ref. [39], there are limits on this provided by measurements in proton-proton (and proton-antiproton) collisions at the LHC and Tevatron. With this in mind, it is worth examining whether the theoretical uncertainties in the modeling of the QCD-initiated process can indeed account for the discrepancy between the data and theory, under the assumption that this process is indeed the dominant source of the observed background. The key fact here is that the observed background ratio is found to be $\sim 50\%$ of the LbyL component in the signal region, while from Table III we can see that the dominant incoherent component is expected to be at the percent level, i.e., over an order of magnitude lower than this.

As we might expect, it is indeed very hard to justify such a discrepancy on the grounds of a theoretical uncertainty of this order. A principle element of this is the existing CDF measurement of exclusive diphoton production [41]. In the original analysis it is observed that the Durham model predictions are in rather good agreement with this, assuming

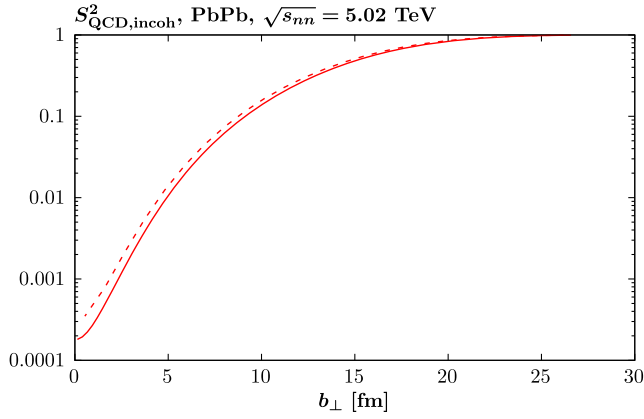


FIG. 12. Predicted survival factor for incoherent gluon-initiated diphoton production in PbPb collisions, at $\sqrt{s_{nn}} = 5.02$ TeV, as a function of the nucleon-nucleon impact parameter, $b_{\perp}$. The dashed curve corresponds to the extreme and unrealistic case of dividing the nucleon-nucleon inelastic cross section by a factor of 2, and is shown for the sake of illustration. The result corresponds to a suitable generalization of Eq. (33), as described in the text.

a suitable LO PDF set is taken, as it is here. Taking the latest SuperChic MC implementation, and with all shared theoretical ingredients with the heavy ion case (PDF choice, scale, etc.) kept the same, we predict a central cross section of 1.1 pb, which is in good agreement with the CDF measurement of 2.5 ± 0.6 pb (where we have roughly added the experimental uncertainties in quadrature for brevity), if slightly below it. The Tevatron c.m.s. energy here is 1.96 TeV, which is only a factor of just over 2 less than the nucleon-nucleon c.m. system energy relevant for the ATLAS and CMS measurements. Hence, while there will clearly be some extrapolation uncertainty between these two energies, and otherwise differences from the particular cuts, etc., this gives us very good grounds for believing that the underlying nucleon-nucleon cross section is well modeled.

Now, we are interested here in the case of heavy ion collisions, and so there is certainly an additional source of uncertainty due to the modeling of the ion-ion cross section in terms of the nucleon-nucleon cross section, in essence due to the modeling of the underlying ion-ion survival factor. Focusing on the case of purely incoherent production (although for the coherent and mixed cases the situation is similar), this survival factor suppression, which is given approximately by Eq. (32), is indeed rather significant, at the $\sim 10^{-4}$ level. This is demonstrated in an alternative manner in Fig. 12, where the effective survival factor as a function of the nucleon-nucleon impact parameter is shown; the average suppression, which lies close to the $b_{\perp} \sim 0$ region, due to the short-range nature of the QCD interaction, can indeed be seen to be at this level. This plot then demonstrates the extent to which a significantly higher survival factor, and certainly one that is increased by the ~ 1 order of magnitude or more required to match the ATLAS and CMS data, is ruled out on rather general grounds. In particular, this is only achieved by

going to impact parameters at the ~ 5 fm level, which is clearly well outside the range of QCD. Another way to demonstrate this is shown by the dashed curve, which comes from the extreme and unrealistic case of dividing the nucleon-nucleon inelastic cross section by a factor of 2. In even this case, the survival factor in the low $b_{\perp}$ region increases somewhat, but clearly not sufficiently; this serves as a clear indication that any reasonable variation of the underlying model parameters is not going to provide the necessary source of uncertainty. We in addition recall that as discussed above the approach taken here to the calculation of the ion-ion survival factor 1 is taken to be as aggressive as is reasonably possible in terms of the size of the predicted incoherent cross section, further underlining the above conclusions.

As a final comment on this, we note that in the original analysis of Ref. [39] the result of simply scaling the nucleon-nucleon cross section by a factor of $\sim A^2 R^4$, where the R^4 factor accounts for nuclear shadowing effects, was shown. If we simply scale the predicted nucleon-nucleon cross section by a factor of A^2 , then for the CMS event selection we find that the predicted incoherent cross section is comparable in size, if somewhat larger than, the coherent PI signal. Therefore, in order to match the observed CMS background would require rather little suppression from ion-ion interactions. In terms of nuclear shadowing effects, we note that these are not included in Table III; in other words a free proton PDF is used throughout, but if an appropriate nuclear PDF is used then the predicted cross section is further reduced.

V. SUMMARY AND OUTLOOK

In this paper we have for the first time provided a complete treatment of incoherent photon-initiated production in ultraperipheral heavy ion collisions, focussing on the dilepton and diphoton final states. In the former case we have compared to the ATLAS measurement [3] of dielectron production and found that our predictions match the data very well. In the latter case we have shown that this does not provide the dominant contribution to the observed background to the purely coherent LbyL signal. We have in addition revisited the case of QCD-initiated diphoton production and have found that a similar conclusion holds, as we will summarize below.

The new modeling of incoherent PI production, and the updated modeling of the QCD-initiated contributions to the LbyL final state will be included in the SuperChic MC generator [42], which is publicly available at [42].

We present below a summary of the implications of the above results for our understanding of the ATLAS [2] and CMS [7] measurements of LbyL scattering, and in particular the larger acoplanarity background to this process. We recall that this is extracted using a template fit to the diphoton acoplanarity, making use of the fact that the signal is highly peaked at low values of acoplanarity and therefore

in principle well separated from the background, which extends to higher values. The size of the background in the $\alpha < 0.01$ signal region is found to be rather large in terms of the number of events, composing $\sim 50\%$ of the signal (somewhat less for ATLAS, and somewhat more for CMS).

Without considering any theoretical input, as a first point of comparison we can consider the measurement of dielectron production by ATLAS [3]. This in particular applies a similar (though not identical) template fitting technique to extract the coherent UPC signal, but also provides measurements of the cross section fraction due to the higher acoplanarity tail that is not described by the coherent signal modeling. Across the entire acoplanarity region this is found to be $\sim 4\%$ in the full fiducial region, and hence in the lower $\alpha < 0.01$ region would be lower still.

We note that the kinematic regions and c.m. system energies are similar between the dielectron and LbyL measurements. Therefore, based on the above information alone, it is clear that the source of high acoplanarity background in the LbyL case must be dominantly of a different nature to that in the dielectron case. This has been explicitly verified in this paper, where we have shown that the size and shape in acoplanarity of the dielectron background is well described by incoherent PI production, where one of the ions interacts incoherently and breaks up. The predictions for this background in the LbyL case are as expected at the same order, and in fact after accounting for the precise kinematic cuts lower still. These therefore lie over an order of magnitude below the background event fractions measured by ATLAS and CMS.

The question is then what differences there are between the dielectron and LbyL processes that can lead to the differing size of the higher acoplanarity background in the two cases. One factor is that the final state in the LbyL case can be coupled (indirectly) to gluons, and hence be produced via a QCD-initiated mechanism at the same loop order as the PI case, while this is not the case for dielectron production. A second factor is that the diphoton and dielectron final states themselves are different, such that different production topologies enter in the incoherent PI case; for diphoton production it is of particular note that the LbyL signal proceeds via a loop-induced process and is therefore relatively suppressed. We have considered both possibilities here.

In particular, we have revisited the modelling of QCD-initiated production and find that no realistic modeling of this process, and account of the theoretical uncertainties in it can lead to a background of the size observed by ATLAS and CMS, which lie about an order of magnitude above theoretical expectations. The size of the nucleon-nucleon cross section is in particular constrained by earlier CDF measurement of this process [41] to be of the order of that in our theoretical predictions, such that to match the ATLAS and CMS observations would essentially require

no additional suppression from the fact that most of these nucleon-nucleon interactions will lead to ion-ion overlap and an inelastic final state. This is certainly at odds with the short range nature of the QCD interaction.

We have also considered for the first time the impact of the quark emission topology to the incoherent PI process, where the interacting quark in the incoherent ion emits the final-state photons directly. Given this bypasses the need for an intermediate loop as in the default LbyL process, it enters at a lower order in α and hence might play a role; it is for this reason that the relative contribution could be enhanced relative to the dielectron case. However, on general grounds we find this is not expected to be the case, and we have verified this with an explicit calculation. Related to this, we have also commented on the possibility of purely exclusive two-photon photoproduction off the nucleon. In general this is expected to again be suppressed, being proportional to the valence generalized quark PDFs, although a remaining possibility remains that due to the related inelastic channel, a full analysis of which is left to future work.

Therefore, the reason for the increased size of the broad acoplanarity background in the LbyL case remains unclear. Given the size of this background, this is certainly an unsustainable situation with respect to future measurements. Even if the template fitting technique is viewed as a purely data-driven method, the accuracy of the extrapolation into the signal region and the dependence on this on the other kinematics of the process will clearly depend on this background being accurately modeled. As the current available models for the various components do not match the observed normalization of this background, it is hard to argue convincingly that this is the case.

In the future, there are approaches that can be used to mitigate this. An obvious possibility is study to LbyL scattering in the $0n0n$ ZDC class, for which the above incoherent (QCD and PI) mechanisms should not be present. If a reasonable high acoplanarity background persists for this ZDC class then it would certainly point to some other source of background being present. Further to this, an additional LHC measurement of diphoton production in pp collisions would provide a cross-check of the so-far only measurement of this process, by the CDF Collaboration, and hence of the modeling of the QCD-initiated background. In addition, providing measurements of the higher acoplanarity background, differential in the final-state kinematics, would assist in the modeling of this background. With such further analysis, we may hope to shed some light on this challenging issue.

ACKNOWLEDGMENTS

The author thanks Iwona Grabowska-Bold for initial interesting discussions and for motivating this study, and Valery Khoze and Misha Ryskin for reading through the manuscript and for their many useful comments. The

author also thanks the Science and Technology Facilities Council (STFC), part of U.K. Research and Innovation, for support via Grant Award No. ST/T000856/1.

DATA AVAILABILITY

The data that support the findings of this article are openly available [42]; embargo periods may apply.

APPENDIX: STRUCTURE FUNCTION INPUTS

In this appendix, we describe the inputs for the inelastic and elastic structure functions entering the photon density matrix (3) in Sec. II B. For an ion beam we are only interested in the case of elastic photon emission, in which case the structure function F_1 is given in terms of the magnetic form factor of the ion, which is suppressed by $\sim Z^2$ with respect to the electric form factor contribution, F_2 , and can be safely dropped. We in particular recall that for incoherent production we are considering the case where one ion continues to interact elastically and hence from Eq. (1) this is suppressed by a factor of $\sim Z$ at the cross section level. That is, the magnetic form factor contribution remains parametrically suppressed with respect to incoherent production, as well as coherent. We therefore have that

$$\begin{aligned} F_2^{\text{el,A}}(x_{B,i}, Q_i^2) &= F_p^2(Q_i^2) G_E^2(Q_i^2) \delta(1 - x_{B,i}), \\ F_1^{\text{el,A}}(x_{B,i}, Q_i^2) &= 0. \end{aligned} \quad (\text{A1})$$

Here G_E is the ‘‘Sachs’’ form factor of the proton. That is, we have included a term due to the form factor of the protons within the ion; numerically this has a negligible impact, as the ion form factor falls much more steeply; however, we include this for completeness. The ion form factor $F_p(Q^2)$ is given in terms of the proton density in the ion, and is calculated as described in Ref. [11].

For elastic emission from a proton we have

$$\begin{aligned} F_2^{\text{el,p}}(x_{B,i}, Q_i^2) &= F_E(Q_i^2) \delta(1 - x_{B,i}), \\ F_1^{\text{el,A}}(x_{B,i}, Q_i^2) &= G_M^2(Q_i^2), \end{aligned}$$

where

$$F_E(Q_i^2) = \frac{4m_p^2 G_E^2(Q_i^2) + Q_i^2 G_M^2(Q_i^2)}{4m_p^2 + Q_i^2}, \quad (\text{A2})$$

and G_M is the magnetic Sachs form factor of the proton. For these, we use the fit from the A1 Collaboration [43]. Although elastic emission from a neutron is nonzero and can be accounted for following, e.g., the approach of Ref. [44], we have verified that this provides a negligible contribution and hence it can be safely omitted in the current study.

For inelastic emission, we use CEBAF large acceptance spectrometer (CLAS) data [45] on inelastic structure functions in the resonance $W^2 < 3.5 \text{ GeV}^2$ region, primarily concentrated at lower Q^2 due to the W^2 kinematic requirement; the HERMES fit [46] to the inelastic low $Q^2 < 1 \text{ GeV}^2$ structure functions in the continuum $W^2 > 3.5 \text{ GeV}^2$ region; inelastic high $Q^2 > 1 \text{ GeV}^2$ structure functions for which the perturbative quantum chromodynamics prediction in combination with PDFs determined from a global fit provide the strongest constraint (we take the ZM-VFNS (zero mass variable flavour number scheme) at NNLO in QCD predictions for the structure functions as implemented in APFEL [47], with the MSHT20qed_nnlo PDFs [27] throughout). All of the above inputs are precisely as in earlier studies; see, e.g., Refs. [12,13].

The latter discussion applies to the case of inelastic emission from a proton. For the neutron case we follow the approach of Ref. [44], namely for the resonant region we take the explicit CLAS fit to the inelastic neutron structure functions, while for the continuum region we reweight the structure functions by the charge weighted singlet of the neutron to the proton PDFs.

-
- [1] G. Aad *et al.* (ATLAS Collaboration), *Phys. Rev. C* **104**, 024906 (2021).
 - [2] G. Aad *et al.* (ATLAS Collaboration), *J. High Energy Phys.* **03** (2021) 243; 050(E) (2021) 50.
 - [3] G. Aad *et al.* (ATLAS Collaboration), *J. High Energy Phys.* **06** (2023) 182.
 - [4] G. Aad *et al.* (ATLAS Collaboration), *Phys. Rev. Lett.* **131**, 151802 (2023).
 - [5] A. Tumasyan *et al.* (CMS Collaboration), *Phys. Rev. Lett.* **131**, 151803 (2023).
 - [6] A. M. Sirunyan *et al.* (CMS Collaboration), *Phys. Rev. Lett.* **127**, 122001 (2021).
 - [7] A. Hayrapetyan *et al.* (CMS Collaboration), *J. High Energy Phys.* **08** (2025) 006.
 - [8] G. Aad *et al.* (ATLAS Collaboration), *arXiv:2504.07795*.
 - [9] J. Adam *et al.* (STAR Collaboration), *Phys. Rev. Lett.* **127**, 052302 (2021).
 - [10] M. Klusek-Gawenda, V. P. Goncalves, and A. Szczurek, *Phys. Lett. B* **867**, 139614 (2025).

- [11] L. A. Harland-Lang, V. A. Khoze, and M. G. Ryskin, *Eur. Phys. J. C* **79**, 39 (2019).
- [12] L. A. Harland-Lang, M. Tasevsky, V. A. Khoze, and M. G. Ryskin, *Eur. Phys. J. C* **80**, 925 (2020).
- [13] S. Bailey and L. A. Harland-Lang, *Phys. Rev. D* **105**, 093010 (2022).
- [14] L. Harland-Lang, J. Jaeckel, and M. Spannowsky, *Phys. Lett. B* **793**, 281 (2019).
- [15] A. Bodek, [arXiv:2111.03631](https://arxiv.org/abs/2111.03631).
- [16] Y.-S. Tsai, *Rev. Mod. Phys.* **46**, 815 (1974); **49**, 421(E) (1977).
- [17] K. Hencken, D. Trautmann, and G. Baur, *Z. Phys. C* **68**, 473 (1995).
- [18] L. A. Harland-Lang, V. A. Khoze, M. G. Ryskin, and W. J. Stirling, *Eur. Phys. J. C* **69**, 179 (2010).
- [19] L. A. Harland-Lang, *Phys. Rev. D* **107**, 093004 (2023).
- [20] L. A. Harland-Lang, [arXiv:2506.03264](https://arxiv.org/abs/2506.03264).
- [21] L. A. Harland-Lang, V. A. Khoze, and M. G. Ryskin, *Eur. Phys. J. C* **76**, 9 (2016).
- [22] L. A. Harland-Lang, V. A. Khoze, and M. G. Ryskin, *Eur. Phys. J. C* **76**, 255 (2016).
- [23] L. A. Harland-Lang, *J. High Energy Phys.* **03** (2020) 128.
- [24] S. R. Klein, J. Nystrand, J. Seger, Y. Gorbunov, and J. Butterworth, *Comput. Phys. Commun.* **212**, 258 (2017).
- [25] T. Sjöstrand *et al.*, *Comput. Phys. Commun.* **191**, 159 (2015).
- [26] A. Ogrodnik, Measurement of photon-induced processes in heavy-ion collisions with the ATLAS detector, PhD thesis, AGH-UST, Cracow, 2022.
- [27] T. Cridge, L. A. Harland-Lang, A. D. Martin, and R. S. Thorne, *Eur. Phys. J. C* **82**, 90 (2022).
- [28] R. Abdul Khalek, R. Gauld, T. Giani, E. R. Nocera, T. R. Rabemananjara, and J. Rojo, *Eur. Phys. J. C* **82**, 507 (2022).
- [29] L. A. Harland-Lang, V. A. Khoze, and M. G. Ryskin, *SciPost Phys.* **11**, 064 (2021).
- [30] P. E. Bosted and V. Mamyan, [arXiv:1203.2262](https://arxiv.org/abs/1203.2262).
- [31] M. Sajjad Athar, A. Fatima, and S. K. Singh, *Prog. Part. Nucl. Phys.* **129**, 104019 (2023).
- [32] K. J. Ozeren and W. J. Stirling, *J. High Energy Phys.* **11** (2005) 016.
- [33] Z. Bern, A. De Freitas, L. J. Dixon, A. Ghinculov, and H. L. Wong, *J. High Energy Phys.* **11** (2001) 031.
- [34] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, H.-S. Shao, T. Stelzer, P. Torrielli, and M. Zaro, *J. High Energy Phys.* **07** (2014) 079.
- [35] R. Frederix, S. Frixione, V. Hirschi, D. Pagani, H.-S. Shao, and M. Zaro, *J. High Energy Phys.* **07** (2018) 185; **11** (2021) 85.
- [36] A. Pedrak, B. Pire, L. Szymanowski, and J. Wagner, *Phys. Rev. D* **96**, 074008 (2017); **100**, 039901(E) (2019).
- [37] O. Grocholski, B. Pire, P. Sznajder, L. Szymanowski, and J. Wagner, *Phys. Rev. D* **105**, 094025 (2022).
- [38] O. Grocholski, B. Pire, P. Sznajder, L. Szymanowski, and J. Wagner, *Phys. Rev. D* **104**, 114006 (2021).
- [39] L. A. Harland-Lang, V. A. Khoze, M. G. Ryskin, and W. J. Stirling, *Int. J. Mod. Phys. A* **29**, 1430031 (2014).
- [40] S. Bailey, T. Cridge, L. A. Harland-Lang, A. D. Martin, and R. S. Thorne, *Eur. Phys. J. C* **81**, 341 (2021).
- [41] T. Aaltonen *et al.* (CDF Collaboration), *Phys. Rev. Lett.* **108**, 081801 (2012).
- [42] <https://github.com/LucianHL/SuperChic>.
- [43] J. C. Bernauer *et al.* (A1 Collaboration), *Phys. Rev. C* **90**, 015206 (2014).
- [44] L. A. Harland-Lang, A. D. Martin, R. Nathvani, and R. S. Thorne, *Eur. Phys. J. C* **79**, 811 (2019).
- [45] M. Osipenko *et al.* (CLAS Collaboration), *Phys. Rev. D* **67**, 092001 (2003).
- [46] A. Airapetian *et al.* (HERMES Collaboration), *J. High Energy Phys.* **05** (2011) 126.
- [47] V. Bertone, S. Carrazza, and J. Rojo, *Comput. Phys. Commun.* **185**, 1647 (2014).