

Violation of universal operator growth hypothesis in $\mathcal{W}_3$ conformal field theories

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We show that operator growth in large-central-charge conformal field theories with $\mathcal{W}_3$ symmetry can violate the universal operator growth hypothesis once the Liouvillian is enlarged to probe the higher-spin generators. For the generalized Liouvillian $\mathcal{L} = \kappa_1(L_1 + L_{-1}) + \kappa_2(W_2 + W_{-2})$, we compute the Lanczos coefficients in the descendant module of a heavy primary and find several classes with faster-than-linear growth in the descendant level N , including maximally violating sectors with asymptotic behavior $b_N \sim N^2$. This superlinear growth exceeds the conjectured bound and renders the Krylov complexity divergent. We further show that the same quadratic asymptotic growth already arises in the global $SL(3, \mathbb{R})$ subalgebra, indicating that the violation is rooted in the extended higher-rank symmetry itself. Our results demonstrate that extended $\mathcal{W}$ symmetries can qualitatively modify operator growth and evade conventional bounds on information scrambling.

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Introduction. Quantum complexity has emerged as a useful probe of operator growth, information spreading, and ergodicity in quantum many-body systems and quantum field theories [1]. In this context, Krylov complexity (K complexity) provides a quantitative measure of how an initially simple operator spreads in the Krylov basis generated by successive action of the Liouvillian [2–4]. In two-dimensional conformal field theories (CFTs), operator growth is particularly natural to study because the state-operator correspondence translates the problem into the growth of descendant states in a given module [5]. A central conjecture in this framework is the universal operator growth hypothesis (OGH), according to which the Lanczos coefficients (LC) b_N grow at most linearly with the Krylov level N in chaotic systems [2]. This linear growth, in turn, implies at most exponential growth of K complexity.

Most existing studies of the OGH have focused either on standard lattice models and other many-body systems [6–13] or on CFT settings where the dynamics is governed by the Virasoro algebra [5,14–21]. It is therefore natural to

ask whether the same bound continues to hold in theories with extended symmetry. The physical relevance of extended algebras such as $\mathcal{W}$ algebras stretches far beyond the formal classification of two-dimensional CFTs [22,23]. They are quite relevant for a variety of strongly correlated condensed matter and statistical systems. For instance, higher-spin current algebras organize edge excitations in incompressible quantum Hall fluids [24,25], while higher-spin symmetries also emerge in Sachdev–Ye–Kitaev (SYK)-like models [26]. Furthermore, $\mathcal{W}$ algebras play an important role in critical statistical systems, notably manifesting as $\mathcal{W}_3$ symmetry in the critical three-state Potts model [27] and in parafermionic models [28]. Determining whether higher-spin symmetries preserve or qualitatively alter the universal growth bound is therefore an important step toward understanding operator growth beyond the Virasoro paradigm with direct relevance to condensed matter and statistical systems.

In a CFT with only Virasoro symmetry, a natural choice of Liouvillian is $L = \kappa(L_1 + L_{-1})$, and this choice underlies the known results for Liouville theory [5]. In a theory with $\mathcal{W}_3$ symmetry, however, restricting to the Virasoro generators fails to probe the enlarged symmetry algebra. To capture the effect of the spin-3 generators on operator growth, we enlarge the Liouvillian to $\mathcal{L} = \kappa_1(L_1 + L_{-1}) + \kappa_2(W_2 + W_{-2})$. This is the minimal choice that directly incorporates the higher-spin sector; using $W_{\pm 1}$ instead leads to the same qualitative conclusions.

In this paper, we study operator growth in a large-central-charge CFT with $\mathcal{W}_3$ symmetry using this generalized Liouvillian. Working in the descendant module of a heavy

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primary, we compute the LC and show that the OGH is generically violated. In particular, we identify several classes of LC with faster-than-linear growth, including two classes whose asymptotic behavior saturates at $b_N \sim N^2$, where N is the descendent level. This superlinear growth exceeds the conjectured universal bound and implies that the corresponding K complexity diverges. We further show that the same quadratic asymptotic growth already appears in the global $SL(3, \mathbb{R})$ subalgebra, indicating that the violation is rooted in the extended higher-rank character of the symmetry algebra itself rather than in the details of full infinite-dimensional $\mathcal{W}_3$ algebra. Our results therefore show that extended $\mathcal{W}$ symmetry can qualitatively modify operator growth and evade known bounds on information scrambling.

Lanczos coefficients in $\mathcal{W}_3$ CFT. A $\mathcal{W}_3$ Toda field theory [22,23,29–31] is a two-dimensional irrational CFT that involves a spin-3 conserved current $W(z)$ and the stress tensor $T(z)$ in the presence of two background charges Q_1 and Q_2 at infinity. We only consider the holomorphic sector; the same story repeats for the antiholomorphic sector. The $\mathcal{W}_3$ Toda field theory admits a free field realization in terms of two scalar fields, φ_1 and φ_2 with background charge. The spin-2 current $T(z)$ and the spin-3 current $W(z)$ written in terms of φ_1 and φ_2 take the form

$$T(z) = -\frac{1}{2}[(\partial\varphi_1)^2 + (\partial\varphi_2)^2] - (Q_1\partial^2\varphi_1 + Q_2\partial^2\varphi_2), \quad (1)$$

$$W(z) = -iA \left\{ \frac{1}{3}(\partial\varphi_1)^3 - \partial\varphi_1(\partial\varphi_2)^2 + (Q_1\partial\varphi_1\partial^2\varphi_1 - 2Q_2\partial\varphi_1\partial^2\varphi_2 - Q_2\partial\varphi_2\partial^2\varphi_2) + \left(\frac{1}{3}Q_1^2\partial^3\varphi_1 - Q_1Q_2\partial^3\varphi_2 \right) \right\}, \quad (2)$$

where $A = 2/\sqrt{261}$. The scalar fields φ_1 and φ_2 can be expanded in terms of their oscillator modes, which we denote as α_m^1 and α_m^2 , respectively. The oscillators satisfy the commutation relation

$$[\alpha_m^i, \alpha_n^j] = m\delta_{ij}\delta_{m+n,0}. \quad (3)$$

Although the Virasoro generators are at most quadratic in α_m^i , the modes W_n of $W(z)$ contain cubic terms in α_m^i . As a result, the commutator of Virasoro generators with the oscillator modes is linear in the oscillators, but that of the W_3 generators is quadratic in the oscillators.

Computation of the LC depends on the choice of the Liouvillian. Any CFT can be studied purely in terms of the Virasoro representation theory. This amounts to ignoring any additional symmetry of the theory and its representations. If we do not incorporate any additional symmetry, the behavior of the LC will be identical to that observed in the Liouville CFT. This choice of the Liouvillian therefore cannot capture the operator growth due to new symmetry

generators. To capture this effect, we have to judiciously choose a modified Liouvillian. The minimal such choice is $\mathcal{L} = \kappa_1(L_1 + L_{-1}) + \kappa_2(W_2 + W_{-2})$. We could have chosen $W_{\pm 1}$ instead of $W_{\pm 2}$, but there are a couple of reasons for this choice. Firstly, $W_{\pm 2}$ commutes with $L_{\pm 1}$ with correlated signs, i.e., $[W_{\pm 2}, L_{\pm 1}] = 0$, while $[W_{\pm 1}, L_{\pm 1}] \neq 0$, and secondly, $W_{\pm 2}$, like $L_{\pm 1}$ are the highest global generators of that symmetry. Nonetheless, the alternative choice of the Liouvillian, $\mathcal{L}_1 = \kappa_1(L_1 + L_{-1}) + \kappa_2(W_1 + W_{-1})$, does not alter the conclusion. We will work with $W_{\pm 2}$, because it provides a greater contrast compared to the Liouvillian with only Virasoro generators.

The LC are a measure of the rate of operator growth. We start with a heavy primary state generated by a heavy operator $\Phi(z)$ which forms a representation of the $\mathcal{W}_3$ symmetry. To evaluate operator growth, we need to look at the descendant states in the heavy operator module. These descendant states can be created using L_{-n} , or W_{-n} , $n > 0$. The Krylov subspace is generated by L_{-1} and W_{-2} ; however, the resulting representation does not lead to orthonormal states. In the free field realization, descendant states are expressed in terms of action of oscillator creation operators on the primary state, which is an orthonormal basis. Thus, the oscillator realization of the states in the Krylov subspace naturally constitutes the Krylov basis. In the oscillator basis, a generic descendant state at level N , denoted by $\Phi_{\{m_j\}\{n_p\}}$, can be expressed in terms of the oscillator modes α_n^1 and α_n^2 as

$$\Phi_{\{m_j\}\{n_p\}} = \frac{1}{\mathcal{N}_{\{m_j\}\{n_p\}}} (\alpha_{-1}^1)^{m_1} (\alpha_{-2}^1)^{m_2} \cdots (\alpha_{-1}^2)^{n_1} (\alpha_{-2}^2)^{n_2} \cdots |\Phi\rangle, \quad (4)$$

where the set of non-negative integers $\{\{m_j\}, \{n_p\}\}$ satisfying $\sum_j j m_j + \sum_p p n_p = N$ represent generalized two-partition of integer N . The normalization constant is given by

$$\mathcal{N}_{\{m_j\}\{n_p\}} = \left[\prod_j m_j! j^{m_j} \right]^{1/2} \left[\prod_p n_p! p^{n_p} \right]^{1/2}. \quad (5)$$

To obtain LC, it is sufficient to consider the action of the operator $\kappa_1 L_{-1} + \kappa_2 W_{-2}$ in the descendant state $\Phi_{\{m_j\}\{n_p\}}$ at level N . The action of remaining generators follows from the relations $L_n = L_{-n}^\dagger$ and $W_n = W_{-n}^\dagger$ among the Virasoro and spin-3 algebra generators, respectively. The action of W_{-2} on an arbitrary descendant state $\Phi_{\{m_j\}\{n_p\}}$ gives a long expression, which is provided for completeness in Eq. (S7) of Supplemental Material [32]. This action gives rise to 19 classes of LC, but not all of them violate the upper bound; among them, ten coefficients have a faster-than-linear growth in the large N limit. For the discussion in the main text, the most relevant ones are two classes, which we will define momentarily, whose large- N growth is N^2 .

Violation of universal operator growth hypothesis. Out of the 19 classes of LC, two classes denoted as $b^I := b_{\{m_j\}\{n_p\} \rightarrow \{m_{j \neq k}, m_k - 1\}(\{n_{p \neq k+2}, n_{k+2} + 1\})}$ and $b^{II} := b_{\{m_j\}\{n_p\} \rightarrow \{m_{j \neq l+2}, m_{l+2} + 1\}(\{n_{p \neq l}, n_l - 1\})}$ maximally violate the conjectured upper bound. These coefficients are obtained by acting W_{-2} on a generic descendant state $\Phi_{\{m_j\}\{n_p\}}$:

$$b^I = -2iA Q_2 \kappa_2 \sqrt{k(k+1)^2(k+2)m_k(n_{k+2}+1)}, \quad (6)$$

$$b^{II} = 2iA Q_2 \kappa_2 \sqrt{l(l+1)^2(l+2)n_l(m_{l+2}+1)}. \quad (7)$$

In particular, we identify a specific class of descendant states whose modules lead to this violation. At any level N , a descendant state denoted in terms of two sets of integers $\{\{m_j\}, \{n_p\}\} = \{\{0, \dots, 0, \sqrt{N}, 0, \dots, 0\}, \{0, \dots, 0\}\}$ or $\{\{0, \dots, 0\}, \{0, \dots, 0, \sqrt{N}, 0, \dots, 0\}\}$, where the entry $m_j = \sqrt{N}$ appears exactly at position $j = \sqrt{N}$ or in $n_p = \sqrt{N}$ at $p = \sqrt{N}$ but not both and all other $\{m_j\}$ and $\{n_p\}$ are zero will lead to the violation of the conjectured upper bound in both b^I and b^{II} . Other coefficients also violate the linear growth bound, but the dominant violation is seen in b^I and b^{II} .

A large class of states can be shown to maximally violate the bound. To see this, let us consider a state at a level $N \gg 1$ in the module of the $\mathcal{W}_3$ algebra, labeled by $\{\{m_j\}, \{n_p\}\}$. In this set we will choose the subset $\{n_p\} = \{0, \dots, 0\}$ and all nonvanishing entries appear only in the subset $\{m_j\}$, and the nonvanishing entries are centered around $j = N^\alpha$ with a breadth of N^γ in the values of j and the degeneracy $m_j = N^\beta$ at each j value, such that $\alpha + \beta + \gamma = 1$ and $\alpha > 0, \beta > 0, \gamma > 0$. In the large- N limit, LC b^I and b^{II} grow as $N^{2\alpha+(\beta/2)}$. Whenever $\alpha - \beta/2 - \gamma > 0$, either the growth of b^I or of b^{II} is faster than N and hence these coefficients violate the conjectured bound. Since α, β , and γ satisfy only one constraint, the states that violate the growth bound are quite generic. The degeneracy $\{m_j\}$ of this state reduces with every action of W_{-2} , but at the same time it increases the degeneracy on the $\{n_p\}$ side. The successive action of W_{-2} results in the switch of the growth pattern from b^I to b^{II} and back. Therefore, either b^I or b^{II} grows faster than N at any step. If the action of W_{-2} is continued *ad infinitum*, then the growth rate of LC approaches N^2 asymptotically. Since the LC growth rate saturates at N for CFTs with Virasoro symmetry and saturates at N^2 for CFTs with $\mathcal{W}_3$ symmetry, it is tempting to conjecture that, for CFTs with $\mathcal{W}_n$ symmetry, the growth of LC will saturate at N^{n-1} .

This conclusion is further reinforced by the global subalgebra. Restricting the analysis to the global symmetry algebra $SL(3, \mathbb{R})$, which is the global part of the $\mathcal{W}_3$ algebra, we again find quadratic asymptotic growth of the LC [32]. Therefore, the violation of the OGH is not merely a consequence of the infinite-dimensional or nonlinear

structure of the full $\mathcal{W}_3$ algebra. Rather, it already arises at the level of the extended higher-rank global symmetry itself.

Typical descendants. Here we address how fast the typical states grow in the regime of high descendant levels. Correlation functions in typical high-energy states appear thermal. In particular, we use the definition of typical states of [35]. In $\mathcal{W}_3$ Toda field theory, descendants are eigenstates of number operator $\alpha_{-j}^1 \alpha_j^1 + \alpha_{-p}^2 \alpha_p^2$ with eigenvalue $jm_j + pn_p$. For a typical high-level descendant of a heavy primary, the occupation numbers m_j and n_p of the generalized integer two-partition $\sum_{j=1}^N jm_j + \sum_{p=1}^N pn_p = N$ are distributed according to the Boltzmann distribution and their expectation values are $\langle m_j \rangle = \frac{q^j}{1-q^j}$ and $\langle n_p \rangle = \frac{q^p}{1-q^p}$, with $q = e^{-\pi/\sqrt{3N}}$. There are three distinct large- N behaviors of LC for submodules involving typical descendants focusing on transition amplitudes with $k, l, r, q \ll N$ in Eq. (S7) of Supplemental Material [32]. These are $N^{1/4}$, $N^{1/2}$, and $N^{3/4}$. The former two were also observed in [5] for Liouville CFT, whereas the latter one, $N^{3/4}$, is novel.

K complexity. We will now investigate the effect of this growth of LC on the K complexity. Since we are interested in understanding the effect of the term $W_2 + W_{-2}$ on the K complexity, we set $\kappa_1 = 0$ in the Liouvillian. The time evolution of a state is then given by $e^{i\kappa_2 t(W_{-2} + W_2)}|h, w\rangle$, where $|h, w\rangle$ denotes the highest weight state with conformal weight h and $\mathcal{W}_3$ charge w . Using the Baker-Campbell-Hausdorff decomposition, this state can be expressed as

$$\begin{aligned} |\mathcal{O}(t)\rangle &= e^{i\kappa_2 t(W_{-2} + W_2)}|h, w\rangle \\ &= e^{\alpha_- W_{-2}} e^{\alpha_0 L_0} e^{\alpha_+ W_2} |h, w\rangle. \end{aligned} \quad (8)$$

To determine the coefficients α_- , α_+ , and α_0 , one may either use the matrix representations of $W_{\pm 2}$ and L_0 [36] or perform a series expansion on both sides of the Baker-Campbell-Hausdorff decomposition and compare the coefficients of the powers of $W_{\pm 2}$ and L_0 . Carrying out this computation gives

$$\alpha_+ = \alpha_- = \frac{i}{4} \tan(4\kappa_2 t), \quad \alpha_0 = \log \sec(4\kappa_2 t). \quad (9)$$

Since $|h, w\rangle$ is the highest-weight state, it is annihilated by W_2 ; hence, the time-evolved state simplifies to

$$|\mathcal{O}(t)\rangle = e^{\alpha_0 h} e^{\alpha_- W_{-2}} |h, w\rangle. \quad (10)$$

Expanding the exponential involving W_{-2} gives

$$|\mathcal{O}(t)\rangle = \sum_{n=0}^{\infty} i^n \varphi_n(t) |h + 2n, w\rangle, \quad (11)$$

where the Krylov wave functions $\varphi_n(t)$ are given by

$$\varphi_n(t) = \frac{1}{4^n n!} \sqrt{\frac{(2n)! \Gamma(2h + 2n)}{\Gamma(2h)}} \frac{\tan^n(4\kappa_2 t)}{\cos^h(4\kappa_2 t)}. \quad (12)$$

The Krylov complexity for the $\mathcal{W}_3$ CFT is then given by

$$\begin{aligned} K(t) &= \sum_{n=0}^{\infty} n |\varphi_n(t)|^2 \\ &= \sum_{n=0}^{\infty} \frac{n(2n)!}{4^{2n} (n!)^2} \frac{\Gamma(2h + 2n)}{\Gamma(2h)} \frac{\tan^{2n}(4\kappa_2 t)}{\cos^{2h}(4\kappa_2 t)}. \end{aligned} \quad (13)$$

The above series diverges, indicating that the Krylov complexity diverges for the $\mathcal{W}_3$ CFT.

Discussion and outlook. Our analysis shows that the universal operator growth hypothesis, which is respected in Virasoro CFTs, can be violated once the Liouvillian is enlarged to probe higher-spin generators. The key point is not merely the infinite dimensionality or nonlinearity of the $\mathcal{W}_3$ algebra, but the presence of an extended higher-rank symmetry sector. In particular, while Virasoro CFTs exhibit at most linear growth of Lanczos coefficients with descendant level N , the $\mathcal{W}_3$ case exhibits asymptotic growth up to N^2 . Moreover, the same quadratic growth already appears in the global $SL(3, \mathbb{R})$ subalgebra. This shows that the violation is rooted in the extended higher-rank symmetry itself. These results naturally suggest that, in CFTs with $\mathcal{W}_n$ symmetry, the maximal growth may scale as N^{n-1} .¹

¹This mirrors the pattern found by Perlmutter [37], who showed that irrational $\mathcal{W}_n$ -symmetric CFTs violate the Maldacena-Shenker-Stanford chaos bound with $\lambda_L = (n-1) \cdot 2\pi/\beta$, where the higher-spin current of spin n dominates the out-of-time-ordered correlator in the Regge limit and contributes a factor $(n-1)$ beyond the maximal-chaos value. That the same $(n-1)$ scaling appears in the Lanczos coefficients suggests a common structural origin in the higher-rank algebra.

At first sight, the enhancement of operator growth in a theory with more symmetry may appear counterintuitive. The resolution is that the enlarged symmetry also greatly enlarges the descendant module accessible to the dynamics. Thus, there is a competition between the additional conserved charges, which tend to constrain the dynamics, and the rapid growth of the accessible Hilbert-space sector generated by the higher-spin operators. Our results indicate that the latter effect dominates.

Gaiotto and Verlinde [38] showed that the partition function of the double-scaled Sachdev–Ye–Kitaev (DSSYK) model is identical to the Schur index of the $\mathcal{N} = 2$ SU(2) supersymmetric Yang–Mills theory [39,40]. The Alday–Gaiotto–Tachikawa correspondence for 4D SU(2) theories and Liouville theory [41,42] was generalized, by Wyllard [43], to SU(N) theories and $\mathcal{W}_N$ Toda field theories. It would be interesting to extend our results to higher-rank gauge theories in 4D and generalized DSSYK models, using these correspondences. In particular, it will be nice to identify the generalized DSSYK model, which captures the spectrum of the Schur sector of SU(3) quiver gauge theories and connects it to the W_3 Toda field theory studied here.

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