

Chiral Cherenkov radiation in the presence of a time-dependent chiral chemical potential

Kirill Tuchin *Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA*

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The photon production process $f \rightarrow f + \gamma$ in the presence of a time-dependent chiral chemical potential μ_5 is studied. The validity of the adiabatic approximation is demonstrated in the ultrarelativistic limit. Analytical expressions for the photon spectrum are derived for two models of the chiral magnetic conductivity $b_0 \propto \mu_5$: (i) $b_0(t) = A_1 + B_1 \tanh(t/\tau)$ and (ii) $b_0(t) = A_2(t/\tau)/(1 + t^2/\tau^2)$. It is known that, for constant b_0 , photon emission may exhibit a resonance for one-photon polarization depending on the sign of b_0 . It is shown that, in model (i), up to two resonances may occur, depending on the sign of the asymptotic values $b_0(\infty)$ and $b_0(-\infty)$. No resonances are observed in model (ii). Numerical calculations are performed using parameters relevant to quark-gluon plasma, and the universality of the results is discussed.

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I. INTRODUCTION

One of the most intriguing features of chiral media is the chiral magnetic effect whereby electric current is induced in the direction of the external magnetic field [1–5]. The corresponding transport coefficient b_0 , known as the chiral magnetic conductivity, is proportional to the chiral chemical potential μ_5 , which reflects the net chirality in the medium. The chiral imbalance can have various sources. In quark-gluon plasma, it may be induced by the parallel chromoelectric and chromomagnetic fields in the initial stage following a heavy-ion collision [6] or by a sphaleron transition at a later stage [5]. The phenomenological manifestation of the chiral magnetic effect is electric charge separation with respect to the collision plane of the heavy ions [4]. Recent experimental measurements have produced encouraging signs of this charge separation [7]. Another manifestation of the chiral magnetic effect is the chiral Cherenkov radiation, in which a circularly polarized photon is emitted by a charge's fermion [8,9]. This process, kinematically prohibited in the empty space, becomes possible owing to the momentum supplied by the chiral magnetic current. The chiral Cherenkov radiation contributes to the parton energy loss and the jet structure, which can be directly observed in relativistic heavy-ion experiments.

The time evolution of the chiral chemical potential is governed by three processes. The first is the abovementioned sphaleron transition, a stochastic process that generates the chiral charge. At very high temperatures, the transition time is estimated to be significantly shorter than the lifetime of the quark-gluon plasma [10,11]. However, at phenomenologically

relevant temperatures, these timescales are comparable. The second process is a gradual relaxation of the chiral chemical potential due to the inelastic processes breaking the chiral symmetry [12]. The third process is chirality transfer to the electromagnetic field via the inverse chiral cascade [13–20]. These processes impart time dependence to the chiral chemical potential, which, in turn, impacts the chiral magnetic effect, the charge separation [21], and the chiral Cherenkov radiation. The latter is the subject of this paper.

The chiral Cherenkov radiation is the excitation of the electromagnetic field in a chiral medium by a charged fermion. If the time dependence of b_0 is neglected, the eigenstates of the electromagnetic field are circularly polarized monochromatic plane waves with a dispersion relation dependent on the photon polarization. The transition amplitude is then proportional to the δ function, which ensures energy conservation. However, when b_0 has a nontrivial time dependence, the δ function gives way to a distribution proportional to the Fourier image of the photon amplitude. The relationship between the photon production rate and this amplitude is provided by perturbation theory.

The main goal of this paper is to compute the photon production rate, given the time dependence of b_0 . The precise form of b_0 is unknown. It is therefore advantageous to employ models that both provide a qualitative description of its time evolution and are amenable to analytical methods. Two models are examined: (i) $b_0 = A_1 + B_1 \tanh(t/\tau)$ and (ii) $b_0 = A_2(t/\tau)/(1 + t^2/\tau^2)$, where A_1, B_1, A_2 , and $\tau > 0$ are real numbers. Model (i) describes a transition between states characterized by different asymptotic values of b_0 , whereas model (ii) describes a burst of chirality with a finite duration. It will be shown that the models exhibit qualitatively distinct spectra, differing from the spectrum associated with constant b_0 .

The paper is structured as follows. In Sec. II, the general formalism for computing the photon production rate at the leading order of perturbation theory in the ultrarelativistic approximation is reviewed. The main formula for the

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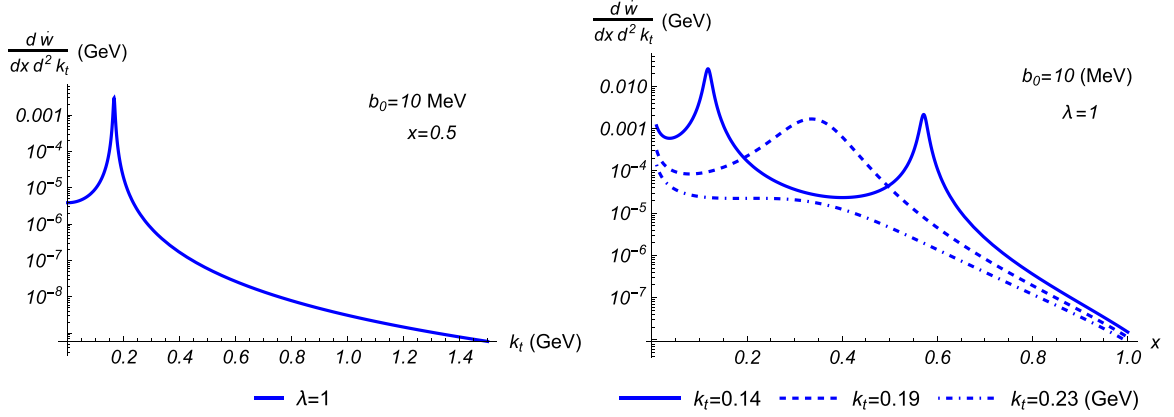


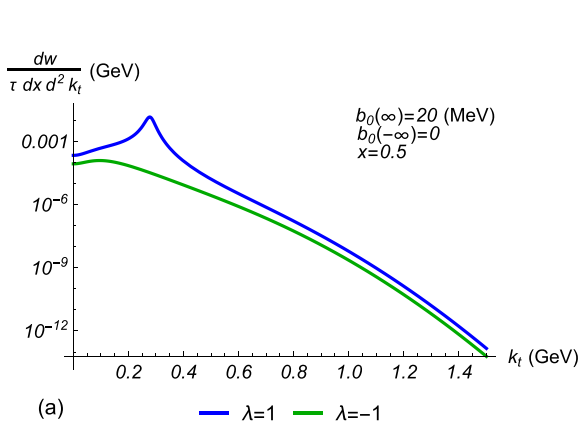
FIG. 1. The photon spectrum for a constant $b_0 = 10$ MeV. The incident fermion energy $E = 20$ GeV, mass $m = 0.3$ GeV, and unit charge. The Lorentzian parameter $\epsilon = 1$ MeV. Only the right-handed photons $\lambda = 1$ are emitted.

transition probability is Eq. (13). The photon amplitude $a(t)$ satisfies a second-order differential equation, Eq. (3), which is solved for a given $b_0(t)$. Section III lays out the benchmark results for the photon rate at constant b_0 . Figure 1 shows the corresponding photon spectrum. The exact solution to Eq. (3) is known only in a few relevant cases. Fortunately, all applications to relativistic heavy-ion collisions involve photons whose momenta k are so large that one can use the adiabatic approximation, thereby obtaining a general expression for the field amplitude in Sec. IV. The main results are derived in Secs. V and VI where the photon spectra for the models (i) and (ii) are obtained and analyzed. The results are exhibited in Figs. 2–5. The final section summarizes and discusses the results.

II. PHOTON RADIATION PROBABILITY

The electromagnetic field, in the radiation gauge $A^0 = 0$ and $\nabla \cdot \mathbf{A} = 0$, is described by the vector potential $\mathbf{A}$ obeying the following equation:

$$\nabla^2 \mathbf{A} - \partial_t^2 \mathbf{A} + b_0(t) \nabla \times \mathbf{A} = 0. \quad (1)$$



It is assumed that the chiral magnetic conductivity b_0 is a given function of time unaffected by the radiation. The wave function of a photon of energy ω and momentum $\mathbf{k}$ is a plane-wave solution to Eq. (1):

$$A(t, \mathbf{r}) = \frac{1}{\sqrt{V}} a(t) \epsilon_\lambda e^{i\mathbf{k} \cdot \mathbf{r}}, \quad (2)$$

where the polarization vector ϵ_λ , $\lambda = \pm 1$, satisfies the condition $i\hat{\mathbf{k}} \times \epsilon_\lambda = \lambda \epsilon_\lambda$, which implies that photons must be circularly polarized. Substituting Eq. (2) into Eq. (1), one finds that the amplitude $a(t)$ is governed by the equation

$$\ddot{a}(t) + \Omega^2(t) a(t) = 0, \quad (3)$$

where

$$\Omega^2(t) = k^2 - \lambda b_0(t) k. \quad (4)$$

The photon wave function will be normalized to one photon per unit volume as usual.

Denote the energy and momentum of the incident and final fermions as E , $\mathbf{p}$ and E' , $\mathbf{p}'$ respectively. The wave function of

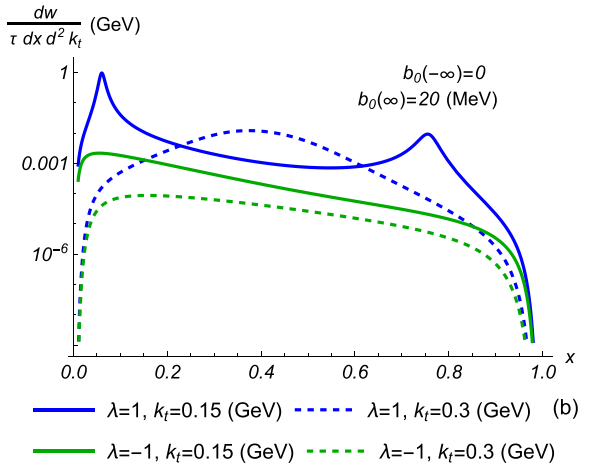


FIG. 2. The photon spectrum for $b_0(t) = A_1 + B_1 \tanh \frac{t}{\tau}$, with $A_1 = B_1 = 10$ MeV and $\tau = 25 \text{ GeV}^{-1} \approx 5 \text{ fm}/c$. The incident fermion energy $E = 20$ GeV, mass $m = 0.3$ GeV, and unit charge.

an incident fermion with momentum $\mathbf{p}$ and spin s reads

$$\psi_i(t, \mathbf{r}) = \frac{1}{\sqrt{2EV}} u_s(\mathbf{p}) e^{-iEt + i\mathbf{p} \cdot \mathbf{r}}, \quad (5)$$

where $E = \sqrt{p^2 + m^2}$. The wave function of the final fermion ψ_f is described by a similar expression. The photon radiation amplitude is given by

$$S_{fi} = -ie \int \bar{\psi}_f \boldsymbol{\gamma} \cdot \mathbf{A}^* \psi_i d^3x dt = -ie \frac{\bar{u}_{s'}(\mathbf{p}') \boldsymbol{\gamma} \cdot \boldsymbol{\epsilon}_\lambda^* u_s(\mathbf{p})}{2V^{3/2} \sqrt{EE'}} \times (2\pi)^3 \delta(\mathbf{p} - \mathbf{p}' - \mathbf{k}) \tilde{a}^*(E - E'), \quad (6)$$

where I define

$$\tilde{a}(E - E') = \int_{-\infty}^{+\infty} e^{-i(E' - E)t} a(t) dt. \quad (7)$$

Given a bounded function $b_0(t)$, the convergence of the integral in Eq. (7) as assured by inserting a positive infinitesimal parameter ϵ as follows:

$$\tilde{a}(E - E') = \int_0^\infty e^{-i(E' - E - i\epsilon)t} a(t) dt + \int_{-\infty}^0 e^{-i(E' - E + i\epsilon)t} a(t) dt. \quad (8)$$

The photon radiation probability is then calculated as

$$dw = \frac{1}{2} \sum_{\lambda, s, s'} |S_{fi}|^2 \frac{V d^3 p'}{(2\pi)^3} \frac{V d^3 k}{(2\pi)^3} = \frac{e^2}{8(2\pi)^3 E} \sum_{\lambda} |\tilde{a}(E - E')|^2 \times d^2 k_\perp \frac{dx}{1-x} \text{Tr}[(\not{p} + m) \not{\epsilon}_\lambda^* (\not{p}' + m) \not{\epsilon}_\lambda], \quad (9)$$

where x denotes the fraction of the incident fermion momentum carried away by the photon. Assuming that the incident fermion moves along the z axis, $x = k_z/p_z$ and $k^2 = x^2 p_z^2 + k_\perp^2$, where $\mathbf{k}_\perp$ is the photon's transverse momentum. The product of the photon polarization vectors can be written as

$$\epsilon_\lambda^{\mu*} \epsilon_\lambda^\nu = \begin{cases} \frac{1}{2} \left(\delta^{ij} - \frac{k^i k^j}{k^2} \right) + \frac{i\lambda}{2} \epsilon^{ij\ell} \frac{k^\ell}{k}, & \mu = i, \nu = j, \\ 0, & \mu\nu = 0, \end{cases} \quad (10)$$

where no summation over λ is implied.

In most applications the incident fermion can be regarded as ultrarelativistic, meaning that $p_z \gg k_\perp, m$. In this approximation,

$$E = p_z \left(1 + \frac{m^2}{2p_z^2} \right), \quad (11)$$

$$E' = (1-x)p_z \left(1 + \frac{m^2 + k_\perp^2}{2(1-x)^2 p_z^2} \right), \quad (12)$$

and the differential probability (9) becomes

$$\frac{dw}{d^2 k_\perp dx} = \frac{1}{8(2\pi)^3 E} \sum_{\lambda} |\tilde{a}(E - E')|^2 \frac{2e^2}{x^2 (1-x)^2} [k_\perp^2 (2 - 2x + x^2) + m^2 x^4]. \quad (13)$$

The problem of computing the photon spectrum thus reduces to determining the Fourier image (8) of the field amplitude $a(t)$.

III. PHOTON SPECTRUM AT CONSTANT b_0

It is instructive to first consider the particular case of constant b_0 that was discussed before [8,9,22,23]. In this case the positive frequency solution of Eq. (3) is

$$a = \frac{1}{\sqrt{2\omega}} e^{-i\omega t}, \quad (14)$$

where ω is the photon energy given by

$$\omega = \sqrt{k^2 + M^2} \approx x p_z \left(1 + \frac{M^2 + k_\perp^2}{2x^2 p_z^2} \right), \quad (15)$$

and I introduced a new parameter, $M^2 = -\lambda b_0 k$. It follows then from Eq. (7) that

$$\tilde{a}(E - E') = 2\pi \delta(\omega + E' - E). \quad (16)$$

Using Eqs. (11) and (12), the argument of the δ function can be written as

$$\omega + E' - E = \frac{k_\perp^2 + M^2(1-x) + m^2 x^2}{2x(1-x)p_z}. \quad (17)$$

The inverse of this expression is the coherence length of the photon emission: $\ell_c = 1/|\omega + E' - E|$. It signifies the distance traversed by the fermion as it radiates a photon carrying the fraction x of its momentum, the transverse momentum $\mathbf{k}_\perp$, and the polarization λ . Substituting Eqs. (17) and (16) into Eq. (13) yields the differential photon radiation rate $d\dot{w}$ at constant b_0 [8]:

$$\frac{d\dot{w}}{dx d^2 k_\perp} = \frac{e^2}{8\pi^2 E x} \left[\lambda E b_0 \left(1 - x + \frac{x^2}{2} \right) - m^2 x \right] \times \delta[k_\perp^2 - \lambda b_0 E x (1-x) + m^2 x^2]. \quad (18)$$

The photon spectrum (18) is plotted in Fig. 1 for a set of parameters relevant in relativistic heavy-ion collisions. The δ function is approximated by the Lorentzian: $\delta(x) \approx \epsilon/[\pi(\epsilon^2 + x^2)]$. The peaks of the spectrum correspond to the values of $k_\perp$ and x at which the argument of the δ function in Eq. (18) vanishes. At those values the coherence length ℓ_c is divergent (in Fig. 1 and the following figures it is regulated by ϵ), as it is in the classical Cherenkov radiation. The radiated photons are right- or left-handed circularly polarized if $b_0 > 0$ or $b_0 < 0$, respectively.

Throughout the paper, the following color coding is employed: right-handed photons are plotted in blue, left-handed photons are plotted in green, and their overlap is plotted in turquoise.

IV. ADIABATIC APPROXIMATION

Equation (3) cannot be analytically solved for an arbitrary function $b_0(t)$. However, for a sufficiently slow time-variation, the solution can be obtained in the adiabatic approximation. In this section, a general formula for the amplitude $a(t)$ based

on this approximation is derived. First, introduce an auxiliary function $R(t)$ defined as

$$a(t) = e^{iR(t)}. \quad (19)$$

Substituting Eq. (19) into Eq. (3), one finds that it satisfies the equation

$$i\ddot{R} - \dot{R}^2 + \Omega^2 = 0. \quad (20)$$

The adiabatic approximation consists of treating $R(t)$ as a slowly varying function such that

$$|\ddot{R}| \ll \dot{R}^2. \quad (21)$$

To develop the adiabatic expansion, introduce a bookkeeping parameter α into Eq. (20):

$$i\alpha\ddot{R} - \dot{R}^2 + \Omega^2 = 0, \quad (22)$$

and substitute in it the expansion

$$R = \sum_{j=0}^{\infty} \alpha^j R_j. \quad (23)$$

By collecting the terms of the same order in α , one obtains equations that govern the functions R_0 and R_1 :

$$\dot{R}_0^2 = \Omega^2, \quad (24)$$

$$i\ddot{R}_0 = 2\dot{R}_0\dot{R}_1. \quad (25)$$

Solving these equations and plugging the result back into Eq. (19) yields the desired result, after setting $\alpha = 1$:

$$a(t) = \frac{1}{\sqrt{2\Omega}} e^{-i \int_0^t \Omega(t') dt'}. \quad (26)$$

The sign in the exponent corresponds to the outgoing wave as appropriate for the emitted photon. Expanding Ω in the ultrarelativistic approximation $k \gg b_0$,

$$\Omega(t) \approx k - \frac{\lambda b_0(t)}{2} \quad (27)$$

leads to the final result for the amplitude:

$$a(t) = \frac{1}{\sqrt{2k}} e^{-ikt + \frac{i\lambda}{2} \int_0^t b_0(t') dt'}. \quad (28)$$

It is valid as long as Eq. (21) is satisfied, which is equivalent to the following condition:

$$|\dot{b}_0| \ll k_z^2. \quad (29)$$

Given $b_0(t)$, one can compute the amplitude $a(t)$ using Eqs. (28) and its Fourier transform (8). The latter can be carried out analytically in a number of cases, but in general one has to rely on numerical methods. The next section deals with two examples which can be solved analytically.

V. PHOTON SPECTRUM FOR $b_0(t) = A_1 + B_1 \tanh \frac{t}{\tau}$

In this section, I consider the following model for the time evolution of the chiral magnetic conductivity:

$$b_0(t) = A_1 + B_1 \tanh \frac{t}{\tau}. \quad (30)$$

The initial and final values of $b_0(t)$ are $b_0(-\infty) = A_1 - B_1$ and $b_0(\infty) = A_1 + B_1$, respectively. The transition time between them is controlled by the parameter τ .

A. Exact solution for $a(t)$

Substituting Eq. (30) into Eq. (4), one obtains the function $\Omega(t)$:

$$\Omega^2(t) = k^2 - \lambda k A - \lambda k B \tanh \frac{t}{\tau}. \quad (31)$$

The exact solution of Eq. (3) with Ω given by Eq. (31) was obtained in Ref. [24]. It is most readily expressed in terms of the asymptotic values of Ω as follows:

$$\Omega_{\text{in}} = \sqrt{k^2 - \lambda k(A_1 - B_1)}, \quad (32)$$

$$\Omega_{\text{out}} = \sqrt{k^2 - \lambda k(A_1 + B_1)}, \quad (33)$$

$$\Omega_{\pm} = \frac{1}{2}(\Omega_{\text{out}} \pm \Omega_{\text{in}}). \quad (34)$$

In the ultrarelativistic approximation, expand Eqs. (32), (33), and (34) at large k_z :

$$\Omega_{\text{in}} = k_z - \frac{\lambda}{2}(A_1 - B_1) + \mathcal{O}(k_z^{-1}), \quad (35)$$

$$\Omega_{\text{out}} = k_z - \frac{\lambda}{2}(A_1 + B_1) + \mathcal{O}(k_z^{-1}), \quad (36)$$

$$\Omega_+ = k_z - \frac{\lambda A_1}{2}, \quad (37)$$

$$\Omega_- = -\frac{\lambda B_1}{2}. \quad (38)$$

The solution to Eq. (3) describing the emitted photon, namely, the one that has positive frequency at the future infinity, reads as follows:

$$\begin{aligned} a(t) &= \frac{1}{\sqrt{2\Omega_{\text{out}}}} e^{-i\Omega_+ t - i\Omega_- \tau \ln[2 \cosh \frac{t}{\tau}]} \\ &= {}_2F_1 \left[1 + i\Omega_- \tau, i\Omega_- \tau; 1 + i\Omega_{\text{out}} \tau; \frac{1}{2} \left(1 - \tanh \frac{t}{\tau} \right) \right] \\ &\approx \frac{1}{\sqrt{2k_z}} e^{-i \left(k_z - \frac{\lambda A_1}{2} \right) t + i \frac{\lambda B_1}{2} \tau \ln[2 \cosh \frac{t}{\tau}]} \\ &= {}_2F_1 \left[1 - \frac{i\lambda B_1 \tau}{2}, -\frac{i\lambda B_1 \tau}{2}; 1 + ik_z \tau; \frac{1}{2} \left(1 - \tanh \frac{t}{\tau} \right) \right]. \end{aligned} \quad (39)$$

Using Euler's hypergeometric transformation ${}_2F_1(\alpha, \beta; \gamma; z) = (1-z)^{\gamma-\alpha-\beta} {}_2F_1(\gamma-\alpha, \gamma-\beta; \gamma; z)$ and assuming, consistently with the ultrarelativistic approximation, that $k_z \tau \gg 1$ and $k_z \gg |B_1|$, so that $\gamma \gg \alpha, \beta$, it emerges that the hypergeometric function in Eq. (40) reduces to unity.

B. Adiabatic approximation

The adiabatic approximation (28) precisely reproduces the exact solution (40), which can be seen by substituting Eq. (30)

into Eq. (28) and replacing $k \approx k_z$:

$$a(t) = \frac{1}{\sqrt{2k_z}} e^{-i\left(k_z - \frac{\lambda A_1}{2}\right)t + i\frac{\lambda B_1}{2}\tau \ln[2 \cosh \frac{t}{\tau}]}. \quad (41)$$

According to Eq. (29), this formula is valid when the parameters satisfy $k_z^2 \tau \gg |B_1|$. Thus, the adiabatic approximation accurately describes the time evolution of the electromagnetic field in the ultrarelativistic limit, which spares one seeking exact solutions to Eq. (3).

To calculate the Fourier image of the amplitude using Eq. (8), cast it in the following form:

$$\tilde{a}(E - E') = \frac{2}{\sqrt{2k_z}} \int_0^\infty \cos\left[\left(E - E' - k_z + \frac{\lambda A_1}{2}\right)t\right] \times e^{-\left(i\frac{\lambda B_1}{2}\tau + \epsilon\right) \ln[2 \cosh \frac{t}{\tau}]} dt. \quad (42)$$

I use the fact that, at $t \rightarrow \infty$, the infinitesimal term in the exponent becomes $-\epsilon t/\tau$ as prescribed by Eq. (8) (up to the nonessential factor $1/\tau$ that simplify redefines ϵ). Employing Eqs. (35)–(38), Eq. (42) may be written as

$$\tilde{a}(E - E') = \frac{2}{\sqrt{2\Omega_{\text{out}}}} \int_0^\infty \cos[(E - E' - \Omega_+)t] \times e^{-(i\Omega_- \tau + \epsilon) \ln[2 \cosh \frac{t}{\tau}]} dt. \quad (43)$$

Performing the Fourier transform using Eq. 1.9(5) of Ref. [25] yields

$$\tilde{a}(E - E') = \frac{1}{\sqrt{2\Omega_{\text{out}}}} \frac{2^{i\Omega_- \tau - 1} \tau}{\Gamma(i\Omega_- \tau)} \Gamma\left(\frac{i\tau}{2}(E - E' - \Omega_{\text{in}} - i\epsilon)\right) \times \Gamma\left(\frac{i\tau}{2}(-E + E' + \Omega_{\text{out}} - i\epsilon)\right). \quad (44)$$

Substituting Eq. (44) into Eq. (13) furnishes the photon spectrum:

$$\begin{aligned} \frac{dw}{dx d^2 k_\perp} &= \frac{1}{16(2\pi)^3 E^2} \sum_\lambda \frac{2e^2}{x^3(1-x)^2} \\ &\times [k_\perp^2(2-2x+x^2) + m^2 x^4] \frac{\tau^2}{4|\Gamma(i\Omega_- \tau)|^2} \\ &\times \left| \Gamma\left(\frac{i\tau}{2}(E - E' - \Omega_{\text{in}} - i\epsilon)\right) \right. \\ &\times \left. \Gamma\left(\frac{i\tau}{2}(-E + E' + \Omega_{\text{out}} - i\epsilon)\right) \right|^2. \end{aligned} \quad (45)$$

The explicit dependence of the Γ -function arguments on photon momentum is as follows:

$$E' - E + \Omega_{\text{in}} = \frac{k_\perp^2 - \lambda x(1-x)p_z(A_1 - B_1) + m^2 x^2}{2x(1-x)p_z}, \quad (46)$$

$$E' - E + \Omega_{\text{out}} = \frac{k_\perp^2 - \lambda x(1-x)p_z(A_1 + B_1) + m^2 x^2}{2x(1-x)p_z}. \quad (47)$$

These expressions vanish at certain values of $k_\perp$ and x , which correspond to the resonances in the spectrum. The height and the width of these resonances are regulated by the parameter ϵ .

To plot the photon spectrum, select the values of the parameters relevant to the phenomenology of quark-gluon plasma produced in relativistic heavy-ion collisions. The chiral magnetic conductivity is estimated to be within the range of 10–100 MeV [26–29]. Assuming that the time dependence of b_0 is predominantly due to sphaleron transitions, the typical value of τ is $1/\alpha_s^2 T$ [10], in the range of 1–10 fm, shorter than the plasma's lifetime.

Figures 2–4 present the photon spectrum for typical parameters. They describe the photon spectrum for various values of $b_0(\infty)$ and $b_0(-\infty)$. The resonances correspond to vanishing of expressions (46) and (47), which occurs only if $\lambda(A_1 - B_1) = \lambda b_0(-\infty) > 0$ and $\lambda(A_1 + B_1) = \lambda b_0(\infty) > 0$, respectively. The positions of the resonances satisfy the equations

$$k_\perp^2 - \lambda x(1-x)E(A_1 \mp B_1) + m^2 x^2 = 0. \quad (48)$$

Clearly, a particular polarization is enhanced depending on the signs of asymptotic values $b_0(\pm\infty)$. When one of those values vanishes, only single resonance appears in the $k_\perp$ spectrum [see Figs. 2(a) and 3(a)]. The two polarizations overlap in Fig. 3(b) because $\lambda b_0(-\infty) = -\lambda b_0(\infty)$. In Fig. 4 one can see two distinct resonances in the right-handed photon spectrum because both asymptotic values are positive. There are twice as many resonances in the x spectrum as compared to the $k_\perp$ spectrum as is evident from Eqs. (46) and (47). This is seen in Fig. 2(b).

The resonant value of $k_\perp$ at a given x is

$$K_\perp = \sqrt{\lambda x(1-x)E(A_1 \mp B_1) - m^2 x^2}, \quad (49)$$

provided that

$$0 < x \leq \frac{\lambda E(A_1 \mp B_1)}{\lambda E(A_1 \mp B_1) + m^2}, \quad (50)$$

while the resonant values of x at a given $k_\perp$ are

$$X_\alpha = \frac{\lambda E(A_1 \mp B_1) + (-1)^\alpha \sqrt{E^2(A_1 \mp B_1)^2 - 4[m^2 + \lambda E(A_1 \mp B_1)]k_\perp^2}}{2[m^2 + \lambda E(A_1 \mp B_1)]}, \quad \alpha = 1, 2, \quad (51)$$

provided that

$$0 < k_\perp^2 \leq \frac{E^2(A_1 \mp B_1)^2}{4[m^2 + \lambda E(A_1 \mp B_1)]}. \quad (52)$$

C. Sudden transition

The adiabatic approximation does not require τ to be very large, as was shown in the previous subsection. In fact, it can be rather small as long as the condition $k_z^2 \tau \gg |B_1|$ is fulfilled. In particular, τ can be much smaller than the coherence length

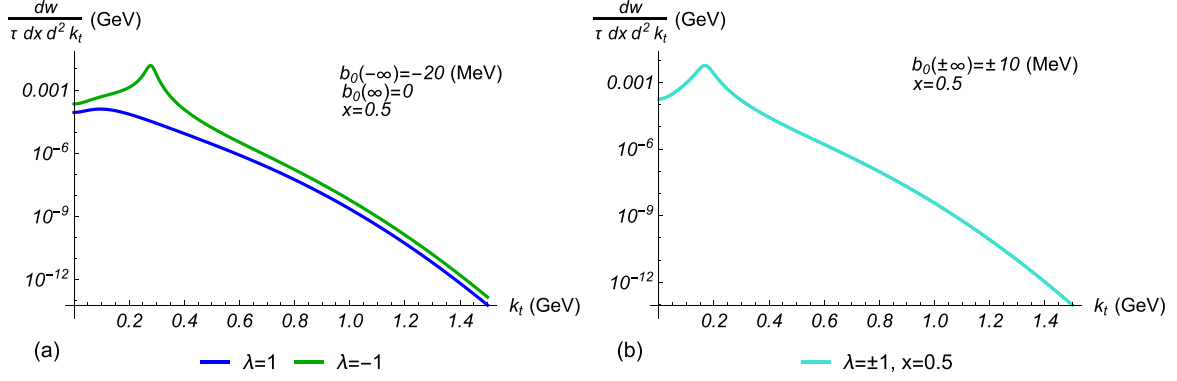


FIG. 3. The photon spectrum for $b_0(t) = A_1 + B_1 \tanh \frac{t}{\tau}$, with (a) $A_1 = -B_1 = -10$ MeV, (b) $A_1 = 0, B_1 = 10$ MeV, and $\tau = 25 \text{ GeV}^{-1} \approx 5 \text{ fm}/c$. The incident fermion energy $E = 20$ GeV, mass $m = 0.3$ GeV, and unit charge. The turquoise color indicates that the spectrum is independent of the photon's polarization.

ℓ_c defined in Sec. III for constant b_0 . In the present case, $1/\tau$ must be much larger than either of two expressions, Eqs. (46) and (47). This condition is certainly satisfied near the resonances. It also holds for not too large values of $k_\perp$. One can refer to such a scenario as a sudden transition. Taking the limit $\tau \rightarrow 0$ in the exponent of Eq. (41) yields

$$a(t) = \frac{1}{\sqrt{2\Omega_{\text{out}}}} [e^{-i\Omega_{\text{out}}t} \theta(t) + e^{-i\Omega_{\text{in}}t} \theta(-t)]. \quad (53)$$

Its Fourier transform (7) is

$$\tilde{a}(E - E') = \frac{1}{\sqrt{2\Omega_{\text{out}}}} \left\{ \int_0^\infty e^{i(E-E'-\Omega_{\text{out}}+i\epsilon)t} dt + \int_{-\infty}^0 e^{i(E-E'-\Omega_{\text{in}}-i\epsilon)t} dt \right\} \quad (54)$$

$$= \frac{i}{\sqrt{2\Omega_{\text{out}}}} \left\{ \frac{1}{E - E' - \Omega_{\text{out}} + i\epsilon} - \frac{1}{E - E' - \Omega_{\text{in}} - i\epsilon} \right\}. \quad (55)$$

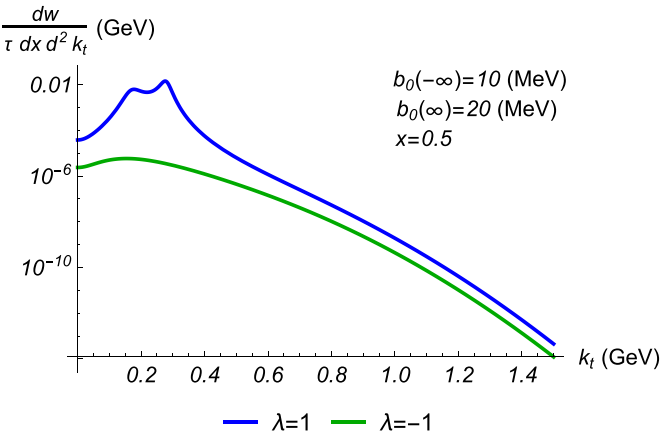


FIG. 4. The photon spectrum for $b_0(t) = A_1 + B_1 \tanh \frac{t}{\tau}$, with $A_1 = 15$ MeV, $B_1 = 5$ MeV, and $\tau = 25 \text{ GeV}^{-1} \approx 5 \text{ fm}/c$. The incident fermion energy $E = 20$ GeV, mass $m = 0.3$ GeV, and unit charge.

In particular, if b_0 is constant, then $B_1 = 0$, $\Omega_{\text{out}} = \Omega_{\text{in}}$, and Eq. (55) reduces to Eq. (16) as can be seen by using the following formula:

$$\frac{1}{x \pm i\epsilon} = \mp i\pi \delta(x) + P \frac{1}{x}. \quad (56)$$

VI. PHOTON SPECTRUM FOR $b(t) = A_2(t/\tau)/(1 + t^2/\tau^2)$

In this section, consider a scenario in which b_0 evolves in such a way that its value at distant past and future vanish. It can be described by the following model:

$$b(t) = \frac{A_2 t}{\tau(1 + \frac{t^2}{\tau^2})}. \quad (57)$$

Unlike the previous model, the exact solution to Eq. (3) with Eq. (57) is not known. I therefore employ the adiabatic approximation that was developed in Sec. IV and successfully used in the previous section. Accordingly, plugging Eq. (57) into Eq. (28) yields

$$a(t) = \frac{1}{\sqrt{2k}} e^{-ikt} \left(1 + \frac{t^2}{\tau^2}\right)^{\frac{i\lambda\tau A_2}{4}}. \quad (58)$$

Its Fourier transform is given by

$$\begin{aligned} \tilde{a}(E - E') &= \frac{1}{\sqrt{2k}} \int_0^\infty e^{-i(E'-E+k-i\epsilon)t} \left(1 + \frac{t^2}{\tau^2}\right)^{\frac{i\lambda\tau A_2}{4}} dt \\ &\quad + \frac{1}{\sqrt{2k}} \int_{-\infty}^0 e^{-i(E'-E+k+i\epsilon)t} \left(1 + \frac{t^2}{\tau^2}\right)^{\frac{i\lambda\tau A_2}{4}} dt \end{aligned} \quad (59)$$

$$\begin{aligned} &= \frac{2}{\sqrt{2k}} \int_0^\infty \left(1 + \frac{t^2}{\tau^2}\right)^{\frac{i\lambda\tau A_2}{4}} \\ &\quad \times \cos[(E' - E + k)t] e^{-\epsilon t} dt. \end{aligned} \quad (60)$$

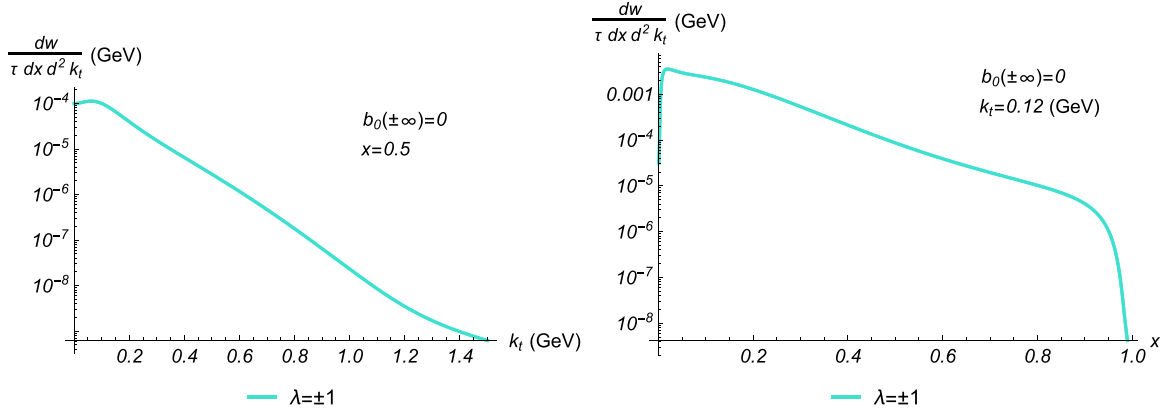


FIG. 5. The photon spectrum for $b_0(t) = A(t/\tau)/(1 + t^2/\tau^2)$, with $A_2 = 10$ MeV and $\tau = 25 \text{ GeV}^{-1} \approx 5 \text{ fm}/c$. The incident fermion energy $E = 20$ GeV, mass $m = 0.3$ GeV, and unit charge. The spectrum does not depend on the photon's polarization, which is reflected in turquoise color, and does not exhibit any resonances.

This integral can be taken using MATHEMATICA 14.2 package and the result expressed in terms of the function:

$$F_\nu(x) = x^{\frac{1}{2}+i\nu} \left[\nu J_{-\frac{1}{2}-i\nu}(x^*) \Gamma(-i\nu) - J_{\frac{1}{2}+i\nu}(x^*) \Gamma(1-i\nu) \sinh(\pi\nu) + \nu \cosh(\pi\nu) \Gamma(-i\nu) \mathbf{H}_{\frac{1}{2}+i\nu}(x^*) \right], \quad (61)$$

where $\nu = \frac{\lambda\tau A_2}{4}$ and $\mathbf{H}$ is Struve's function. Thus,

$$\tilde{a}(E - E') = \frac{\pi^{\frac{3}{2}} 2^{-\frac{1}{2}+i\nu}}{\sqrt{2k} \tau^{2i\nu}} \frac{(y^2 + \epsilon^2)^{-\frac{1}{2}-i\nu}}{\cosh(\pi\nu) \sinh(\pi\nu) \Gamma(1-i\nu) \Gamma(-i\nu)} \{F_\nu[(iy + \epsilon)\tau] + F_\nu[(-iy + \epsilon)\tau]\}, \quad (62)$$

where

$$y = E' - E + k = \frac{k_\perp^2 + m^2 x^2}{2x(1-x)p_z}. \quad (63)$$

In the sudden transition limit $\tau \rightarrow 0$, Eq. (62) vanishes. Indeed,

$$\lim_{\tau \rightarrow 0} \tilde{a}(E - E') = \frac{i}{\sqrt{2k}} \left(\frac{1}{E' - E + k + i\epsilon} - \frac{1}{E' - E + k - i\epsilon} \right) = \frac{1}{\sqrt{2k}} 2\pi \delta(y) = 0, \quad (64)$$

because $y > 0$. Substituting Eq. (62) into Eq. (13) furnishes the photon spectrum:

$$\begin{aligned} \frac{dw}{dx d^2 k_\perp} &= \frac{1}{16(2\pi)^3 E^2} \sum_\lambda \frac{2e^2}{x^3(1-x)^2} [k_\perp^2(2-2x+x^2) + m^2 x^4] \\ &\times \frac{\pi\tau}{2 \cosh^2(\pi\nu)} \frac{|F_\nu[(iy + \epsilon)\tau] + F_\nu[(-iy + \epsilon)\tau]|^2}{y^2 + \epsilon^2}. \end{aligned} \quad (65)$$

The numerical result representing the parameters of the quark-gluon plasma is shown in Fig. 5. The most conspicuous feature of the spectrum is the equality of the photon polarizations indicating that the λ dependence cancels out. This happens because $|F_\nu + F_{-\nu}|^2$ is an even function of ν and, consequently, of λ . Also, it is noteworthy that the resonances are absent, which aligns with the observations in the preceding section, where it was observed that the emergence of the resonances is contingent upon finite values of at least one of the asymptotics of b_0 .

VII. SUMMARY

The chiral Cherenkov mechanism of photon radiation was investigated using two models of the time evolution of the chiral magnetic conductivity $b_0(t)$. The first model

$b_0(t) = A_1 + B_1 \tanh(t/\tau)$ describes a transition of b_0 between two asymptotically constant values $b_0(\pm\infty)$. This mechanism allows the production of only circularly polarized photons. In general, the cross sections for the two polarizations are significantly different. The main feature of the $k_\perp$ spectrum is the potential emergence of one or two resonances. The necessary conditions of the resonance emergence are the fulfillment of either or both of the following conditions: $\lambda b_0(\infty) > 0$ and $\lambda b_0(-\infty) > 0$. The positions of the resonances are given by Eqs. (49)–(52). The typical spectra are presented in Figs. 2–4. In contrast, only a single resonance may appear when b_0 is constant as illustrated in Fig. 1.

The second model represents a pulse of chirality with a finite duration: $b_0(t) = A_2(t/\tau)/(1 + t^2/\tau^2)$. In this case, there are no resonances, and the two polarizations have identical spectra.

The two models investigated in this paper suggest that the time dependence of the chiral chemical potential diminishes the average polarization of the chiral Cherenkov radiation as compared to the constant b_0 case. The chiral Cherenkov resonance, appearing in the figures as a peak at a certain $k_\perp$ (or two peaks at the corresponding x 's), gives the largest contribution to the rate and emerges if $b_0(\pm\infty) \neq 0$. It also determines the average polarization. The mathematical structure of the Fourier image of the amplitude $a(t)$ suggests that these conclusions may hold universally. Indeed, the parameter ϵ that controls the convergence of the time integral at $t \rightarrow \pm\infty$ in Eq. (8) also sets the resonance height, which is proportional to $1/\epsilon$. The resonance appears in any model in which b_0 is finite at either the remote past or the future.

The present calculations significantly relied on the adiabatic approximation developed in Sec. IV. This approximation enabled us to analytically compute the photon amplitude and its Fourier transform. It holds when $k_z^2 \gg |b_0|/\tau$. For b_0 in the range of 10–100 MeV [26–29] and τ in the range of 1–10 fm [10,11], it implies conservatively that $k \gg 100$ MeV. The validity of the adiabatic approximation in a wide range of photon energies further indicates that the present main results—the qualitative structure of the photon spectrum—are

not model dependent. Nevertheless, further investigation of the model space, potentially employing numerical methods, would be of great interest.

I implicitly assumed that the medium lifetime and spatial dimensions are much larger than any timescale or distance scale associated with sphaleron transitions and the process $f \rightarrow f + \gamma$. This assumption might be in tension with the relativistic heavy-ion phenomenology. It would be interesting to investigate how the finite dimensions of the realistic quark-gluon plasma impact the results presented in this paper.

The present analysis focused exclusively on applications to the quark-gluon plasma. Nevertheless, the developed formalism is equally applicable to describe photon radiation emanating from cosmic rays in the background of the axion field [30] or from electrons in Weyl semimetals [31].

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DATA AVAILABILITY

No data were created or analyzed in this study.

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