

Melting of heavy quarkonia in QGP using deep neural networks

Mohammad Yousuf Jamal^{1,*}, Fu-Peng Li^{1,2,3,4,†}, Long-Gang Pang^{1,2,‡} and Guang-You Qin^{1,2,§}

¹Key Laboratory of Quark and Lepton Physics (MOE) & Institute of Particle Physics, *Central China Normal University*, Wuhan 430079, China

²Artificial Intelligence and Computational Physics Research Center, *Central China Normal University*, Wuhan 430079, China

³Key Laboratory of Nuclear Physics and Ion-beam Application (MOE) & Institute of Modern Physics, *Fudan University*, Shanghai 200433, China

⁴Shanghai Research Center for Theoretical Nuclear Physics, NSFC and *Fudan University*, Shanghai 200438, China



(Received 23 September 2025; revised 23 December 2025; accepted 20 February 2026; published 18 March 2026)

Machine learning techniques have emerged as powerful tools for tackling nonperturbative challenges in quantum chromodynamics. In this study, we introduce a data-driven framework employing deep neural networks to systematically predict the temperature-dependent behavior of the screening mass $m_D(T)$ and the strong coupling constant $\alpha_s(T)$ within a quark-gluon plasma medium. These medium-sensitive quantities are subsequently employed to compute the thermal widths $\Gamma_n(T)$ and binding energies $E_B(T)$ of heavy quarkonia states, specifically charmonia and bottomonia, by numerically solving the Schrödinger equation with medium-modified heavy quark potentials. To estimate the dissociation temperatures T_d of various quarkonia states, we employ two complementary dissociation criteria: the conventional one, where $2E_B(T_d) = \Gamma_n(T_d)$, and an additional lower bound criterion defined by $E_B(T_d) = 3T_d$. This dual-criterion approach provides a more constrained and physically motivated estimate of the temperature range over which quarkonia states dissolve in the quark-gluon plasma (QGP) environment. Our machine learning-enhanced predictions show excellent agreement with available lattice quantum chromodynamics results, especially for the ground states $\Upsilon(1S)$ and J/ψ , and offer new perspectives on the sequential suppression pattern detected in relativistic heavy-ion collision experiments. Overall, this work advances the quantitative description of quarkonium suppression and demonstrates the prospect of modern machine learning methods to bridge theoretical predictions and experimental observations, thereby contributing significantly to QGP tomography.

DOI: [10.1103/z3kh-zr1t](https://doi.org/10.1103/z3kh-zr1t)

I. INTRODUCTION

Ultrarelativistic heavy-ion collisions at RHIC and the LHC have established that the quark-gluon plasma (QGP) behaves as a nearly perfect fluid, exhibiting strong collective flow and an exceptionally low shear viscosity to entropy density ratio [1]. This remarkable finding stands in stark contrast to early theoretical expectations of a weakly interacting gas of quarks and gluons, emphasizing the necessity for precision probes to unravel the emergent properties of this strongly coupled medium. Heavy quarkonia ($Q\bar{Q}$ bound states) have proven to be particularly sensitive indicators of deconfinement and the QGP's screening behavior [2–5], with their suppression

patterns encoding information on color screening, parton density, and medium-induced dissociation mechanisms [6,7].

Being color-singlet states composed of heavy quark-antiquark pairs ($c\bar{c}$, $b\bar{b}$), quarkonia occupy a unique position in quantum chromodynamics (QCD) phenomenology [8–10]. Their production occurs predominantly in the initial hard scatterings of a collision, allowing them to probe the earliest stages of the medium before significant collective evolution ensues. The interplay of perturbative and nonperturbative scales in QCD, ranging from the large heavy quark mass to confinement-driven string dynamics [11–13], has inspired a diverse array of models for quarkonium production and suppression, including the color evaporation model [14,15], color singlet and octet mechanisms [16,17], and recombination scenarios [18,19].

The foundational idea, proposed by Matsui and Satz [2], states that color screening in the QGP weakens the confining force between the heavy quark and antiquark, resulting in quarkonium dissociation. This picture has evolved through refinements incorporating thermal broadening, dynamical screening, and real-time effects [20–22]. In practice, medium modifications to the heavy quark potential, often formulated as a screened Cornell-type potential [23–26], play a central role, modified via dielectric permittivity using the hard thermal loop approximation [27–30]. This gives us a medium

*Contact author: yousufjml5@gmail.com

†Contact author: fppli@fudan.edu.cn

‡Contact author: lgpang@ccnu.edu.cn

§Contact author: guangyou.qin@ccnu.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

modified complex potential between $Q\bar{Q}$ bound states. The real part governs the temperature-dependent binding energy $E_B(T)$, while the imaginary part yields the thermal decay width $\Gamma_n(T)$,

In the standard approach, quarkonium dissociation is identified by equating the thermal width to twice the binding energy, $\Gamma_n(T_d) = 2E_B(T_d)$ where T_d denotes the dissociation temperature [8,31]. To further refine this estimate, we additionally impose a physically motivated lower bound criterion: the state is considered too weakly bound to survive when its binding energy drops to the order of the mean thermal energy of the medium, i.e., $E_B(T_d) = 3T_d$ [21,32]. The simultaneous use of both criteria constrains the possible dissociation temperature window, yielding a more robust estimate consistent with lattice and spectral function analyses.

A key innovation in this work is the use of machine learning (ML) to enhance the determination of the Debye screening mass $m_D(T)$ and the strong coupling $\alpha_s(T)$, crucial inputs for the in-medium potential. By training a deep neural network (DNN) on lattice QCD results, we obtain nonperturbative, data-driven estimates of m_D and α_s , which in turn lead to improved predictions for $E_B(T)$, $\Gamma_n(T)$, and hence, T_d for various charmonium and bottomonium states. This ML-augmented framework bridges the rigor of first-principles calculations with the computational efficiency of modern data science tools, providing a powerful approach for QGP tomography.

The remainder of this paper is organized as follows. Section II outlines the theoretical framework, including the ML-based extraction of the Debye mass and the medium-modified potential. Section III presents our results for the dissociation temperatures using both dissociation criteria, and compares them with lattice QCD and some other available data. Section IV summarizes our conclusions and provides an outlook for future developments. Some discussions on numerical analysis related to the current work are presented in Appendix B.

II. FORMALISM

The behavior of heavy quarkonia in a hot QCD medium crucially depends on how the interquark potential is modified by color screening and medium-induced dissipation. This section details the theoretical framework employed in this work, including the construction of the in-medium quarkonium potential, the machine-learning-based determination of the Debye mass, and the extraction of binding energies, thermal widths, and dissociation temperatures using both the conventional and lower-bound criteria.

A. Quarkonium potential in the hot QGP medium

In vacuum, the static quark-antiquark interaction is well described by the Cornell potential [23,24], which captures both the short-range Coulombic attraction and long-range confining force:

$$V_{\text{vac}}(r) = -\frac{\alpha}{r} + \sigma r, \quad (1)$$

where α is the effective strong coupling constant and σ denotes the string tension. At finite temperature, screening effects in the QGP medium weaken both components of the potential. Following the approach in Refs. [21,32], we introduce medium modifications via the dielectric permittivity $\epsilon(k)$ in momentum space:

$$\tilde{V}(k, T) = \frac{\tilde{V}_{\text{vac}}(k)}{\epsilon(k)}. \quad (2)$$

The Fourier transform of the vacuum Cornell potential is given by

$$\tilde{V}_{\text{vac}}(k) = -\sqrt{\frac{2}{\pi}} \left(\frac{\alpha}{k^2} + \frac{2\sigma}{k^4} \right). \quad (3)$$

The in-medium potential is obtained by inverse Fourier transform of Eq. (2) where the complex form of $\epsilon(k)$ can be found in Ref. [33]. Due to the complex form of $\epsilon(k)$, the medium-modified potential also splits into real and imaginary parts. The real potential is obtained as

$$\text{Re}[V(r, T)] = -\alpha \frac{e^{-m_D r}}{r} + \frac{2\sigma}{m_D} (1 - e^{-m_D r}) - \sigma r e^{-m_D r}, \quad (4)$$

$$\text{Im}[V(r, T)] = \text{Im}V_1(r, T) + \text{Im}V_2(r, T), \quad (5)$$

where

$$\begin{aligned} \text{Im}V_1(r, T) &= -2\alpha T \int_0^\infty \frac{dz}{(z^2 + 1)^2} \left(1 - \frac{\sin(m_D r z)}{m_D r z} \right), \\ \text{Im}V_2(r, T) &= \frac{4\sigma T}{m_D^2} \int_0^\infty \frac{dz}{z(z^2 + 1)^2} \left(1 - \frac{\sin(m_D r z)}{m_D r z} \right). \end{aligned} \quad (6)$$

The presence of an imaginary part accounts for Landau damping and inelastic scattering processes that broaden the quarkonium state in the medium [8,27].

The screening mass $m_D(T)$ modifies both the short-range Coulomb term and the long-range string term of the potential. Specifically, it introduces an exponential damping that weakens the attractive force at large distances, reflecting color screening in the deconfined medium. The Coulombic term becomes Yukawa-like, while the linear confining term is partially screened and suppressed.

It is important to note that the Debye screening mass $m_D(T)$ is not just an algebraic function of the quasiparton masses, but is more deeply rooted in the underlying thermal structure of the medium. In hot QCD, the Debye mass originates from the static limit of the longitudinal gluon self-energy and is fundamentally expressed in terms of the thermal distribution functions of quarks and gluons as

$$\begin{aligned} m_D^2 &= -4\pi\alpha_s(T) \left(2N_c \int \frac{d^3p}{(2\pi)^3} \partial_p f_g(\mathbf{p}) \right. \\ &\quad \left. + N_f \int \frac{d^3p}{(2\pi)^3} \partial_p (f_q(\mathbf{p}) + f_{\bar{q}}(\mathbf{p})) \right). \end{aligned} \quad (7)$$

In quasiparticle models, these distribution functions $f_{g,q,\bar{q}}(p)$ are modified to include medium effects, typically by introducing temperature-dependent effective masses $m_{g/q}(T)$ in the dispersion relation $E(p) = \sqrt{p^2 + m^2(T)}$

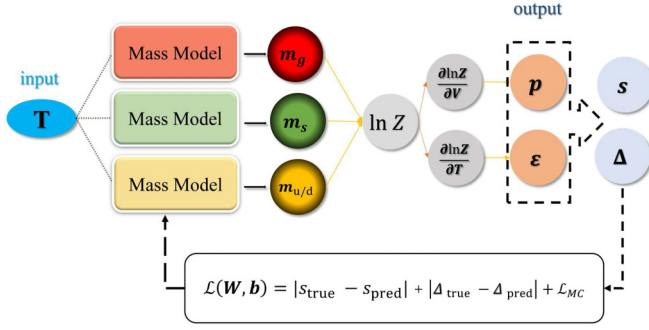


FIG. 1. Sketch of the DNN used to extract the temperature-dependent quasiparton masses $m_{u/d}(T)$, $m_s(T)$, and $m_g(T)$ [38,39]. The symbols $\mathbf{W}$ and $\mathbf{b}$ denote the trainable weights and biases of the neural network.

or by adding fugacity-like corrections. As a result, the Debye mass becomes implicitly dependent on the quasiparticle properties through these modified distributions. While our approach computes $m_D(T)$ using a formula derived from leading-order hard thermal loop (HTL) theory, the inputs, specifically the quasiparton masses obtained via machine learning, already encode nontrivial thermal corrections [34–36]. Therefore, the Debye mass in this framework effectively captures the collective response of the QGP medium and remains tightly connected to the quasiparticle description.

B. Machine learning determination of Debye mass and strong coupling

A crucial input parameter for the medium-modified potential is the Debye mass $m_D(T)$, which incorporates non-perturbative effects beyond leading-order HTL calculations [37]. In this work, we utilize a deep-learning quasiparton gas model (DLQPM) to extract $m_D(T)$ and the running coupling constant $\alpha_s(T)$ directly from lattice QCD data [38–40]. The training methodology comprises the following steps:

- (1) *Architecture.* The framework of DLQPM, sketched in Fig. 1, is designed to reconstruct the QCD equation of state (EoS) at zero chemical potential. The model incorporates three mass subnetworks for light u/d quarks, strange quarks, and gluons, respectively. Each subnetwork is implemented as an eight-layer residual neural network (ResNet) with swish activation functions and 32 neurons per layer.
- (2) *Training set.* We employ 50 numerical data points from HotQCD calculations [40], covering the temperature range $T/T_c \in [1, 3]$, where the critical temperature is taken as $T_c = 0.15$ GeV.
- (3) *Objective.* Given the quasiparton masses, the EoS is obtained from the partition function. The network parameters (weights and biases) are optimized by minimizing the mean squared error between the model predictions and lattice QCD results for the entropy density $s(T)$ and the trace anomaly $\Delta(T) = \varepsilon(T) - 3p(T)$. In addition, we impose high-temperature

constraints inspired by HTL calculations through the loss term $\mathcal{L}_{MC}$, which is activated at $T > 2.5 T_{\text{cut}}$,

$$\mathcal{L}_{MC} = \left| R_{g/q} - \frac{3}{2} \right| + \left| \frac{m_s - m_{u/d}}{\bar{m}_s - \bar{m}_{u/d}} - 1 \right|,$$

$$R_{g/q} = \frac{m_{g,T>2.5T_{\text{cut}}}}{m_{u/d,T>2.5T_{\text{cut}}}} = \sqrt{\frac{3}{2} \left(\frac{N_c}{3} + \frac{N_f}{6} \right)}, \quad (8)$$

where $m_{u/d}$, m_s , and m_g are the effective masses output by the DLQPM, and $\bar{m}_{u/d}$, $\bar{m}_s$ denote the colored corresponding current masses ($\bar{m}_s = 95$ MeV). We set $T_{\text{cut}} = T_c$, and the choice $T > 2.5 T_{\text{cut}}$ defines a high-temperature region where HTL expectations are reliable and can be used to constrain the asymptotic behavior of the masses. After 5×10^4 training epochs, the total loss converges to values of order 10^{-5} .

- (4) $m_D(T)$ and $\alpha_s(T)$. Once the quasiparton masses are obtained using the DLQPM, the strong coupling constant $\alpha_s(T)$ is determined from the thermal gluon and light-quark masses [41]:

$$m_g^2(T) = \frac{1}{6} g_s^2(T) \left[\left(N_c + \frac{1}{2} N_f \right) T^2 \right],$$

$$m_{u,d}^2(T) = \frac{N_c^2 - 1}{8 N_c} g_s^2(T) T^2, \quad (9)$$

leading to

$$g_s^2(T) = \frac{m_g^2(T) + m_{u,d}^2(T)}{\frac{1}{6} \left[\left(N_c + \frac{1}{2} N_f \right) T^2 \right] + \frac{N_c^2 - 1}{8 N_c} T^2}, \quad (10)$$

and $\alpha_s(T) = g_s^2(T)/(4\pi)$.

The relations above connect the quasiparton thermal masses to the temperature-dependent strong coupling $\alpha_s(T)$, which in turn provides a key input for the Debye screening mass. Physically, the Debye mass $m_D(T)$ increases with temperature as a consequence of the growing density of color charges in the deconfined medium. This enhancement reflects the QGP's increasing ability to screen long-range color interactions, thereby weakening the confining potential between quarks and antiquarks at elevated temperatures. Now using these inputs, $m_D(T)$ can be computed via the following gauge-invariant relation:

$$m_D(T) = T \sqrt{4\pi \alpha_s(T) \left(\frac{N_c}{3} + \frac{N_f}{6} \right)}. \quad (11)$$

To quantify the uncertainty associated with the DLQPM extraction, we retrain the model ten times, obtaining ten distinct sets of quasiparton masses. From these, we compute the mean and variance of $m_D(T)$ and $\alpha_s(T)$, and propagate them through the potential to the binding energies and thermal widths. The resulting error bands shown in the figures of Sec. III therefore reflect the spread over these independent trainings. Additional training diagnostics, including the evolution of the total loss and learning rate with the epoch, are presented in Appendix A. As a validation of the DLQPM, in Fig. 2 we compare the entropy density s/T^3 and the trace anomaly $\Delta(T)$ obtained from the network with the HotQCD

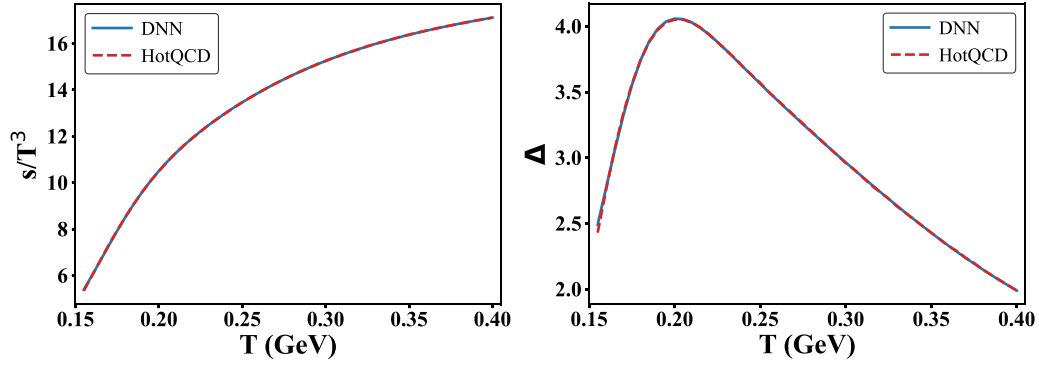


FIG. 2. Entropy density s/T^3 (left) and trace anomaly $\Delta = \varepsilon - 3p$ (right) as functions of temperature, comparing the DLQPM reconstruction with HotQCD lattice results [40].

data [40]. The excellent agreement over the range $T/T_c \in [1, 3]$ demonstrates that the learned quasiparton masses reproduce the lattice equation of state. In Fig. 3 we show the resulting effective masses $m_{u/d}(T)$, $m_s(T)$, and $m_g(T)$ and confront them with representative quasiparticle model parametrizations [42,43]. The overall consistency, including the expected hierarchy between light and strange quarks, suggests that the DLQPM effectively captures the main medium effects encoded in these models. The corresponding strong coupling $g^2(T)$, reconstructed from Eq. (10), is displayed in Fig. 4 and compared with standard parametrizations; the modest differences at intermediate temperatures can be traced back to distinct implementation choices in the underlying quasiparticle fits.

While our framework is data-driven and anchored to lattice QCD results via ML training, the analytic forms used to extract $\alpha_s(T)$ and $m_D(T)$ incorporate leading-order HTL structures. Thus, the approach captures nonperturbative trends through ML optimization but still relies on perturbative-inspired expressions. Therefore, we interpret the nonperturbative nature of the model in a practical,

hybrid sense—combining perturbative inputs with lattice-constrained ML outputs to enhance the authenticity of the potential. This ML-derived Debye mass exhibits a smooth transition to lattice QCD predictions, thereby ensuring that the medium screening and damping effects are fundamentally grounded in nonperturbative QCD data rather than being dependent solely on perturbative approximations [21,32]. In particular, this data-driven approach ensures that the potential’s temperature dependence is anchored to first-principles lattice QCD results, especially in the crossover region ($1.0 \leq T/T_c \leq 1.5$), where conventional perturbative methods are known to break down [21,31]. The resulting potential then serves as input for the Schrödinger equation (Sec. II C) to compute binding energies and widths.

C. Schrödinger equation and numerical solutions

The temperature-dependent binding energies $E_B(T)$ and radial wave functions $\psi_n(r)$ are obtained by solving the time-independent radial Schrödinger equation using the real part of

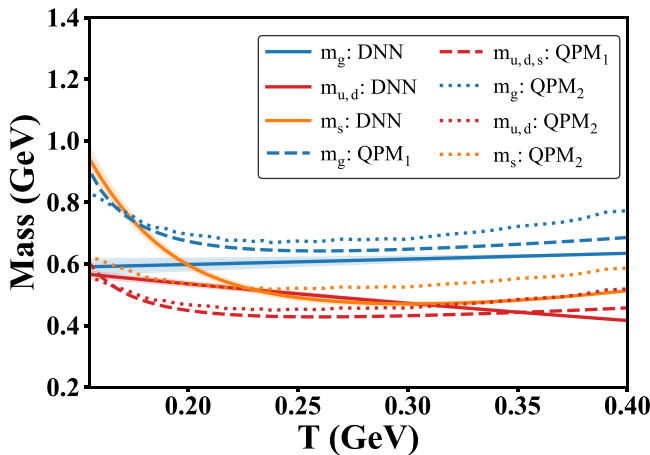


FIG. 3. Temperature dependence of the effective quasiparton masses for light quarks, strange quarks, and gluons obtained from the present DLQPM, compared with representative quasiparticle model results from Refs. [42,43].

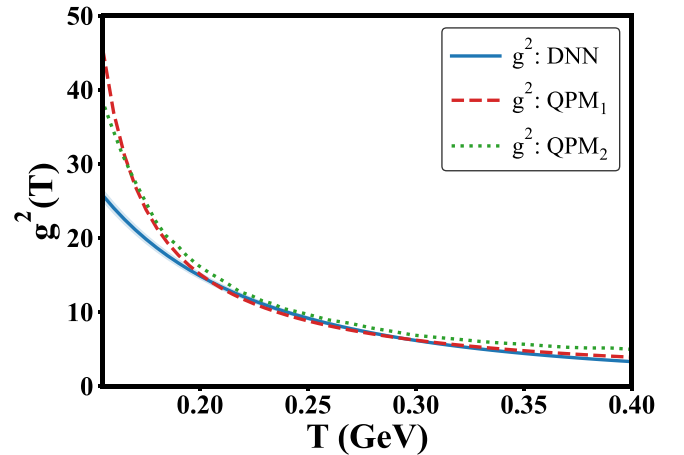


FIG. 4. Temperature dependence of the strong coupling $g^2(T)$ extracted from the present DLQPM, compared with representative quasiparticle model parametrizations from Refs. [42,43].

the in-medium heavy-quark potential:

$$\left[-\frac{1}{2\mu_{Q\bar{Q}}} \frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{2\mu_{Q\bar{Q}}r^2} + \text{Re}[V(r, T)] \right] \psi_n(r) = E_n(T) \psi_n(r), \quad (12)$$

where $\mu_{Q\bar{Q}} = m_Q/2$ is the reduced mass of the heavy quark-antiquark pair and ℓ is the orbital angular momentum quantum number (taken $\ell = 0$ for the S -wave ground and first excited states). To accurately resolve the delicate balance between short-range Coulomb attraction and long-range screening, we employ a finite difference method (FDM) with second-order central difference stencils and Neumann boundary conditions at $r = r_{\min}$ and $r = r_{\max}$. The radial coordinate is discretized into $N = 4000$ grid points extending up to $r_{\max} = 30$ fm to ensure convergence even at low binding energies near the dissociation point. The resulting large sparse matrix eigenvalue problem is solved iteratively for each temperature step using a combination of bisection and inverse iteration, allowing us to track both the ground and excited states continuously as functions of $T/T_c \in [1, 3]$. This approach ensures robust extraction of weakly bound states near the deconfinement crossover, where the binding energies become comparable to the thermal energy scale. We have included Appendix B, which provides a detailed introduction to the FDM and its comparison with alternative numerical approaches.

D. Thermal width

The thermal width $\Gamma_n(T)$ characterizes the in-medium decay rate, or equivalently the inverse lifetime, of a quarkonium state embedded in the QGP. It provides a quantitative measure of the extent to which a bound state becomes unstable due to interactions with the thermal medium. Physically, this width reflects the spectral broadening of quarkonium states arising from medium-induced processes such as dissociation and scattering with thermal partons.

In the potential model framework, the interaction between a heavy quark and antiquark is described by a complex potential, where the real part governs the binding and the imaginary part encodes dissipative effects due to the QGP. The imaginary component acts as a perturbative contribution to the otherwise real Cornell-type potential. Within the formalism of first-order quantum mechanical perturbation theory, this imaginary term induces a finite decay width for the bound state [27,44]. The thermal width is thus computed as the expectation value of the imaginary part of the potential over the normalized quarkonium wave function:

$$\Gamma_n(T) = - \int_0^\infty 4\pi r^2 |\psi_n(r)|^2 \text{Im}[V(r, T)] dr. \quad (13)$$

This formulation captures how the imaginary part of the in-medium potential introduces a thermal decay channel for the heavy quark-antiquark pair. The imaginary potential arises from processes like Landau damping and gluodissociation, which represent the scattering of the quarkonium with the thermal gluons and quarks of the surrounding plasma. These interactions lead to decoherence and eventual dissociation of the bound state, manifesting as a broadening of its spectral peak. Here, $\psi_n(r)$ is the radial wave function corresponding to

the n th quarkonium state, obtained by solving the Schrödinger equation using the real part of the temperature-dependent complex potential. The term $\text{Im}[V(r, T)]$, provided explicitly in Eq. (6), encapsulates the dissipative features of the QGP medium. The negative sign in Eq. (13) ensures that a negative imaginary potential translates to a positive decay width, consistent with physical intuition.

The magnitude of $\Gamma_n(T)$ governs how rapidly a quarkonium state is likely to decay or dissolve under thermal fluctuations. A larger thermal width corresponds to a shorter survival time, making the bound state more susceptible to melting. As the temperature of the QGP increases, both the screening of the color potential and the inelastic damping effects become more pronounced, enhancing $\text{Im}[V(r, T)]$ and thereby increasing $\Gamma_n(T)$. When the thermal width becomes comparable to or exceeds the binding energy, the bound state is effectively destroyed. In this study, we systematically evaluate $\Gamma_n(T)$ for both ground and excited states of charmonium and bottomonium. These widths, in conjunction with the corresponding binding energies, are used to determine the dissociation temperatures of the quarkonium states based on well-motivated physical criteria.

E. Dissociation criteria

A precise determination of the dissociation temperature T_d for quarkonium states is crucial for interpreting their sequential suppression patterns in heavy-ion collisions [2,32]. In this study, we adopt two well-motivated and complementary criteria to estimate the temperature range over which a given quarkonium state melts in the quark-gluon plasma. The first is the conventional thermal width criterion, which asserts that a resonance ceases to exist as a distinguishable bound state once its thermal decay width becomes comparable to its binding energy [8,27,31]. Following standard practice, we employ the quantitative condition $\Gamma_n(T_d) = 2E_B(T_d)$. Physically, this implies that the mean time for thermal dissociation due to Landau damping and gluodissociation is on the order of the inverse binding time, leading to significant broadening of the quarkonium spectral function and the disappearance of a well-defined peak [44].

However, it is well-known that potential models can underestimate thermal fluctuations, especially near the crossover region where nonperturbative effects persist. To address this, we incorporate an additional lower-bound criterion for dissociation, inspired by the argument in Refs. [21,32,45]. A quarkonium state is considered too weakly bound to survive if its binding energy drops to the scale of the average kinetic energy of thermal partons. For an ultrarelativistic medium, this scale is approximately $3T$, leading to the condition $E_B(T_d) = 3T_d$. This lower bound accounts for the fact that even if the width is small, a shallow binding renders the state highly susceptible to break-up via thermal collisions or screening. It is important to note that $E_B(T_d) = 3T_d$ corresponds to the average kinetic energy of a thermal excitation in the weak-coupling (Stefan-Boltzmann) limit. In quasiparticle models, the thermal masses tend to reduce the kinetic energy, so this criterion acts as a conservative lower bound.

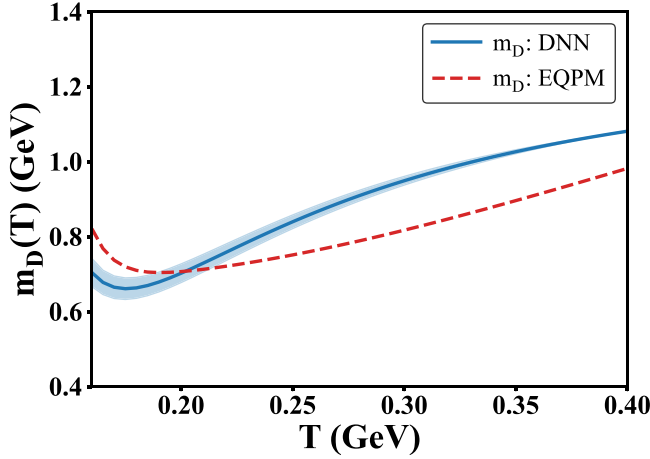


FIG. 5. Screening mass as a function of temperature obtained from our DNN and EQPM. See the text for full details.

Both conditions are solved numerically, applied to the temperature-dependent binding energy and thermal width obtained from the Schrödinger solution. This dual-criterion framework yields a physically constrained temperature window for quarkonium dissociation, providing insight into how robustly each state survives at different QGP temperatures. The extracted dissociation temperatures for J/ψ , $\psi(2S)$, $\Upsilon(1S)$, and $\Upsilon(2S)$ are presented and compared with lattice QCD and experimental observations in Sec. III.

III. RESULTS AND DISCUSSION

We now present our results for the dissociation temperatures of various heavy quarkonia states within the QGP, based on an ML framework which combines lattice-QCD-informed Debye screening masses and running coupling $\alpha_s(T)$ with a robust potential model. Figure 5 displays the temperature dependence of the Debye mass extracted via our DNN training. We trained the DNN 10 times and propagated the variance from $m_D(T)$ through to the observables. The smooth rise with increasing temperature ensures a consistent input to the in-medium potential. We have compared our results for the screening mass with predictions from the effective

fugacity quasiparticle model (EQPM). In this model, nonideal medium effects are incorporated via a temperature-dependent fugacity. Our findings show a consistent agreement between the two. For further details on the EQPM, we refer the reader to Refs. [52–55]. The behavior of the real and imaginary parts of the potential is shown in Figs. 6 and 7. The real part demonstrates the expected screening: at larger separations or higher temperatures, the binding weakens significantly, reflecting color screening in the deconfined medium [31,46]. Meanwhile, the magnitude of the imaginary part grows monotonically with both interquark distance and temperature, encoding Landau damping and quark-antiquark scattering off thermal gluons [27].

We solve the radial Schrödinger equation with this ML-enhanced complex potential to extract the temperature-dependent binding energy $E_B(T)$ and thermal width $\Gamma(T)$ for each quarkonium state. Figures 8–11 present the detailed evolution of these quantities for representative charmonium and bottomonium states under both dissociation conditions. In the case of the width criterion, the curves for $2E_B$ and Γ are shown together as functions of T/T_c . Their intersection indicates the temperature above which the thermal fluctuations are strong enough to overcome the binding within the lifetime of the resonance, signaling effective melting [31]. This behavior is clearly visible in Figs. 8 and 10: for each state, the crossing point shifts to lower temperatures for weaker binding, reflecting the sequential melting hierarchy. However, the lower-bound criterion shown in Figs. 9 and 11 compares the binding energy to the average thermal energy of the medium, approximated by $3T$ [21]. Once E_B drops below this threshold, the bound state is unlikely to survive collisions with the surrounding deconfined medium. This provides a conservative estimate for the minimum temperature at which a given quarkonium state can persist. Finally, Table I consolidates the resulting dissociation temperatures obtained by both criteria for each quarkonium state and compares them to a range of benchmark predictions from other potential models, lattice extractions, and effective field theory analyses [33,47,48,50,51]. The combined width-based upper-bound and lower-bound thermal estimate provides a credible bracket for the melting point of each state, capturing the known experimental hierarchy of suppression observed in heavy-ion collisions at RHIC and LHC.

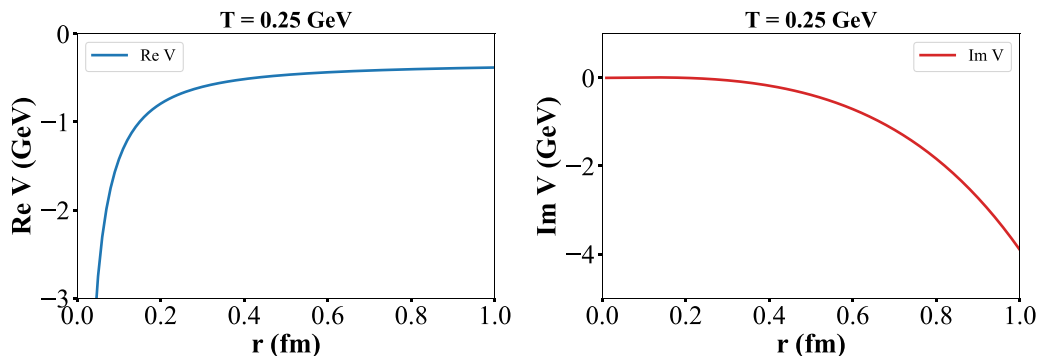


FIG. 6. Left: Real part of the heavy quark potential as a function of interquark separation r at fixed temperatures, $T = 0.25$ GeV. Right: Corresponding imaginary part of the potential vs r , capturing Landau damping and medium-induced broadening effects.

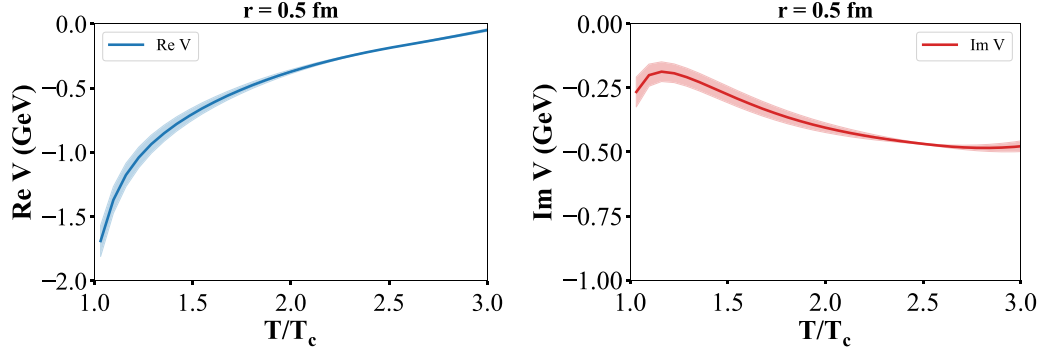


FIG. 7. Temperature dependence of the heavy quark potential at a fixed interquark distance, $r = 0.5$ fm. Left: Real part showing the gradual weakening of binding with increasing temperature. Right: Imaginary part illustrating the growth of thermal broadening as the QGP becomes hotter.

For the tightly bound $\Upsilon(1S)$ ground state, our model predicts a dissociation temperature range of approximately $T_d \in [1.38, 1.99]T_c$, bounded from below by the $E_B = 3T$ criterion and from above by the $\Gamma = 2E_B$ width condition.

This substantial survival window reflects the deeply bound nature of the state, which resists color screening and thermal broadening even at temperatures well above the deconfinement threshold. This is consistent with CMS measurements

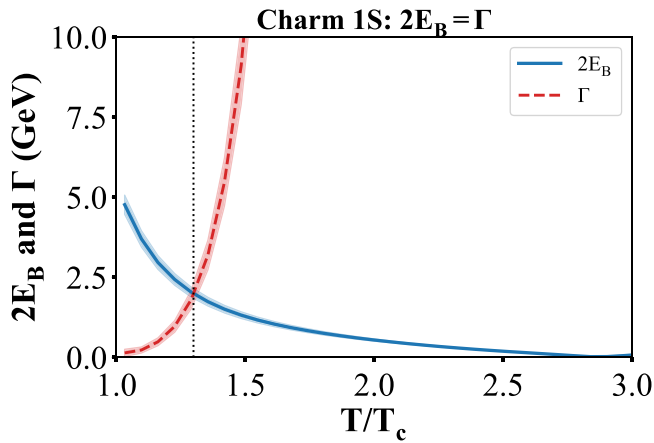


FIG. 8. Binding energy and thermal width for charmonium 1S-states (J/ψ) as a function of temperature. The dissociation temperature is obtained using the $2E_B = \Gamma$ criterion.

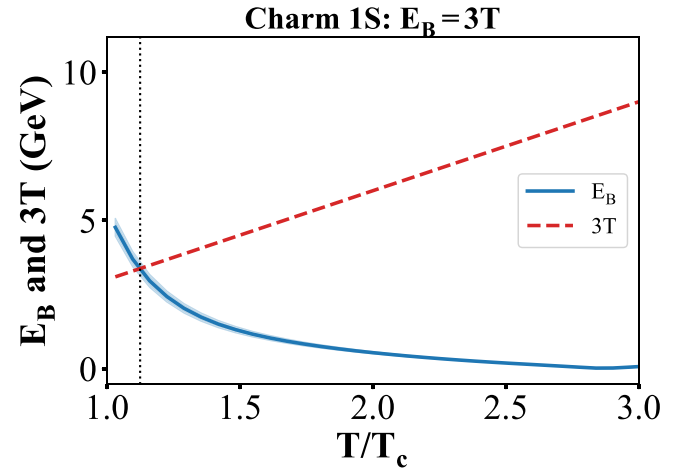


FIG. 9. Binding energy for charmonium 1S-states (J/ψ) as a function of temperature. The dissociation temperature is obtained using the lower bound criterion $E_B = 3T$.

TABLE I. Comparison of the dissociation temperatures T_d for selected quarkonium states. The first two rows show the results of this work using the thermal width criterion ($2E_B = \Gamma$) and the lower bound criterion ($E_B = 3T$). The lower rows present representative theoretical predictions from the literature [31,33,46–51]. All temperatures are in units of the QCD critical temperature T_c .

Method/Reference	Dissociation Temperatures T_d (in units of T_c)			
	$\Upsilon(1S)$	$\Upsilon(2S)$	J/ψ	$\psi(2S)$
This work: ($2E_B = \Gamma$)	1.99	1.29	1.30	≤ 1.00
This work: ($E_B = 3T$)	1.38	1.10	1.13	≤ 1.00
Mocsy and Petreczky [31]	2.00	1.20	1.20	≤ 1.00
Digal <i>et al.</i> [46]	2.31	1.10	1.10	< 1.00
Satz [47]	> 4.00	1.60	2.10	1.12
Blaschke <i>et al.</i> [48]	2.25	1.05	1.20	< 1.00
Rethika <i>et al.</i> [49]	0.77	0.82	1.47	1.62
Meng <i>et al.</i> [50]	5.81	1.56	2.06	1.13
Jamal <i>et al.</i> [33]	2.96	1.47	1.52	< 1.00
Agotiya <i>et al.</i> [51]	2.60	2.10	1.90	1.70

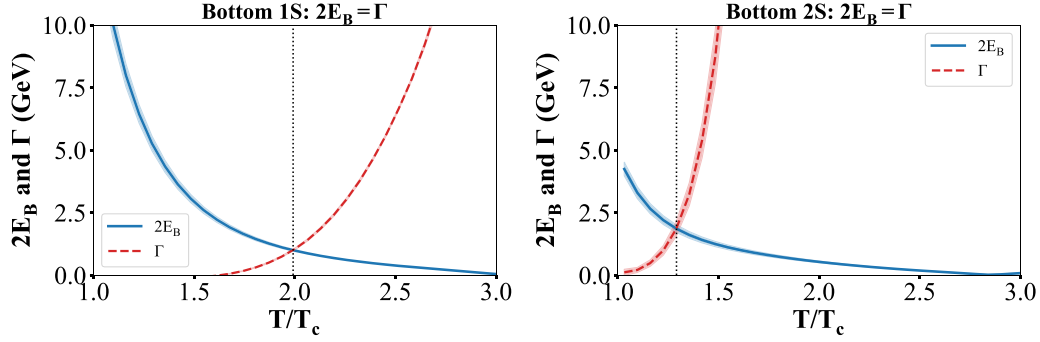


FIG. 10. Binding energy and thermal width for bottomonium states as a function of temperature. The dissociation temperature is obtained using the $2E_B = \Gamma$ criterion. The left panel shows the $\Upsilon(1S)$ state, while the right panel shows the $\Upsilon(2S)$ or Υ' state.

that show a suppressed but nonzero nuclear modification factor $R_{AA} \approx 0.5$ for $\Upsilon(1S)$ even in the most central (0–10%) Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV [56]. In contrast, the excited bottomonium state $\Upsilon(2S)$ exhibits a much lower dissociation threshold, with $T_d \in [1.10, 1.29] T_c$. This relatively narrow survival window implies that $\Upsilon(2S)$ mesons are significantly more vulnerable to medium effects, owing to their weaker binding and larger spatial extent. This theoretical result coheres with LHC data from CMS [56,57], which show a dramatic suppression of $\Upsilon(2S)$ production, with $R_{AA}(\Upsilon(2S)) \approx 0.12$, nearly an order of magnitude lower than that of $\Upsilon(1S)$. Such differential suppression between the ground and excited states provides one of the clearest experimental signatures of sequential quarkonium melting in a thermalized QGP, affirming the predictive strength of our potential-based model enhanced with machine-learned screening dynamics.

For charmonium, our calculations yield a dissociation temperature range for the ground state J/ψ of approximately $T_d \in [1.13, 1.30] T_c$. This moderate survival window reflects the fact that, while the J/ψ is more tightly bound than its excited counterparts, its binding energy is still significantly smaller than that of $\Upsilon(1S)$, rendering it more sensitive to color screening and collisional broadening. This theoretical range aligns well with spectral function results from lattice QCD studies [31] and recent phenomenological analyses that include nonperturbative corrections. Experimentally, the ALICE and CMS collaborations have reported a clear but

incomplete suppression of the J/ψ yield in heavy-ion collisions. Specifically, the nuclear modification factor $R_{AA}(J/\psi)$ remains at about 0.6–0.7 for semi-central Pb–Pb events at the LHC [58], indicating that a nonnegligible fraction of J/ψ states persist through the QGP phase. Moreover, the measured elliptic flow v_2 of J/ψ mesons is finite, typically around 0.05–0.1 [59], supporting the idea that some fraction of J/ψ production originates from regeneration via recombination of deconfined charm quarks in the late stages of QGP evolution [58]. Our calculated survival window, therefore, naturally accommodates both the direct production of primordial J/ψ that survive color screening and the regenerated component that forms near hadronization.

The excited charmonium state $\psi(2S)$, however, shows a much weaker binding and thus a very limited survival probability in the QGP. In our framework, both dissociation criteria predict T_d near or below the critical temperature, T_c , confirming that even mild thermal fluctuations can completely screen its binding potential. Since our framework is not applicable to the confined region, we can not comment on that. However, this is consistent with the striking experimental observation by the CMS collaboration [60] that the $\psi(2S)$ is significantly more suppressed than the J/ψ in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, with an R_{AA} well below 0.2 in central collisions. The enhanced suppression relative to the ground state underscores the significance of the binding energy hierarchy and the medium-induced imaginary potential in explaining the sequential melting pattern.

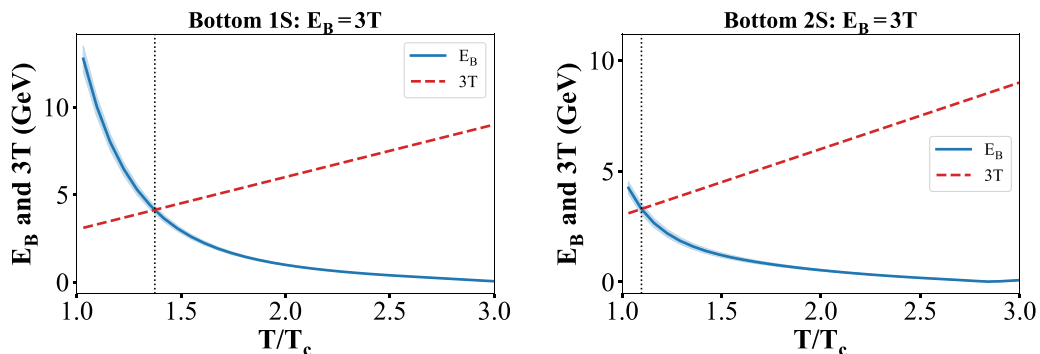


FIG. 11. Binding energy for bottomonium states as a function of temperature. The dissociation temperature is obtained using the lower bound criterion $E_B = 3T$. The left panel shows the $\Upsilon(1S)$ state, and the right panel shows the $\Upsilon(2S)$ or Υ' state.

Taken together, our predicted hierarchy based on the ranges reads

$$T_d(\Upsilon(1S)) > T_d(J/\psi) \sim T_d(\Upsilon(2S)) > T_d(\psi(2S)),$$

which aligns with the sequential suppression pattern observed in R_{AA} measurements at RHIC and LHC [56,58],

$$R_{AA}(\Upsilon(1S)) > R_{AA}(J/\psi) \sim R_{AA}(\Upsilon(2S)) > R_{AA}(\psi(2S)),$$

and illustrate how the dissociation range captured by the dual criteria ($\Gamma = 2E_B$ and $E_B = 3T$) constrains the expected suppression hierarchy and its temperature dependence. Notably, our ML-enhanced potential tightens the theoretical uncertainties by providing a nonperturbative screening mass, thus bridging lattice QCD constraints with continuum phenomenology. We present a comparison of our results at different grid sizes in Appendix B 5 to verify numerical stability. Overall, these results support the interpretation of quarkonia as sensitive QGP thermometers. The approach naturally provides an improved baseline for interpreting quarkonium suppression at RHIC and LHC, reproduces the observed trends in suppression and flow observables, and offers a systematic way to extend to P-wave quarkonia, finite-momentum effects, and eventually to open quantum system treatments that can predict time-resolved R_{AA} and v_2 in dynamic, evolving media [61]. Such developments will further solidify heavy quarkonia as precision probes for the emergent collective properties of the QGP.

IV. SUMMARY AND CONCLUSIONS

In this work, we have developed a robust, data-driven framework to investigate the in-medium behavior and dissociation characteristics of heavy quarkonia in the QGP. To achieve this, we employed an ML approach using a DNN trained on lattice QCD data to obtain a reliable, nonperturbative estimate of the Debye screening mass $m_D(T)$ and the strong coupling constant $\alpha_s(T)$. These ML-improved quantities were then used to construct a medium-modified Cornell potential that accurately incorporates both color screening and dynamical Landau damping effects through a complex-valued potential. Solving the radial Schrödinger equation with this in-medium potential, we extracted the temperature-dependent binding energies and thermal widths for different quarkonium states, J/ψ , $\psi(2S)$, $\Upsilon(1S)$, and $\Upsilon(2S)$. The thermal width, derived as a first-order perturbative estimate from the imaginary part of the potential, quantifies the decay rate due to medium-induced interactions such as gluodissociation and scattering with thermal gluons.

To determine the dissociation temperatures, we implemented two complementary physical criteria. The first, the conventional width criterion ($2E_B = \Gamma$), identifies the point at which the resonance peak broadens sufficiently to lose its bound state character, considered as an upper bound [31]. The second, the lower bound criterion ($E_B = 3T$), asserts that a quarkonium state cannot survive once its binding energy falls below the scale of typical thermal fluctuations in the medium [21]. This dual-criterion approach provides a more constrained and physically motivated estimate of the dissociation temperature range for each state. Our results suggest

that the ground-state bottomonium $\Upsilon(1S)$ may remain intact up to approximately $1.99 T_c$, reinforcing its status as a reliable probe of the hottest phases of the QGP. In contrast, the excited bottomonium state $\Upsilon(2S)$ is expected to survive only up to a considerably lower temperature near $1.29 T_c$. For charmonia, the J/ψ is predicted to dissolve around $1.3 T_c$, whereas the more weakly bound $\psi(2S)$ state melts close to the critical temperature. These findings are consistent with the sequential suppression patterns observed experimentally in heavy-ion collisions at RHIC and the LHC.

A detailed comparison with various theoretical estimates from the literature demonstrates that our ML-augmented framework yields dissociation temperatures in good agreement with lattice QCD constraints and modern potential model predictions, while also reducing dependence on purely perturbative approximations. Overall, this study highlights the utility of integrating ML with potential model analyses to bridge nonperturbative lattice data and phenomenological quarkonium observables.

A. Future prospects

While the present study provides a comprehensive estimate of quarkonium dissociation temperatures using an ML-enhanced potential model, several avenues remain open for further refinement and deeper insights. First, an immediate extension of this work will involve solving the full time-independent Schrödinger equation with the potential itself modeled directly by a deep neural network. By training the DNN to learn the entire complex-valued in-medium potential from lattice QCD correlators or spectral functions, one can bypass analytic ansätze and capture more subtle nonperturbative effects beyond the conventional dielectric model. This data-driven approach promises a more flexible and accurate description of quarkonium binding and broadening at all temperatures.

Second, an important improvement will be to generalize the current static potential framework to an open quantum system treatment. In such a formalism, the heavy quarkonium is treated as an evolving subsystem interacting with a thermalized QGP environment, described dynamically via stochastic Langevin or Lindblad equations. This would allow for a first-principles computation of real-time decoherence, non-Markovian effects, and medium-induced transitions between bound and unbound states, providing a more realistic picture of quarkonium survival and regeneration in heavy-ion collisions.

In addition to these methodological advancements, future studies will systematically extend the present analysis to include P-wave states, explore finite momentum effects, and incorporate dynamical quark masses with explicit chiral symmetry restoration. Such improvements will further strengthen the connection between theory and the wealth of high-precision quarkonium data from RHIC, LHC, and forthcoming facilities, such as the electron-ion collider.

ACKNOWLEDGMENTS

This work was supported in part by Natural Science Foundation of China (NSFC) under Grants No. 12225503 and No.

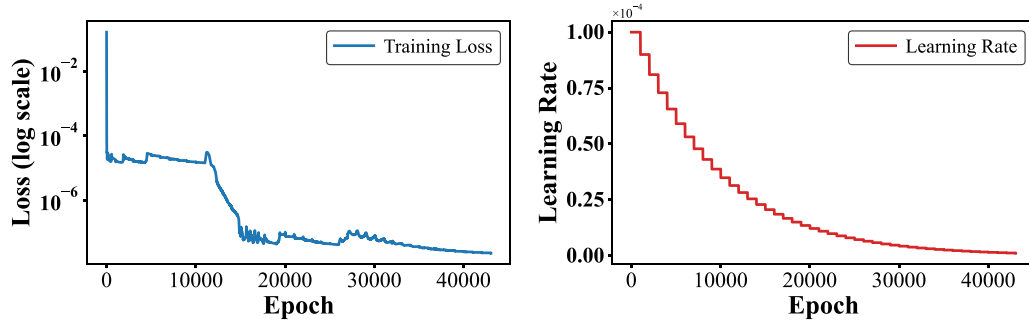


FIG. 12. Training loss (upper panel) and learning rate (lower panel) as functions of epoch.

12435009, and by National Key Research and Development Program of China under Grant No. 2020YFE0202002. The work is also partly supported by the Cross Research Project of Fundamental Research Funds for Central Universities of Central China Normal University in 2025: “Advanced Detection and Artificial Intelligence at the Frontiers of Physics” (Grant No. 30101250317). We gratefully acknowledge the extensive computing resources provided by the Nuclear Science Computing Center at Central China Normal University (NSC³).

DATA AVAILABILITY

The data that support the findings of this article are openly available [67].

APPENDIX A: DNN DETAILS

In Fig. 12 we display the training diagnostics of the DLQPM, namely the total loss and the learning rate as functions of the training epoch. The loss decreases monotonically and saturates after approximately 5×10^4 epochs, indicating stable convergence of the optimization, while the learning-rate schedule ensures a smooth approach to the minimum. This behavior is consistent across the independent trainings used for the uncertainty analysis described in Sec. II B.

APPENDIX B: NUMERICAL PERFORMANCE OF FDM

This section outlines the FDM employed in our study and compares it with other numerical approaches. In addition to standard benchmarks, we include a direct comparison of different grid sizes within our current results, illustrating both the accuracy and convergence properties of the solver.

1. Numerical method (we apply)

We solve the s -wave radial Schrödinger equation on a uniform grid in coordinate space. The radial coordinate is discretized between $r_{\min}$ and $r_{\max}$ with N points, and the second derivative is approximated by a centered finite-difference scheme. This leads to a real, symmetric, tridiagonal Hamiltonian matrix, subject to Dirichlet boundary conditions at the endpoints of the grid [62–65]. The lowest eigenvalues and eigenfunctions are then obtained with an iterative sparse-matrix eigensolver, which is well suited for large tridiagonal systems [66].

The real part of the in-medium potential enters directly in the Hamiltonian, together with the centrifugal contribution for higher orbital angular momenta (though we focus here on $l = 0$ states). The imaginary part of the potential is used afterwards to estimate thermal widths from the normalized wave functions. To control numerical instabilities near $r = 0$, a small-distance regulator is applied when evaluating the kernels.

The discretization error of this scheme decreases quadratically with the grid spacing, so energy levels converge as the inverse square of the number of grid points. In practice, we vary the resolution N while keeping the domain $[r_{\min}, r_{\max}]$ fixed, and check that the dissociation observables are stable. Unless otherwise noted, calculations use $r_{\min} = 0.01$ fm, $r_{\max} = 30.01$ fm, and a baseline resolution of $N = 4000$ grid points, with additional runs at coarser and finer grids to verify convergence.

2. Benchmarks with other potentials having analytical spectra

To validate the solver and its units, we first consider simple potentials with known spectra. The harmonic oscillator (HO) and Coulomb problems serve as analytic benchmarks, while

TABLE II. Benchmark ground-state energies computed with the finite-difference solver and comparison to exact values when available.

Potential	Parameters	N	$r_{\max}$ [fm]	Ground-state energy [GeV]		
				Numerical	Exact	Rel. error
Harmonic oscillator	$\omega = 0.30$	2000	15.0	0.449986	0.450000	3.2×10^{-5}
Coulomb (vacuum)	$\alpha = 0.40$	2000	15.0	−0.059998	−0.060000	3.9×10^{-5}
Woods-Saxon	$V_0 = 0.50, R = 2.0, a = 0.5$	2000	15.0	−0.378654	—	—

TABLE III. Grid-refinement study at fixed $r_{\max} = 15$ fm. The observed slopes of $|E(2N) - E(N)|$ versus N are close to -2 , consistent with the second-order stencil. Richardson extrapolations and internal error bars are quoted in the text.

Potential	Ground-state energy E_0 [GeV] vs grid size N			
	$N = 500$	$N = 1000$	$N = 2000$	$N = 4000$
Harmonic oscillator	0.449756	0.449940	0.449986	0.449997
Coulomb (vacuum)	-0.059968	-0.059992	-0.059998	-0.059999
Woods-Saxon	-0.378669	-0.378657	-0.378654	-0.378653

a Woods-Saxon (WS) well provides a more realistic test case without closed-form solutions. The corresponding potentials are as follows:

$$V_{\text{HO}}(r) = \frac{1}{2} \mu \omega^2 (r/\hbar c)^2, \quad E_0^{\text{HO}} = \frac{3}{2} \omega, \quad (\text{B1})$$

$$V_C(r) = -\alpha \frac{\hbar c}{r}, \quad E_0^C = -\frac{\mu \alpha^2}{2}, \quad (\text{B2})$$

$$V_{\text{WS}}(r) = -\frac{V_0}{1 + \exp[(r - R)/a]} \quad (\text{no closed form}), \quad (\text{B3})$$

The corresponding potentials and ground-state energies are listed in Table II. In all cases, the finite-difference solver reproduces the analytic results to very high accuracy, confirming both the discretization scheme and our unit conventions. The Woods-Saxon problem illustrates the stability of the method even when exact energies are unavailable.

3. Convergence with grid size N

We next assess numerical convergence by refining the uniform grid in a fixed spatial domain. As expected for a second-order stencil, the errors decrease quadratically with $1/N$. Table III summarizes the ground-state energies at successive refinements, while Fig. 13 shows the log-log behavior of residuals. The observed slopes closely follow the theoretical -2 trend, and Richardson extrapolation provides values

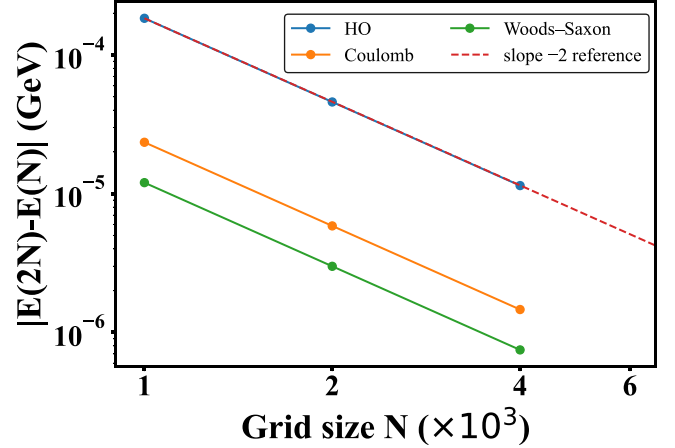


FIG. 13. Grid-refinement residuals $|E(2N) - E(N)|$ as a function of N (log-log scale). The dashed line indicates a reference slope of -2 . The three series (HO, Coulomb, WS) follow the N^{-2} trend.

indistinguishable from the exact results. The Woods-Saxon benchmark likewise exhibits rapid stabilization, with negligible change once N exceeds a few thousand.

4. Reporting practice for in-medium potentials

For realistic in-medium heavy-quark potentials, no analytic reference exists. In these cases, we accompany quoted binding energies with systematic convergence checks: (i) refinement in the number of grid points to confirm the expected N^{-2} scaling, and (ii) variation of the spatial cutoff $r_{\max}$ to control finite-volume effects. Dirichlet boundary conditions are imposed consistently. With these safeguards, the residual numerical uncertainty is much smaller than the physical variation of the binding energy across the relevant temperature range.

5. Convergence check with grid size N of current numerical analysis

The objective of the analysis is to examine how the dissociation temperatures of selected quarkonia states vary with

TABLE IV. Dissociation temperatures T_d/T_c vs grid size N for selected quarkonia. Lower row in each cell shows the absolute difference relative to $N = 500$.

State/Criterion (lower row: $ \Delta $ vs $N = 500$)	Dissociation temperatures T_d/T_c ($T_c = 0.15$ GeV) vs grid size N							
	$N = 500$	$N = 1000$	$N = 1500$	$N = 2000$	$N = 3000$	$N = 4000$	$N = 5000$	$N = 6000$
$\Upsilon(1S) (2E_B = \Gamma)$	3.319	2.616	2.253	2.125	2.029	1.992	1.969	1.956
$ \Delta $ vs $N = 500$	0.000	0.703	1.065	1.194	1.289	1.327	1.350	1.363
$\Upsilon(1S) (E_B = 3T)$	2.490	1.883	1.606	1.488	1.405	1.375	1.359	1.349
$ \Delta $ vs $N = 500$	0.000	0.607	0.884	1.002	1.085	1.116	1.131	1.141
$\Upsilon(2S) (2E_B = \Gamma)$	1.813	1.641	1.485	1.392	1.317	1.291	1.274	1.264
$ \Delta $ vs $N = 500$	0.000	0.171	0.327	0.421	0.496	0.521	0.539	0.549
$\Upsilon(2S) (E_B = 3T)$	1.286	1.254	1.196	1.150	1.113	1.097	1.092	1.087
$ \Delta $ vs $N = 500$	0.000	0.033	0.091	0.136	0.174	0.189	0.194	0.199
$J/\psi (2E_B = \Gamma)$	2.332	1.573	1.415	1.359	1.317	1.299	1.291	1.284
$ \Delta $ vs $N = 500$	0.000	0.758	0.917	0.972	1.015	1.033	1.040	1.048
$J/\psi (E_B = 3T)$	1.901	1.352	1.206	1.160	1.135	1.125	1.120	1.115
$ \Delta $ vs $N = 500$	0.000	0.549	0.695	0.740	0.766	0.776	0.781	0.786

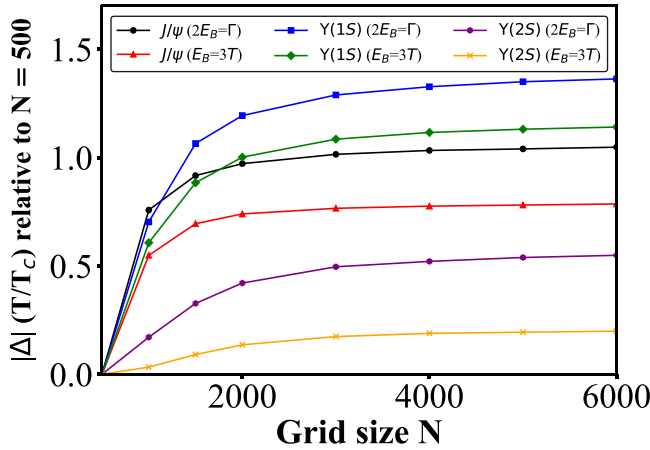


FIG. 14. Comparison of dissociation temperatures T_d/T_c , with $T_c = 0.15$ GeV for various quarkonia states and criteria vs grid size N .

the numerical grid size used in the computation. The states of interest include J/ψ (charmonium ground state), $\Upsilon(1S)$ (bottomonium ground state), and $\Upsilon(2S)$ (bottomonium first excited state). For each state, two physical dissociation criteria are considered: (1) $2E_B = \Gamma$ and (2) $E_B = 3T$, where E_B denotes the binding energy and Γ the thermal width.

The dissociation temperatures are expressed in units of the critical temperature, T_d/T_c , with $T_c = 0.15$ GeV. The benchmark grid chosen is $N = 500$, and the results from larger grids are compared against it. The absolute difference $|\Delta|$ with respect to the benchmark is used as a measure of numerical convergence.

The results shown in Table IV indicate that:

- (1) For all states and criteria, the dissociation temperature decreases systematically as the grid size increases. This behavior reflects the fact that coarse grids tend to overestimate the binding, thus delaying the dissociation.
- (2) The difference $|\Delta|$ grows substantially for finer grids, stabilizing for $N \gtrsim 3000$. This demonstrates the need for large grid sizes to achieve accurate and stable dissociation predictions.
- (3) The ground state bottomonium, $\Upsilon(1S)$, exhibits the largest differences with increasing grid size. This sensitivity highlights the stronger binding of bottom quarkonia and the numerical challenges in resolving their dissociation accurately.
- (4) The excited bottomonium state, $\Upsilon(2S)$, shows smaller but still noticeable variations, consistent with its weaker binding and earlier dissociation.
- (5) The charmonium state J/ψ exhibits intermediate sensitivity, lying between the bottomonium ground and excited states.

Overall, the analysis confirms that grid resolution is important in numerical studies, specifically in the current study of dissociation temperature calculations. The Fig. 14 shows that the benchmarks at small grid sizes ($N = 500$) are insufficient for quantitative precision. Only at higher resolutions does the dissociation temperature converge to stable values, providing reliable physical insight into the quarkonia melting pattern in the quark-gluon plasma.

-
- [1] U. W. Heinz, Concepts of heavy-ion physics, [arXiv:hep-ph/0407360](#).
 - [2] T. Matsui and H. Satz, J/ψ suppression by quark-gluon plasma formation, *Phys. Lett. B* **178**, 416 (1986).
 - [3] L. McLerran, The physics of the quark-gluon plasma, *Rev. Mod. Phys.* **58**, 1021 (1986).
 - [4] K. Adcox, *et al.*, (PHENIX Collaboration), Formation of dense partonic matter in relativistic nucleus–nucleus collisions at RHIC: Experimental evaluation by the PHENIX Collaboration, *Nucl. Phys. A* **757**, 184 (2005).
 - [5] I. Nilima, M. Hasan, B. K. Singh, and M. Y. Jamal, Quarkonia dissociation at finite magnetic field in the presence of momentum anisotropy, *Eur. Phys. J. C* **84**, 160 (2024).
 - [6] M.-C. Chu and T. Matsui, Dynamic Debye screening for a heavy-quark-antiquark pair traversing a quark-gluon plasma, *Phys. Rev. D* **39**, 1892 (1989).
 - [7] Y. Koike, Energy loss of high energy partons traversing a quark-gluon plasma, *AIP Conf. Proc.* **243**, 916 (1992).
 - [8] Y. Burnier, M. Laine, and M. Vepsäläinen, Quarkonium dissociation in the presence of a small momentum space anisotropy, *Phys. Lett. B* **678**, 86 (2009).
 - [9] A. Dumitru, Y. Guo, and M. Strickland, Imaginary part of the static gluon propagator in an anisotropic (viscous) QCD plasma, *Phys. Rev. D* **79**, 114003 (2009).
 - [10] Pooja, M. Y. Jamal, P. P. Bhaduri, M. Ruggieri, and S. K. Das, $c\bar{c}$ and $b\bar{b}$ suppression in the glasma, *Phys. Rev. D* **110**, 094018 (2024).
 - [11] M. B. Voloshin, Charmonium, *Prog. Part. Nucl. Phys.* **61**, 455 (2008).
 - [12] N. Brambilla, *et al.*, Heavy quarkonium: Progress, puzzles, and opportunities, *Eur. Phys. J. C* **71**, 1534 (2011).
 - [13] P. C. Patrignani, T. K. Pedlar, and J. L. Rosner, Recent results in bottomonium, *Annu. Rev. Nucl. Part. Sci.* **63**, 21 (2013).
 - [14] A. D. Frawley, T. Ullrich, and R. Vogt, Heavy flavor in heavy-ion collisions at RHIC and RHIC II, *Phys. Rep.* **462**, 125 (2008).
 - [15] J. F. Amundson, O. J. P. Éboli, E. M. Gregores, and F. Halzen, Quantitative tests of color evaporation: Charmonium production, *Phys. Lett. B* **390**, 323 (1997).
 - [16] E. L. Berger and D. L. Jones, Inelastic photoproduction of J/ψ and Υ by gluons, *Phys. Rev. D* **23**, 1521 (1981).
 - [17] L. Adamczyk, *et al.* (STAR Collaboration), J/ψ production at high transverse momenta in $p + p$ and Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, *Phys. Lett. B* **722**, 55 (2013).
 - [18] C. Silvestre (the PHENIX Collaboration), PHENIX first measurement of the J/ψ elliptic flow parameter v_2 in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, *J. Phys. G* **35**, 104136 (2008).

- [19] C. R. Singh, M. Y. Jamal, and R. Sahoo, Impact of medium anisotropy on quarkonium dissociation and regeneration, *Eur. Phys. J. C* **84**, 891 (2024).
- [20] A. Mocsy and P. Petreczky, Heavy quarkonia survival in potential model, *Eur. Phys. J. C* **43**, 77 (2005).
- [21] V. Agotiya, V. Chandra, and B. K. Patra, Dissociation of quarkonium in a hot QCD medium: Modification of the interquark potential, *Phys. Rev. C* **80**, 025210 (2009).
- [22] J. Sebastian, M. Y. Jamal, and N. Haque, Liénard-Wiechert potential of a heavy quarkonium moving in QGP medium, *Phys. Rev. D* **107**, 054040 (2023).
- [23] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, Charmonium: The model, *Phys. Rev. D* **17**, 3090 (1978).
- [24] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, Charmonium: Comparison with experiment, *Phys. Rev. D* **21**, 203 (1980).
- [25] H. S. Chung, J. Lee, and D. Kang, Cornell potential parameters for S-wave heavy quarkonia, *J. Korean Phys. Soc.* **52**, 1151 (2008).
- [26] I. Nilima, B. K. Singh, and M. Y. Jamal, Quarkonia mass spectra and thermodynamical properties under the effect of finite magnetic field and quark condensate, *Eur. Phys. J. Plus* **139**, 1126 (2024).
- [27] M. Laine, O. Philipsen, P. Romatschke, and M. Tassler, Real-time static potential in hot QCD, *J. High Energy Phys.* **054** (2007).
- [28] M. Strickland and D. Bazow, Thermal bottomonium suppression at RHIC and LHC, *Nucl. Phys. A* **879**, 25 (2012).
- [29] M. Margotta, K. McCarty, C. McGahan, M. Strickland, and D. Yager-Elorriaga, Quarkonium states in a complex-valued potential, *Phys. Rev. D* **83**, 105019 (2011); **84**, 069902(E) (2011).
- [30] L. Thakur, U. Kakade, and B. K. Patra, Dissociation of quarkonium in a complex potential, *Phys. Rev. D* **89**, 094020 (2014).
- [31] A. Mocsy and P. Petreczky, Color screening melts quarkonium, *Phys. Rev. Lett.* **99**, 211602 (2007).
- [32] S. Digal, P. Petreczky, and H. Satz, String breaking and quarkonium dissociation at finite temperatures, *Phys. Lett. B* **514**, 57 (2001).
- [33] M. Y. Jamal, I. Nilima, V. Chandra, and V. K. Agotiya, Dissociation of heavy quarkonia in an anisotropic hot QCD medium in a quasiparticle model, *Phys. Rev. D* **97**, 094033 (2018).
- [34] T. Song, H. Berrehrah, D. Cabrera, W. Cassing, and E. Bratkovskaya, Charm production in Pb + Pb collisions at energies available at the CERN Large Hadron Collider, *Phys. Rev. C* **93**, 034906 (2016).
- [35] M. L. Sambataro, V. Greco, G. Parisi, and S. Plumari, Quasi particle model vs lattice QCD thermodynamics: Extension to $N_f = 2 + 1 + 1$ flavors and momentum dependent quark masses, *Eur. Phys. J. C* **84**, 881 (2024).
- [36] O. Soloveva, A. Palermo, and E. Bratkovskaya, Extraction of the microscopic properties of quasiparticles using deep neural networks, *Phys. Rev. C* **110**, 034908 (2024).
- [37] P. Levai and U. W. Heinz, Massive gluons and quarks and the equation of state obtained from SU(3) lattice QCD, *Phys. Rev. C* **57**, 1879 (1998).
- [38] F.-P. Li, H.-L. Lü, L.-G. Pang, and G.-Y. Qin, Deep-learning quasi-particle masses from QCD equation of state, *Phys. Lett. B* **844**, 138088 (2023).
- [39] F.-P. Li, L.-G. Pang, and G.-Y. Qin, QCD equation of state at finite μ using deep learning assisted quasi-parton model, *Phys. Lett. B* **868**, 139692 (2025).
- [40] A. Bazavov, *et al.* (HotQCD Collaboration), Equation of state in $(2 + 1)$ -flavor QCD, *Phys. Rev. D* **90**, 094503 (2014).
- [41] S. Plumari, W. M. Alberico, V. Greco, and C. Ratti, Recent thermodynamic results from lattice QCD analyzed within a quasiparticle model, *Phys. Rev. D* **84**, 094004 (2011).
- [42] F.-L. Liu, W.-J. Xing, X.-Y. Wu, G.-Y. Qin, S. Cao, and X.-N. Wang, QLBT: A linear Boltzmann transport model for heavy quarks in a quark-gluon plasma of quasi-particles, *Eur. Phys. J. C* **82**, 350 (2022).
- [43] V. Mykhaylova, M. Bluhm, K. Redlich, and C. Sasaki, Quark-flavor dependence of the shear viscosity in a quasiparticle model, *Phys. Rev. D* **100**, 034002 (2019).
- [44] N. Brambilla, J. Ghiglieri, A. Vairo, and P. Petreczky, Static quark-antiquark pairs at finite temperature, *Phys. Rev. D* **78**, 014017 (2008).
- [45] C.-Y. Wong, Heavy quarkonia in quark-gluon plasma, *Phys. Rev. C* **72**, 034906 (2005).
- [46] S. Digal, P. Petreczky, and H. Satz, Quarkonium feed-down and sequential suppression, *Phys. Rev. D* **64**, 094015 (2001).
- [47] H. Satz, Colour deconfinement and quarkonium binding, *J. Phys. G* **32**, R25 (2006).
- [48] D. Blaschke, O. Kaczmarek, E. Laermann, and V. Yudinchev, Heavy quark potential and quarkonia dissociation rates, *Eur. Phys. J. C* **43**, 81 (2005).
- [49] K. T. Rethika, C. D. Ravikumar, and V. M. Bannur, Heavy quarkonium properties at finite temperature in strongly coupled quark gluon plasma, *Few-Body Syst.* **62**, 10 (2021).
- [50] Q. Meng, Q. Wu, P. Cheng, J. Ping, and H. Zong, Calculation of dissociation temperature of quarkonium using Gaussian expansion method, *Chin. Phys. C* **42**, 083103 (2018).
- [51] V. K. Agotiya, V. Chandra, M. Y. Jamal, and I. Nilima, Dissociation of heavy quarkonium in hot QCD medium in a quasiparticle model, *Phys. Rev. D* **94**, 094006 (2016).
- [52] A. Kumar, M. Y. Jamal, V. Chandra, and J. R. Bhatt, Collective excitations of a hot anisotropic QCD medium with the Bhatnagar-Gross-Krook collisional kernel within an effective description, *Phys. Rev. D* **97**, 034007 (2018).
- [53] M. Y. Jamal, S. Mitra, and V. Chandra, Optical properties of an anisotropic hot QCD medium, *J. Phys. G* **47**, 035107 (2020).
- [54] M. Y. Jamal, Collective modes in the anisotropic collisional hot QCD medium at finite chemical potential, *Eur. Phys. J. Plus* **139**, 46 (2024).
- [55] M. Y. Jamal, S. Mitra, and V. Chandra, Collective excitations of hot QCD medium in a quasiparticle description, *Phys. Rev. D* **95**, 094022 (2017).
- [56] S. Chatrchyan, *et al.* (the CMS Collaboration), Study of high- p_T charged particle suppression in PbPb compared to pp collisions at $\sqrt{s_{NN}} = 2.76$ TeV, *Eur. Phys. J. C* **72**, 1945 (2012).
- [57] A. M. Sirunyan, *et al.* (the CMS Collaboration), Measurement of nuclear modification factors of $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ mesons in PbPb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, *Phys. Lett. B* **790**, 270 (2019).
- [58] J. Adam, *et al.* (ALICE Collaboration), J/ψ suppression at forward rapidity in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, *Phys. Lett. B* **766**, 212 (2017).
- [59] V. Khachatryan, *et al.* (CMS Collaboration), Suppression and azimuthal anisotropy of prompt and nonprompt J/ψ production in PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, *Eur. Phys. J. C* **77**, 252 (2017).

- [60] A. M. Sirunyan, *et al.*, (CMS Collaboration), Relative modification of prompt $\psi(2S)$ and J/ψ Yields from pp to PbPb Collisions at $\sqrt{sNN} = 5.02$ TeV, *Phys. Rev. Lett.* **118**, 162301 (2017).
- [61] N. Brambilla, M. A. Escobedo, J. Soto, and A. Vairo, Heavy quarkonium suppression in a fireball, *Phys. Rev. D* **97**, 074009 (2018).
- [62] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes: The Art of Scientific Computing*, 3rd ed. (Cambridge University Press, Cambridge, UK, 2007).
- [63] J. M. Thijssen, *Computational Physics*, 2nd ed. (Cambridge University Press, Cambridge, UK, 2007).
- [64] R. J. LeVeque, *Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-state and Time-dependent Problems* (SIAM, Philadelphia, PA, 2007).
- [65] U. M. Ascher, R. M. M. Mattheij, and R. D. Russell, *Numerical Solution of Boundary Value Problems for Ordinary Differential Equations* (SIAM, Philadelphia, PA, 1995).
- [66] R. B. Lehoucq, D. C. Sorensen, and C. Yang, *ARPACK Users' Guide: Solution of Large-scale Eigenvalue Problems with Implicitly Restarted Arnoldi Methods* (SIAM, Philadelphia, PA, 1998).
- [67] F.-P. Li, The package of deep-learning quasiparton gas model, <https://github.com/leefp29/DL4QPM> (2023).