

# Existence of nuclear modifications of the nucleon longitudinal-transverse structure-function ratio

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It has been assumed that nuclear modification does not exist in the longitudinal-transverse structure-function ratio  $R_N = F_L^N / (2xF_1^N)$  in lepton deep inelastic scattering. This assumption is widely used in obtaining structure functions of the “nucleon” from nuclear data such as the deuteron ones. However, nuclear modifications do exist theoretically at least in medium- and large- $x$  regions because nucleons in a nucleus move in any direction, which is not necessarily the longitudinal direction of the virtual-photon or weak-boson momentum in lepton scattering. Because of this transverse motion, the nucleon’s transverse and longitudinal structure functions should mix with each other in nuclei with the mixture probability proportional to the nucleon’s transverse momentum squared  $\vec{p}_T^2/Q^2$ . In this work, numerical results are explicitly shown regarding such nuclear modifications in the deuteron. These nuclear modifications are important for determining precise structure functions of the nucleon. Furthermore, modifications of  $R_N$  should be investigated also at small  $x$  by the future electron-ion collider to find interesting gluon dynamics in nuclei. Hopefully, this nuclear effect of  $R_N$  could be found by future experimental measurements at lepton accelerator facilities.

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## I. INTRODUCTION

Lepton deep inelastic scattering from the nucleon has been investigated from the 1970s, and now the details have become clear about the internal structure of the nucleon in the unpolarized parton distribution functions (PDFs). The polarized PDFs have been also determined and the polarized gluon part is expected to be determined by future electron-ion colliders (EICs) [1–3]. In recent years, such studies have been extended to investigate three-dimensional structure functions in terms of transverse-momentum-dependent parton distribution functions [4], generalized parton distributions (GPDs) [5], and generalized distribution amplitudes (or  $s$ -channel GPDs) [6]. Currently, these experimental studies are actively done mainly at the Thomas Jefferson National Accelerator Facility (JLab) [7], and the projects will be continued at the EICs.

In charged-lepton deep inelastic scattering from the unpolarized nucleon, there are two structure functions,  $F_1^N$  and  $F_2^N$ . Here,  $N$  indicates the nucleon and it is explicitly written in this paper to distinguish them from the nuclear ( $A$ ) and deuteron ( $D$ ) structure functions. The function  $F_1^N$  is the structure function for the transverse polarization of the virtual photon, and the function  $F_2^N$  has components of both the transverse and

longitudinal polarizations. The longitudinal structure function  $F_L^N$  is defined as  $F_L^N = (1 + Q^2/\nu^2)F_2^N - 2xF_1^N$  in terms of  $F_1^N$  and  $F_2^N$  with the four-momentum transfer  $q$  ( $Q^2 = -q^2$ ), the energy transfer  $\nu$ , and the Bjorken scaling variable  $x = Q^2/(2M_N\nu)$ , where  $M_N$  is the nucleon mass. The longitudinal-transverse structure-function ratio  $R$  is then given by  $R_N = F_L^N/(2xF_1^N)$  [8].

Nuclear modifications of the structure function  $F_2^N$  are now well known from small  $x$  to large  $x$  for many nuclei. At small  $x$ , there are negative modifications due to nuclear shadowing. At medium  $x$ , nuclear binding and possible nucleon’s internal modifications contribute. At large  $x$ , the nuclear modification increases due to the nucleon’s Fermi motion in a nucleus [9].

On the other hand, there is no experimental evidence on the nuclear modification of the ratio  $R_N$  at this stage as reported in 2001 and 2002 [10,11]. The longitudinal-transverse ratios were measured also at JLab in 2007 [12]; however, a significant nuclear effect was not clear by considering experimental errors. Therefore, it is taken as granted in analyzing nuclear data, e.g., for the deuteron, that the nuclear modification does not exist for the ratio  $R_N$  in order to obtain the structure functions of the “nucleon”. This assumption is used, for example, in obtaining the polarized structure function  $g_1$  for the neutron from the measurements of the deuteron and  $^3\text{He}$ . In a similar way, the nucleon’s function  $R_N$  is used even for heavy nuclei in studying short-range correlations.

To be precise, this assumption is not right theoretically as it was proposed in 2003 [13]. Because a nucleon in a nucleus moves in any direction, which is not necessarily the longitudinal direction along the virtual photon momentum, due to

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its Fermi motion, the transverse and longitudinal structure function could mix. This topic could become interesting again in the near future because a new experimental study of  $R_N$  [14] is scheduled to run in 2026 [15] and because the polarized deuteron experiment will start soon at JLab [16]. Therefore, it is worth shedding light on this nuclear modification topic by using a reliable theoretical model. Although I focus on the medium- and large- $x$  regions in this work, the small- $x$  nuclear modification should also be an interesting topic because  $F_L^A$  is related to gluon dynamics in nuclei as investigated at HERA (Hadron-Elektron-Ringanlage) for the nucleon [17] and it will be investigated at the EICs [1–3].

In this paper, I show numerically that such nuclear modifications of  $R_N$  exist in the deuteron. These nuclear modifications need to be considered for obtaining precise structure functions of the “nucleon” from nuclear data. This article consists of the following. First, the theoretical formalism is shown for the nuclear modifications of the longitudinal-transverse ratio  $R_N$ . Then, numerical results are shown and they are summarized.

## II. FORMALISM

The cross section of charged-lepton deep inelastic scattering from a nucleon or a nucleus is given by the lepton tensor  $L^{\mu\nu}$  and the hadron tensor  $W_{\mu\nu}^{A,N}$  as

$$\frac{d\sigma}{dE'_\ell d\Omega'_\ell} = \frac{|\vec{p}'_\ell|}{|\vec{p}_\ell|} \frac{\alpha^2}{(q^2)^2} L^{\mu\nu}(p_\ell, q) W_{\mu\nu}^{A,N}(p_{A,N}, q), \quad (1)$$

where  $E'_\ell$  and  $\Omega'_\ell$  are the final charged-lepton energy and solid angle,  $p_\ell$  and  $p'_\ell$  are the initial and final charged-lepton momenta,  $q$  is the momentum transfer  $q = p_\ell - p'_\ell$ ,  $p_A$  and  $p_N$  are the nuclear and nucleon momenta, respectively, and  $\alpha$  is the fine-structure constant of the quantum electrodynamics. The hadron tensor of the charged-lepton scattering is given in general by two structure functions,  $W_1^{A,N}$  and  $W_2^{A,N}$ , for the unpolarized nucleon and nuclei.

The hadron tensor for the polarized photon with the helicity  $\lambda$  is given by [8]

$$W_{\lambda}^{A,N}(p_{A,N}, q) = \varepsilon_{\lambda}^{\mu*} \varepsilon_{\lambda}^{\nu} W_{\mu\nu}^{A,N}(p_{A,N}, q), \quad (2)$$

where  $\varepsilon_{\lambda}^{\mu}$  is the photon polarization vector. Then, the transverse and longitudinal structure functions  $W_T^{A,N}$  and  $W_L^{A,N}$  are defined by these helicity functions,  $W_1^{A,N}$  and  $W_2^{A,N}$ , as

$$W_T^{A,N} = \frac{W_{\lambda=+1}^{A,N} + W_{\lambda=-1}^{A,N}}{2} = W_1^{A,N},$$

$$W_L^{A,N} = W_{\lambda=0}^{A,N} = \left(1 + \frac{v_{A,N}^2}{Q^2}\right) W_2^{A,N} - W_1^{A,N}, \quad (3)$$

where  $v_N$  and  $v_A$  are defined by

$$v_A = \frac{p_A \cdot q}{\sqrt{p_A^2}} = v,$$

$$v_N = \frac{p_N \cdot q}{\sqrt{p_N^2}}. \quad (4)$$

Instead of  $W_{1,2,T,L}$ , the structure functions  $F_{1,2,T,L}$  are often used. They are related with each other by the following relations:

$$F_1^{A,N} = F_T^{A,N} = \sqrt{p_{A,N}^2} W_1^{A,N},$$

$$F_2^{A,N} = \frac{p_{A,N} \cdot q}{\sqrt{p_{A,N}^2}} W_2^{A,N},$$

$$F_L^{A,N} = \left(1 + \frac{Q^2}{v_{A,N}^2}\right) F_2^{A,N} - 2x_{A,N} F_1^{A,N}. \quad (5)$$

From these structure functions, the longitudinal-transverse structure-function ratio is defined as

$$R_{A,N}(x_A, Q^2) = \frac{F_L^{A,N}(x_{A,N}, Q^2)}{2x_{A,N} F_1^{A,N}(x_{A,N}, Q^2)}. \quad (6)$$

Here, the variables  $x_A$  and  $x_N$  are defined in terms of the Bjorken scaling variable  $x$  and the momentum fraction  $y$  as

$$x_A = \frac{Q^2}{2p_A \cdot q}, \quad x_N = \frac{Q^2}{2p_N \cdot q} = \frac{x}{y},$$

$$x = \frac{Q^2}{M_N v}, \quad y = \frac{p_N \cdot q}{M_N v}, \quad (7)$$

where  $M_A$  is the nuclear mass.

A conventional way to theoretically describe the nuclear structure functions at medium and large  $x$  is to use the convolution formalism. Then, the nuclear hadron tensor is given by the nucleonic one convoluted with the nucleon momentum distribution  $S(p_N)$ , which is called the spectral function, as

$$W_{\mu\nu}^A(p_A, q) = \int d^4 p_N S(p_N) W_{\mu\nu}^N(p_N, q). \quad (8)$$

A simple form of the spectral function is given by the shell model as

$$S(p_N) = \sum_i |\phi_i(\vec{p}_N)|^2 \delta(p_N^0 - M_A + \sqrt{M_{A-i}^2 + \vec{p}_N^2}). \quad (9)$$

Here,  $\phi_i$  is the wave function of the nucleon  $i$ , and the separation energy  $\varepsilon_i$  and  $M_{A-i}$  are related by  $\varepsilon_i = (M_{A-i} + M_N) - M_A$ . I average over all the nucleons for calculating the average separation energy,  $\varepsilon_i \rightarrow \langle \varepsilon \rangle$ . By using a nonrelativistic approximation for  $\sqrt{M_{A-i}^2 + \vec{p}_N^2}$ ,  $p_N^0$  and  $\langle \varepsilon \rangle$  are related by

$$p_N^0 = M_N - \langle \varepsilon \rangle - \frac{\vec{p}_N^2}{2M_{A-1}}, \quad (10)$$

with the replacement  $M_{A-i} \rightarrow M_{A-1}$  [18,19]. The spectral function contains the binding and Fermi-motion effects by the change of the nucleon's effective mass. Here, the nucleons are off-shell; however, the structure functions of the on-shell nucleons are used. Meson-exchange currents could also contribute, but they are neglected in this work.

Applying the projection operators  $\hat{P}_{1,2}^{\mu\nu}$  to  $W_{\mu\nu}^A$ , I obtain the nuclear structure functions  $W_{1,2}^A$  in the convolution model. From  $W_{1,2}^A$ , the transverse and longitudinal structure functions

$F_1^A$  and  $F_L^A$  are obtained in the convolution form as

$$\begin{pmatrix} F_2^A(x_A, Q^2) \\ 2x_A F_1^A(x_A, Q^2) \\ F_L^A(x_A, Q^2) \end{pmatrix} = \int_x^A dy \begin{pmatrix} f_{22}(y) & 0 & 0 \\ 0 & f_{11}(y) & f_{1L}(y) \\ 0 & f_{L1}(y) & f_{LL}(y) \end{pmatrix} \times \begin{pmatrix} F_2^N(x/y, Q^2) \\ 2\frac{x}{y} F_1^N(x/y, Q^2) \\ F_L^N(x/y, Q^2) \end{pmatrix}. \quad (11)$$

Here, the nucleon's light-cone momentum distributions  $f_{22}$ ,  $f_{LL}$ ,  $f_{L1}$ ,  $f_{1L}$ , and  $f_{11}$  are defined by [13]

$$\begin{aligned} f_{22}(y) &= \int_0^\infty dp_{N\perp} 2\pi p_{N\perp} y \frac{M_{Nv}}{|\vec{q}|} |\phi(\vec{p}_N)|^2 \\ &\quad \times \left[ \frac{2(x/y)^2 \vec{p}_{N\perp}^2}{(1 + Q^2/v^2)Q^2} + \left( 1 + \frac{2(x/y)p_{N\parallel}}{\sqrt{Q^2 + v^2}} \right)^2 \right], \\ f_{LL}(y) &= f_{11}(y) \\ &= \int_0^\infty dp_{N\perp} 2\pi p_{N\perp} y \frac{M_{Nv}}{|\vec{q}|} |\phi(\vec{p}_N)|^2 \left( 1 + \frac{\vec{p}_{N\perp}^2}{\tilde{p}_N^2} \right), \\ f_{L1}(y) &= f_{1L}(y) \\ &= \int_0^\infty dp_{N\perp} 2\pi p_{N\perp} y \frac{M_{Nv}}{|\vec{q}|} |\phi(\vec{p}_N)|^2 \frac{\vec{p}_{N\perp}^2}{\tilde{p}_N^2}, \end{aligned} \quad (12)$$

where  $\tilde{p}_\mu = p_\mu - (p \cdot q)q_\mu/q^2$ . Then, the longitudinal-transverse ratio  $R_A$  of a nucleus is calculated by Eq. (6). This convolution model has been used for studying the structure function  $F_2^A$ . Here, I assume it is valid also for  $F_1^A$  and  $F_L^A$ . Higher-twist effects are not taken into account. Here,  $\tilde{p}_N^2$  is expressed by  $Q^2$  as

$$\tilde{p}_N^2 = \frac{Q^2}{4x_N^2} \left( 1 + \frac{4x_N^2 p_N^2}{Q^2} \right) \simeq \frac{Q^2}{4x_N^2} \left( 1 + \frac{4x_N^2 M_N^2}{Q^2} \right). \quad (13)$$

Because of the transverse motion of the nucleons in a nucleus, the transverse and longitudinal structure functions of the nucleon ( $F_1^N$ ,  $F_L^N$ ) mix with each other in Eqs. (11) and (12), and the mixture coefficient is proportional to the transverse momentum squared  $\vec{p}_{N\perp}^2$ . The mixture factor  $\vec{p}_{N\perp}^2/\tilde{p}_N^2$  is proportional to  $\vec{p}_{N\perp}^2/Q^2$ . Because of this admixture, I definitely can claim that the nuclear modifications of  $R_N$  do exist, although there is no experimental finding at this stage. Because of the factor  $\vec{p}_{N\perp}^2/Q^2$ , this mixture does not occur in the Bjorken scaling limit  $Q^2 \rightarrow \infty$ .

However, one may note that the nuclear modifications of  $R_N$  exist even at  $Q^2 \rightarrow \infty$  through the convolution integrals of Eq. (11) as shown numerically later. This modification occurs because the  $x_N$ -dependent functional forms are different between  $F_1^N$  and  $F_L^N$ , although the light-cone momentum distributions of the nucleon,  $f_{LL}(y)$  and  $f_{11}(y)$ , are same. In Eq. (11), the structure function  $F_2^A$  is expressed only by  $F_2^N$  and the mixing with other structure functions does not exist; however, there are correction factors  $\vec{p}_{N\perp}^2/Q^2$  and  $p_{N\parallel}/\sqrt{Q^2 + v^2}$  in  $f_{22}(y)$  of Eq. (12). By the convolution

description for the nuclear structure functions, I found that there are two sources of the nuclear modifications of  $R_N$ . One is the admixture of  $F_1^N$  and  $F_L^N$  in a nuclear medium, and the other is by the convolution with the nucleon's momentum distributions  $f_{11}(y)$  and  $f_{LL}(y)$ . In this work, the nuclear modifications are actually shown numerically for the deuteron by taking an appropriate deuteron wave function  $\phi_i$  in Eq. (12).

Next, I explain comparisons with related works. There were works regarding convolution model calculations on nuclear structure functions of light nuclei including the deuteron. For example, the functions  $F_2^A$  were calculated for the deuteron,  $^3\text{He}$ , and  $^3\text{H}$  in Ref. [20]. The functions  $F_T^A$ ,  $F_2^A$ , and  $F_3^A$  were investigated also by including target-mass corrections (TMCs), off-shell effects, and pion contributions in Ref. [21]. These nuclear binding, Fermi-motion, TMCs, and off-shell effects are shown in detail in comparison with the SLAC and JLab experimental data [22]. The nuclear-binding and Fermi-motion parts of these works are essentially the same as the work of Ref. [13] and this work as shown in Eqs. (11) and (12) by considering the factor in Eq. (13). It is noteworthy that the nuclear modifications of  $R_N$  are affected by the TMCs. In Ref. [23], different prescriptions of the TMCs are shown numerically by using the operator product expansion (OPE), the collinear factorization of Ellis, Furmanski, and Petronzio (EFP),  $\xi$  scaling, and the Accardi and Qiu (AQ) method.

### III. RESULTS

In order to calculate the nuclear modifications of  $R_N$  by using Eqs. (6), (11), and (12), I need to supply the information on the used functions. First, the structure function of the nucleon  $F_2^N$  is calculated by the PDFs of MSTW08 [24] in the leading order of the running coupling constant  $\alpha_s$ . The function  $R_N$  of the nucleon is given by the SLAC parametrization of 1990 [25]. Using these functions,  $F_1^N$  is calculated by  $F_1^N = (1 + Q^2/v^2) F_2^N/[2x(1 + R_N)]$ . As for the wave function  $\phi$  of the deuteron, the Bonn wave function [26] is used.

With these preparations, the nuclear structure functions  $F_2^A$ ,  $F_1^A$ , and  $F_L^A$  are calculated, and their nuclear modifications are shown in Fig. 1 at  $Q^2 = 5 \text{ GeV}^2$ . The nuclear modifications of the transverse structure function ( $F_1^D/F_1^N$ ) are very similar to the ones of  $F_2^D/F_2^N$ . However, the modifications of the longitudinal structure function ( $F_L^D/F_L^N$ ) are very different. Especially, the modification dip in the medium- $x$  region ( $x \sim 0.5$ ) is not large. This significant difference comes from the mixture effects  $\vec{p}_{N\perp}^2/\tilde{p}_N^2$  ( $\sim \vec{p}_{N\perp}^2/Q^2$ ) in Eq. (12). In fact, if these mixing terms are terminated, the longitudinal modification curve  $F_L^D/F_L^N$  is very similar to the ones of the transverse and  $F_2$  ones ( $F_1^D/F_1^N$ ,  $F_2^D/F_2^N$ ). Because the nuclear modifications of  $F_L^N$  are so different from the ones of  $F_2^N$ , one should not assume that they are the same in handling nuclear data which contain  $F_L^A$ .

Next, I show the nuclear modifications in the form of the longitudinal-transverse ratio  $R_A/R_N$  by using Eq. (6) in Fig. 2 at  $Q^2 = 1, 5$ , and  $100 \text{ GeV}^2$ . The modifications are shown by the solid curves, and the dotted ones are obtained by terminating the mixing terms. It means that  $\vec{p}_{N\perp}^2/\tilde{p}_N^2 \rightarrow 0$  is taken in the expressions of  $f_{LL}(y)$ ,  $f_{11}(y)$ ,  $f_{L1}(y)$ , and  $f_{1L}(y)$

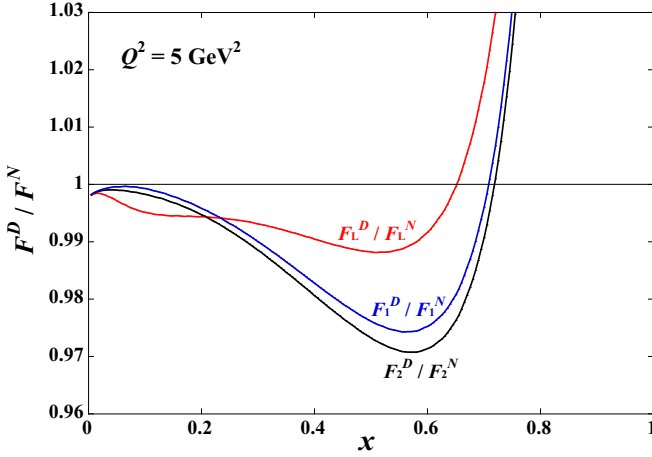


FIG. 1. Nuclear modifications of the structure functions  $F_1$ ,  $F_2$ , and  $F_L$  at  $Q^2 = 5 \text{ GeV}^2$  in the deuteron.

of Eq. (12) and that the replacement  $[\dots] \rightarrow 1$  is taken in  $f_{22}(y)$ . Because the major source of the nuclear modifications of  $R_N$  is the longitudinal-transverse mixing ( $\sim \vec{p}_{N\perp}^2/Q^2$ ), the modifications are large at small  $Q^2$  ( $\sim 1 \text{ GeV}^2$ ). As the  $Q^2$  increases as  $Q^2 = 1 \rightarrow 5 \rightarrow 100 \text{ GeV}^2$ , these modifications become smaller. However, the significant modifications still exist even at large  $Q^2$ . In fact, I notice that the modifications are noticeable at  $Q^2 = 100 \text{ GeV}^2$  even without the mixing terms as shown by the dotted curve of  $Q^2 = 100 \text{ GeV}^2$ , and such modifications are obtained through the convolution with the nucleon's light-cone momentum distribution  $f_{LL}(y)$ . At smaller  $Q^2$  ( $= 1, 5 \text{ GeV}^2$ ), the nuclear modifications in  $R_N$  are not large if there is no mixing effect as shown by the dotted curves. One should note that the function  $R_N$  is not accurately determined especially at large  $x$ , so that the current numerical results at large  $x$  could have large uncertainties at this stage.

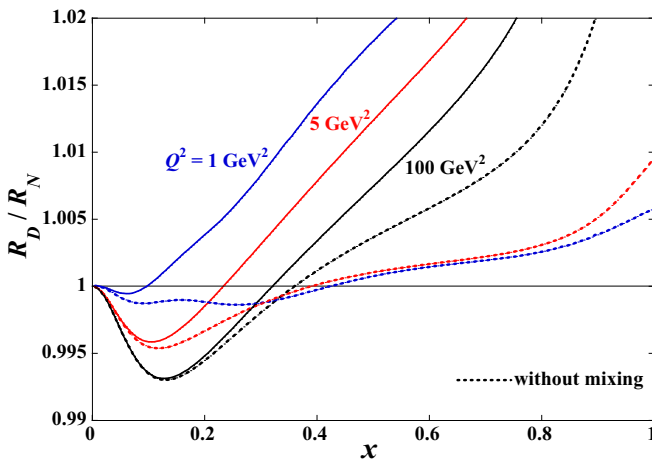


FIG. 2. Nuclear modifications of the longitudinal-transverse structure-function ratio  $R_N$  at  $Q^2 = 1, 5$ , and  $100 \text{ GeV}^2$  in the deuteron. The dotted curves are obtained by terminating the mixing terms.

The nucleon's PDFs and structure functions have been determined by partially using nuclear data including the deuteron ones. In fact, the deuteron is often used for determining the nucleon's, especially neutron's, structure functions by assuming that the nuclear modification does not exist in the longitudinal-transverse structure-function ratio  $R_N$ . However, it is obviously not appropriate according to the results shown in this work. As shown in Fig. 1, the nuclear modifications of the deuteron's structure functions are of the order of a few percent. The modifications of  $R_N$  are of similar magnitude or less in Fig. 2. The effects may be larger at large  $x$ ; however, it is difficult to predict the large- $x$  region of  $R_D$  due to the possible uncertainty of  $R_N$  itself. Obviously experimental efforts are needed to accurately measure not only the functions  $R_A$  for various nuclei but also the  $R_N$  for the nucleon including the large- $x$  region.

In comparison with the actual experimental data, one should note that the TMCs could affect the modification curves of Fig. 2 especially at small  $Q^2$ . Modifications of the nucleon's structure functions by using the different prescriptions of the TMCs, namely, the OPE, EFP,  $\xi$  scaling, and AQ methods at  $Q^2 = 2 \text{ GeV}^2$  [23]. For the deuteron, the large Fermi-motion effects are reduced by these TMCs depending on the prescription. If the TMCs are used for both the deuteron's and the nucleon's structure functions, their effects partially cancel with each other in taking the deuteron/nucleon ratios. Therefore, the careful and detailed comparisons should be made with the data in the future by including the TMCs especially for comparison with the JLab deuteron data in the near future [14–16].

As mentioned in the end of Sec. II, there are various prescriptions of the TMCs, for example, the OPE, EFP,  $\xi$  scaling, and AQ methods. According to Ref. [23], the  $\xi$  scaling and the AQ prescriptions suppress the Fermi-motion rise in the nuclear structure functions at large  $x$ , and the OPE and EFP methods suppress more significantly. In order to illustrate such an effect, numerical results are shown by taking the  $\xi$  scaling as an example. The  $\xi$  scaling formulas are given in Ref. [23], where one may note that the TMCs have different overall factors among  $F_1^{A,N}$ ,  $F_L^{A,N}$ , and  $F_2^{A,N}$ . The nuclear modifications of  $R_N$  are also changed. The TMCs of the  $\xi$  scaling are shown by the dashed curves for the nuclear modification  $R_A/R_N$  in Fig. 3. As shown in Ref. [23], the TMCs suppress the Fermi-motion rise and such effects are large at small  $Q^2$  ( $= 1 \text{ GeV}^2$ ) because of the  $M_N^2/Q^2$  corrections. Since the TMCs depend greatly on the used prescriptions, careful analyses should be done in the future in comparison with the deuteron data.

This kind of study could affect the polarized PDFs of the nucleon. As the longitudinally polarized PDFs become accurate, one needs to be careful about the modifications of  $R_N$  in nuclei because the deuteron and  $^3\text{He}$  data are used for extracting the polarized PDFs without such modifications. In the longitudinally polarized charged-lepton nucleon deep inelastic scattering, experimental data are shown by the spin asymmetry  $A_1$ . It is expressed as  $A_1 \simeq g_1^N/F_1^N = g_1^N 2x_N(1 + R_N)/[(1 + Q^2/v_N^2)F_2^N]$  in terms of the function ratio  $R_N$ . If the target is the polarized deuteron or  $^3\text{He}$ , one needs to use  $R_D$  or  $R_{^3\text{He}}$ . However, in the current global analyses, it has been assumed that there is no nuclear modification, namely,

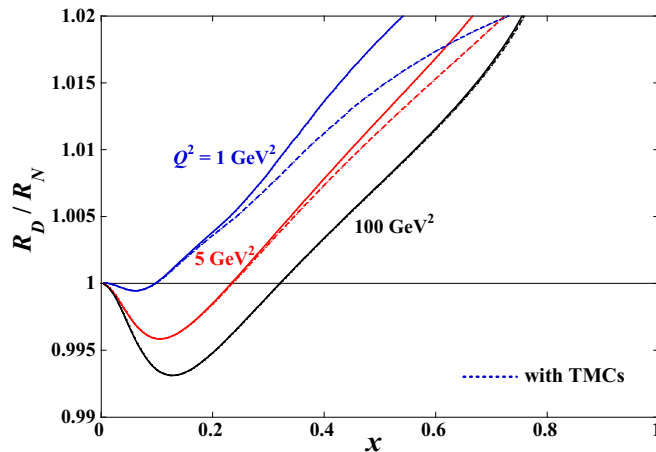


FIG. 3. Nuclear modifications of the longitudinal-transverse structure-function ratio  $R_N$  at  $Q^2 = 1, 5$ , and  $100 \text{ GeV}^2$  in the deuteron. The dashed curves are the modifications with the TMCs given by the  $\xi$  scaling.

$R_D = R_{\text{He}} = R_N$ . This assumption is not appropriate, as the polarized PDFs become accurate, by considering the current research results of  $R_D$  in Fig. 2. Needless to say, if the target is a larger nucleus, the nuclear modifications become more significant and they should be taken into account. In order to obtain accurate unpolarized and polarized structure functions of the nucleon and then to apply them to other high-energy hadron reactions, it is important to consider the nuclear modifications of  $R_N$ . The predicted nuclear modifications of  $R_N$  could be measured at JLab [7,14] and at the future electron-ion colliders [1–3].

One of the recent major achievements of JLab is the finding of short-range correlations (SRCs) in nucleon-nucleon interactions by using electron-nucleus scattering [27]. In the SRC measurements, used nuclei range from the deuteron as the smallest nucleus to a large nucleus of lead ( $^{208}\text{Pb}$ ). The electron-nucleus cross section is generally expressed by  $F_1^A$  and  $F_2^A$ , although the experimental kinematical region is different from the deep inelastic one. Obviously, nuclear modifications should be large in the longitudinal-transverse ratio  $R_A$ ; however, they are neglected in the experimental analysis of the SRCs and simply the nucleon's ratio  $R_N$  has been used [28]. Therefore, the nuclear-modification issue should be readdressed for the longitudinal-transverse ratio by throwing away the preconception that such a modification does not exist.

Although I focused the theoretical studies in this work at medium and large  $x$  for discussing the nuclear modification effects from the nuclear binding and Fermi motion, the nuclear effect of  $R_N$  should be interesting also at small  $x$ . In fact, the  $F_L$  data of HERA at small  $x$  is related to the gluon distribution in the nucleon [17]. Therefore, the nuclear modification of  $R_N$  is an interesting quantity to investigate gluon dynamics in nuclei. The studies regarding the nuclear modifications of  $R_N$  contain not only the standard nuclear physics, in terms of nuclear binding and Fermi motion, but also possibly the gluon saturation physics in nuclei. Therefore, such studies could lead to another interesting new project at future EICs because the nuclear gluon dynamics is an important aspect.

#### IV. SUMMARY

Using the standard convolution description for the structure functions of the deuteron, I calculated nuclear modifications of  $F_2^N$ ,  $F_1^N$ , and  $F_L^N$ . Because the nucleons in the deuteron move in any direction which is generally different from the longitudinal direction defined by the virtual photon momentum in the charged-lepton scattering, the longitudinal structure function  $F_L^N$  and the transverse one  $F_1^N$  mix with each other in the deuteron. Because of this mixture, the nuclear modification of  $F_L^N$  is very different from the nuclear modifications of  $F_2^N$  and  $F_1^N$ . In addition, the modification of  $R_N$  comes from the convolution integrals with the nucleon's momentum distributions. These mechanisms led to significant nuclear modifications of  $R_N$  in the deuteron. It has been assumed until now that the nuclear modification does not exist in the function  $R_N$ ; therefore, our theoretical predictions should be tested by future experiments.

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#### DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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