

Magnetic properties of the hadron resonance gas with physical magnetic moments

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(Received 22 May 2025; revised 6 August 2025; accepted 11 September 2025; published 14 October 2025)

We study magnetic properties of the hadron resonance gas in the presence of a strong ($0 \leq B \leq 0.15 \text{ GeV}^2$) magnetic field, using physical values of the magnetic moments of hadrons, i.e., including their anomalous parts. The values of these moments are taken from experiment, or when unavailable, from theoretical estimates. We evaluate the conserved charge susceptibilities, finding the expected sizable effects of the anomalous magnetic moments, in particular for the octet baryons, such as the proton and neutron, where they are exceptionally large. We also study in detail the large effects of the magnetic moments of the $\Delta(1232)$ states, for which various theoretical estimates and experimental values differ significantly. We compare our model results with the lattice quantum chromodynamics data and find reasonable agreement within the model uncertainty.

DOI: [10.1103/th4j-whrv](https://doi.org/10.1103/th4j-whrv)

I. INTRODUCTION

The behavior of the hadronic medium under strong magnetic fields (B) has been a topic of active research, due to its relevance in hot systems formed in the early stages of ultrarelativistic nuclear collisions as well as in the early universe (for reviews, see, e.g., [1–4]), or in the cold neutron star matter (for a review, see, e.g., [5]). On the lattice [6–14], one can study the effects of B in a thermally equilibrated hot quantum chromodynamics (QCD) medium in a static setup, where B is spatially uniform and time-independent (unlike in the collisions, where the system is dynamic), which has clear advantages for modeling and understanding of the basic features. One is therefore limited to systems in the vicinity of the vanishing baryon, strangeness, or charge chemical potentials. Such systems, with nearly zero net charge densities, are formed in the midrapidity region of the ultra-relativistic heavy-ion collisions at the highest BNL Relativistic Heavy Ion Collider (RHIC) or the CERN Large Hadron Collider (LHC) energies. Various susceptibilities, studied in this paper, bear significance to the investigation of the phase transitions and the critical point of QCD [15,16], which have been actively studied on the lattice (see, e.g., [17–19]), importantly also in the presence of B [13,14]. A notable associated phenomenon to mention here is the magnetic catalysis [20–22], proving nontrivial magnetic properties of the QCD vacuum itself.

In a hot hadronic matter at vanishing chemical potentials, the QCD quark and gluon degrees of freedom dominate above the crossover temperature $T_c \sim 155 \text{ MeV}$. Below T_c , a remarkably good description of the thermodynamic properties of the medium is achieved within the hadron resonance gas (HRG) model [23–28] (for the historical development and more references see, e.g., [29]). Its most prominent success (at $B = 0$) is the description of the hadron multiplicities measured in the ultrarelativistic nuclear collisions (for a review, see [30]). When amended with flow, HRG describes the features of the soft p_T spectra as well [29,31]. Importantly, the model also reproduces to expected accuracy various charge (baryon, strangeness, electric charge) susceptibilities (diagonal and nondiagonal) as functions of T at $B = 0$ [32–36], matching smoothly to the lattice QCD simulations up to T_c .

The susceptibilities from the lattice QCD at B up to 0.8 GeV^2 and for $T = 145, 155$, and 165 MeV were recently presented in [14], extending the results of [13]. In these works, the comparisons to HRG were also made. The HRG analysis of [13] was further developed in dedicated studies of [37,38]. In all these papers, following the work of [8,9,11], the effects of anomalous magnetic moments of hadrons were neglected. In particular, for spin-1/2 hadrons the Landé g factors were taken as $g = 2Q$, where Q is the electric charge. This calls for improvement, as one expects large effects in the baryon susceptibilities from the nucleons, where the g factor of the proton $g_p \simeq 2 \times 2.79 \simeq 5.6$ (a very large value compared to 2) and for the neutron $g_n \simeq -2 \times 1.91 \simeq -3.8$ (the neutron's B -dependent contribution was neglected in the above-mentioned HRG studies).

In fact, the anomalous magnetic moments of hadrons were investigated much earlier within HRG in the context of susceptibilities by Bhattacharyya, Ghosh, Ray, and Samanta [39], prior to the availability of the lattice QCD data. These authors apparently used the formula for the spin- $\frac{1}{2}$ particles with anomalous magnetic moments [40], but details were not given.

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In this paper we return to the issue of the diagonal and nondiagonal susceptibilities in HRG, investigating the role of the anomalous magnetic moments of hadrons and comparing the results to the now available lattice QCD data. We start by carefully discussing (Sec. II) the inclusion of anomalous magnetic moments, where a rigorous theoretical treatment is available only for elementary spin- $\frac{1}{2}$ states [40], whereas the treatment of higher spin hadrons must be approximate. In this context, we list the magnetic properties of the lowest mass hadrons, exhibiting, i.e., a large departure of the g factors of the octet baryons from the nonanomalous $2Q$ value. We also discuss till what value of B the model is applicable. Section III contains a glossary of the used standard thermodynamics formulas and the details of the version of HRG used in this work. In Sec. IV we present a simple estimate of the role of the anomalous magnetic moments in susceptibilities, and pass to their detailed anatomy in Sec. V. Our results are presented in Sec. VI, first for $B = 0$, to check how HRG with its choice of the hadron states works in this “conventional” case, and then passing to the non-zero B case. As expected, large effects of the anomalous moments are found compared to the $g = 2Q$ assumption, leading to improvements in the comparison to the lattice data [13,14] in the studied domain of $B \leq 0.15$ GeV². For completeness we also discuss the effect of anomalous magnetic moment on hadron multiplicities in Sec. VII.

Throughout this paper we work in the natural units $e = \hbar = c = k_B = 1$.

II. ENERGY SPECTRA OF HADRONS IN A UNIFORM MAGNETIC FIELD

In the presence of a constant uniform magnetic field B (assumed to be aligned with the z -axis), two competing effects appear: The Landau diamagnetism [41] from the motion of a charged particle of mass M and charge Q , and the paramagnetism due to nonzero intrinsic magnetic moments of hadrons with spin $s > 0$ (present for both charged and neutral baryons, and charged mesons). Nonrelativistically, the Landau levels appearing via the minimal substitution in the nonrelativistic kinetic term $p^2/(2M)$ have the form [41]

$$\epsilon_{l,p_z} = M + \frac{p_z^2}{2M} + \frac{B|Q|}{2M}(2l+1), \quad (1)$$

where $l = 0, 1, \dots$ labels the level. A paramagnetic interaction adds the term

$$\epsilon_p = -\vec{\mu} \cdot \vec{B} \quad (2)$$

to each Landau level with $\vec{\mu}$ denoting the magnetic moment and $\vec{B}$ the magnetic field. Further,

$$\mu = \mu_M g s_z, \quad \mu_M = \frac{1}{2M}, \quad (3)$$

where μ_M is the *natural magneton* (reflecting the mass of a given particle), $s_z = -s, \dots, s$ is the spin projection, and g denotes the Lande g factor for the hadron. Since the experimental values of μ are by convention defined for the highest value of spin, $s_z = s$, one has

$$\mu_{s_z=s} \equiv \mu_{\text{exp}} = \mu_M g s. \quad (4)$$

TABLE I. Magnetic moments and the corresponding g factors from Eq. (5) for the lowest mass hadrons, collected from various experimental, lattice QCD, and theoretical estimates. In the reference column the corresponding sources are listed. In case of multiple values of μ given, the value used in our calculation is indicated in bold. While the value of the anomalous magnetic moments for resonances are in general complex, we quote and use the real values.

Hadron species	μ/μ_N	g [Eq. (5)]	Reference
$\rho^+(775)$	1.94 (1) 2.21 2.37	1.60(1) 1.82 1.96	LQCD [42] χ PT [43] QM [44]
$K^{*+}(892)$	2.4 (2) 2.19	2.3(2) 2.08	LQCD [45] QM [44]
$K^{*0}(896)$	−0.183	−0.175	QM [44]
$p(938)$	2.793	5.586	PDG [46]
$n(939)$	−1.913	−3.831	PDG [46]
$\Lambda^0(1115)$	−0.613(4)	−1.458(9)	PDG [46]
$\Sigma^+(1189)$	2.458 (10)	6.232(25)	PDG [46]
$\Sigma^0(1192)$	0.65 0.791	1.65 2.011	χ PT [47] QM [46]
$\Sigma^-(1197)$	−1.160(25)	−2.96(63)	PDG [46]
$a_1^+(1230)$	1.7 (2) 1.44	2.2(2) 1.89	LQCD [48] QM [44]
$\Delta^{++}(1232)$	3.7-7.5 6.14 (51) 5.24 (18) 4.97 (89)	3.23-6.56 5.37(45) 4.58(16) 4.34(78)	PDG [46] Lopez <i>et al.</i> [49] LQCD [50] χ PT [51]
$\Delta^+(1232)$	$2.7^{+1.0}_{-1.3} \pm 1.5 \pm 3$ 2.6 (5)	$2.36^{+0.87}_{-1.14}$ 2.27(4)	PDG [46] χ PT [51]
$\Delta^0(1232)$	0.02(12)	0.017(100)	χ PT [51]
$\Delta^-(1232)$	−2.48(32)	−2.17(28)	χ PT [51]
$\Xi^-(1321)$	−0.651(3)	−1.834(8)	PDG [46]
$\Xi^0(1321)$	−1.250(14)	−3.503(39)	PDG [46]
$\Sigma^{*+}(1383)$	2.55 (26) 1.76(38)	2.50(25) 1.73(37)	LQCD [52] χ PT [51]
$\Sigma^{*0}(1384)$	0.27 (5) −0.02(3)	0.26(5) −0.02(3)	LQCD [52] χ PT [51]
$\Sigma^{*-}(1387)$	−2.02 (18) −1.85(38)	−1.99(18) −1.82(37)	LQCD [52] χ PT [51]
$N^+(1440)$	0.48	1.47	χ PT [53]
$N^0(1440)$	0.27	0.84	χ PT [53]
$\Xi^{*-}(1530)$	−1.68 (12) −1.90(47)	−1.82(13) −2.07(51)	LQCD [52] χ PT [51]
$\Xi^{*0}(1530)$	0.46 (7) −0.42(13)	0.50(8) −0.46(14)	LQCD [52] χ PT [51]
$\Omega^-(1672)$	−2.02(5)	−2.40(6)	PDG [46]

Typically, the experimental values for all states are quoted in nuclear magnetons $\mu_N = 1/(2m_p)$, hence from Eqs. (3) and (4)

$$g_{\text{exp}} = \frac{\mu_{\text{exp}}}{s\mu_M} = \frac{\mu_{\text{exp}}}{s\mu_N} \frac{M}{m_p}. \quad (5)$$

The values of the magnetic moments and the corresponding g factors for the lowest mass states used in this paper are collected in Table I (other states used are taken with $g = 2Q$). We recall that because the magnetic moment operator is odd

under the charge conjugation, the magnetic moments of particles and antiparticles are equal and opposite. In particular, the magnetic moments of neutral mesons vanish.

To recapitulate the above part, in the nonrelativistic treatment the formula for the energy level of a charged hadron of any spin is

$$E_{l,p_z}^{\text{nr}} = M + \frac{p_z^2}{2M} + \frac{B}{2M} [|Q|(2l+1) - gs_z], \quad (6)$$

whereas for a neutral hadron

$$E_p^{\text{nr}} = M + \frac{p^2}{2M} - \frac{B}{2M} gs_z. \quad (7)$$

However, in ultrarelativistic nuclear collisions the large temperatures imply large momenta of hadrons, typically of a few hundred MeV, hence for a successful phenomenology of HRG it is required to use the relativistic kinematics. Below, we discuss one-by-one the subsequent spin states, keeping in mind that the full hadron list contains states up to as large a value as $s = \frac{11}{2}$, with the higher s states suppressed by mass, but enhanced by their spin degeneracy factor $(2s+1)$.

The spin-0 states obviously have no magnetic moment, as follows from the Wigner-Eckart theorem. In this case the minimum coupling in the Klein-Gordon equation yields, for the charged mesons of charge Q ,

$$\epsilon_{l,p_z} = \sqrt{M^2 + p_z^2 + B|Q|(2l+1)}, \quad (s=0, \text{ charged}), \quad (8)$$

and obviously $\epsilon_p = \sqrt{M^2 + p^2}$, independent of B , for the neutral mesons.

For the spin- $\frac{1}{2}$ Dirac states (describing, i.a., the octet baryons) with the nonminimal (anomalous) coupling of the form $\mu_M \kappa \bar{\psi} \sigma_{\mu\nu} F^{\mu\nu} \psi$, the eigenvalue problem was first solved by Tsai and Yildiz [40] with the result [we use the convention where $g = 2(Q + \kappa)$]

$$\epsilon_{l,p_z} = \sqrt{(\sqrt{M^2 + B(2l+1-2Qs_z)} - 2\mu_M B \kappa s_z)^2 + p_z^2} \quad (s = \frac{1}{2}, Q = \pm 1). \quad (9)$$

For $Q = 0$, an elementary calculation yields

$$\epsilon_{p,p_z} = \sqrt{(\sqrt{M^2 + p^2 - p_z^2} - 2\mu_M B \kappa s_z)^2 + p_z^2} \quad (s = \frac{1}{2}, Q = 0). \quad (10)$$

For the spin-1 states, the energy levels take a simple form only for the case $g = 2Q$, which in this case requires a non-minimal coupling in addition to the minimum substitution in the Proca field Lagrangian (cf. Eq. (17) of [40] with their $\kappa = 1$, and $g = 1 + \kappa$ for the assumed $Q = 1$). This is in agreement with the general conviction discussed in [54] that at the tree level $g = 2$ (or, in general $2Q$, when charges different from 1 are admitted) for point-like particles of any spin s , rather than the earlier expectations of $g = Q/s$ [55]. For the spin-1 mesons, as seen from Table I, the g factor is indeed close to 2. Thus we adopt Eq. (17) of [40], supplementing it with a small correction accounting for the departure of g from

the $2Q$ value in the nonrelativistic form of Eq. (6), namely

$$\epsilon_{l,p_z} = \sqrt{M^2 + p_z^2 + B[|Q|(2l+1) - 2Qs_z]} - B\mu_M(g - 2Q)s_z, \quad (s = 1, Q \neq 0). \quad (11)$$

For the spin- $\frac{3}{2}$ Rarita-Schwinger states with $g = 2Q$, it was argued in [56] that the energy spectrum is given by $\epsilon_{l,p_z} = \sqrt{M^2 + p_z^2 + B[|Q|(2l+1) - 2Qs_z]}$. Extension to account for the anomalous case is difficult (see, e.g., the early work [57]). Fortunately, for the $\Delta(1232)$ states listed in Table I, g is close to the $2Q$ value. To include a small departure of g from this value, similarly to the spin-1 case above, we take

$$\epsilon_{l,p_z} = \sqrt{M^2 + p_z^2 + B[|Q|(2l+1) - 2Qs_z]} - B\mu_M(g - 2Q)s_z, \quad (s = \frac{3}{2}, Q \neq 0). \quad (12)$$

For the neutral case we use, correspondingly,

$$\epsilon_p = \sqrt{M^2 + p^2} - B\mu_M gs_z, \quad (s = \frac{3}{2}, Q = 0). \quad (13)$$

For hadrons of spins higher than $\frac{3}{2}$, which are heavier from the lower spin states [the lightest state is $a_2(1320)$], we use, with the lack of detailed knowledge and guided by the lower-spin cases, the expressions

$$\epsilon_{l,p_z} = \sqrt{M^2 + p_z^2 + B|Q|(2l+1) - B\mu_M gs_z} \quad (s > \frac{3}{2}, Q \neq 0) \quad (14)$$

and

$$\epsilon_p = \sqrt{M^2 + p^2} - B\mu_M gs_z, \quad (s > \frac{3}{2}, Q = 0), \quad (15)$$

assuming $g = 2Q$.

The policy of including the hadron states in HRG in the uniform magnetic field according to the above formulas is in our view the best phenomenological approach that we have at hand. It is approximate in several aspects: First, the hadrons are not point-like, hence their structure such as the mass may additionally depend on B . Second, the g factors for finite-width resonances are complex numbers, so the formulas are valid for the zero-width case. Also, the position of a resonance and its width in general can depend on B . Finally, at very large values of B the square of the energies of the low Landau levels may turn negative, exhibiting instability [8] and indicating the limits of applicability of the approach. For instance, the lowest Landau level for Δ^{++} with $s_z = \frac{3}{2}$ turns imaginary at $B \simeq 0.3 \text{ GeV}^2$, providing a rough upper limit for applicability of the approach with structureless hadrons.

Taking a measure of a basic excitation energy in the nucleon as $m_\Delta - m_N$ and requesting that the magnetic interaction in the proton be smaller,

$$\mu_p B \lesssim m_\Delta - m_p, \quad (16)$$

yields the applicability condition

$$B \lesssim 0.2 \text{ GeV}^2, \quad (17)$$

more stringent than the one following from the instability, discussed above.

Concerning the magnetic effects on the mass to the second order in low B , one can make an estimate for the nucleon making use of the known values of the magnetic polarizabilities. The relevant formula here is

$$\Delta M = -\frac{1}{2}\beta B^2, \quad (18)$$

where β denotes the magnetic polarizability. For the proton and neutron we have $\beta_p = 2.5(4) \times 10^{-4} \text{ fm}^3$ and $\beta_n = 3.7(1.2) \times 10^{-4} \text{ fm}^3$, which at our top B value of 0.15 GeV^2 leads to mass shifts $\sim 0.5 \text{ MeV}$, hence completely negligible at temperatures of the order of T_c . However, the conclusion is no longer valid at stronger B fields, where higher-order effect may take over. In fact, estimates made within the Skyrme model show that in strong magnetic fields the deformation effects lead to a substantial increase of the nucleon and Δ masses [58,59]. Such effects, not attempted to be incorporated here, would quench the susceptibilities at larger values of B , as well as would help with the instability problem mentioned above.

III. HRG IN A UNIFORM MAGNETIC FIELD

In this section we provide the standard expressions (see, e.g., [60]) for the thermodynamics of HRG, needed in our work. The thermal partial pressure of the individual species of HRG can be written as [8,37,38]

$$P = -\eta T(2s+1) \int \frac{d^3p}{(2\pi)^3} \ln[1 - \eta f(E, T, \mu)], \quad (19)$$

where $f(E, T, \mu)$ is the distribution function

$$f(E, T, \mu) = \frac{1}{\exp\left(\frac{E-\mu}{T}\right) + \eta} \quad (20)$$

with $\mu = \mu_B B + \mu_S S + \mu_Q Q$ representing the total chemical potential of a single hadron, and $\eta = +1$ or -1 , depending on whether we deal with a fermion or a boson. In the presence of a magnetic field, for charged hadrons the phase space integral is written as the sum over the Landau levels and the spin states [2]

$$(2s+1) \int \frac{d^3p}{(2\pi)^3} \rightarrow \frac{B|Q|}{2\pi^2} \sum_{l=0}^{\infty} \sum_{s_z=-s}^s \int_0^{\infty} dp_z, \quad (21)$$

while for the neutral hadrons

$$(2s+1) \int \frac{d^3p}{(2\pi)^3} \rightarrow \frac{1}{2\pi^2} \sum_{s_z=-s}^s \int_0^{\infty} p^2 dp. \quad (22)$$

Then for the charged particles, the partial pressure P takes the form

$$P = -\eta T \frac{B|Q|}{2\pi^2} \sum_{l=0}^{\infty} \sum_{s_z=-s}^s \int_0^{\infty} dp_z \ln[1 - \eta f], \quad (23)$$

whereas for neutral particles

$$P = -\eta T \frac{1}{2\pi^2} \sum_{s_z=-s}^s \int_0^{\infty} p^2 dp \ln[1 - \eta f]. \quad (24)$$

The leading order conserved charge (dimensionless) susceptibilities are defined as

$$\chi_{Q_1 Q_2} = \left. \frac{\partial^2(P/T^4)}{\partial(\mu_{Q_1}/T) \partial(\mu_{Q_2}/T)} \right|_T, \quad (25)$$

where $Q_1, Q_2 = \{B, S, Q\}$ indicate the type of the conserved charge. Using Eqs. (23) and (24) in Eq. (25) immediately yields

$$\chi_{Q_1 Q_2} = \frac{Q_1 Q_2 B |Q|}{2\pi^2 T^3} \sum_{l=0}^{\infty} \sum_{s_z=-s}^s \int_0^{\infty} dp_z f(1 - \eta f) \quad (26)$$

for the charged particles and

$$\chi_{Q_1 Q_2} = \frac{Q_1 Q_2}{2\pi^2 T^3} \sum_{s_z=-s}^s \int_0^{\infty} p^2 dp f(1 - \eta f) \quad (27)$$

for the neutral case.

A remark regarding how we handle numerically the calculation of the susceptibilities of Eqs. (26) and (27) is in order. At a given T and B , a certain number of Landau levels (up to $l_{\max} \sim 1/B$) effectively saturates the infinite sum. We thus sum as many levels as needed to achieve convergence, which for any $B > 0$ is straightforward. For $B = 0$ strictly, one of course uses the formulas without the magnetic field following Eqs. (19) and (20).

For a critical review of the HRG model and its modifications, the reader is referred to, e.g., [12]. Despite the caveats discussed there, the success of the HRG model in the description of both the hadron multiplicities in ultrarelativistic nuclear collisions, as well as the properties of the hot medium in its validity range of $100 \text{ MeV} \lesssim T \lesssim 150 \text{ MeV}$, places it as an established phenomenological tool or even as a benchmark in certain comparisons to data.

We use the hadron list QMHRG2020 [12,13], which contains an extended list of resonance states based on the quark model (including the so-called missing states) compared to the Particle Data Group listings [46]. The PDG list, which has less states, fails to reproduce with an expected accuracy the lattice data at zero magnetic field [12], hence QMHRG2020 is preferable.

The magnetic moments of the lowest mass hadrons are collected in Table I. We note the large anomalous magnetic moments of the octet baryons (N , Λ , Σ , and Ξ), and the approximate proximity to the $g = 2Q$ values for the decuplet baryons (Δ , Σ^* , Ξ^* , and Ω). We also remark that for the Roper state, $N^+(1440)$, the g factor is somewhat below 2. For the ρ^+ , K^{*+} , and a_1^+ mesons, the corresponding g factors are close to 2. These results support the view that except for the octet baryons, which are spin- $\frac{1}{2}$ and for which we have a formula with an exact treatment of the anomalous magnetic moments, the remaining hadrons abide to the $g \simeq 2Q$ case, which can be treated to a good accuracy with the formulas collected in Sec. II.

IV. RELEVANCE OF PHYSICAL MAGNETIC MOMENTS OF HADRONS

In the nonrelativistic treatment of Eq. (6), which we apply in this section for the simplicity of the discussion, and neglecting, for clarity, the effects of statistics in final expressions (i.e., setting $\eta^2 = 1$ and $\eta = 0$ after evaluation of the derivatives with respect to the chemical potentials), one easily gets the formula for the generic susceptibility,

$$\chi_{Q_1 Q_2} = \frac{Q_1 Q_2 B |Q| e^{-\frac{M}{T}} \sqrt{MT} \sinh\left(\frac{Bg(2s+1)}{4MT}\right)}{4\sqrt{2}\pi^{3/2} T^3 \sinh\left(\frac{B|Q|}{2MT}\right) \sinh\left(\frac{Bg}{4MT}\right)}. \quad (28)$$

Here, we write it for the electric charge $Q \neq 0$, but the limit of $Q \rightarrow 0$ gives the same expression for the neutral particles as well. Several interesting features can be inferred from this formula.

For $s = 0$ states, such as the pion or kaon, Eq. (28) behaves, as a function of B , as $B|Q|/\sinh(B|Q|/2MT)$, which is monotonically *decreasing* to zero. On the other hand, for $s > 0$, the last factors in the numerator and denominator of Eq. (28) yield an increase. The change of the behavior from decreasing to increasing occurs around $s \sim |Q/g|$. More precisely, from the expansions at low B ,

$$\chi_{Q_1 Q_2} = \frac{Q_1 Q_2 M^{3/2} (2s+1) e^{-\frac{M}{T}}}{2\sqrt{2}\pi^{3/2} T^{3/2}} \times \left[1 + \frac{B^2 (g^2 s(s+1) - Q^2)}{24M^2 T^2} \right] + \mathcal{O}\left(\frac{B^4}{M^2 T^2}\right), \quad (29)$$

we conclude that the function is concave when $g^2 s(s+1) < Q^2$, and convex otherwise. The above conclusions on the behavior of $\chi_{Q_1 Q_2}(B)$ remain qualitatively valid for the relativistic case and with the inclusion of the Bose-Einstein or Fermi-Dirac statistics.

There is one more important observation we want to underscore. Let us introduce

$$\Delta\chi_{Q_1 Q_2}(B) = \chi_{Q_1 Q_2}(B) - \chi_{Q_1 Q_2}(0), \quad (30)$$

which represents the difference between susceptibility at nonzero B and $B = 0$. Due to the subtraction of the $B = 0$ contribution, such a quantity shows the influence of a physical g more prominently. As long as B is sufficiently small [i.e., keeping the terms up to $\mathcal{O}(B^2)$], from Eq. (29) we can see that for a given hadron the departure of $\chi_{Q_1 Q_2}$ from its value at $B = 0$ is controlled by the prefactor $g^2 s(s+1) - Q^2$. With the prescription $g = 2Q$, this prefactor becomes $[4s(s+1) - 1]Q^2$.

The change between using the physical values of g and $g = 2Q$ can thus be quantified through the ratio

$$R_{Q_1 Q_2}(B) = \frac{\Delta\chi_{Q_1 Q_2}(B)_{g \text{ phys.}}}{\Delta\chi_{Q_1 Q_2}(B)_{g=2Q}}, \quad (31)$$

which is independent of B at low B . For a single state, according to the previous expressions, it is (nonrelativistically and without statistics)

$$R_{Q_1 Q_2}(B) \simeq \frac{\left[\frac{g^2}{Q^2} s(s+1) - 1\right]}{[4s(s+1) - 1]}. \quad (32)$$

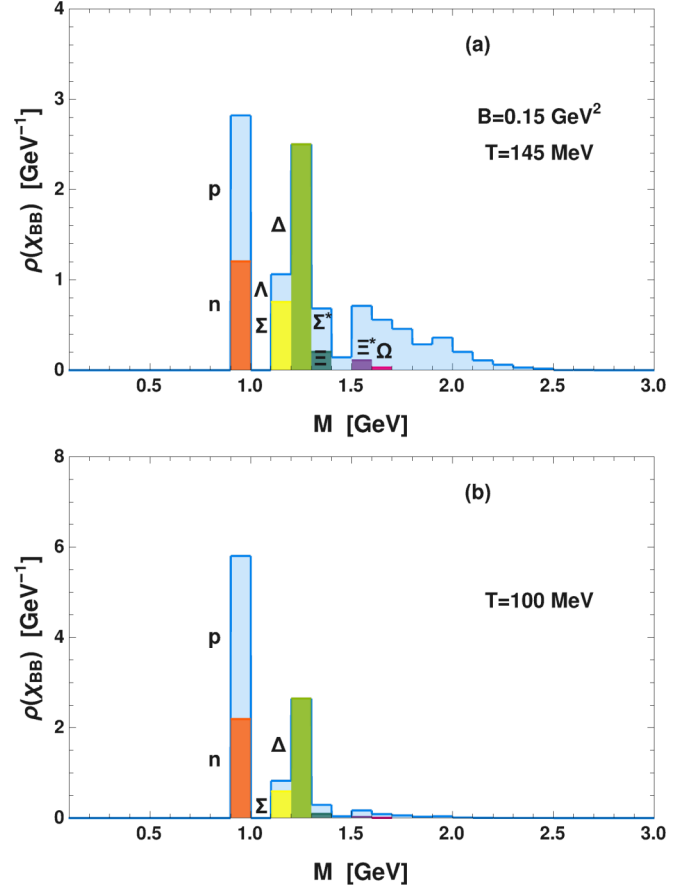


FIG. 1. Normalized histogram density for the baryon number susceptibility χ_{BB} in HRG, plotted as a function of the hadron mass M at $T = 145$ MeV (a) and $T = 100$ MeV (b), and at magnetic field strength $B = 0.15$ GeV². The HRG contribution in each bin is shown in light blue color, while the individual contributions from baryon octet and decuplet states are shown in different colors and are labeled accordingly.

For the case of χ_{BB} , the ratio R_{BB} is dominated by the nucleons and the $\Delta(1232)$ resonances, which are the lightest baryons (cf. the following discussion of Fig. 1). For the proton $g_p \simeq 5.6$, and Eq. (32) for this state yields an enhancement as large as $R \simeq 10$. It is to be mentioned that with the use of proper statistics and relativistic formula as discussed in Sec. II, this factor is even larger, $R \sim 14$. For the neutron $g_n \simeq -3.8$, and the relative contribution of χ_{BB} of the neutron to the proton is about 0.5, hence substantial. We recall that since the neutron has no charge, its corresponding contribution is not included in the $g = 2Q$ prescription at all. For Δ^{++} , using the value of g from Table I compared to $2Q = 4$, leads to an enhancement by about 20%, hence moderate.

V. ANATOMY OF THE SUSCEPTIBILITIES

For a better perspective and to highlight the key motivation behind this paper, we look closer at the anatomy of $\chi_{Q_1 Q_2}$, breaking it into the contributions of the subsequent states, binned with width of 100 MeV in the mass M . The resulting histogram densities (distributions in M), denoted by $\rho(\chi_{Q_1 Q_2})$,

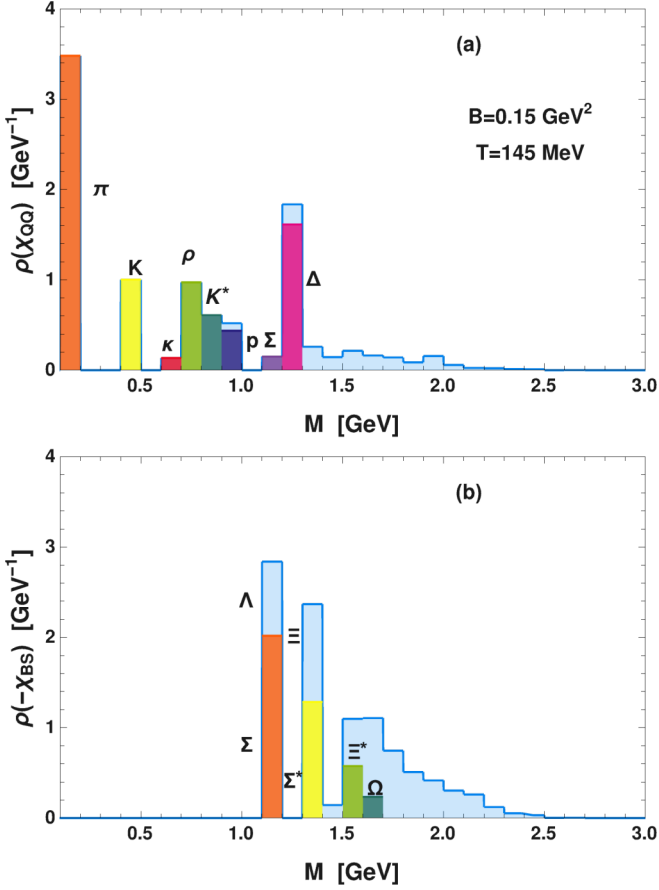


FIG. 2. Same as Fig. 1, but for the electric charge susceptibility χ_{QQ} and the nondiagonal baryon-strangeness susceptibility χ_{BS} at $T = 145$ MeV.

are normalized as $\int dM \rho(\chi_{Q_1 Q_2}) = 1$. Figure 1 shows $\rho(\chi_{BB})$ at $B = 0.15 \text{ GeV}^2$ for two different temperatures, $T = 145$ MeV and 100 MeV, where the contributions from individual octet and decuplet baryons are shown in different colors. At $T = 145$ MeV [panel (a)], which is a value close to the crossover phase transition, the dominant contribution to χ_{BB} comes from the nucleons, followed by the Δ resonance states, amounting jointly to about $\sim 50\%$ of the total, while the rest $\sim 50\%$ can be attributed to the other states, including a $\sim 10\%$ contributions from the Λ and Σ baryons. It is instructive to note that with the use of the physical values of g (as given in Table I), compared to the $g = 2Q$ prescription, the proton contribution to $\chi_{BB}(T)$ is enhanced by a factor of ~ 2 , and for the neutron, by more than 50%, which is very substantial. The neutron takes about $\sim 40\%$ of the total nucleon contribution. For the Δ , the analogous enhancement is around $\sim 20\%$.

As the temperature decreases, the higher mass states get suppressed with the thermal factors, hence the nucleons largely dominate. As seen from panel (b), for $T = 100$ MeV the nucleons alone contribute almost $\sim 60\%$ of the total, the Δ resonances yield $\sim 25\%$ of the total, while the rest is essentially saturated with the remaining octet states.

Similar histograms are presented in Fig. 2 for the electric charge susceptibility $\rho(\chi_{QQ})$ in panel (a) and for the

nondiagonal baryon-strangeness susceptibility $-\chi_{BS}$ (note the inverted sign) in panel (b), at $T = 145$ MeV for both cases. For χ_{QQ} , the dominant contribution, naturally, comes from the pions ($\sim 35\%$), followed by the Δ states ($\sim 18\%$). Together, π , Δ , ρ and K yield almost 75% of the total. The large contribution of the Δ , despite the larger mass, comes from the electric charge of Δ^{++} and large spin and isospin degeneracy. As the temperature goes down (not shown), the pions dominate more and more, taking $\sim 70\%$ at $T = 100$ MeV.

On the other hand, for χ_{BS} of panel (b), the dominant contribution comes from Σ and Σ^* , taking more than 30% of the total, followed by the Ξ and Λ baryons. As expected, in the anatomy of the strangeness susceptibility, χ_{SS} (not shown), the kaons provide the dominant contribution.

Returning to the distributions of the susceptibilities χ_{BB} , χ_{QQ} , and also χ_{BQ} (not shown), we note that the contribution of the Δ resonance to these quantities is large. At the same time, the accuracy of the determination of magnetic moments of the decuplet states is not precise, hence Δ brings in a model uncertainty to the HRG predictions.

VI. COMPARISON TO THE LATTICE QCD DATA

A. Susceptibilities at $B = 0$

We first study the susceptibilities in the absence of the magnetic field, i.e., for $B = 0$, in order to see how HRG works in this basic case, before switching on the $B > 0$ effects. Such a study was already done in [12], but we independently repeat it here for completeness. Figure 3 shows the leading order diagonal and non-diagonal conserved charge susceptibilities as functions of T . The HRG model calculations (red lines) are compared with two sets of the lattice QCD data, one from [12] and the other one digitized from [14]. Both sets of data agree with HRG in the fiducial range of $T \lesssim 150$ MeV within a few percent. It should be noted that for $-\chi_{BS}$, χ_{QS} , and χ_{SS} , HRG exhibits a somewhat better agreement with the data from [12] than from [14]. Beyond ~ 155 MeV there is an anticipated departure of the the model results from the data, which bend over to the quark-gluon plasma (QGP) phase results. The quality of the agreement between HRG and lattice data is somewhat worse in case of χ_{QQ} , where the HRG calculation overestimates the lattice results by about 8%. This is an expected behavior of χ_{QQ} at $B = 0$, also seen in past studies [12,13,37,38], and not a primary focus of this paper. However this discrepancy at $B = 0$ is propagated to $B > 0$ case, as we discuss in the next section.

B. Susceptibilities at nonzero B

This section presents the main results of our paper, namely, the dependence of the conserved charge susceptibilities on the magnetic field with physical g factors used in HRG. We restrict ourselves to sufficiently small magnitudes of the magnetic field, up to $B \sim 0.15 \text{ GeV}^2$, which satisfies the approximate condition (17), allowing for the treatment of hadrons as structureless particles. The magnetic moments of hadrons are incorporated according to the policy stated at the end of Sec. III.

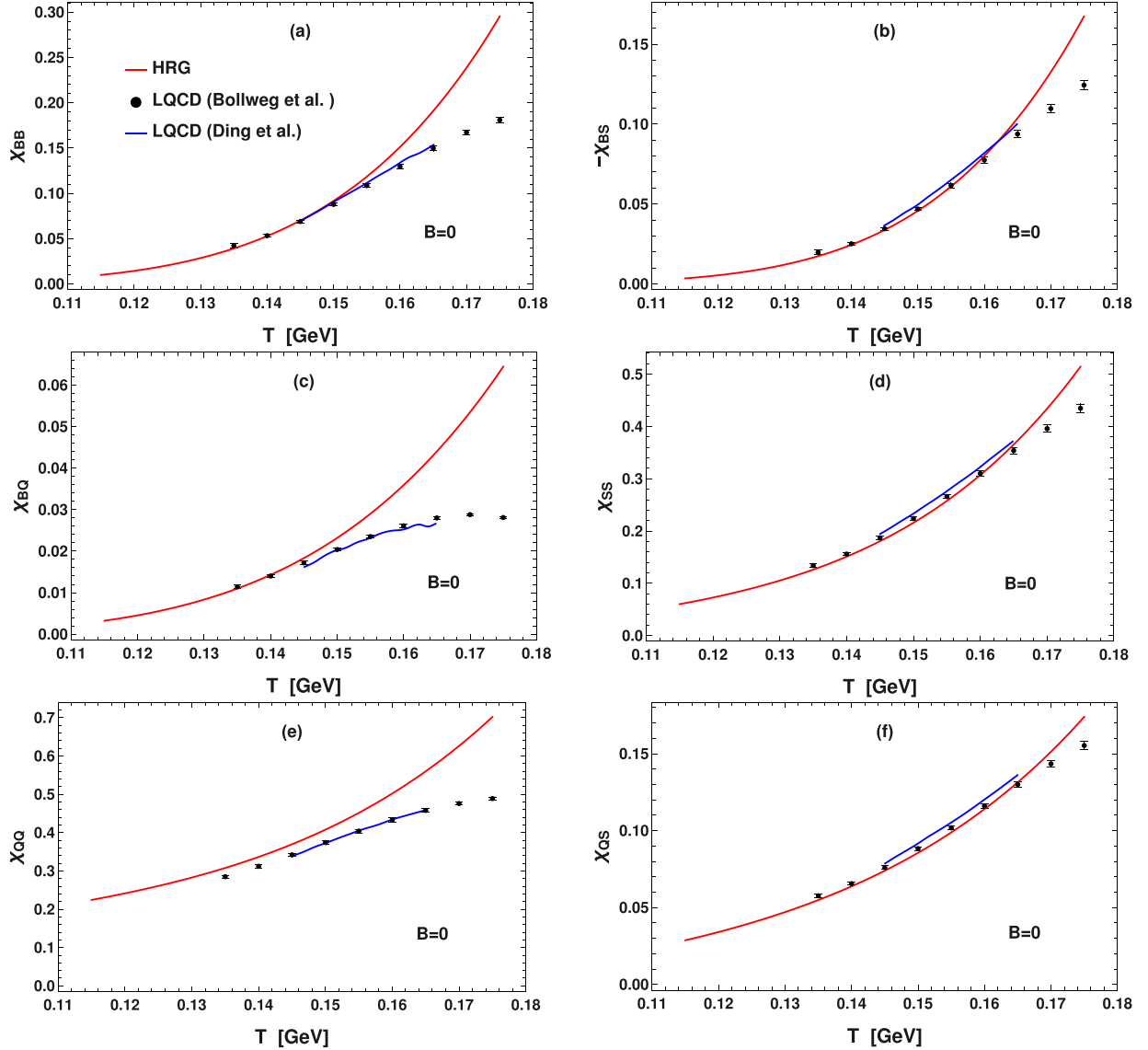


FIG. 3. Diagonal and nondiagonal conserved charge susceptibilities at zero magnetic field, plotted as functions of the temperature T . The red solid line represents our HRG model calculation, the black symbols denote the lattice data from [12], and the blue curve represents the lattice data digitized from [14].

We start with panel (a) of Fig. 4, which presents the baryon susceptibility $\chi_{BB}(B)$. The legends specify various curves and bands in the figure. The orange bands represent the susceptibilities from the lattice [14], which were obtained by digitizing the plots. The additional data points at $B = 0$ from [12] are indicated in black. For a better reference, we draw horizontal black lines at the values of the model susceptibilities at $B = 0$. The solid magenta lines represent the HRG calculation with the $g = 2Q$ (or $\kappa = 0$) prescription, as used in [37,38].

The other bands and lines in Fig. 4(a) show various predictions of HRG with the physical values of the g factors. The magnetic moments (g factors) for the nucleons are precisely measured [46], therefore contain negligible uncertainty. The next dominant contributions comes from the $\Delta(1232)$ for which we lack a detailed determination [46]. For this reason

we probe different values of g for the $\Delta(1232)$ isobars, as listed in the appropriate entries in Table I. The green band in Fig. 4(a) shows the result with the χ PT values [51] with errors for the g of all the $\Delta(1232)$ charge states, as given in Table I. The blue band corresponds to the very wide range of the PDG [46] data for $g_{\Delta^{++}}$ (cf. Table I). As for the other charge states of Δ , the g factors are experimentally poorly known (or unknown), hence for these states we take the χ PT values while using the PDG range for Δ^{++} . The same recipe is used for the case of the most recent experimental value of Lopez *et al.* [49], indicated with the dotted line. Finally, the dashed line, passing through the middle of the green band, uses the lattice QCD prediction [50] for the Δ^{++} g factor.

We note that this kaleidoscope of HRG calculations is in reasonable agreement with the lattice data, in particular for $T = 145$ MeV. We note that the χ PT band touches the data

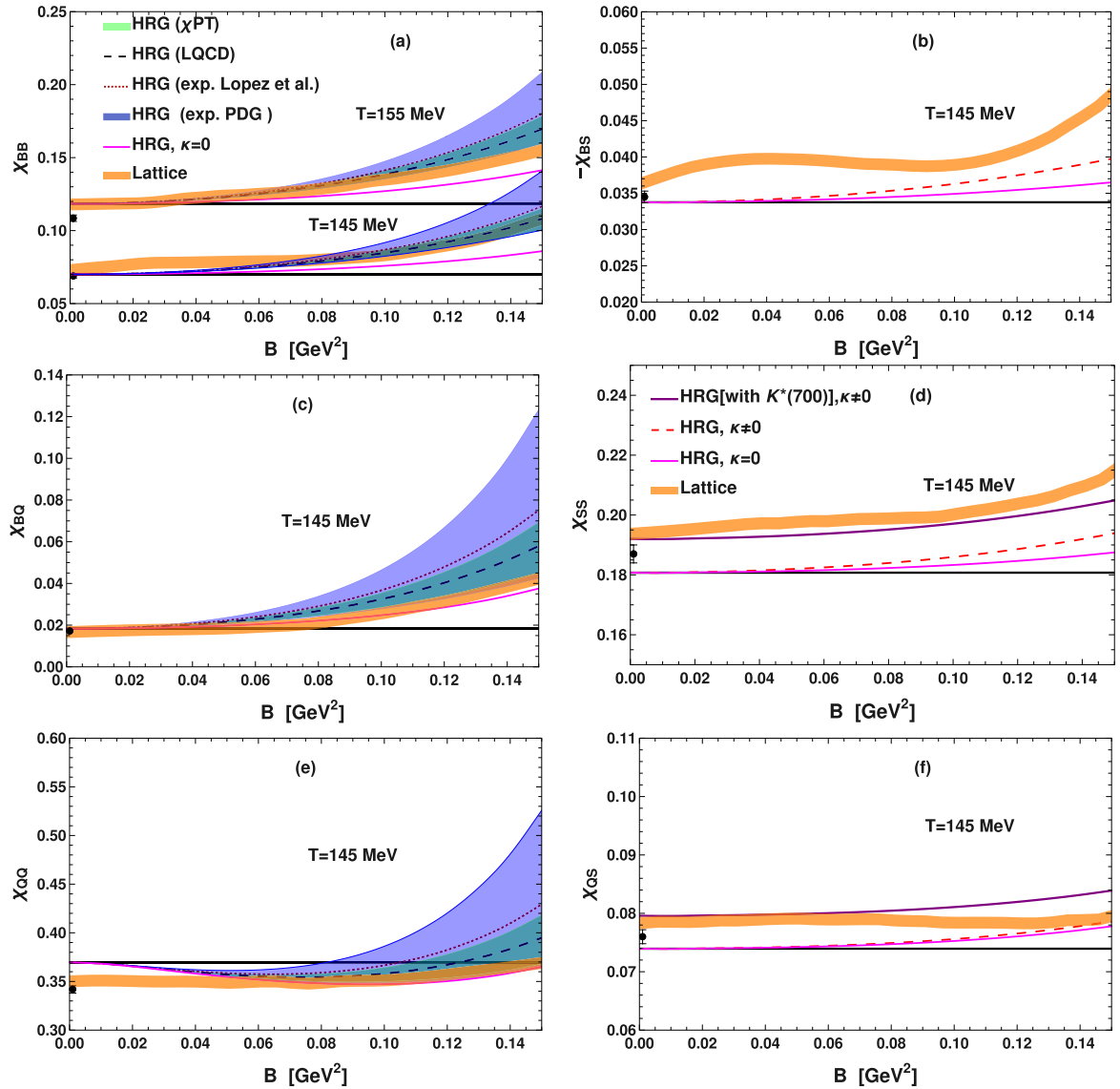


FIG. 4. Conserved charge susceptibilities, plotted as functions of the magnetic field B . The orange bands represent the lattice QCD data digitized from [14]. The black points correspond to the data at $B = 0$ from [12]. The horizontal black lines indicate the values of our model susceptibilities at $B = 0$, plotted for a reference. The solid magenta lines show the HRG calculation with the $g = 2Q$ (or $\kappa = 0$) prescription. The remaining bands and lines show various predictions of HRG with hadrons having the physical values of the g factors, but for different values of g for the Δ isobars, as indicated in the legends and specified in Table I (see the text for details). This affects the plots in (a), (c), and (e), but not for (b), (d), or (f), where Δ does not contribute and where the HRG results with physical g factors are drawn with dashed red lines. The solid purple lines in (d) and (f) represent the results with the addition of $K^*(700)$ states in the hadron list.

band. Importantly, the discussed HRG estimates are significantly above the magenta solid line indicating the $g = 2Q$ prescription [37,38].

For the other susceptibilities in Fig. 4, we only provide the results for $T = 145$ MeV case, which is in a better agreement with the data than for $T = 155$ MeV.

In panel (c) we display $\chi_{BQ}(B)$, with a qualitative behavior and the agreement with the data similar to panel (a), but with even wider uncertainty bands from the errors in the g values of the $\Delta(1232)$. This is mainly due to the charge $Q = 2$ enhancement from Δ^{++} [cf. Eq. (26)].

In panel (e) we show $\chi_{QQ}(B)$. The interesting behavior of the model observed here is that when B is being increased,

χ_{QQ} first encounters a mild decrease, followed by an increase at larger B , with a dip around $B = 0.10 \text{ GeV}^2$. The decrease is due to the $s = 0$ states (pions, kaons, etc.) for which χ_{QQ} decreases with B , as implied by Eq. (28). The lattice QCD data, however, do not exhibit such a clear nonmonotonicity. The discrepancy at $B = 0$ follows from Fig. 3(e).

Panels (b), (d), and (f) show the corresponding results for $-\chi_{BS}$, χ_{SS} , and χ_{QS} , respectively. Since in these cases the Δ states do not contribute, we plot only one HRG curve (red dashed line) with the central values of the g factors from Table I. In general, these quantities show a weaker dependence on the magnetic field B than the previously discussed cases. The data for $-\chi_{BS}$ in panel (b) exhibits a nonmonotonic

behavior in B , which is not observed in the HRG model calculations. For χ_{QS} , the data seems flat compared to the model. We note a few percent mismatch at $B = 0$, which follows from Fig. 3. This can be improved for χ_{SS} and χ_{QS} as we discuss below.

For χ_{SS} and χ_{QS} , the model predictions depend on the inclusion of the $K^*(700)$ state, as first considered in [61], where it was shown that the attraction in the S wave $\pi - K$ isospin $I = 1/2$ channel is partially canceled with the repulsion in the S wave $I = 3/2$ channel. As a result, a net 30% contribution of the $K^*(700)$ state, treated as a wide resonance, is left [61]. We apply this prescription here. The inclusion of $K^*(700)$ improves significantly the results for χ_{SS} and χ_{QS} , in particular near $B = 0$, as seen from Figs. 4(d) and 4(f).

To summarize this section, we note that the effect of the anomalous moments on χ_{BB} , χ_{BQ} , χ_{BS} , and χ_{SS} , when compared to the results at $B = 0$ indicated with the solid black line in Fig. 4, is substantial (about 100% compared to the $\kappa = 0$ lines), and improves the comparison with the lattice QCD data.

VII. EFFECT OF ANOMALOUS MAGNETIC MOMENTS ON PARTICLE MULTIPLICITIES

In the previous sections, devoted to susceptibilities (corresponding to second derivatives of the pressure with respect to chemical potentials), we focused on comparisons of HRG with lattice QCD for a system in a thermal equilibrium and, most importantly, in a stationary and uniform magnetic field. On the other hand, the description of the effects of magnetic field on the particle yields (corresponding to first derivatives of the pressure with respect to chemical potentials) measured in ultrarelativistic nuclear collisions is more difficult for several reasons. First, during a collision the magnetic field is not static, but has a time dependence resembling a short flash originating from the moving initial charged projectiles and the final receding spectators (for estimates of the magnitude see, e.g., [62]), accompanied with a long lived but an order of magnitude weaker magnetic field generated in the process of the collision via the eddy currents induced in the fireball [63,64]. The magnetic field is not spatially uniform [62], but exhibits a profile a few fm wide. Moreover, the fireball is expanding, which further complicates the dynamics, with the Hall effect appearing in addition to the Faraday induction [65], leading to possible flow signatures of the magnetic field.

The strength of the magnetic field is, naturally, larger in peripheral collisions than in central collisions, where it can only result from fluctuations (cf. Fig. 6 in [62]). This opens a possibility of explaining the centrality behavior of the proton to pion ratio measured at the LHC by the ALICE Collaboration [66,67], where the p/π^+ ratio raises by a factor of ~ 1.3 from central to peripheral collisions. In the scenario considered in [37,38], the effect may be explained if a large magnetic field ($B > 0.1$ GeV) occurs in peripheral collisions.

Here, we discuss the possible role of the physical magnetic moments when amended to the argumentation of [37,38]. For charged particles of a given species, the primordial multiplicities in thermal equilibrium at chemical freeze-out are given

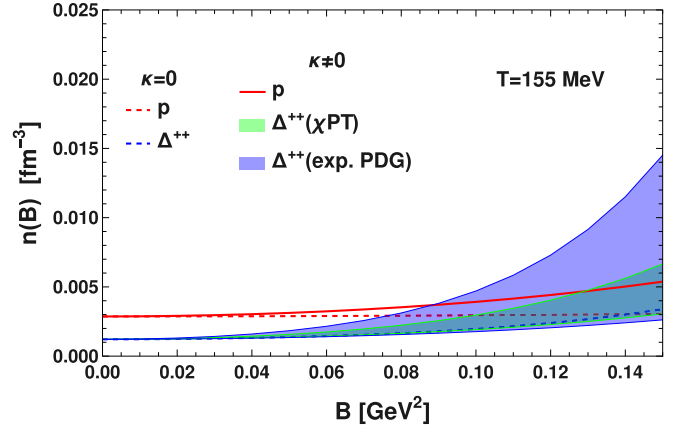


FIG. 5. Proton and $\Delta^{++}(1232)$ primordial multiplicities in HRG, plotted as functions of the magnetic field B at $T = 155$ MeV. The case without the anomalous magnetic moments ($\kappa = 0$) is indicated with dashed lines while the solid lines represent the $\kappa \neq 0$ scenario. The bands for Δ^{++} correspond to the uncertainty of its anomalous magnetic moments, analogically to Fig. 4.

by the formula

$$n(B) = \frac{B|Q|}{2\pi^2} \sum_{l=0}^{\infty} \sum_{s_z=-s}^s \int_0^{\infty} dp_z f(B), \quad (33)$$

following from Eq. (19). We have indicated explicitly the dependence of the distribution functions on B , which enters via the formulas for the energy spectra given in Sec. II. A derivation analogous to the one leading to Eq. (29) leads, under the assumption of large M , the following expansions at low B :

$$n(B) = \frac{(TM)^{3/2}(2s+1)e^{-\frac{M}{T}}}{2\sqrt{2}\pi^{3/2}} \times \left[1 + \frac{B^2(g^2s(s+1) - Q^2)}{24M^2T^2} \right] + \mathcal{O}\left(\frac{B^4}{M^2T^2}\right). \quad (34)$$

Thus, in the large M approximation, the leading relative dependence on B is of the same form as for the susceptibilities of Eq. (29). We note that the relative B^2 term strongly depends on the g factor and spin, while it is suppressed with M and T .

An exact calculation with Eq. (33) at $T = 155$ MeV gives the result displayed in Fig. 5, here for the case of the proton and $\Delta^{++}(1232)$. We note an increase of the proton primordial yield with B (solid red line), with about 50% increment at $B = 0.1$ GeV² compared to the value at $B = 0$. This is much larger than for the case without the anomalous magnetic moment (dashed red line). However, the primordial protons take only ~ 20 – 25 % of the total yield (which includes the resonance decays) [68]. Decays of $\Delta(1232)$ supply another ~ 20 – 30 %, which involves a strong B dependence (cf. Fig. 5), albeit with a large uncertainty. Other important feed-down contributions to the proton yield, including the one from $\Lambda(1115)$, have a very weak B dependence. Together with the fact that the π^+ yield is very weakly dependent on B [37,38], the anomalous magnetic moments of baryons reduce the estimate for the magnetic field needed to explain the ALICE p/π^+ data by

about 10–30 % from the value $\sim 0.12 \text{ GeV}^2$ obtained within HRG in [37,38], down to values $0.08 - 0.11 \text{ GeV}^2$. The uncertainty comes from the error in the $\Delta(1232)$ magnetic moments.

Thus, even with the inclusion of the anomalous magnetic moment for the baryons, the value of the magnetic field needed to explain the ALICE data remains large in the light of the results of [63,64], brought up at the beginning of this section. Note, however, the crude nature of the approximations used, shared with [37,38]. In particular, the resonance feeding must be affected by the magnetic field via modification of the branching ratios, as discussed in [37] in the context of detailed balance. Moreover, the branching ratios must depend on spin, since the energies of particles do depend on their spin. Therefore, a more appropriate dynamical modeling of the issue of particle yields in the presence of a magnetic field is desired.

VIII. SUMMARY

We have evaluated the diagonal and non-diagonal susceptibilities related to the conserved charges in the hadron resonance gas model in the presence of a uniform magnetic field in the range $0 \leq B \leq 0.15 \text{ GeV}^2$. A new feature is a systematic inclusion of the physical values for the magnetic moments of hadrons. Accounting for the anomalous parts of the magnetic moments is particularly important for the octet baryons, where the Lande g factors differ very significantly from the value $2Q$ following from the minimum substitution prescription. Since the B^2 effects in the susceptibilities are roughly proportional to g^2 , this amounts to a large enhancement. Note that the neutral baryons also contribute, due to their anomalous magnetic moments.

In our comparison to the lattice QCD data, we have included the large uncertainty of the g factors of the $\Delta(1232)$ isobar (from experimental, lattice QCD, and theoretical estimates), whose contributions are particularly important for the nonstrange susceptibilities due to the large spin and isospin degeneracy. We have found that at $T = 145 \text{ MeV}$ the agreement of the model susceptibilities as functions of B with the lattice QCD data is very reasonable. Importantly, it improves with the inclusion of the physical g factors. As a result, HRG with a proper list of hadron states and the proper g factors is capable of reproducing the B dependence of the (diagonal and nondiagonal) susceptibilities for temperatures $\sim 150 \text{ MeV}$ and $0 \leq B \leq 0.15 \text{ GeV}^2$. For yet stronger magnetic fields, modification effects in the structure of the ground-state hadrons and their resonances (describing the interactions) must be incorporated.

We have also reviewed the formulas for the energy spectra of point-like particles in magnetic moment up to spin $\frac{3}{2}$, used in the phenomenological studies of this paper.

Finally, we have discussed the issue of obtaining the proton yields in ultrarelativistic nuclear collisions, taking into account the anomalous magnetic moments.

ACKNOWLEDGMENTS

The authors thank Michal Marczenko for valuable discussions during the revision of the manuscript. The support from the Polish National Science Center Grant No. 2023/51/B/ST2/01625 is acknowledged.

DATA AVAILABILITY

The data that support the findings of this article are openly available [12,14]; embargo periods may apply.

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