

Quarkyonic quark-meson coupling model for nuclear and neutron matter

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We unite the dual quarkyonic model with the quark-meson coupling (QMC) model to construct a novel nuclear model based on the quark degrees of freedom, which can cover a wide range of nuclear densities, from low density to the crossover region. In the model, the relativistic, Gaussian quark wavefunction is used to describe the nucleon structure. We first evaluate the energy density, chemical potential, pressure, and sound velocity within the ideal Fermi gas picture. In this case, those physical quantities are discontinuous or divergent at the quark saturation density, where the quarkyonic phase emerges. To remove such singular behavior, we next introduce an infrared regulator, and combine the dual quarkyonic model and the QMC model to include the nuclear interaction, which we call the quarkyonic quark-meson coupling (QQMC) model. In this model, the quark saturation density depends strongly on the nucleon size. For example, when $r_p = 0.6(0.8)$ fm, where r_p is the root-mean-square radius of the proton, the quark saturation density is about $3.6(1.5) \times \rho_0$ in symmetric nuclear matter, where ρ_0 is the nuclear saturation density. Furthermore, the nuclear interaction plays an important role in considering physical quantities quantitatively. In fact, the QQMC model can produce sound velocity which is consistent with that inferred from the observed data of several neutron stars. Furthermore, pressure in symmetric or pure neutron matter deduced from the experiments of heavy-ion collisions at high energy can be explained by the QQMC model as well. We discuss in detail the formulation for the QQMC model and the physical quantities calculated by the model.

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I. INTRODUCTION AND PRELIMINARIES

For more than a decade, the masses and radii of neutron stars (NSs), together with the constraint on the tidal deformability from the binary neutron-star merger event, GW170817 [1–3], have been receiving much attention theoretically as well as experimentally, because these observations can provide decisive tests for understanding the equations of state (EOSs) for cold, dense nuclear matter. In particular, members of the neutron-star population with the heaviest masses are quite important, because, from them, we can obtain information on baryonic and/or exotic compositions in EOSs, which is not accessible through terrestrial experiments. The observed heavy pulsars are listed as [4] PSR J1614–2230 ($M_P \approx 1.91M_\odot$) [5], PSR J0348+0432 ($M_P \approx 2.01M_\odot$) [6], PSR J0740+6620 ($M_P \approx 2.08M_\odot$) [7], and PSR J0952–0607

($M_P \approx 2.35M_\odot$) [8], where M_P ($M_\odot$) is the mass of pulsar (sun). Furthermore, the precise measurements of NS radii have been performed with the NICER telescope observation [9–13].

One of the important findings from the NS observations concerns the stiffness of NS matter. As the nuclear density, ρ_N , increases, conventional nuclear calculations usually lead to a *softening* of EOSs for NS matter, because new degrees of freedom like hyperons, etc., are opened at high densities. However, the result from NS observations tells us that the EOS is rapidly *stiffened* slightly above the nuclear saturation density, ρ_0 . This discrepancy is called the *hyperon puzzle*, and a tremendous amount of research in this field has been reported (see, for example, Refs. [14–17]).

Among those various approaches, the idea of quarkyonic matter, which is originally based on the $1/N_c$ expansion [18] and hadron-quark continuity [19,20], seems one of the promising ways to settle the hyperon puzzle. For review and recent developments, see Refs. [21–23].

We first explain the concept of the quarkyonic picture very intuitively, which may provide a clue to better understand quarkyonic matter from the nuclear point of view. A nucleon at low energy may be approximately viewed as composed of a hard core surrounded by meson clouds. In the case of an isolated nucleon, the quarks inside a nucleon are confined forever. However, in nuclear matter, it is *not* the case. In general,

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the quark wavefunction has a relatively long tail even if it is confined. In matter, because there are many nucleons, the quark wavefunction can overlap with other quark wavefunctions confined to other nucleons. When the nuclear density is very dilute, the matter is conventional and is best viewed in terms of nucleons only. Thus, the probability of such overlap can be ignored and the quark momentum distribution, f_Q , is simply characterized by quark compositions inside isolated nucleons. However, with increasing ρ_N , the quarks with low momenta, whose wavefunctions are spread out, can hop from one nucleon to others through the overlap region. Such quarks can behave as delocalized low-momentum quark modes rather than asymptotically free quarks. However, they still feel the confinement potential. It is just like conduction electrons in metal. Such a phase is a novel concept and is called *soft deconfinement* in Ref. [20].

As the density increases further, the overlap region is extended more, and it starts to affect the momentum distributions of quarks, $f_Q(q)$, and nucleons, $f_N(k)$, in matter. (Hereafter, the letters q and k are used exclusively for quarks and nucleons, respectively.) Then the distribution, $f_Q(q)$, increases and eventually reaches the upper bound in Fermi statistics, i.e., $f_Q(q) = 1$ at $q = 0$. We call this density the *quark saturation density*, ρ_{sat} . Above ρ_{sat} , the levels with quark momenta between $q = 0$ and q_b are fully occupied, i.e., the bulk Fermi sea, where q_b is the upper momentum of the filled states, and the matter turns out to be *quarkyonic* from ordinary nuclear matter. In this region, because $f_Q(q)$ is fully occupied, additional quarks with the momenta $0 \leq q \leq q_b$ are not permitted due to the Pauli blocking at the quark level. This fact surely reduces the nucleon momentum distribution, $f_N(k)$, with *low* momentum, because slow nucleons have relatively more low-momentum quarks which are excluded by the Pauli blocking.

The quarkyonic phase thus emerges in the region of soft deconfinement (or in the crossover region), and it is characterized by percolation of the quark wavefunction at densities *lower* than the threshold density for *hard deconfinement* at which the overlap of repulsive hard cores of nucleons is created; that is, it is the usual concept of deconfinement in which the confinement potential melts down. In other words, the quarkyonic phase may be viewed as a precursor of hard deconfinement.

Recently, Fujimoto *et al.* [24,25] have constructed a novel model of quarkyonic matter, which is called the ideal dual quarkyonic (IdylliQ) model (see also Ref. [26]). In the IdylliQ model, the nuclear interaction is ignored, and the nucleon is composed of quarks whose wavefunction in momentum space, $\varphi(q)$, is supposed to be a Yukawa-type function,

$$\varphi(q) = \frac{2\pi^2}{\Lambda^3} \frac{e^{-q/\Lambda}}{q/\Lambda}, \quad (1)$$

with the typical momentum scale $\Lambda \sim \Lambda_{\text{QCD}}$. Then, collecting quark contributions from each nucleon generates the quark distribution in nuclear matter. For symmetric nuclear matter, it is described by the sum rule (or duality relation)

$$f_Q(q) = \int_k \varphi\left(q - \frac{k}{N_c}\right) f_N(k), \quad (2)$$

where the normalization of φ is $\int_k \varphi \equiv \int d^3\mathbf{k}/(2\pi)^3 \varphi = 1$, Q designates up (u) or down (d), and N_c is the color number.

Actually, in the IdylliQ model, by choosing the Yukawa-type wavefunction, Eq. (1), the sum rule can be transformed into the differential equation, which enables us to solve the IdylliQ model exactly [24]. By solving it with appropriate boundary conditions, the nucleon momentum distribution in the quarkyonic phase is obtained analytically as a sum of two segments,

$$f_N(k) = \frac{1}{N_c^3} \theta(k_b - k) + \theta(k_s - k) \theta(k - k_b), \quad (3)$$

with $k_b = N_c q_b$. The first term is the underoccupied bulk part with the order of $1/N_c^3$, while the second one is the nucleon distribution in the shell structure at $k_b \leq k \leq k_s$ with $k_s (= N_c q_s)$ the upper bound. The quark momentum distribution in the quarkyonic phase is calculated analytically as well [24]. This fact realizes that, above the quark saturation density, pressure, P , and the sound velocity, v_s , in matter are enhanced considerably.

At the same time, the IdylliQ model has two excellent advantages: (a) duality holds, and (b) a problem with the mismatched Fermi momenta of u and d quarks in the quarkyonic phase, which emerges in other quarkyonic models, can be naturally solved. In particular, duality is important, and it tells us that both the nuclear density and the energy density of matter can be expressed by either the nucleon or quark momentum distribution.

In Ref. [25], expanding this idea to the matter including hyperon degrees of freedom, the importance of the quark substructure in neutron matter is investigated.

In contrast, in the general case where the wavefunction, φ , is not Yukawa type and the nuclear interaction is involved, the situation is more complex. Basically, the momentum distributions, $f_Q(q)$ and $f_N(k)$, should be determined so as to minimize the energy density by optimizing the distributions under the constraints of Fermi statistics and the sum rule, Eq. (2). It may be a cumbersome task practically. Furthermore, the sum rule is *global*, i.e., f_Q is a functional of f_N , which thus makes the model nontrivial. The difficulty lies in the reconstruction of $f_N(k)$ from a given $f_Q(q)$ [27]. At present, a systematic theoretical method for solving this problem has not yet been established. Recall that, in the IdylliQ model, the momentum distributions can be calculated by solving the differential equation, which is *local*.

So far, in quarkyonic models, the nuclear interaction is not considered. Thus, it is urgently desirable to consider the effect of interactions among baryons to make the quarkyonic model more realistic. In Ref. [28], Koch *et al.* have first performed such an investigation using the IdylliQ model with the interaction due to σ and π meson exchanges. In their calculation, the quarkyonic phase begins below the nuclear saturation density, ρ_0 (see also Refs. [29,30]).

However, even below ρ_0 , using the quark-meson coupling (QMC) model, it has been indicated that subhadronic degrees of freedom are essential to understand the properties of the nuclear medium and finite nuclei [31–33]. The QMC model is originally based on a mean-field description of

nonoverlapping nucleon (or baryon) bags bound by the self-consistent exchange of scalar and vector mesons in the isoscalar and isovector channels [34,35]. The model is extended to investigate the properties of finite nuclei, in which, using the Born-Oppenheimer approximation to describe the interacting quark-meson system, one can derive the effective equation of motion for the nucleon (or baryon), as well as the self-consistent equations for the meson mean fields [31,32,36]. In Ref. [37], using the Miller-Spencer correlation function, the short-range quark-quark correlation is introduced into the QMC model phenomenologically and it is found that the saturation curve for symmetric nuclear matter eventually turns out to be stiff at high density. Furthermore, by using naive dimensional analysis, it is possible to see that the QMC model can provide remarkably natural coupling constants and hence the model itself is regarded as a natural effective field theory for nuclei [38].

The model can be applied to various finite nuclei, including strange and exotic hypernuclei [39,40]. In Ref. [32], the extensive application to these topics with a discussion of similarities and differences between the QMC and Skyrme energy density functionals are presented in detail. It is also of great interest that the QMC model predicts a variation of the nucleon form factors in nuclear matter [41], which affects certainly the analysis of electron scattering off nuclei [42], including the polarization transfer in quasielastic $A(\bar{e}, e'\bar{p})$ reaction (A1 Collaboration) [43] and the EMC (European Muon Collaboration) effect [44–47].

The QMC model has also been applied to the EOSs for massive NSs and the hyperon puzzle [48–50], and has predicted that, in the EOS for NSs, the cascade Ξ^- first appears around $\rho_N/\rho_0 \approx 3$ and other hyperons are all suppressed until $\rho_N/\rho_0 \approx 4.5$. A more phenomenological method to include short-range repulsion may be based on the excluded volume effect [51]. This method was recently used in the framework of the QMC model [52] and the Nambu-Jona-Lasinio (NJL)-type model [53].

Notably, the heart of the QMC model is the scalar polarizability of a baryon in matter, i.e., the variation of the quark scalar density inside the nucleon caused by the condensed, scalar mean field, which can naturally describe nuclear saturation and induce many-body forces [54]. Therefore, hadrons are polarized in medium even below ρ_0 , because they have quark substructure. The medium modification of the nucleon has been examined by lattice calculation (NPLQCD Collaboration) [55] as well.

The present paper is aimed at unifying the dual quarkyonic picture and the QMC model to construct a new nuclear model based on the quark degrees of freedom, which is valid from low density to a crossover region (or soft-deconfinement region), where the transition from baryonic to quark matter begins. Therefore, the new model includes the effect of Pauli blocking at the quark level as well as the scalar polarizability of nucleons in medium.

In the IdylliQ model, the single quark momentum distribution in a nucleon is chosen to be the Yukawa-type function, Eq. (1), specifically, while in the QMC model the quark wavefunction is usually given by the bag model. Therefore, first of all, it is necessary to prepare a unified setup for the quark

wavefunction. In this paper, we adopt a Gaussian function as the quark wavefunction in a nucleon, which is given by the relativistic confinement potential of the scalar-vector harmonic oscillator (HO) type. The QMC model with the relativistic HO potential is sometimes referred to as the quark mean field (QMF) model [56,57] or the modified quark-meson coupling (MQMC) model [58]. This quark model has the useful feature that all quantities of interest can be basically calculated analytically.

We first consider a dual quarkyonic model in an ideal Fermi gas. It is thus identical to the IdylliQ model, but the wavefunction is Gaussian, instead of the Yukawa-type one. We call this the Gaussian quarkyonic (GQ) model. This indicates that, since the Gaussian wavefunction is relativistic, due to the lower component, the quark saturation density turns out to be higher than in the case where a nonrelativistic wavefunction is used. In the GQ model, we can find that ρ_{sat} is never lower than ρ_0 .

In the GQ model, since it is naive, we encounter the following three problems. As previously stated, because the quark wavefunction is not Yukawa type, it is unavailable to use the differential equation derived in the IdylliQ model, and it is thus very hard to find the exact momentum distributions of quarks, f_Q , and nucleons, f_N , simultaneously. This is the first problem. However, from the discussion in Refs. [21,27], we can deem that Eq. (3) is still a good guess even in the GQ model. The second one is how the two characteristic momenta in the quarkyonic phase, k_b and k_s , should be determined. To settle this issue, we choose the same boundary condition as in the IdylliQ model, because it should be satisfied in any case, regardless of the quark wavefunction. Using those assumptions, we can estimate the nucleon momentum distribution, f_N . Once f_N is obtained, due to duality, we can calculate various physical quantities (energy density, chemical potential, pressure, sound velocity) over the wide range of ρ_N . However, as in the IdylliQ model, they show singular behavior at the quark saturation density, which should be remedied. This is the last problem. To control it, we introduce an infrared regulator, which may be regarded as an effective parametrization of the smearing of the Fermi surface caused by quark-exchange and gluonic interactions among neighboring nucleons. Then, we can obtain the physical quantities in symmetric nuclear and pure neutron matter, which behave continuously or smoothly at ρ_{sat} .

Next, we construct a dual quarkyonic model including the nuclear interaction, which is called the quarkyonic quark-meson coupling (QQMC) model, by unifying the GQ model and the QMC model. As explained previously, because the QMC model can be viewed as an extension of quantum hadrodynamics (QHD) to include the quark degrees of freedom, the physical quantities like the nuclear density, energy density, etc., can be expressed by those of a relativistic Fermi gas of nucleons with in-medium mass, M_N^* . Furthermore, both quark and nucleon degrees of freedom couple to the common mean scalar and vector fields. Thus, we assume that duality still holds in the QMC framework, and we construct the QQMC model in terms of the nucleon degrees of freedom only.

Here, it should be notable that the quark saturation density in matter, ρ_{sat}^* , becomes lower than the value in the GQ model,

because a nucleon in matter is swelled by the attractive, scalar field. However, we can confirm that ρ_{sat}^* is still larger than ρ_0 . This is important and indicates that the QQMC model below ρ_{sat}^* is entirely consistent with the present status of nuclear experiments at low energy. It is, however, remarkable that, in symmetric nuclear matter, ρ_{sat}^* is not so far from ρ_0 , i.e., $\rho_{\text{sat}}^* \simeq 1.5\rho_0$, when the nucleon radius in free space is 0.8 fm.

Finally, using the QQMC model, we again calculate the energy density, chemical potential, pressure, and sound velocity over the range $0 \leq \rho_N/\rho_0 < 5-10$. In particular, the QQMC model can provide the sound velocity that is consistent with that inferred from the observed neutron-star data by using the neural network model or Bayesian inference analysis. Furthermore, the pressure calculated by the QQMC model lies within the range deduced from the experimental flow data in heavy-ion collisions at high energy and the kaon production data. As the quark saturation density relies on the size of the nucleon, r_p , various physical quantities also depend on it accordingly. From the present consideration, as expected, the choice of $r_p = 0.6-0.8$ fm seems most suitable for describing dense nuclear matter in the QQMC model. The QQMC model provides a unified framework in which both quark-level Pauli blocking and medium modification of nucleon structure are incorporated simultaneously. This combination is essential for describing dense matter in the crossover region between hadronic and quark degrees of freedom.

Here is an outline. In Sec. II, using the Gaussian quark wavefunction, we first study the quarkyonic matter in an ideal Fermi gas. We discuss the feature of the Gaussian quarkyonic model, and calculate the energy density, chemical potential, pressure, and sound velocity in symmetric or pure neutron matter. Furthermore, we consider a minimal correction to the Gaussian quarkyonic model to remove the singular behavior at the quark saturation density. In Sec. III B, we briefly review the quark-meson coupling model with a harmonic oscillator potential, and build a new, quark-based nuclear model, in which we incorporate the QMC model and the Gaussian quarkyonic model. We then discuss the results of the present model in detail. Finally, in Sec. IV, we give our summary and discussion. In the Appendix, we apply a Sommerfeld expansion to the Gaussian quarkyonic model.

II. QUARKYONIC MATTER IN IDEAL FERMI GAS

In this section, we study a quarkyonic model with the relativistic, Gaussian quark wavefunction. We here neglect the nuclear interaction, and consider the effect of Pauli blocking at the quark level only. We call it the Gaussian quarkyonic (GQ) model. Using the GQ model, we calculate several physical quantities in symmetric nuclear matter (SNM) and in pure neutron matter (PNM).

A. Relativistic quark model of nucleon

We want to use a Gaussian function as the quark wavefunction in a nucleon, which is given by the relativistic confinement potential of the scalar-vector HO type,

$$U(r) = \frac{c}{2}(1 + \gamma_0)r^2, \quad (4)$$

with the strength parameter c . It is well known that the Dirac equation with the HO potential, Eq. (4), can be solved analytically [59–61]. In free space, the lowest-state solution for an isosymmetric ($Q = u = d$) quark is given by

$$\psi_Q(\mathbf{r}) = \frac{1}{\pi^{3/4}a^{3/2}} \sqrt{\frac{2\lambda^2 a^2}{2\lambda^2 a^2 + 3}} \left(i\vec{\sigma} \cdot \hat{\mathbf{r}} \frac{1}{\lambda a} \left(\frac{\mathbf{r}}{a} \right) \right) e^{-r^2/2a^2} \chi, \quad (5)$$

$$\lambda = \epsilon + m, \quad a^2 = \frac{1}{\sqrt{c\lambda}} = \frac{3}{\epsilon^2 - m^2}, \quad (6)$$

where m is the constituent quark mass, a is the width, and χ is the quark spinor. Here, the single-particle quark energy, ϵ , is determined by

$$\sqrt{\epsilon + m}(\epsilon - m) = 3\sqrt{c}. \quad (7)$$

Note that, in the limit $\lambda \rightarrow \infty$, this wavefunction turns out to be the nonrelativistic (NR) one.

The zero-order energy of the nucleon, E_N^0 , is then simply given by a sum of the quark energies, $E_N^0 = 3\epsilon$. Now we should take into account some corrections to the nucleon energy such as the spin correlations, E_N^{spin} , due to the quark-gluon and quark-pion interactions [62,63], and the center of mass (c.m.) correction, $E_N^{\text{c.m.}}$. Here we do not calculate the spin correlation explicitly, but we assume that it can be treated as a constant parameter which is fixed so as to reproduce the nucleon mass [56,57]. The c.m. correction to the spurious motion can be calculated analytically [61]:

$$E_N^{\text{c.m.}} = \frac{77\epsilon + 31m}{3a^2(3\epsilon + m)^2}. \quad (8)$$

The nucleon mass in vacuum is thus given by

$$M_N = E_N^0 + E_N^{\text{spin}} - E_N^{\text{c.m.}}, \quad (9)$$

which is equal to the proton or neutron mass ($M_N = M_p = M_n$), because any breaking of isospin symmetry is not included. We also find the root-mean-square charge radius of the proton including the c.m. correction as [61]

$$r_p \equiv \langle r^2 \rangle_p^{1/2} = \sqrt{\frac{11\epsilon + m}{(3\epsilon + m)(\epsilon^2 - m^2)}}. \quad (10)$$

In this quark model, we have three parameters, c , m , and E_N^{spin} . We then choose the quark mass as follows: case 1, $m = 250$ MeV; case 2, $m = 300$ MeV; and case 3, $m = 350$ MeV. The remaining two parameters, c and E_N^{spin} , are determined so as to reproduce the nucleon mass, $M_N = 939$ MeV, and the proton charge radius, $r_p = 0.6, 0.7, 0.8, 0.9$ fm. Note that the experimental value of the proton charge radius is 0.8409 ± 0.0004 fm [64], while r_p in the present calculation may correspond to the charge radius of valence quarks. In Table I, the parameters and the quark energies are summarized.

Using the Gaussian quark wavefunction, Eq. (5), the single quark momentum distribution in a single nucleon is calculated as

$$\varphi(\mathbf{q}) = \pi^{3/2}a^3 \left(\frac{16\lambda^2 a^2}{2\lambda^2 a^2 + 3} \right) \left(1 + \frac{q^2}{\lambda^2} \right) e^{-a^2 q^2}, \quad (11)$$

where the second term on the right-hand side comes from the lower component of the quark wavefunction. This satisfies the

TABLE I. Parameters in the quark model and the quark saturation density, ρ_{sat} . The nuclear saturation density is chosen to be $\rho_0 = 0.15 \text{ fm}^{-3}$. The quark saturation density is listed in the last two columns (see Sec. II B 3).

Case	m^a (MeV)	r_p^a (fm)	a (fm)	c (fm $^{-3}$)	ϵ	E_N^0 (MeV)	E_N^{spin} (MeV)	$E_N^{c.m.}$ (MeV)	ρ_{sat}/ρ_0	
									SNM	PNM
1	250	0.6	0.567	2.121	653.0	1959.1	-539.2	480.9	4.372	3.370
		0.7	0.665	1.230	571.8	1715.3	-382.5	393.8	2.648	2.042
		0.8	0.764	0.761	512.7	1538.2	-270.3	328.9	1.714	1.321
		0.9	0.863	0.496	468.4	1405.2	-187.4	278.8	1.167	0.900
2	300	0.6	0.570	1.922	670.2	2010.5	-616.3	455.2	4.175	3.219
		0.7	0.669	1.102	592.3	1776.8	-468.9	368.8	2.526	1.947
		0.8	0.769	0.675	536.3	1608.8	-365.0	304.8	1.634	1.260
		0.9	0.869	0.436	494.7	1484.0	-289.1	255.9	1.114	0.859
3	350	0.6	0.574	1.750	691.0	2072.9	-703.6	430.3	4.010	3.092
		0.7	0.673	0.993	616.6	1849.7	-565.6	345.1	2.426	1.871
		0.8	0.774	0.603	563.7	1691.1	-469.6	282.5	1.572	1.212
		0.9	0.874	0.387	524.8	1574.5	-400.3	235.2	1.073	0.828

^aInput.

normalization condition, $\int_q \varphi(q) = 1$. Throughout the present paper, we use this single quark momentum distribution.

B. Gaussian quarkyonic model

In the dual quarkyonic model, the nuclear density, ρ_N , and the energy density, ϵ_N , are expressed by either the nucleon, $f_N(k)$, or quark momentum distribution, $f_Q(q)$:

$$\rho_N = \gamma \int_k f_N(k) = \gamma \int_q f_Q(q), \quad (12)$$

$$\epsilon_N[f_N] = \epsilon_Q[f_Q], \quad \epsilon_N[f_N] = \gamma \int_k E_N(k) f_N(k),$$

$$\epsilon_Q[f_Q] = \gamma \int_q E_Q(q) [N_c f_Q(q)], \quad (13)$$

with $\gamma = 4$ (2) the spin-isospin degeneracy for SNM (PNM), E_N the nucleon energy, and E_Q the quark energy. This duality is ensured by the sum rule, Eq. (2). However, as explained in Sec. I, it is difficult to calculate the exact momentum distributions, $f_N(k)$ and $f_Q(q)$, because the quark wavefunction is chosen to be Gaussian. Instead, we determine the momentum distributions as in the following sections.

1. Nuclear density

The nuclear density, ρ_N , in SNM or PNM is expressed by

$$\rho_N = \frac{\gamma}{2\pi^2} \int dk k^2 f_N(k). \quad (14)$$

Here, we note that $f_N(k) = f_p(k) = f_n(k)$ for SNM, while $f_N(k) = f_n(k)$, $f_p(k) = 0$, for PNM, where $f_{p(n)}(k)$ is the momentum distribution for protons (neutrons). The nuclear density is also described in terms of the quark momentum distribution f_Q as in Eq. (12).

2. Presaturation region

Because, below the quark saturation density, the nuclear matter is normal, the momentum distribution in Eq. (14) is

simply given by the usual Fermi distribution

$$f_N(k) = \theta(k_s - k), \quad (15)$$

where $k_s = k_F$ (and k_F is the Fermi momentum).

In SNM, the dual momentum-space distribution for isosymmetric light quarks with a fixed color is expressed by the sum rule [see Eq. (2)]

$$f_Q(q, k_s) = \int_k \varphi\left(\mathbf{q} - \frac{\mathbf{k}}{N_c}\right) f_N(k), \quad (16)$$

which is less than unity in the presaturation region. Then, we can calculate f_Q analytically,

$$\begin{aligned} f_Q(q, k_s) = & \frac{N_c^3}{\sqrt{\pi}} \left[\text{Er}(\bar{q}) + \text{Er}(-\bar{q}) - \text{Er}(\bar{k}_s + \bar{q}) - \text{Er}(\bar{k}_s - \bar{q}) \right. \\ & + \frac{1}{2\bar{q}} \left(1 + \frac{2\bar{k}_s^2 + 1}{2\bar{\lambda}^2 + 3} \right) (e^{-(\bar{k}_s + \bar{q})^2} - e^{-(\bar{k}_s - \bar{q})^2}) \\ & \left. + \frac{\bar{k}_s}{2\bar{\lambda}^2 + 3} (e^{-(\bar{k}_s + \bar{q})^2} + e^{-(\bar{k}_s - \bar{q})^2}) \right], \end{aligned} \quad (17)$$

where $\bar{q} \equiv aq$, $\bar{\lambda} \equiv a\lambda$, and $\bar{k} \equiv ak/N_c$ are dimensionless variables. We here define the error function as

$$\text{Er}(x) \equiv \int_x^\infty dt e^{-t^2}. \quad (18)$$

In contrast, in PNM, there are only neutrons (udd) in matter, and so the saturation of d quarks first occurs because the u -quark levels are half-occupied compared with the d -quark levels. Therefore, the sum rule for PNM reads [25]

$$f_d(q, k_s) = B_d^n \int_k \varphi\left(\mathbf{q} - \frac{\mathbf{k}}{N_c}\right) f_n(k), \quad (19)$$

where the factor $B_d^n = 2/3$ is the baryon number of d quarks in a neutron. It can be calculated analytically as well.

3. At quark saturation density

At quark saturation density, the quark momentum distribution first reaches unity at $q = 0$. In SNM, the nucleon

momentum k_{sat} at ρ_{sat} is given by the saturation condition

$$g_Q(k_{\text{sat}}) = \frac{N_c^3}{\sqrt{\pi}} \left[\sqrt{\pi} - 2\text{Er}(\bar{k}_{\text{sat}}) - 2\bar{k}_{\text{sat}} \left(1 + \frac{2\bar{k}_{\text{sat}}^2}{2\bar{\lambda}^2 + 3} \right) e^{-\bar{k}_{\text{sat}}^2} \right] = 1, \quad (20)$$

where we define $g_Q(k) \equiv f_Q(0, k)$. Similarly, in PNM, the neutron momentum k_{sat} at ρ_{sat} is calculated by

$$g_d(k_{\text{sat}}) = 1, \quad (21)$$

with $g_d(k) \equiv f_d(0, k)$.

In the last two columns of Table I, we show the quark saturation densities in SNM and in PNM. In the NR limit, we find that, when $r_p = 0.8$ fm, $\rho_{\text{sat}}/\rho_0 \simeq 1.27$ in SNM. Here, we take $\rho_0 = 0.15$ fm⁻³. In contrast, when $m = 300$ MeV and $r_p = 0.8$ fm, the present relativistic calculation predicts $\rho_{\text{sat}}/\rho_0 \simeq 1.63$, which is larger than in the NR result. This discrepancy is caused by the lower component of the relativistic wavefunction, which enhances the higher-momentum part in the quark wavefunction, $\varphi(\mathbf{q})$. Furthermore, the quark saturation density in SNM is larger than that in PNM by about 30%. Such a tendency is seen in the IdylliQ model as well, although the difference in the IdylliQ model is tiny. We note that, in Ref. [29], using the transverse momentum densities for the valence and sea quarks in a nucleon, ρ_{sat}/ρ_0 is estimated to be about 1.13.

The quark saturation density strongly depends on the size of nucleon. This strong dependence originates from the width of the quark wavefunction, which controls the population of low-momentum quark modes and thus determines the onset of quark-level Pauli blocking. However, the dependence on the quark mass is approximately linear and weak; i.e., fixing r_p , the difference between ρ_{sat} in case 1 and that in case 3 is less than 10%. Hence, hereafter, we consider case 2, i.e., $m = 300$ MeV, only.

4. Postsaturation region

Above the quark saturation density, the low momentum levels of quarks are fully occupied, where the quarks behave like Fermi gas but still feel the confinement force, i.e., soft deconfinement. Such a region is denoted by $0 \leq q \leq q_b$, and we posit $f_{Q(d)}^{< q_b}(q) = \theta(q_b - q)$ for SNM (PNM). Correspondingly, the dual nucleon momentum distribution should be strongly suppressed by the quark Pauli blocking. As explained in Sec. I, Eq. (3) may be still a good guess even in the GQ model [27]. We thus assume that $f_N^{< k_b}(k) = \frac{1}{N_c^3} \theta(k_b - k)$ for SNM and $f_n^{< k_b}(k) = \frac{1}{B_d^3 N_c^3} \theta(k_b - k)$ for PNM, which are the underoccupied bulk parts ($0 \leq k \leq k_b = N_c q_b$). Here, the suppression factor, $1/N_c^3$, can be calculated by Eq. (2) with the wavefunction in the limit $a \rightarrow \infty$, because quarks in the bulk Fermi sea are free. Alternatively, this factor can be found through the discussion of phase space density [27].

However, for the higher-momentum states of quarks, i.e., $q > q_b$, the levels are not fully occupied, and thus the quark momentum distribution is expected to be expressed by $\theta(q - q_b) f_{Q(d)}^{> q_b}(q)$, where $f_{Q(d)}^{> q_b}(q = q_b) = 1$ and $f_{Q(d)}^{> q_b}(q)$ is a decreasing function from unity. Thus, we assume that

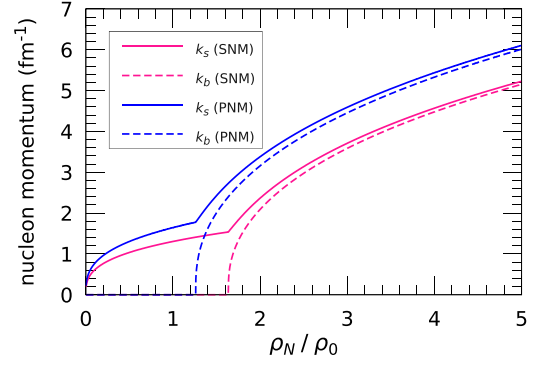


FIG. 1. Density dependence of k_s and k_b in the case of $r_p = 0.8$ fm.

$f_{Q(d)}^{> q_b}(q) = f_{Q(d)}(q - q_b, k_{\text{sat}})$, as discussed in Ref. [27]. In this region, because the Pauli blocking at the quark level does not work, the nucleon momentum distribution is not suppressed and it should form the ordinary shell structure, i.e., $f_N^{> k_b}(k) = \theta(k_s - k)\theta(k - k_b)$, where k_s is the upper bound given by ρ_N . It is thus expected that the nucleon momentum distributions for SNM and PNM can be expressed, respectively, as

$$f_N(k) = \frac{1}{N_c^3} \theta(k_b - k) + \theta(k_s - k)\theta(k - k_b), \quad (22)$$

$$f_n(k) = \frac{1}{B_d^3 N_c^3} \theta(k_b - k) + \theta(k_s - k)\theta(k - k_b). \quad (23)$$

Now, the problem is how we should determine the momenta at the boundary, k_b , and at the upper bound, k_s . Substituting the momentum distributions, Eqs. (22) and (23), into the sum rules, Eqs. (16) and (19), respectively, we can calculate the quark momentum distributions in the postsaturation region. Then, at $q = 0$, their values should be unity:

$$g_Q(k_s) - \beta g_Q(k_b) = 1 \quad \text{for SNM}, \quad (24)$$

$$g_d(k_s) - \beta' g_d(k_b) = 1 \quad \text{for PNM}, \quad (25)$$

with $\beta = 1 - 1/N_c^3$ and $\beta' = 1 - 1/(B_d^3 N_c^3)$. Note that, when $k_s = k_{\text{sat}}$, k_b vanishes. The boundary conditions, Eqs. (24) and (25), are the *necessary* conditions, and they should hold in any case, regardless of the quark wavefunction, $\varphi(\mathbf{q})$. Furthermore, they are identical to the condition that sets the relation between k_b and k_s in the IdylliQ model [24]. Thus, in this paper we posit these boundary conditions to determine k_b and k_s . In Fig. 1, we show the density dependence of k_b and k_s in SNM and in PNM. The rapid rise of the momenta, k_b and k_s , occurs beyond ρ_{sat} , which leads to singular behavior of thermodynamic quantities. Furthermore, in Fig. 2, we illustrate the quark and nucleon momentum distributions, $f_Q(q)$ and $f_N(k)$, in SNM.

C. Initial results

Using the GQ model, we evaluate the energy density, chemical potential, pressure, and sound velocity from the viewpoint of nucleon matter.

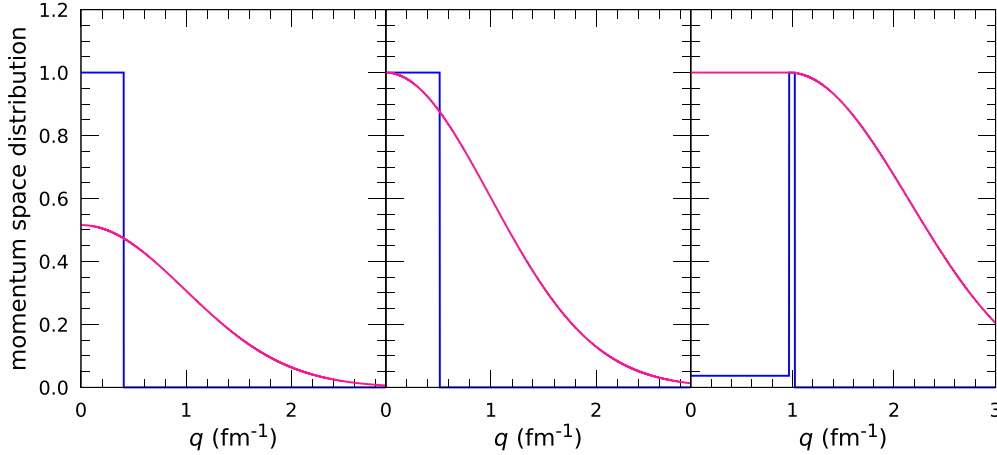


FIG. 2. Quark and nucleon momentum distributions in symmetric nuclear matter. We use the GQ model with $r_p = 0.8$ fm. The left-hand, middle, and right-hand panels are for $\rho_N/\rho_{\text{sat}} = 0.5, 1.0$, and 1.5 , respectively. The distributions, $f_Q(q)$ and $f_N(k)$ (with $k = qN_c$), are denoted by the red and blue lines, respectively.

1. Symmetric nuclear matter

First, the nuclear density, Eq. (14), reads

$$\rho_N^{\text{below}} = \frac{2}{3\pi^2} k_s^3 \quad (k_s = k_F), \quad (26)$$

$$\rho_N^{\text{above}} = \frac{2}{3\pi^2} (k_s^3 - \beta k_b^3), \quad (27)$$

where the superscripts "below" and "above" indicate $\rho_N < \rho_{\text{sat}}$ and $\rho_N > \rho_{\text{sat}}$, respectively. Because, in the limit $\rho_N \rightarrow \rho_{\text{sat}}$, k_b approaches zero, the density is continuous at ρ_{sat} . The derivative of the density with respect to k_s is then given by

$$\frac{\partial \rho_N^{\text{below}}}{\partial k_s} = \frac{2}{\pi^2} k_s^2, \quad (28)$$

$$\frac{\partial \rho_N^{\text{above}}}{\partial k_s} = \frac{2}{\pi^2} k_s^2 \left[1 - \beta \frac{k_b^2}{k_s^2} \frac{\partial k_b}{\partial k_s} \right] = \frac{2}{\pi^2} k_s^2 \left[1 - \frac{k_b^2}{k_s^2} \frac{\phi(k_s)}{\phi(k_b)} \right]. \quad (29)$$

Here, the derivative of k_b with respect to k_s is calculated by the boundary condition, Eq. (24), as

$$\frac{\partial k_b}{\partial k_s} = \frac{1}{\beta} \frac{\phi(k_s)}{\phi(k_b)} = \frac{1}{\beta} \frac{r_1(k_s) \bar{k}_s^2}{r_1(k_b) \bar{k}_b^2} e^{-\bar{k}_s^2 + \bar{k}_b^2}, \quad (30)$$

where we define

$$\phi(k) \equiv \frac{\partial}{\partial \bar{k}} g_Q(k) = \frac{N_c^3}{\sqrt{\pi}} [4r_1(k) \bar{k}^2] e^{-\bar{k}^2}, \quad (31)$$

with

$$r_n(k) = 1 - \frac{(2n+1) - 2\bar{k}^2}{2\bar{\lambda}^2 + 3}. \quad (32)$$

Note that $g_Q(k)$ and $\phi(k)$ are dimensionless.

In Eq. (29), the second term on the right-hand side has the factor k_b^2 coming from the Jacobian, but it is completely canceled by the factor k_b^{-2} in Eq. (30). This is an outstanding characteristic in the case of the Gaussian quark wavefunction. Thus, unfortunately, for chemical potential, pressure, or sound velocity, the continuity at ρ_{sat} does *not* hold.

We next calculate the energy density, ϵ_N . We find

$$\epsilon_N^{\text{below}} = \frac{2}{\pi^2} \int_0^{k_s} dk k^2 E_N(k) = \frac{2}{\pi^2} I(M_N, k_s), \quad (33)$$

$$\begin{aligned} \epsilon_N^{\text{above}} &= \frac{2}{\pi^2} \left(\int_0^{k_s} -\beta \int_0^{k_b} \right) dk k^2 E_N(k) \\ &= \frac{2}{\pi^2} [I(M_N, k_s) - \beta I(M_N, k_b)], \end{aligned} \quad (34)$$

with $E_N(k) = \sqrt{M_N^2 + k^2}$ and

$$\begin{aligned} I(M, x) &= \int_0^x dt t^2 \sqrt{M^2 + t^2} \\ &= \frac{1}{8} \left[x(M^2 + 2x^2) \sqrt{M^2 + x^2} \right. \\ &\quad \left. + M^4 \log \left(\frac{M}{x + \sqrt{M^2 + x^2}} \right) \right]. \end{aligned} \quad (35)$$

Therefore, the energy density is continuous at ρ_{sat} . However, the first derivatives are given by

$$\frac{\partial \epsilon_N^{\text{below}}}{\partial k_s} = \frac{2}{\pi^2} k_s^2 E_N(k_s), \quad (36)$$

$$\begin{aligned} \frac{\partial \epsilon_N^{\text{above}}}{\partial k_s} &= \frac{2}{\pi^2} k_s^2 E_N(k_s) \left[1 - \beta \frac{k_b^2 E_N(k_b)}{k_s^2 E_N(k_s)} \frac{\partial k_b}{\partial k_s} \right] \\ &= \frac{2}{\pi^2} k_s^2 E_N(k_s) \left[1 - \frac{k_b^2 E_N(k_b)}{k_s^2 E_N(k_s)} \frac{\phi(k_s)}{\phi(k_b)} \right]. \end{aligned} \quad (37)$$

So, the derivative has a gap at ρ_{sat} .

The chemical potential, $\mu_N = \partial \epsilon_N / \partial \rho_N$, is similarly evaluated as

$$\mu_N^{\text{below}} = E_N(k_s), \quad \frac{\partial \mu_N^{\text{below}}}{\partial k_s} = \frac{k_s}{E_N(k_s)}, \quad (38)$$

$$\mu_N^{\text{above}} = E_N(k_s) \frac{1 - \frac{k_b^2 E_N(k_b)}{k_s^2 E_N(k_s)} \frac{\phi(k_s)}{\phi(k_b)}}{1 - \frac{k_b^2}{k_s^2} \frac{\phi(k_s)}{\phi(k_b)}}. \quad (39)$$

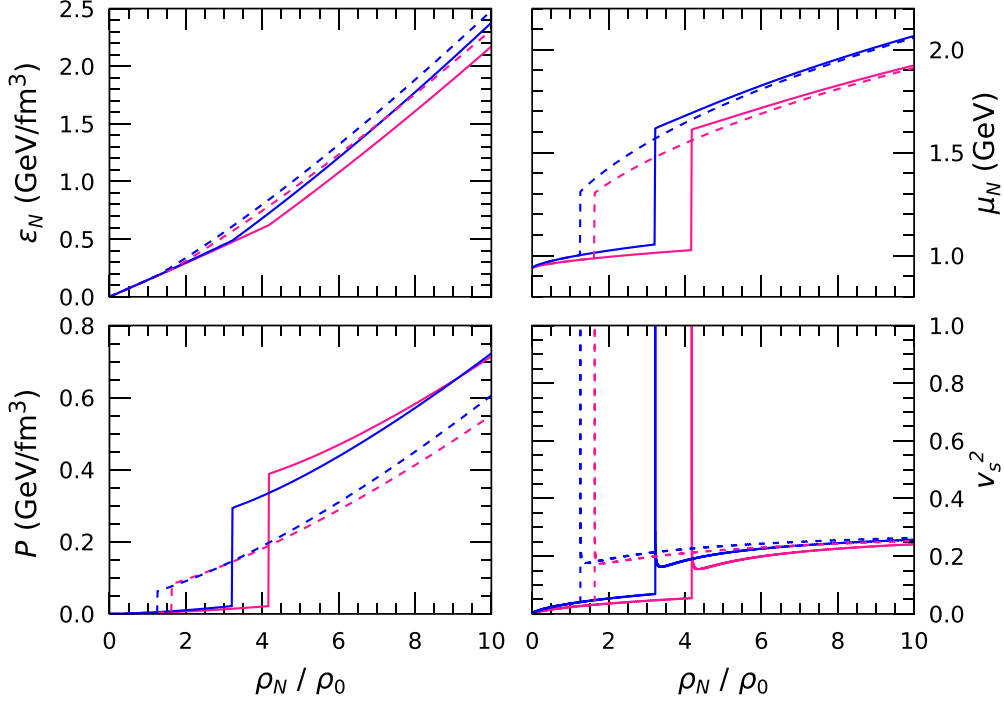


FIG. 3. Energy density, chemical potential, pressure, or sound velocity as a function of ρ_N . The solid and dashed red lines are respectively for $r_p = 0.6$ and 0.8 fm in SNM, while the solid and dashed blue lines are respectively for $r_p = 0.6$ and 0.8 fm in PNM.

The ratio of μ_N^{above} to μ_N^{below} gives the gap of chemical potential at ρ_{sat} , and it is

$$\frac{\mu_N^{\text{above}}}{\mu_N^{\text{below}}} = \frac{1 - \left[\frac{r_1(k_s)E_N(k_b)}{r_1(k_b)E_N(k_s)} \right] e^{-\bar{k}_s^2 + \bar{k}_b^2}}{1 - \left[\frac{r_1(k_s)}{r_1(k_b)} \right] e^{-\bar{k}_s^2 + \bar{k}_b^2}} > 1. \quad (40)$$

We can calculate pressure, P , by using ρ_N , ϵ_N , and μ_N :

$$P = \mu_N \rho_N - \epsilon_N. \quad (41)$$

Therefore, pressure has a gap at ρ_{sat} as well.

Finally, the (squared) sound velocity, v_s^2 , is estimated by

$$v_s^2 = \frac{\rho_N}{\mu_N} \left(\frac{\partial \mu_N}{\partial \rho_N} \right). \quad (42)$$

Below ρ_{sat} , the sound velocity reads

$$v_s^2{}^{\text{below}} = \frac{k_s^2}{3E_N(k_s)^2}. \quad (43)$$

To calculate $v_s^2{}^{\text{above}}$, we need the second derivative, $\partial^2 k_b / \partial k_s^2$, which is again derived from the boundary condition, Eq. (24). We then obtain

$$\frac{\partial^2 k_b}{\partial k_s^2} = \frac{1}{\beta} \left[\frac{\phi'(k_s)}{\phi(k_b)} - \beta \left(\frac{\partial k_b}{\partial k_s} \right)^2 \frac{\phi'(k_b)}{\phi(k_b)} \right] \xrightarrow{k_b \rightarrow 0} k_b^{-5}, \quad (44)$$

with

$$\phi'(k) = \frac{N_c^3}{\sqrt{\pi}} 8\bar{k} [r_1(k) - r_2(k)\bar{k}^2] e^{-\bar{k}^2}. \quad (45)$$

Thus, the sound velocity reads

$$v_s^2{}^{\text{above}} = \frac{\rho_N^{\text{above}}}{\mu_N^{\text{above}}} \left(\frac{\partial \mu_N^{\text{above}}}{\partial k_s} \right) / \left(\frac{\partial \rho_N^{\text{above}}}{\partial k_s} \right), \quad (46)$$

where

$$\frac{\partial \mu_N^{\text{above}}}{\partial k_s} = \mu_N^{\text{above}} \times \left(\frac{A'}{A} - \frac{B'}{B} \right), \quad (47)$$

with

$$A = k_s^2 E_N(k_s) - \beta k_b^2 E_N(k_b) \frac{\partial k_b}{\partial k_s}, \quad (48)$$

$$B = k_s^2 - \beta k_b^2 \frac{\partial k_b}{\partial k_s}, \quad (49)$$

$$A' = 2k_s E_N(k_s) + \frac{k_s^3}{E_N(k_s)} - \beta \left[k_b^2 E_N(k_b) \frac{\partial^2 k_b}{\partial k_s^2} + \left(2k_b E_N(k_b) + \frac{k_b^3}{E_N(k_b)} \right) \left(\frac{\partial k_b}{\partial k_s} \right)^2 \right], \quad (50)$$

$$B' = 2k_s - \beta \left[k_b^2 \frac{\partial^2 k_b}{\partial k_s^2} + 2k_b \left(\frac{\partial k_b}{\partial k_s} \right)^2 \right]. \quad (51)$$

In Fig. 3, we show ϵ_N , μ_N , P , and v_s^2 in the case of $r_p = 0.6$ or 0.8 fm. At ρ_{sat} , the energy density is continuous but is not smooth, while there is a large gap in the chemical potential or pressure, which may not be permitted in thermodynamics. The sound velocity diverges at ρ_{sat} , as seen in the IdylliQ model.

2. Pure neutron matter

For PNM, the neutron density reads

$$\rho_n^{\text{below}} = \frac{1}{3\pi^2} k_s^3 \quad (k_s = k_F), \quad (52)$$

$$\rho_n^{\text{above}} = \frac{1}{3\pi^2} (k_s^3 - \beta' k_b^3), \quad (53)$$

and the energy density is

$$\epsilon_n^{\text{below}} = \frac{1}{\pi^2} \int_0^{k_s} dk k^2 E_n(k) = \frac{1}{\pi^2} I(M_N, k_s), \quad (54)$$

$$\begin{aligned} \epsilon_n^{\text{above}} &= \frac{1}{\pi^2} \left(\int_0^{k_s} -\beta' \int_0^{k_b} \right) dk k^2 E_n(k) \\ &= \frac{1}{\pi^2} [I(M_N, k_s) - \beta' I(M_N, k_b)], \end{aligned} \quad (55)$$

with $E_n(k) = \sqrt{M_N^2 + k^2}$. Then, replacing N_c^3 with $\frac{2}{3}N_c^3$, we can calculate μ_n , P , and v_s^2 as in the case of SNM. Figure 3 depicts ϵ_n , μ_n , P , and v_s^2 in PNM as well.

D. Gaussian quarkyonic model with minimal corrections

As seen in Sec. II C, we have encountered the singular behavior of physical quantities in the naïve GQ model. We thus need to somehow remedy such behavior. Because the cause of the gap and divergence at the quark saturation density comes from $\phi(k_b)$ in Eq. (30), which is related to $\partial g_{Q(d)}(k_b)/\partial k_b$, a minimal correction to cure this problem can be provided with a smearing of the sharp Fermi surface, $\theta(k_b - k)$, in the nucleon distributions, Eqs. (22) and (23). Such a smearing should be originally evaluated by the quark and gluon exchanges among nucleons, but it is extremely difficult to handle them in a multibaryon system. Instead, for example, we could apply a Sommerfeld expansion to blur the Fermi surface phenomenologically (for details, see the Appendix).

In this paper, however, we take another, more practical prescription to remove the singular behavior in the present model. Here, we want to modify k_b^2 in the denominator of Eq. (30) so that the power of k_b is weaker. In the method of the Sommerfeld expansion, the power of k_b is eventually weakened (see the Appendix). To do so, we introduce an infrared regulator to the boundary conditions, Eqs. (24) and (25). Then, we take the simplest choice,

$$\alpha_v(k_b) = 1 + \left(\frac{w}{k_b}\right)^\nu \rightarrow \infty (1) \text{ as } k_b \rightarrow 0 (\infty), \quad (56)$$

with w a width parameter and ν a positive, real number. The boundary conditions are thus redefined by

$$g_Q(k_s) - \beta \alpha_v(k_b) g_Q(k_b) = 1 \quad \text{for SNM}, \quad (57)$$

$$g_d(k_s) - \beta' \alpha_v(k_b) g_d(k_b) = 1 \quad \text{for PNM}. \quad (58)$$

Note that, when $w = 0$, $\alpha_v = 1$ (without the regulator). Because $g_{Q(d)}(k_b)$ behaves like $\mathcal{O}(k_b^3)$ when $k_b \rightarrow 0$, the parameter ν should be taken as $0 < \nu < 3$. Furthermore,

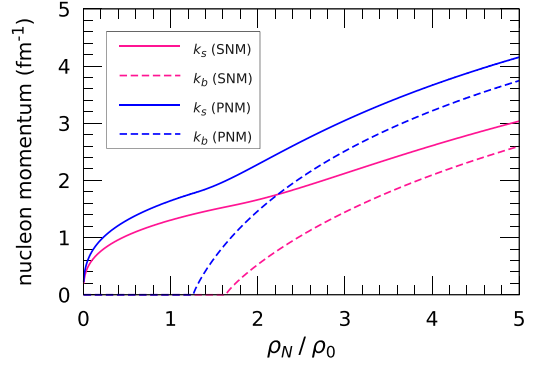


FIG. 4. Density dependence of k_s and k_b in the case of $r_p = 0.8$ fm, $\nu = 1.8$, and $w = 0.25$ GeV.

we find

$$\alpha'_v(k_b) \equiv \frac{d}{dk_b} \alpha_v(k_b) = -\frac{\nu}{\bar{w}} \left(\frac{w}{k_b}\right)^{\nu+1}, \quad (59)$$

$$\alpha''_v(k_b) \equiv \frac{d^2}{dk_b^2} \alpha_v(k_b) = \frac{\nu(\nu+1)}{\bar{w}^2} \left(\frac{w}{k_b}\right)^{\nu+2}, \quad (60)$$

where $\bar{w} (\equiv aw/N_c)$, α_v , α'_v , and α''_v are all dimensionless. Figure 4 depicts the momenta, k_b and k_s , calculated with the modified boundary conditions, Eqs. (57) and (58). We notice that the momenta increase gradually beyond the quark saturation density, compared with the result without the regulator.

In the case of SNM, using the modified boundary condition, the first derivative, $\partial k_b / \partial k_s$, reads

$$\frac{\partial \bar{k}_b}{\partial \bar{k}_s} = \frac{\partial k_b}{\partial k_s} = \frac{\phi(k_s)}{\beta[\alpha'_v(k_b)g_Q(k_b) + \alpha_v(k_b)\phi(k_b)]} \xrightarrow{k_b \rightarrow 0} k_b^{\nu-2}, \quad (61)$$

which ensures that the physical quantities (except v_s^2) are continuous at the quark saturation density. Similarly, using the modified boundary condition again, the second derivative reads

$$\begin{aligned} \frac{\partial^2 \bar{k}_b}{\partial \bar{k}_s^2} &= \left(\frac{N_c}{a}\right) \frac{\partial^2 k_b}{\partial k_s^2} \\ &= \frac{\phi'(k_s) - \beta \left(\frac{\partial \bar{k}_b}{\partial \bar{k}_s}\right)^2 X(k_b)}{\beta[\alpha'_v(k_b)g_Q(k_b) + \alpha_v(k_b)\phi(k_b)]} \xrightarrow{k_b \rightarrow 0} k_b^{2\nu-5}, \end{aligned} \quad (62)$$

with

$$\begin{aligned} X(k_b) &\equiv \frac{\partial^2}{\partial k_b^2} \alpha_v(k_b) g_Q(k_b) \\ &= \alpha''_v(k_b) g_Q(k_b) + 2\alpha'_v(k_b) \phi(k_b) + \alpha_v(k_b) \phi'(k_b). \end{aligned} \quad (63)$$

Therefore, to make the sound velocity continuous and make μ_N and P smooth at ρ_{sat} , the parameter, ν , is further restricted to be $\frac{3}{2} < \nu < 3$ [see Eqs. (50) and (51)]. For PNM, it is necessary to replace β with β' . Now, we can calculate ϵ_N , μ_N , P , and v_s^2 , as in Sec. II C, replacing $\partial k_b / \partial k_s$ and $\partial^2 k_b / \partial k_s^2$ with the modified ones. In Fig. 5, we present the result in SNM and PNM. As expected, ϵ_N , μ_N , P , and v_s^2 are all continuous, and ϵ_N , μ_N , and P are smooth at ρ_{sat} .

Furthermore, Figs. 6 and 7 respectively show the dependence of μ_N and v_s^2 on ν and w in detail. It is seen that, in the chemical potential, as the width parameter, w , is larger,

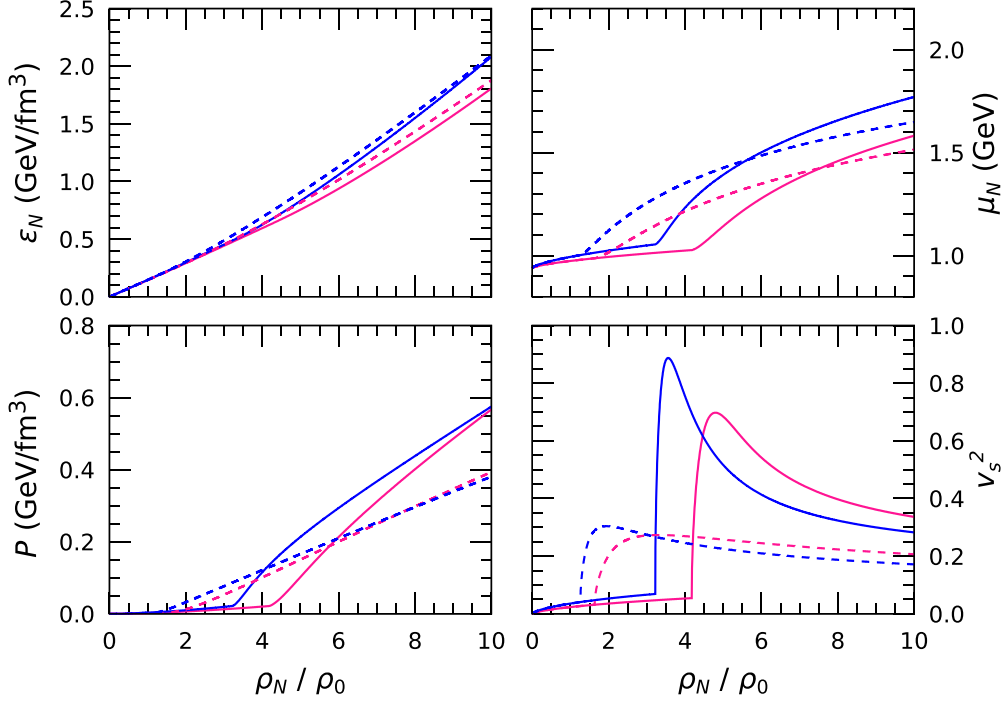


FIG. 5. Energy density, chemical potential, pressure, and sound velocity in the case of $v = 1.8$ and $w = 0.25$ GeV. The solid and dashed red lines are respectively for $r_p = 0.6$ and 0.8 fm in SNM, while the solid and dashed blue lines are respectively for $r_p = 0.6$ and 0.8 fm in PNM.

μ_N beyond ρ_{sat} is more suppressed, and that the larger power, v , tames the rapid rise at ρ_{sat} more. We can see a similar trend in v_s^2 as well. The sound velocity with $w = 0.1$ GeV does not diverge at ρ_{sat} , but overshoots the upper limit. The same occurs in the case of $r_p = 0.6$ fm and $w = 0.2$ GeV. The parameter, w , broadens the peak of the sound velocity, while v enhances the height of the peak and hinders the rapid increase of the velocity around ρ_{sat} .

III. QUARKYONIC QUARK-MESON COUPLING MODEL

We here build a new, quark-based nuclear model, in which we incorporate the GQ model into the QMC model. The two models are combined with each other through the Gaussian quark wavefunction, Eq. (5). We call it the quarkyonic quark-meson coupling (QQMC) model.

A. Brief review of the quark-meson coupling model

In this section, we briefly review the quark-meson coupling (QMC) model with the Gaussian quark wavefunction.

We consider the mean fields of σ , ω , and ρ mesons, which interact with the confined quarks, in uniformly distributed, asymmetric nuclear matter. Let the mean-field values for the σ , ω (the time component), and ρ (the time component in the third direction of isospin) fields be $\bar{\sigma}$, $\bar{\omega}$, and $\bar{\rho}$, respectively.

Under the relativistic HO potential, Eq. (4), the Dirac equation for the quark field ψ_j ($j = u$ or d) is given by [61]

$$\left[i\gamma \cdot \partial - (m - V_s) - \gamma_0 V_0 - \frac{c}{2}(1 + \gamma_0)r^2 \right] \psi_j(\mathbf{r}, t) = 0, \quad (64)$$

where $V_s = g_\sigma^q \bar{\sigma}$ and $V_0 = g_\omega^q \bar{\omega} + \tau_{3j} g_\rho^q \bar{\rho}$ [$\tau_{3j} = \pm 1$ for (u) quark] with the quark-meson coupling constants, g_σ^q , g_ω^q , and

g_ρ^q . Note that the isoscalar σ meson couples to the u and d quarks equally. We respectively define the effective quark mass and the effective single-particle quark energy as $m^* \equiv m - V_s = m - g_\sigma^q \bar{\sigma}$ and $\epsilon^* \equiv \epsilon_j - V_0 = \epsilon_j - g_\omega^q \bar{\omega} \mp g_\rho^q \bar{\rho}$ for (u) quark, where ϵ_j is the eigenenergy of Eq. (64). The static, lowest-state wavefunction in matter is presented by

$$\psi_j(\mathbf{r}, t) = \exp[-i\epsilon_j t] \psi_Q(\mathbf{r}). \quad (65)$$

The wavefunction $\psi_Q(\mathbf{r})$ is then given by Eqs. (5) and (6), in which ϵ , m , λ , and a are respectively replaced with ϵ^* , m^* , λ^* , and a^* , and the effective energy ϵ^* is determined by $\sqrt{\epsilon^* + m^*}(\epsilon^* - m^*) = 3\sqrt{c}$ [see Eq. (7)]. We assume that the strength parameter, c , of the confinement potential does not change in matter.

The zero-order energy of the nucleon in matter is thus given by $E_N^{0*} = 3\epsilon^*$, and we have the effective nucleon mass in matter ($M_N^* = M_p^* = M_n^*$),

$$M_N^* = E_B^{0*} + E_N^{spin} - E_N^{c.m.*}. \quad (66)$$

Here, we assume that the spin correlation E_N^{spin} does not change in matter, and the c.m. correction $E_N^{c.m.*}$ is given by Eq. (8) with ϵ^* and m^* , instead of ϵ and m .

For describing asymmetric nuclear matter, we now start from the following Lagrangian density in the mean-field approximation [36],

$$\mathcal{L} = \bar{\psi}_N [i\gamma \cdot \partial - M_N^*(\bar{\sigma}) - g_\omega \gamma_0 \bar{\omega} - g_\rho \gamma_0 \tau_3 \bar{\rho}] \psi_N - \frac{m_\sigma^2}{2} \bar{\sigma}^2 + \frac{m_\omega^2}{2} \bar{\omega}^2 + \frac{m_\rho^2}{2} \bar{\rho}^2 - \frac{g_2}{3} \bar{\sigma}^3, \quad (67)$$

with ψ_N the nucleon field, and τ_3 the third component of the Pauli matrix. The nucleon-meson coupling constants, g_σ , g_ω ,

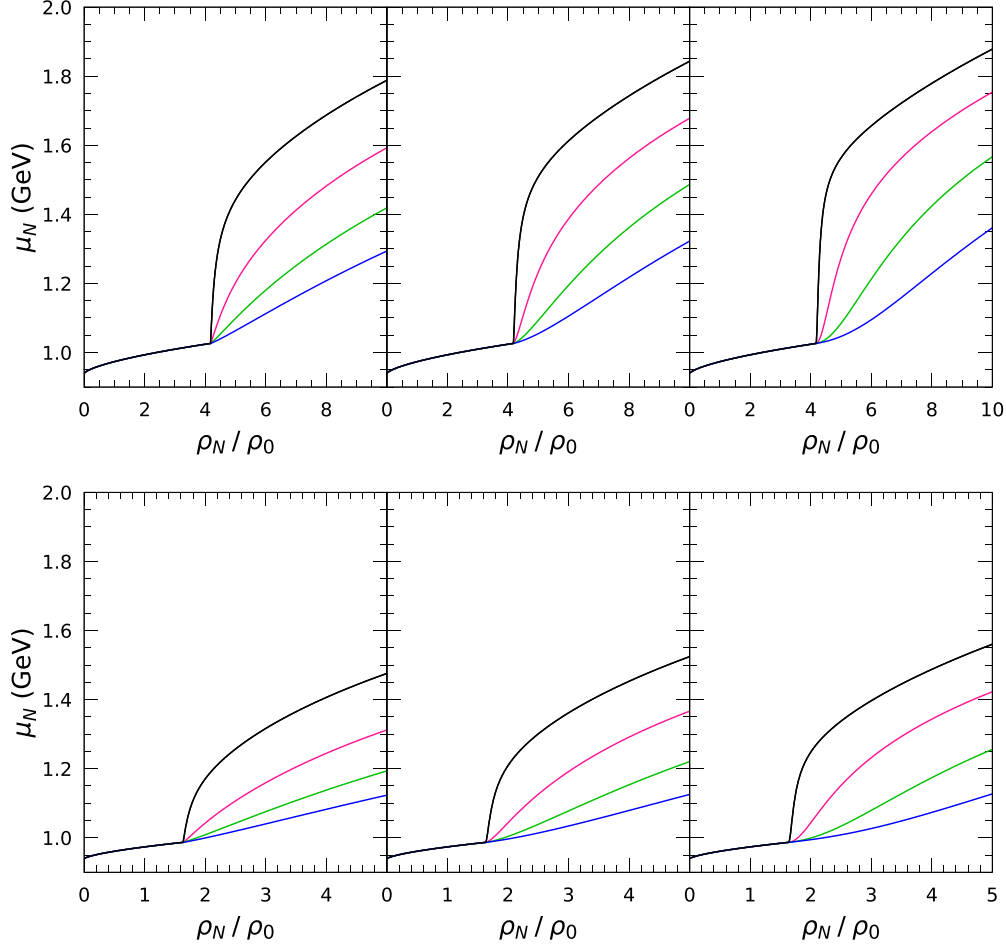


FIG. 6. Dependence of chemical potential on ν and w in SNM. Top, $r_p = 0.6$ fm; bottom, $r_p = 0.8$ fm. The left-hand, middle, and right-hand panels are for $\nu = 1.6, 1.8$, and 2.0 , respectively. The black, red, green, and blue lines denote $w = 0.1, 0.2, 0.3$, and 0.4 GeV, respectively.

and g_ρ , are respectively related to the quark-meson coupling constants as $g_\sigma = 3g_\sigma^q$, $g_\omega = 3g_\omega^q$, and $g_\rho = g_\rho^q$. The meson masses are taken to be $m_\sigma = 550$ MeV, $m_\omega = 783$ MeV, and $m_\rho = 770$ MeV. We add the last term to the Lagrangian, which is the nonlinear, self-coupling term of the σ meson, in order to reproduce the properties of nuclear matter as discussed later. Here, we do not include the nonlinear term $\frac{1}{4}g_2\sigma^4$, because it plays similar roles as $\frac{1}{3}g_2\sigma^3$.

The total energy per nucleon of asymmetric nuclear matter is then obtained by

$$\begin{aligned} \epsilon_{tot}/\rho_N = & \epsilon_N^*/\rho_N + g_\omega\bar{\omega} + g_\rho\left(\frac{\rho_3}{\rho_N}\right)\bar{\rho} \\ & + \frac{1}{2\rho_N}(m_\sigma^2\bar{\sigma}^2 - m_\omega^2\bar{\omega}^2 - m_\rho^2\bar{\rho}^2) + \frac{1}{3\rho_N}g_2\bar{\sigma}^3, \end{aligned} \quad (68)$$

where

$$\epsilon_N^* = \frac{1}{\pi^2} \int dk k^2 E_N^*(k) [f_p(k) + f_n(k)], \quad (69)$$

with $E_N^*(k) = \sqrt{M_N^{*2} + k^2}$. Note that $f_{i=p,n}(k) = \theta(k_{F_i} - k)$ with k_{F_i} the Fermi momentum for protons or neutrons, which

is related to the density of protons (neutrons), $\rho_{p(n)}$, through $\rho_{p(n)} = k_{F_{p(n)}}^3/(3\pi^2)$. The total nucleon density is given by $\rho_N = \rho_p + \rho_n$, and the difference in proton and neutron densities is defined by $\rho_3 \equiv \rho_p - \rho_n$. The binding energy per nucleon, ϵ_b , is given by $\epsilon_b(\rho_N) = \epsilon_{tot}/\rho_N - M_N$.

The mean-field values of the meson fields satisfy [61]

$$(m_\sigma^2 + g_2\bar{\sigma})\bar{\sigma} = -\left(\frac{\partial M_N^*}{\partial \bar{\sigma}}\right)(\rho_p^s + \rho_n^s) \equiv g_\sigma G_\sigma(\bar{\sigma})(\rho_p^s + \rho_n^s), \quad (70)$$

$$m_\omega^2\bar{\omega} = g_\omega\rho_N, \quad (71)$$

$$m_\rho^2\bar{\rho} = g_\rho\rho_3, \quad (72)$$

where Eq. (70) is the gap equation for the nucleon mass in matter. Here, $\rho_{i=p(n)}^s$ is the scalar density of protons (neutrons) in matter,

$$\rho_i^s = \frac{1}{\pi^2} \int dk k^2 \frac{M_N^*}{\sqrt{M_N^{*2} + k^2}} f_i(k) = \frac{M_N^*}{\pi^2} J(M_N^*, k_{F_i}), \quad (73)$$

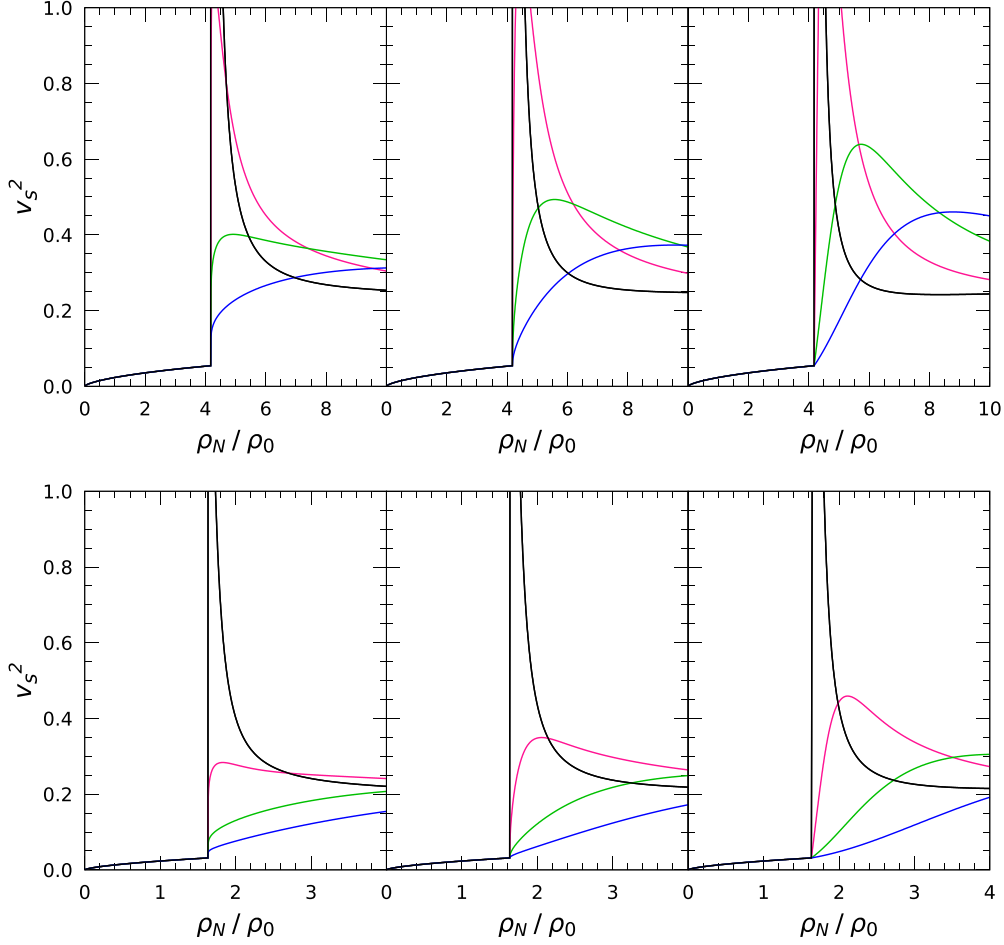


FIG. 7. Dependence of sound velocity on v and w in SNM. Top, $r_p = 0.6$ fm; bottom, $r_p = 0.8$ fm. The left-hand, middle, and right-hand panels are for $v = 1.6, 1.8$, and 2.0 , respectively. The black, red, green, and blue lines denote $w = 0.1, 0.2, 0.3$, and 0.4 GeV, respectively.

with

$$J(M, x) = \int_0^x dt \frac{t^2}{\sqrt{M^2 + t^2}} = \frac{1}{2} \left[x\sqrt{M^2 + x^2} + M^2 \log \left(\frac{M}{x + \sqrt{M^2 + x^2}} \right) \right]. \quad (74)$$

Furthermore, $G_\sigma(\bar{\sigma})$ is the isoscalar, Lorentz scalar polarizability [61]

$$G_\sigma(\bar{\sigma}) \equiv S^*(\bar{\sigma}) - \frac{1}{3} \frac{\partial}{\partial m^*} E_N^{c.m.*}, \quad (75)$$

where

$$S^*(\bar{\sigma}) = \int_N d\mathbf{r} \bar{\psi}_Q(\mathbf{r}) \psi_Q(\mathbf{r}) = \frac{\partial \epsilon^*}{\partial m^*} = \frac{\epsilon^* + 3m^*}{3\epsilon^* + m^*}, \quad (76)$$

$$\frac{1}{3} \frac{\partial}{\partial m^*} E_N^{c.m.*} = \frac{1}{27} \left[\frac{2(\epsilon^* S^* - m^*)(77\epsilon^* + 31m^*)}{(3\epsilon^* + m^*)^2} + \frac{(\epsilon^{*2} - m^{*2})(77S^* + 31)}{(3\epsilon^* + m^*)^2} - 2 \frac{(\epsilon^{*2} - m^{*2})(77\epsilon^* + 31m^*)(3S^* + 1)}{(3\epsilon^* + m^*)^3} \right]. \quad (77)$$

Now, we have four parameters, g_σ , g_ω , g_ρ , and g_2 . The coupling constants, g_σ and g_ω , are determined so as to fit the binding energy, $\epsilon_b(\rho_0) = -16$ MeV, at the nuclear saturation density in equilibrium, symmetric nuclear matter. We fit the ρ -nucleon coupling constant so as to reproduce the bulk symmetry energy of symmetric nuclear matter, $E_{sym}(\rho_0) = 32.0$ MeV. Finally, the nonlinear coupling constant, g_2 , is used to fit the nuclear incompressibility, $K_0 = 240$ MeV.

The coupling constants and the properties of nucleons and nuclear matter in the case 2 are, respectively, listed in Tables II and III. At ρ_0 , the root-mean-square charge radius of a proton, r_p^* , increases by about 5% compared with the radius in vacuum, r_p . This swelling is consistent with the experimental data (see Ref. [41]). We also calculate the slope and the curvature

TABLE II. Coupling constants in case 2. For details, see the text.

r_p (fm)	g_σ	g_ω	g_ρ	g_2 (fm ⁻¹)
0.6	11.00	7.47	4.36	22.78
0.7	10.49	7.56	4.36	23.81
0.8	10.11	7.65	4.35	24.65
0.9	9.83	7.74	4.35	25.32

TABLE III. Properties of in-medium nucleons and symmetric nuclear matter at the nuclear saturation density, ρ_0 , in case 2. Here, L and K_{sym} are respectively the slope and the curvature parameters of nuclear symmetry energy. The quark saturation densities in matter are listed in the last two columns (see Sec. III B).

r_p	r_p^*	m^*	ϵ^*	M_N^*	E_N^{0*}	$E_N^{c.m.*}$	L	K_{sym}	$\rho_{\text{sat}}^*/\rho_0$	
(fm)					(MeV)				SNM	PNM
0.6	0.627	223.5	620.4	776.5	1861.2	468.4	86.4	-18.1	3.594	2.826
0.7	0.733	226.7	541.6	773.8	1624.9	382.2	86.5	-17.5	2.212	1.740
0.8	0.840	228.9	484.7	771.1	1454.1	318.0	86.6	-16.9	1.459	1.146
0.9	0.946	230.5	442.1	768.6	1326.4	268.7	86.7	-16.3	1.012	0.794

parameters of nuclear symmetry energy, L and K_{sym} , which are related to astrophysical multimessenger observations of neutron stars [65,66].

The binding energy per nucleon and pressure are shown in Fig. 8. Together with the results of the QMC model, those of the QQMC model (see Sec. III B) are illustrated in the figure as well. Figure 9 illustrates the mean-field value for the σ meson in SNM. The increase of the scalar field becomes slower at high density, compared with that at low density, which generates the nuclear saturation with a moderate value of incompressibility, K_0 [34]. In Fig. 10, we present the density dependence of the effective quark mass and that of the effective nucleon mass in SNM. Figure 11 depicts the scalar polarizability, G_σ . In matter, the quark mass is lighter, i.e., *more relativistic*, than the mass in vacuum. Therefore, the lower component in the quark wavefunction is enhanced, and the quark scalar density in a nucleon in matter is more reduced, i.e., $G_\sigma(0) > G_\sigma(\bar{\sigma})$, which softens the incompressibility, K_0 .

B. Quarkyonic quark-meson coupling model

In this section, we construct the QQMC model for symmetric nuclear matter and pure neutron matter, in which the nuclear interaction is involved at the level of the mean-field approximation. Furthermore, in the QMC model, as in QHD, the physical quantities such as the nuclear density, the energy density, etc., in matter can be expressed by those of a relativistic Fermi gas of nucleons of mass M^* and both quark and nucleon degrees of freedom couple to the same mean

scalar and vector fields. This implies that duality remains valid below the onset of hard deconfinement. Thus, we assume duality and formulate the QQMC model in terms of the nucleon degrees of freedom only.

First, the quark saturation density, ρ_{sat}^* , in SNM is determined by Eq. (20) with m^* , ϵ^* , and a^* , instead of m , ϵ , and a :

$$g_Q^*(k_{\text{sat}}^*) = \frac{N_c^3}{\sqrt{\pi}} \left[\sqrt{\pi} - 2\text{Er}(\bar{k}_{\text{sat}}^*) - 2\bar{k}_{\text{sat}}^* \left(1 + \frac{2\bar{k}_{\text{sat}}^{*2}}{2\bar{\lambda}^{*2} + 3} \right) e^{-\bar{k}_{\text{sat}}^{*2}} \right] = 1, \quad (78)$$

with $\bar{\lambda}^* = a^*\lambda^*$ and $\bar{k}^* = a^*k^*/N_c$ dimensionless variables. Then, we find $\rho_{\text{sat}}^* = 2\bar{k}_{\text{sat}}^{*3}/(3\pi^2)$, which is smaller than ρ_{sat} in the GQ model, because the quark wavefunction in matter is more spread out than in free space, i.e., $a^* > a$. For PNM, Eq. (21) with m^* , ϵ^* , and a^* provides the quark saturation density. In the last two columns of Table III, the quark saturation density is summarized. In fact, ρ_{sat}^* is roughly 10% smaller than ρ_{sat} .

Once the quark saturation density is found, we can realize the model in the pre- and postsaturation regions as in Sec. IID. In the presaturation region, i.e., $\rho_N < \rho_{\text{sat}}^*$, the matter can be well described by the QMC model itself.

In the postsaturation region, i.e., $\rho_N > \rho_{\text{sat}}^*$, we introduce the depletion phase in the nucleon momentum distribution. The boundary for the unoccupied bulk part, k_b^* , and the upper bound of the nucleon momentum, k_s^* , in SNM are evaluated by Eq. (27), in which k_b and k_s are respectively replaced with

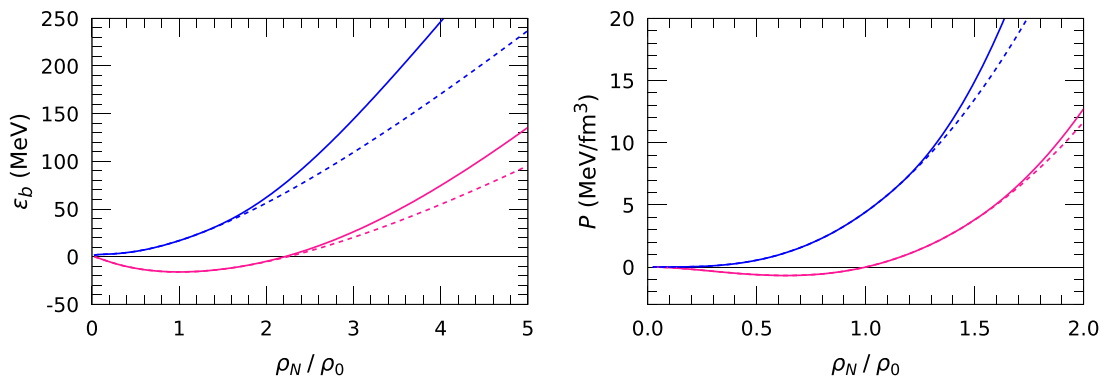


FIG. 8. Binding energy per nucleon, ϵ_b , and pressure, P , in the case of $r_p = 0.8$ fm, $v = 2.0$, and $w = 0.4$ GeV. The red solid (dashed) lines denote the results of the QQMC (QMC) model in SNM, while the blue solid (dashed) lines denote those of the QQMC (QMC) model in PNM.

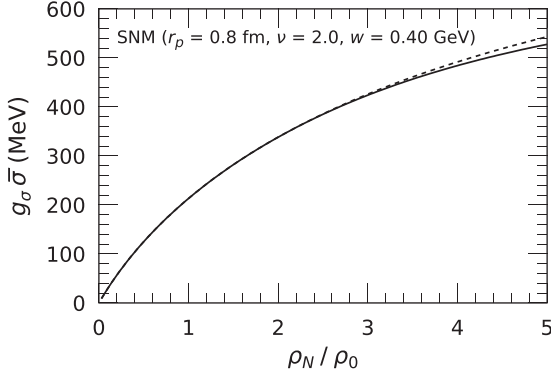


FIG. 9. Density dependence of the sigma field, $g_\sigma \bar{\sigma}$, in the case of $r_p = 0.8$ fm. The solid (dashed) line shows the result of the QQMC (QMC) model.

k_b^* and k_s^* , and by the modified boundary condition, Eq. (57) with k_b^* and k_s^* :

$$g_Q^*(k_s^*) - \beta \alpha_v(k_b^*) g_Q^*(k_b^*) = 1. \quad (79)$$

For PNM, Eqs. (53) and (58) with m^* , ϵ^* , and a^* similarly generate the two momenta, k_b^* and k_s^* . Figure 12 depicts the density dependence of k_b^* and k_s^* . In the calculation of the QQMC model, as discussed in the end of this section, we adjust the parameters in the regulator so as to reproduce the sound velocity inferred from the observed data of neutron stars and pressure determined by the experiment of heavy-ion collisions (HICs) at high energy. We here take $v = 2.0$ and $w = 0.4$ GeV. By choosing the large values of the parameters, the rapid rise of the momenta beyond ρ_{sat} can be completely controlled.

Using Eqs. (22) and (23) with k_b^* and k_s^* , the total energy per nucleon in matter is given by Eq. (68) with

$$\epsilon_N^* = \frac{2}{\pi^2} [I(M_N^*, k_s^*) - \beta I(M_N^*, k_b^*)] \quad \text{for SNM}, \quad (80)$$

$$\epsilon_N^* = \frac{1}{\pi^2} [I(M_N^*, k_s^*) - \beta' I(M_N^*, k_b^*)] \quad \text{for PNM}. \quad (81)$$

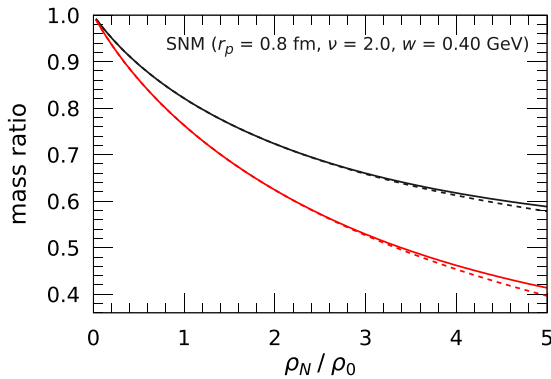


FIG. 10. Density dependence of mass ratio, M_N^*/M_N (black lines) or m^*/m (red lines), in the case of $r_p = 0.8$ fm. The solid (dashed) lines show the results of the QQMC (QMC) model.

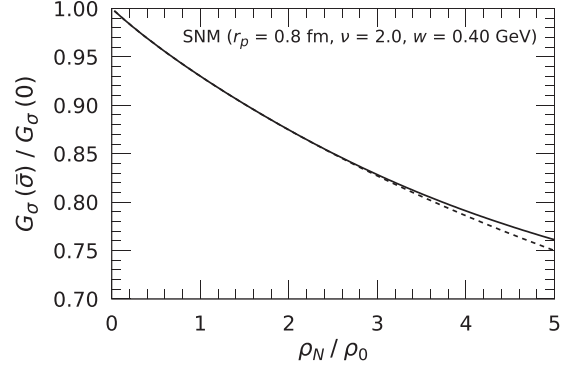


FIG. 11. Density dependence of the scalar polarizability, $G_\sigma(\bar{\sigma})/G_\sigma(0)$, in the case of $r_p = 0.8$ fm. The solid (dashed) line denotes the result of the QQMC (QMC) model.

For the mean field of the σ meson, Eq. (70) turns out to be

$$\begin{aligned} & (m_\sigma^2 + g_2 \bar{\sigma}) \bar{\sigma} \\ &= g_\sigma G_\sigma(\bar{\sigma}) \left(\frac{2M_N^*}{\pi^2} \right) [J(M_N^*, k_s^*) - \beta J(M_N^*, k_b^*)] \quad \text{for SNM}, \end{aligned} \quad (82)$$

$$\begin{aligned} & (m_\sigma^2 + g_2 \bar{\sigma}) \bar{\sigma} \\ &= g_\sigma G_\sigma(\bar{\sigma}) \left(\frac{M_N^*}{\pi^2} \right) [J(M_N^*, k_s^*) - \beta' J(M_N^*, k_b^*)] \quad \text{for PNM}. \end{aligned} \quad (83)$$

The mean field of the ω meson in SNM is calculated by Eq. (71), in which ρ_N is given by Eq. (27) with k_b^* and k_s^* , instead of k_b and k_s . In the case of PNM, the value of $\bar{\omega}$ is evaluated similarly, using Eq. (53) with k_b^* and k_s^* . In contrast, the mean field of the ρ meson in SNM vanishes because $\rho_3 = 0$, while, in PNM, it is calculated by Eq. (72), where ρ_3 is given by Eq. (53) with k_b^* and k_s^* .

The binding energy per nucleon, pressure, the mean-field value for the σ meson, the density dependencies of the effective quark mass and of the effective nucleon mass, and the scalar polarizability in the QQMC model are respectively illustrated in Figs. 8, 9, 10, and 11. We note that the difference

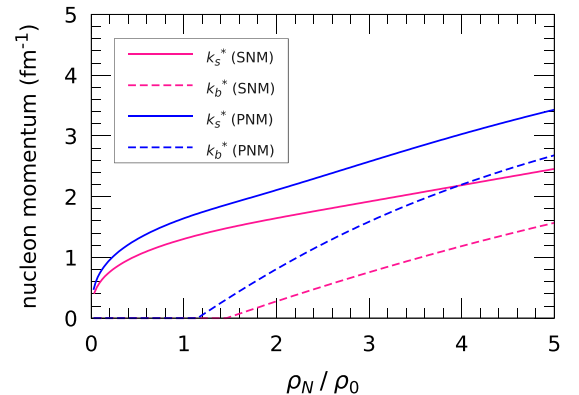


FIG. 12. Density dependence of k_s^* and k_b^* in the case of $r_p = 0.8$ fm, $v = 2.0$, and $w = 0.4$ GeV.

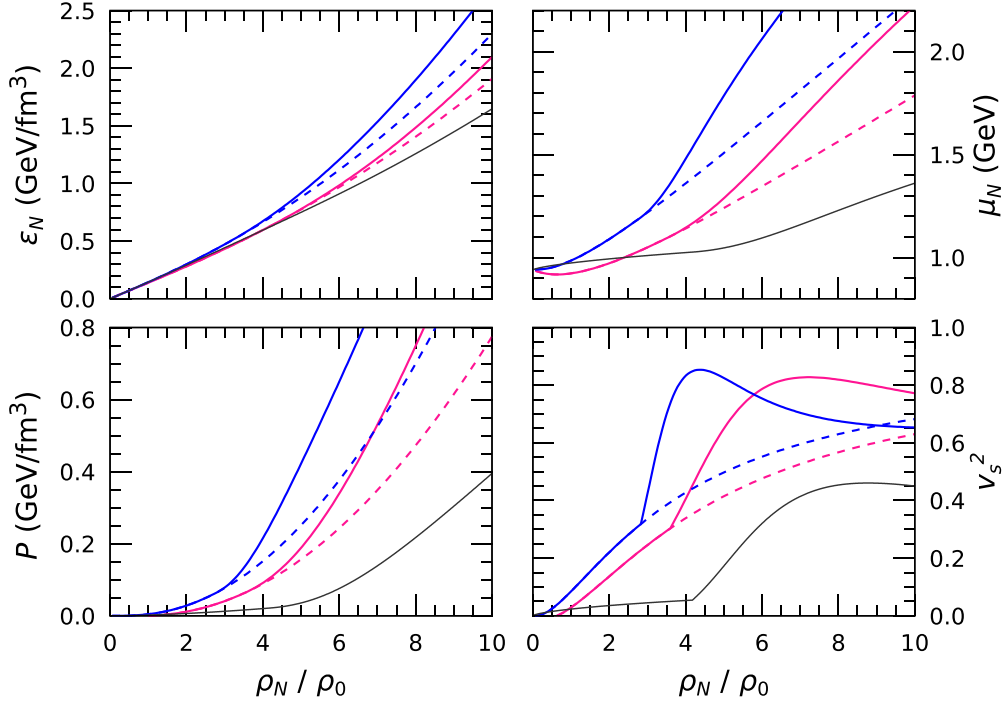


FIG. 13. Energy density, chemical potential, pressure, and sound velocity in the case of $r_p = 0.6$ fm, $v = 2.0$, and $w = 0.4$ GeV. The red solid (dashed) lines denote the results of the QQMC (QMC) model in SNM, while the blue solid (dashed) lines denote those of the QQMC (QMC) model in PNM. For comparison, we also show the results of the GQ model by the black lines.

between those quantities in the QMC and QQMC models is small.

Once we obtain the total energy density, ϵ_{tot} , as a function of the nuclear density, ρ_N , we can calculate the chemical potential, pressure, and sound velocity numerically by

$$\begin{aligned} \mu_N &= \frac{\partial \epsilon_{tot}}{\partial \rho_N}, \quad P = \mu_N \rho_N - \epsilon_{tot}, \\ v_s^2 &= \frac{\rho_N}{\mu_N} \frac{\partial \mu_N}{\partial \rho_N} = \frac{\rho_N}{\mu_N} \frac{\partial^2 \epsilon_{tot}}{\partial \rho_N^2}. \end{aligned} \quad (84)$$

Figures 13 and 14 present the results of the QQMC and QMC models in the cases of $r_p = 0.6$ and 0.8 fm, respectively. For comparison, we add the results of the GQ model (see Sec. II D), which do not involve any nuclear interactions except for the quark Pauli blocking, to the figures. It is seen that the nuclear interaction is very important to describe the properties of nuclear matter quantitatively, and that, as expected, the effect of Pauli blocking at the quark level enhances the chemical potential, pressure, and the sound velocity beyond ρ_{sat} .

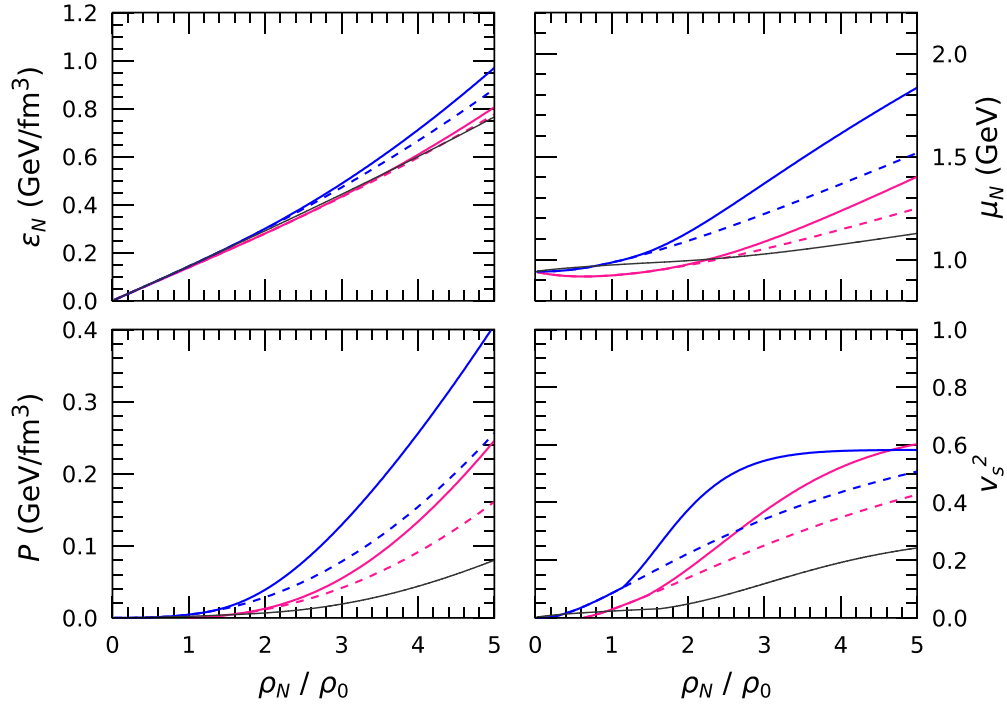
As for the sound velocity, it strongly depends on the nucleon size. The density dependence of v_s^2 is deduced from several neutron-star data by using the neural network model [67], in which v_s^2 shows the characteristic feature, i.e., it increases rapidly around $\rho_N / \rho_0 \approx 4.0$ and reaches the maximum ($v_s^2 \approx 0.5$ – 0.8), and then it decreases gradually beyond that density. However, such behavior cannot be explained by the QMC model, in which v_s^2 increases monotonically with increasing ρ_N . In contrast, in the case of the QQMC model with $r_p = 0.6$ fm (see Fig. 13), v_s^2 in PNM reaches the maximum,

at which its value exceeds 0.8, around $\rho_N / \rho_0 \approx 4.0$, and then decreases gradually. Because a neutron star is composed of neutrons with a fraction of protons, it can be expected that the maximum value of v_s^2 in neutron-star matter turns out to be slightly smaller than the value in PNM. Thus, the sound velocity calculated by the QQMC model is consistent with the result of v_s^2 inferred from the observed neutron-star data.

However, in Refs. [16,68], the chemical potential, pressure, and sound velocity are extracted from Bayesian inference analysis of the observed data of neutron stars. Then, the analysis tells us that the sound velocity behaves differently than in Ref. [67], and that it is enhanced rapidly around $\rho_N / \rho_0 \approx 2.0$ – 3.0 , reaches the maximum ($v_s^2 \approx 0.6$), and is saturated beyond $\rho_N / \rho_0 \approx 3.0$. This is very close to the present result with $r_p = 0.8$ fm in PNM (see Fig. 14).

Therefore, in the QQMC model, the value of r_p should lie within the range 0.6 – 0.8 fm. We here note that the behavior of v_s^2 observed in neutron stars can be explained by the OMEG family [15,66], particularly the set of OMEG1, in relativistic mean-field theory as well.

Furthermore, we can compare the present results with the experimental data of pressure extracted from HICs at high energy. Figures 15 and 16 show such comparisons. In the figures, the shaded areas represent the constraints from elliptical flow data [69,70] and kaon production data [71,72]. In both cases $r_p = 0.6$ and 0.8 fm, pressure calculated by the QMC or QQMC model is consistent with the experimental constraints. We note that pressure at high density is mainly generated by the repulsive force due to the exchanges of ω and ρ mesons, and that the effect of quarkyonic phase contributes to pressure additionally.

FIG. 14. Same as in Fig. 13, but for the case of $r_p = 0.8$ fm.

IV. SUMMARY AND DISCUSSION

We have constructed a novel, practical nuclear model based on the quark degrees of freedom, namely, the quarkyonic quark-meson coupling (QQMC) model, in which not only

the variation of nucleon structure but also the effect of Pauli blocking at the quark level are taken into account. In the QQMC model, a relativistic, Gaussian quark wavefunction is adopted to describe the nucleon structure.

We summarize the present study as follows:

- (1) The quark saturation density, ρ_{sat} , is very sensitive to the quark wavefunction. In the present calculation,

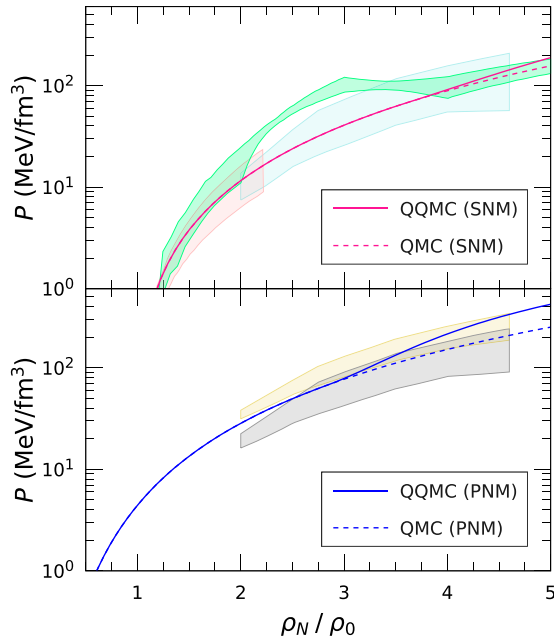


FIG. 15. Pressure in SNM and PNM as a function of ρ_N / ρ_0 . We take $r_p = 0.6$ fm, $v = 2.0$, and $w = 0.4$ GeV. In SNM, the green and light blue areas are for the flow data of HICs [69,70], while the pink area is for the kaon production data [71,72]. In PNM, the yellow and gray areas are for the flow data of HICs [69].

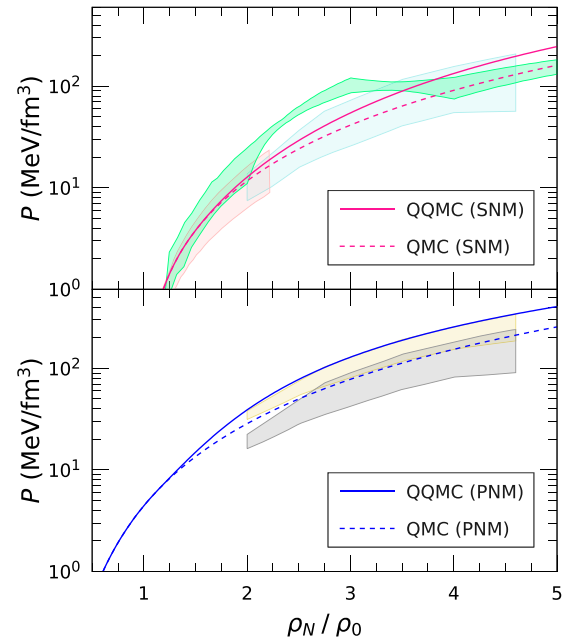


FIG. 16. Same as in Fig. 15 but for $r_p = 0.8$ fm, $v = 2.0$, and $w = 0.4$ GeV.

we have chosen a relativistic Gaussian wavefunction. Owing to the lower component of the relativistic wavefunction, the value of ρ_{sat} turns out to be higher than that in the NR case. Furthermore, it is also important to consider the c.m. correction to the estimate of a nucleon radius properly. For example, in the case of SNM in the ideal Fermi gas with $r_p = 0.8$ fm, we found $\rho_{\text{sat}} = 1.63\rho_0$, which is higher than in the NR case by about 28%. In contrast, when the nuclear interaction is included, the quark saturation density is reduced because the nucleon size is swollen in matter. In the case of $r_p = 0.8$ fm in SNM, we found $\rho_{\text{sat}}^* = 1.46\rho_0$, which is lower than the value of 1.63 by about 10%. Thus, the quark saturation density in SNM becomes eventually larger by about 15%, compared with the value in the NR case. We note that ρ_{sat}^* in PNM is roughly 22% lower than that in SNM. In the present calculation, ρ_{sat}^* in SNM is *never* below the nuclear saturation density, ρ_0 , which is consistent with the present status of nuclear experiments.

- (2) The quarkyonic phase is characterized by two momenta, k_b and k_s , where the former defines the underoccupied bulk part at low momentum and the latter gives the upper bound of the shell structure at high momentum. In the present paper, to determine those two momenta, we have used the same boundary condition as in the IdylliQ model. Then, using the Gaussian quark wavefunction, we have first evaluated the energy density, chemical potential, pressure, and sound velocity within the ideal Fermi gas picture. In this naïve GQ model, the chemical potential and pressure are discontinuous and the sound velocity diverges at ρ_{sat} . As a minimal correction to remedy this singular behavior, we have adopted a regulator which smears the sharp Fermi surface, $\theta(k_b - k)$, in the nucleon momentum distribution. Then, optimizing the regulator, we have found that the chemical potential and pressure are smoothed at ρ_{sat} , and that the sound velocity becomes continuous at ρ_{sat} .
- (3) We have next combined the GQ model and the QMC model in order to introduce the nuclear interaction; it is called the QQMC model. The QQMC model is a nuclear model based on the quark degrees of freedom, and it is applicable over a wide range of nuclear densities from low density to the crossover region where the transition from baryonic to quark matter occurs. The new model includes the effect of Pauli blocking at the quark level as well as the effect of scalar polarizability of nucleons in the nuclear medium. Therefore, the quarks feel the scalar field condensed in matter, which induces many-body forces. Then, we have calculated the physical quantities of the energy density, chemical potential, pressure, and sound velocity, which include the effects of nuclear interaction.
- (4) We emphasize that the inclusion of nuclear interaction in the quarkyonic model is quite important to consider the physical quantities quantitatively. In fact, the QQMC model can provide the sound velocity, which is consistent with that inferred from the observed

neutron-star data by using the neural network model or Bayesian inference analysis. Furthermore, pressure in PNM and SNM calculated with the QQMC model is within the range deduced from the experiments of HICs at high energy. As the quark saturation density heavily relies on the size of the nucleon, various physical quantities also depend on it accordingly. From the consideration of the sound velocity, the choice of $r_p = 0.6\text{--}0.8$ fm seems most suitable for describing dense nuclear matter in the QQMC model.

There are still some problems left. The first one concerns the boundary condition to determine the two momenta, k_b and k_s . Although in the present study we have assumed a condition which is the same as in the IdylliQ model, we need further investigation to make sure whether such a condition is *sufficient* or not. At the same time, it is also necessary to find a general rule for constructing the quark momentum distribution, $f_Q(q)$, in the postsaturation region, which satisfies Fermi statistics and the minimization of energy. Further rigorous investigation on this issue is urgent. The second one relates to the singular behavior of physical quantities around the quark saturation density. In the present calculations, we have used an *ad hoc* regulator to remove it. However, such singular behavior should be basically resolved by the fundamental dynamics, i.e., QCD (for example, see Refs. [73,74]). Those are the future issues.

Finally, it may be possible to include temperature in the QQMC model and to apply the model to EOSs for neutron stars with hyperons [25,75].

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DATA AVAILABILITY

There are no publicly available research data or software supporting this article. Requests for further information or data should be sent to the authors.

APPENDIX: SOMMERFELD EXPANSION FOR A GAUSSIAN QUARKYONIC MODEL

In nuclear physics, for example, the Fermi function

$$f(z) = \frac{1}{1 + e^z}, \quad z = \frac{r - R}{\delta}, \quad (\text{A1})$$

with R the half-density radius and δ the surface thickness, is often used to describe a shape for the radial density distribution of a nucleus. However, it is more convenient to use the

symmetrized Fermi function defined by [76]

$$\begin{aligned} f_s(r, R) &\equiv f((r - R)/\delta) + f(-(r + R)/\delta) - 1 \\ &= f((r - R)/\delta) - f((r + R)/\delta) \\ &= \frac{\sinh(R/\delta)}{\cosh(r/\delta) + \cosh(R/\delta)}, \end{aligned} \quad (\text{A2})$$

where $f_s(r, R) = f_s(-r, R)$, because it can avoid some difficulties appearing in actual calculations. Here, instead of the usual Fermi function, $f(z)$, we consider minimal corrections to the GQ model by using the symmetrized Fermi function, $f_s(r, R)$.

First, we formulate a Sommerfeld expansion generally. We consider the integral

$$K[h, p] = \int_0^\infty dk h(k) f_s(k, p), \quad (\text{A3})$$

with $h(k)$ any smooth function. Defining

$$H(k) \equiv \int_0^k dk' h(k'), \quad H(0) = 0, \quad (\text{A4})$$

we integrate Eq. (A3) by parts, so that

$$\begin{aligned} K[h, p] &= - \int_0^\infty dk H(k) \frac{\partial}{\partial k} f_s(k, p) \\ &= - \int_0^\infty dk H(k) \frac{\partial}{\partial k} [f(x - y) - f(x + y)] \\ &= - \int_0^\infty dk H(k) \left[\frac{\partial u}{\partial k} f'(u) - \frac{\partial v}{\partial k} f'(v) \right] \\ &= - \frac{1}{\delta} \int_0^\infty dk H(k) [f'(u) - f'(v)], \end{aligned} \quad (\text{A5})$$

with $x = k/\delta$, $y = p/\delta$, $u = x - y = (k - p)/\delta$, and $v = x + y = (k + p)/\delta$. Then, $H(k)$ is expanded around $k = p$ up to $\mathcal{O}((k - p)^2)$,

$$H(k) \simeq H(p) + \delta u h(p) + \frac{1}{2} \delta^2 u^2 h'(p), \quad (\text{A6})$$

with $h'(p) = \frac{d}{dk} h(k)|_{k=p}$. Thus, we find

$$\begin{aligned} K[h, p] &\simeq - \frac{1}{\delta} H(p) \int_0^\infty dk [f'(u) - f'(v)] \\ &\quad - h(p) \int_0^\infty dk u [f'(u) - f'(v)] \\ &\quad - \frac{\delta}{2} h'(p) \int_0^\infty dk u^2 [f'(u) - f'(v)], \end{aligned} \quad (\text{A7})$$

where the integrals on the right-hand side are evaluated as

$$\chi_0^-(y) \equiv - \frac{1}{\delta} \int_0^\infty dk [f'(u) - f'(v)] = f_s(0, y) = \tanh\left(\frac{y}{2}\right), \quad (\text{A8})$$

$$\chi_1^-(y) \equiv - \frac{1}{\delta} \int_0^\infty dk u [f'(u) - f'(v)] = \frac{2y}{1 + e^y}, \quad (\text{A9})$$

$$\begin{aligned} \chi_2^-(y) &\equiv - \frac{1}{\delta} \int_0^\infty dk u^2 [f'(u) - f'(v)] \\ &= \frac{\pi^2}{3} S_2(y) + 4y \log(1 + e^{-y}), \end{aligned} \quad (\text{A10})$$

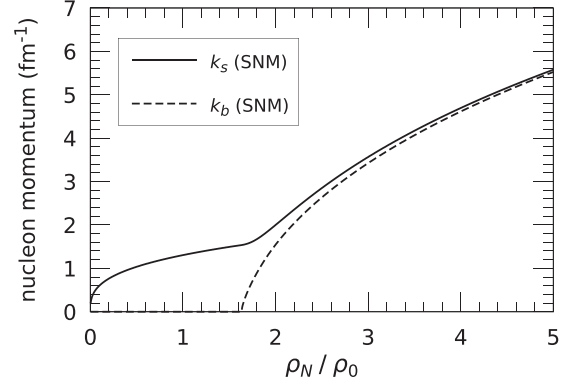


FIG. 17. Momenta, k_s and k_b , in the Sommerfeld expansion. We choose $r_p = 0.8$ fm and $\delta = 35$ MeV.

with

$$\begin{aligned} S_2(y) &\equiv \frac{12}{\pi^2} \int_0^{y/2} dt \frac{t^2}{\cosh^2 t} \\ &\simeq \frac{y^3}{y^3 + (19y^2 - 17y + 40)e^{-0.6y}} \quad (0 \leq S_2(y) \leq 1). \end{aligned} \quad (\text{A11})$$

Note that, because of the symmetrized Fermi function, the integrals (except S_2) can be performed without any approximation. Eventually, up to $\mathcal{O}(\delta^2)$, the Sommerfeld expansion gives

$$K[h, p] \simeq \chi_0^-(y) H(p) + \delta \chi_1^-(y) h(p) + \frac{1}{2} \delta^2 \chi_2^-(y) h'(p). \quad (\text{A12})$$

Similarly, we can find the first and second derivatives of $K[h, p]$ up to $\mathcal{O}(\delta^2)$ as

$$\frac{\partial}{\partial p} K[h, p] \simeq h(p) + \delta \chi_1^+(y) h'(p) + \frac{1}{2} \delta^2 \chi_2^+(y) h''(p), \quad (\text{A13})$$

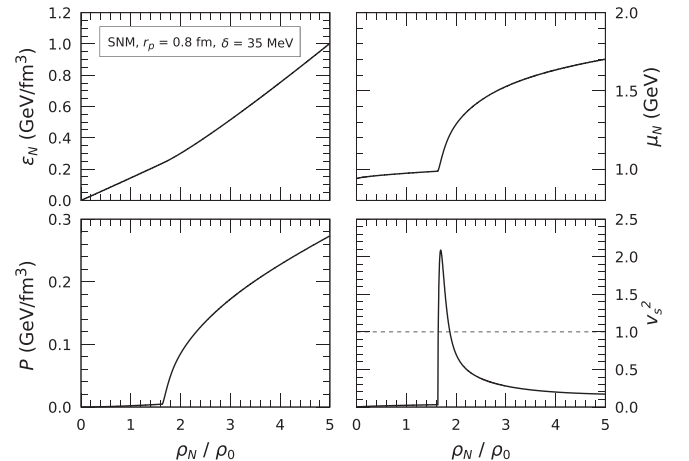


FIG. 18. Energy density, chemical potential, pressure, and sound velocity in the Sommerfeld expansion.

with

$$\chi_1^+(y) \equiv -\frac{1}{\delta} \int_0^\infty dk u [f'(u) + f'(v)] = 2 \log(1 + e^{-y}), \quad (\text{A14})$$

$$\begin{aligned} \chi_2^+(y) &\equiv -\frac{1}{\delta} \int_0^\infty dk u^2 [f'(u) + f'(v)] \\ &= \frac{\pi^2}{3} - 4y \log(1 + e^{-y}), \end{aligned} \quad (\text{A15})$$

and

$$\begin{aligned} \frac{\partial^2}{\partial p^2} K[h, p] &\simeq \chi_0^-(y) h'(p) + \delta \chi_1^-(y) h''(p) \\ &\quad + \frac{1}{2} \delta^2 \chi_2^-(y) h'''(p). \end{aligned} \quad (\text{A16})$$

Next, using Eqs. (A12), (A13), and (A16), we consider the minimal corrections to the GQ model; that is, we want to modify the first and second derivatives, $\partial k_b / \partial k_s$ and $\partial^2 k_b / \partial k_s^2$, in the naïve GQ model so as to remove the singular behavior at the quark saturation density. Here, we focus on the case of SNM only. The PNM case can be handled similarly as well.

In the boundary condition, Eq. (24), we replace $g_Q(k_b)$ with $K[h, p]$. In Eq. (A12), in the limit $\delta \rightarrow 0$, $K[h, p]$ approaches $H(p)$, so that we set $p \rightarrow k_b$, $H \rightarrow g_Q(k_b)$, and $h \rightarrow \phi$ [see Eq. (31)]. Therefore, the boundary condition with the corrections reads

$$g_Q(k_s) - \beta K[\phi, k_b] = 1, \quad (\text{A17})$$

and

$$\begin{aligned} K[\phi, k_b] &\simeq \chi_0^-(k_b/\delta) g_Q(k_b) + \bar{\delta} \chi_1^-(k_b/\delta) \phi(k_b) \\ &\quad + \frac{1}{2} \bar{\delta}^2 \chi_2^-(k_b/\delta) \phi'(k_b), \end{aligned} \quad (\text{A18})$$

where $\bar{\delta} = a\delta/N_c$ is dimensionless and ϕ' is given by Eq. (45). Figure 17 depicts the momenta, k_b and k_s , calculated with the Sommerfeld expansion, Eq. (A17). We can see that the rapid rise of the momenta appearing in the naïve GQ model is softened at ρ_{sat} .

Furthermore, the first derivative of $K[\phi, k_b]$ with respect to $\bar{k}_b$ is given by Eq. (A13):

$$\begin{aligned} \frac{\partial K[\phi, k_b]}{\partial \bar{k}_b} &\simeq \phi(k_b) + \bar{\delta} \chi_1^+(k_b/\delta) \phi'(k_b) \\ &\quad + \frac{1}{2} \bar{\delta}^2 \chi_2^+(k_b/\delta) \phi''(k_b). \end{aligned} \quad (\text{A19})$$

Thus, we obtain the first derivative, $\partial k_b / \partial k_s$, as

$$\begin{aligned} \frac{\partial k_b}{\partial k_s} &= \frac{\partial \bar{k}_b}{\partial \bar{k}_s} \\ &= \frac{\phi(k_s)}{\beta[\phi(k_b) + \bar{\delta} \chi_1^+(k_b/\delta) \phi'(k_b) + \frac{1}{2} \bar{\delta}^2 \chi_2^+(k_b/\delta) \phi''(k_b)]}, \end{aligned} \quad (\text{A20})$$

with

$$\phi''(k) = \frac{N_c^3}{\sqrt{\pi}} [8r_1(k) - 5r_2(k) \bar{k}^2 + 2r_3(k) \bar{k}^4] e^{-\bar{k}^2}. \quad (\text{A21})$$

We notice that, as $k_b \rightarrow 0$, the denominator in Eq. (A20) is $\mathcal{O}(1)$ and thus the singular behavior at ρ_{sat} vanishes.

To calculate the sound velocity, we need the second derivative, $\partial^2 k_b / \partial k_s^2$, which should be modified as well. Using Eqs. (A16), (A17), and (A20), we find

$$\begin{aligned} \frac{\partial^2 \bar{k}_b}{\partial \bar{k}_s^2} &= \left(\frac{N_c}{a} \right) \frac{\partial^2 k_b}{\partial k_s^2} \\ &= \frac{\phi'(k_s) - \beta \left(\frac{\partial \bar{k}_b}{\partial \bar{k}_s} \right)^2 \frac{\partial^2}{\partial \bar{k}_b^2} K[\phi, k_b]}{\beta[\phi(k_b) + \bar{\delta} \chi_1^+(k_b/\delta) \phi'(k_b) + \frac{1}{2} \bar{\delta}^2 \chi_2^+(k_b/\delta) \phi''(k_b)]}, \end{aligned} \quad (\text{A22})$$

with

$$\begin{aligned} \frac{\partial^2}{\partial \bar{k}_b^2} K[\phi, k_b] &\simeq \chi_0^-(k_b/\delta) \phi'(k_b) + \bar{\delta} \chi_1^-(k_b/\delta) \phi''(k_b) \\ &\quad + \frac{1}{2} \bar{\delta}^2 \chi_2^-(k_b/\delta) \phi'''(k_b), \end{aligned} \quad (\text{A23})$$

and

$$\phi'''(k) = \frac{N_c^3}{\sqrt{\pi}} (-16) \bar{k} [6r_2(k) - 9r_3(k) \bar{k}^2 + 2r_4(k) \bar{k}^4] e^{-\bar{k}^2}. \quad (\text{A24})$$

We again notice that the denominator in Eq. (A22) is $\mathcal{O}(1)$ as $k_b \rightarrow 0$. Now, all we need is prepared. Along the line of Sec. II C, we can calculate ρ_N , ϵ_N , μ_N , P , and v_s^2 by using Eqs. (A20) and (A22), instead of Eqs. (30) and (44). Figure 18 illustrates the results. We take $\delta = 35$ MeV and $r_p = 0.8$ fm. All the calculated quantities are continuous at ρ_{sat} . Furthermore, the sound velocity does not diverge. However, it overshoots the upper limit. A larger value of δ may, of course, suppress more the sound velocity around ρ_{sat} , but it turns out to be negative at very high density. Therefore, the present expansion up to $\mathcal{O}(\delta^2)$ may be insufficient, and we should include higher-order contributions beyond $\mathcal{O}(\delta^2)$ to obtain a satisfactory result. It may be very intricate.

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