

Isomeric states in odd-*A* tantalum isotopes with $N = 106$ – 116

A. Kardan,¹ P. M. Walker², and A. K. Jain³¹*Yerevan State University, 0025 Yerevan, Armenia*²*Department of Physics, University of Surrey, Guildford GU2 7XH, United Kingdom*³*Department of Physics, IIT Roorkee, Roorkee, Uttarakhand 247667, India*

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The structure of high- K isomeric states in the odd- A tantalum ($Z = 73$) isotopes is investigated systematically within the cranked Nilsson–Strutinsky (CNS) and cranked Nilsson–Strutinsky–Bogoliubov (CNSB) frameworks, spanning neutron numbers $N = 106$ – 116 . The low-lying one-proton isomers are identified with the $9/2^-$ [514], $5/2^+$ [402], and $1/2^+$ [411] Nilsson orbitals, while the two-quasineutron configurations $K^\pi = 8^-, 10^+$, and 10^- are shown to play a decisive role in driving multiquasiparticle isomer formation across the chain. High- K isomers are predicted in ^{189}Ta at low excitation energies, presenting a strong motivation for future experiments. The CNSB calculations predict a transition from stable prolate shapes to triaxiality and shape coexistence in odd- A Ta isotopes near $N = 114$ – 116 .

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I. INTRODUCTION

High- K isomers arise from the coupling of nucleons occupying high- j Nilsson orbitals near the Fermi surface, forming states with a large projection K of the total angular momentum on the nuclear symmetry axis. The large difference in K between the isomeric state and lower-lying levels strongly hinders electromagnetic decay. As a consequence, high- K isomers may exhibit half-lives ranging from nanoseconds to years. Because the K quantum number is closely connected to the underlying axial deformation, high- K isomers are highly sensitive to the deformed mean field, pairing correlations, and residual interactions that govern the single-particle structure near the Fermi surface [1–4].

The tantalum ($Z = 73$) isotopic chain occupies a strategic position in the nuclear chart. Situated in the rare-earth deformed region between the $Z = 50$ and $Z = 82$ proton shell closures, tantalum nuclei lie in a mass region where the high- j proton $h_{11/2}$ intruder orbital and the neutron $i_{13/2}$ intruder level are active near the Fermi surface, creating conditions that are particularly favorable for the formation of high- K , multiquasiparticle isomers [5,6]. Experimentally, a rich spectrum of one-, three-, and five-quasiparticle isomeric configurations has been identified in the lighter odd- A tantalum isotopes [5,7], and the prominent two-neutron $K^\pi = 8^-, 10^+$, and 10^- isomeric configurations have been highlighted as benchmark structures across this mass region [1,8–10]. More recently, neutron-rich isotopes $^{185-189}\text{Ta}$, populated through projectile fragmentation and multinucleon-transfer reactions, have

become accessible through storage-ring mass measurements and isomeric decay spectroscopy [11–15], extending experimental information toward increasingly neutron-rich systems and necessitating systematic theoretical investigation.

The tantalum isotopes also lie in a region of pronounced structural evolution. With increasing neutron number toward $N \approx 116$ – 118 , the well-deformed prolate shapes characteristic of midshell rare-earth nuclei are expected to evolve toward oblate or γ -soft configurations [16,17]. In neighboring even-even hafnium ($Z = 72$), tungsten ($Z = 74$) and osmium ($Z = 76$) isotopes, competing prolate and oblate minima in the potential-energy surfaces (PESs) give rise to shape coexistence and enhanced γ softness [17–19]. In odd- A nuclei, the presence of a blocked quasiparticle can further modify the equilibrium deformation and pairing correlations, making the structure of high- K states particularly sensitive to shape polarization effects. Despite extensive experimental information, a systematic microscopic description of the evolution of one- and multiquasiparticle isomeric configurations across the prolate-to-oblate transitional region in odd- A tantalum isotopes is still lacking.

In our previous works [19,20], we demonstrated that combining the paired cranked Nilsson–Strutinsky–Bogoliubov (CNSB) [21,22] and unpaired cranked Nilsson–Strutinsky (CNS) [23,24] frameworks leads to a good understanding of the formation of nuclear isomers and provides a clear picture of their underlying structure. In the present work, we extend this approach to perform a systematic investigation of high- K isomeric states in the odd- A tantalum isotopes $^{179-189}\text{Ta}$ within these two frameworks. The excitation energies are calculated using the CNSB framework, while the CNS model is employed to analyze the underlying structures and configurations of the identified states. Particular attention is devoted to low-lying one-proton, three-quasiparticle (3-qp), and five-quasiparticle (5-qp) isomeric configurations, whose evolution

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across the isotopic chain reflects the interplay between the changing neutron Fermi surface and the onset of the prolate-to-oblate shape transition.

The paper is organized as follows: Section II presents the theoretical frameworks, including the CNS and CNSB formalisms. Section III presents the results for one-proton isomers, while Sec. IV discusses the 3-qp and 5-qp isomeric configurations across the isotopic chain. Section V examines the prolate-oblate shape transition and its influence on the calculated PES structures. Finally, the main conclusions are summarized in Sec. VI.

II. THEORETICAL FRAMEWORKS

The CNS and CNSB approaches are one-dimensional cranking methods built on the same underlying single-particle picture. Nucleons move independently in a modified-oscillator (MO) potential $H_{\text{MO}}(\varepsilon_2, \gamma, \varepsilon_4)$ that rotates uniformly about a fixed principal axis (the x axis) with angular frequency ω [25]. In the CNS formalism, the cranked Hamiltonian is

$$H_{\text{CNS}} = H_{\text{MO}}(\varepsilon_2, \gamma, \varepsilon_4) - \omega j_x, \quad (1)$$

where the second term describes the Coriolis coupling to collective rotation. The total energies are obtained through the Strutinsky renormalization method [26], as a sum of a macroscopic, rotating-liquid-drop energy E_{RLD} , calculated with the Lublin–Strasbourg drop (LSD) model [27], and a microscopic shell-correction energy evaluated from Eq. (1). The total energy is minimized self-consistently with respect to the deformation parameters ε_2 , γ , and ε_4 at each spin value and for each configuration, using the standard Nilsson single-particle parameters of Ref. [25]. The triaxiality parameter γ is defined such that axial symmetry is recovered at $\gamma = 0^\circ, \pm 60^\circ, -120^\circ$, with collective rotation taking place about the shortest, intermediate, and longest principal axes for $\gamma \in [0^\circ, 60^\circ]$, $[0^\circ, -60^\circ]$, and $[-60^\circ, -120^\circ]$, respectively.

The CNS Hamiltonian, Eq. (1), neglects pairing correlations; this is a good approximation only at spins high enough that pairing has essentially vanished. Because the isomers discussed in this work occur at comparatively low spin, where pairing correlations are not negligible, we use the paired CNSB extension [21,22] for all quantitative excitation energy results. In CNSB, a monopole pairing term is added to Eq. (1),

$$H_{\text{CNSB}} = H_{\text{CNS}} - GP^\dagger P, \quad (2)$$

where $P^\dagger$ and P are the monopole pair creation and annihilation operators and G is the pairing strength parameter. The pairing term $H_G = -GP^\dagger P$ is linearized in the standard way to a mean-field pairing potential $H_\Delta = -\Delta(P^\dagger + P)$ [21,25], and, together with a particle-number constraint, the cranking Hamiltonian becomes

$$h_{\text{CNSB}} = H_{\text{CNS}} - \Delta(P^\dagger + P) - \lambda \hat{N}, \quad (3)$$

where $\hat{N}$ is the particle-number operator and λ is the corresponding Lagrange multiplier (Fermi energy). Equation (3) is solved separately for protons and neutrons through a Hartree–Fock–Bogoliubov (HFB) transformation, giving quasiparticle wave functions that depend on the pairing gap Δ and Fermi

energy λ , in addition to the rotational frequency and deformation parameters. The total energy in the laboratory frame is evaluated with an approximate particle-number projection,

$$E(\omega, \Delta, \lambda, \varepsilon_2, \gamma, \varepsilon_4) = \frac{\langle \Phi | (H_{\text{CNSB}} + \omega j_x) P^N | \Phi \rangle}{\langle \Phi | P^N | \Phi \rangle}, \quad (4)$$

where P^N is the particle-number projection operator [28]. The projected energy of Eq. (4) is then minimized with respect to the pairing parameters Δ and λ and to the deformation parameters at each spin. Following Ref. [29], the pairing strength G is additionally scaled down by a factor of 0.95 relative to its bare BCS value, which corrects for the systematic overestimation of pairing correlations that particle-number projection otherwise introduces relative to plain (unprojected) BCS calculations.

While CNSB is used for excitation energies, the absence of pairing in Eq. (1)—combined with the simplicity of the modified-oscillator basis—makes CNS the better-suited tool for identifying and labeling the quasiparticle configurations underlying each state, through the diabatic blocking procedure [30]. In CNS, the principal quantum number of the rotating modified-oscillator potential, $N_{\text{rot}} \equiv \mathcal{N}$, is preserved (neglecting pairing and small off-shell matrix elements), and at small and intermediate deformations the orbitals within each $\mathcal{N}$ -shell can further be separated into low- j and high- j groups [31,32]. A CNS configuration can therefore be specified not only by the number of particles occupying orbitals of given parity and signature, but also by the number of low- j and high- j particles in each $\mathcal{N}$ -shell. This allows a given multiquasiparticle excitation to be tracked diabatically across all (virtual) crossings, from low to high spin. In CNSB, by contrast, only the total parity π and signature α are preserved for protons and neutrons separately, giving, in general, 16 combinations of the form $(\pi_p, \alpha_p)(\pi_n, \alpha_n)$. As a result, individual CNS configurations can no longer always be tracked as a fixed quasiparticle content, so comparison between the two approaches is most direct for states that remain yrast within a fixed $(\pi_p, \alpha_p)(\pi_n, \alpha_n)$ block.

Because the cranked Hamiltonian, Eq. (1), is defined in a rotating, noninertial frame, it does not commute with the total angular-momentum operator $\hat{I}^2$, and total spin I is therefore *not* an exact quantum number of the cranking model. What Eq. (1) *does* conserve are the two discrete symmetries introduced above: parity π , and the signature α , defined as the eigenvalue of a 180° rotation about the cranking axis, $e^{-i\pi j_x}$, whose eigenvalues are $e^{-i\pi\alpha}$ with $\alpha = +\frac{1}{2}$ or $-\frac{1}{2}$ for a single particle (and correspondingly integer or half-integer, for the total signature $\alpha_{\text{tot}} = \sum_i \alpha_i$) [23]. Parity and signature determine which spin values a given configuration can physically carry. For an even-mass nucleus, $\alpha_{\text{tot}} = 0$ restricts the configuration to $I = 0, 2, 4, \dots$, while $\alpha_{\text{tot}} = 1$ restricts it to $I = 1, 3, 5, \dots$; for odd-mass nuclei, $\alpha_{\text{tot}} = +\frac{1}{2} \Rightarrow I = \frac{1}{2}, \frac{5}{2}, \frac{9}{2}, \dots$ and $\alpha_{\text{tot}} = -\frac{1}{2} \Rightarrow I = \frac{3}{2}, \frac{7}{2}, \frac{11}{2}, \dots$. Each configuration is specified by the parity and signature quantum numbers of the occupied proton and neutron orbitals separately, written as $(\pi_p, \alpha_p)(\pi_n, \alpha_n)$.

Within a fixed (π, α) configuration, an actual value of spin is then assigned at each rotational frequency ω through the

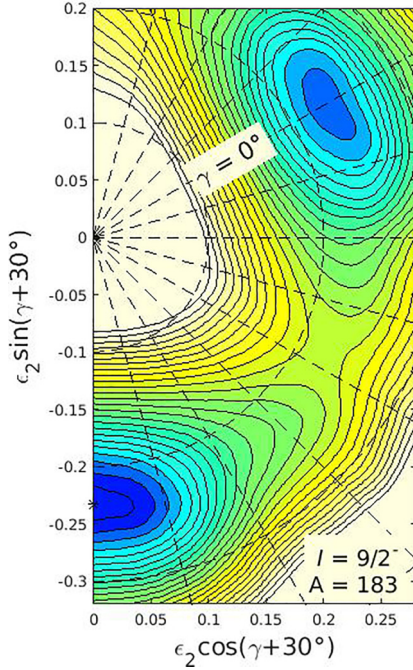


FIG. 2. Calculated CNSB PESs versus quadrupole deformation ε_2 and the triaxiality parameter γ of ^{183}Ta for the $I = 9/2^-$. Contour lines are separated by 0.2 MeV, and the γ plane is marked at 15° intervals. Dark regions represent low energy, and absolute minima are labeled with stars.

and $1/2^+[411]$ proton configurations to the ground state $I^\pi = 7/2^+$, creates favorable conditions for the formation of low-lying isomeric states, as will be discussed in the following sections.

$E1$ transitions in deformed nuclei are intrinsically hindered, often by several orders of magnitude relative to Weisskopf single-particle estimates, due to structural effects and selection rules [34]. This hindrance implies that the $9/2^-$ state may exhibit a pronounced isomeric character. The $9/2^-[514]$ and $7/2^+[404]$ orbitals differ by $\Delta N = 1$ and $\Delta n_z = 1$, violating the approximate selection rules $\Delta N = 0$, $\Delta n_z = 0$ that govern allowed $E1$ transitions in the Nilsson scheme. Similarly, the $5/2^+[402]$ and $1/2^+[411]$ orbitals near the Fermi surface may give rise to isomeric states via hindered transitions to the $7/2^+[404]$ ground state. It should be noted, however, that these orbitals are not pure, which may influence the degree of hindrance.

A. The $9/2^-$ isomeric state

A low-lying quasiparticle state with spin-parity $I^\pi = 9/2^-$ is found in the present calculations at a prolate shape with $\gamma = -120^\circ$ as illustrated in Fig. 2 for ^{183}Ta . The calculated excitation energies are listed in Table I and compared with the experimental data.

The extremely low excitation energy and the persistence of this state across these nuclei indicate a robust shell structure associated with the $9/2[514]$ proton orbital, originating from the $h_{11/2}$ spherical shell-model orbital, in agreement with Refs. [11,36]. This state is a feature of the low-energy

spectra of tantalum isotopes; however, its character as a long-lived isomer changes with the addition of neutrons toward $N = 116$. As shown in Table I, the half-life of the $9/2^-$ isomer, where measured, decreases with increasing excitation energy. All observed decays proceed via $E1$ transitions to the ground state, consistent with the single-particle orbital structure shown in Fig. 1. While the state is not seen as a long-lived isomer in isotopes heavier than ^{185}Ta , spectroscopic data do identify the $9/2^-$ bandhead at 245 keV and 283 keV in ^{187}Ta and ^{189}Ta , respectively [13,15], where it decays rapidly to the ground state.

In experiments, the excitation energy of the $9/2^-$ bandhead increases systematically with neutron number for the $A > 179$ isotopes: 6 keV $\rightarrow$ 73 keV $\rightarrow$ 176 keV $\rightarrow$ 245 keV $\rightarrow$ 283 keV. The CNSB calculations reproduce the experimentally observed increase in excitation energy with increasing neutron number; however, the calculated trend exhibits a more gradual slope. In ^{179}Ta , the $7/2^+$ state is the experimental ground state, while the $9/2^-$ state lies only a few keV lower in the calculations, giving rise to an almost degenerate doublet structure. Since the gamma-ray decay probability is proportional to $E_\gamma^{2\lambda+1}$, the half-life drops substantially as the state moves higher in energy. By $A = 187$, the state deexcites so rapidly that it is no longer classified as a “long-lived isomer,” making it harder to distinguish from prompt transitions [13,15].

B. The $1/2^+$ and $5/2^+$ isomeric states

The next potential isomeric state obtained in the calculations has spin-parity $I^\pi = 5/2^+$ and is associated with the $p5/2[402]$ orbital. Within the CNSB calculations, this configuration appears as a low-lying state with excitation energies listed in Table II. With increasing neutron number, the excitation energy of the $5/2^+$ state decreases gradually, consistent with experimental observations.

A further one-proton isomeric state with spin-parity $I^\pi = 1/2^+$ is observed, built on the $p1/2[411]$ Nilsson orbital. In the CNSB calculations, this $1/2^+$ state is also built on the configuration $p1/2[411]$ and is calculated at an excitation energy close to experimental data. Table II summarizes the experimental and deduced properties of the low-lying $1/2^+$ and $5/2^+$ proton configurations. In the lighter isotopes, the $5/2^+$ state lies lower in energy than the $1/2^+$ state, such that the $1/2^+$ isomer decays to the $5/2^+$ state. As the mass number increases toward the $N = 114$ region, the excitation energy of the $1/2^+$ configuration decreases and eventually falls below that of the $5/2^+$ configuration in ^{185}Ta . This behavior is also reproduced by the calculations, which yield a small energy separation between the two configurations. The enhanced isomeric character of the $1/2^+$ state can be understood in terms of structural hindrance effects and the violation of Nilsson selection rules (e.g., $\Delta n_z = 1$), which lead to small transition probabilities [37]. In addition, the decay of the $1/2^+$ state proceeds via an $E2$ transition to the $5/2^+$ level, which is relatively slow compared with the $M1$ transition from the $5/2^+$ state to the $7/2^+$ level. This contributes to the observed difference in the lifetimes of these configurations. The $1/2^+$ and $5/2^+$ states have not been observed experimentally in the isotopes $^{187,189}\text{Ta}$, although the calculations predict both

TABLE II. Calculated excitation energies of the quasiparticle $1/2^+$ and $5/2^+$ states and their configuration in $^{179-189}\text{Ta}$ isotopes. The labels $p1/2^+$ and $p5/2^+$ denote the proton $1/2^+[411]$ and $5/2^+[402]$ Nilsson configurations, respectively. Experimental excitation energies, half-lives, and decay modes are also shown. Experimental data are taken from Ref. [35] for $^{179,181}\text{Ta}$, Ref. [37] for ^{183}Ta , and Refs. [35,36] for ^{185}Ta .

Isotope	J^π	Configuration	E_{calc} (keV)	E_{expt} (keV)	$T_{1/2}$
^{179}Ta	$1/2^+$	$p1/2^+$	564	520.23	0.28(8) μs
	$5/2^+$	$p5/2^+$	446	238.56	65(10) ns
^{181}Ta	$1/2^+$	$p1/2^+$	640	615.19	18.5(4) μs
	$5/2^+$	$p5/2^+$	495	482.168	10.8(1) ns
^{183}Ta	$(1/2^+)$	$p1/2^+$	603	$459.1 + x^a$	41_{-4}^{+6} μs
	$(5/2^+)$	$p5/2^+$	488	459.1	Prompt, Short ^b
^{185}Ta	$(3/2^+)^c$	$p1/2^+$	533	406.0	0.9(3) μs
	$(5/2^+)$	$p5/2^+$	482	418	Prompt, Short
^{187}Ta	$1/2^+$	$p1/2^+$	266		
	$5/2^+$	$p5/2^+$	353		
^{189}Ta	$1/2^+$	$p1/2^+$	24		
	$5/2^+$	$p5/2^+$	261		

^a $x \leq 80$ keV [37].

^bObserved as prompt relative to ^{183}Hf β decay [37].

^c $3/2$ member of the $1/2^+$ band; the $I = 1/2$ and $3/2$ members have a small energy spacing [36].

configurations to have lower energies than those observed in the lighter isotopes.

IV. MULTIQUASIPARTICLE ISOMERIC STATES

Many isomeric states have been observed in tantalum isotopes, which are understood to originate predominantly from multi-quasiparticle (MQP) excitations. These MQP isomers are most prominently found in a region characterized by axially symmetric prolate shapes with relatively stable deformation, as established by experimental systematics. However, with increasing mass number and approaching $N = 116$, the occurrence of these specific conditions becomes progressively less likely. This motivates a continued and systematic investigation of MQP isomers in heavier tantalum isotopes, where their presence may still be expected, but they are likely to be more weakly populated and experimentally challenging to identify.

For the discussion of MQP isomers below, we use the shorthand notation for quasiparticle configurations, in which p and n denote a proton and a neutron quasiparticle, respectively, with a superscript giving the number of quasiparticles of each type. Thus p^3 denotes a three-proton quasiparticle configuration, pn^2 a configuration with one proton and two neutron quasiparticles, and so on.

A. The $19/2^+$ isomeric state

Experimentally, the lowest-spin 3-qp isomeric state in odd-A tantalum isotopes has been identified as a $19/2^+$ state. Such a state has been observed in ^{183}Ta at 1332 keV [38], and more recently in ^{189}Ta at 1309 keV [15].

Within the CNSB calculations, the lowest-lying $19/2^+$ state is found in the $(\pi_p, \alpha_p)(\pi_n, \alpha_n) = (-, +1/2)(-, 1)$ configuration by coupling the proton orbital $p9/2^-[514]$ to a

neutron $I^\pi = 5^-$ structure. As seen in the neutron single-particle orbitals diagram in Fig. 1, in the present mass region the $I^\pi = 5^-$ neutron configuration can be formed either by the two-quasineutron combination $n1/2^-[510]n9/2^+[624]$ or by $n1/2^-[510]n11/2^+[615]$, both involving orbitals originating from the neutron $i_{13/2}$ subshell.

Each n -quasiparticle configuration gives rise to a multiplet of $2^{(n-1)}$ states, which are split by the nucleon-nucleon interactions. These residual two-body interactions, treated as a sum over all possible two-quasiparticle combinations, are included in the present calculations. These interactions are orbital-dependent nucleon-nucleon contributions and are determined from empirical data on two-quasiparticle states in neighboring even-even and odd-odd nuclei [39]. The corresponding interaction energies are incorporated within the framework of the Gallagher–Moszkowski (G–M) coupling scheme [40,41], which accounts for the splitting of MQP configurations and plays an important role in determining the relative energies of these structures. For the configuration involving the $n9/2^+[624]$ orbital, the residual interaction contribution is calculated to be about +8 keV, whereas for the $n11/2^+[615]$ configuration it is approximately –221 keV. This additional binding, together with the upward movement of the neutron Fermi level as the neutron number increases, makes the $n11/2^+[615]$ -based configuration increasingly favored for the $19/2^+$ isomeric state in the tantalum isotopes. A $K^\pi = 5^-$ state based on the $n1/2^-[510]n11/2^+[615]$ configuration has also been identified experimentally in the neighboring even-even nuclei, ^{184}W [42] and ^{182}Hf [20].

In ^{181}Ta , the calculated $19/2^+$ state is located at an excitation energy of 924 keV. Based on the neutron single-particle levels shown in Fig. 1, where the $1/2[510]$ and $9/2[624]$ orbitals lie close to the Fermi surface, one may expect a $17/2^+$ state with the configuration $p9/2[514]n1/2^-[510]n9/2^+[624]$ to occur at a lower or at least comparable excitation energy.

Our calculations predict such a $17/2^+$ state in ^{181}Ta at an excitation energy of 955 keV, with a residual interaction energy of approximately -161 keV. It should be noted that insufficient experimental data are available to determine the residual interaction energy between the $p9/2[514]$ and $n9/2[624]$ orbitals. Consequently, this interaction has been estimated from the average of the available G–M energy splittings [43]. With this consideration, the $17/2^+$ state is predicted to be a favored 3-qp isomeric state in ^{181}Ta . The corresponding PESs for both the $17/2^+$ and $19/2^+$ states exhibit pronounced minima at approximately $(\varepsilon_2, \gamma) \approx (0.24, -120^\circ)$, supporting their interpretation as deformed 3-qp isomeric states.

A similar energy minimum is also obtained at the $19/2^+$ in ^{183}Ta , where the same configuration is calculated at 1357 keV. In contrast, the corresponding state in ^{179}Ta is predicted at a considerably higher excitation energy of 1718 keV, which may reduce the probability for its experimental population and observation. Although no firm experimental evidence for a $19/2^+$ isomer has yet been reported in $^{185,187}\text{Ta}$, the CNSB calculations predict pronounced energy trapping for such states, at energies 1301 keV and 1111 keV, respectively, indicating that they may exhibit isomeric character. In ^{189}Ta , a $19/2^+$ isomer was recently observed at an excitation energy of 1309 keV [15], while the corresponding CNSB calculation places this state at 930 keV.

B. The $21/2^-$ isomeric state

The $21/2^-$ excitation represents one of the characteristic MQP structures in odd- A tantalum isotopes $A = 179$ – 189 and exhibits an interesting evolution of configuration character with increasing neutron number. The $21/2^-$ isomer is also observed in the lighter odd- A isotopes $^{173,175,177}\text{Ta}$ [5], indicating that this excitation represents a persistent structural feature in the lighter members of the isotopic chain.

In the present calculations, low-lying $I^\pi = 21/2^-$ states are systematically obtained across the Ta isotopic chain in three different configurations, $(\pi_p, \alpha_p)(\pi_n, \alpha_n) = (-, +1/2)(+, 0)$, $(+, +1/2)(-, 0)$, and $(+, -1/2)(-, 1)$. These three configurations are found to compete in energy; see Fig. 3, with their relative ordering depending on the neutron number. The first configuration corresponds to the three-quasiproton configuration $p5/2[402]p7/2[404]p9/2[514]$. The second is associated with the mixed proton-neutron configuration $p5/2[402]n7/2[514]n9/2[624]$, built on the neutron 8^- state identified in the $N = 106$ isotones (see Sec. IV C). Based on Fig. 1 and the evolution of the neutron Fermi level with increasing neutron number, the third configuration can originate from two different neutron configurations. For the lighter Ta isotopes, the $p7/2[404]n7/2[503]n7/2[633]$ configuration is favored, whereas with increasing neutron number the $p7/2[404]n3/2[512]n11/2[615]$ configuration becomes energetically preferred. The latter configuration is further stabilized by the attractive G–M residual interaction energy of -139 keV, compared with the positive contribution of $+218$ keV for the former configuration.

In the lighter isotope ^{179}Ta , the lowest $21/2^-$ state is calculated to be of p^3 character. The neutron subsystem remains

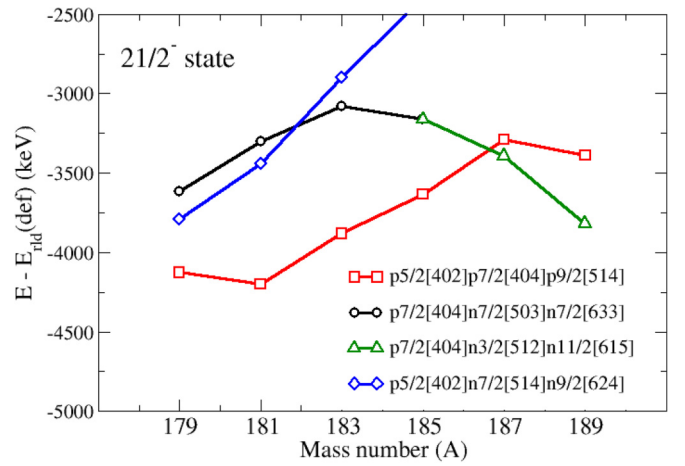


FIG. 3. Low-lying $21/2^-$ states in tantalum isotopes with $A = 179$ – 189 . Energies are calculated relative to a rotating liquid drop energy reference (see Sec. IV C) and include the G–M residual interaction energies.

in its vacuum configuration, while the negative parity arises entirely from the three-quasiproton structure. The calculated excitation energy, including residual interaction effects, is $E_x = 1252$ keV, in good agreement with the experimental value of 1252.6 keV. Despite this agreement, the experimentally determined gyromagnetic factor of the $21/2^-$ isomer in ^{179}Ta suggests that the state is not purely of proton origin and contains admixtures of quasiproton and quasineutron configurations [5]. Consistently, as seen in Fig. 3 the relatively small energy separation between the p^3 and two pn^2 states provides conditions for configuration mixing, supporting the experimental indication of a more complex structure for the observed isomer in ^{179}Ta .

Moving to heavier isotopes, the same competing $21/2^-$ configurations are obtained in $^{181,183,185}\text{Ta}$ within the CNSB calculations. In these nuclei, the lowest state retains the p^3 character with calculated excitation energies of 1243, 1200, and 1281 keV, respectively. The $21/2^-$ isomer is experimentally observed at an energy of 1483.43 keV in ^{181}Ta [44]. To our knowledge, no experimental evidence for a mixed configuration has been reported in this isotope. The observed $21/2^-$ state in ^{183}Ta at 1477 keV exhibits relatively fast $E1$ decay to the $19/2^+$ isomer [38]. Experimentally, ^{185}Ta exhibits a long-lived $21/2^-$ isomer at 1273.5 keV with a half-life of 17(2) ms, decaying to the $p9/2^-$ band through K -forbidden transitions, which indicates a well-preserved axial symmetry [11].

In ^{187}Ta , the situation evolves further. The lowest $21/2^-$ state is calculated at 1323 keV with a configuration of mixed proton-neutron character, $p7/2[404]n3/2[512]n11/2[615]$, although still close to the p^3 configuration in energy. In ^{187}Ta , the lowest-lying experimental 3-qp isomer shifts to a $25/2^-$ state at 1778 keV [13]. Within the CNSB framework, a low-lying $25/2^-$ state with the configuration $p5/2[402]n11/2[615]n9/2[505]$ is calculated at 1681 keV.

Finally, in ^{189}Ta ($N = 116$), a noncollective $21/2^-$ state has been observed experimentally at 1444 keV with a short

TABLE III. Calculated and experimental values of the excitation energy, and the structure of the 3-qp and 5-qp potential and observed isomeric states in $^{179-189}\text{Ta}$. Experimental data for the excitation energies are taken from Refs. [15,35].

$I_n^\pi = 8^-, 7/2[514]9/2[624]$					
Isotope	n -qp	Spin	Proton configuration	E_{calc} (keV)	E_{expt} (keV)
^{179}Ta	3-qp	$25/2^+$	$9/2[514]$	1309	1317.2
	3-qp	$23/2^-$	$7/2[404]$	1231	1328.0
	5-qp	$37/2^+$	$5/2[402]7/2[404]9/2[514]$	2579	2639.5
	5-qp	$33/2^-$	$1/2[541]7/2[404]9/2[514]$	2819	2792.8
$I_n^\pi = 10^+, 9/2[624]11/2[615]$					
^{181}Ta	3-qp	$29/2^-$	$9/2[514]$	2462	2230
	3-qp	$27/2^+$	$7/2[404]$	2754	
	5-qp	$41/2^-$	$5/2[402]7/2[404]9/2[514]$	3670	
^{183}Ta	3-qp	$29/2^-$	$9/2[514]$	2308	2475
	3-qp	$27/2^+$	$7/2[404]$	2829	
	5-qp	$41/2^-$	$5/2[402]7/2[404]9/2[514]$	3523	3870
^{185}Ta	3-qp	$29/2^-$	$9/2[514]$	2205	
	3-qp	$27/2^+$	$7/2[404]$	2784	
	5-qp	$41/2^-$	$5/2[402]7/2[404]9/2[514]$	3248	
$I_n^\pi = 10^-, 9/2[505]11/2[615]$					
^{187}Ta	3-qp	$29/2^+$	$9/2[514]$	1461	
	3-qp	$27/2^-$	$7/2[404]$	1547	
	5-qp	$41/2^+$	$5/2[402]7/2[404]9/2[514]$	2758	2933
^{189}Ta	3-qp	$29/2^+$	$9/2[514]$	749	
	3-qp	$27/2^-$	$7/2[404]$	861	
	5-qp	$41/2^+$	$5/2[402]7/2[404]9/2[514]$	2092	

half-life of 160 ns. It has been interpreted as decaying via a K -allowed $E1$ transition to a $19/2^+$ intrinsic state [15]. This interpretation is consistent with our calculations, which predict a low-lying $21/2^-$ state at 1141 keV with the mixed configuration $p7/2^+[404]n3/2^-[512]n11/2^+[615]$. Furthermore, the calculated $19/2^+$ intrinsic state at 930 keV in this isotope (see Sec. IV A) is also in agreement with the experimental observation.

As the neutron number increases, the “yrast traps” (the most energetically favorable high-spin states) can occur at higher angular momentum. In the following section we study higher- K isomers in tantalum isotopes.

C. The isomeric states at higher spins

The systematics of high- K MQP isomeric structures in $^{179-189}\text{Ta}$ exhibit a clear and coherent evolution driven by changes in the underlying neutron configurations as a function of neutron number. Our findings on the energy and structure of these states in odd- A tantalum isotopes are listed and compared with experimental data in Table III.

In ^{179}Ta with $N = 106$, the dominant two-quasineutron configuration is $n7/2[514]n9/2[624]$ ($K_n^\pi = 8^-$). As shown in Fig. 1, the $n7/2[514]$ orbital lies below the $N = 106$ shell gap and originates mainly from the $h_{9/2}$ orbital, while the $n9/2[624]$ lies above the gap and originates from the $i_{13/2}$ sub-shell. Coupling this neutron configuration with the $p9/2[514]$, $p7/2[404]$, or with the three-quasiproton configuration $p5/2[402]p7/2[404]p9/2[514]$, gives rise to a sequence of

energetically favored 3-qp and 5-qp isomeric states $25/2^+$, $23/2^-$, and $37/2^+$, respectively. As listed in Table III, the neutron 8^- configuration can also couple to the three-quasiproton configuration $p1/2[541]p7/2[404]p9/2[514]$, producing a favored 5-qp $33/2^-$ state at 2819 keV. The $p1/2[541]$ orbital originates mainly from the $h_{9/2}$ subshell and decreases rapidly in energy with increasing deformation, becoming low-lying around $\varepsilon_2 \approx 0.24$ (it does not appear in Fig. 1). These configurations are well reproduced in our calculations and are generally consistent with the experimentally observed isomers, confirming the robustness of this neutron structure near midshell. This is also consistent with the well-established systematics of the $K^\pi = 8^-$ two-quasineutron configuration $n7/2^-[514]n9/2^+[624]$ in the $N = 106$ isotonic chain. This configuration defines a systematic sequence of $K^\pi = 8^-$ isomers extending from the well-deformed neutron-rich isotope ^{172}Dy to the neutron-deficient isotope ^{188}Pb , where the presence of the isomer signals a prolate minimum in an otherwise spherical nucleus. The half-life of the isomer varies by seven orders of magnitude across this chain—from 11 s in ^{176}Yb to 0.8 μs in ^{188}Pb —as a result of the increasing transition energy of the main $E1$ decay branch and the gradual reduction of deformation approaching the $Z = 82$ shell closure [8,9,45,46].

A change in the dominant neutron configuration is observed in ^{181}Ta . The $n9/2[624]n11/2[615]$ ($K_n^\pi = 10^+$) neutron alignment becomes low in energy, reflecting the evolution of the single-particle spectrum with increasing neutron number. Within this framework, the present CNS and

CNSB calculations predict a $29/2^-$ 3-qp isomer at an excitation energy of 2462 keV, associated with the configuration $p9/2[514]n9/2[624]n11/2[615]$. This result shows reasonable agreement with the experimentally observed state at 2228 keV. This behavior is consistent with the well-established systematics of the even-even $N = 108$ isotones in this mass region. In ^{182}W ($Z = 74$, $N = 108$), a $K^\pi = 10^+$ isomer based on the same $n9/2^+[624]n11/2^+[615]$ configuration is firmly established at $E_x \approx 2230$ keV with $T_{1/2} = 1.3$ μs [47], and an analogous 10^+ isomer is observed in ^{184}Os at $E_x \approx 2366$ keV with $T_{1/2} = 23$ ns [48]. In contrast, the corresponding state at 2424 keV in ^{180}Hf ($Z = 72$, $N = 108$) does not acquire isomeric character experimentally. The 10^+ bandhead lies above the $K^\pi = 8^-$ two-quasiparticle (2-qp) band and decays to it via weakly K -forbidden $E1$ transitions with $T_{1/2} < 2$ ns [49], below the isomeric threshold, although a theoretical isomeric character for this state had been anticipated [20].

An alternative configuration may arise from coupling the neutron 10^+ state to the $7/2^+$ proton orbital, giving rise to a $27/2^+$ state. However, this configuration is energetically unfavorable, since it contains three unfavored 2-qp couplings according to the G–M rules, with a corresponding residual interaction energy of $E_{\text{res}} = +328$ keV. Consistent with this, the configuration is calculated in our model at an excitation energy above 2700 keV in $^{181-185}\text{Ta}$ isotopes; see Table III.

As the neutron number increases further, the Fermi level shifts and leads to a reorganization of the relevant neutron orbitals. In particular, the $n9/2[624]$ orbital moves away from the Fermi surface, while the $n9/2[505]$ and $n11/2[615]$ orbitals become energetically competitive; see Fig. 1. Consequently, in $^{187,189}\text{Ta}$ the configuration $n9/2[505]n11/2[615]$ ($K_n^\pi = 10^-$) emerges as the dominant neutron structure. This configuration forms the basis for a new set of high- K isomeric states in the heavier isotopes. This configuration is characterized by a noncollective structure with $\gamma = -120^\circ$. As discussed in Ref. [19], this state constitutes a primary building block for potential isomeric structures in the region $Z = 72-80$ and $N \approx 116$. Coupling this neutron configuration to the proton orbital $p9/2[514]$ leads to the formation of a $29/2^+$ isomeric state, which is calculated to form an energy trap at 1461 keV in ^{187}Ta and 749 keV in ^{189}Ta . Although these states have not yet been experimentally observed, their relatively low excitation energies suggest that they are promising candidates for future experimental identification, and long half-lives (> 1 s) may be anticipated, especially for ^{189}Ta . In contrast, such configurations are not expected to form isomers in the lighter isotopes, where the $n9/2[505]$ orbital lies further from the Fermi surface and does not support the required quasiparticle alignment.

The presence of the $n9/2[505]n11/2[615]$ configuration also enables the formation of additional 3-qp states through coupling with the $p7/2[404]$ orbital, giving rise to $27/2^-$ states. In ^{187}Ta , this state is calculated at 1547 keV, in reasonable agreement with the experimentally observed level at 1793 keV. In ^{187}Ta , two different spin assignments $I^\pi = 25/2^-$ and $27/2^-$ have been suggested for a long-lived ($T_{1/2} \approx 7$ s) isomer [13,50], and further data would be valuable. In ^{189}Ta , similar configurations remain energetically

favorable (at significantly lower energy than in ^{187}Ta), although no clear experimental counterparts have yet been identified. The evolution toward heavier isotopes also facilitates the formation of higher-order configurations. In particular, a 5-qp isomer in this chain is observed in ^{187}Ta with $I^\pi = (41/2^+)$. This state can be interpreted as the coupling of the favored 3-quasiprotone configuration $p5/2[402]p7/2[404]p9/2[514]$ ($K_p^\pi = 21/2^-$) with the 2-quasiprotone configuration $n9/2[505]n11/2[615]$ ($K_n^\pi = 10^-$). The resulting state is calculated at 2758 keV, in reasonable agreement with the experimental value of 2933 keV reported in Ref. [50]. It should be noted that spectroscopic information for this state remains limited, and the spin-parity assignment $I^\pi = 41/2^+$ is derived from the theoretical calculations [50]. A corresponding $41/2^+$ state is also predicted in our calculations in ^{189}Ta at 2092 keV, although it has not yet been observed experimentally. The ^{187}Ta isomer is known to exhibit an unusually long half-life of 136 s [14], and the significantly lower excitation energy predicted for the analogous state in ^{189}Ta suggests an even longer half-life.

Overall, the present results demonstrate a systematic evolution of neutron configurations from $n7/2[514]n9/2[624]$ to $n9/2[624]n11/2[615]$, and finally to $n9/2[505]n11/2[615]$ with increasing neutron number. This progression reflects the changing single-particle structure near the Fermi surface and provides a consistent framework for understanding the occurrence of high- K isomeric states in tantalum isotopes. Similar trends are observed or predicted in neighboring Hf and W nuclei, supporting the generality of this configuration evolution in the $A \approx 180$ mass region [20,49,51–53].

Figure 4 shows the lowest experimental and calculated energies of the observed and potential 1-qp, 3-qp, and 5-qp isomeric states in each isotope listed in Tables I and III. The excitation energies are plotted relative to a rotating liquid drop (RLD) reference energy, $E_{\text{rld}}(I)$, rather than to the more traditional rigid-rotor reference $E_{\text{ref}} = CI(I+1)$ [23], where I is the nuclear spin and C is a constant that depends on the mass number of the nucleus. The RLD reference is obtained by minimizing the macroscopic liquid-drop energy itself, at each spin I , with respect to nuclear shape. We use the LSD model [27] for the static part, together with a rigid-body moment of inertia evaluated at that same equilibrium shape with a radius parameter $r_0 = 1.16$ fm and a surface diffuseness $a = 0.6$ fm [24]. Physically, this reference tracks the shape evolution of an unpaired, structureless liquid drop with spin: it is spherical at $I = 0$, with a correspondingly small moment of inertia, and becomes increasingly oblate as I increases, which raises its rigid-body moment of inertia. Because this shape and moment-of-inertia evolution is built directly into the reference, $E - E_{\text{rld}}$ isolates the microscopic (shell and pairing) contribution to the excitation energy far more cleanly than a fixed-moment-of-inertia reference does; for more detail, see Sec. IV C in Ref. [54]. This provides a smooth energy scale for high-spin configurations and allows meaningful comparison of MQP states across different isotopes. All experimental and calculated energies for a given nucleus are evaluated relative to the same $E_{\text{rld}}(I)$ and are subsequently renormalized to the ground-state energy, so that the plotted quantity is a genuine excitation energy above the $I = 7/2^+$ state. Within

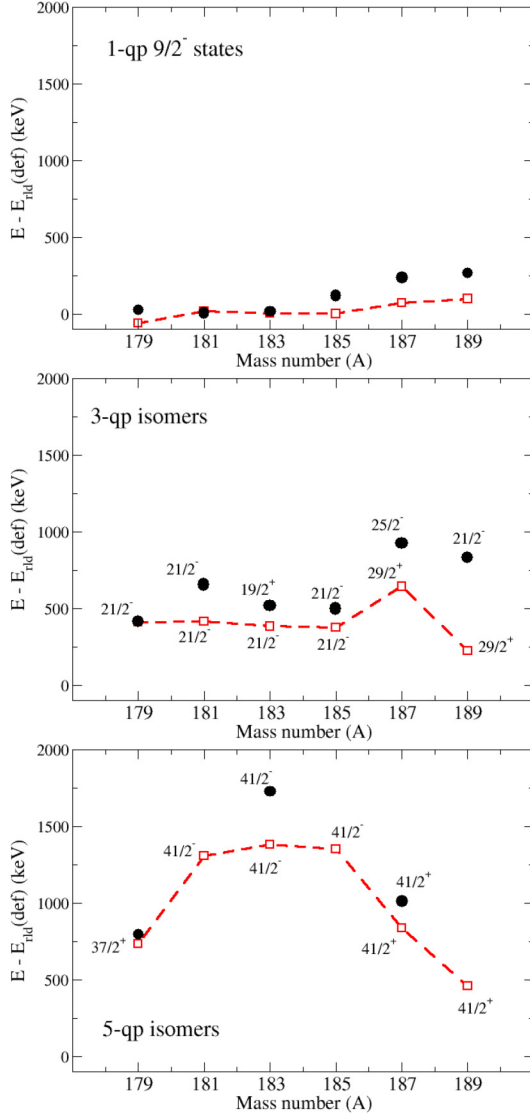


FIG. 4. Observed (filled circles) and calculated (open squares) energy of the lowest 1-qp (upper panel), 3-qp (middle panel), and 5-qp (lower panel) states relative to the rotating liquid drop energies, renormalized with respect to the ground-state energy, in Ta isotopes ($A = 179$ – 189). The data points are labeled with their K^π values. The energy and configuration of these states are listed in Tables I and III.

this common framework, lower excitation energies relative to E_{rld} generally correspond to longer half-lives of the high- K isomeric states, and the calculations reproduce the observed energies reasonably for states with identical spin and parity assignments.

In Fig. 4, an upward trend with increasing mass number in the calculated $9/2^-$ excitation energies is observed. A similar overall behavior is also seen in the experimental data, although with some quantitative differences. As shown in Table I, beyond ^{185}Ta , the $9/2^-$ state loses its isomeric character. This is primarily attributed to the substantially higher excitation energies of 245 and 283 keV in ^{187}Ta and ^{189}Ta , respectively, which accelerate the $E1$ decay rate through its strong E_γ^3 dependence and shorten the associated half-life.

Figure 4 also shows that the lowest experimental and calculated 3-qp states in $^{179,181}\text{Ta}$ and ^{185}Ta have the same spin and parity, $K^\pi = 21/2^-$, whereas they differ in the other three isotopes. In ^{183}Ta , the lowest 3-qp state is calculated to be $21/2^-$, while experimentally the $21/2^-$ state is found to be 145 keV above the $19/2^+$ isomer [38]. For the high- K states in the heavier isotopes ($^{187,189}\text{Ta}$), the calculated energies indicate a restabilization of certain configurations associated with the $n9/2[505]n11/2[615]$ neutron alignment, leading to relatively low-lying 3-qp and 5-qp structures in ^{189}Ta . This results in a lowest 3-qp state with $K^\pi = 29/2^+$, whereas experiments identify the $25/2^-$ [13] and $21/2^-$ [15] states as the lowest 3-qp states in ^{187}Ta and ^{189}Ta , respectively. The predicted excitation energies for the lowest 3-qp and 5-qp states in ^{189}Ta are notably smaller than those in ^{187}Ta , placing these configurations lower in energy relative to a rotor and suggesting longer half-lives, which would make their experimental identification a considerable spectroscopic challenge.

V. SHAPE EVOLUTION IN TANTALUM ISOTOPES

The region around neutron number $N = 116$ has long been an interesting area for studying the transition between prolate and oblate nuclear shapes. In the heavier isotopic chains, particularly platinum ($Z = 78$) and osmium ($Z = 76$), this transition has been characterized through decades of spectroscopic investigation. The platinum isotopes have become something of a canonical reference for shape coexistence and γ -softness in this mass region, with the evolution of low-lying energy levels and $E2$ transition rates providing compelling evidence for a structural change that sharpens dramatically near $N = 116$ [55]. The osmium isotopes similarly display a well-documented shift in ground-state deformation across this neutron number, reinforced by both Coulomb excitation measurements and beyond-mean-field calculations [56–58]. Despite the knowledge available for higher- Z nuclei, the situation becomes less clear toward tantalum ($Z = 73$), where the coupling of the odd proton to the collective core adds further complexity to the theoretical description.

The rotational band populated in the isomeric decay of ^{189}Ta ($N = 116$) exhibits pronounced signature splitting in the moment of inertia, in contrast with the smoother behavior observed in nearby nuclei such as ^{185}Ta [15]. In axially symmetric nuclei, approximate conservation of the K quantum number leads to large reduced hindrance factors (f_v) for K -forbidden decays, typically $f_v \approx 30$ – 300 [2]. In the Ta isotopes, however, f_v decreases with increasing neutron number, from approximately 71 in ^{185}Ta [11] and approximately 27 in ^{187}Ta [13] to approximately 7 in ^{189}Ta [15]. This trend indicates a progressive weakening of axial symmetry toward $N = 116$ and motivates a systematic investigation of the shape evolution in Ta isotopes within the present approach.

In Fig. 5, the CNSB PESs are plotted for the $(\pi_p, \alpha_p)(\pi_n, \alpha_n) = (-, +1/2)(+, 0)$ configuration at $33/2^-$. As observed in the figure, the $33/2^-$ state exhibits an axial prolate shape in ^{179}Ta with $\gamma = -120^\circ$. With increasing neutron number toward $N = 116$, the energy minimum gradually shifts toward triaxial shapes with $\gamma \approx -85^\circ$, still close to the noncollective prolate energy. It exhibits a shape coexis-

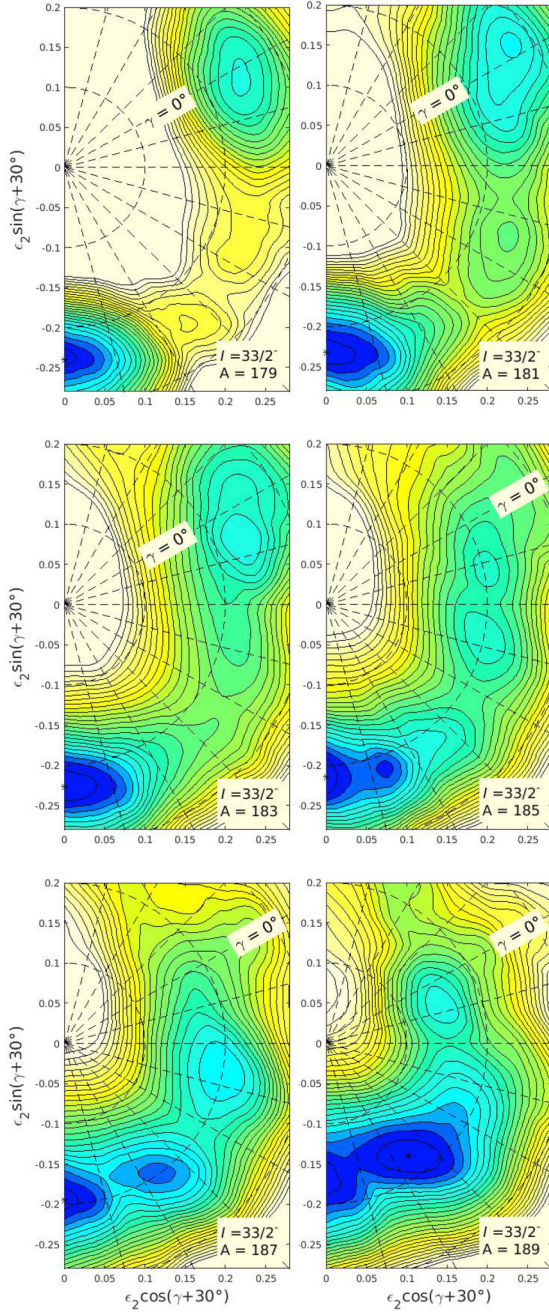


FIG. 5. Calculated CNSB PESs versus quadrupole deformation ε_2 and the triaxiality parameter γ of tantalum isotopes with $A = 179$ – 189 for the $I = 33/2^-$ state. Contour lines are separated by 0.2 MeV, and the γ plane is marked at 15° intervals. Dark regions represent low energy, and absolute minima are labeled with dots.

tence between the triaxial minimum and a residual prolate configuration in the heaviest isotope in our calculations, ^{189}Ta . Concurrently, the quadrupole deformation parameter ε_2 decreases from approximately 0.24 and gradually approaches 0.15 . This trend indicates a gradual softening of the potential toward triaxiality with increasing neutron number. The hexadecapole deformation parameter is $\varepsilon_4 \approx 0.06$ for the isotopes $A = 179$ – 187 where the minimum stays at a noncollective

prolate shape. A striking discontinuity then appears at ^{189}Ta , where ε_4 drops sharply to 0.027 , coinciding with the migration of the energy minimum toward the triaxial region at $\gamma \approx -85^\circ$. Such a reduction in the hexadecapole deformation parameter accompanying the shape transition from a noncollective prolate to a collective triaxial configuration has been theoretically reported in the $N \approx 116$ mass region [19].

Our calculations show that, for the heaviest isotope, ^{189}Ta where the yrast state has a triaxial shape, the neutron configuration at $33/2^-$ state has the structure $13/2[606] 11/2[615]$, corresponding to $I_n^\pi = 12^+$. The proton configuration $9/2[514]$ corresponds to $I_p^\pi = 9/2^-$, giving rise to the total spin-parity state $33/2^-$. Breaking of a neutron pair and the occupation of the $n13/2[606]$ and $n11/2[615]$ orbitals by the two unpaired quasiparticles result in a noticeable reduction of the neutron pairing gap Δ_n from approximately 0.9 MeV in the ground state to approximately 0.6 MeV in the $33/2^-$ state. The proton pairing gap remains comparable to that of the ground state.

A similar trend is also observed in the PESs of the configuration $(\pi_p, \alpha_p)(\pi_n, \alpha_n) = (+, -1/2)(+, 0)$ with a proton $7/2[404]$ configuration combining with the neutron $13/2[606] 11/2[615]$ configuration to make $I^\pi = 31/2^+$.

It is noteworthy that all dominant Nilsson orbitals contributing to the MQP configurations identified in Table III— $p7/2^+[404]$, $p9/2^-[514]$, $n9/2^+[624]$, $n11/2^+[615]$, and $n7/2^-[514]$ —carry large projections of angular momentum onto the nuclear symmetry axis ($\Omega \geq 7/2$), reflecting a deformation-aligned (DAL) character. In the DAL regime, the angular momentum of the quasiparticle configuration is generated by intrinsic alignment with the symmetry axis, in contrast with the rotation-aligned (RAL) scheme where nucleons align their angular momentum with the rotation axis perpendicular to the symmetry axis [25]. As the neutron number increases toward $N \approx 114$ – 116 , the competition between the RAL and DAL structures intensifies, reflecting the softening of the prolate PESs and signaling the onset of the prolate-to-oblate shape transition in this region.

VI. SUMMARY AND CONCLUSIONS

A systematic investigation of high- K isomeric states in the odd- A tantalum isotopes ^{179} – ^{189}Ta ($N = 106$ – 116) has been performed within the cranked Nilsson–Strutinsky (CNS) and cranked Nilsson–Strutinsky–Bogoliubov (CNSB) frameworks, covering one-proton, 3-qp, and 5-qp configurations.

In the present calculations, the single-quasiproton isomeric states, including the $9/2^-$ [514], $5/2^+$ [402], and $1/2^+$ [411] configurations, are well reproduced across the isotopic chain. The lowest-lying $19/2^+$ isomeric states are formed by coupling $p9/2^-[514]$ to the $n1/2^-[510]n11/2^+[615]$ configuration, where the large attractive residual interaction of approximately -221 keV plays a decisive role in stabilizing these states. For the $21/2^-$ isomers, the dominant configuration evolves from a pure three-quasiproton structure toward the pn^2 configuration $p7/2^+[404]n3/2^-[512]n11/2^+[615]$ with increasing neutron number, accompanied by probable configuration mixing that reflects the growing influence of neutron degrees of freedom in the heavier isotopes.

The present work shows the decisive role played by the two-quasineutron configurations $K^\pi = 8^-, 10^+$, and 10^- in driving the formation of MQP isomers across the isotopic chain. The systematic evolution of these neutron configurations with increasing neutron number governs which MQP states form energy traps and thereby acquire isomeric character, providing a unifying structural interpretation of the isomeric systematics across $^{179-189}\text{Ta}$.

Of particular significance are the predicted $K^\pi = 29/2^+$ 3-qp and $K^\pi = 41/2^+$ 5-qp isomers in ^{189}Ta , arising from the configurations $p9/2^- [514]$ and $p9/2^- [514]p7/2^+ [404]p5/2^+ [402]$ combining with $n9/2^- [505]n11/2^+ [615]$, respectively. Both states are calculated to lie at low excitation energies relative to rotor energies, suggesting significantly extended half-lives that place their experimental identification as a considerable spectroscopic challenge and a motivation for new experiments.

Finally, the CNSB PESs for the odd-A Ta isotopes at medium spins reveal a gradual evolution of nuclear shapes with increasing neutron number. While the lighter isotopes ($A \lesssim 187$) are characterized by stable noncollective prolate minima, a clear transition toward triaxial deformation emerges at $N \approx 114$, where the energy minimum shifts to $\gamma \approx -85^\circ$ and shape coexistence appears in ^{189}Ta . This structural evolution is accompanied by a reduction in ε_2 , a pronounced drop in ε_4 , and a weakening of pairing correlations due to the breaking of a neutron pair and occupation of the $n13/2[606]$ and $n11/2[615]$ orbitals. The dominant MQP configurations

are found to be strongly deformation-aligned, although increasing neutron number enhances the competition between deformation-aligned and rotation-aligned coupling schemes. Overall, these results indicate a softening of the prolate potential and the onset of triaxiality and shape transition in the $N \approx 114-116$ region.

The present calculations reproduce the main features of the available experimental level schemes in the neutron-rich tantalum mass region. Since the calculations are based on the Nilsson potential, the predicted excitation energies depend on the adopted Nilsson single-particle parameters. Further optimization of these parameters for this mass region is expected to improve the quantitative agreement with experiment and refine the predicted excitation energies.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this paper. Requests for further information or data should be sent to the authors.

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