

# Signs of nonmonotonic finite-volume corrections to $g_A$

Zack Hall <sup>1,2</sup> Dimitra A. Pefkou <sup>3,2</sup> Aaron S. Meyer <sup>4</sup> Thomas R. Richardson <sup>3,2</sup> Raúl A. Briceño <sup>3,2</sup> M. A. Clark <sup>5</sup>  
 Martin Hoferichter <sup>6</sup> Emanuele Mereghetti <sup>7</sup> Henry Monge-Camacho <sup>8</sup> Colin Morningstar <sup>9</sup> Amy Nicholson <sup>1</sup>  
 Pavlos Vranas <sup>4,2</sup> and André Walker-Loud <sup>2,3,4</sup>

<sup>1</sup>*Department of Physics and Astronomy, University of North Carolina, Chapel Hill, North Carolina 27516-3255, USA*

<sup>2</sup>*Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*

<sup>3</sup>*Department of Physics, University of California, Berkeley, California 94720, USA*

<sup>4</sup>*Lawrence Livermore National Laboratory, Livermore, California 94550, USA*

<sup>5</sup>*NVIDIA Corporation, 2701 San Tomas Expressway, Santa Clara, California 95050, USA*

<sup>6</sup>*Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, University of Bern, Sidlerstrasse 5, 3012 Bern, Switzerland*

<sup>7</sup>*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*

<sup>8</sup>*National Center For Computational Sciences Oak Ridge National Laboratory, Oak Ridge, Tennessee, USA*

<sup>9</sup>*Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA*



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We study finite-volume (FV) corrections to determinations of  $g_A$  via lattice quantum chromodynamics (QCD) using analytic results and numerical analysis. We observe that  $SU(2)$  heavy Baryon chiral perturbation theory does not provide an unambiguous prediction for the sign of the FV correction, which is not surprising when one also considers large- $N_c$  constraints on the axial couplings. We further show that nonmonotonic FV corrections are naturally allowed when one considers either including explicit  $\Delta$ -resonance degrees of freedom or one works to higher orders in the chiral expansion. We investigate the potential impact of these FV corrections with a precision study of  $g_A$  using models of FV corrections that are monotonic and nonmonotonic. Using lattice QCD data that is approximately at the 1% level of precision, we do not see significant evidence of nonmonotonic corrections. Looking forward to the next phase of lattice QCD calculations, we estimate that calculations that are between the 0.1% and 1% level of precision may be sensitive to these FV artifacts. Finally, we present an update of the CalLat prediction of  $g_A$  in the isospin limit with subpercent precision,  $g_A^{\text{QCD}} = 1.2674(96)$ .

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## I. INTRODUCTION

The nucleon axial coupling,  $g_A$ , is a critical quantity for determining many nuclear physics processes, from the primordial abundances of hydrogen and helium formed during big bang nucleosynthesis [1], to our theoretical understanding of solar fusion [2], and to relating precision measurements of nuclear  $\beta$  decay rates to Standard Model (SM) parameters [3]. Measurements of  $g_A$  from the  $\beta$ -asymmetry of ultracold neutron decays have yielded an experimental value at the 0.1% precision level, but with some tension between the results [4–13] that go into the Particle Data Group (PDG) average [14]. The most precise measurement has a total uncertainty of 0.04% [11].

There is currently a  $\sim 4\sigma$  discrepancy between beam [15–18] and bottle [19–27] measurements of the neutron lifetime ( $\tau_n$ ), which depends upon  $g_A$ , inspiring suggestions

of a light dark-matter particle [28]. However, a recent beam measurement at J-PARC [29] has been found to be consistent with the bottle measurements. Reference [30] points out that the bottle measurements are consistent with recent measurements of  $g_A$ . A theoretical prediction of  $g_A$  with a 0.2% precision could discriminate between the two lifetime measurements with the same precision as the beam measurements, using the relation [31,32],

$$\tau_n(1 + 3\lambda^2)|V_{ud}|^2 = 4902.3(1.3) \text{ s}, \quad (1.1)$$

where  $\lambda = g_A/g_V$  and  $V_{ud}$  is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element.

However, the value of  $g_A$  in this formula, and as reported by experiments and in the PDG, contains radiative quantum electrodynamics (QED) corrections. While many of these corrections are accounted for [3,33–40], it has recently been pointed out that there are previously unrealized structure-dependent QED corrections to  $g_A$  that may be as large as 2% [41]. This amount is estimated with  $SU(2)$  heavy baryon chiral perturbation theory (HB $\chi$ PT) [42,43] and thus dependent upon unknown low-energy constants (LECs). This means that presently lattice quantum chromodynamics (LQCD) calculations of  $g_A$  [44–50], which are performed in the isospin limit, cannot be directly

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compared to the experimental determination within the current percent-level of LQCD precision. References [51,52] propose strategies, such as LQCD + QED calculations that can be used to determine these QED corrections and thus determine the unknown LECs.

The importance of obtaining subpercent theoretical predictions for  $g_A$  is further exemplified in the context of precision tests of the first row of the CKM matrix. In the SM,  $\Delta_{\text{CKM}} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0$ , yet there currently exists a  $3\sigma$  deficit from unitarity [53] which could be due to beyond-the-SM (BSM) physics. Precision measurements of  $\tau_n$  and  $g_A$  are critical to render the determination of  $|V_{ud}|$  from neutron decay competitive with superallowed  $\beta$  decays [54–56]. While the aforementioned QED corrections to  $g_A$  do not impact the determination of  $|V_{ud}|$ , since they cancel in Eq. (1.1), comparison of LQCD predictions of  $g_A$  to the experimental measurements places competitive constraints on BSM right-handed charged currents (RHCCs) [57]. A recent study considering high-energy collider observables, low-energy charged-current processes, and electroweak precision observables has shown that, within an SM effective field theory (EFT) framework, scenarios including RHCCs are most preferred for explaining this unitarity tension [58]. At present, LQCD calculations of  $g_A$  allow one to test RHCCs at the level of  $\epsilon_R = -0.2(1.2) \times 10^{-2}$  [41,45,46] (using the normalization of Ref. [59]), while the unitarity deficit points to  $\epsilon_R = -0.69(27) \times 10^{-3}$  [53], strongly motivating further improvements.

Originally,  $g_A$  served as an important benchmark quantity to demonstrate that LQCD could precisely reproduce important, known experimental results of relevance to nuclear physics [60]. In recent years, it has become a key quantity for precision tests of the SM and for testing our low-energy understanding of QCD through stringent constraints on LECs of HB $\chi$ PT [61].

Motivated by a desire for subpercent precision results for  $g_A$ , we explore the effects of the finite-volume (FV) on LQCD calculations of this quantity and examine the behavior of the results as a function of volume ( $L^3$ ) and pion mass ( $m_\pi$ ). For sufficiently large volumes,  $m_\pi L \gg 1$ , artifacts due to the volume are exponentially suppressed,  $\mathcal{O}(e^{-m_\pi L})$  [62]. As a result, one might naïvely expect these systematic effects to be simple to parameterize and monotonically vanish as  $L \rightarrow \infty$ . However, prior to more recent results, there was a systematic deficit in LQCD results for  $g_A$  as compared with the PDG value which became more discrepant as  $m_\pi$  was reduced towards the physical pion mass. This led to speculation that  $g_A$  had particularly large FV corrections [63] (which was refuted in Ref. [64]) which may be the source of this discrepancy [65] and that  $m_\pi L > 6$  may be required to keep FV corrections  $\lesssim 1\%$  [66]. It is now understood that this discrepancy arose from uncontrolled excited state contamination [44,45,67].

Because the precision of LQCD calculations of  $g_A$  has increased, we scrutinize FV effects as they become more relevant as statistical uncertainties decrease. In particular, we note that the sign of the approach to infinite volume (IV) at heavy pion mass differs with the expectation from HB $\chi$ PT suggesting that these effects might be *nonmonotonic* with

respect to variations  $m_\pi$  and/or  $m_\pi L$ . With this context in mind, our main goal is to understand the potential shortcomings of previous treatments of FV that have ignored potential nonmonotonic FV effects and to present a new strategy to arrive at a final uncertainty that accounts for this FV systematic. We investigate the pion mass dependence of such FV trends and explore the sensitivity of a final quoted result for  $g_A$  to the use of various IV extrapolation formulae, including those predicted by HB $\chi$ PT. We take the opportunity to update the CalLat prediction from Refs. [45,68].

## II. THEORETICAL EXPECTATIONS AND NUMERICAL OBSERVATIONS

### A. Numerical observations of FV corrections

In Ref. [45], CalLat observed the central values of  $g_A$  trending upwards with decreasing volume at  $m_\pi \approx 220$  MeV, consistent with the leading FV prediction from  $SU(2)$  HB $\chi$ PT without explicit  $\Delta$  degrees of freedom (HB $\chi$ PT( $\Delta$ )) [69], though the trend was not statistically significant (see the upper panel of Fig. 1). The previously unpublished data at  $m_\pi \approx 310$  MeV plotted in the middle panel of Fig. 1 shows the opposite trend with mild statistical significance. In both cases, the precision of these results is also compatible with no FV dependence at the  $\sim 1\sigma$  level. All CalLat data used in this study is tabulated in Sec. IV. RQCD has computed  $g_A$  at several volumes (and lattice spacings in the range  $a \simeq 0.064$ – $0.086$  fm) for  $m_\pi \simeq 285$  MeV, showing a statistically significant trend that the FV corrections to  $g_A$  are negative at this pion mass [48]. This is seen on the bottom panel of Fig. 1, while the plotted RQCD results are listed in Table I.

### B. Expectations from $\chi$ PT and large $N_c$

From an EFT perspective, it is not surprising that the FV corrections to  $g_A$  are more complex than for other quantities, particularly in comparison with meson quantities. First, the chiral expansion for nucleon quantities is a series in  $\epsilon_\pi$ , defined as

$$\epsilon_\pi \equiv \frac{m_\pi}{\Lambda_\chi}, \quad \Lambda_\chi \equiv 4\pi F_\pi, \quad (2.1)$$

where  $F_\pi$  is the pion decay constant. For pions, the expansion is in powers of  $\epsilon_\pi^2$  and so higher-order chiral corrections are relatively more important for nucleons.

Second, several resonances are strongly coupled to the nucleon and of relatively low mass excitation. In particular, the mass gap to the  $\Delta$  resonance at the physical point is small,  $\Delta \equiv m_\Delta - m_N \simeq 293$  MeV, and decreases as the pion mass increases [70]. The  $\Delta$  resonances are also strongly coupled to the nucleon, both through QCD dynamics as well as through large electroweak couplings that govern  $N \rightarrow \Delta$  matrix elements. Therefore, the  $\Delta$ s make significant contributions to many quantities, adding another channel of important radiative virtual corrections [43,71–75].

Third, for  $g_A$ , as for other quantities, the large- $N_c$  expansion places constraints on radiative corrections. These constraints do not need to be imposed but emerge naturally from the Feynman rules in  $SU(2)$  HB $\chi$ PT with  $\Delta$  degrees of freedom

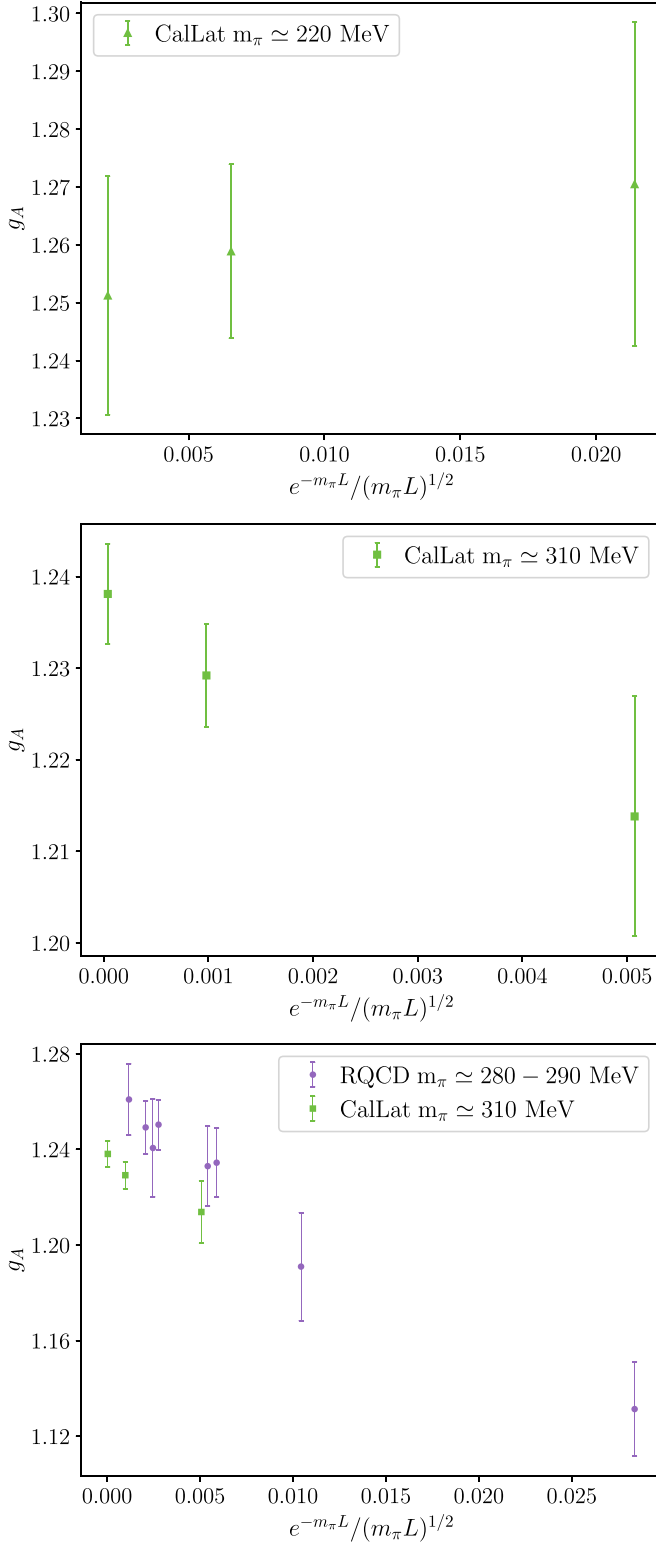


FIG. 1. Top: We plot the results from Ref. [45] for  $m_\pi \simeq 220$  MeV and  $a \simeq 0.12$  fm. Middle: The data point at the smallest volume with  $m_\pi \simeq 310$  MeV and  $a \simeq 0.12$  fm from Ref. [45], with two new ensembles at larger volumes analyzed for this work. Bottom: The volume dependence of data published by RQCD [48] at  $m_\pi \simeq (280-290)$  MeV.

(HB $\chi$ PT( $\Delta$ )). This theory has an additional small parameter to control the perturbative expansion and is sometimes referred to as the small-scale expansion [76],

$$\epsilon_\Delta = \frac{\Delta}{\Lambda_\chi}. \quad (2.2)$$

For  $g_A$ , there are significant cancellations between contributions with virtual  $\Delta$ -resonance states and those with virtual nucleon states [43]. These cancellations are evident with the use of phenomenological values of the LECs and become manifest when the large- $N_c$  expansion is used to relate them [71,73,77–79]. These cancellations also hold for the FV corrections, as we will now discuss.

In  $SU(2)$  HB $\chi$ PT( $\Delta$ ), the expression for  $g_A$  at next-to-leading order (NLO) in the chiral expansion is [69,80]<sup>1</sup>

$$\begin{aligned} g_A^{(\Delta)} = & g_0 \left[ 1 - \epsilon_\pi^2 \ln \epsilon_\pi^2 \right] + \epsilon_\pi^2 \left( 4\tilde{d}_{16}^{r,\Delta} - g_0^3 \right) \\ & - 2\epsilon_\pi^2 \ln \epsilon_\pi^2 \left[ g_0^3 + \frac{1}{9} g_0 g_{N\Delta}^2 + \frac{25}{81} g_\Delta^2 g_{N\Delta}^2 \right] \\ & - R\left(\frac{m_\pi^2}{\Delta^2}\right) g_{N\Delta}^2 \left[ \epsilon_\pi^2 \frac{32}{27} g_0 + \epsilon_\Delta^2 \left( \frac{76}{27} g_0 + \frac{100}{81} g_\Delta \right) \right] \\ & + \frac{32}{27} g_0 g_{N\Delta}^2 \epsilon_\pi^2 \left[ 1 + \ln \frac{4\epsilon_\Delta^2}{\epsilon_\pi^2} + \frac{\pi \epsilon_\pi}{\epsilon_\Delta} \right] \\ & + \frac{2}{81} g_{N\Delta}^2 \epsilon_\pi^2 (9g_0 + 25g_\Delta) [1 + \ln(4\epsilon_\Delta^2)], \end{aligned} \quad (2.3)$$

where the nonanalytic function is

$$R(z) = \begin{cases} \sqrt{1-z} \ln\left(\frac{1-\sqrt{1-z}}{1+\sqrt{1-z}}\right) + \ln(4/z), & z \leq 1 \\ 2\sqrt{z-1} \arctan(z) + \ln(4/z), & z > 1 \end{cases}, \quad (2.4)$$

and we have defined  $\tilde{d}_{16}^{r,\Delta} = (4\pi F)^2 d_{16}^{r,\Delta}$ . Each of the axial couplings,  $g_0$ ,  $g_{N\Delta}$ , and  $g_\Delta$ , represent the leading-order (LO) contribution to the axial charges of the nucleon, the nucleon-to- $\Delta$  transition, and  $\Delta$ , respectively. As such, they all scale like  $N_c$  [77,84,85]. Because  $g_A$  scales like  $N_c$ , and the radiative corrections scale like  $g^3/F^2 \sim N_c^2$  ( $F^2 \sim N_c$ ) with  $g \in \{g_0, g_\Delta, g_{N\Delta}\}$ , all corrections with three powers of these axial couplings must exactly cancel in the large- $N_c$  limit.

One can consider the large- $N_c$  expansion about  $N_c \rightarrow \infty$  or about large but finite  $N_c$ , which leads to two different relations at LO in the large- $N_c$  expansion between the axial couplings [86]:

$$N_c = \infty: \quad \Delta = 0, \quad g_{N\Delta} = \frac{3}{2}g_0, \quad g_\Delta = -\frac{9}{5}g_0, \quad (2.5)$$

$$N_c \neq \infty: \quad \Delta = 0, \quad g_{N\Delta} = \frac{6}{5}g_0, \quad g_\Delta = -\frac{9}{5}g_0. \quad (2.6)$$

Both the former [75,77,78,84,87] and the latter [73,79,85,88–91] commonly appear in the literature. The difference between these two is formally  $\mathcal{O}(N_c^{-2})$ . Using the expansion about  $N_c \rightarrow \infty$ , Eq. (2.5), the cancellation between all terms that

<sup>1</sup>This expression is derived in Appendix A from that given in Ref. [69].  $d_{16}^{r,\Delta}$  is defined in such a way that for  $\Delta \rightarrow \infty$  it coincides with  $d_{16}'$  in the conventions of Refs. [81–83]. The  $\Delta$  parameters are related to those of Ref. [75] by  $g_{N\Delta} = \sqrt{2}h_A$  and  $g_\Delta = -g_1$ .

scale as  $g_0^3$  is explicit. This can be verified by inserting these relations into Eq. (2.3), yielding the result

$$\lim_{N_c \rightarrow \infty} g_A^{(\Delta)} = g_0 + \epsilon_\pi^2 \left[ -g_0 \ln \epsilon_\pi^2 + 4\tilde{d}_{16}^{r,\Delta} \right]. \quad (2.7)$$

This expression is contrasted with that in  $SU(2)$  HB $\chi$ PT( $\Delta$ ) at same order in  $\epsilon_\pi$ ,

$$g_A^{(\Delta)} = g_0 + \epsilon_\pi^2 \left[ -g_0(1 + 2g_0^2) \ln \epsilon_\pi^2 + 4\tilde{d}_{16}^r - g_0^3 \right]. \quad (2.8)$$

As  $g_0 \simeq 1.2$ , the coefficient of the  $\epsilon_\pi^2 \ln \epsilon_\pi^2$  term in HB $\chi$ PT( $\Delta$ ) is much larger than one would expect starting from HB $\chi$ PT( $\Delta$ ). That LQCD results for  $g_A$  show such mild pion mass dependence is evidence that the large- $N_c$  expansion provides at least a good qualitative guide for understanding chiral

corrections to  $g_A$  as it naturally reduces the size of corrections at a given order through the large- $N_c$ -predicted opposite-sign contributions from virtual  $\Delta$  corrections [43,79,80,92].

Similar constraints hold when considering FV corrections, which at NLO are given by  $\delta_{\text{FV}}^{(2)} + \delta_{\text{FV}}^{(2,\Delta)}$  [69], where

$$\delta_{\text{FV}}^{(2)} = g_0 \frac{8}{3} \epsilon_\pi^2 \left[ g_0^2 F_1^{(2)}(m_\pi L) + F_3^{(2)}(m_\pi L) \right], \quad (2.9)$$

$$\delta_{\text{FV}}^{(2,\Delta)} = g_0 \frac{8}{3} \epsilon_\pi^2 \left[ g_{N\Delta}^2 \left( 1 + \frac{25}{81} \frac{g_\Delta}{g_0} \right) F_2^{(2)}(m_\pi L, \Delta L) + g_{N\Delta}^2 F_4^{(2)}(m_\pi L, \Delta L) \right], \quad (2.10)$$

and the FV functions  $F_i^{(2)}$  are defined in Ref. [69],

$$\begin{aligned} F_1^{(2)}(x) &= \sum_{\vec{n} \neq 0} \left[ K_0(x|\vec{n}|) - \frac{K_1(x|\vec{n}|)}{x|\vec{n}|} \right], \\ F_3^{(2)}(x) &= -\frac{3}{2} \sum_{\vec{n} \neq 0} \frac{K_1(x|\vec{n}|)}{x|\vec{n}|}, \\ F_2^{(2)}(x, y) &= -\sum_{\vec{n} \neq 0} \left[ \frac{K_1(x|\vec{n}|)}{x|\vec{n}|} + \frac{y^2 - x^2}{x^2} K_0(x|\vec{n}|) - \frac{y}{x^2} \int_x^\infty dz \frac{2z K_0(z|\vec{n}|) + (y^2 - x^2)|\vec{n}| K_1(z|\vec{n}|)}{\sqrt{z^2 + y^2 - x^2}} \right], \\ F_4^{(2)}(x, y) &= \frac{8}{9} \sum_{\vec{n} \neq 0} \left[ \frac{K_1(x|\vec{n}|)}{x|\vec{n}|} - \frac{\pi e^{-x|\vec{n}|}}{2y|\vec{n}|} - \frac{y^2 - x^2}{x^2 y} \int_x^\infty dz \frac{z K_0(z|\vec{n}|)}{\sqrt{z^2 + y^2 - x^2}} \right]. \end{aligned} \quad (2.11)$$

$K_n(x)$  are modified Bessel functions of the second kind. As with the IV corrections, a careful implementation of the large- $N_c$  limit at fixed quark mass, Eq. (2.5), leads to an exact cancellation of all corrections that scale like  $g_0^3$ . In this limit, one would predict that the leading FV correction to  $g_A$  is given by

$$\delta_{\text{FV}}^{(2)} + \delta_{\text{FV}}^{(2,\Delta)} \rightarrow -4\epsilon_\pi^2 g_0 \sum_{\vec{n} \neq 0} \frac{K_1(m_\pi L|\vec{n}|)}{m_\pi L|\vec{n}|}. \quad (2.12)$$

In contrast, the leading FV correction in HB $\chi$ PT( $\Delta$ ) is

$$\begin{aligned} \delta_{\text{FV}}^{(2)} &= 4\epsilon_\pi^2 g_0 \sum_{\vec{n} \neq 0} \left[ \frac{2}{3} g_0^2 K_0(m_\pi L|\vec{n}|) \right. \\ &\quad \left. - \left( 1 + \frac{2}{3} g_0^2 \right) \frac{K_1(m_\pi L|\vec{n}|)}{m_\pi L|\vec{n}|} \right]. \end{aligned} \quad (2.13)$$

For values of  $m_\pi L \gtrsim 3$  and  $g_0 \simeq 1.2$ , the first term is larger than the second term and so the predicted FV correction is positive, while the leading prediction from HB $\chi$ PT( $\Delta$ ) after application of the large- $N_c$  expansion is negative for any value of  $m_\pi L$ .

The full NLO prediction for the HB $\chi$ PT( $\Delta$ ) FV correction will lie somewhere between these limits. We estimate them as follows. First, we plot the predicted FV correction from HB $\chi$ PT( $\Delta$ ) as a function of  $\epsilon_\pi$  while setting  $m_\pi L = 4$  and  $g_0 = 1.20(05)$ , see Fig. 2. Next, we set  $\Delta = 293$  MeV and then approximate  $g_{N\Delta}$  and  $g_\Delta$  according to Eq. (2.5) while adding an extra uncertainty from the large  $N_c$  expansion,

e.g.,  $g_{N\Delta} = \frac{3}{2}(g_0 + (0, 1/N_c^2))$ . We plot the same correction except using the finite  $N_c$  relation  $g_{N\Delta} = \frac{6}{5}(g_0 + (0, 1/N_c^2))$ , Eq. (2.6) and, finally, the prediction with Eq. (2.5) and  $\Delta = 0$ . The spread of these predictions demonstrates that our present knowledge is insufficient to predict the sign of the NLO FV correction without further input. It would be ideal to have LQCD calculations of the  $N \rightarrow \Delta$  axial transition matrix elements and the  $\Delta$  axial charge [93,94], which could be used to

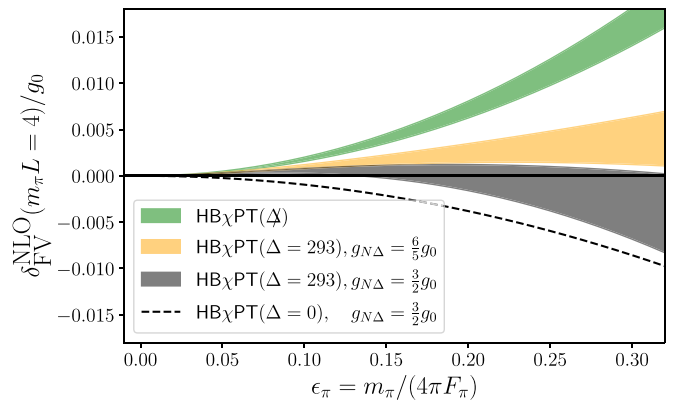


FIG. 2. We plot the predicted NLO FV corrections to  $g_A$  using  $SU(2)$  HB $\chi$ PT( $\Delta$ ) and the large- $N_c$  expansion, with two different large- $N_c$  relations for  $g_{N\Delta}$ . We also plot the predicted FV correction from HB $\chi$ PT( $\Delta$ ) and the large- $N_c$  expansion also with  $\Delta = 0$ . All corrections are plotted with  $m_\pi L = 4$ .

make more precise predictions for these FV corrections, to be compared with more precise LQCD results of  $g_A$  at different volumes and pion masses. To this end, we note that also the full next-to-next-to-leading-order ( $N^2$ LO) results including  $\Delta$  degrees of freedom and FV corrections are available, see Appendix B for the complete expressions.

### III. QUANTIFYING THE FV UNCERTAINTY

When accounting for FV corrections to  $g_A$ , the strategy used by most groups is to use the asymptotic form of the leading prediction from  $SU(2)$  HB $\chi$ PT( $\Delta$ )

$$\delta_{\text{FV}} = c_V m_\pi^2 \frac{e^{-m_\pi L}}{\sqrt{m_\pi L}}, \quad (3.1)$$

which can be obtained from Eq. (2.13) by expanding the Bessel functions for large argument and replacing the coefficient predicted by HB $\chi$ PT( $\Delta$ ) with  $c_V$ . This parameter is then determined by fitting results at heavier than physical pion mass, typically in a global analysis simultaneously with discretization corrections as well as quark mass dependence [44,46,48,49]. Under the assumption that the FV corrections are nonmonotonic versus pion mass, an important question to resolve is at what level of precision will the above model introduce an error?

In order to estimate the potential impact of the FV model, we perform the following exercise: First, we pick a specific model to describe the pion mass dependence and two models of the FV corrections, one that is monotonic and one that supports nonmonotonicity. In order to create this model, we use  $SU(2)$  HB $\chi$ PT( $\Delta$ ) at  $N^2$ LO, for which  $g_A$  is given by

$$g_A = g_0 + \delta_\chi^{(2)} + \delta_\chi^{(3)} + \delta_{\text{FV}}^{(2)} + \delta_{\text{FV}}^{(3)}, \quad (3.2)$$

where the superscript,  $n$ , denotes the order in  $\epsilon_\pi^n$  at which a term contributes (see also Appendix B for a summary of all corrections). The IV chiral corrections are given by [81]

$$\begin{aligned} \delta_\chi^{(2)} &= \epsilon_\pi^2 \left[ -g_0(1 + 2g_0^2) \ln \epsilon_\pi^2 + 4\tilde{d}_{16}^r - g_0^3 \right], \\ \delta_\chi^{(3)} &= \epsilon_\pi^3 g_0 \frac{2\pi}{3} \left[ 3(1 + g_0^2) \frac{4\pi F}{M_0} + 4(2\tilde{c}_4 - \tilde{c}_3) \right], \end{aligned} \quad (3.3)$$

where the LECs are defined as

$$F = \lim_{m_\pi \rightarrow 0} F_\pi, \quad \tilde{c}_i = (4\pi F) c_i, \quad \tilde{d}_i^r = (4\pi F)^2 d_i^r, \quad (3.4)$$

such that  $\tilde{c}_i$  and  $\tilde{d}_i$  are dimensionless. Note that the use of  $\epsilon_\pi$  in these expressions, rather than  $m_\pi/(4\pi F)$ , induces higher-order corrections beginning at  $\mathcal{O}(\epsilon_\pi^4)$ , which should be accounted for if the next-to-next-to-next-to-leading ( $N^3$ LO) expression is used to fit  $g_A$  results (see Refs. [45,95]).

The NLO FV corrections are given in Eq. (2.9), while the  $N^2$ LO ones are

$$\begin{aligned} \delta_{\text{FV}}^{(3)} &= \epsilon_\pi^3 g_0 \frac{2\pi}{3} \left\{ g_0^2 \frac{4\pi F}{M_0} F_1^{(3)}(m_\pi L) \right. \\ &\quad \left. - \left[ \frac{4\pi F}{M_0} (3 + 2g_0^2) + 4(2\tilde{c}_4 - \tilde{c}_3) \right] F_3^{(3)}(m_\pi L) \right\}, \end{aligned} \quad (3.5)$$

TABLE I. Data from Ref. [48] shown in Fig. 1(c), and used in the FV study of Sec. III A. We also estimate the value of  $\epsilon_\pi$  for each CLS ensemble by scaling  $\epsilon_\pi$  on our a12m310XL ensemble with the ratio of pion masses where we convert our pion mass to MeV units using the scale setting in Ref. [96]:  $\epsilon_\pi = \epsilon_\pi^{\text{a12m310XL}} \times (m_\pi/m_\pi^{\text{a12m310XL}})$ .

| Ensemble | $m_\pi$ [MeV] | $\epsilon_\pi$ | $m_\pi L$ | $a$ [fm] | $g_A$     |
|----------|---------------|----------------|-----------|----------|-----------|
| N101     | 281           | 0.2216         | 5.86      | 0.086    | 1.261(15) |
| N451     | 289           | 0.2279         | 5.34      | 0.076    | 1.249(11) |
| N401     | 287           | 0.2263         | 5.30      | 0.076    | 1.241(20) |
| N450     | 287           | 0.2263         | 5.30      | 0.076    | 1.250(10) |
| N201     | 287           | 0.2263         | 4.49      | 0.064    | 1.234(14) |
| N200     | 286           | 0.2255         | 4.47      | 0.064    | 1.233(17) |
| H105     | 281           | 0.2216         | 3.91      | 0.086    | 1.191(23) |
| S201     | 290           | 0.2287         | 3.02      | 0.064    | 1.131(20) |

with

$$\begin{aligned} F_1^{(3)}(x) &= \sum_{\vec{n} \neq \vec{0}} \frac{K_{\frac{1}{2}}(x|\vec{n}|)}{\sqrt{\frac{\pi}{2}x|\vec{n}|}} x|\vec{n}| = \sum_{\vec{n} \neq \vec{0}} e^{-x|\vec{n}|}, \\ F_3^{(3)}(x) &= \sum_{\vec{n} \neq \vec{0}} \frac{K_{\frac{3}{2}}(x|\vec{n}|)}{\sqrt{\frac{\pi}{2}x|\vec{n}|}} = \sum_{\vec{n} \neq \vec{0}} \frac{e^{-x|\vec{n}|}}{x|\vec{n}|}. \end{aligned} \quad (3.6)$$

With these functional pieces defined, we proceed to study the potential impact of different choices in modeling the FV corrections.

#### A. FV sensitivity study: A cautionary tale

To study the impact of the FV model used, we undertake the following exercise:

- (1) We begin by fitting the CalLat results at lattice spacing  $a \simeq 0.12$  fm provided in Sec. IV to the IV expression,

$$g_A = g_0 + \delta_\chi^{(2)} + \delta_\chi^{(3)}, \quad (3.7)$$

utilizing only the largest volume at a given pion mass. We use this fit to provide an estimate for the LECs

$$g_0^{\text{FV}}, \quad \tilde{c}_{34}^{\text{FV}} = 2\tilde{c}_4^{\text{FV}} - \tilde{c}_3^{\text{FV}}, \quad (3.8)$$

that we will use to estimate the FV corrections. The Gaussian distributed priors used in this analysis are listed in Table II along with their corresponding posterior values. In  $\delta_\chi^{(3)}$ , we set  $M_0 = 939$  MeV and  $F = 92.3$  MeV as the difference using these values and their respective chiral-limit values is higher order in the chiral expansion. We focus on a single lattice spacing to isolate the FV effects.

- (2) We use the RQCD results [48] at  $m_\pi \simeq (280\text{--}290)$  MeV, listed in Table I and shown in the bottom panel of Fig. 1, to construct two models of FV corrections:

- (1) Monotonic FV model: Using the value of  $g_0^{\text{FV}}$  from step one, we fit the RQCD data to the model

$$g_A^{285} = g_0^{285} + f_2 \delta_{\text{FV}}^{(2)}(g_0^{\text{FV}}), \quad (3.9)$$

with priors and posteriors for  $g_0^{285}$  and  $f_2$  given in Table II.

TABLE II. For each step in Sec. III A, we list the fit priors (top) and posteriors (bottom) used in the FV study comparison between monotonic and nonmonotonic  $\chi$ PT models.

| Step   |             | $g_0$       | $\tilde{d}_{16}^r$ | $\tilde{c}_{34}$ |
|--------|-------------|-------------|--------------------|------------------|
|        |             |             |                    |                  |
| Step 1 | Eq. (3.7)   | 1.00(50)    | -1(10)             | 1(10)            |
|        |             | 1.212(20)   | -4.55(51)          | 0.52(42)         |
| Step 2 | Eq. (3.9)   | $g_A^{285}$ | $f_2$              | $f_3$            |
|        |             | 1.27(3)     | -3.4(2.0)          |                  |
|        | Eq. (3.10)  | 1.2613(65)  | -4.18(66)          |                  |
|        |             | 1.27(3)     |                    | 6(3)             |
| Step 3 | Eq. (3.11a) | 1.2398(54)  |                    | 10.4(1.4)        |
|        |             | $g_0$       | $\tilde{d}_{16}^r$ | $\tilde{c}_{34}$ |
|        | Eq. (3.11b) | 1.0(5.0)    | -1(10)             | 1(10)            |
|        |             | 1.248(19)   | -5.39(49)          | 1.12(37)         |
|        |             | 1.0(5.0)    | -1(10)             | 1(10)            |
|        |             | 1.219(19)   | -4.77(48)          | 0.68(38)         |

- (2) Nonmonotonic FV model: Using the values of  $g_0^{\text{FV}}$  and  $\tilde{c}_{34}^{\text{FV}}$  from step 1, as well as the same values of  $M_0$  and  $F$ , we fit the RQCD data to the model

$$g_A^{285} = g_0^{285} + \delta_{\text{FV}}^{(2)}(g_0^{\text{FV}}) + f_3 \delta_{\text{FV}}^{(3)}(g_0^{\text{FV}}, \tilde{c}_{34}^{\text{FV}}), \quad (3.10)$$

where  $f_3$  priors and posterior values are listed in Table II.

- (3) We then fit the full  $a \simeq 0.12$  fm data set listed in Table III with two models

$$g_A^{\text{mono}} = g_0 + \delta_{\chi}^{(2)}(g_0^{\text{IV}}) + \delta_{\chi}^{(3)}(g_0^{\text{IV}}, \tilde{c}_{34}^{\text{IV}}) + f_2 \delta_{\text{FV}}^{(2)}(g_0^{\text{FV}}), \quad (3.11a)$$

$$g_A^{\text{non-mono}} = g_0 + \delta_{\chi}^{(2)}(g_0^{\text{IV}}) + \delta_{\chi}^{(3)}(g_0^{\text{IV}}, \tilde{c}_{34}^{\text{IV}}) + \delta_{\text{FV}}^{(2)}(g_0^{\text{FV}}) + f_3 \delta_{\text{FV}}^{(3)}(g_0^{\text{FV}}, \tilde{c}_{34}^{\text{FV}}), \quad (3.11b)$$

where the LECs in the IV and FV functions are kept separate, with the FV values determined in step 1 used in the FV functions and the values of  $f_2$  or  $f_3$  taken from step 2. The resulting analysis is used to predict  $g_A$  at the physical pion mass for which we find

$$g_A^{(\text{III.11a})} = 1.286(12), \\ g_A^{(\text{III.11b})} = 1.266(12), \quad (3.12)$$

showing tension between these models for the final prediction even at the  $\mathcal{O}(1\%)$  uncertainty level.

- (4) We repeat step 3, while artificially decreasing the statistical uncertainty on the  $a \simeq 0.12$  fm data by a common rescaling factor and compare the prediction from these two models as a function of the final total uncertainty on the predicted value of  $g_A$  at the physical pion mass and IV. The results from this study are plotted in Fig. 3.

This exercise demonstrates that the choice of FV model can have a statistically significant systematic impact on the predicted final value of  $g_A$ . For results at the percent level and more precise, it is important to incorporate a set of models with sufficient flexibility to support nonmonotonic FV corrections and compare the extrapolation with monotonic FV models. Otherwise, one may invariably introduce an unquantified systematic uncertainty that is comparable to, or larger than, the statistical uncertainty.

This particular study was performed under the assumption that the RQCD results could be used to determine the

TABLE III. Results of  $\epsilon_\pi$ ,  $m_\pi L$ ,  $a/w_0$ , and  $g_A$  on each ensemble used in this work. The results in the top part of the table are from Ref. [45] and the four ensembles in the bottom are updated (a12m130) or new. The ensembles generated by CalLat used the MILC code with the a12m310L, a12m310XL, and a15m135XL ones also using QUDA [97,98]. We also list the source (first publication) where each ensemble was used.

| Ensemble  | Source     | $\epsilon_\pi$ | $m_\pi L$ | $a/w_0$     | $g_A$      |
|-----------|------------|----------------|-----------|-------------|------------|
| a15m400   | Ref. [45]  | 0.30374(54)    | 4.8       | 0.9201(12)  | 1.2154(60) |
| a12m400   | Ref. [45]  | 0.29840(52)    | 5.8       | 0.73335(43) | 1.2162(97) |
| a09m400   | Ref. [45]  | 0.29819(54)    | 5.8       | 0.53516(77) | 1.2097(80) |
| a15m350   | Ref. [45]  | 0.27412(52)    | 4.2       | 0.9101(11)  | 1.197(14)  |
| a12m350   | Ref. [45]  | 0.27063(69)    | 5.1       | 0.72701(58) | 1.235(14)  |
| a09m350   | Ref. [45]  | 0.26949(58)    | 5.1       | 0.52910(73) | 1.227(15)  |
| a15m310   | Ref. [99]  | 0.24957(36)    | 3.8       | 0.90261(73) | 1.215(12)  |
| a12m310   | Ref. [99]  | 0.24486(51)    | 4.5       | 0.72134(62) | 1.214(13)  |
| a09m310   | Ref. [99]  | 0.24619(44)    | 4.5       | 0.52449(61) | 1.235(11)  |
| a15m220   | Ref. [100] | 0.18084(31)    | 4.0       | 0.88849(63) | 1.274(14)  |
| a12m220   | Ref. [100] | 0.18220(44)    | 4.3       | 0.70942(55) | 1.259(15)  |
| a12m220L  | Ref. [100] | 0.18155(42)    | 5.4       | 0.70972(35) | 1.251(21)  |
| a12m220S  | Ref. [100] | 0.18418(57)    | 3.3       | 0.71078(81) | 1.270(28)  |
| a09m220   | Ref. [100] | 0.18197(37)    | 4.7       | 0.51578(32) | 1.2519(85) |
| a12m310L  | This work  | 0.24295(40)    | 6.0       | 0.72084(46) | 1.2292(57) |
| a12m310XL | Ref. [96]  | 0.24318(39)    | 9.0       | 0.72124(31) | 1.2381(55) |
| a12m130   | Ref. [100] | 0.11343(32)    | 3.9       | 0.70368(30) | 1.266(13)  |
| a15m135XL | Ref. [101] | 0.11486(20)    | 4.9       | 0.88347(31) | 1.273(13)  |

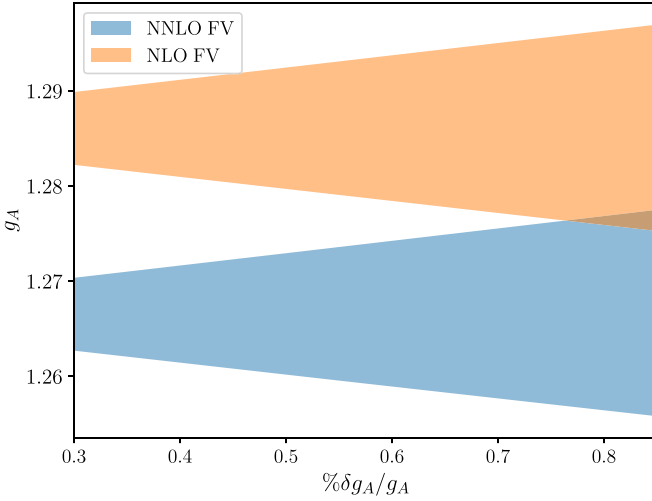


FIG. 3. Using the strategy outlined in the FV study, we plot the value of  $g_A$ , as function of the relative percent uncertainty,  $\delta g_A/g_A$ . The top band is determined by utilizing Eq. (3.11a) with NLO FV corrections, while the bottom band includes corrections up to N<sup>2</sup>LO, Eq. (3.11b).

coefficients of the FV model. A more realistic study would add these models in a global analysis, including discretization effects. One challenge in isolating the FV corrections to  $g_A$  is that they are comparable in size to the observed discretization effects. In principle, these corrections should be independent through the separation of short- and long-distance scales of the problem. In practice, because the FV corrections are assessed by fitting coefficients multiplying FV correction terms, their contributions become entangled with discretization corrections through the analysis procedure and the two effects can influence each other, including changing the sign of the leading FV or discretization corrections depending upon the model for these systematic effects.

#### IV. UPDATED $g_A$ RESULT WITH MDWF/HISQ

We update the results in Ref. [45], which utilized a mixed lattice action [102,103] with  $N_f = 2 + 1 + 1$  highly improved staggered quarks (HISQ) [104] in the sea and Möbius domain wall fermion (MDWF) [105,106] quarks in the valence sector. The gauge links were smoothed with gradient flow [107,108] prior to using them in the Dirac operator, resulting in our MDWF/HISQ action [109]. Some of the HISQ ensembles were generated by the MILC Collaboration [99,100], while the rest were generated by the CalLat Collaboration [45,96,101] including a new ensemble first used in this work.

Table III lists the resulting values of  $g_A$ ,  $\epsilon_\pi$ ,  $m_\pi L$ , and  $a/w_0$  on each ensemble needed for the extrapolation analysis, where  $w_0$  is the gradient flow scale defined in Ref. [110]. The short-hand naming key [111] indicates the approximate lattice spacing and pion mass on the given ensemble, with a12m130 indicating a lattice spacing of  $a \simeq 0.12$  fm and a pion mass of  $m_\pi \simeq 130$  MeV, for example. For details of the input parameters of the action, see Refs. [96,101,109]. The analysis of the correlation functions is described in Ref. [45]: A constant is fit to the  $m_{\text{res}}$  correlation function and multistate

fits are performed to the pion correlation function to determine  $m_\pi$  and  $F_\pi$  on each ensemble; the values of  $\hat{g}_A$  and  $\hat{g}_V$ , which give  $g_A = (Z_A/Z_V)(\hat{g}_A/\hat{g}_V)$ , with  $Z_A/Z_V - 1 \lesssim 1 \times 10^{-4}$  [45], are determined with a multistate fit to the nucleon two-point correlation function and the “Feynman-Hellmann” correlation functions described in Ref. [112]. Further details of the correlator analysis will be provided in a forthcoming publication.

The results in the upper part of the table are from Ref. [45]. The results on the a12m310L and a12m310XL ensemble are new to this work and enable a volume study at  $m_\pi \simeq 310$  MeV in addition to that possible with the a12m220 set of ensembles. The results on the a12m130 and a15m135XL ensembles were first reported in Ref. [68]. For a12m130, we changed the quark smearing from that in Ref. [45] to that in Ref. [101] and increased the number of sources per configuration from 3 to 32. The result a15m135XL is obtained from a newly generated ensemble [96,101] and uses 8 sources per configuration on 1000 configurations.

In order to predict  $g_A^{\text{QCD}}$  from QCD in the isospin-symmetric, physical-pion-mass, IV, and continuum limit, we follow the strategy in Ref. [45]. We use Bayes model averaging in order to have a data-driven extrapolation/interpolation. For a given model, the Bayes *evidence* is used as a relative probability that said model is correct, given the data. In the limit that all distributions are Gaussian, the evidence can be computed analytically and is given by the Gaussian Bayes factor (GBF). These GBF factors can then be used to perform a weighted average over all models considered in the analysis. For further details, see Ref. [45]. A more detailed discussion of this and similar strategies is presented in Ref. [113].

#### A. Models considered

We do not assume that  $SU(2)$  HB $\chi$ PT( $\Delta$ ) is correct or not, and so in addition to using the EFT-derived chiral extrapolation formula, we also use a set of polynomials in  $\epsilon_\pi$  (Taylor expansions about  $\epsilon_\pi = 0$ ) to parameterize the pion mass dependence. Similarly, we do not assume that the EFT-derived FV corrections provide the correct description of the FV effects or not, and so we use a set of models of FV corrections that are both fixed by  $SU(2)$  HB $\chi$ PT( $\Delta$ ) as well as ones where the functional form comes from the EFT, but the numerical prefactors do not, as well as models that are untethered to  $\chi$ PT and only use our universal knowledge that leading FV corrections are set by the pion cloud effects [114] and that pions are derivatively coupled to other hadrons, leading to  $\epsilon_\pi^2$  prefactors. Another advantage of the Bayesian analysis is that when the FV corrections are modeled with free coefficients, as well as the discretization corrections, there are flat directions in the  $\chi^2$  that are lifted by the prior contributions to the augmented  $\chi_{\text{aug}}^2$  [115]

$$\chi_{\text{aug}}^2 = \chi^2 + \sum_i \left( \frac{\mu_i - \tilde{\mu}_i}{\sigma_{\mu_i}} \right)^2, \quad (4.1)$$

where  $\tilde{\mu}_i$  and  $\sigma_{\mu_i}$  are the prior mean and width for parameter  $\mu_i$ . In addition to the Bayes model averaging strategy, we also perform a frequentist analysis, which yields consistent individual and final model average results.

For the  $\chi$ PT models, we only consider the expressions from  $SU(2)$  HB $\chi$ PT( $\Delta$ ). This is because our numerical results are not sufficient to constrain the new parameters describing the axial  $N \rightarrow \Delta$  transitions or the axial coupling of the  $\Delta$ , and we wish to minimize the influence from phenomenological input. For the quantitative  $\chi$ PT model, we use the expressions given in Eqs. (2.9) and (3.5).

The quark-mass dependence of  $g_A$  is parameterized using four models,

$$g_{A,\chi} = g_0 + \delta_\chi^{(2)} + \delta_\chi^{(3)}, \quad (4.2a)$$

$$g_{A,\text{pol}}^{(1)} = c_0 + c_1\epsilon_\pi + c_2\epsilon_\pi^2, \quad (4.2b)$$

$$g_{A,\text{pol}}^{(2)} = c_0 + c_2\epsilon_\pi^2 + c_3\epsilon_\pi^3, \quad (4.2c)$$

$$g_{A,\text{pol}}^{(3)} = c_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4. \quad (4.2d)$$

In the above we include the HB $\chi$ PT( $\Delta$ ) prediction together with simple Taylor expansions in different powers of  $\epsilon_\pi^n$ , as was done in Refs. [45,68]. We keep the total number of parameters  $c_i$  fixed across the polynomial models. Equation (4.2c) was not used in the previous work. This model can be considered as HB $\chi$ PT( $\Delta$ ) with the chiral logarithms set to zero.

Meanwhile, the nonmonotonic FV corrections models used are

$$g_{A,\text{FV}} = \delta_{\text{FV}}^{(2)} + \delta_{\text{FV}}^{(3)}, \quad (4.3a)$$

$$g_{A,\text{FV}}^{f_2} = f_2\delta_{\text{FV}}^{(2)} + \delta_{\text{FV}}^{(3)}, \quad (4.3b)$$

$$g_{A,\text{FV}}^{f_3} = \delta_{\text{FV}}^{(2)} + f_3\delta_{\text{FV}}^{(3)}, \quad (4.3c)$$

$$g_{A,\text{FV}}^{f_2,f_3} = f_2\delta_{\text{FV}}^{(2)} + f_3\delta_{\text{FV}}^{(3)}. \quad (4.3d)$$

The above models use the full  $N^2$ LO expressions for the FV corrections, and in the case of the later three include different choices of unknown coefficients  $f_2$  and  $f_3$ . The purpose of these latter three models is to use  $\chi$ PT to guide the analytic form of the IV corrections, but allow for the strength of the FV corrections to become decoupled from the values of the LECs needed to describe the IV pion mass dependence of model  $g_{A,\chi}$ . We augment this set with four additional  $\chi$ PT-agnostic models

$$g_{A,\text{FV}}^{\text{mon-1}} = f_2\epsilon_\pi^2 F_1^{(2)}, \quad (4.4a)$$

$$g_{A,\text{FV}}^{\text{mon-2}} = f_2\epsilon_\pi^2 F_3^{(2)}, \quad (4.4b)$$

$$g_{A,\text{FV}}^{\text{mon-3}} = f_2\epsilon_\pi^2 F_1^{(3)}, \quad (4.4c)$$

$$g_{A,\text{FV}}^{\text{mon-4}} = f_2\epsilon_\pi^2 F_3^{(3)}, \quad (4.4d)$$

which are all monotonic by construction, and have their  $m_\pi L$  dependence parameterized by the FV functions defined in Eqs. (2.11) and (3.6). The purpose of these additional models is to diagnose whether statistically significant evidence of the nonmonotonic behavior predicted by  $\chi$ PT, which we observed as a trend in the  $a \simeq 0.12$  fm data and the RQCD data in the previous section, can be detected in a global analysis of our full dataset.

Finally, the discretization effects are parameterized up to  $\mathcal{O}(a^4)$  in the Symanzik expansion [116,117]. For this mixed action with valence fermions that respect chiral symmetry, the leading discretization effects begin at  $\mathcal{O}(a^2)$  [118]. We

parameterize them as

$$g_{A,a}^{(2)} = a_2\epsilon_a^2, \quad (4.5a)$$

$$g_{A,a}^{(4)} = a_2\epsilon_a^2 + b_4\epsilon_a^2\epsilon_\pi^2 + a_4\epsilon_a^4, \quad (4.5b)$$

with

$$\epsilon_a^2 = \frac{a^2}{4w_0^2}. \quad (4.6)$$

The values for  $\epsilon_a$  for the ensembles used besides a12m310L were previously determined in Ref. [96].

We consider these two models for the discretization dependence together with all combinations of the pion mass and FV dependence listed above, resulting in 64 different models. We fit the parameters of each model by  $\chi^2$  minimization using the `lsqfit` package, and include a discussion on the choice of Bayesian priors in Appendix C. We use the posterior distributions to interpolate to the physical point. We take the FLAG definition of the isospin symmetric point [119]

$$m_\pi^{\text{iso,phys}} = 135 \text{ MeV},$$

$$F_\pi^{\text{iso,phys}} = \frac{130.5}{\sqrt{2}} = 92.3 \text{ MeV},$$

$$\epsilon_\pi^{\text{iso,phys}} = 0.1164. \quad (4.7)$$

The resulting central values and errors for  $g_A$  are model-averaged weighted by the Bayes factors of the corresponding model fit as was done previously [45,68].

## B. Final results and comparison with other works

The final result we obtain for  $g_A = 1.2674(96)$ , with a percentage uncertainty of 0.76%,

$$g_A = 1.2674(79)^s(28)^\chi(05)^{\text{FV}}(38)^a(26)^M = 1.2674(96). \quad (4.8)$$

The uncertainty breakdown into the statistical (s), chiral extrapolation ( $\chi$ ), FV, and discretization (a) is done by differentiating the final posterior prediction at the physical point with respect to the relevant priors. The statistical uncertainty is with respect to the  $g_A$  data and  $\epsilon_\pi$ , while the FV uncertainty arises from the  $f_2$  and  $f_3$  coefficients when relevant, for example. The final model-selection uncertainty arises from the variance of the models with respect to the Bayes model average value; see Refs. [45,101] for further details.

The model-averaged chiral fit to  $g_A$  is plotted in the continuum and IV limits as a function of  $\epsilon_\pi$  in Fig. 4, along with the individual ensemble values extrapolated to the continuum and to IV. The individual model results including the priors and posteriors for all parameters along with the corresponding Bayes factors are listed in Tables VI and VII of Appendix D.

In Fig. 5, we plot the updated  $g_A$  and compare it against other LQCD calculations. On the lower part of the plot, we include the latest FLAG [119] averages which, just like all LQCD results, correspond to QCD-only predictions. In Ref. [41], it was estimated that pion-induced radiative corrections to  $g_A$  are

$$g_A^{\text{PDG}} = g_A^{\text{QCD}}(1 + \delta_{\text{RC}}), \quad \delta_{\text{RC}} = 2.0(6) \times 10^{-2}. \quad (4.9)$$

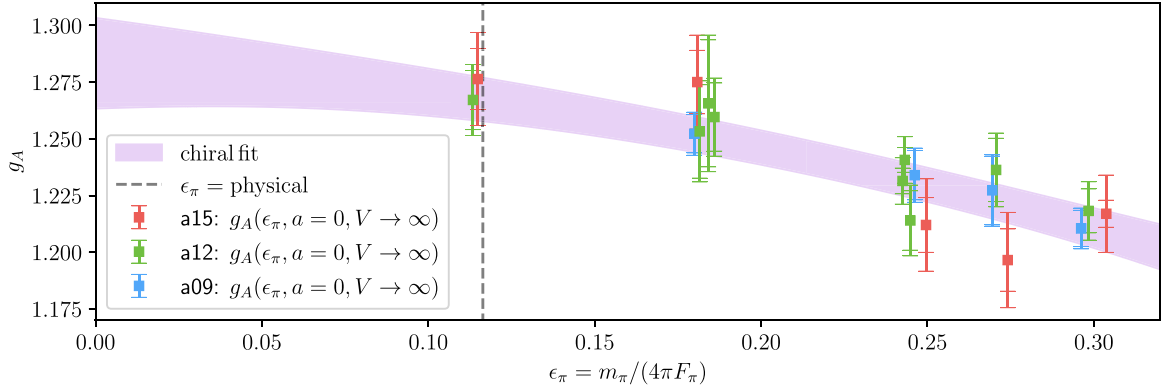


FIG. 4. Our final fit for the pion mass dependence of  $g_A$  model-averaged over all 64 models considered, as described in Sec. IV A, plotted along the values of  $g_A$  extrapolated to the continuum and IV limits. The outer error bars correspond to the uncertainty of the extrapolated results, while the inner ones to the uncertainties of the original values listed in Table III. Some of the data have been slightly shifted in the  $x$  axis for visibility.

We propagate these estimates to the FLAG  $N_f = 2 + 1$  and  $N_f = 2 + 1 + 1$  predictions, and plot the resulting values, denoted as FLAG + QED<sup>†</sup>, on the bottom of the plot, just above the PDG value  $g_A = 1.2754(13)$  [14].

At our current LQCD precision, the FV dependence of our data is small and modeling these corrections contributes the least to the final uncertainty. The result of a global analysis using just the  $\chi$ PT and  $\chi$ PT-inspired nonmonotonic models of Eq. (4.3) is  $g_A^{\text{non-mon}} = 1.2672(96)$ , which is consistent with the value  $g_A^{\text{mon}} = 1.2674(96)$  obtained by the monotonic FV models of Eq. (4.4).

### C. $SU(2)$ HB $\chi$ PT( $\Delta$ ) extrapolations

It is interesting to examine the  $SU(2)$  HB $\chi$ PT( $\Delta$ ) extrapolations in more detail. This is both because this theory serves as a foundation for many modern applications of chiral EFT to predicting the spectrum and properties of nuclei, see Ref. [135] for a recent review. It also enables us to more stringently test our low-energy understanding of nuclear physics by examining the convergence properties of the theory as well as comparing the LECs determined from comparisons with LQCD versus experimental data.

The first interesting observation is that the  $SU(2)$  HB $\chi$ PT( $\Delta$ ) extrapolations have the lowest likelihood of any of the extrapolation models, as can be seen from the logGBF values in Table VII. In fact, the strict  $SU(2)$  HB $\chi$ PT( $\Delta$ ) model without the  $f_2$  or  $f_3$  FV coefficients [those using Eq. (4.2a) for  $m_\pi$  dependence and Eq. (4.3a) for FV corrections] is the least favored of all fits. At the same time, it should be noted that the final extrapolated value is perfectly consistent with all other models. What likely drives these fits to have a lower likelihood than the Taylor expansion fits is that the  $\epsilon_\pi^2 \ln \epsilon_\pi^2$  have large coefficients, and thus the counterterm contributions and higher-order corrections must cancel against each other in order to produce such a mild pion mass dependence. This is contrasted with the Taylor models which all have  $\mathcal{O}(1)$  or smaller coefficients in the expansion, and thus provide a rapidly converging expansion to describe the numerical results. It will be interesting to see if an extrapolation with explicit  $\Delta$ s, that exhibits milder pion

mass dependence, is more favored. Such an extrapolation with only LQCD input will require calculations of  $N \rightarrow \Delta$  and  $\Delta \rightarrow \Delta$  axial matrix elements. See Ref. [94] for promising first steps in this direction.

The second interesting observation concerns the combination of LECs  $2c_4 - c_3$  that appears in Eq. (3.3). A recent phenomenological determination from N<sup>3</sup>LO analysis of pion-nucleon ( $\pi N$ ) scattering data [136,137] found the following values for the LECs  $c_3$  and  $c_4$ ,<sup>2</sup>

$$\begin{aligned} c_3 &= -5.61(6) \text{ GeV}^{-1}, \\ c_4 &= 4.26(4) \text{ GeV}^{-1}, \\ 2c_4 - c_3 &= 14.1(1) \text{ GeV}^{-1}. \end{aligned} \quad (4.10)$$

In our analysis using Eq. (4.2a) to describe the pion mass dependence of  $g_A$ , we utilized the dimensionless versions of these LECs, Eq. (3.4). From Table VII, one sees that the posterior values of  $\tilde{c}_{34}$  for the  $SU(2)$  HB $\chi$ PT( $\Delta$ ) fit models range from 0.76 to 0.81. Converting to  $\text{GeV}^{-1}$  using  $F_\pi = 92.3 \text{ MeV}$ , the value of  $2c_4 - c_3$  determined from this analysis

<sup>2</sup>The counting of chiral orders refers to the nontrivial ones, and since chiral corrections to  $g_A$  start at  $\mathcal{O}(\epsilon_\pi^2)$ , the N<sup>3</sup>LO results from Refs. [136,137] are consistent with the N<sup>2</sup>LO corrections to  $g_A$ . This distinction matters since the indicated errors, propagated from the  $\pi N$  input, are subleading compared to the differences between determinations at different chiral orders.

TABLE IV. Convergence pattern of  $g_A$  at the physical point (upper table). Dimensionless prefactors of nonanalytic contributions in Eq. (3.3) and LECs (lower table).

| $\delta_\chi^{(n)}(\epsilon_\pi^{\text{phys}})$ | $g_0$     | $\delta_\chi^{(2)}$ | $\delta_\chi^{(3)}$         |                                    |
|-------------------------------------------------|-----------|---------------------|-----------------------------|------------------------------------|
|                                                 | 1.226(15) | -0.007(11)          | 0.0507(53)                  |                                    |
| Source                                          | $g_0$     | $g_0(1 + 2g_0^2)$   | $4\tilde{d}_{16}^r - g_0^3$ | $\delta_\chi^{(3)}/\epsilon_\pi^3$ |
| Phenomenology                                   | 1.2(1)    | 4.7(1.0)            | —                           | 187(18)                            |
| LQCD analysis                                   | 1.22(2)   | 4.9(0.2)            | -21(2)                      | 32(3)                              |

becomes

$$2c_4 - c_3 = (0.66-0.70) \text{ GeV}^{-1}, \quad (4.11)$$

which is 20 times smaller than the phenomenological estimate above. The small priors given to  $\tilde{c}_{34}$  are not the source of this discrepancy. These values were chosen to optimize the Bayes factor of the fits. If instead a prior width of 50 is used, then the posterior value of  $\tilde{c}_{34} = 0.85(25)$  or similar is found.

This discrepancy for the LECs observed here is a particularly extreme case, but similar effects have been encountered in other examples, including the determination from  $\pi N$  scattering in the first place [75,136,137]. That is, the large values of  $c_3$  and  $c_4$  entering at subleading loop level generate sizable corrections that can differ from case to case, and therefore do not necessarily drop out when comparing observables. In the  $\pi N$  case, this implies that the  $\text{HB}\chi\text{PT}(\Delta)$  theory is not even accurate enough to relate consistently the expansion of the amplitude around subthreshold and threshold kinematics. The situation improves appreciably when the  $\Delta$  resonance is included as an explicit degree of freedom [75], as the large loop corrections are reduced, but, for reasons currently not understood, also when resumming higher orders in the  $1/M_0$  expansion into a covariant formulation. Understanding this convergence behavior is important, as the LECs are used extensively to predict two-body nuclear currents and three-body nuclear forces [138–140], see, e.g., Ref. [141] for a discussion how to account for  $c_i$  chiral uncertainties in the axial current. The present work suggests that  $g_A$  is another quantity particularly sensitive to this issue. Future work that includes the  $\Delta$  resonance as an explicit degree of freedom, and/or employs covariant formulations of baryon  $\chi\text{PT}$ , could thus provide valuable new perspectives. The discrepancy in the values of these  $c_i$  is also important for  $\text{HB}\chi\text{PT}$  analyses of the nucleon form factors. For example, Ref. [142] used the phenomenological values as input to chiral extrapolations of the nucleon axial form factor. It is not presently known how sensitive these results would be to significant changes in the values of  $c_3$  and/or  $c_4$ . Understanding the convergence of the chiral expansion is also important for the radiative corrections to  $g_A$ , which, at NLO, depend on the combination  $c_4 - c_3$  [41]. Also in this case, it will be important to extend the calculation to include the  $\Delta$  resonance and to thoroughly study the convergence.

It is also interesting to examine the size of the coefficients of the nonanalytic terms in Eq. (3.3) as well as the resulting order-by-order convergence of  $g_A$ . The magnitude of the dimensionless prefactors in front of the nonanalytic contributions provides a measure of how sensitive a given quantity may be to infrared pion loop effects. These coefficients should be  $\mathcal{O}(1)$  if the breakdown scale of the theory were really  $\Lambda_\chi \simeq 4\pi F \simeq M_0$ , while larger values, e.g., due to enhancements by factors of  $\pi$  or resonance contributions in the LECs, indicate a lower breakdown scale of the theory. In Table IV, we show the convergence pattern at the physical pion mass and also these dimensionless prefactors and LECs. One striking observation is how large the prefactor of the  $\delta_\chi^{(3)}$  term is, particularly with the use of the phenomenological values of  $c_3$  and  $c_4$ , but also with those determined from the analysis of

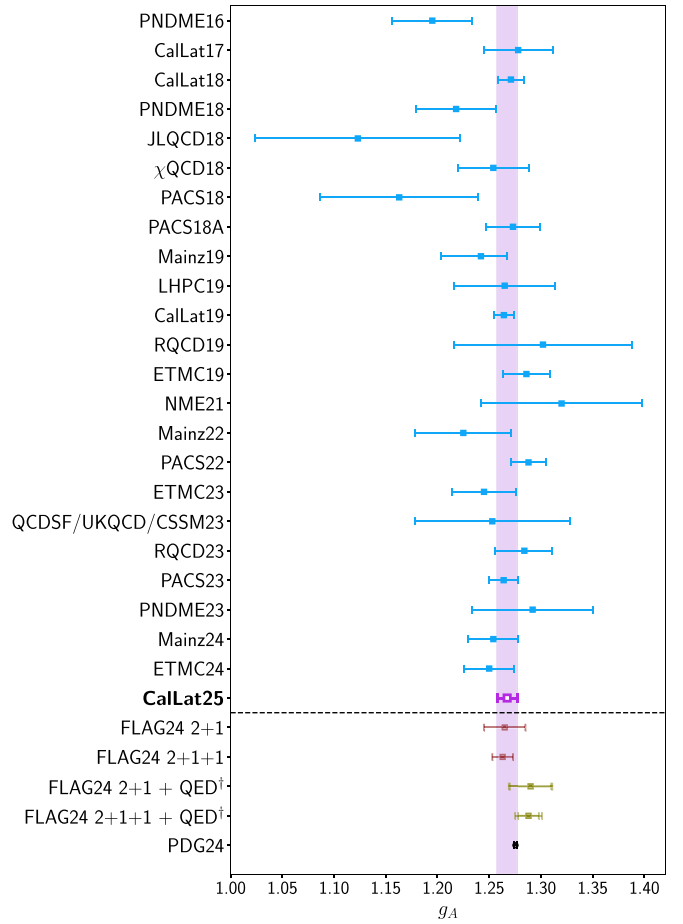


FIG. 5. Comparison of our final value for  $g_A$  against several LQCD collaborations [44–50,68,120–134], the FLAG average [119], the FLAG average modified to include an estimate of the QED corrections [41], and the PDG average [14].

the LQCD results. We also note that at the physical pion mass, the analytic and nonanalytic contributions to  $\delta_\chi^{(2)}$  nearly cancel each other out, leaving a convergence pattern where  $|\delta_\chi^{(3)}| > 5 \times |\delta_\chi^{(2)}|$  rather than a natural pattern of  $|\delta_\chi^{(3)}| \simeq \epsilon_\pi |\delta_\chi^{(2)}|$ . In Fig. 6, we plot the contributions to  $g_A$  normalized by the

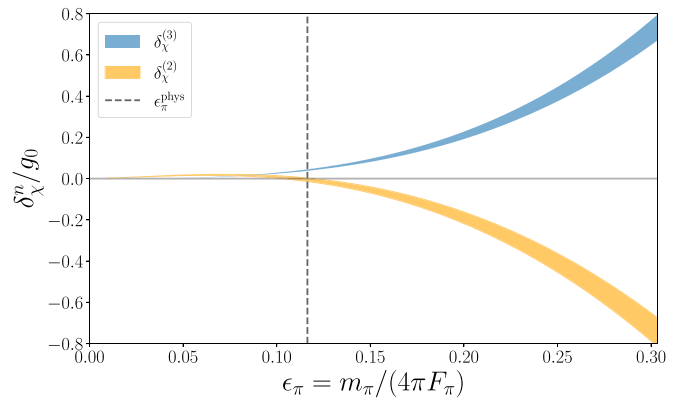


FIG. 6. We plot the resulting chiral contributions to  $g_A$  normalized by  $g_0$  as a function of the pion mass. The dashed line marks the physical point.

LO result. The NLO contribution,  $\delta_\chi^{(2)}$ , changes sign near the physical pion mass. For values of  $\epsilon_\pi$  slightly smaller than  $\epsilon_\pi^{\text{phys}}$  and larger, the convergence pattern does not look healthy:  $|\delta_\chi^{(3)}| > |\delta_\chi^{(2)}|$  in this region and for  $\epsilon_\pi \gtrsim \epsilon_\pi^{\text{phys}}$  these contributions are of opposite sign and become comparable in magnitude to  $g_0$ .

#### D. Comparison with other chiral extrapolations

Extrapolations of  $g_A$  in the literature fall into two broad groups: (1) Extrapolations that are a subset of that performed in this work [44,45,67,120,125,142,143] and (2) groups that include explicit  $\Delta$  degrees of freedom and also fix some LECs from phenomenological extractions [60,79,80,92,144–146]. It is worth pointing out that first results have been obtained with only near-physical pion masses [50]. In all cases, when the FV corrections have been implemented, it has only been with the functional form arising from the leading FV correction, Eq. (3.1), or variants of this form that only track the  $e^{-m_\pi L}$  dependence.

Extrapolations in the first group similarly noted that the observed  $m_\pi$  dependence in LQCD results for  $g_A$  are not well described by the NLO  $SU(2)$  HB $\chi$ PT( $\Delta$ ) formula, while a simple polynomial provided a good description. The inclusion of the  $N^2\text{LO } m_\pi^3$  term enables a controlled extrapolation but with strong competition between terms [45,49], as also noted in this work. Reference [92] performed a simultaneous extrapolation also with the axial form factor along with some LECs taken from phenomenology.

In contrast, Refs. [79,80,92,144–146] have emphasized the importance of including explicit  $\Delta$  degrees of freedom, with Refs. [79,80,92] highlighting the cancellation between virtual nucleon and  $\Delta$  loops that is predicted in the large  $N_c$  limit, which improves the convergence of the chiral expansion of  $g_A$ . Reference [146] also noted that the  $\Delta$  extrapolation function for  $g_A$  derived with the relativistic formulation of baryon  $\chi$ PT does not alleviate the tension between predictions of  $g_A$  using phenomenological LECs and the LQCD results at  $m_\pi > m_\pi^{\text{phys}}$ . References [144,145] performed a simultaneous extrapolation with the axial form factor as well. Reference [145] predicts a discontinuous function of both  $M_N$  and  $g_A$  versus  $m_\pi$  at  $m_\pi^{\text{discont.}} = 373_{-12}^{+18}$  MeV. There is currently no LQCD evidence for such a discontinuity, but this could be investigated with a large volume LQCD calculation closer to this predicted  $m_\pi^{\text{discont.}}$  point.

In all cases including explicit  $\Delta$ , these extrapolations require the use of phenomenologically determined LECs as there is not sufficient LQCD results to constrain them, in particular the LECs describing the  $N \rightarrow \Delta$  transition and the axial charge of the  $\Delta$ . In this work, we are interested in a complementary question of understanding both the convergence pattern of baryon  $\chi$ PT as well as comparing and contrasting the values of LECs determined from QCD calculations with those determined phenomenologically. Utilizing phenomenologically determined LECs would not allow for such a comparison. Until there are LQCD results for the  $N \rightarrow \Delta$  and  $\Delta \rightarrow \Delta$  axial matrix elements, including the  $N\pi$  initial/final states required to interpret the FV matrix elements [147–150], it will not be possible to reliably determine these

axial LECs from LQCD calculations. Further, before tackling a more extensive analysis including the axial form factor, which requires another small parameter, the momentum transfer  $Q^2$ , our focus is first on understanding the axial coupling.

#### V. DISCUSSION AND CONCLUSION

A theoretical prediction of  $g_A$  with a subpercent precision can contribute to tests of the SM, as well as put constraints on BSM right-handed currents [57] that offer a possible explanation for the first-row CKM unitarity tension [58]. At this level of precision, systematic uncertainties from FV and discretization corrections must be scrutinized more carefully to ensure they remain as small as or smaller than the statistical precision. LQCD calculations of  $g_A$  all have small discretization errors that are currently compatible with a zero slope with respect to powers of the lattice spacing [44–50], while some of them display statistically significant FV corrections. Therefore, in this work, we have investigated FV corrections in greater detail than has been previously explored in the literature.

Our previous results [45] on three volumes at  $m_\pi \simeq 220$  MeV showed a trend, albeit not statistically significant, that is consistent with the predicted FV correction from  $SU(2)$  HB $\chi$ PT( $\Delta$ ), Eq. (2.9). Subsequently, we generated new results at different volumes at  $m_\pi \simeq 310$  MeV which showed the opposite trend. Further, the RQCD results at  $m_\pi \simeq 285$  MeV also showed this opposite trend with statistical significance, see Fig. 1, hinting at the possibility of nonmonotonic FV corrections. As discussed in Sec. II B, nonmonotonic FV corrections to  $g_A$  would not be surprising as there are significant cancellations from radiative corrections to  $g_A$  between virtual contributions involving nucleons and  $\Delta$ -resonance states. These cancellations are predicted from large- $N_c$  considerations and the cancellations also manifest in the predicted FV corrections.

In Sec. IV, we performed a thorough analysis of our results, Table III, using various models for the pion mass dependence and various models for the FV corrections. We report an updated value of  $g_A$ , Eq. (4.8), with a 0.76% relative uncertainty. One interesting finding from our analysis using  $SU(2)$  HB $\chi$ PT( $\Delta$ ) to describe the pion mass dependence of  $g_A$  is that the value of the LEC  $2c_4 - c_3$  we determine is an order of magnitude smaller than the state-of-the-art phenomenological determinations [137]. Going forward, it is important to understand this discrepancy as the phenomenological LECs are utilized to make predictions for properties of nuclei through modern applications of chiral EFT [135]. As in similar cases [75], studies including the  $\Delta$  resonance as an explicit degree of freedom and/or using covariant formulations could shed light on the convergence properties of the chiral expansion in the baryon sector. A more global analysis of various properties of the nucleon including  $M_N$ ,  $g_A$ ,  $M_\Delta$ , axial form factors, etc., could provide more clarity on this issue. Such studies will have to wait until LQCD calculations of  $N \rightarrow \Delta$  and  $N \rightarrow N\pi$  become more common such that the LECs contributing to these quantities can be constrained from LQCD results.

In the FV correction, we included models that were monotonic by construction and models that allow for nonmonotonic behavior. With our current level of precision, our numerical

results cannot distinguish between these two types of FV corrections. In Sec. III A, we studied the potential sensitivity of using monotonic versus nonmonotonic FV models under the assumption that the numerical results are sufficiently precise to constrain the FV corrections, finding that as the total uncertainty is reduced below a percent, the FV systematic can become a larger uncertainty than the statistical uncertainty. We conclude that going forward, for subpercent determinations of  $g_A$ , it is important to utilize models of FV corrections that support nonmonotonic as well as monotonic behavior, at least if the final result depends upon heavier than physical pion mass results to perform all extrapolations.

As noted in Sec. I, LQCD results for  $g_A$  must not be compared to the PDG value at their current quoted precision because of unaccounted for QED corrections, estimated to be 2% using  $SU(2)$  HB $\chi$ PT( $\Delta$ ) and phenomenological values of some of the LECs [41]. The estimate relied upon Naïve dimensional analysis [151] as one of the LECs has not been constrained. Therefore, to take advantage of the present and future precision of LQCD results of  $g_A$ , it is necessary to carry out a determination of these QED corrections with LQCD + QED calculations, as suggested for example in Ref. [51,52].

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### DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

### APPENDIX A: NLO $\chi$ PT FORMULA FOR $g_A^{(\Delta)}$

The FV corrections to  $g_A$  in  $SU(2)$  HB $\chi$ PT( $\Delta$ ) that arise at NLO were derived in Ref. [69]. The IV formulae were also provided and, as written there, given by

$$\begin{aligned}\delta_\chi^{(2)} &= -i \frac{2}{3F^2} \left[ \frac{3}{2} g_0 R_1(m_\pi, \mu) + 4g_0^3 J_1(m_\pi, 0, \mu) \right], \\ \delta_\chi^{(2,\Delta)} &= -i \frac{2}{3F^2} \left[ 4 \left( g_{N\Delta}^2 g_0 + \frac{25}{81} g_{N\Delta}^2 g_\Delta \right) J_1(m_\pi, \Delta, \mu) \right. \\ &\quad \left. - \frac{32}{9} g_{N\Delta}^2 g_0 N_1(m_\pi, \Delta, \mu) \right],\end{aligned}\quad (\text{A1})$$

where the nonanalytic functions are defined as

$$\begin{aligned}iR_1(m, \mu) &= \frac{m^2}{(4\pi)^2} \ln \frac{m^2}{\mu^2}, \\ iJ_1(m, \Delta, \mu) &= \frac{3}{4} \frac{\Delta^2}{(4\pi)^2} \left[ \left( \frac{m^2}{\Delta^2} - 2 \right) \ln \frac{m^2}{\mu^2} + 2 \sqrt{1 - \frac{m^2}{\Delta^2}} \ln \frac{1 - \sqrt{1 - \frac{m^2}{\Delta^2}}}{1 + \sqrt{1 - \frac{m^2}{\Delta^2}}} \right], \\ iN(m, \Delta, \mu) &= \frac{3}{4} \frac{\Delta^2}{(4\pi)^2} \left[ \left( \frac{m^2}{\Delta^2} - \frac{2}{3} \right) \ln \frac{m^2}{\mu^2} + \frac{2\pi}{3} \frac{m^3}{\Delta^3} + \frac{2}{3} \left( 1 - \frac{m^2}{\Delta^2} \right)^{3/2} \ln \frac{1 - \sqrt{1 - \frac{m^2}{\Delta^2}}}{1 + \sqrt{1 - \frac{m^2}{\Delta^2}}} \right],\end{aligned}\quad (\text{A2})$$

and one can show the latter two functions have the limit

$$iN_1(m, 0, \mu) = iJ_1(m, 0, \mu) = \frac{3}{4} \frac{m^2}{(4\pi)^2} \ln \frac{m^2}{\mu^2}. \quad (\text{A3})$$

It is convenient to define subtracted functions for the purpose of chiral extrapolations:

$$\begin{aligned} \bar{J}_1(m, \Delta, \mu) &= J_1(m, \Delta, \mu) - J_1(m, 0, \mu), \\ \bar{N}_1(m, \Delta, \mu) &= N_1(m, \Delta, \mu) - N_1(m, 0, \mu), \end{aligned} \quad (\text{A4})$$

such that the parameter  $g_0$  in the extrapolation functions is  $g_0 = \lim_{m_\pi \rightarrow 0} g_A$ . This amounts to a renormalization of  $g_0$  in the Lagrangian, which we call  $\hat{g}_0$ ,

$$g_0 = \hat{g}_0(\mu) + \frac{4\Delta^2}{(4\pi F)^2} \ln \frac{\Delta^2}{\mu^2} \frac{\hat{g}_{N\Delta}^2}{81} (57\hat{g}_0 + 25\hat{g}_\Delta). \quad (\text{A5})$$

The NLO loop contribution to  $g_A$  can then be expressed as

$$\begin{aligned} \delta_\chi^{(2)} + \delta_\chi^{(2,\Delta)} \Big|_{\text{loop}} &= - (g_0 + 2g_0^3) \epsilon_\pi^2 \ln \frac{m_\pi^2}{\mu^2} \\ &\quad - \frac{8}{3} \left( g_{N\Delta}^2 g_0 + \frac{25}{81} g_{N\Delta}^2 g_\Delta \right) \frac{i\bar{J}_1(m_\pi, \Delta, \mu)}{\Lambda_\chi^2} \\ &\quad + \frac{64}{27} g_0 g_{N\Delta}^2 \frac{i\bar{N}_1(m_\pi, \Delta, \mu)}{\Lambda_\chi^2}, \end{aligned} \quad (\text{A6})$$

where

$$\begin{aligned} \frac{i\bar{J}_1(m_\pi, \Delta, \mu)}{\Lambda_\chi^2} &= \frac{3}{4} \epsilon_\Delta^2 \left[ \frac{m_\pi^2}{\Delta^2} \ln \frac{m_\pi^2}{\mu^2} + 2R\left(\frac{m_\pi^2}{\Delta^2}\right) \right], \\ \frac{i\bar{N}_1(m_\pi, \Delta, \mu)}{\Lambda_\chi^2} &= \frac{3}{4} \epsilon_\pi^2 \ln \frac{m_\pi^2}{\mu^2} + \frac{\pi}{2} \epsilon_\pi^2 \frac{m_\pi}{\Delta} + \frac{1}{2} \epsilon_\pi^2 \ln \frac{4\Delta^2}{m_\pi^2} \\ &\quad + \frac{1}{2} (\epsilon_\Delta^2 - \epsilon_\pi^2) R\left(\frac{m_\pi^2}{\Delta^2}\right), \end{aligned} \quad (\text{A7})$$

and  $R(z)$  is defined in Eq. (2.4). Finally, we redefine  $\tilde{d}_{16}^{r,\Delta}$  such that the decoupling of the  $\Delta$  resonance is explicit in the limit  $\Delta \rightarrow \infty$  and so that our definition agrees with Refs. [81–83]. Inserting these into the previous expression and subtracting all corrections proportional to  $g_{N\Delta}^2$  in the heavy- $\Delta$  limit yields Eq. (2.3).

## APPENDIX B: N<sup>2</sup>LO FV FUNCTIONS WITH EXPLICIT $\Delta$ s

The full N<sup>2</sup>LO result including  $\Delta$  degrees of freedom and FV corrections can be expressed as

$$\begin{aligned} g_A &= g_0 + \delta_\chi^{(2)} + \delta_\chi^{(3)} + \delta_{\text{FV}}^{(2)} + \delta_{\text{FV}}^{(3)} + \delta_\chi^{(2,\Delta)} + \delta_\chi^{(3,\Delta)} + \delta_{\text{FV}}^{(2,\Delta)} \\ &\quad + \delta_{\text{FV}}^{(3,\Delta)}, \end{aligned} \quad (\text{B1})$$

where the terms without “ $\Delta$ ” superscript refer to the IV chiral and FV corrections in  $SU(2)$  HB $\chi$ PT( $\Delta$ ) as indicated, while the last four terms refer to the additional effects in  $(\Delta)$ . The individual corrections are given by

$$\begin{aligned} \delta_\chi^{(2)} &= \epsilon_\pi^2 [-g_0(1 + 2g_0^2) \ln \epsilon_\pi^2 + 4\tilde{d}_{16}^r - g_0^3], \\ \delta_\chi^{(3)} &= \epsilon_\pi^3 g_0 \frac{2\pi}{3} \left[ 3(1 + g_0^2) \frac{4\pi F}{M_0} + 4(2\tilde{c}_4 - \tilde{c}_3) \right], \\ \delta_{\text{FV}}^{(2)} &= \frac{8}{3} \epsilon_\pi^2 \sum_{\vec{n} \neq \vec{0}} \left[ g_0^3 K_0(x_n) - \frac{g_0(3 + 2g_0^2)}{2x_n} K_1(x_n) \right], \quad x_n = m_\pi L |\vec{n}|, \\ \delta_{\text{FV}}^{(3)} &= -\epsilon_\pi^3 g_0 \frac{2\pi}{3} \sum_{\vec{n} \neq \vec{0}} \left[ \frac{4\pi F}{M_0} (3 + g_0^2(2 - x_n)) + 4(2\tilde{c}_4 - \tilde{c}_3) \right] \frac{e^{-x_n}}{x_n}, \\ \delta_\chi^{(2,\Delta)} &= -2\epsilon_\pi^2 \ln \epsilon_\pi^2 \left[ \frac{1}{9} g_0 g_{N\Delta}^2 + \frac{25}{81} g_\Delta g_{N\Delta}^2 \right] - R\left(\frac{m_\pi^2}{\Delta^2}\right) g_{N\Delta}^2 \left[ \epsilon_\pi^2 \frac{32}{27} g_0 + \epsilon_\Delta^2 \left( \frac{76}{27} g_0 + \frac{100}{81} g_\Delta \right) \right] \\ &\quad + \frac{32}{27} g_0 g_{N\Delta}^2 \epsilon_\pi^2 \left[ 1 + \ln \frac{4\epsilon_\Delta^2}{\epsilon_\pi^2} + \frac{\pi \epsilon_\pi}{\epsilon_\Delta} \right] + \frac{2}{81} g_{N\Delta}^2 \epsilon_\pi^2 (9g_0 + 25g_\Delta) [1 + \ln(4\epsilon_\Delta^2)], \\ \delta_\chi^{(3,\Delta)} &= -\frac{2g_{N\Delta}^2 \epsilon_\Delta}{243} \frac{4\pi F}{M_0} \left[ (63g_0 - 25g_\Delta) \epsilon_\pi^2 + 20(9g_0 + 5g_\Delta) \epsilon_\Delta^2 + \frac{36\sqrt{2}}{g_{N\Delta}} \frac{M_0}{4\pi F} (13\tilde{b}_4 + 12\tilde{b}_5) (\epsilon_\Delta^2 - \epsilon_\pi^2) \right] R\left(\frac{m_\pi^2}{\Delta^2}\right) \\ &\quad + \frac{4g_{N\Delta}^2 \epsilon_\pi^2 \epsilon_\Delta}{243} \frac{4\pi F}{M_0} \left[ 5(9g_0 + 5g_\Delta) + \frac{9\sqrt{2}}{g_{N\Delta}} \frac{M_0}{4\pi F} (13\tilde{b}_4 + 12\tilde{b}_5) \right] \left[ 1 + \ln \frac{4\epsilon_\Delta^2}{\epsilon_\pi^2} \right], \\ \delta_{\text{FV}}^{(2,\Delta)} &= \sum_{\vec{n} \neq \vec{0}} \left[ -\frac{\epsilon_\pi^3}{\epsilon_\Delta} g_0 g_{N\Delta}^2 \frac{32\pi}{27} \frac{e^{-x_n}}{x_n} - \epsilon_\pi^2 g_{N\Delta}^2 \frac{8}{243} (81g_0 + 25g_\Delta) \int_0^\infty d\lambda \lambda [3K_0(y_n) - y_n K_1(y_n)] \right. \\ &\quad \left. + \frac{\epsilon_\pi^3}{\epsilon_\Delta} g_0 g_{N\Delta}^2 \frac{64}{27} \int_0^\infty d\lambda \beta \left[ K_0(y_n) - \frac{1}{y_n} K_1(y_n) \right] \right], \quad y_n = x_n \sqrt{\beta}, \quad \beta = 1 + \lambda^2 + 2\lambda\xi, \quad \xi = \frac{\Delta}{m_\pi}, \end{aligned}$$

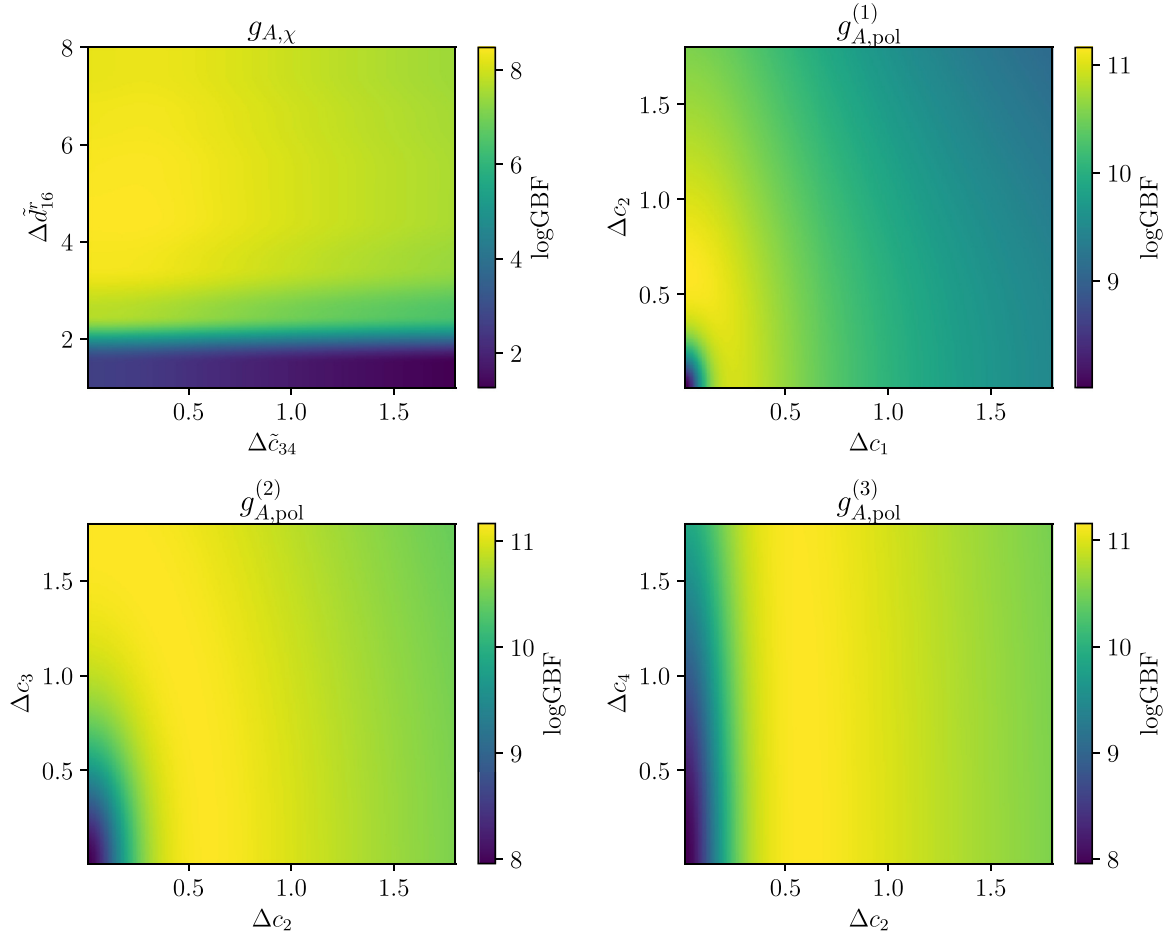


FIG. 7. The log of the Bayes factors obtained by fitting the largest volume  $a12$  data using the models of Eqs. (4.2) plotted as functions of the prior width of their free parameters.

$$\begin{aligned}
 \delta_{\text{FV}}^{(3,\Delta)} = & \sum_{\vec{n} \neq \vec{0}} \left[ \epsilon_{\pi}^3 g_{N\Delta}^2 (81g_0 + 25g_{\Delta}) \frac{4}{243} \frac{4\pi F}{M_0} \int_0^{\infty} d\lambda \lambda \left[ 2\xi K_0(y_n) - \frac{\lambda y_n}{\beta} (3\beta - 2\xi^2) K_1(y_n) \right. \right. \\
 & \left. \left. + \lambda \left( \frac{y_n}{\beta} \right)^2 (\beta^2 - \xi^3 \lambda + \beta \lambda (\lambda + 3\xi)) K_2(y_n) \right] \right. \\
 & \left. + \epsilon_{\pi}^3 g_0 g_{N\Delta}^2 \frac{32}{27} \frac{4\pi F}{M_0} \int_0^{\infty} d\lambda \left[ (\beta - 4\xi \lambda - 3\lambda^2) K_0(y_n) + \left( \frac{2\beta}{y_n} - \frac{y_n(\beta + \xi \lambda)^2}{\beta} \right) K_1(y_n) \right] \right. \\
 & \left. - \epsilon_{\pi}^3 g_{N\Delta} \frac{16\sqrt{2}}{27} (13\tilde{b}_4 + 12\tilde{b}_5) \int_0^{\infty} d\lambda \beta \left[ K_0(y_n) - \frac{1}{y_n} K_1(y_n) \right] \right], \quad (\text{B2})
 \end{aligned}$$

with  $\tilde{b}_i = (4\pi F)b_i$  and  $b_i$  in the conventions of Ref. [75], where large- $N_c$  estimates are derived, leading to  $|b_4 \pm b_5| \leq 5 \text{ GeV}^{-1}$ .  $\delta_{\text{FV}}^{(2)}$  and  $\delta_{\text{FV}}^{(2,\Delta)}$  agree with Eq. (2.10),  $\delta_{\chi}^{(2)}$  and  $\delta_{\chi}^{(2,\Delta)}$  combine to Eq. (2.3). The N<sup>2</sup>LO FV corrections including  $\Delta$ s were derived in the context of Refs. [162,163]. For the N<sup>2</sup>LO IV expressions, see also Refs. [152,164].

### APPENDIX C: CHOICE OF BAYESIAN PRIORS IN GLOBAL FITS

The global fits for  $g_A$  are performed with functional forms consisting of three terms, parameterizing the quark mass, FV, and discretization dependence. We include four possible choices for the quark mass [Eqs. (4.2)], eight for the FV [Eqs. (4.3) and (4.4)], and two for the

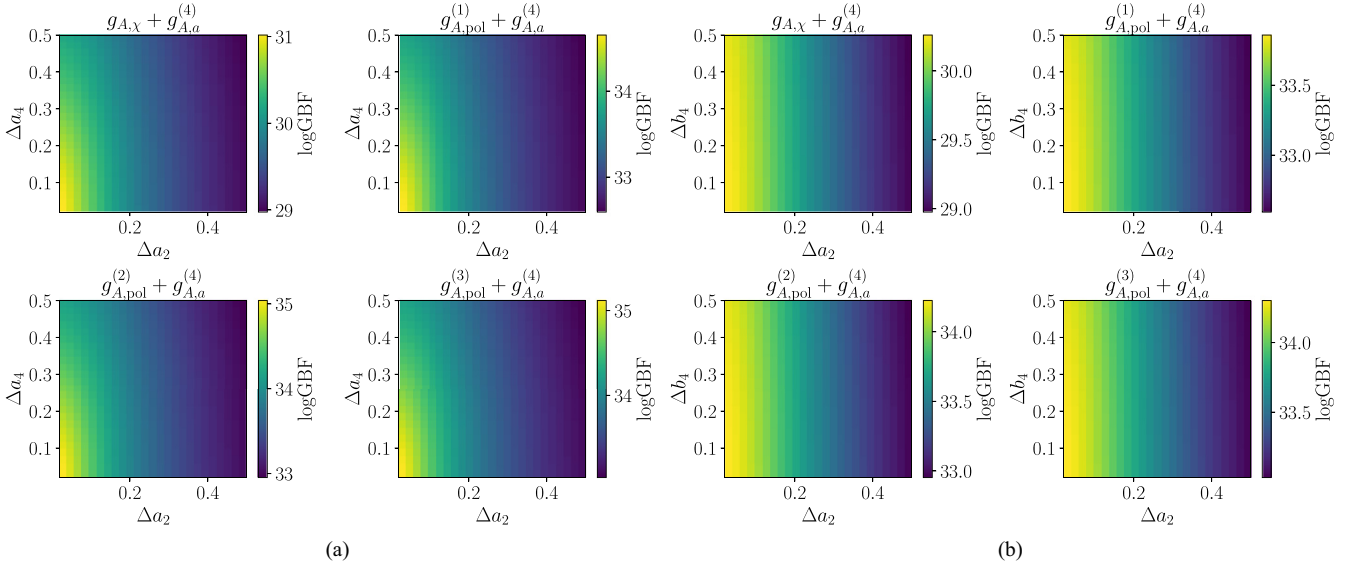


FIG. 8. The log of the Bayes factors obtained by fitting the data at all lattice spacings (keeping the largest volume ensemble for the pion masses for which multiple volumes are available) using the models of Eqs. (4.2) plus the discretization dependence model  $g_{A,a}^{(4)}$  defined in Eq. (4.5b). In (a) we fix  $\Delta b_4 = 0.5$  and vary  $a_2$  and  $a_4$ , while in (b) we fix  $\Delta a_4 = 0.5$  and vary  $a_2$  and  $b_4$ .

discretization dependence [Eqs. (4.5)]. The large number of parameters for each of the 64 models considered motivates us to follow a Bayesian approach, and to carefully consider the effect of the choice of priors to the fit results. Our procedure for selecting the priors is described below, while the summary of the final model results for  $g_A$ , along with the priors, model weights, and posteriors are listed in Appendix D.

Our knowledge about the physical value of  $g_A$  motivates us to set the prior of the parameters  $g_0$  and  $c_0$  of Eq. (4.2) to 1.25(10). For the remaining parameters, we set the central values of the priors to 0, and follow the steps below, motivated by the empirical Bayes criterion, to choose the corresponding prior widths. We also note that for all parameters, varying our choices of the prior widths by  $\mathcal{O}(1)$  factors does not affect our final results for  $g_A$ .

- (1) *Chiral dependence:* We first isolate the parameters of the mass-dependence models of Eq. (4.2) by considering fits to the  $a_{12}$  data only, choosing the largest volume at the pion masses for which ensembles at multiple volumes are available. We vary the prior widths of the pairs of parameters,  $(\tilde{d}_{16}^r, \tilde{c}_{34})$  for  $g_{A,\chi}$ ,  $(c_1, c_2)$  for  $g_{A,\text{pol}}^{(1)}$ ,  $(c_2, c_3)$  for  $g_{A,\text{pol}}^{(2)}$ , and  $(c_2, c_4)$  for  $g_{A,\text{pol}}^{(3)}$ , and study the resulting Bayes factors. For  $g_{A,\chi}$ , there is a maximum in the region we explore for  $(\Delta \tilde{d}_{16}^r, \Delta \tilde{c}_{34})$ . For the three Taylor models, there is a maximum with respect to  $\Delta c_2$ , while the dependence on the second parameter of each is small and thus we conservatively set the corresponding priors to those chosen for  $\Delta c_2$ . Our final prior widths choices are well within the regions where the largest Bayes factors are observed,

as seen in the plots of Fig. 7,  $(\Delta \tilde{d}_{16}^r, \Delta \tilde{c}_{34}) = (5, 1)$ ,  $(\Delta c_1, \Delta c_2) = (0.5, 0.5)$ ,  $(\Delta c_2, \Delta c_3) = (1, 1)$ , and  $(\Delta c_2, \Delta c_4) = (1, 1)$ .

- (2) *Discretization dependence:* Next, the priors widths for the mass-dependence parameters are frozen and prior widths for parameters with lattice spacing dependence are varied. Our dataset exhibits minimal dependence on the finite lattice spacing, making it impossible to choose optimal prior widths of the parameters  $a_2$ ,  $a_4$ , and  $b_4$  defined in Eqs. (4.5) using the empirical Bayes criterion. However, we observe that the central value and error of our final model-averaged estimate for  $g_A$  is identical for any choice of  $\Delta a_2 \gtrsim 0.5$ , including those larger by several orders of magnitude. As  $\Delta a_2$  decreases, the final error becomes smaller and the fit output Bayes factor grows, but simultaneously the probability of the model shrinks by an amount that cannot be quantified. Therefore we choose to set  $\Delta a_2 = 0.5$ . The dependence of the Bayes factor on the width of the higher-order parameter  $b_4$  is much milder, as seen in on the right side of Fig. 8, therefore we conservatively set the prior  $\Delta b_4 = \Delta a_2 = 0.5$ . The left side of Fig. 8 shows the dependence of the Bayes factor on the prior width of  $a_4$  and  $a_2$  at fixed  $b_2$ , and it is clear that smaller values of  $\Delta a_4$  are preferred compared to  $\Delta b_4$ , so we set its prior to half of  $\Delta b_4$ ,  $\Delta a_4 = 0.25$ .
- (3) *FV dependence:* After having chosen prior widths for the chiral and discretization part of the models as described above, the only parameters that remain are those that are introduced solely by the FV models of Eqs. (4.3) and (4.4),  $f_2$  and  $f_3$ . We again employ the empirical Bayes criterion and perform fits to the full

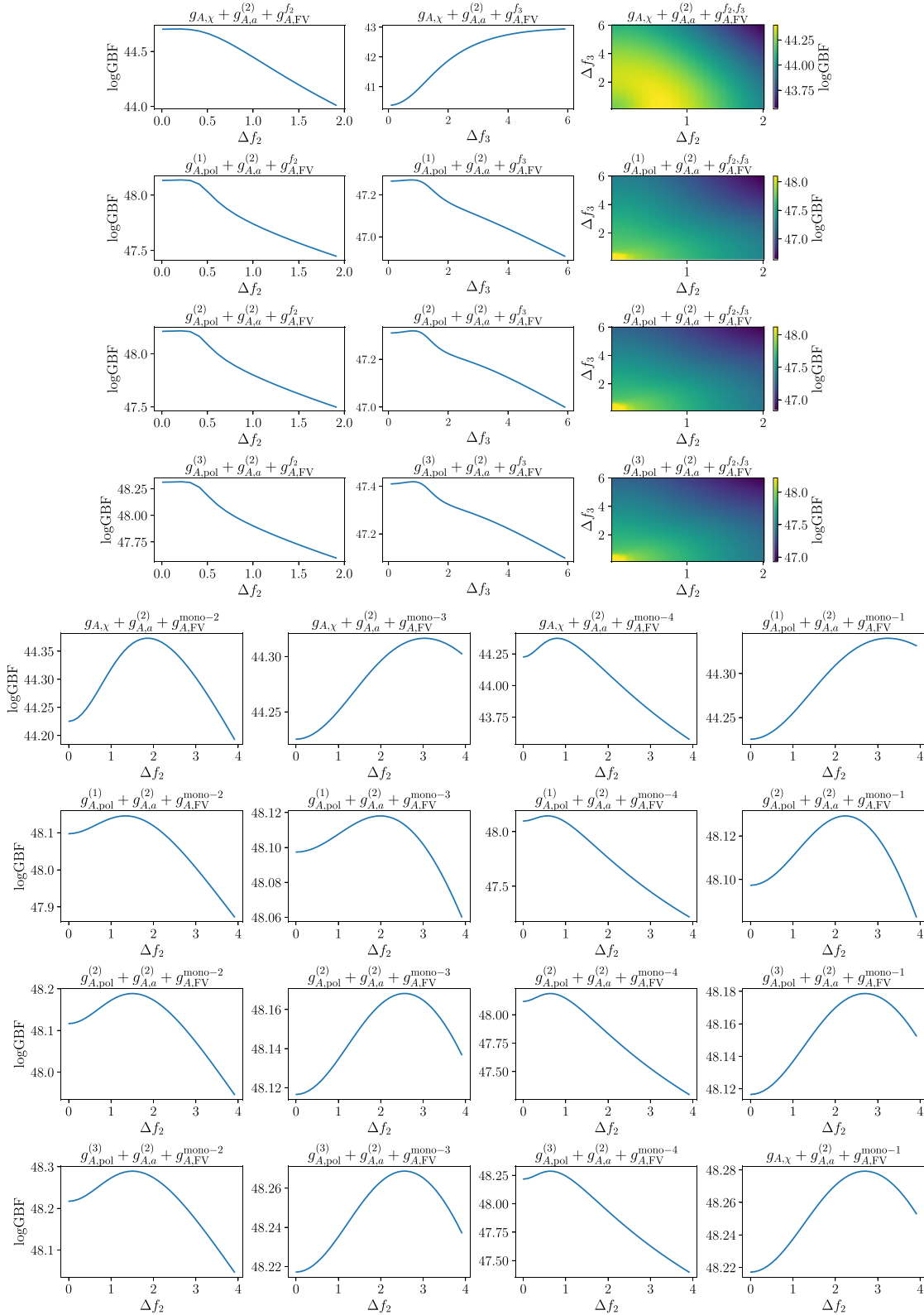


FIG. 9. The log of the Bayes factors obtained by fitting the data at all lattice spacings (keeping the largest volume ensemble for the pion masses for which multiple volumes are available) using the models of Eqs. (4.2) and Eq. (4.5a) plus the FV models of Eqs. (4.3) (upper) and Eqs. (4.4) (lower) that depend on  $f_2$  or/and  $f_3$ , the parameters whose prior width we vary.

TABLE V. The extrapolated  $g_A$  values of the 64 models included in the analysis along with the augmented  $\chi^2_{\text{aug}}$  per degree of freedom of each fit,<sup>a</sup> the log Gaussian Bayes factors representing the log-likelihood distribution, logGBF, and the normalized model weights,  $P(M_k|D)$ . Models are sorted by descending weight.

| Model             | Chiral | FV     | Disc.  | $\chi^2_{\text{aug}}/\text{d.o.f}$ | logGBF | $P(M_k D)$            | $P(g_A M_k)$ |
|-------------------|--------|--------|--------|------------------------------------|--------|-----------------------|--------------|
| poly3_F21_a2      | (4.2d) | (4.4a) | (4.5a) | 1.03                               | 48.27  | $3.12 \times 10^{-2}$ | 1.2655(79)   |
| poly3_F33_a2      | (4.2d) | (4.4d) | (4.5a) | 0.67                               | 48.27  | $3.11 \times 10^{-2}$ | 1.2660(79)   |
| poly3_F23_a2      | (4.2d) | (4.4b) | (4.5a) | 1.01                               | 48.26  | $3.09 \times 10^{-2}$ | 1.2661(79)   |
| poly3_F21_a4      | (4.2d) | (4.4a) | (4.5b) | 0.67                               | 48.21  | $2.91 \times 10^{-2}$ | 1.2657(97)   |
| poly3_F33_a4      | (4.2d) | (4.4d) | (4.5b) | 0.63                               | 48.20  | $2.89 \times 10^{-2}$ | 1.2659(96)   |
| poly3_F23_a4      | (4.2d) | (4.4b) | (4.5b) | 0.71                               | 48.19  | $2.87 \times 10^{-2}$ | 1.2659(96)   |
| poly2_F21_a2      | (4.2c) | (4.4a) | (4.5a) | 0.63                               | 48.17  | $2.82 \times 10^{-2}$ | 1.2655(82)   |
| poly2_F33_a2      | (4.2c) | (4.4d) | (4.5a) | 0.70                               | 48.17  | $2.81 \times 10^{-2}$ | 1.2660(81)   |
| poly2_F23_a2      | (4.2c) | (4.4b) | (4.5a) | 0.76                               | 48.16  | $2.79 \times 10^{-2}$ | 1.2660(81)   |
| poly1_F33_a2      | (4.2b) | (4.4d) | (4.5a) | 0.63                               | 48.13  | $2.70 \times 10^{-2}$ | 1.2718(90)   |
| poly1_F21_a2      | (4.2b) | (4.4a) | (4.5a) | 0.75                               | 48.12  | $2.67 \times 10^{-2}$ | 1.2713(91)   |
| poly1_F23_a2      | (4.2b) | (4.4b) | (4.5a) | 0.63                               | 48.12  | $2.67 \times 10^{-2}$ | 1.2718(90)   |
| poly2_F21_a4      | (4.2c) | (4.4a) | (4.5b) | 0.64                               | 48.11  | $2.64 \times 10^{-2}$ | 1.2657(98)   |
| poly2_F33_a4      | (4.2c) | (4.4d) | (4.5b) | 0.70                               | 48.10  | $2.62 \times 10^{-2}$ | 1.2659(98)   |
| poly2_F23_a4      | (4.2c) | (4.4b) | (4.5b) | 0.64                               | 48.09  | $2.60 \times 10^{-2}$ | 1.2659(98)   |
| poly1_F33_a4      | (4.2b) | (4.4d) | (4.5b) | 0.70                               | 48.05  | $2.51 \times 10^{-2}$ | 1.271(11)    |
| poly1_F21_a4      | (4.2b) | (4.4a) | (4.5b) | 0.79                               | 48.05  | $2.49 \times 10^{-2}$ | 1.271(11)    |
| poly1_F23_a4      | (4.2b) | (4.4b) | (4.5b) | 0.72                               | 48.04  | $2.48 \times 10^{-2}$ | 1.271(11)    |
| poly3_F31_a2      | (4.2d) | (4.4c) | (4.5a) | 0.78                               | 47.93  | $2.22 \times 10^{-2}$ | 1.2651(79)   |
| poly3_NNLOf2_a2   | (4.2d) | (4.3b) | (4.5a) | 0.72                               | 47.90  | $2.15 \times 10^{-2}$ | 1.2654(81)   |
| poly3_F31_a4      | (4.2d) | (4.4c) | (4.5b) | 0.70                               | 47.87  | $2.09 \times 10^{-2}$ | 1.2655(97)   |
| poly3_NNLOf2_a4   | (4.2d) | (4.3b) | (4.5b) | 0.75                               | 47.84  | $2.01 \times 10^{-2}$ | 1.2658(97)   |
| poly2_F31_a2      | (4.2c) | (4.4c) | (4.5a) | 0.70                               | 47.83  | $2.01 \times 10^{-2}$ | 1.2650(82)   |
| poly2_NNLOf2_a2   | (4.2c) | (4.3b) | (4.5a) | 0.75                               | 47.80  | $1.95 \times 10^{-2}$ | 1.2654(83)   |
| poly2_F31_a4      | (4.2c) | (4.4c) | (4.5b) | 0.82                               | 47.77  | $1.90 \times 10^{-2}$ | 1.2655(98)   |
| poly1_F31_a2      | (4.2b) | (4.4c) | (4.5a) | 0.69                               | 47.76  | $1.87 \times 10^{-2}$ | 1.2707(91)   |
| poly1_NNLOf2_a2   | (4.2b) | (4.3b) | (4.5a) | 0.82                               | 47.74  | $1.83 \times 10^{-2}$ | 1.2710(92)   |
| poly2_NNLOf2_a4   | (4.2c) | (4.3b) | (4.5b) | 0.69                               | 47.74  | $1.82 \times 10^{-2}$ | 1.2657(99)   |
| poly3_NNLO_a2     | (4.2d) | (4.3a) | (4.5a) | 0.69                               | 47.73  | $1.80 \times 10^{-2}$ | 1.2656(80)   |
| poly3_NNLOf2f3_a2 | (4.2d) | (4.3d) | (4.5a) | 0.75                               | 47.71  | $1.78 \times 10^{-2}$ | 1.2649(80)   |
| poly1_F31_a4      | (4.2b) | (4.4c) | (4.5b) | 0.69                               | 47.69  | $1.75 \times 10^{-2}$ | 1.271(11)    |
| poly1_NNLOf2_a4   | (4.2b) | (4.3b) | (4.5b) | 0.75                               | 47.67  | $1.71 \times 10^{-2}$ | 1.271(11)    |
| poly3_NNLO_a4     | (4.2d) | (4.3a) | (4.5b) | 0.81                               | 47.66  | $1.68 \times 10^{-2}$ | 1.2654(97)   |
| poly3_NNLOf2f3_a4 | (4.2d) | (4.3d) | (4.5b) | 0.74                               | 47.65  | $1.67 \times 10^{-2}$ | 1.2654(97)   |
| poly2_NNLO_a2     | (4.2c) | (4.3a) | (4.5a) | 0.81                               | 47.62  | $1.63 \times 10^{-2}$ | 1.2656(82)   |
| poly2_NNLOf2f3_a2 | (4.2c) | (4.3d) | (4.5a) | 0.74                               | 47.61  | $1.61 \times 10^{-2}$ | 1.2648(83)   |
| poly1_NNLOf2f3_a2 | (4.2b) | (4.3d) | (4.5a) | 0.72                               | 47.56  | $1.54 \times 10^{-2}$ | 1.2704(92)   |
| poly2_NNLO_a4     | (4.2c) | (4.3a) | (4.5b) | 0.77                               | 47.56  | $1.52 \times 10^{-2}$ | 1.2654(98)   |
| poly2_NNLOf2f3_a4 | (4.2c) | (4.3d) | (4.5b) | 0.72                               | 47.55  | $1.51 \times 10^{-2}$ | 1.2654(99)   |
| poly1_NNLO_a2     | (4.2b) | (4.3a) | (4.5a) | 0.77                               | 47.55  | $1.51 \times 10^{-2}$ | 1.2713(91)   |
| poly1_NNLOf2f3_a4 | (4.2b) | (4.3d) | (4.5b) | 0.85                               | 47.50  | $1.43 \times 10^{-2}$ | 1.271(11)    |
| poly1_NNLO_a4     | (4.2b) | (4.3a) | (4.5b) | 0.71                               | 47.48  | $1.40 \times 10^{-2}$ | 1.271(11)    |
| poly3_NNLOf3_a2   | (4.2d) | (4.3c) | (4.5a) | 0.85                               | 47.42  | $1.32 \times 10^{-2}$ | 1.2662(79)   |
| poly3_NNLOf3_a4   | (4.2d) | (4.3c) | (4.5b) | 0.71                               | 47.35  | $1.23 \times 10^{-2}$ | 1.2658(97)   |
| poly2_NNLOf3_a2   | (4.2c) | (4.3c) | (4.5a) | 0.71                               | 47.32  | $1.20 \times 10^{-2}$ | 1.2661(82)   |
| poly1_NNLOf3_a2   | (4.2b) | (4.3c) | (4.5a) | 0.77                               | 47.27  | $1.14 \times 10^{-2}$ | 1.2720(91)   |
| poly2_NNLOf3_a4   | (4.2c) | (4.3c) | (4.5b) | 0.71                               | 47.25  | $1.12 \times 10^{-2}$ | 1.2658(98)   |
| poly1_NNLOf3_a4   | (4.2b) | (4.3c) | (4.5b) | 0.77                               | 47.20  | $1.07 \times 10^{-2}$ | 1.271(11)    |
| chPT_NNLOf2_a2    | (4.2a) | (4.3b) | (4.5a) | 0.80                               | 44.45  | $6.80 \times 10^{-4}$ | 1.266(10)    |
| chPT_NNLOf2_a4    | (4.2a) | (4.3b) | (4.5b) | 0.74                               | 44.39  | $6.41 \times 10^{-4}$ | 1.266(11)    |
| chPT_F21_a2       | (4.2a) | (4.4a) | (4.5a) | 0.80                               | 44.37  | $6.31 \times 10^{-4}$ | 1.267(11)    |
| chPT_F21_a4       | (4.2a) | (4.4a) | (4.5b) | 0.74                               | 44.31  | $5.93 \times 10^{-4}$ | 1.267(12)    |

TABLE V. (*Continued.*)

| Model               | Chiral | FV     | Disc.  | $\chi^2_{\text{aug}}/\text{d.o.f}$ | logGBF | $P(M_k D)$            | $P(g_A M_k)$ |
|---------------------|--------|--------|--------|------------------------------------|--------|-----------------------|--------------|
| chPT_F33_a2         | (4.2a) | (4.4d) | (4.5a) | 0.72                               | 44.31  | $5.93 \times 10^{-4}$ | 1.268(11)    |
| chPT_NNLOf2f3_a2    | (4.2a) | (4.3d) | (4.5a) | 0.77                               | 44.30  | $5.88 \times 10^{-4}$ | 1.267(11)    |
| chPT_F23_a2         | (4.2a) | (4.4b) | (4.5a) | 0.72                               | 44.30  | $5.85 \times 10^{-4}$ | 1.268(11)    |
| chPT_F33_a4         | (4.2a) | (4.4d) | (4.5b) | 0.77                               | 44.25  | $5.59 \times 10^{-4}$ | 1.267(12)    |
| chPT_NNLOf2f3_a4    | (4.2a) | (4.3d) | (4.5b) | 0.85                               | 44.24  | $5.54 \times 10^{-4}$ | 1.267(12)    |
| chPT_F23_a4         | (4.2a) | (4.4b) | (4.5b) | 0.71                               | 44.24  | $5.52 \times 10^{-4}$ | 1.267(12)    |
| chPT_F31_a2         | (4.2a) | (4.4c) | (4.5a) | 0.85                               | 44.09  | $4.78 \times 10^{-4}$ | 1.266(11)    |
| chPT_F31_a4         | (4.2a) | (4.4c) | (4.5b) | 0.71                               | 44.03  | $4.50 \times 10^{-4}$ | 1.266(12)    |
| chPT_NNLOf3_a2      | (4.2a) | (4.3c) | (4.5a) | 0.71                               | 42.94  | $1.51 \times 10^{-4}$ | 1.267(10)    |
| chPT_NNLOf3_a4      | (4.2a) | (4.3c) | (4.5b) | 0.77                               | 42.91  | $1.46 \times 10^{-4}$ | 1.266(12)    |
| chPT_NNLO_a4        | (4.2a) | (4.3a) | (4.5b) | 0.71                               | 41.56  | $3.80 \times 10^{-5}$ | 1.267(12)    |
| chPT_NNLO_a2        | (4.2a) | (4.3a) | (4.5a) | 0.77                               | 41.51  | $3.61 \times 10^{-5}$ | 1.270(10)    |
| Bayes Model Average |        |        |        |                                    |        |                       | 1.2674(96)   |

<sup>a</sup>Given the ambiguity of counting degrees of freedom with priors,  $\chi^2_{\text{aug}}$  is not suited for quantitative statistical inference. They still provide a nice qualitative heuristic for the goodness of fit, and are therefore often reported, but not used.

dataset using the 48 of the 64 models that depend on  $f_2$  and/or  $f_3$ , and varying the corresponding prior widths. In this procedure, we set the priors to the remaining parameters,  $g_0$ ,  $\tilde{c}_{34}$ ,  $\tilde{d}_{16}^r$ ,  $c_0$ ,  $c_1$ ,  $c_2$ ,  $c_3$ ,  $c_4$ ,  $a_2$ ,  $a_4$ , and  $b_4$  to the values described in the previous steps, with the exception of the  $g_0$  that shows up in the FV part of the polynomial models, which is set to 1.2(5) since, in contrast to the  $\chi$ PT models, it is not tightly constrained by the chiral dependence of the model. Based on the results for the Bayes factor shown in Fig. 9, we choose to set  $\Delta f_2 = 1$  for the nonmonotonic models,  $\Delta f_2 = 2$  for the monotonic models,  $\Delta f_3 = 6$  for the  $g_{A,\chi} + g_{A,\text{FV}}^{f_3}$  model, and  $\Delta f_3 = 1$  for the rest of the

models. We only include the results for the discretization model  $g_{A,a}^{(2)}$  in the figures, but visually identical results are obtained when using the other discretization model of Eq. (4.5),  $g_{A,a}^{(4)}$ .

#### APPENDIX D: GLOBAL FIT MODEL RESULTS

In this appendix, tables are provided summarizing the final choices for fit results. Table V provides the log Gaussian Bayes factors and model weights along with the posterior distributions for  $g_A$ . Table VI provides the list of priors for the fits used in this manuscript. Table VII gives the final posterior distributions for the LECs that have been fit in each model.

TABLE VI. Priors used in the model fits. The models are ordered according to the order in which the model Eqs. (4.2), (4.3), (4.4), and (4.5) appear in the main text.

| Chiral | FV     | Disc.  | $g_0$    | $\tilde{c}_{34}$ | $\tilde{d}_{16}^r$ | $c_0$    | $c_1$  | $c_2$  | $c_3$ | $c_4$ | $f_2$ | $f_3$ | $a_2$  | $a_4$   | $b_4$  |
|--------|--------|--------|----------|------------------|--------------------|----------|--------|--------|-------|-------|-------|-------|--------|---------|--------|
| (4.2a) | (4.3a) | (4.5a) | 1.25(10) | 0(1)             | 0(5)               |          |        |        |       |       |       |       | 0(0.5) |         |        |
|        |        | (4.5b) | 1.25(10) | 0(1)             | 0(5)               |          |        |        |       |       |       |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(10) | 0(1)             | 0(5)               |          |        |        |       |       | 0(1)  |       | 0(0.5) |         |        |
|        | (4.3b) | (4.5b) | 1.25(10) | 0(1)             | 0(5)               |          |        |        |       |       | 0(1)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(10) | 0(1)             | 0(5)               |          |        |        |       |       |       | 0(6)  | 0(0.5) |         |        |
|        | (4.3c) | (4.5b) | 1.25(10) | 0(1)             | 0(5)               |          |        |        |       |       |       | 0(6)  | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(10) | 0(1)             | 0(5)               |          |        |        |       |       | 0(1)  | 0(1)  | 0(0.5) |         |        |
|        | (4.3d) | (4.5b) | 1.25(10) | 0(1)             | 0(5)               |          |        |        |       |       | 0(1)  | 0(1)  | 0(0.5) | 0(0.25) | 0(0.5) |
|        | (4.4a) | (4.5a) | 1.25(10) | 0(1)             | 0(5)               |          |        |        |       |       | 0(2)  |       | 0(0.5) |         |        |
|        |        | (4.5b) | 1.25(10) | 0(1)             | 0(5)               |          |        |        |       |       | 0(2)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(10) | 0(1)             | 0(5)               |          |        |        |       |       | 0(2)  |       | 0(0.5) |         |        |
|        |        | (4.5b) | 1.25(10) | 0(1)             | 0(5)               |          |        |        |       |       | 0(2)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(10) | 0(1)             | 0(5)               |          |        |        |       |       | 0(2)  |       | 0(0.5) |         |        |
|        |        | (4.5b) | 1.25(10) | 0(1)             | 0(5)               |          |        |        |       |       | 0(2)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(10) | 0(1)             | 0(5)               |          |        |        |       |       | 0(2)  |       | 0(0.5) |         |        |
|        |        | (4.5b) | 1.25(10) | 0(1)             | 0(5)               |          |        |        |       |       | 0(2)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        | (4.3a) | (4.5a) | 1.25(50) | 0(1)             |                    | 1.25(10) | 0(0.5) | 0(0.5) |       |       |       |       | 0(0.5) |         |        |
|        |        | (4.5b) | 1.25(50) | 0(1)             |                    | 1.25(10) | 0(0.5) | 0(0.5) |       |       |       |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(50) | 0(1)             |                    | 1.25(10) | 0(0.5) | 0(0.5) |       |       | 0(1)  |       | 0(0.5) |         |        |
|        | (4.3b) | (4.5b) | 1.25(50) | 0(1)             |                    | 1.25(10) | 0(0.5) | 0(0.5) |       |       | 0(1)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(50) | 0(1)             |                    | 1.25(10) | 0(0.5) | 0(0.5) |       |       |       | 0(1)  | 0(0.5) |         |        |
|        | (4.3c) | (4.5b) | 1.25(50) | 0(1)             |                    | 1.25(10) | 0(0.5) | 0(0.5) |       |       |       | 0(1)  | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(50) | 0(1)             |                    | 1.25(10) | 0(0.5) | 0(0.5) |       |       | 0(1)  | 0(1)  | 0(0.5) |         |        |
|        | (4.3d) | (4.5b) | 1.25(50) | 0(1)             |                    | 1.25(10) | 0(0.5) | 0(0.5) |       |       | 0(1)  | 0(1)  | 0(0.5) | 0(0.25) | 0(0.5) |
| (4.2b) | (4.4a) | (4.5a) |          |                  |                    | 1.25(10) | 0(0.5) | 0(0.5) |       |       | 0(2)  |       | 0(0.5) |         |        |
|        |        | (4.5b) |          |                  |                    | 1.25(10) | 0(0.5) | 0(0.5) |       |       | 0(2)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) |          |                  |                    | 1.25(10) | 0(0.5) | 0(0.5) |       |       | 0(2)  |       | 0(0.5) |         |        |
|        | (4.4b) | (4.5b) |          |                  |                    | 1.25(10) | 0(0.5) | 0(0.5) |       |       | 0(2)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) |          |                  |                    | 1.25(10) | 0(0.5) | 0(0.5) |       |       | 0(2)  |       | 0(0.5) |         |        |
|        | (4.4c) | (4.5b) |          |                  |                    | 1.25(10) | 0(0.5) | 0(0.5) |       |       | 0(2)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) |          |                  |                    | 1.25(10) | 0(0.5) | 0(0.5) |       |       | 0(2)  |       | 0(0.5) |         |        |
|        | (4.4d) | (4.5b) |          |                  |                    | 1.25(10) | 0(0.5) | 0(0.5) |       |       | 0(2)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        | (4.3a) | (4.5a) | 1.25(50) | 0(1)             |                    | 1.25(10) |        | 0(1)   | 0(1)  |       |       |       | 0(0.5) |         |        |
|        |        | (4.5b) | 1.25(50) | 0(1)             |                    | 1.25(10) |        | 0(1)   | 0(1)  |       |       |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(50) | 0(1)             |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(1)  |       |       | 0(0.5) |         |        |
|        | (4.3b) | (4.5b) | 1.25(50) | 0(1)             |                    | 1.25(10) |        | 0(1)   | 0(1)  |       | 0(1)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(50) | 0(1)             |                    | 1.25(10) |        | 0(1)   | 0(1)  |       |       | 0(1)  | 0(0.5) |         |        |
|        | (4.3c) | (4.5b) | 1.25(50) | 0(1)             |                    | 1.25(10) |        | 0(1)   | 0(1)  |       |       | 0(1)  | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(50) | 0(1)             |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(1)  | 0(1)  | 0(1)  | 0(0.5) |         |        |
|        | (4.3d) | (4.5b) | 1.25(50) | 0(1)             |                    | 1.25(10) |        | 0(1)   | 0(1)  |       | 0(1)  | 0(1)  | 0(0.5) | 0(0.25) | 0(0.5) |
| (4.2c) | (4.4a) | (4.5a) |          |                  |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(2)  |       |       | 0(0.5) |         |        |
|        |        | (4.5b) |          |                  |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(2)  |       |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) |          |                  |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(2)  |       |       | 0(0.5) |         |        |
|        | (4.4b) | (4.5b) |          |                  |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(2)  |       |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) |          |                  |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(2)  |       |       | 0(0.5) |         |        |
|        | (4.4c) | (4.5b) |          |                  |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(2)  |       |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) |          |                  |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(2)  |       |       | 0(0.5) |         |        |
|        | (4.4d) | (4.5b) |          |                  |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(2)  |       |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        | (4.3a) | (4.5a) | 1.25(50) | 0(1)             |                    | 1.25(10) |        | 0(1)   | 0(1)  |       |       |       | 0(0.5) |         |        |
|        |        | (4.5b) | 1.25(50) | 0(1)             |                    | 1.25(10) |        | 0(1)   | 0(1)  |       |       |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(50) | 0(1)             |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(1)  |       |       | 0(0.5) |         |        |
|        | (4.3b) | (4.5b) | 1.25(50) | 0(1)             |                    | 1.25(10) |        | 0(1)   | 0(1)  |       | 0(1)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(50) | 0(1)             |                    | 1.25(10) |        | 0(1)   | 0(1)  |       |       | 0(1)  | 0(0.5) |         |        |
|        | (4.3c) | (4.5b) | 1.25(50) | 0(1)             |                    | 1.25(10) |        | 0(1)   | 0(1)  |       |       | 0(1)  | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(50) | 0(1)             |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(1)  | 0(1)  | 0(1)  | 0(0.5) |         |        |
|        | (4.3d) | (4.5b) | 1.25(50) | 0(1)             |                    | 1.25(10) |        | 0(1)   | 0(1)  |       | 0(1)  | 0(1)  | 0(0.5) | 0(0.25) | 0(0.5) |
|        | (4.4a) | (4.5a) |          |                  |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(2)  |       |       | 0(0.5) |         |        |
|        |        | (4.5b) |          |                  |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(2)  |       |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) |          |                  |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(2)  |       |       | 0(0.5) |         |        |
|        | (4.4b) | (4.5b) |          |                  |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(2)  |       |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) |          |                  |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(2)  |       |       | 0(0.5) |         |        |
|        | (4.4c) | (4.5b) |          |                  |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(2)  |       |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) |          |                  |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(2)  |       |       | 0(0.5) |         |        |
|        | (4.4d) | (4.5b) |          |                  |                    | 1.25(10) |        | 0(1)   | 0(1)  | 0(2)  |       |       | 0(0.5) | 0(0.25) | 0(0.5) |

TABLE VI. (*Continued.*)

| Chiral | FV     | Disc.  | $g_0$    | $\tilde{c}_{34}$ | $\tilde{d}_{16}^r$ | $c_0$    | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $f_2$ | $f_3$ | $a_2$  | $a_4$   | $b_4$  |
|--------|--------|--------|----------|------------------|--------------------|----------|-------|-------|-------|-------|-------|-------|--------|---------|--------|
| (4.2d) | (4.3a) | (4.5a) | 1.25(50) | 0(1)             |                    | 1.25(10) |       | 0(1)  |       | 0(1)  |       |       | 0(0.5) |         |        |
|        |        | (4.5b) | 1.25(50) | 0(1)             |                    | 1.25(10) |       | 0(1)  |       | 0(1)  |       |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(50) | 0(1)             |                    | 1.25(10) |       | 0(1)  |       | 0(1)  | 0(1)  |       | 0(0.5) |         |        |
|        | (4.3b) | (4.5b) | 1.25(50) | 0(1)             |                    | 1.25(10) |       | 0(1)  |       | 0(1)  | 0(1)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(50) | 0(1)             |                    | 1.25(10) |       | 0(1)  |       | 0(1)  |       | 0(1)  | 0(0.5) |         |        |
|        | (4.3c) | (4.5b) | 1.25(50) | 0(1)             |                    | 1.25(10) |       | 0(1)  |       | 0(1)  |       | 0(1)  | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) | 1.25(50) | 0(1)             |                    | 1.25(10) |       | 0(1)  |       | 0(1)  | 0(1)  | 0(1)  | 0(0.5) |         |        |
|        | (4.3d) | (4.5b) | 1.25(50) | 0(1)             |                    | 1.25(10) |       | 0(1)  |       | 0(1)  | 0(1)  | 0(1)  | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) |          |                  |                    | 1.25(10) |       | 0(1)  |       | 0(1)  | 0(2)  |       | 0(0.5) |         |        |
|        | (4.4a) | (4.5b) |          |                  |                    | 1.25(10) |       | 0(1)  |       | 0(1)  | 0(2)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) |          |                  |                    | 1.25(10) |       | 0(1)  |       | 0(1)  | 0(2)  |       | 0(0.5) |         |        |
|        | (4.4b) | (4.5b) |          |                  |                    | 1.25(10) |       | 0(1)  |       | 0(1)  | 0(2)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) |          |                  |                    | 1.25(10) |       | 0(1)  |       | 0(1)  | 0(2)  |       | 0(0.5) |         |        |
|        | (4.4c) | (4.5b) |          |                  |                    | 1.25(10) |       | 0(1)  |       | 0(1)  | 0(2)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        | (4.5a) |          |                  |                    | 1.25(10) |       | 0(1)  |       | 0(1)  | 0(2)  |       | 0(0.5) |         |        |
|        | (4.4d) | (4.5b) |          |                  |                    | 1.25(10) |       | 0(1)  |       | 0(1)  | 0(2)  |       | 0(0.5) | 0(0.25) | 0(0.5) |
|        |        |        |          |                  |                    |          |       |       |       |       |       |       |        |         |        |

TABLE VII. Posterior values of the parameters from each model fit. The models are ordered according to the order in which the model Eqs. (4.2), (4.3), (4.4), and (4.5) appear in the main text.

| Chiral | FV     | Disc.  | $g_0$     | $\tilde{c}_{34}$ | $\tilde{d}_{16}^r$ | $c_0$     | $c_1$     | $c_2$     | $c_3$ | $c_4$ | $f_2$     | $f_3$     | $a_2$      | $a_4$       | $b_4$     |
|--------|--------|--------|-----------|------------------|--------------------|-----------|-----------|-----------|-------|-------|-----------|-----------|------------|-------------|-----------|
| (4.2a) | (4.3a) | (4.5a) | 1.226(15) | 0.81(24)         | -4.95(33)          |           |           |           |       |       |           |           | -0.025(50) |             |           |
|        |        | (4.5b) | 1.224(15) | 0.82(24)         | -4.93(33)          |           |           |           |       |       |           |           | 0.012(88)  | -0.10(24)   | -0.13(48) |
|        |        | (4.5a) | 1.221(15) | 0.76(24)         | -4.85(33)          |           |           |           |       |       | -0.47(52) |           | 0.033(54)  |             |           |
|        |        | (4.5b) | 1.221(15) | 0.76(24)         | -4.85(33)          |           |           |           |       |       | -0.46(53) |           | 0.035(89)  | 0.004(243)  | -0.04(48) |
|        | (4.3b) | (4.5a) | 1.223(15) | 0.79(25)         | -4.92(33)          |           |           |           |       |       |           | 5.6(2.9)  | 0.022(54)  |             |           |
|        |        | (4.5b) | 1.222(16) | 0.79(25)         | -4.90(34)          |           |           |           |       |       |           | 5.5(2.9)  | 0.041(89)  | -0.05(24)   | -0.10(48) |
|        | (4.3c) | (4.5a) | 1.222(15) | 0.77(24)         | -4.87(33)          |           |           |           |       |       | -0.60(54) | 0.16(97)  | 0.030(54)  |             |           |
|        |        | (4.5b) | 1.221(16) | 0.77(24)         | -4.86(34)          |           |           |           |       |       | -0.60(54) | 0.17(97)  | 0.031(89)  | 0.004(243)  | -0.03(48) |
|        | (4.4a) | (4.5a) | 1.222(15) | 0.76(24)         | -4.86(33)          |           |           |           |       |       | -1.4(1.4) |           | 0.022(53)  |             |           |
|        |        | (4.5b) | 1.221(16) | 0.77(24)         | -4.86(34)          |           |           |           |       |       | -1.4(1.4) |           | 0.028(89)  | -0.009(243) | -0.04(48) |
|        | (4.4b) | (4.5a) | 1.222(15) | 0.77(24)         | -4.87(33)          |           |           |           |       |       | 1.1(1.7)  |           | 0.012(51)  |             |           |
|        |        | (4.5b) | 1.222(16) | 0.77(24)         | -4.86(34)          |           |           |           |       |       | 1.1(1.7)  |           | 0.022(88)  | -0.02(24)   | -0.05(48) |
|        | (4.4c) | (4.5a) | 1.221(15) | 0.76(24)         | -4.85(33)          |           |           |           |       |       | -0.98(75) |           | 0.033(55)  |             |           |
|        |        | (4.5b) | 1.221(16) | 0.76(24)         | -4.85(34)          |           |           |           |       |       | -0.98(76) |           | 0.034(89)  | 0.007(244)  | -0.03(48) |
|        | (4.4d) | (4.5a) | 1.222(15) | 0.77(24)         | -4.87(33)          |           |           |           |       |       | -1.2(1.7) |           | 0.012(51)  |             |           |
|        |        | (4.5b) | 1.222(16) | 0.77(24)         | -4.86(34)          |           |           |           |       |       | -1.1(1.7) |           | 0.022(88)  | -0.02(24)   | -0.05(48) |
| (4.2b) | (4.3a) | (4.5a) | 0.79(38)  | 0.50(95)         |                    | 1.302(22) | -0.24(17) | -0.20(38) |       |       |           |           | 0.009(52)  |             |           |
|        |        | (4.5b) | 0.79(38)  | 0.49(95)         |                    | 1.301(23) | -0.24(17) | -0.20(38) |       |       |           |           | 0.015(89)  | -0.01(24)   | -0.04(48) |
|        |        | (4.5a) | 1.27(46)  | 0.30(95)         |                    | 1.301(22) | -0.24(17) | -0.21(38) |       |       | -0.51(61) |           | 0.024(53)  |             |           |
|        |        | (4.5b) | 1.27(46)  | 0.31(95)         |                    | 1.301(23) | -0.23(17) | -0.21(39) |       |       | -0.51(61) |           | 0.023(89)  | 0.01(24)    | -0.02(48) |
|        | (4.3b) | (4.5a) | 0.75(39)  | 0.23(99)         |                    | 1.303(22) | -0.24(17) | -0.19(38) |       |       |           | 0.41(97)  | 0.003(51)  |             |           |
|        |        | (4.5b) | 0.75(39)  | 0.23(99)         |                    | 1.302(23) | -0.24(17) | -0.19(38) |       |       |           | 0.41(97)  | 0.011(89)  | -0.02(24)   | -0.04(48) |
|        | (4.3c) | (4.5a) | 1.37(44)  | -0.04(1.00)      |                    | 1.301(22) | -0.24(17) | -0.18(38) |       |       | -0.40(59) | -0.14(98) | 0.028(54)  |             |           |
|        |        | (4.5b) | 1.37(44)  | -0.04(1.00)      |                    | 1.301(23) | -0.24(17) | -0.18(39) |       |       | -0.40(59) | -0.14(98) | 0.023(89)  | 0.02(24)    | -0.01(48) |
|        | (4.3d) | (4.5a) |           |                  |                    | 1.302(22) | -0.24(17) | -0.19(38) |       |       | -1.2(1.4) |           | 0.016(51)  |             |           |
|        |        | (4.5b) |           |                  |                    | 1.302(23) | -0.24(17) | -0.19(39) |       |       | -1.2(1.4) |           | 0.018(89)  | 0.001(242)  | -0.03(48) |
|        | (4.4b) | (4.5a) |           |                  |                    | 1.303(22) | -0.24(17) | -0.19(38) |       |       | 1.0(1.7)  |           | 0.007(50)  |             |           |
|        |        | (4.5b) |           |                  |                    | 1.302(23) | -0.24(17) | -0.19(39) |       |       | 1.0(1.7)  |           | 0.013(88)  | -0.01(24)   | -0.03(48) |
|        | (4.4c) | (4.5a) |           |                  |                    | 1.301(22) | -0.24(17) | -0.19(38) |       |       | -0.85(75) |           | 0.026(53)  |             |           |
|        |        | (4.5b) |           |                  |                    | 1.301(23) | -0.24(17) | -0.19(39) |       |       | -0.86(76) |           | 0.023(89)  | 0.02(24)    | -0.02(48) |

TABLE VII. (*Continued.*)

| Chiral | FV     | Disc.  | $g_0$    | $\tilde{c}_{34}$ | $\tilde{d}_{16}'$ | $c_0$     | $c_1$     | $c_2$    | $c_3$ | $c_4$    | $f_2'$    | $f_3$     | $a_2$     | $a_4$       | $b_4$     |
|--------|--------|--------|----------|------------------|-------------------|-----------|-----------|----------|-------|----------|-----------|-----------|-----------|-------------|-----------|
| (4.2c) | (4.5a) | (4.5a) |          |                  | 1.303(22)         | -0.24(17) | -0.19(38) |          |       |          | -1.0(1.7) |           | 0.007(50) |             |           |
|        | (4.4d) | (4.5b) |          |                  | 1.302(23)         | -0.24(17) | -0.18(39) |          |       |          | -1.0(1.7) |           | 0.013(88) | -0.01(24)   | -0.03(48) |
|        | (4.5a) | (4.5a) | 0.79(38) | 0.52(95)         | 1.276(10)         |           | -0.76(32) | 0.06(88) |       |          |           |           | 0.019(52) |             |           |
|        | (4.3a) | (4.5b) | 0.79(38) | 0.52(95)         | 1.275(12)         |           | -0.75(33) | 0.06(88) |       |          |           |           | 0.021(88) | 0.005(242)  | -0.04(48) |
|        | (4.5a) | (4.5a) | 1.29(46) | 0.32(96)         | 1.275(10)         |           | -0.75(32) | 0.05(88) |       |          | -0.51(59) |           | 0.035(53) |             |           |
|        | (4.3b) | (4.5b) | 1.29(46) | 0.32(96)         | 1.276(12)         |           | -0.75(33) | 0.05(88) |       |          | -0.51(59) |           | 0.029(89) | 0.03(24)    | -0.02(48) |
|        | (4.5a) | (4.5a) | 0.75(39) | 0.27(99)         | 1.276(10)         |           | -0.76(32) | 0.07(88) |       |          |           | 0.46(97)  | 0.013(51) |             |           |
|        | (4.3c) | (4.5b) | 0.75(39) | 0.27(99)         | 1.276(12)         |           | -0.76(33) | 0.07(88) |       |          |           | 0.46(97)  | 0.017(88) | -0.001(242) | -0.04(48) |
|        | (4.5a) | (4.5a) | 1.38(44) | -0.03(1.00)      | 1.275(10)         |           | -0.75(32) | 0.08(88) |       |          | -0.41(58) | -0.09(98) | 0.038(54) |             |           |
|        | (4.3d) | (4.5b) | 1.38(44) | -0.03(1.00)      | 1.275(12)         |           | -0.74(33) | 0.08(88) |       |          | -0.42(59) | -0.09(98) | 0.029(89) | 0.04(24)    | -0.01(48) |
|        | (4.5a) | (4.5a) |          |                  | 1.276(10)         |           | -0.76(32) | 0.08(88) |       |          | -1.3(1.4) |           | 0.027(51) |             |           |
|        | (4.4a) | (4.5b) |          |                  | 1.276(12)         |           | -0.75(33) | 0.08(88) |       |          | -1.3(1.4) |           | 0.023(88) | 0.02(24)    | -0.03(48) |
|        | (4.5a) | (4.5a) |          |                  | 1.276(10)         |           | -0.76(32) | 0.08(88) |       |          | 1.1(1.7)  |           | 0.018(50) |             |           |
|        | (4.4b) | (4.5b) |          |                  | 1.276(12)         |           | -0.76(33) | 0.08(88) |       |          | 1.1(1.7)  |           | 0.018(88) | 0.006(241)  | -0.04(48) |
|        | (4.5a) | (4.5a) |          |                  | 1.275(10)         |           | -0.75(32) | 0.08(88) |       |          | -0.89(75) |           | 0.037(53) |             |           |
|        | (4.5b) | (4.5b) |          |                  | 1.275(12)         |           | -0.75(33) | 0.07(88) |       |          | -0.90(76) |           | 0.029(89) | 0.03(24)    | -0.02(48) |
|        | (4.4c) | (4.5a) |          |                  | 1.276(10)         |           | -0.76(32) | 0.08(88) |       |          | -1.1(1.7) |           | 0.018(50) |             |           |
|        | (4.4d) | (4.5b) |          |                  | 1.276(12)         |           | -0.76(33) | 0.08(88) |       |          | -1.1(1.7) |           | 0.019(88) | 0.007(241)  | -0.04(48) |
|        | (4.5a) | (4.5a) | 0.79(38) | 0.52(95)         | 1.2758(91)        |           | -0.75(15) |          |       | 0.13(96) |           |           | 0.019(51) |             |           |
|        | (4.3a) | (4.5b) | 0.79(38) | 0.52(95)         | 1.276(11)         |           | -0.75(17) |          |       | 0.13(96) |           |           | 0.021(88) | 0.005(242)  | -0.04(48) |
|        | (4.5a) | (4.5a) | 1.29(46) | 0.32(96)         | 1.2756(92)        |           | -0.75(15) |          |       | 0.13(96) | -0.51(59) |           | 0.035(53) |             |           |
| (4.3c) | (4.3b) | (4.5b) | 1.29(46) | 0.32(96)         | 1.276(11)         |           | -0.75(17) |          |       | 0.12(96) | -0.51(59) |           | 0.029(89) | 0.03(24)    | -0.02(48) |
|        | (4.5a) | (4.5a) | 0.75(39) | 0.27(99)         | 1.2764(90)        |           | -0.75(15) |          |       | 0.14(96) |           | 0.46(97)  | 0.013(51) |             |           |
|        | (4.5b) | (4.5b) | 0.75(39) | 0.27(99)         | 1.276(11)         |           | -0.75(16) |          |       | 0.14(96) |           | 0.46(97)  | 0.017(88) | -0.002(241) | -0.04(48) |
|        | (4.5b) | (4.5b) |          |                  |                   |           |           |          |       |          |           |           |           |             |           |

TABLE VII. (Continued.)

| Chiral | FV     | Disc.  | $g_0$    | $\tilde{c}_{34}$ | $\tilde{d}_{16}^r$ | $c_0$      | $c_1$ | $c_2$     | $c_3$ | $c_4$    | $f_2$     | $f_3$     | $a_2$     | $a_4$      | $b_4$     |
|--------|--------|--------|----------|------------------|--------------------|------------|-------|-----------|-------|----------|-----------|-----------|-----------|------------|-----------|
| (4.2d) | (4.3d) | (4.5a) | 1.38(44) | -0.03(1.00)      |                    | 1.2748(91) |       | -0.73(15) |       | 0.15(96) | -0.41(58) | -0.09(98) | 0.038(53) |            |           |
|        |        | (4.5b) | 1.38(44) | -0.03(1.00)      |                    | 1.275(11)  |       | -0.73(17) |       | 0.15(96) | -0.42(59) | -0.09(98) | 0.029(89) | 0.04(24)   | -0.01(48) |
|        | (4.4a) | (4.5a) |          |                  |                    | 1.2756(90) |       | -0.74(15) |       | 0.14(96) | -1.3(1.4) |           | 0.027(51) |            |           |
|        |        | (4.5b) |          |                  |                    | 1.276(11)  |       | -0.74(16) |       | 0.14(96) | -1.3(1.4) |           | 0.023(88) | 0.02(24)   | -0.03(48) |
|        | (4.4b) | (4.5a) |          |                  |                    | 1.2762(90) |       | -0.75(15) |       | 0.14(96) | 1.1(1.7)  |           | 0.018(50) |            |           |
|        |        | (4.5b) |          |                  |                    | 1.276(11)  |       | -0.75(16) |       | 0.14(96) | 1.1(1.7)  |           | 0.018(88) | 0.006(241) | -0.04(48) |
|        | (4.4c) | (4.5a) |          |                  |                    | 1.2750(90) |       | -0.74(15) |       | 0.14(96) | -0.89(75) |           | 0.037(53) |            |           |
|        |        | (4.5b) |          |                  |                    | 1.275(11)  |       | -0.74(16) |       | 0.14(96) | -0.90(76) |           | 0.029(89) | 0.03(24)   | -0.02(48) |
|        | (4.4d) | (4.5a) |          |                  |                    | 1.2762(90) |       | -0.75(15) |       | 0.15(96) | -1.1(1.7) |           | 0.018(50) |            |           |
|        |        | (4.5b) |          |                  |                    | 1.276(11)  |       | -0.75(16) |       | 0.15(96) | -1.1(1.7) |           | 0.019(88) | 0.007(241) | -0.04(48) |

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