

Addendum to “Electromagnetic moments of ground and excited states calculated in heavy odd- N open-shell nuclei”

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This is an addendum to the article “Electromagnetic moments of ground and excited states calculated in heavy odd- N open-shell nuclei” published in *Phys. Rev. C* **113**, 024306 (2026). We update Table I by adding the column reporting the works by E. F. Zhou *et al.*

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Table I summarizes the work of theoretical groups that have previously studied nuclear electromagnetic moments using different versions of nuclear density functional theory (DFT). In this Addendum, we added a column describing the work by Zhou *et al.* [15–17]. Together with Minkov *et al.* [5], these are the only implementations so far that use parity breaking and parity restoration, with another such application already on the way [19]. Zhou *et al.* [15–17] determined the electromagnetic moments using nuclear wave functions without (^{25}Mg) and with (^{21}Ne , ^{129}Xe , ^{199}Hg , ^{225}Ra , ^{229}Th , and ^{229}Pa) parity symmetry breaking. After finding the self-consistent relativistic mean-field plus BCS (RMF + BCS) solutions, they restored the angular-momentum, particle-

number, and parity symmetries and mixed states corresponding to various collective constraints. However, they also used conserved signature and time-reversal symmetries, along with the equal-filling approximation, thereby neglecting the mean-field time-odd fields and angular-momentum polarizations of the core.

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Data availability. No data were created or analyzed in this study.

TABLE I. Tabular comparison of the differences of work done by other groups and our own, detailing the methods and symmetries imposed.

	Bally <i>et al.</i> Refs. [1–3]	Bonneau <i>et al.</i> , Minkov <i>et al.</i> Refs. [4,5]	Co' <i>et al.</i> Ref. [6]	Borrajio and Egido Refs. [7,8]	Li and Meng Ref. [9]	Péru <i>et al.</i> Ref. [10]	Sassarini <i>et al.</i> , Bonnard <i>et al.</i> , Wibowo <i>et al.</i> Refs. [11–13]	Ryssens <i>et al.</i> Ref. [14]	Zhou <i>et al.</i> Refs. [15–17]	Nakada and Iwata Ref. [18]
Nuclei	²⁵ Mg,Xe ¹⁹⁷ Au	$A \approx 50, 100$ $A \approx 178, 236$ ²²⁹ Th	Near doubly magic	Mg isotopes	$A \approx 16, 40$ $A \approx 208$	Hg	Near doubly magic Open-shell Hg-Pb Open-shell Sn-Gd	In	²⁵ Mg ²¹ Ne ¹²⁹ Xe ¹⁹⁹ Hg ²²⁵ Ra ²²⁹ Th ²²⁹ Pa	Near doubly magic
HF			✓		✓		✓			✓
HF+BCS		✓								
RMF+BCS					✓				✓	
HFB	✓			✓		✓	✓	✓		
S.P. operator	✓	✓	MEC	✓	MEC	✓	✓	✓	✓	✓
Effective g factors		✓				✓				
Core contribution	Microscopic	Microscopic	Perturbative	Microscopic	Perturbative	Model	Microscopic	Microscopic	Microscopic	Microscopic
Collective mixing	✓			✓				✓	✓	
Blocking	✓	✓		✓		✓	✓	✓	✓	
Skyrme	SLyMR0	SLyIII, SLyIII.0.8					UNEDF1, SLy4, SkO'	BSkG		
Gogny			D1S, D1M	D1S		D1M	D1S			
Yukawa										M3Y-P6
Regularized							N ³ LO			
Lagrangian					PK1, PC-F1				PC-F1, PC-PK1	
Spin-spin	Native	Native	None	None	None	None	g'_0 adjusted	Native	None	Native
HO basis		Cylindrical	Spherical	Spherical	Spherical	Deformed	3D		Cylindrical, 3D	
Oscillator shells		13		8	10	19	16		8, 12	
Gaussian basis										Spherical
Spatial coordinates	3D							3D		
Conserved parity	✓	✓ or not	✓	✓	✓	✓	✓	✓		✓
Conserved signature	✓		✓	✓	✓	✓		✓	✓	
Conserved T -reversal			✓	✓	✓	✓			✓	
Spherical states			✓		✓					
Axial states		✓				✓	✓		✓	✓
Triaxial states ^a	✓			✓				✓	✓	
Reference frame	AMP+PNP	Intrinsic	Laboratory	AMP+PNP	Laboratory	Intrinsic	AMP	Intrinsic	AMP+PNP+ π	Intrinsic

^aSee the original publications regarding the employed relationship between the conserved signature and triaxial shapes.

- [1] B. Bally, B. Avez, M. Bender, and P.-H. Heenen, Beyond mean-field calculations for odd-mass nuclei, *Phys. Rev. Lett.* **113**, 162501 (2014).
- [2] B. Bally, G. Giacalone, and M. Bender, Structure of ^{128,129,130}Xe through multi-reference energy density functional calculations, *Eur. Phys. J. A* **58**, 187 (2022).
- [3] B. Bally, G. Giacalone, and M. Bender, The shape of gold, *Eur. Phys. J. A* **59**, 58 (2023).
- [4] L. Bonneau, N. Minkov, D. D. Duc, P. Quentin, and J. Bartel, Effect of core polarization on magnetic dipole moments in deformed odd-mass nuclei, *Phys. Rev. C* **91**, 054307 (2015).
- [5] N. Minkov, A. Pálffy, P. Quentin, and L. Bonneau, Skyrme-Hartree-Fock-BCS approach to ^{229m}Th and neighboring nuclei, *Phys. Rev. C* **110**, 034327 (2024).
- [6] G. Co', V. De Donno, M. Anguiano, R. N. Bernard, and A. M. Lallena, Electric quadrupole and magnetic dipole moments of odd nuclei near the magic ones in a self-consistent approach, *Phys. Rev. C* **92**, 024314 (2015).
- [7] M. Borrajio and J. L. Egido, A symmetry-conserving description of odd nuclei with the Gogny force, *Eur. Phys. J. A* **52**, 277 (2016).
- [8] M. Borrajio and J. L. Egido, Ground-state properties of even and odd Magnesium isotopes in a symmetry-conserving approach, *Phys. Lett. B* **764**, 328 (2017).
- [9] J. Li and J. Meng, Nuclear magnetic moments in covariant density functional theory, *Front. Phys.* **13**, 132109 (2018).
- [10] S. Péru, S. Hilaire, S. Goriely, and M. Martini, Description of magnetic moments within the Gogny

- Hartree-Fock-Bogolyubov framework: Application to Hg isotopes, *Phys. Rev. C* **104**, 024328 (2021).
- [11] P. L. Sassarini, J. Dobaczewski, J. Bonnard, and R. F. Garcia Ruiz, Nuclear DFT analysis of electromagnetic moments in odd near doubly magic nuclei, *J. Phys. G: Nucl. Part. Phys.* **49**, 11LT01 (2022).
- [12] J. Bonnard, J. Dobaczewski, G. Danneaux, and M. Kortelainen, Nuclear DFT electromagnetic moments in heavy deformed open-shell odd nuclei, *Phys. Lett. B* **843**, 138014 (2023).
- [13] H. Wibowo, B. C. Backes, J. Dobaczewski, R. P. de Groote, A. Nagpal, A. Sánchez-Fernández, X. Sun, and J. L. Wood, Electromagnetic moments in the Sn-Gd region determined within nuclear DFT, *J. Phys. G: Nucl. Part. Phys.* **52**, 065104 (2025).
- [14] W. Ryssens, G. Scamps, S. Goriely, and M. Bender, Skyrme-Hartree-Fock-Bogoliubov mass models on a 3D mesh: II. Time-reversal symmetry breaking, *Eur. Phys. J. A* **58**, 246 (2022).
- [15] E. F. Zhou, X. Y. Wu, and J. M. Yao, Multireference covariant density-functional theory for the low-lying states of odd-mass nuclei, *Phys. Rev. C* **109**, 034305 (2024).
- [16] E. F. Zhou, J. M. Yao, J. Engel, and J. Meng, Effects of beyond-mean-field correlations on nuclear Schiff moments, *arXiv:2507.01369*.
- [17] E. F. Zhou and J. M. Yao, Microscopic study of low-lying states in odd-mass nuclei for atomic electric dipole moment searches, *arXiv:2511.05984*.
- [18] H. Nakada and H. Iwata, Magnetic dipole moments adjacent to doubly magic nuclei in self-consistent mean-field theory with realistic spin-isospin and tensor forces, *Phys. Rev. C* **113**, 014315 (2026).
- [19] A. Restrepo-Giraldo, J. Dobaczewski, J. Bonnard, and X. Sun, Radiative decay of the ^{229}Th isomer determined within nuclear DFT, *arXiv:2602.02429*.