

Erratum: Quantum magic and multipartite entanglement in the structure of nuclei [Phys. Rev. C **111**, 034317 (2025)]

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Incorrect values of J_z were used in some of the calculations of the n -tangles of some nuclei, namely, ^{14}N , ^{26}Na , ^{27}Na , ^{29}Mg , ^{28}Al , ^{32}Al , ^{34}Cl , ^{38}K , and their mirrors. These resulted in incorrect values for the associated n -tangles displayed in Fig. 6 of Sec. IV B, and in Tables I to XII of Appendix A in the original paper. The values of magic (stabilizer Rényi entropies) are correct. However, the above error resulted in incorrect values of J entered in Tables IX to XV of Appendix A for these nuclei.

Below we provide in Fig. 1 a corrected version of Fig. 6 of the original paper. In Tables I, II, III, IV we provide the corrected affected lines for Tables I to XII of Appendix A of the original paper (n -tangles). Finally the following Tables V, VI, VII, and VIII show the corrected lines for Tables IX to XV of Appendix D of the original paper. In the tables we highlight the modified entries in bold font. The lines not shown here are correct in the original paper.

The conclusions of the paper remain unchanged.

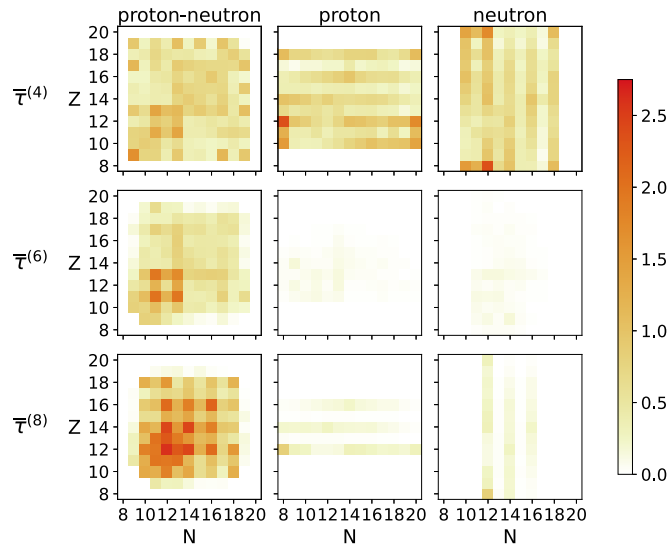


FIG. 1. Values of the proton-neutron, pure proton, and pure neutron summed n -tangles, $\bar{\tau}_{\pi\nu}^{(n)}$, $\bar{\tau}_{\pi}^{(n)}$, $\bar{\tau}_{\nu}^{(n)}$, as defined in Eq. (9), for sd -shell nuclei.

TABLE I. Summed values of n -tangles, $\bar{\tau}_{\pi\nu}^{(n)}$, $\bar{\tau}_{\pi}^{(n)}$ and $\bar{\tau}_{\nu}^{(n)}$, for $n = 2, 4, 6, 8$, in the $J_z = J$ ground states of the p -shell nuclei carbon, nitrogen, and oxygen. The implicit isospin symmetry of the Hamiltonian implemented in BIGSTICK gives rise to exact relations between the n -tangles among different nuclei. All entries result from exact calculations.

Z	N	A	J	Summed 2-tangle		Summed 4-tangle			Summed 6-tangle			Summed 8-tangle		
				$\overline{\tau}_{\pi}^{(2)}$	$\overline{\tau}_{\nu}^{(2)}$	$\overline{\tau}_{\pi\nu}^{(4)}$	$\overline{\tau}_{\pi}^{(4)}$	$\overline{\tau}_{\nu}^{(4)}$	$\overline{\tau}_{\pi\nu}^{(6)}$	$\overline{\tau}_{\pi}^{(6)}$	$\overline{\tau}_{\nu}^{(6)}$	$\overline{\tau}_{\pi\nu}^{(8)}$	$\overline{\tau}_{\pi}^{(8)}$	$\overline{\tau}_{\nu}^{(8)}$
Nitrogen (N)														
2+5	2+5	14	1	0.0320	0.0320	0.1984	0	0	0	0	0	0	0	0

TABLE II. Summed values of n -tangles, $\bar{\tau}_{\pi\nu}^{(n)}$, $\bar{\tau}_{\pi}^{(n)}$ and $\bar{\tau}_{\nu}^{(n)}$, for $n = 4, 6, 8$, in the $J_z = J$ ground states of the sd -shell nuclei sodium, magnesium, and aluminum. The implicit isospin symmetry of the Hamiltonian implemented in BIGSTICK gives rise to exact relations between the n -tangles among different nuclei. All entries result from exact calculations.

Z	N	A	J	Summed 4-tangle			Summed 6-tangle			Summed 8-tangle		
				$\overline{\tau}_{\pi\nu}^{(4)}$	$\overline{\tau}_{\pi}^{(4)}$	$\overline{\tau}_{\nu}^{(4)}$	$\overline{\tau}_{\pi\nu}^{(6)}$	$\overline{\tau}_{\pi}^{(6)}$	$\overline{\tau}_{\nu}^{(6)}$	$\overline{\tau}_{\pi\nu}^{(8)}$	$\overline{\tau}_{\pi}^{(8)}$	$\overline{\tau}_{\nu}^{(8)}$
Sodium (Na)												
8+3	8+7	26	3	0.3973	0.5497	0.3315	0.3833	0.0160	0.0298	0.6267	0	0.0210
	8+8	27	5/2	0.4554	0.5816	0.6287	0.4351	0.0183	0.0185	1.0569	0	0.1090
Magnesium (Mg)												
8+4	8+9	29	3/2	0.4318	0.8205	0.2194	0.6411	0.0451	0.0109	0.7398	0.2047	0
Aluminum (Al)												
8+5	8+7	28	3	0.8923	0.4251	0.3974	0.8015	0.0523	0.0926	1.1091	0.0238	0.0272
	8+11	32	1	1.0592	0.1640	0	0.2626	0.0084	0	0.1406	0.0039	0

TABLE III. Summed values of n -tangles, $\bar{\tau}_{\pi\nu}^{(n)}$, $\bar{\tau}_{\pi}^{(n)}$, and $\bar{\tau}_{\nu}^{(n)}$, for $n = 4, 6, 8$, in the $J_z = J$ ground states of the sd -shell nuclei silicon, phosphorus, and sulfur. The implicit isospin symmetry of the Hamiltonian implemented in BIGSTICK gives rise to exact relations between the n -tangles among different nuclei. All entries result from exact calculations.

Z	N	A	J	Summed 4-tangle			Summed 6-tangle			Summed 8-tangle		
				$\overline{\tau}_{\pi\nu}^{(4)}$	$\overline{\tau}_{\pi}^{(4)}$	$\overline{\tau}_{\nu}^{(4)}$	$\overline{\tau}_{\pi\nu}^{(6)}$	$\overline{\tau}_{\pi}^{(6)}$	$\overline{\tau}_{\nu}^{(6)}$	$\overline{\tau}_{\pi\nu}^{(8)}$	$\overline{\tau}_{\pi}^{(8)}$	$\overline{\tau}_{\nu}^{(8)}$
Phosphorus (P)												
8+7	8+3	26	3	0.3973	0.3315	0.5497	0.3833	0.0298	0.0160	0.6267	0.0210	0
	8+5	28	3	0.8923	0.3974	0.4251	0.8015	0.0926	0.0523	1.1091	0.0272	0.0238
Sulfur (S)												
8+8	8+3	27	5/2	0.4554	0.6287	0.5816	0.4351	0.0185	0.0183	1.0569	0.1090	0

TABLE IV. Summed values of n -tangles, $\bar{\tau}_{\pi\nu}^{(n)}$, $\bar{\tau}_{\pi}^{(n)}$, and $\bar{\tau}_{\nu}^{(n)}$, for $n = 4, 6, 8$, in the $J_z = J$ ground states of the sd -shell nuclei chlorine, argon, and potassium. The implicit isospin symmetry of the Hamiltonian implemented in BIGSTICK gives rise to exact relations between the n -tangles among different nuclei. All entries result from exact calculations.

Z	N	A	J	Summed 4-tangle			Summed 6-tangle			Summed 8-tangle		
				$\overline{\tau}_{\pi\nu}^{(4)}$	$\overline{\tau}_{\pi}^{(4)}$	$\overline{\tau}_{\nu}^{(4)}$	$\overline{\tau}_{\pi\nu}^{(6)}$	$\overline{\tau}_{\pi}^{(6)}$	$\overline{\tau}_{\nu}^{(6)}$	$\overline{\tau}_{\pi\nu}^{(8)}$	$\overline{\tau}_{\pi}^{(8)}$	$\overline{\tau}_{\nu}^{(8)}$
Chlorine (Cl)												
8+9	8+4	29	3/2	0.4318	0.2194	0.8205	0.6411	0.0109	0.0451	0.7398	0	0.2047
	8+9	34	3	0.6965	0.3182	0.3182	0.5353	0.0097	0.0097	0.3775	0	0
Potassium (K)												
8+11	8+5	32	1	1.0592	0	0.1640	0.2626	0	0.0084	0.1406	0	0.0039
	8+11	38	3	0.0382	0	0	0	0	0	0	0	0

TABLE V. Measures of magic in the $J_z = J$ ground states of the p -shell nuclei carbon, nitrogen, and oxygen. The implicit isospin symmetry of the Hamiltonian implemented in BIGSTICK gives rise to exact relations between the measures of magic among different nuclei, as indicated in the entry. The second to last column indicates the technique used in the evaluation: “Exact” indicates an exact computation, while “PSIZE-MCMC” indicates evaluation using the PSize-MCMC algorithm, in which case the last column indicates the $\hat{R}$ value.

Z	N	A	J	$\mathcal{M}_{\text{lin}}$	$\mathcal{M}_1$	$\mathcal{M}_2$	Method
Nitrogen (N)							
2+5	2+5	14	1	0.2298	0.6863	0.3767	Exact

TABLE VI. Measures of magic in the $J_z = J$ ground states of the sd -shell nuclei sodium, magnesium and aluminium. The implicit isospin symmetry of the Hamiltonian implemented in BIGSTICK gives rise to exact relations between the measures of magic among different nuclei, as indicated in the entry. The second to last column indicates the technique used in the evaluation: “Exact” indicates an exact computation, while “MC” indicates evaluation using the PSize-MCMC algorithm, in which case the last column indicates the $\hat{R}$ value.

Z	N	A	J	$\mathcal{M}_{\text{lin}}$	$\mathcal{M}_1$	$\mathcal{M}_2$	Method	$\hat{R}$
Sodium (Na)								
8+3	8+7	26	3	0.9372(7)	8.469(25)	3.995(15)	MC	1.01
	8+8	27	5/2	0.9492(5)	8.370(20)	4.300(15)	MC	1.01
Magnesium (Mg)								
8+4	8+9	29	3/2	0.9738(2)	9.178(14)	5.256(11)	MC	1.01
Aluminium (Al)								
8+5	8+7	28	3	0.9743(0)	10.1967(10)	5.2814(17)	MC	1.00
	8+11	32	1	0.8072(3)	4.7921(74)	2.3749(25)	MC	1.00

TABLE VII. Measures of magic in the $J_z = J$ ground states of the sd -shell nuclei silicon, phosphorus, and sulfur. The implicit isospin symmetry of the Hamiltonian implemented in BIGSTICK gives rise to exact relations between the measures of magic among different nuclei, as indicated in the entry. The second to last column indicates the technique used in the evaluation: “Exact” indicates an exact computation, while “MC” indicates evaluation using the PSize-MCMC algorithm, in which case the last column indicates the $\hat{R}$ value.

Z	N	A	J	$\mathcal{M}_{\text{lin}}$	$\mathcal{M}_1$	$\mathcal{M}_2$	Method	$\hat{R}$
Phosphorus (P)								
8+7	8+3	26	3	0.9372(7)	8.469(25)	3.995(15)	MC	(²⁶ Na)
	8+5	28	3	0.9750(1)	10.54(4)	5.32(5)	MC	(²⁸ Al)
Sulfur (S)								
8+8	8+3	27	5/2	0.9492(5)	8.370(20)	4.300(15)	MC	(²⁷ Na)

TABLE VIII. Measures of magic in the $J_z = J$ ground states of the sd -shell nuclei chlorine, argon, and potassium. The implicit isospin symmetry of the Hamiltonian implemented in BIGSTICK gives rise to exact relations between the measures of magic among different nuclei, as indicated in the entry. The second to last column indicates the technique used in the evaluation: “Exact” indicates an exact computation, while “MC” indicates evaluation using the PSize-MCMC algorithm, in which case the last column indicates the $\hat{R}$ value.

Z	N	A	J	$\mathcal{M}_{\text{lin}}$	$\mathcal{M}_1$	$\mathcal{M}_2$	Method	$\hat{R}$
Chlorine (Cl)								
8+9	8+4	29	3/2	0.9738(2)	9.178(14)	5.256(11)	MC	(²⁹ Mg)
	8+9	34	3	0.8792(0)	6.2101(60)	3.0492(7)	MC	1.00
Potassium (K)								
8+11	8+5	32	1	0.8072(3)	4.7921(74)	2.3749(25)	MC	(³² Al)
	8+11	38	3	0.0545(0)	0.2251(3)	0.0809(0)	MC	1.00