

Probing nuclear structure with the Balitsky-Kovchegov equation in full impact-parameter dependence

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Building on the newly available solution of the Balitsky-Kovchegov (BK) equation with the full impact-parameter dependence, we extend the study of parton evolution from proton to nuclear targets. Since a key part of the scientific program for future experimental facilities such as the EIC is to study gluon dynamics and shed new light on the phenomenon of parton saturation, we present predictions for key processes, such as deep-inelastic scattering or the diffractive production of vector mesons, on a variety of nuclear targets. Besides the standard BK equation, we employ its linearized version to identify a promising channel to search for gluon saturation in the nuclear collisions. Furthermore, we implement a tetrahedral model of oxygen to search for deviations from the standard, isotropic, Woods-Saxon approach. In addition to the future colliders, the presented results are also of interest for the current studies of nuclear vector-meson production at the LHC.

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I. INTRODUCTION

The structure of nuclei, specifically its gluon component, has been studied extensively in the perturbative regime of QCD. Besides nuclear deep-inelastic scattering (DIS), the coherent photoproduction of vector mesons was shown a long time ago to be a sensitive probe of the nuclear color field [1,2]. Such processes have been studied in the past and current experimental facilities, such as HERA [3–5] and LHC [6–9], and are of great interest for future electron-ion facilities such as EIC [10,11] or LHeC [12]. Both the recent measurements and those to come have renewed interest in predictions of such nuclei-induced processes [13–15].

Compared to previous facilities, EIC will, for the first time, provide electron-ion collisions over a larger range of the virtuality-Bjorken- x plane [11]. The kinematic region to be explored is important for studying parton saturation, a phenomenon necessary for a consistent description of hadron structure. In addition to extending the kinematic region, new data should be available for a variety of light and heavy nuclear targets [16].

These new data will shed light on the difference between the gluon structure of nuclear and proton targets. At low Bjorken x , the nuclear structure function per nucleon is smaller than that of the proton. This suppression, known as shadowing, has been interpreted in phenomenological studies

as a consequence of multiple scattering [17,18]. Additional contributions may arise from gluon recombination due to the overlap of gluon fields from different nucleons [19,20]. At a certain scale, the gluon recombination balances splitting and the gluon density ceases to grow with increasing interaction energy. Such a regime is referred to as gluon saturation. Quantitatively, the evolution of the gluon density in a fast-moving frame is described by nonlinear evolution equations. One of the most commonly used evolution equations in this context is the Balitsky-Kovchegov (BK) equation [21,22], which has been used to describe phenomena observed in proton collisions. This has been done using the impact-parameter-independent framework (e.g., Ref. [23]), and was later extended to a framework including the dependence on the impact-parameter magnitude [24,25] and even the relative angle between the dipole size and impact parameter [26–28]. The BK evolution equation has also been used to evolve the gluon structure of the nucleus in Refs. [13,29], and compared to the more commonly used approach via the Glauber model in the framework with explicit dependence on the magnitude of the impact parameter.

The work presented in this paper builds on the recently published solution of the target-rapidity BK equation with the full impact-parameter dependence for the proton target, presented in Refs. [27,28], and extends it to nuclear targets according to the procedure described in Refs. [13,29]. Our solution provides theoretical predictions for processes connected to saturation physics, to be measured at the EIC and other future experimental facilities. We chose a palette of nuclear targets of potential interest for EIC, namely C, O, Ca, Fe, Cu, Au, and Pb. Since oxygen-oxygen and proton-oxygen collisions are on the current LHC agenda, and since various models exist for describing the nuclear structure, we also adopted an α -cluster approach for the oxygen profile alongside the standard Woods-Saxon distribution. This model

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is based on Refs. [30,31], and we provide its implementation as a Python PyPI library [32].

The results of the nuclear BK equation are used to predict the nuclear structure functions, the nuclear modification factor R_{pA} , and the cross sections of J/ψ vector-meson photoproduction. In all cases, good agreement with available experimental data is found.

To study the onset of saturation effects in nuclear targets in a quantitative way, we introduce a suppression of the nonlinear term in the BK evolution equation. In schematic form, the BK equation reads $\partial_\eta N = K \otimes (N - \kappa N^2)$, where the second term on the right-hand side, proportional to N^2 , encodes the nonlinear dynamics. The scalar regulator $\kappa \in (0, 1)$ controls the strength of the saturation effects such that in the limiting cases one either recovers the original BK equation ($\kappa = 1$), or gets its linearized version ($\kappa = 0$), equivalent to the BFKL equation [33–36].

By comparing theoretical predictions from the standard and linearized BK equations, we identify nuclear vector-meson production, and especially its differential cross section, as a particularly promising channel to probe saturation effects.

The work is organized as follows: Sec. II presents an overview of the BK formalism and its extension to nuclear targets, together with a description of the approach to modeling the tetrahedral oxygen target. In Sec. III, we compare the dipole amplitudes obtained from the standard and linearized BK equation for proton and lead targets, Sec. IV presents the predictions for the nuclear structure functions and the vector-meson production cross sections and show the comparison to data, and Sec. V briefly summarizes the presented work.

II. FORMALISM OVERVIEW

The Balitsky-Kovchegov equation [21,22,37–39]

$$\begin{aligned} & \frac{\partial N(\vec{r}, \vec{b}, \eta)}{\partial \eta} \\ &= \int d\vec{r}_1 K(\vec{r}, \vec{r}_1, \vec{r}_2) (N(\vec{r}_1, \vec{b}_1, \eta_1) + N(\vec{r}_2, \vec{b}_2, \eta_2) \\ & \quad - N(\vec{r}, \vec{b}, \eta) - \kappa N(\vec{r}_1, \vec{b}_1, \eta_1) N(\vec{r}_2, \vec{b}_2, \eta_2)) \end{aligned} \quad (1)$$

provides a phenomenological tool to incorporate gluon saturation within the color dipole picture. It evolves $N(\vec{r}, \vec{b}, \eta)$, the amplitude for the interaction between the color field of the target and a quark-antiquark dipole of size $\vec{r}$ and impact parameter $\vec{b}$, with respect to η , the rapidity of the target. This relates to the Bjorken x as $\eta = \ln(x_0/x)$ with $x_0 = 0.01$. Both the dipole size and the impact parameter are two-dimensional (2D) vectors in the light-cone transverse plane with sizes $r \equiv |\vec{r}|$ and $b \equiv |\vec{b}|$. The terms on the right-hand side of the equation correspond to the parent dipole $(\vec{r}, \vec{b}, \eta)$ and two daughter dipoles $(\vec{r}_1, \vec{b}_1, \eta_1)$ and $(\vec{r}_2, \vec{b}_2, \eta_2)$. The rapidities η_1, η_2 introduce a nonlocality to the equation via

$$\eta_i = \eta - \max \left\{ 0, \ln \left(\frac{r^2}{r_i^2} \right) \right\}. \quad (2)$$

TABLE I. Parameters of the solution to the BK including explicit dependence on (r, b, θ) according to Ref. [28]. Parameters were obtained for quark masses $m_{uds} = 0.1$ GeV, $m_c = 1.27$ GeV and $m_b = 4.18$ GeV.

Q_{s0}^2	γ	c	B	C
0.496 GeV ²	1.25	1	3.4 GeV ²	5

The kernel K in the collinearly improved target-rapidity framework reads [40]

$$K(\vec{r}, \vec{r}_1, \vec{r}_2) = \frac{\bar{\alpha}_s}{2\pi} \frac{r^2}{r_1^2 r_2^2} \left(\frac{r^2}{\min(r_1^2, r_2^2)} \right)^{\pm \bar{\alpha}_s A_1}, \quad (3)$$

where the constant $A_1 = 11/12$. The sign in exponent is positive whenever $r < \min(r_1, r_2)$ and negative otherwise. The coupling constant is defined as $\bar{\alpha}_s = (N_C/\pi)\alpha_s$ with $\alpha_s = \alpha_s(\min(r, r_1, r_2))$ the running coupling constant evaluated in the variable-number-of-flavors scheme, see Ref. [25]. The free parameter of the coupling constant is the infrared regulator C , and the number of colors is $N_C = 3$.

The solution of (target-rapidity, collinearly improved) BK equation in various levels of approximation has been demonstrated to be a viable tool for describing experimental data for various processes [such as DIS, vector-meson production, deeply virtual Compton scattering (DVCS), etc.] off a proton target [25,40,41]. This equation has recently been solved, including the dependence on three out of the four total spatial degrees of freedom, i.e., dependence on the dipole size r , impact parameter size b , and their relative angle θ , using initial conditions of the form

$$\begin{aligned} N(r, b, \theta, \eta = 0) \\ = 1 - \exp \left(-\frac{1}{4} (Q_{s0}^2 r^2)^\gamma T_p(b, r) (1 + c \cos(2\theta)) \right) \end{aligned} \quad (4)$$

with

$$T_p(b, r) = \exp \left(-\frac{b^2 + (r/2)^2}{2B} \right). \quad (5)$$

Here, Q_{s0}^2, γ, c, B are free parameters chosen to describe available experimental data for proton-induced processes such as DIS or production of various vector mesons, as described in more detail in Refs. [27,28]. The parameter values in Table I differ from those in Refs. [27,28], as additional experimental data have been used for their extraction, namely DVCS and the electroproduction of various vector mesons off a proton target.

Due to the computational complexity of the three-dimensional (3D) BK equation, the parameters in Table I do not come from a full fitting procedure, but rather from an estimate based on a coarse scanning of the parameter space. The palette of experimental measurements used for this estimate consists of proton structure functions [4], differential and total cross sections of vector-meson production (ϕ [42,43], ρ [42,44–47], ω [48,49], J/ψ [42,43,50–54], $\psi(2S)$ [51,55], and $\Upsilon(1S)$ [56–60]) and DVCS [61], all of them in virtuality range $Q^2 \in [0, 35]$ GeV², resulting in $\chi^2/\text{ndof} = 6.4$. While this χ^2 may seem large, it should be noted that this study

is simultaneously taking into account a large variety of experimental observables rather than a narrow set of data for one particular process. Given that several of the included data sets have very narrow experimental uncertainties, while the present framework is expected to carry at least a few-percent systematic theory uncertainty (e.g., missing higher-order effects), an χ^2 close to unity is not expected; substantially smaller values would likely indicate that theory uncertainties are being effectively absorbed into the fit rather than genuinely predicted. Furthermore, no K factor has been introduced to improve the data description.

A. Nuclear BK equation

As this newly obtained solution was only validated against available experimental data probing the structure of the proton, we aim to extend the palette of phenomenological predictions to processes including nuclear targets. The transition from the proton BK equation to the nuclear one follows the approach of Refs. [13,29] and consists of two modifications of the initial condition. First, the A -dependent form of saturation scale is used,

$$Q_{s0}^2 \rightarrow Q_{s0,A}^2 = 0.1 \text{ GeV}^2 A^{1/3}, \quad (6)$$

where A is the nucleus mass number, and second, the profile function, modeling the proton as a Gaussian, is replaced with the nuclear thickness functions, normalized such that

$$T_p(b, r) \rightarrow T_A(b, r) = \frac{T(\sqrt{b^2 + r^2/4})}{\max(T(\sqrt{b^2 + r^2/4}))}. \quad (7)$$

In most cases, the nuclear profile is modeled according to the Woods-Saxon (two-point Fermi) distribution for the corresponding nucleus [62]

$$T(\tilde{b} = \sqrt{b^2 + r^2/4}) = \int_{-\infty}^{+\infty} dz \frac{\rho_0}{1 + \exp[(\sqrt{\tilde{b}^2 + z^2} - R)/a]}, \quad (8)$$

where $\max(T(\tilde{b})) = T(0)$. For very light nuclei, the distribution is modified, namely to the three-point Fermi distribution for oxygen and to the modified harmonic-oscillator distribution for carbon [62]. In case of oxygen, an additional model is used, approximating the nucleus as a tetrahedral cluster of α particles. This model leads to the geometry where the nuclear profile is not maximal at the center, but its maximum is shifted to the outer shell of the nucleus.

In addition to the aforementioned changes with respect to the proton solution in Ref. [28], the upper bound of the dipole-amplitude sampling range is extended to $r_{\max} = 10^4 \text{ GeV}^{-1}$, $b_{\max} = 10^4 \text{ GeV}^{-1}$, as can be seen in Fig. 3. This is to address that, in the interaction with large nuclei, a significant contribution comes from larger dipoles than in the case of protons. The remaining numerical setup for the BK equation solution is the same as in Ref. [28].

B. Tetrahedral oxygen

The simulation of the oxygen nucleus as a cluster of α particles was performed by combining phenomenological models

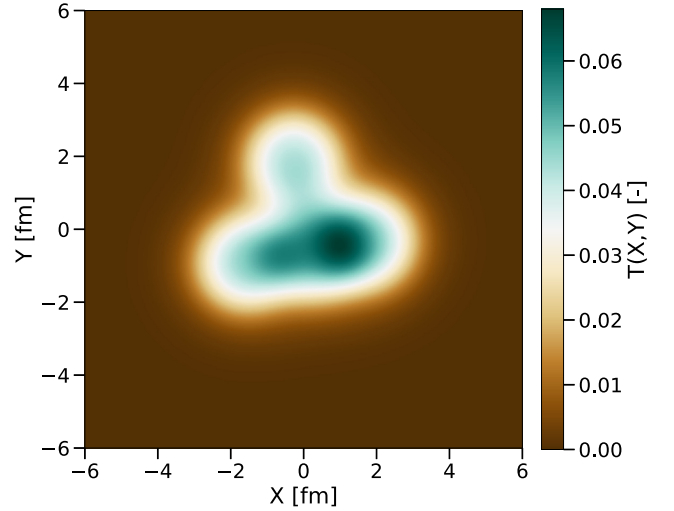


FIG. 1. An example density profile obtained from a single random orientation of the oxygen α -cluster tetrahedron.

of its internal tetrahedral composition and a parametrization of the helium density, obtained from elastic electron-scattering experiments [31,62].

We started by randomly orienting a tetrahedron with a side of 3.42 fm that represents the α cluster forming the oxygen nucleus. This tetrahedron size has been shown to reproduce the ^{16}O rms radius of 2.699 fm [30] as discussed in more detail in Ref. [31].

We imposed a stochastic uncertainty on the exact structure of the tetrahedron by applying a normal smearing on the position of the vertices of the tetrahedron in 3D with $\sigma = 0.1 \text{ fm}$ in order not to assume this structure to be absolutely rigid event by event.

Having generated the spatial tetrahedron structure, we projected its vertices onto the transverse plane, obtaining the centers of the helium nuclei in 2D. Their density was then modeled with the experimentally obtained parametrization available in Table V of Ref. [62]. We integrated this distribution over the longitudinal direction, solving for the transverse density profile of the oxygen nucleus; an example shown in Fig. 1.

In order to obtain a thickness function of the α -clustered oxygen, we took an average over 5000 independent random setups of such a nucleus. The resulting thickness functions are shown in Fig. 2.

III. BK EQUATION SOLUTIONS

The solution of the BK equation for the proton and nuclear targets is shown in Fig. 3. For the case of the nonlinear version of the equation with $\kappa = 1$, the evolution does not inflate the amplitude indefinitely, but rather modifies its morphology with a slower propagation speed of the wave front. This alteration of the evolution speed was reported previously in the work focusing on the evolution in the magnitude of the vectors $\vec{r}$ and $\vec{b}$ [25]. In contrast, the amplitude obtained by solving the linearized version of the BK equation grows steadily during

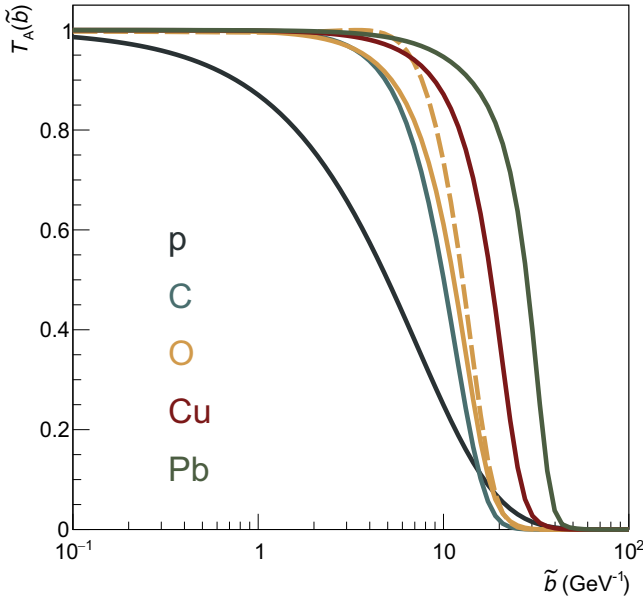


FIG. 2. The Gaussian profile of the proton (shown in black), and partially integrated Woods-Saxon profiles of carbon, copper, and lead (teal, red, and green, respectively). Two versions of the oxygen profile are shown. The full ochre curve is obtained using the Woods-Saxon distribution, and the dashed ochre curve corresponds to the averaged tetrahedral oxygen formed by clusters of α particles.

the evolution. This behavior has a significant impact on the observables calculated within both limiting cases.

Note that, while in the particular phase-space point shown in Fig. 3, the peak value of the dipole amplitude decreases between the initial condition and $\eta = 4$, the physically relevant evolution takes place at smaller values of r , to the left of the peak. Values of $r \geq 1/\Lambda_{\text{QCD}}$ correspond to the nonperturbative region, and this part of phase-space is suppressed by the convolution with (mostly perturbatively calculated) wave

functions in calculations of measurable observables. Here, the amplitude increases steadily, and the evolution resembles the standard “traveling wave” known from solutions of the only- r -dependent BK equation. Furthermore, it is this low- r region that has the dominant effect on the observables such as the structure functions, which consistently increase with rapidity, despite the decrease of the amplitude peak. This feature is natural to impact-parameter-dependent solutions of the BK equation and has been reported in Ref. [25].

IV. OBSERVABLES

To study the effect of saturation in the evolution of hadron structure, two processes are considered here: deep-inelastic scattering and coherent vector-meson photoproduction.

A. Nuclear structure functions

For deep-inelastic scattering, the studied observable is the structure function $F_2(x, Q^2)$. It is given by the cross sections of the interaction between the virtual photon γ^* and the target nucleus A summed over possible photon polarizations (L, T) and flavors f of the contributing quarks [63,64]:

$$F_2^{p,A}(x, Q^2) = \sum_f \frac{Q^2}{4\pi^2 \alpha_{\text{em}}} [\sigma_{L,f}^{\gamma^* p,A}(Q^2, x_f) + \sigma_{T,f}^{\gamma^* p,A}(Q^2, x_f)], \quad (9)$$

where α_{em} is the electromagnetic coupling constant and

$$x_f = \frac{x}{1 + 4 \frac{m_f^2}{Q^2}} = \frac{x_0 e^{-\eta}}{1 + 4 \frac{m_f^2}{Q^2}}, \quad (10)$$

with m_f the mass of a quark with flavor f . Virtuality of the exchanged photon is denoted as Q^2 .

The photon-nucleus cross sections are connected to the dipole amplitude by a convolution with the light-front photon

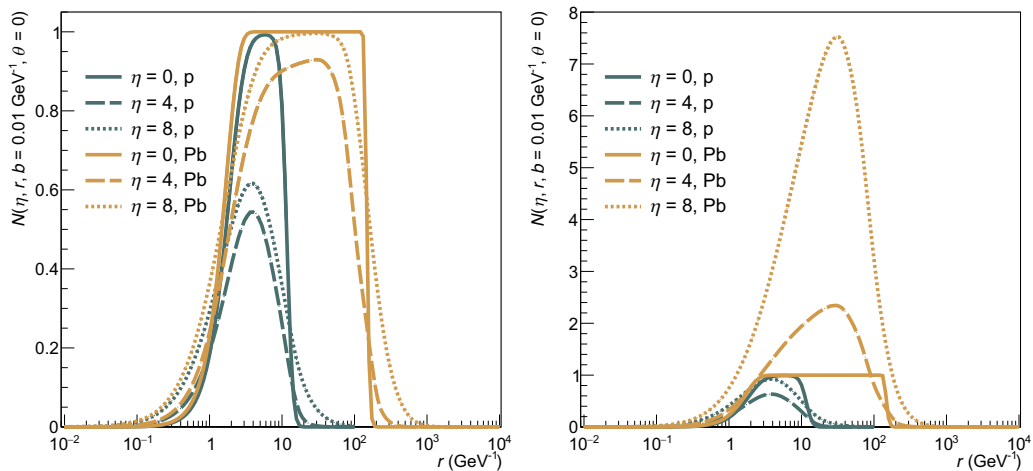


FIG. 3. Comparison of the dipole amplitudes at the initial condition (full) and evolved to rapidity 4 (dashed) and 8 (dotted) for a proton (teal) and lead (ochre) target. The shown amplitudes are central (i.e., at small impact parameter $b = 0.01 \text{ GeV}^{-1}$) and at $\theta = 0$ for the standard BK equation $\kappa = 1$ (left panel) and its linearized version $\kappa = 0$ (right panel).

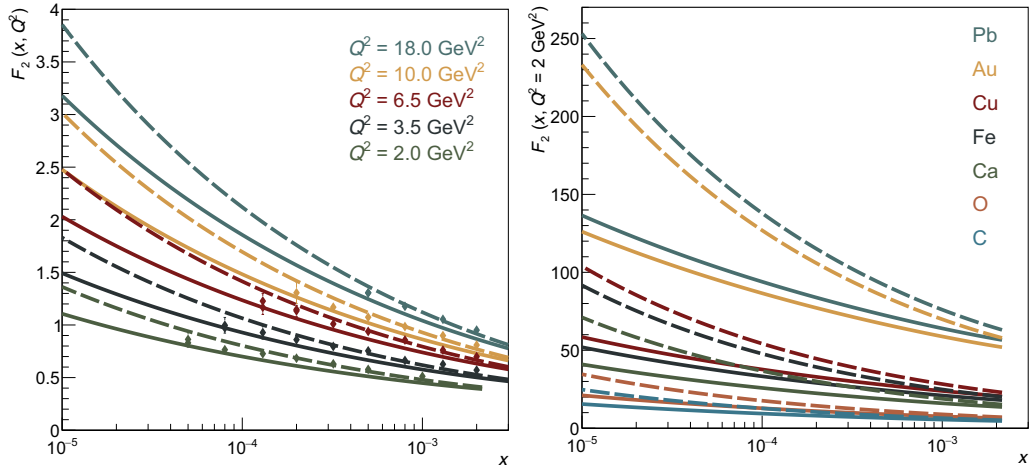


FIG. 4. The $F_2(x, Q^2)$ structure functions of proton (left panel) and nuclear (right panel) targets from the full-impact-parameter-dependent BK equation. The available data from HERA [4] are shown for the proton target. Results for the tetrahedral oxygen are not shown in these plots as they are indistinguishable from the Woods-Saxon profile. The full curves correspond to the standard BK equation ($\kappa = 1$), the dashed ones come from the linearized version ($\kappa = 0$).

wave functions as

$$\sigma_{L,T,f}^{\gamma^*p,A}(Q^2, x_f) = 2\pi \int dr r \int dz |\psi_{L,T,f}(r, z, Q^2)|^2 \times \int d\vec{b} 2N^{p,A}(r, b, \theta, \eta), \quad (11)$$

with

$$|\psi_{L,f}(r, z, Q^2)|^2 = \frac{3\alpha_{em}}{2\pi^2} e_f^2 4Q^2 z^2 (1-z)^2 K_0^2(r\epsilon),$$

$$|\psi_{T,f}(r, z, Q^2)|^2 = \frac{3\alpha_{em}}{2\pi^2} e_f^2 [(z^2 + (1-z)^2)\epsilon^2 K_1^2(r\epsilon) + m_f^2 K_0^2(r\epsilon)],$$

where $K_{0,1}(r\epsilon)$ are Bessel functions, z is the fraction of photon momentum carried by one of the quarks from the dipole and $\epsilon = \sqrt{z(1-z)Q^2 + m_f^2}$. Following Eq. (9), the structure functions of protons and a variety of nuclei were calculated

and are shown in Fig. 4 for both limiting cases of the evolution equation, i.e., with $\kappa = 0$ (dashed) and $\kappa = 1$ (solid). The proton results for different scales are validated against the available experimental data from HERA [4]. One can see that both the nonlinear and linearized BK model describe the available data on the proton target reasonably well, as measurements at much lower Bjorken x would be needed for their clear distinction. The nuclear results are shown in the right panel of Fig. 4, ranging from light to heavy nuclei. Here, the difference between the nonlinear and linearized model grows significantly with the mass number A . The reason is that in heavy nuclei, gluon clouds from different nucleons have a higher chance of overlapping and recombining, which in turn leads to stronger suppression from the nonlinear component of the BK equation.

In Fig. 5, predictions for the nuclear modification factor $R_{pA} = F_2^A/AF_2$ are shown for two values of virtuality $Q^2 = 2 \text{ GeV}^2$ (left panel) and $Q^2 = 18 \text{ GeV}^2$ (right panel) for

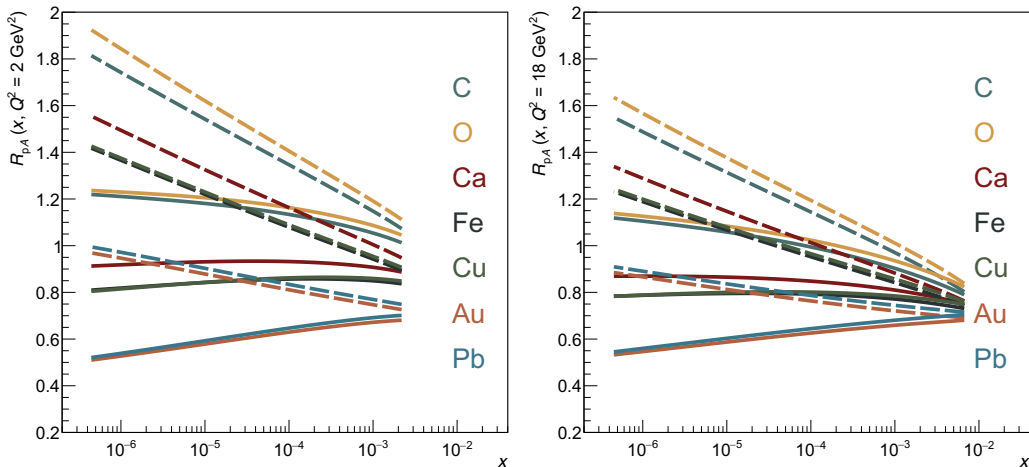


FIG. 5. The nuclear modification factor for virtualities $Q^2 = 2 \text{ GeV}^2$ (left panel) and $Q^2 = 18 \text{ GeV}^2$ (right panel). The full curves correspond to the standard BK equation ($\kappa = 1$), while the dashed ones come from the linearized version ($\kappa = 0$).

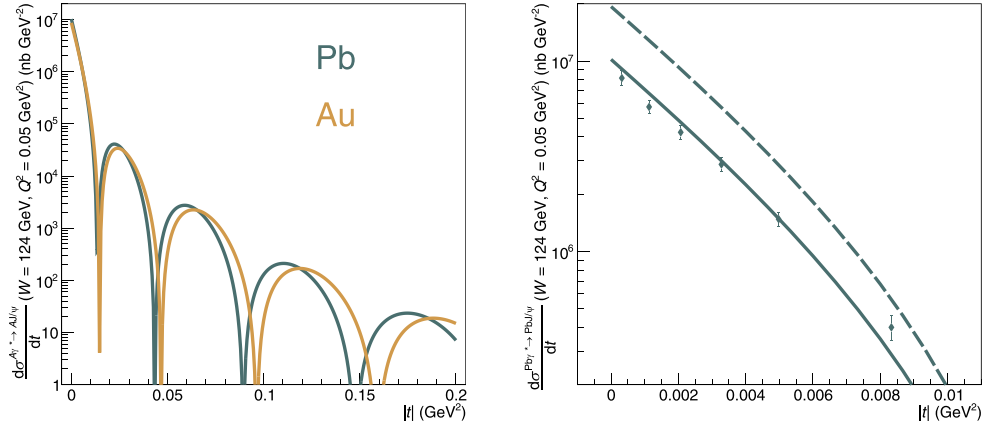


FIG. 6. Differential cross sections of diffractive coherent J/ψ photoproduction as a function of Mandelstam t for heavy nuclei. Predictions for available data from ALICE [71] are shown for the lead target in the right panel with the full curve. The dashed curve corresponds to the linearized version of the BK equation with $\kappa = 0$.

both the nonlinear and linearized model. The nuclear modification factors obtained from the linearized version of BK (dashed) show no suppression and are enhanced for all nuclei in the whole range of Bjorken x . For the nonlinear version, the strongest suppression of the nuclear modification factor can be seen in heavy nuclei, confirming the importance of collective effects in densely packed systems. Note that the amount of suppression does not scale precisely with A and, e.g., $R_{pO} > R_{pC}$. This is because the thickness function enters the initial condition normalized such that $\max(T(\tilde{b})) = 1$, emphasising its morphology rather than magnitude. The normalization of the nuclear profile is then compensated by the explicit dependence of the saturation scale on A , leading to the correct low-energy limit of the nuclear modification factor with $R_{pA} \rightarrow 1$.

B. Vector-meson production

For coherent vector-meson production, both the total and the Mandelstam- t differential cross section of J/ψ photoproduction have been studied. In general, the differential cross section of the diffractive exclusive vector-meson production as a function of the Mandelstam $|t|$, photon virtuality Q^2 , and the photon-ion center-of-mass energy W is given by the corresponding transverse and longitudinal amplitudes

$$\frac{d\sigma^{p,A}}{d|t|}(|t|, Q^2, W) = \frac{1}{16\pi} \sum_{T,L} (1 + \beta_{T,L}^2) R_{T,L}^2 |\mathcal{A}_{T,L}^{p,A}|^2, \quad (12)$$

which are in turn connected to the dipole scattering amplitude using the optical theorem as

$$\begin{aligned} \mathcal{A}^{p,A} &= i \int dr 2\pi r \int_0^1 \frac{dz}{4\pi} \int d\vec{b} (\Psi_V^\dagger \Psi)_{T,L} \\ &\times e^{-i[\vec{b} - (\frac{1}{2} - z)\vec{r}]\vec{\Delta}} 2N^{p,A}(r, b, \theta, \eta). \end{aligned} \quad (13)$$

The magnitude of $\vec{\Delta}$ is given by $\vec{\Delta}^2 = |t|$ and $(\Psi_V^\dagger \Psi)_{T,L}$ is the overlap between the wave functions of a virtual photon and the vector meson V :

$$(\Psi_V^\dagger \Psi)_T = e_f \frac{N_C}{\pi z(1-z)} (m_f^2 K_0(\epsilon r) \Phi_T(r, z)$$

$$-(z^2 + (1-z)^2) \epsilon K_1(\epsilon r) \partial_r \Phi_T(r, z)),$$

$$\begin{aligned} (\Psi_V^\dagger \Psi)_L &= e_f \frac{N_C}{\pi} 2Qz(1-z) K_0(\epsilon r) \\ &\times \left(m_V \Phi_L(r, z) + \delta \frac{m_f^2 - \nabla_r^2}{m_V z(1-z)} \Phi_L(r, z) \right), \end{aligned} \quad (14)$$

see, e.g., Refs. [65,66]. Here e_f is an effective charge that corresponds to the choice of the vector meson, $\nabla_r^2 = \frac{1}{r} \partial_r + \partial_r^2$ and δ is a switch that enables including the nonlocal part of the wave function introduced in Refs. [67,68]. The scalar part of the vector-meson wave function $\Phi_{T,L}(r, z)$ is obtained using the boosted Gaussian model [67,68] with the same parameter setup as in Ref. [28]. The corrections for the real part of the amplitude $\beta_{T,L}^2$ [69] and skewedness effects $R_{T,L}^2$ [70] are calculated in the same way as in Ref. [28]. The rapidity relates to Bjorken x as

$$\eta = \ln \left(\frac{x_0}{x} \right) = \ln \left(x_0 \frac{W^2 + Q^2}{m_V^2 + Q^2} \right), \quad (15)$$

where m_V is the mass of the emergent vector meson.

The available solutions of the BK equation have been used to obtain predictions for the coherent J/ψ photoproduction differential cross sections for $W = 124$ GeV as shown in the left panel of Fig. 6 for heavy nuclei. The expected series of dips is present, with the dip positions slightly changing based on a particular nucleus.

In the right panel of Fig. 6, one can see that the model with included saturation (nonlinear BK) correctly describes available data from ALICE [71], and can also be used for other nuclei. The linearized version of the BK equation shows a sizable discrepancy with the data. Compared to the DIS case, the logarithmic difference is, however, constant with $|t|$, suggesting that this dependence is not sensitive to the nonlinear/linearized scenario at fixed energy. Note that no additional free parameter has been added to the model when moving from the proton target to the nuclear one.

The predictions for the J/ψ differential photoproduction cross sections are shown in Fig. 7 for midmass nuclei (left panel) and light nuclei (right panel). The location of the

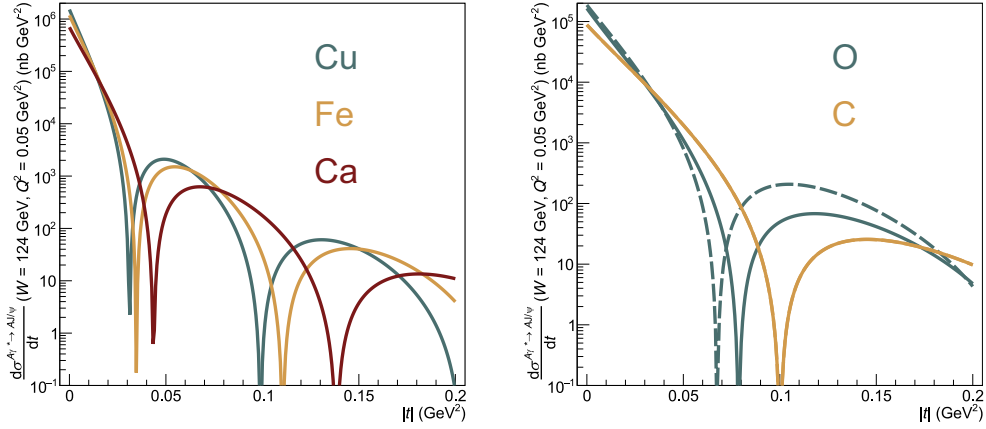


FIG. 7. Differential cross sections of the diffractive J/ψ photoproduction as a function of Mandelstam t for midmass (left) and light (right) nuclei. The tetrahedral oxygen target is shown with the dashed curve.

diffractive dips reflects the different sizes of target nuclei, where the larger the nucleus, the earlier in $|t|$ the dips appear. The positions also move more significantly for lighter nuclei than we have seen for heavy nuclei. In the case of oxygen, the difference between the three-point Fermi model (solid) and the tetrahedral helium model (dashed) is sizable at large $|t|$, while at the level of the first dip, the models are not different enough to be distinguished by the foreseen experiments.

The predictions for the J/ψ total photoproduction cross sections as a function of W , the energy of the γ - A system, are shown in the left panel of Fig. 8 for various nuclei. The total cross section is calculated as an integral of the differential one over Mandelstam $|t|$ over the interval $[0, 0.2]$. The Pb predictions are compared to measurements from ALICE [9], showing a good agreement of the model with the available data. The dashed curves correspond to the linearized version of BK with $\kappa = 0$. In contrast to the DIS case, the difference between nonlinear/linearized approaches is significant even

at low energies, and in order to describe available data, the nonlinear suppression is important.

The predictions for the cross sections of differential J/ψ photoproduction off Pb are shown as a function of W in the right panel of Fig. 8 for three specific bins in $|t|$. The clear difference between the nonlinear and linearized model can be seen at $|t| \rightarrow 0$, where the nonlinear model predicts a drop in the cross section, while the linearized one only results in growth regardless of $|t|$. At the same time, the total cross section for the nonlinear model in the left panel of Fig. 8 rises with energy, suggesting that the fall-off of the contribution at $|t| \rightarrow 0$ is compensated by the rise of the contribution at larger values of $|t|$. This behavior demonstrates the crucial role of multidifferential measurements in elucidating the origin of saturation effects.

V. SUMMARY

In this paper, we have extended the solutions of the Balitsky-Kovchegov equation with the dependence on the

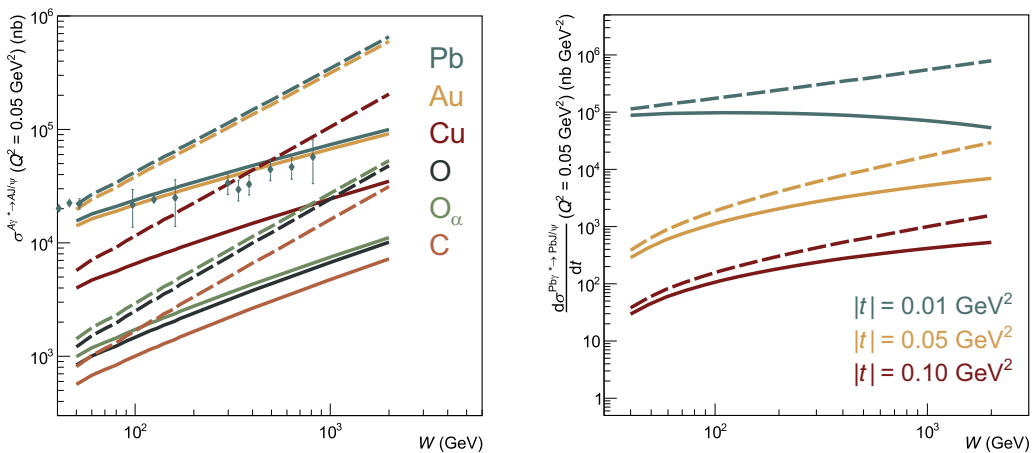


FIG. 8. Total (left) and differential (right) cross sections for J/ψ photoproduction as functions of the γ - A center-of-mass energy W . The differential cross sections are shown for three fixed bins in $|t|$. The total cross sections are integrated over $|t| \in [0, 0.2]$ and are presented for various nuclei, together with available measurements from ALICE [9] and CMS [72] (both for lead). Solid curves correspond to the standard BK equation ($\kappa = 1$), while dashed curves represent the linearized case ($\kappa = 0$).

full impact parameter, recently published in Ref. [28], from proton to nuclear targets. This was done solely by modifying the initial condition of the evolution, without presenting any additional free parameters. The dipole scattering amplitude includes nuclear effects based on the gluon component of the hadronic structure, and the behavior of these effects comes purely from the dynamics of the BK equation. Here, the main studied phenomenon is the gluon saturation manifesting itself at low Bjorken x . The resulting amplitude has been used to calculate observables from deep-inelastic scattering (structure function F_2) and from vector-meson production (cross sections of coherent J/Ψ photoproduction), all for a variety of nuclei. The observables have been confronted with available data, showing a good agreement. Predictions for different nuclei have been presented, being of potential interest for future experimental facilities such as EIC and LHeC, as well as for the current LHC program.

Moreover, a simplified version of the BK equation without the nonlinear term has been studied. This linearized version resembles the BFKL evolution equation, and even though the agreement with the available data for proton targets is reasonable in both cases, the difference is a lot more apparent for nuclear targets, allowing for the finding of an experimentally accessible observable that allows the discrimination between the models. The linearized version of the BK equation predicts a too steep rise of the cross-sections with energy (or equivalently with decreasing Bjorken- x). This rise does not allow for a reasonable prediction of the nuclear modification factor in the case of DIS and a correct data description in the case of the coherent J/Ψ photoproduction cross sections. Potentially,

the most prominent signature showing an altered behavior of the nonlinear and linearized BK model is the W distribution of J/Ψ at different Mandelstam t . For the linearized version, the cross section rises with energy at all values of $|t|$ while the nonlinear BK evolution predicts a drop in the cross section at small $|t|$.

Finally, the effect of the alternative density profile for oxygen, composed of a tetrahedral combination of four helium nuclei, has been studied and compared to the more standard three-point Fermi distribution. While the alternative model predicts a peak of the transverse density shifted more to the outer part of the nucleus, the effect on the studied observables is not pronounced. The only measurable difference between the two models is for the differential J/Ψ photoproduction cross sections at large $|t|$, where each of the models predicts a different position of the dips.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [32], embargo periods may apply.

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