

Reconstructing sources of rare particles by femtoscopic correlations

Liang Zhang (张良) ^{1,2} Song Zhang (张松) ^{1,3,*} Kai-Jia Sun (孙开佳),^{1,3,†} and Yu-Gang Ma (马余刚) ^{1,3,4,‡}

¹Key Laboratory of Nuclear Physics and Ion-Beam Application (MOE), Institute of Modern Physics, Fudan University, Shanghai 200433, China

²Shanghai Institute of Applied Physics (SINAP), Shanghai, China

³Shanghai Research Center for Theoretical Nuclear Physics, NSFC and Fudan University, Shanghai 200438, China

⁴School of Physics, East China Normal University, Shanghai 200241, China



(Received 7 March 2026; revised 9 April 2026; accepted 20 May 2026; published 22 June 2026)

Measurement of particle emission source is a fundamental objective of femtoscopy in high-energy nuclear collisions. Conventional analyses rely on Gaussian parametrizations of pair emission sources, which makes the extraction of single-particle emission sources challenging, particularly for rare particles. Here, we introduce a novel statistical reconstruction method that allows extracting information of the target source relative to a data-constrained reference source instead of the Gaussian assumption. The correlation function is expressed as an ensemble average over the single-particle-conditioned correlation kernel, defined as the particle-by-particle contribution to the correlation function conditioned by the target particles. For particles with rare yields, the particle-by-particle distribution of this kernel can be transformed into event-by-event extraction and becomes experimentally accessible, enabling a direct statistical reconstruction of the emission source of single particles, instead of inferring a pair source. We apply this method to reconstruct J/ψ source via p - J/ψ correlations, using HAL QCD-derived NJ/ψ potentials in $\sqrt{s} = 13.6$ TeV pp collisions simulated with EPOS4HQ. The reconstructed source reproduces the key characteristics and this new approach achieves a systematic uncertainty of approximately 13% based on EPOS4 simulation.

DOI: [10.1103/kpdh-t7hs](https://doi.org/10.1103/kpdh-t7hs)

I. INTRODUCTION

Hanbury Brown and Twiss (HBT) pioneered identical-particle intensity interferometry to measure the apparent angular diameter of remote stars [1]. In the 1960s, this concept was extended to high-energy physics [2], where HBT interferometry has since developed into a powerful tool for exploring the femtometer-scale spatiotemporal structure of particle-emission sources [3,4] in intermediate-energy [5,6] and high-energy [7–9] heavy-ion collisions, which provides an ideal venue to study properties of nuclear matter [10–15].

At such microscopic scales, correlation analyses must account for final-state interactions. This inclusion allows the method to be extended to nonidentical particle pairs [16–24]. In the context of relativistic heavy-ion collisions, this methodology is commonly referred to as “femtoscopia” [7–9,20,21]. It is widely employed to characterize the geometry of

emission sources [4,17,25–32] and dynamics of system evolution [33–38] and, more recently, to probe the nature of hadronic interactions at low relative momenta [16,39–44], including studies of exotic hadrons and hadronic resonant states [11,45–52].

The experimental measurements from the ALICE Collaboration have demonstrated that the emission sources of π , K , p , and Λ in 13-TeV pp collisions at the CERN Large Hadron Collider (LHC) originate from a common hadron-emission source [27,28]. These findings naturally raise the question of whether the emission sources of short-lived charm hadrons exhibit a similar spatial structure. Since charm hadrons are mainly produced at the early stage of the collision, their emission sources are expected to be more compact than those of longer-lived particles mentioned above. More fundamentally, conventional femtoscopy accesses source information only through two-particle observables, such that the single-particle emission source of charm hadrons is not directly measurable. As these hadrons are emitted during a nonequilibrium state, the commonly adopted Gaussian parametrizations should not be assumed *a priori*. Consequently, the extraction of charm-hadron emission sources within the standard femtoscopy framework remains an open problem.

With the continuous improvement of experimental precision, particle source extraction in femtoscopy has moved beyond traditional Gaussian parametrizations [4,7,25]. Since the particle emission source is inferred from measured two-particle correlation functions, source reconstruction

*Contact author: song_zhang@fudan.edu.cn

†Contact author: kjsun@fudan.edu.cn

‡Contact author: mayugang@fudan.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

constitutes an ill-posed inverse problem, motivating the development of various inverse approaches, including machine-learning techniques [31], optical deblurring algorithms [29], and regularization methods such as Tikhonov regularization [30]. While these methods significantly improve the reconstruction of femtoscopic sources, they are all formulated at the level of two-particle (pair) sources that are directly accessible through correlation measurements. In contrast, the direct extraction of a source of single particles, especially for rare particles, remains an open problem, as the single-particle emission information is implicitly folded into pair observables.

In the present study, we introduce a statistical reconstruction method, which reformulates femtoscopy as an inverse problem for emission sources of single particles. The method is based on expressing the measured two-particle correlation as an ensemble average over single-particle-conditioned correlation kernels, whose distribution encodes the underlying source of target particles. This reconstruction cannot be achieved within conventional source imaging approaches, which invert only the source of pairs. In practice, the reconstruction requires (i) a well-characterized emission source of an abundant reference hadron (e.g., pions or protons) which can be obtained by phenomenological models, and (ii) a constrained interaction potential, provided by effective field theories or lattice QCD.

As a proof of concept, we demonstrate the method through EPOS4HQ simulations [53–58], where the J/ψ emission source is reconstructed via p - J/ψ correlations with NJ/ψ potentials derived from HAL QCD calculations [59]. In this work, we focus on validating the stability and statistical uncertainty of the reconstruction method under controlled input conditions.

This paper is structured as follows. Section II A introduces our statistical reconstruction method. Section III shows the reconstruction of the J/ψ source in EPOS4HQ simulations to validate the method. Finally, a summary is provided in Sec. IV.

II. STATISTICAL RECONSTRUCTION METHOD

A. Methodology

The two-particle momentum correlation function constitutes the fundamental observable in femtoscopy. It is defined as [7]

$$C(\mathbf{k}^*) = \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 S_1(\mathbf{r}_1) S_2(\mathbf{r}_2) |\psi_{\mathbf{k}^*}(\mathbf{r}_1 - \mathbf{r}_2)|^2, \quad (1)$$

where $\mathbf{k}^* = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2)$ denotes the relative momentum in the pair rest frame, and $\mathbf{r}_i$ are the particle coordinates. $S_1(\mathbf{r}_1)$ and $S_2(\mathbf{r}_2)$ denote the normalized emission sources of single particles, while $|\psi_{\mathbf{k}^*}(\mathbf{r}_1 - \mathbf{r}_2)|^2$ incorporates contributions from quantum statistics (QS) (for identical particles) and final-state interactions. This formalism provides the basis for extracting source properties from experimental data, allowing a quantitative investigation of the space-time structure of particle-emitting sources.

In conventional approaches, particle emission sources are typically modeled as Gaussian distributions

[4,25,41–43]. Recently, the ALICE Collaboration proposed the resonance source model [27,28], which incorporates a universal Gaussian core for primordially produced particles, along with an exponential tail from resonance-decay contributions. The two-particle Gaussian source can be expressed as

$$S_{12}(r) = \frac{1}{(4\pi R^2)^{3/2}} \exp\left(-\frac{r^2}{4R^2}\right). \quad (2)$$

This Gaussian parametrization continues to be widely adopted in femtoscopic analyses [28,39,43,60,61]. Motivated by these developments, it is necessary to test the universality of the Gaussian source for early-stage produced particles. In particular, one can investigate the J/ψ emission source via p - J/ψ correlations without assuming a Gaussian form.

Building on the conventional approach, pair source distributions are often inferred by fitting correlation functions with predetermined interaction wave functions. To isolate the source distribution of an individual particle, $S_1(\mathbf{r}_1)$, in Eq. (1), the integral can be rewritten as

$$C(k^*) = \int d\mathbf{r}_1 S_1(\mathbf{r}_1) \tilde{C}_{k^*}(\mathbf{r}_1), \quad (3)$$

where $\tilde{C}_{k^*}(\mathbf{r}_1)$, termed the single-particle-conditioned correlation kernel, represents the correlation between particle 1 located at $\mathbf{r}_1$ and the ensemble of particle 2:

$$\tilde{C}_{k^*}(\mathbf{r}_1) = \int d^3\mathbf{r}_2 S_2(\mathbf{r}_2) |\psi_{k^*}(\mathbf{r}_1 - \mathbf{r}_2)|^2, \quad (4)$$

under the assumption of isotropic emission. For simplicity, the angular distribution of the source is taken to be uniform, and hereafter the angular-averaged source is denoted simply as $S_i(r_i)$.

Equation (3) can be interpreted statistically, which corresponds to the mean value $\langle \tilde{C}_{k^*}(r_1) \rangle$ over a set of samples drawn from $r_1 \sim S_1(r_1)$. The probability of $y = \tilde{C}_{k^*}(r_1)$ can then be expressed as

$$p_{\tilde{C}_{k^*}}(y) = \int S_1(r) \delta[y - \tilde{C}_{k^*}(r)] dr, \quad (5)$$

which describes the transformation of the source distribution $S_1(r)$ into the distribution of y through the functional map $\tilde{C}_{k^*}(r)$. This property is essential, as it enables a direct statistical reconstruction of the emission source of single particles, rather than an indirect inference of an effective source of pairs. It should be noted that the extracted S_1 is not a distribution in absolute spacetime, but a relative distribution obtained after averaging the spatial distribution of particle 2. However, it still contains spatial information about particle 1.

In reversal of the thought of Eq. (5), a reconstruction strategy for source is presented as

$$\begin{aligned} S_1^{(k^*)}(r) &= \int p(y) \delta[r - \tilde{C}_{k^*}^{-1}(y)] dy \\ &= \sum_j [p(y_j) \tilde{C}'_{k^*}(r)]_{y_j = \tilde{C}_{k^*}(r)}. \end{aligned} \quad (6)$$

Due to the potentially nonmonotonicity of the $\tilde{C}_{k^*}(r)$ function, the reconstructed source distribution $S_1^{(k^*)}(r)$ at individual

k^* may deviate from the original source distribution $S_1(r)$. In the following, we show that a coherent combination of reconstructions over multiple relative-momentum bins provides a robust estimate of the underlying source.

This approach fundamentally relies on two essential requirements, the distribution of $y = \tilde{C}_{k^*}(r_1)$ and the analytical calculation of $\tilde{C}_{k^*}(r)$. The latter one is based on the established inputs, a known source of single particles as reference source and a well-constrained interaction potential, as introduced in Sec. I.

In experiments, the correlation function $C(k^*)$ is determined through the normalized ratio [7,27,28]

$$C(k^*) = \mathcal{N} \frac{N^{\text{same}}(k^*)}{N^{\text{mixed}}(k^*)}, \quad (7)$$

where $\mathcal{N}$ is the normalization factor, $N^{\text{mixed}}(k^*)$ represents the distribution of uncorrelated pairs obtained from event mixing, and $N^{\text{same}}(k^*) = \sum_i^{N_{\text{events}}} \sum_j^{N_T^i} n_{ij}^{\text{same}}(k^*)$ is the distribution of correlated pairs from the same event. Here N_{events} denotes the total number of events, N_T^i is the number of target particles to be reconstructed in i th event and $n_{ij}^{\text{same}}(k^*)$ represents the pair number with the momentum k^* including j th target particle in the i th event. The corresponding conditioned kernel $\tilde{C}_{k^*}$ for a target particle in the i th event can then be defined as

$$\tilde{C}_{k^*,ij} = \mathcal{N} N_T^{\text{tot}} \frac{n_{ij}^{\text{same}}(k^*)}{N^{\text{mixed}}(k^*)}. \quad (8)$$

This definition allows the particle-by-particle distribution of $y = \tilde{C}_{k^*}$ to be directly accessed experimentally, providing the statistical input required for reconstructing the emission source of single particles. However, experimentally determining the conditioned kernel is challenging because it is difficult to isolate individual particles for pairing in a given event. Equation (8) also incorporates particle production fluctuations into the kernel fluctuations, which increase the systematic uncertainty.

B. Validation

To validate the feasibility of our statistical reconstruction method, we perform a controlled numerical test. We use pre-defined Gaussian emission sources with radii $R_1 = 0.5$ fm and $R_2 = 1.4$ fm for S_1 and S_2 , correspondingly, and adopt the wave function

$$\psi_{\mathbf{k}^*}(\mathbf{r}) = \sqrt{2} \cos(\mathbf{k}^* \cdot \mathbf{r}). \quad (9)$$

The corresponding analytical correlation function is shown in Fig. 1. We then quantitatively evaluate the reconstruction accuracy by comparing the extracted source profiles against ground-truth inputs.

The validation is initiated by sampling radial coordinates r_1 for particle 1 from the source distribution S_1 . For each sampled coordinate, the $\tilde{C}_{k^*}(r)$, defined in Eq. (4), is evaluated over the range $r \in [0, 15]$ fm. The resulting distribution of $y = \tilde{C}_{k^*}(r)$ at $k^* = 100$ MeV/c is shown as a histogram in Fig. 2(a).

The analytical form of $\tilde{C}_{k^*}(r)$, shown as the blue curve in Fig. 2(b), is discretized on a two-dimensional grid with 200 bins in y and 60 bins in r . Within each bin, the local slope

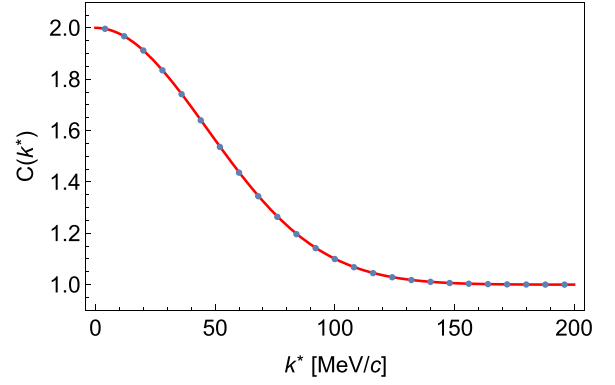


FIG. 1. The analytically computed correlation function derived from Gaussian emission sources with radii $R_1 = 0.5$ fm and $R_2 = 1.4$ fm for S_1 and S_2 , correspondingly, employs the wave function $\psi_{\mathbf{k}^*}(\mathbf{r}) = \sqrt{2} \cos(\mathbf{k}^* \cdot \mathbf{r})$. Blue markers denote candidate momenta selected for reconstruction validation.

$d\tilde{C}_{k^*}(r)/dr$ is calculated. These slopes are used to construct the reconstruction kernel K_{k^*} , where the elements $(K_{k^*})_{ij}$ are set to zero if the curve of $\tilde{C}_{k^*}(r)$ does not pass through the bin (r_i, y_j) . To address regions where $\tilde{C}_{k^*}(r)$ varies rapidly and passes through multiple y -bins within a single r -bin, we implement a multiplicity-averaging procedure:

$$K_{ij} \rightarrow (K_{k^*})_{ij} / n_i^{\text{cross } y\text{-bins}}, \quad (10)$$

where $n_i^{\text{cross } y\text{-bins}}$ counts the number of y -bins traversed by the curve within the i th r -bin. This prevents overcounting of probability when the reconstruction mapping is multivalued.

The reconstructed source distribution at a given k^* is then obtained via

$$S_1^{(k^*)}(r_i) = \frac{K_{k^*} \cdot \mathbf{y}_j^{\text{counts}}}{\sum_j y_j^{\text{counts}}} \Delta r_i, \quad (11)$$

where $\mathbf{y}^{\text{counts}}$ denotes the histogram-count vector of $\tilde{C}_{k^*}(r)$. As demonstrated in Fig. 2(c), the reconstructed distribution agrees excellently with the ground-truth source.

However, not every reconstruction yields a success story. The nonmonotonic behavior of $\tilde{C}_{k^*}(r)$ (Fig. 3) violates the bijectivity of reconstruction mapping. As a result, the reconstructed source distributions exhibit pronounced artifacts. For example, at $k^* = 156$ MeV/c (Fig. 4), spurious peaks arise in reconstruction because $\tilde{C}_{k^*}(r)$ possesses nonzero slopes within $r < 3$ fm [Fig. 3(b)]. To suppress these artifacts, we average reconstructions performed at multiple momenta. This multimomentum averaging amplifies the contribution from the true source and yields a significantly improved profile, as shown in Fig. 4.

III. RECONSTRUCT J/ψ SOURCE FROM EPOS4HQ

A. Model setup

EPOS4 is a comprehensive simulation framework for high-energy particle collisions [53–56]. It provides a fully self-consistent description of both proton-proton (pp) and nucleus-nucleus (AA) collisions by integrating four core

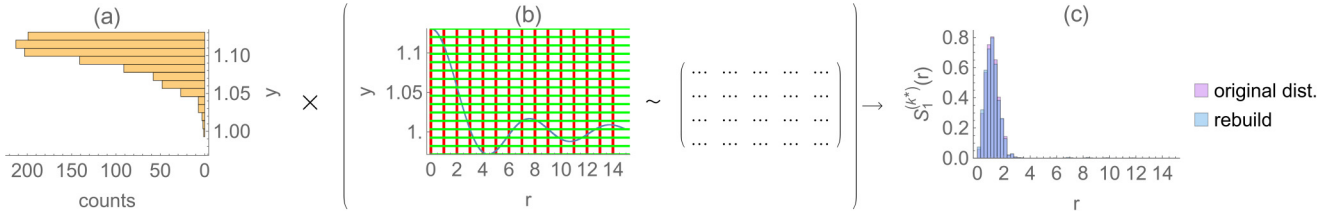


FIG. 2. Flowchart on reconstructing $S_1^{(k^*)}(r)$. Panel (a) shows the histogram of $y = \tilde{C}_{k^*}(r)$, where $k^* = 100$ MeV/c. Panel (b) is the $\tilde{C}_{k^*}(r)$ function and presents the calculation of the reconstruction kernel K_{k^*} . Panel (c) is the reconstruction of $S_1^{(k^*)}(r)$ and comparison with the ground-truth source distribution.

concepts: parallel scattering (multiple partons or nucleons acting parallelly in high-energy collisions), energy conservation, factorization (separating hard processes into parton distribution functions and differential scattering cross sections), and the dynamic saturation effect (screening of low-transverse-momentum particle production by saturation scale). The framework first models the initial particle interactions using an S -matrix approach to handle parallel scatterings. Subsequently, the collision evolution is treated through a sequence of secondary processes, beginning with core corona separation and followed by hydrodynamic evolution to describe collective behavior, microcanonical hadronization to convert the fluid into hadrons, and finally the UrQMD hadron cascade acting as an afterburner to simulate resonant decays and hadronic rescatterings.

In a pure EPOS4 framework, heavy flavor partons are created in primary interactions. Their subsequent interactions with the simulated quark-gluon plasma are incorporated via the dedicated HQ module. This module also enables, under specific conditions, the coalescence of plasma partons with heavy-flavor partons to form heavy-flavor hadrons. The extended version, EPOS4HQ [57,58], is endowed with the capability to describe a wide range of heavy-flavor observables from experiments and remains accessible for further studies.

To compute the correlation function in Eq. (1), we simulate 10^5 pp collisions at $\sqrt{s} = 13.6$ TeV within EPOS4HQ, which provides the spatiotemporal emission sources for protons and J/ψ . Eventwise particle samples are subjected to standard fiducial selections; protons are required to satisfy a pseudorapidity window of $-0.9 < \eta < 0.9$ and a

transverse-momentum range of $0.25 < p_T < 3$ GeV/c, while J/ψ candidates are selected with $-0.9 < \eta < 0.9$ and $p_T > 1$ GeV/c. The p - J/ψ scattering wave function is derived from the N - J/ψ potential obtained via HAL QCD calculations [59], a nonperturbative approach extracting spatial potentials from lattice QCD [62–64]. Final-state correlations are computed using the correlation analysis tool using the Schrödinger equation [65], employing a test particle analysis with event mixing over all simulated collisions.

B. Priori proton source

Since femtoscopic sources are local in momentum space, even high-momentum protons can still contribute to the p - p correlation when their momentum directions are strongly aligned due to collective flow. While the flow effect on J/ψ is weaker, especially in pp collisions, the proton- and J/ψ -emission processes are regarded as uncorrelated, and particle momentum magnitude is the primary criterion for particle selection in the p - J/ψ correlation analysis. However, the proton source is different in p - p and p - J/ψ correlations, and we cannot extract the proton source via p - p correlation for p - J/ψ pair analysis. In this simulation, the relative momentum is restricted to $0 < k^* < 400$ MeV/c. For a pair with momenta p_1 for proton and p_2 for J/ψ , the constraint on k^* imposes a strong kinematic constraint on the opening angle θ_{12} , since $k^* = \frac{1}{2}\sqrt{p_1^2 + p_2^2 - 2p_1p_2\cos\theta_{12}}$. When particles have large momenta, the acceptance of p_1 and p_2 ,

$$\text{Acc}(p_1, p_2) = \int_0^{400} \sin(\theta_{12}) \frac{\partial \theta_{12}}{\partial k} dk/2, \quad (12)$$

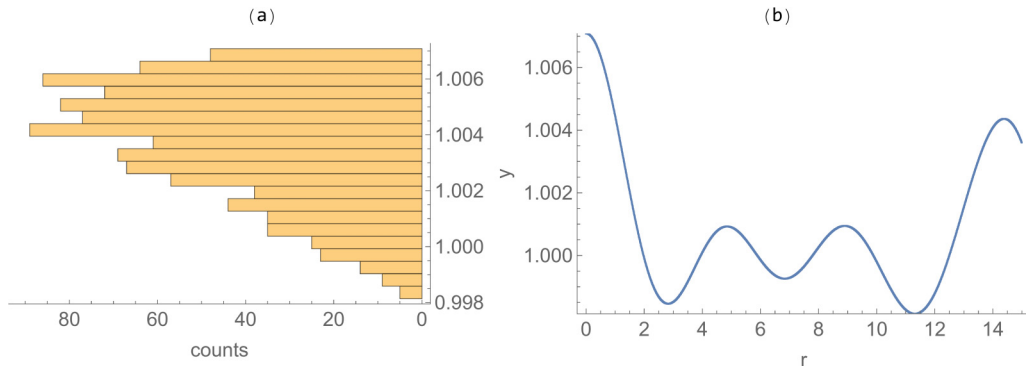


FIG. 3. Panel (a) presents the histogram of $y = \tilde{C}_{k^*}(r)$, where $k^* = 156$ MeV/c. Panel (b) illustrates the nonmonotonic behavior of $\tilde{C}_{k^*}(r)$. This violation of bijectivity induces ambiguity in the reconstruction mapping, resulting in a poorly reconstructed source distribution.

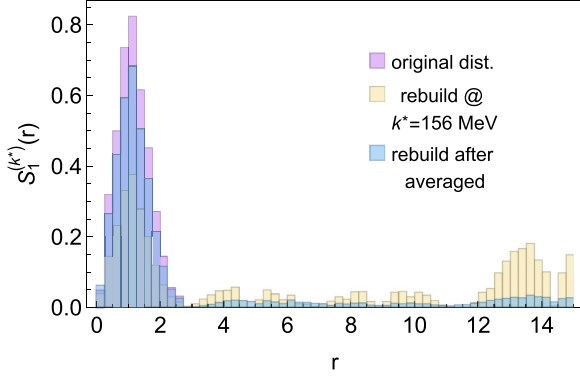


FIG. 4. A bad reconstruction example (yellow) at $k^* = 156$ MeV/c due to the nonmonotonic behavior of $\tilde{C}_{k^*}(r)$ shown in Fig. 3. Averaging on multimomentum reconstructions (blue) can reduce the effects of these fake peaks.

can be small due to a narrow θ_{12} interval. Therefore, pairs with large p_1 and p_2 contribute weakly to the selected k^* region. This provides the practical basis for selecting protons within a controlled momentum window to construct the priori reference source in the present reconstruction framework.

In this simulation, protons with momentum <400 MeV/c are selected as the known reference source. To obtain its spatial distribution, we adopt the Gaussian core + Cauchy tail [27,28],

$$S_p(r) = f \frac{R_C}{\pi^2(R_C^2 + r^2)^2} + (1-f) \frac{1}{\sqrt{4\pi R^2}} \exp\left(-\frac{r^2}{4R^2}\right), \quad (13)$$

to describe the proton source generated by EPOS4, with $R_C \simeq 0.68$ fm, $R \simeq 0.80$ fm, and the mixing fraction $f \simeq 0.51$ (Fig. 5). Although this fitting introduces model dependence into our reconstruction framework, proton sources have been extensively studied in different phenomenological models and are constrained by available experimental data. The model-to-model differences are relatively small and can be incorporated as a systematic uncertainty in the present reconstruction.

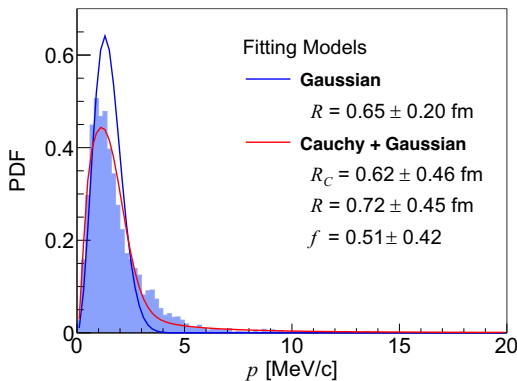


FIG. 5. The protons with momentum <400 MeV/c are selected as the known reference source. The spatial distribution is generated by EPOS4HQ and fitted with Gaussian distribution as well as Eq. (13), where the Gaussian + Cauchy distribution performs better.

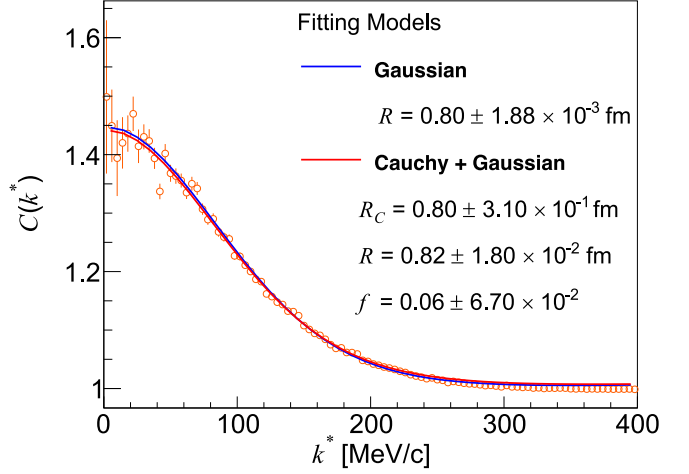


FIG. 6. The p - J/ψ correlation function (orange markers) in $\sqrt{s} = 13.6$ TeV pp collisions, calculated from EPOS4HQ-simulated particle sources and HAL QCD N - J/ψ potentials. Results from two candidate source model fits are also shown with a same fitting range in the p - p correlation. The extracted mixing fraction f is found to be very small, demonstrating that the pair source is well described by a single Gaussian model.

C. Extracting J/ψ source

The p - J/ψ correlation function and the corresponding source fits are presented in Fig. 6. Crucially, the extracted p - J/ψ source exhibits spatial extent significantly smaller than that of the source of proton pairs obtained in Sec. III B, implying that J/ψ mesons are emitted from a more compact region compared to protons. This finding provides direct motivation for applying our statistical reconstruction framework to reconstruct the J/ψ emission source from EPOS4HQ simulations. A conventional fitting strategy is used as a benchmark to compare our methodology (Fig. 6). Since the resulting mixing fraction f of the Gaussian + Cauchy distribution is found to be small in describing p - J/ψ pair source, the pair source can be well described by a single Gaussian component. Adopting the assumption of an uncorrelated emission process, we extract a single- J/ψ Gaussian radius of $R_{J/\psi} \simeq 0.37$ fm, with a single-proton Gaussian source fitted in Fig. 5.

As discussed in Sec. II A, a key distinction between experiment and simulation lies in the nature of the $\tilde{C}_{k^*,i}$ observable. Experimentally measured values [Eq. (8)] are discrete, whereas those computed from simulations [Eq. (4)] are continuous. To emulate the discrete character of experimental data, we discretize the simulated continuous $\tilde{C}_{k^*}$ value via

$$n_i^{\text{same}} = \left\lfloor \frac{\tilde{C}_{k^*} N^{\text{mixed}}(k^*)}{\mathcal{N} N_{\text{events}}^{\text{rare}}(k^*)} \right\rfloor, \quad (14)$$

where $\lfloor x \rfloor$ denotes the floor function, rounding x down to the nearest integer. Within the test-particle method used here, the normalization factor $\mathcal{N} = 1$. The number of uncorrelated pairs $N^{\text{mixed}}(k^*)$ corresponds to the total count of p - J/ψ pairs at the momentum k^* in the EPOS4HQ-generated final-state phase space. $N_{\text{events}}^{\text{rare}}$ denotes the number of events containing one J/ψ . Because the J/ψ yield is rare, the number of J/ψ particles [Eq. (8)] is approximately equal to the number of

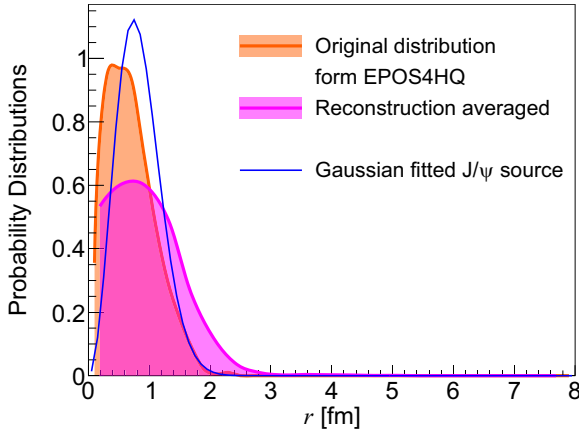


FIG. 7. Comparison between the original J/ψ source and the reconstruction result averaged over $k^* = 82\text{--}238$ MeV/c. The Gaussian parametrizations of the J/ψ source extracted from Fig. 6 are also displayed as a comparison.

events containing J/ψ in Eq. (14). Therefore, fluctuations associated with eventwise particle multiplicity are strongly suppressed. Substituting n_i^{same} back into Eq. (8) yields the discretized $\tilde{C}_{k^*,i}$, which then serves as the input for reconstructing the J/ψ emission source using the statistical reconstruction method.

Following the methodology in Sec. II A, we analytically compute $\tilde{C}_{k^*}(r)$ over $r \in [0, 15]$ fm. The resulting two-dimensional domain of the function on the r - y plane is then discretized into 40 bins along the r axis and coarsely binned along the y axis. The coarse y -binning is chosen to accommodate the spread of the discrete $\tilde{C}_{k^*,i}$ values and is determined by the range of the analytically computed $\tilde{C}_{k^*}(r)$.

Figure 7 shows the reconstructed J/ψ emission source averaged over the momentum range $k^* = 82, \dots, 238$ MeV/c. While the reconstruction exhibits a broader spatial distribution and a more pronounced tail compared to the original EPOS4HQ-simulated J/ψ source, it preserves key characteristics such as the most probable radius and the overall compactness. Notably, the most probable radius agrees better with the original simulation than that obtained from a simple Gaussian fit to the J/ψ source.

In line with our initial conjecture in Sec. I, the reconstructed J/ψ source is more compact than the proton source. Using the fitted proton source together with the reconstructed J/ψ source, we compute the corresponding correlation function (magenta curve in Fig. 8). This reconstructed correlation is weaker than the one derived directly from the EPOS4HQ simulation. The discrepancy originates from the broader spread and enhanced tail structure present in the reconstructed source (Fig. 7).

The systematic uncertainty of the statistical reconstruction method is estimated via

$$\delta_{\text{sys.}} \sim \frac{C_{\text{recon.}}(k^*) - 1}{C(k^*) - 1}, \quad (15)$$

where $C(k^*)$ denotes the p - J/ψ correlation function computed directly from the EPOS4HQ simulation, and $C_{\text{recon.}}(k^*)$ is the

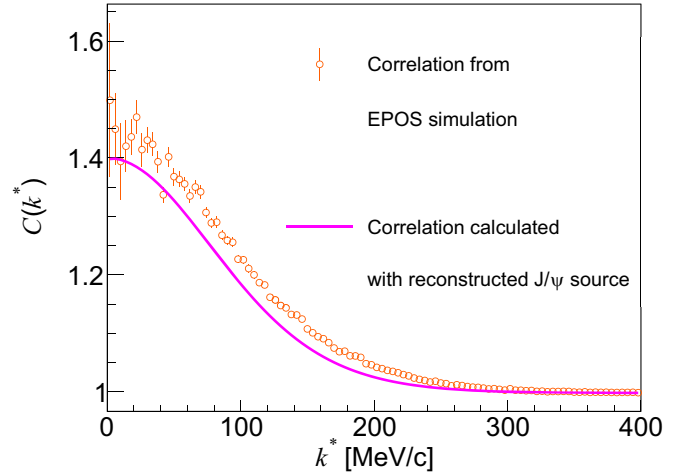


FIG. 8. Comparison of the p - J/ψ correlation function obtained from direct EPOS4HQ simulation with the result reconstructed using the statistical reconstruction method. The momentum interval used in the multimomentum averaging ranges from 82 to 168 MeV/c for k^* . This comparison is used to estimate the systematic uncertainty of the method, yielding an uncertainty of approximately 13% in the present analysis.

result obtained from our reconstruction. This yields a systematic uncertainty of approximately 13% for this J/ψ source reconstruction. One contributor to this systematic error is the discretization of the $\tilde{C}_{k^*}$ values, as well as the simple Gaussian parametrization used for the proton source.

IV. SUMMARY

The present study presents a statistical reconstruction method for reconstructing the emission source of rare particles without the Gaussian parametrization. The method is based on the particle-particle distribution of the single-particle-conditioned correlation kernel, which is more easily experimentally accessible when at most one rare particle is produced per collision event and the fluctuations associated with particle multiplicity are suppressed. This key property enables a direct statistical reconstruction of the source of single particles, rather than an indirect inference from effective pair sources.

By leveraging a well-characterized source of reference particles and a constrained interaction potential, the conditioned kernel function $\tilde{C}_{k^*}(r)$ is computed analytically. Its radial derivative defines the momentum-dependent reconstruction kernel K_{k^*} , through which the spatial extent of the target particle source is obtained from the measured kernel distribution. To suppress reconstruction artifacts arising from nonmonotonic $\tilde{C}_{k^*}(r)$ behavior as r increases, a multimomentum averaging procedure is employed as a stabilization strategy.

We conduct two validation studies, Gaussian source reconstruction and EPOS4HQ-simulated J/ψ source reconstruction, to evaluate the method and quantify its systematic uncertainties. In the Gaussian case, Monte Carlo sampling is used to generate the spatial distribution and compute the

$\tilde{C}_{k^*}$ distribution, with multimomentum averaging yielding an accurate reconstruction. For the more complex EPOS4HQ validation, we use p - J/ψ correlations to reconstruct the J/ψ source. The proton source, extracted by fitting spatial distribution of protons with momentum <400 MeV simulated in EPOS4HQ, serves as the known reference source input. Meanwhile, the HAL QCD-derived N - J/ψ potentials are used to calculate the p - J/ψ correlation and the conditioned kernel $\tilde{C}_{k^*}$. By applying multimomentum averaging, the key characteristics of the reconstructed J/ψ source are preserved, including the most probable radius and intrinsic compactness. A comparison between the directly simulated and reconstructed p - J/ψ correlation functions reveals a systematic uncertainty of approximately 13% for J/ψ source reconstruction.

In summary, the proposed statistical reconstruction method enables reconstruction of rare-particle emission sources without assumptions on the shape. This approach could provide quantitative constraints on source characteristics of rare particles and offer deeper insights into the intermediate processes of both pp and heavy-ion collisions. The methodology

can further serve as a useful tool for future femtoscopy analysis.

ACKNOWLEDGMENTS

This work was supported in part by the National Natural Science Foundation of China under Contracts No. 12275054, No. 12147101, No. 12061141008, No. 12347106, No. 12422509, No. 12375121, and No. 12547102, the National Key R&D Program of China under Grants No. 2024YFA1610802 and No. 2018YFE0104600, the Guangdong Major Project of Basic and Applied Basic Research under Grant No. 2020B0301030008, the Shanghai Pilot Program for Basic Research–Fudan University under Grant No. 21TQ1400100(22TQ006), and the STCSM under Grant No. 23590780100.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

-
- [1] R. H. Brown and R. Q. Twiss, A test of a new type of stellar interferometer on Sirius, *Nature (London)* **178**, 1046 (1956).
 - [2] G. Goldhaber, S. Goldhaber, W. Lee, and A. Pais, Influence of Bose-Einstein statistics on the antiproton-proton annihilation process, *Phys. Rev.* **120**, 300 (1960).
 - [3] G. Kopylov, Like particle correlations as a tool to study the multiple production mechanism, *Phys. Lett. B* **50**, 472 (1974).
 - [4] W. A. Zajc, J. A. Bistirlich, R. R. Bossingham, H. R. Bowman, C. W. Clawson, K. M. Crowe, K. A. Frankel, J. G. Ingersoll, J. M. Kurck, O. Hashimoto, M. Koike, W. J. McDonald, J. P. Miller, P. Truöl, C. J. Martoff, D. L. Murphy, J. O. Rasmussen, J. P. Sullivan, and E. Yoo, Two-pion correlations in heavy ion collisions, *Phys. Rev. C* **29**, 2173 (1984).
 - [5] D. H. Boal, C. K. Gelbke, and B. K. Jennings, Intensity interferometry in subatomic physics, *Rev. Mod. Phys.* **62**, 553 (1990).
 - [6] W. Bauer, C. K. Gelbke, and S. Pratt, Hadronic interferometry in heavy-ion collisions, *Annu. Rev. Nucl. Part. Sci.* **42**, 77 (1992).
 - [7] M. A. Lisa, S. Pratt, R. Soltz, and U. Wiedemann, Femtoscopy in relativistic heavy ion collisions: Two decades of progress, *Annu. Rev. Nucl. Part. Sci.* **55**, 357 (2005).
 - [8] U. Heinz and B. V. Jacak, Two-particle correlations in relativistic heavy-ion collisions, *Annu. Rev. Nucl. Part. Sci.* **49**, 529 (1999).
 - [9] U. A. Wiedemann and U. Heinz, Particle interferometry for relativistic heavy-ion collisions, *Phys. Rep.* **319**, 145 (1999).
 - [10] J. H. Chen *et al.*, Properties of the QCD matter: Review of selected results from the relativistic heavy ion collider beam energy scan (RHIC BES) program, *Nucl. Sci. Tech.* **35**, 214 (2024).
 - [11] Q. Y. Shou, Y. G. Ma, S. Zhang, J.-H. Zhu, Y.-X. Mao, H. Pei, Z.-B. Yin, X.-M. Zhang, D.-C. Zhou, X.-Y. Peng, X.-Z. Bai, Z.-B. Tang, Y.-F. Zhang, and X.-M. Li, Properties of QCD matter: A review of selected results from ALICE experiment, *Nucl. Sci. Tech.* **35**, 219 (2024).
 - [12] P. Lu, R. Kavak, A. Dubla, S. Masciocchi, and I. Selyuzhenkov, Quantification of the low- p_T pion excess in heavy-ion collisions at the LHC and top RHIC energy, *Nucl. Sci. Tech.* **36**, 142 (2025).
 - [13] R. Q. Wang, X. L. Hou, Y. H. Li, J. Song, and F. L. Shao, Production characteristics of light nuclei, hypertritons, and Ω -hypernuclei in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, *Nucl. Sci. Tech.* **36**, 185 (2025).
 - [14] N. Yu, Z. M. Zhang, H. G. Xu, and M. X. Song, Effect of light nuclei on chemical freeze-out parameters at RHIC energies, *Nucl. Sci. Tech.* **36**, 65 (2025).
 - [15] W. J. Dong, X. Z. Yu, S. Y. Ping, X.-T. Wu, G. Wang, H.-Z. Huang, and Z.-W. Lin, Study of baryon number transport dynamics and strangeness conservation effects using Ω -hadron correlations, *Nucl. Sci. Tech.* **35**, 120 (2024).
 - [16] R. Lednicky and V. L. Lyuboshits, Final state interaction effect on pairing correlations between particles with small relative momenta, *Yad. Fiz. (Sov. J. Nucl. Phys.)* **35**, 1316 (1981).
 - [17] S. Pratt, Pion interferometry of quark-gluon plasma, *Phys. Rev. D* **33**, 1314 (1986).
 - [18] M. Bowler, Coulomb corrections to Bose-Einstein corrections have greatly exaggerated, *Phys. Lett. B* **270**, 69 (1991).
 - [19] R. Lednicky and V. L. Lyuboshitz, The influence of final state interaction on two-particle correlations in multiple production of particles and resonances, *Acta Phys. Hung. New Ser. Heavy Ion Phys.* **3**, 93 (1996).
 - [20] R. Lednicky, Femtoscopy with unlike particles, [arXiv:nucl-th/0112011](https://arxiv.org/abs/nucl-th/0112011).
 - [21] R. Lednicky, Progress in correlation femtoscopy, in *Multiparticle Dynamics* (World Scientific, Alushta, Crimea, Ukraine, 2003), pp. 21–26.
 - [22] Y. B. Wei, Y. G. Ma, W. Q. Shen, G. L. Ma, K. Wang, X. Z. Cai, C. Zhong, W. Guo, and J. G. Chen, Exploring binding energy and separation energy dependences of HBT strength, *Phys. Lett. B* **586**, 225 (2004).

- [23] Y. G. Ma, G. H. Liu, X. Z. Cai, D. Q. Fang, W. Guo, W. Q. Shen, W. D. Tian, and H. W. Wang, Hard-photon flow and photon-photon correlation in intermediate-energy heavy-ion collisions, *Phys. Rev. C* **85**, 024618 (2012).
- [24] Y. G. Ma *et al.*, Different mechanism of two-proton emission from proton-rich nuclei ^{23}Al and ^{22}Mg , *Phys. Lett. B* **743**, 306 (2015).
- [25] S. E. Koonin, Proton pictures of high-energy nuclear collisions, *Phys. Lett. B* **70**, 43 (1977).
- [26] J. He, S. Zhang, Y.-G. Ma, J. Chen, and C. Zhong, Clustering structure effect on Hanbury-Brown-Twiss correlation in $^{12}\text{C}+^{197}\text{Au}$ collisions at 200 GeV, *Eur. Phys. J. A* **56**, 52 (2020).
- [27] ALICE Collaboration, Search for a common baryon source in high-multiplicity pp collisions at the LHC, *Phys. Lett. B* **811**, 135849 (2020).
- [28] ALICE Collaboration, Common femtoscopic hadron-emission source in pp collisions at the LHC, *Eur. Phys. J. C* **85**, 198 (2025).
- [29] J. Xu, Z. Qin, R. Zou, D. Si, S. Xiao, B. Tian, Y. Wang, and Z. Xiao, Imaging freeze-out sources and extracting strong interaction parameters in relativistic heavy-ion collisions, *Chin. Phys. Lett.* **42**, 031401 (2025).
- [30] A.-S. Xiong, Q.-W. Yuan, M.-Z. Liu, F.-S. Yu, Z.-W. Liu, and L.-S. Geng, Solving the inverse source problem in femtoscopy with a toy model, *Chin. Phys. C* (2026), doi:10.1088/1674-1137/ae6310.
- [31] L. Wang and J. Zhao, Learning hadron emitting sources with deep neural networks, *Commun. Phys.* **9**, 90 (2026).
- [32] B.-S. Xi, J.-H. Chen, L. Ma, Y.-G. Ma, and T.-T. Wang, Study of the momentum correlation of nucleons in $^{96}\text{Ru}+^{96}\text{Ru}$ and $^{96}\text{Zr}+^{96}\text{Zr}$ collisions at $\sqrt{s_{NN}} = 7.7$ and 200 GeV from a multiphase transport model, *Nucl. Sci. Tech.* **36**, 228 (2025).
- [33] Y. Wang *et al.*, The emission order of hydrogen isotopes via correlation functions in 30 MeV/u Ar+Au reactions, *Phys. Lett. B* **825**, 136856 (2022).
- [34] T.-T. Wang, Simulations of momentum correlation functions of light (anti)nuclei in relativistic heavy-ion collisions at $\sqrt{NN} = 39$ GeV, *Phys. Rev. C* **107**, 014911 (2023).
- [35] F.-H. Qiao, X.-G. Deng, and Y.-G. Ma, Momentum correlation of light nuclei in Au + Au collisions at $\sqrt{s_{NN}} = 2.0 \sim 7.7$ GeV, *Phys. Lett. B* **850**, 138535 (2024).
- [36] T.-T. Wang, Y.-G. Ma, and S. Zhang, Calculation of momentum correlation functions between π , K , and p for several heavy-ion collision systems at $\sqrt{NN} = 39$ GeV, *Phys. Rev. C* **109**, 024912 (2024).
- [37] D.-F. Wang, M.-Y. Chen, Y.-G. Ma, Q.-Y. Shou, S. Zhang, and L. Zheng, Investigating the pion emission source in pp collisions using the AMPT model with subnucleon structure, *Nucl. Sci. Tech.* **36**, 154 (2025).
- [38] Y. G. Ma, Y. B. Wei, W. Q. Shen, X. Z. Cai, J. G. Chen, J. H. Chen, D. Q. Fang, W. Guo, C. W. Ma, G. L. Ma, Q. M. Su, W. D. Tian, K. Wang, T. Z. Yan, C. Zhong, and J. X. Zuo, Surveying the nucleon-nucleon momentum correlation function in the framework of quantum molecular dynamics model, *Phys. Rev. C* **73**, 014604 (2006).
- [39] STAR Collaboration, Measurement of interaction between antiprotons, *Nature (London)* **527**, 345 (2015).
- [40] J. Haidenbauer, Coupled-channel effects in hadron-hadron correlation functions, *Nucl. Phys. A* **981**, 1 (2019).
- [41] L. Fabbietti, V. M. Sarti, and O. V. Doce, Study of the strong interaction among hadrons with correlations at the LHC, *Annu. Rev. Nucl. Part. Sci.* **71**, 377 (2021).
- [42] S. Acharya *et al.* (ALICE Collaboration), First measurement of the Λ - Ξ interaction in proton-proton collisions at the LHC, *Phys. Lett. B* **844**, 137223 (2023).
- [43] S. Acharya *et al.* (ALICE Collaboration), Experimental evidence for an attractive $p-\phi$ interaction, *Phys. Rev. Lett.* **127**, 172301 (2021).
- [44] D. Si *et al.*, Extracting neutron-neutron interaction strength and spatiotemporal dynamics of neutron emission from the two-particle correlation function, *Phys. Rev. Lett.* **134**, 222301 (2025).
- [45] L. Adamczyk *et al.* (STAR Collaboration), $\Lambda\Lambda$ correlation function in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV, *Phys. Rev. Lett.* **114**, 022301 (2015).
- [46] Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, and W. Weise, K^-p correlation function from high-energy nuclear collisions and chiral SU(3) dynamics, *Phys. Rev. Lett.* **124**, 132501 (2020).
- [47] S. Acharya *et al.* (ALICE Collaboration), Accessing the strong interaction between Λ baryons and charged kaons with the femtoscopy technique at the LHC, *Phys. Lett. B* **845**, 138145 (2023).
- [48] ALICE Collaboration, Investigating the $p-\pi^\pm$ and $p-p-\pi^\pm$ dynamics with femtoscopy in pp collisions at $\sqrt{s} = 13$ TeV, *Eur. Phys. J. A* **61**, 194 (2025).
- [49] ALICE Collaboration, Observation of deuteron and antideuteron formation from resonance-decay nucleons, *Nature (London)* **648**, 306 (2025).
- [50] K. J. Sun, Femtoscopy reveals how (anti-)deuteron is formed at the LHC, *Nucl. Sci. Tech.* **37**, 64 (2026).
- [51] M.-Z. Liu, Y.-W. Pan, Z.-W. Liu, T.-W. Wu, J.-X. Lu, and L.-S. Geng, Three ways to decipher the nature of exotic hadrons: Multiplets, three-body hadronic molecules, and correlation functions, *Phys. Rep.* **1108**, 1 (2025).
- [52] J. H. Chen, F. K. Guo, J. J. Wu, B. S. Zou, Y. G. Ma, C. P. Shen, and Q. Y. Shou, Production of exotic hadrons in pp and nuclear collisions, *Nucl. Sci. Tech.* **36**, 55 (2025).
- [53] K. Werner and B. Guiot, Perturbative QCD concerning light and heavy flavor in the EPOS4 framework, *Phys. Rev. C* **108**, 034904 (2023).
- [54] K. Werner, Revealing a deep connection between factorization and saturation: New insight into modeling high-energy proton-proton and nucleus-nucleus scattering in the EPOS4 framework, *Phys. Rev. C* **108**, 064903 (2023).
- [55] K. Werner, Core-corona procedure and microcanonical hadronization to understand strangeness enhancement in proton-proton and heavy ion collisions in the EPOS4 framework, *Phys. Rev. C* **109**, 014910 (2024).
- [56] K. Werner, Parallel scattering, saturation, and generalized Abramovskii-Gribov-Kancheli (AGK) theorem in the EPOS4 framework, with applications for heavy-ion collisions at $\sqrt{s_{NN}}$ of 5.02 TeV and 200 GeV, *Phys. Rev. C* **109**, 034918 (2024).
- [57] J. Zhao, J. Aichelin, P. B. Gossiaux, and K. Werner, Heavy flavor as a probe of hot QCD matter produced in proton-proton collisions, *Phys. Rev. D* **109**, 054011 (2024).

- [58] J. Zhao, J. Aichelin, P. B. Gossiaux, V. Ozvenchuk, and K. Werner, Heavy-flavor hadron production in relativistic heavy ion collisions at energies available at BNL RHIC and at the CERN LHC in the EPOS4HQ framework, *Phys. Rev. C* **110**, 024909 (2024).
- [59] Y. Lyu, T. Doi, T. Hatsuda, and T. Sugiura, Nucleon-charmonium interactions from lattice QCD, *Phys. Lett. B* **860**, 139178 (2025).
- [60] ALICE Collaboration, Unveiling the strong interaction among hadrons at the LHC, *Nature (London)* **588**, 232 (2020).
- [61] S. Acharya *et al.* (ALICE Collaboration), Scattering studies with low-energy kaon-proton femtoscopy in proton-proton collisions at the LHC, *Phys. Rev. Lett.* **124**, 092301 (2020).
- [62] N. Ishii, S. Aoki, and T. Hatsuda, Nuclear force from lattice QCD, *Phys. Rev. Lett.* **99**, 022001 (2007).
- [63] HAL QCD Collaboration, N. Ishii, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, T. Inoue, K. Murano, H. Nemura, and K. Sasaki, Hadron-hadron interactions from imaginary-time Nambu-Bethe-Salpeter wave function on the lattice, *Phys. Lett. B* **712**, 437 (2012).
- [64] S. Aoki and T. Doi, Lattice QCD and baryon-baryon interactions: HAL QCD method, *Front. Phys.* **8**, 307 (2020).
- [65] D. L. Mihaylov, V. Mantovani Sarti, O. W. Arnold, L. Fabbietti, B. Hohlweger, and A. M. Mathis, A femtosopic correlation analysis tool using the Schrödinger equation (CATS), *Eur. Phys. J. C* **78**, 394 (2018).