

Field-theoretical description of the deuteron breakup in the clothed particle representation

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We present a field-theoretical description of the deuteron electrodisintegration reaction $d(e, e'p)n$ induced by unpolarized and polarized electrons. The approach combines the Lehmann-Symanzik-Zimmermann in(out) formalism with the clothed particle representation in the instant form of relativistic dynamics, providing a fully relativistic and gauge-independent framework based on the Fock-Weyl criterion. Within the method of unitary clothing transformations, one and the same transformation that generates the relativistic nucleon-nucleon interaction (the Kharkiv potential) also induces a fresh family of electromagnetic current operators. As a result, one-body and two-body (meson-exchange) currents emerge on a common footing. We compute differential cross sections and polarization observables with the inclusion of final-state interaction effects and meson-exchange current contributions and compare the results with Saclay and Jefferson Lab data as well as with earlier theoretical predictions. The role of relativistic ingredients (one- and two-body currents, Fermi-motion effects, etc.) and the interplay between them are analyzed in several kinematic regimes of the experiments at Saclay and Jefferson Lab.

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I. INTRODUCTION

Explorations of the deuteron electrodisintegration $d(e, e'p)n$ have the eventful history, starting from the first coincidence experiments that have been performed by Stanford [1], Orsay [2], Kharkiv [3] and Saclay [4,5]. We have seen that their torch has been picked up for the 80s and 90s with the MIT-Bates [6], Mainz [7], and NIKHEF [8–10] experiments (see, e.g., our survey in Ref. [11] and references therein) and continued nowadays at Jefferson

Lab [12–16]. Of course, the corresponding results are a source of inspiration for many theoretical studies. We do not intend to give any review of a lot of them. Nevertheless, we would like to remind of the nonrelativistic treatment of the deuteron electrodisintegration by Arenhövel and his colleagues [17] with inclusion of the final-state interaction (FSI) between the outgoing proton and neutron, and effects of meson-exchange currents (MECs). Note also the remarkable work [18]. Furthermore, relativistic descriptions of this reaction have been extensively developed, including the systematic inclusion of relativistic corrections by Arenhövel *et al.* [19,20], the diagrammatic models by Laget [21], the covariant description by Jeschonnek and Van Orden [22], and the studies by Sargsian and Boeglin [23–25].

In addition, let us recall our calculations of the proton angular distributions and polarization observables in the $d(e, e'p)n$ reaction with polarized beam and target [11,26–28]. They have much in common with preceding works, including the same ingredients, i.e., finding the n - p wave functions (WFs) in continuum for a given nucleon-nucleon (NN) potential, FSI and MEC effects.

Recent experiments in deuteron electrodisintegration at high momentum transfers, particularly at Jefferson Lab [14–16,25], have pushed deuteron studies into the relativistic regime, reaching missing neutron momenta k_n up to ~ 1.0 GeV/c. Under such conditions, a nonrelativistic description of the np interaction could be insufficient, requiring

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a more sophisticated approach that incorporates relativistic invariance. Furthermore, it may be necessary to consider sub-nucleonic degrees of freedom.

In this research, a new field-theoretical approach extends the in(out) formalism by Lehmann-Symanzik-Zimmermann (LSZ) [29], being combined with the clothed particle representation (CPR) [30–32]. Under this framework, the NN interaction, MEC, and boost operators are constructed in a consistent manner, ensuring relativistic invariance by satisfying the Poincaré algebra and requiring the current to transform as a true four-vector. We are working in the instant form of relativistic dynamics [33], where only the total Hamiltonian H and the boost operator $\mathbf{B}$ carry interactions. All operators act within a Fock space of clothed particles that incorporate their physical properties, such as observed mass and charge. The relativistic NN potential used in this work (Kharkiv potential) was originally obtained in Ref. [34]. Applications of this potential to describe deuteron properties are given in Ref. [35], while its extension to the three-nucleon (3N) system can be found in Refs. [36–38] and the recent modern exposition [39]. Following our recent work [40], we employ a fully relativistic framework and propose a way toward the gauge-independent (GI) treatment. The latter can be achieved by imposing the Fock-Weyl criterion (details can be found in Ref. [41]).

The electromagnetic current operators used in this work are derived directly from the conserved Noether current of the underlying meson–nucleon field model (we include π , ρ , δ , η , ω , and σ mesons) via its reformulation in terms of clothed particle operators. Within the method of unitary clothing transformation (UCT), the same transformation that generates the relativistic NN interaction operator also induces a new family of current operators. As a result, the one-body, two-body and other many-body current operators emerge on a common footing. In contrast to other approaches, where MECs are often constructed separately from a potential and supplemented with meson form factors, the present two-nucleon currents (MECs) are consistent with the NN potential and the Poincaré group generators. In the course of our studies, we encounter both isoscalar and isovector MECs. The former were first derived in our studies devoted to elastic electron-deuteron scattering [40,42], where only the isoscalar components contribute to the reaction amplitude. In the present work, however, we extend this construction to include the isovector two-nucleon currents as well. The latter becomes essential when describing the deuteron breakup, where both isospin mechanisms contribute to the observables.

This paper is organized as follows. In Sec. II, we introduce the basic notations and definitions necessary for describing the deuteron breakup process. Section III is devoted to the underlying formalism of clothed particles, where we find the essential links between the CPR and in(out) formalism in the quantum field theory (QFT) [43]. This section also details the construction of the deuteron state and the moving np state within the CPR, alongside the derivation of the MEC operators. It concludes with a discussion of the path toward a GI description of the reaction. Finally, in Sec. IV, we present numerical results and give prospects of our approach.

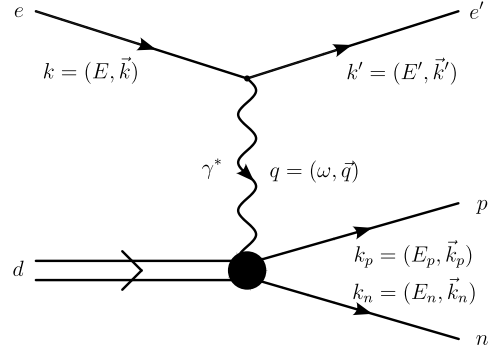


FIG. 1. One-photon-exchange diagram.

II. BASIC EXPRESSIONS AND DEFINITIONS

Within the one-photon-exchange approximation (OPEA) (see Fig. 1) the $d(e, e')n$ cross section for unpolarized particles in the laboratory frame can be written as

$$\sigma_0 \equiv \frac{d^3\sigma}{dE' d\Omega'_e d\Omega_p} = \frac{4\alpha^2 E'^2 R}{(Q^2)^2} l^{\mu\nu} W_{\mu\nu}, \quad (1)$$

$$W_{\mu\nu} = \frac{1}{3} \text{Tr}[\mathcal{F}_\mu(\mathbf{p}_0, \mathbf{q}) \mathcal{F}_\nu^\dagger(\mathbf{p}_0, \mathbf{q})], \quad (2)$$

where neglecting the electron mass $l_{\mu\nu} = (k'_\mu k_\nu + k'_\nu k_\mu - g_{\mu\nu} k' \cdot k)/2E'E$ is the leptonic tensor, $g_{\mu\nu}$ is the metric tensor, $k = (E, \mathbf{k})$ and $k' = (E', \mathbf{k}')$ are the four momenta of the incident and outgoing electrons, $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ is the momentum transfer, $\mathbf{p}_0$ is the relative momentum of the np pair, α the fine-structure constant, and $Q^2 = \mathbf{q}^2 - \omega^2$ with ω is the energy transfer.

The kinematic factor R equals to

$$R = E_p |\mathbf{k}_p| \left[1 - \frac{E_p}{E_n} \left(\frac{|\mathbf{q}|}{|\mathbf{k}_p|} \cos(\widehat{\mathbf{q}, \mathbf{k}_p}) - 1 \right) \right]^{-1}, \quad (3)$$

where E_p (E_n) is the energy of the knocked-out proton (recoil neutron) and $\mathbf{k}_p$ is the proton momentum. Here $\mathcal{F}_\mu(\mathbf{p}_0, \mathbf{q})$ denotes the matrix element

$$\mathcal{F}^\mu(\mathbf{p}_0, \mathbf{q}) = \langle \Psi_{\mathbf{q}, \mathbf{p}_0 S M_S}^{(-)} | J^\mu(0) | \Psi_{1M_d} \rangle, \quad (4)$$

where $J^\mu(0) = (J_0(0), \mathbf{J}(0))$ is the electromagnetic (e.m.) Noether current density operator taken at the space-time point $x = (t, \mathbf{x}) = 0$ sandwiched between the initial deuteron state $|\Psi_{1M_d}\rangle$ at rest and final np -pair state $|\Psi_{\mathbf{q}, \mathbf{p}_0 S M_S}^{(-)}\rangle$ with spin S , its projection M_S , total momentum $\mathbf{q}$, and relative momentum $\mathbf{p}_0$. Unlike

$$F_\mu(\mathbf{p}_0, \mathbf{q}) = \langle \Psi_{\mathbf{p}_0 S M_S}^{(-)} | J_\mu(\mathbf{q}) | \Psi_{M_d} \rangle, \quad (5)$$

from Eq. (3) of Ref. [27] we handle $\mathcal{F}^\mu(\mathbf{p}_0, \mathbf{q})$ defined by Eq. (4), where, in general, the momenta $\mathbf{p}_0$ and $\mathbf{q}$ cannot be separated. Let us consider the coplanar case and introduce the

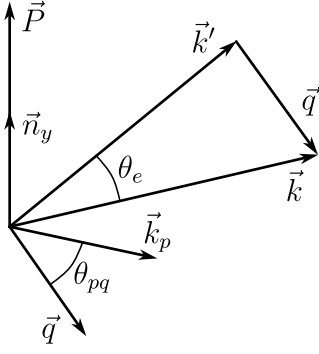


FIG. 2. Proton polarization in the reaction $d(e, e'\bar{p})n$ for the coplanar geometry.

orthonormal basis

$$\mathbf{n}_Z = \frac{\mathbf{q}}{q}, \quad \mathbf{n}_Y = \frac{\mathbf{k} \times \mathbf{k}'}{|\mathbf{k} \times \mathbf{k}'|}, \quad \mathbf{n}_X = \mathbf{n}_Y \times \mathbf{n}_Z, \quad (6)$$

assuming that $\mathbf{n}_X$ and $\mathbf{n}_Y$ form the reaction plane (Fig. 2). According to Ref. [44], one has

$$\begin{aligned} \frac{4\alpha^2 E'^2}{(Q^2)^2} l^{\mu\nu} W_{\mu\nu} = \sigma_M \left\{ \xi^2 W_C + \left(\frac{1}{2}\xi + \eta \right) W_T \right. \\ \left. + (\xi + \eta) W_S + \xi \sqrt{\xi + \eta} W_I \right\}, \\ \xi = \frac{Q^2}{q^2}, \quad \eta = \tan^2 \frac{\theta_e}{2}, \end{aligned} \quad (7)$$

where σ_M is the Mott cross section and θ_e is the electron emission angle.

The structure functions (SFs) W_i ($i = C, T, S, I$) are bilinear combinations of the amplitude (4) and determined in terms of the hadronic tensor. The Coulomb function $W_C = W_{00}$ is determined by the longitudinal current component, while $W_T = 2W_{YY}$ and $W_S = W_{XX} - W_{YY}$ depend on the transverse current component only. The function $W_I = -W_{X0} - W_{0X}$ is the interference term.

It is important to emphasize that, when deriving Eq. (7), the continuity equation

$$q_\mu \mathcal{F}^\mu(\mathbf{p}_0, \mathbf{q}) = 0 \quad (8)$$

was used to eliminate the matrix elements with longitudinal current component.

In the coplanar case the proton polarization

$$\mathbf{P} = \frac{\text{Tr}\{\boldsymbol{\sigma}(1)\mathcal{F}\mathcal{F}^\dagger\}}{\text{Sp}\mathcal{F}\mathcal{F}^\dagger} \quad (9)$$

in the reaction $d(e, e'\bar{p})n$ with unpolarized electrons and deuteron is orthogonal to the reaction plane

$$\mathbf{P} = P\mathbf{n}_Y. \quad (10)$$

By using condition (8) one can show that

$$\begin{aligned} \sigma_0 P = \frac{4\alpha^2 E'^2 R}{(Q^2)^2} l^{\mu\nu} \Sigma_{\mu\nu}^Y \\ = \sigma_M \left\{ \xi^2 \Sigma_C + \left(\frac{1}{2}\xi + \eta \right) \Sigma_T + (\xi + \eta) \Sigma_S + \xi \sqrt{\xi + \eta} \Sigma_I \right\}, \end{aligned} \quad (11)$$

where the polarization SFs Σ_i ($i = C, T, S, I$) are related to the components of the polarization hadronic tensor

$$\Sigma_{\mu\nu} = \frac{1}{3} \text{Tr}\{\boldsymbol{\sigma}(1)\mathcal{F}_\mu(\mathbf{p}_0, \mathbf{q})\mathcal{F}_\nu^\dagger(\mathbf{p}_0, \mathbf{q})\} \quad (12)$$

are related by the same relations as those connecting W_i with $W_{\mu\nu}$. Here $\boldsymbol{\sigma}(1)$ is the Pauli vector for the nucleon labeled by 1. In a theory that preserves both P - and T -invariance, one can show that the proton polarization $\mathbf{P}$ vanishes when the FSI contributions are neglected [27,45].

In the reaction $d(\bar{e}, e'\bar{p})n$ with polarized electrons and unpolarized deuterons the proton polarization $\mathbf{P}_\lambda$ has a component $\mathbf{P}'$ related to the electron helicity λ ,

$$\mathbf{P}_\lambda = \mathbf{P} + \lambda \mathbf{P}', \quad (13)$$

where $\mathbf{P}$ is the polarization vector (10) in case of unpolarized electrons. For the coplanar kinematics (6) the polarization-transfer vector lies in the plane and its components are given by

$$\sigma_0 P'_{X,Z} = \sigma_M R \sqrt{\eta} \left\{ \xi \Sigma_I'^{X,Z} + \sqrt{\eta + \xi} \Sigma_T'^{X,Z} \right\}, \quad P'_Y = 0, \quad (14)$$

where the interference Σ_I' and the transverse Σ_T' SFs are given by

$$\Sigma_I' = i[\Sigma_{0Y} - \Sigma_{Y0}], \quad \Sigma_T' = i[\Sigma_{YX} - \Sigma_{XY}]. \quad (15)$$

Thus, the transferred polarization $\mathbf{P}'$ depends on reaction amplitudes that do not appear in the so-called induced polarization $\mathbf{P}$. In contrast to the latter, the transferred polarization does not vanish when FSI is neglected. This circumstance can be used to extract additional information on the electromagnetic properties of a bound nucleon (e.g., the neutron) under conditions where FSI and MEC effects can be neglected.

III. UNDERLYING FORMALISM

At the beginning, we would like to recall the physical content of the CPR (its systematic exposition can be found in Refs. [31,32,46]).

A. Concept of clothed particles in quantum field theory

In the conventional formulation of QFT the Hamiltonian $H = H(\{\alpha\}) = H_F(\{\alpha\}) + H_I(\{\alpha\})$ is expressed through the set $\{\alpha\}$ of the creation and destruction operators for the bare particles. In such a bare-particle representation (BPR) even the simplest H eigenstates, viz., the physical vacuum and the states of the observed particles, are infinite superpositions of the bare states. Formally, this occurs because the interaction $H_I = V + (\text{renormalization terms})$ contains the so-called bad terms [e.g., of the types $b^\dagger b a^\dagger$ and $b^\dagger d^\dagger a^\dagger$, where $a^\dagger$ ($b^\dagger$, $d^\dagger$) creates a bare meson (nucleon, antinucleon)] that prevent the no-particle (bare vacuum) state and the one-particle states from being the H eigenvectors.

The key idea of the clothing procedure, put forward by Greenberg and Schweber [30] and developed in Refs. [31,32,34,46], is to reformulate the theory in terms of new creation (destruction) operators $\{\alpha_c\}$ that correspond

to the clothed particles, i.e., the particles with their observable properties (physical masses and charges). The transition $\{\alpha\} \Rightarrow \{\alpha_c\}$ is realized via the unitary clothing transformations $W = \exp R$ ($R^\dagger = -R$) that leave the Hamiltonian intact,

$$H = H(\{\alpha\}) = W(\{\alpha_c\})H(\{\alpha_c\})W^\dagger(\{\alpha_c\}) \equiv K(\{\alpha_c\}) \\ = K_F(\{\alpha_c\}) + K_I(\{\alpha_c\}), \quad (16)$$

so one and the same operator H acquires in the CPR a new decomposition into a free part $K_F = H_F(\{\alpha_c\})$ and an interaction K_I between the clothed particles. The generator R is chosen to remove the bad terms from the interaction, which for the first clothing transformation is provided by the condition $[R, H_F(\{\alpha_c\})] = -V_{\text{bad}}$. Here $V \equiv V(\{\alpha_c\})$ denotes a primary interaction expressed through the clothed operators. In the case of interest it is sum of the Yukawa-type (scalar, pseudoscalar and vector) meson–nucleon couplings entered the Lagrangian density $\mathcal{L}_{MN}$. In general, one divides $V = V_{\text{bad}} + V_{\text{good}}$, where the term “good,” as an antithesis of “bad,” is applied to the operators that destroy both the no-particle state (physical vacuum) and the one-particle states (e.g., of the type $b_c^\dagger b_c^\dagger b_c b_c$). In this respect, all the Yukawa-type vertices, being linear in the meson operators, are bad, and the generator R determined by the condition above is of the first order in the coupling constants.

By construction, the physical vacuum (the lowest H eigenstate) coincides with the no-clothed-particle state $|\Omega\rangle$,

$$a_c(\mathbf{k})|\Omega\rangle = b_c(\mathbf{p}\mu)|\Omega\rangle = d_c(\mathbf{p}\mu)|\Omega\rangle = 0, \quad \forall \mathbf{k}, \mathbf{p}, \mu, \quad (17)$$

while the one-clothed-particle states, e.g., $b_c^\dagger(\mathbf{p}\mu)|\Omega\rangle$, are the $H = K(\{\alpha_c\})$ eigenvectors with the physical masses,

$$K(\{\alpha_c\})b_c^\dagger(\mathbf{p}\mu)|\Omega\rangle = E b_c^\dagger(\mathbf{p}\mu)|\Omega\rangle \\ \Rightarrow K_I(\{\alpha_c\})b_c^\dagger(\mathbf{p}\mu)|\Omega\rangle = 0, \quad (18)$$

with energy $E = \sqrt{\mathbf{p}^2 + m^2}$ and observable nucleon mass m . The same is valid for the clothed meson and antinucleon states.

The new interaction K_I is evaluated with the help of the Campbell-Hausdorff expansion

$$H = e^{R(\{\alpha_c\})}[H_F(\{\alpha_c\}) + V(\{\alpha_c\}) \\ + M_{\text{ren}}(\{\alpha_c\}) + V_{\text{ren}}(\{\alpha_c\})]e^{-R(\{\alpha_c\})} \\ = H_F(\{\alpha_c\}) + V(\{\alpha_c\}) + [R, H_F] + [R, V] \\ + \frac{1}{2}[R, [R, H_F]] + \frac{1}{2}[R, [R, V]] \\ + \frac{1}{3!}[R, [R, [R, H_F]]] + \dots \quad (19)$$

Taking into account that $V = V_{\text{bad}}$ for the Yukawa-type couplings and omitting the mass (M_{ren}) and vertex (V_{ren}) renormalization counterterm contributions (the latter are canceled step by step in the course of the clothing procedure [47,48]), one finds

$$K_I(\{\alpha_c\}) = \frac{1}{2}[R, V] + \frac{1}{3}[R, [R, V]] \\ + \frac{1}{8}[R, [R, [R, V]]] + \dots, \quad (20)$$

where every subsequent commutator raises the order in the coupling constants. The second-order term of this series embodies all the two-body interactions between the clothed

particles, viz.,

$$K_I^{(2)}(\{\alpha_c\}) = K(NN \rightarrow NN) + K(\bar{N}\bar{N} \rightarrow \bar{N}\bar{N}) \\ + K(N\bar{N} \rightarrow N\bar{N}) + K(\mathcal{M}N \rightarrow \mathcal{M}N) \\ + K(\mathcal{M}\bar{N} \rightarrow \mathcal{M}\bar{N}) \\ + K(\mathcal{M}\mathcal{M}' \rightarrow N\bar{N}) + K(N\bar{N} \rightarrow \mathcal{M}\mathcal{M}'), \quad (21)$$

where $\mathcal{M}$ ($\mathcal{M}'$) denote the clothed mesons involved. The operator $K(NN \rightarrow NN)$, being of the structure $b_c^\dagger b_c^\dagger b_c b_c$, determines the relativistic NN quasipotential in the one-boson-exchange approximation, i.e., the interaction is generated by the exchange of a single scalar (σ), pseudoscalar (π, η), or vector (ρ, ω) meson between the two nucleons. To regularize the corresponding quasipotential, cutoff functions are introduced at the meson-nucleon vertices. The resulting relativistic quasipotential, referred to as the Kharkiv potential [34], generates both the deuteron bound state and the np scattering states considered in the present paper.

In the higher orders one obtains, along with the corrections to these quasipotentials, the interactions of new types, e.g., the operators $K(NN \rightarrow \mathcal{M}NN)$ of the g^3 order that are responsible for the meson production in nucleon-nucleon collisions [32]. It should be noted, that the first clothing transformation generally generates new bad terms of higher orders through the commutators appearing in the expansion (19). These higher-order bad terms must, in turn, be eliminated by subsequent clothing transformations. Thus, the clothing procedure is iterative and produces a sequence of unitary transformations designed to remove bad terms order by order. Let us stress that, unlike the Okubo-type transformations [49] aimed at the block-diagonalization of H , the clothing procedure does not diagonalize the Hamiltonian. As shown in Ref. [46], $K(\{\alpha_c\})$ acquires a sparse structure, with much more zero matrix elements.

In the instant form of relativistic dynamics, the interactions enter not only the Hamiltonian but also the boost generators. It is important that one and the same transformation W removes the bad terms from all the generators of the Poincaré group simultaneously [31,50], giving rise, in particular, to the decomposition of the boost operator $\mathbf{B} = \mathbf{B}_F + \mathbf{B}_I$ used in Sec. III D. Moreover, being applied to the Noether current density, the same W generates the family of the one-body and many-body current operators (see Sec. III E). We see that this procedure suggests an original way in the renormalization theory [47,48,50], that allows us to reproduce the known Feynman results, at least, in the first order in coupling constants. It would be interesting to know whether such a reproduction in higher orders takes place.

In what follows we allow ourselves to recall a few details concerning the in(out) formalism in QFT [43]. We do it, keeping in mind that this topic is not popular amongst the nuclear and few-body physics community. Meanwhile, this formalism, being combined with the CPR, leads to consistent calculations of the reaction amplitudes and observables for the processes of interest.

B. Reaction amplitudes in the in(out) formalism

In this subsection we will show how within the field-theoretical description the amplitude for an electron scattering off a hadron system in the OPEA can be reduced to the expression

$$\mathcal{T}_{if} = \frac{em_e}{\sqrt{EE'}} \delta(p' + k' - p - k) \bar{u}_e(k') \gamma_\mu u_e(k) \frac{\langle h' | J_h^\mu(0) | h \rangle}{q^2}, \quad (22)$$

where m_e is the mass of physical electron, $u_e(k) \equiv u_e(k\sigma)$ electron's Dirac spinor with 4-momentum k and polarization σ and $J_h^\mu(0)$ is the hadronic current density operator at the space-time point $x = (t, \mathbf{x}) = (0, \mathbf{0})$, that is sandwiched between the initial $|h\rangle$ and final $|h'\rangle$ hadronic states.

In this context, it seems pertinent to address a field model for interacting particles, viz., let us consider the Lagrangian density

$$\mathcal{L} = \mathcal{L}_e + \mathcal{L}_h + \mathcal{L}_\gamma + \mathcal{L}_{\gamma e} + \mathcal{L}_{\gamma h} \quad (23)$$

for interacting electrons, hadrons, and photons, where after Schwinger we have

$$\mathcal{L}_e = \frac{i}{4} [\bar{\psi}_e, \gamma^\mu \partial_\mu \psi_e] - \frac{i}{4} [\partial_\mu \bar{\psi}_e \gamma^\mu, \psi_e] - \frac{m_e^e}{2} [\bar{\psi}_e, \psi_e], \quad (24)$$

$$\mathcal{L}_h = \mathcal{L}_N + \mathcal{L}_M + \mathcal{L}_{MN}, \quad (25)$$

$$\mathcal{L}_N = \frac{i}{4} [\bar{\psi}, \gamma^\mu \partial_\mu \psi] - \frac{i}{4} [\partial_\mu \bar{\psi} \gamma^\mu, \psi] - \frac{m_0}{2} [\bar{\psi}, \psi]. \quad (26)$$

Here the density $\mathcal{L}_M$ consists of the pionic (pseudoscalar) part

$$\mathcal{L}_M = \frac{1}{2} [\partial_\mu \phi_{ps} \partial^\mu \phi_{ps} - \mu_0^2 \phi_{ps}^2] \quad (27)$$

and heavier meson contributions, while $\mathcal{L}_{MN} = \mathcal{L}_s + \mathcal{L}_{ps} + \mathcal{L}_v + \dots$ is the density of the meson-nucleon interaction. Here s , ps , and v stand for scalar, pseudoscalar, and vector couplings. We use notation m_0^e , m_0 for the electron, nucleon bare (trial) masses, respectively, and e_0 for bare electron and proton charge.

Electromagnetic interactions with the lepton current $J_e^\mu = \frac{e_0}{2} [\bar{\psi}_e, \gamma^\mu \psi_e]$ and the hadron current $J_h = J_N + J_M$, where $J_N^\mu = \frac{e_0}{2} [\bar{\psi}, \gamma^\mu \frac{1+\tau^z}{2} \psi]$ are given by

$$\mathcal{L}_{\gamma e} = J_e^\mu A_\mu = J_e A, \quad (28)$$

$$\mathcal{L}_{\gamma h} = J_h A. \quad (29)$$

At last, the Lagrangian density for the radiation field after Stückelberg is defined as

$$\mathcal{L}_\gamma = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\partial A)^2 + \frac{\lambda^2}{2} A^2, \quad (30)$$

with infinitesimally small photon mass λ . $\partial A \equiv \partial_\mu A^\mu$ is the divergence of the e.m. field vector $A = (A^0(x), A^1(x), A^2(x), A^3(x))$.

Note also that these currents originate from the so-called minimal substitution $\partial^\mu \rightarrow \partial^\mu + ie_0 A^\mu$. Introduction of non-minimal currents is a separate problem, but their inclusion does not violate the gauge invariance of the theory.

The corresponding Euler-Lagrange equations

$$(i\partial - m_0)\psi_e = e_0 A \psi_e, \quad (31)$$

$$\bar{\psi}_e(-i\partial - m_0) = e_0 \bar{\psi}_e A, \quad (32)$$

$$(i\partial - m_0^N)\psi = \frac{\partial \mathcal{L}_{MN}}{\partial \bar{\psi}} + e_0 A \frac{1+\tau^z}{2} \psi, \quad (33)$$

$$\bar{\psi}(-i\partial - m_0^N) = \frac{\partial \mathcal{L}_{MN}}{\partial \psi} + e_0 \frac{1+\tau^z}{2} \bar{\psi} A, \quad (34)$$

$$\square A^\mu(x) = J_e^\mu(x) + J_h^\mu(x) \equiv J^\mu(x), \quad (35)$$

where we introduced the symbol $\Phi = \gamma_\mu O^\mu$ for any vector O and $\frac{1+\tau^z}{2}$ is the proton isospin projection operator.

Using these equations we would like to address the reduction technique within the in(out) formalism, in particular, the so-called asymptotic LSZ approach (see, e.g., monographs by Schweber (1961) [51], Goldberger and Watson (1964) [45], Bjorken and Drell (1962) [43]). In this context, relying upon the previous experience (see Ref. [52] and Appendix B in the Ph.D. thesis in Ref. [42]), one can show that the S -matrix for an electron scattering off a hadron system h ,

$$e + h \rightarrow e + h', \quad (36)$$

with the conservation law $k + p = k' + p'$ is given by

$$\begin{aligned} \langle f | S | i \rangle &= \langle e' h'; \text{out} | e h; \text{in} \rangle \\ &= 2\pi i \frac{m_e}{\sqrt{EE'}} e_0 \bar{u}_e(k') \gamma_\mu u_e(k) \\ &\quad \times \frac{\langle h' | J_h^\mu(0) | h \rangle}{q^2} \delta(p' + k' - p - k), \end{aligned} \quad (37)$$

where m_e is the mass of the physical electron, $q^2 = (k' - k)^2 = (p' - p)^2$ is the 4-momentum transferred, $u_e(k) \equiv u_e(k\sigma)$ is the electron's Dirac spinor with 4-momentum k and polarization σ . The matrix element $\langle h' | J_h^\mu(0) | h \rangle$ can be replaced by $\langle h' | J_h^\mu(0) | h \rangle$ since, first, $J_e^\mu(0) = e_0 : \bar{\psi}_e(0) \gamma^\mu \psi(0) :$ with the symbol $:$ for the normal ordering of operators, and, second,

$$\langle h' | : \bar{\psi}_e(0) \gamma^\mu \psi(0) : | h \rangle = 0, \quad (38)$$

with accuracy up to terms of the e^1 order. In addition, assuming that $e_0 - e \propto O(e^3)$ we can resort to the OPEA

$$\begin{aligned} \langle e' h'; \text{out} | e h; \text{in} \rangle &= 2\pi i \frac{m_e}{\sqrt{EE'}} e_0 \bar{u}_e(k') \gamma_\mu u_e(k) \\ &\quad \times \frac{\langle h' | J_h^\mu(0) | h \rangle}{q^2} \delta(p' + k' - p - k), \end{aligned} \quad (39)$$

with the matrix element $\langle h' | J_h^\mu(0) | h \rangle$ between the strong Hamiltonian ($H_S = H_N + H_M + H_{MN}$ in our case) eigenstates.

In this respect, the UCT method [31,46,53] has an advantage allowing us to build up new interaction operators [34] and solve the respective eigenvalue problem nonperturbatively [54]. In the CPR all ingredients of matrix elements

$$\langle \Psi_{\mathbf{q}, \mathbf{p}_0 S M_S}^{(-)} | J_h^\mu(0) | \Psi_{M_d} \rangle, \quad (40)$$

namely, the hadron states and current operator, can be consistently found within the field-theoretical calculations. This is

achieved, in particular, via certain links between the in(out) and clothed particle states.

C. Links between the clothed particle representation and in(out) formalism

Recall that along with the one-clothed-particle states there are other one-particle states which are the H eigenvectors as well. In Ref. [55] there was presented the equivalence theorem that connects the definitions of the S -matrix within the in(out) formalism by LSZ and the CPR approach. It has turned out that the S operators that are determined by the time evolution from a distant past to a distant future, for the two decompositions $H = H(\{\alpha\}) = H_F + H_I$ and $H = K(\{\alpha_c\}) = K_F + K_I$, being sandwiched between the bare states and the corresponding clothed ones are equal to each other. Such a coincidence becomes possible owing to certain isomorphism between the $\{\alpha_c\}$ algebra and the $\{\alpha\}$ algebra once the UCT obey the asymptotic condition

$$\lim_{t \rightarrow \pm\infty} W_D(t) = 1, \quad (41)$$

where the UCT in the Dirac (D) picture $W_D(t) = e^{iH_F t} W e^{-iH_F t}$ and the limit is implied in the strong sense.

As well-known, when evaluating the S matrix in the Heisenberg (H) picture,

$$S_{if} = \langle f; \text{out} | i; \text{in} \rangle \quad (42)$$

one has to deal with the in(out) states (see, e.g., Ref. [45])

$$|\mathbf{k}_1 \cdots \mathbf{k}_n; \text{in(out)}\rangle \equiv a_{\text{in(out)}}^\dagger(\mathbf{k}_1) \cdots a_{\text{in(out)}}^\dagger(\mathbf{k}_n) |\Omega\rangle, \quad (43)$$

in particular, one-particle state

$$|\mathbf{k}; \text{in(out)}\rangle \equiv a_{\text{in(out)}}^\dagger(\mathbf{k}) |\Omega\rangle, \quad (44)$$

where $|\Omega\rangle$ is the physical vacuum. The creation (destruction) in(out) operators $a_{\text{in(out)}}^\dagger$ ($a_{\text{in(out)}}$) meet the same canonical commutation relations that clothed ones $a_c^\dagger$ (a_c) do. By definition, the in(out) states are eigenvectors of the energy-momentum operator $P^\mu = (H, \mathbf{P})$,

$$P^\mu |\mathbf{k}_1 \cdots \mathbf{k}_n; \text{in(out)}\rangle = (k_1^\mu + \cdots + k_n^\mu) |\mathbf{k}_1 \cdots \mathbf{k}_n; \text{in(out)}\rangle, \quad (45)$$

with $k^\mu = (E_{\mathbf{k}}, \mathbf{k})$.

In Appendix A one can find (see also Sec. 4 in Ref. [55] and Ref. [42]) the derivation of important relations between states in the CPR and in(out) formalism for the one particle

$$|\mathbf{k}; \text{in(out)}\rangle \equiv a_{\text{in(out)}}^\dagger(\mathbf{k}) |\Omega\rangle = a_c^\dagger(\mathbf{k}) |\Omega\rangle \quad (46)$$

and two particles

$$\begin{aligned} |\mathbf{k}_1, \mathbf{k}_2; \text{in}\rangle &\equiv a_{\text{in}}^\dagger(\mathbf{k}_1) a_{\text{in}}^\dagger(\mathbf{k}_2) |\Omega\rangle \\ &= \Omega_c^{(+)} a_c^\dagger(\mathbf{k}_1) a_c^\dagger(\mathbf{k}_2) |\Omega\rangle, \end{aligned} \quad (47)$$

$$\begin{aligned} |\mathbf{k}_1, \mathbf{k}_2; \text{out}\rangle &\equiv a_{\text{out}}^\dagger(\mathbf{k}_1) a_{\text{out}}^\dagger(\mathbf{k}_2) |\Omega\rangle \\ &= \Omega_c^{(-)} a_c^\dagger(\mathbf{k}_1) a_c^\dagger(\mathbf{k}_2) |\Omega\rangle, \end{aligned} \quad (48)$$

where $\Omega_c^{(\pm)}$ denotes the Møller operators

$$\Omega_c^{(\pm)} \equiv \lim_{t \rightarrow \mp\infty} \exp(iKt) \exp(-iK_F t). \quad (49)$$

These relations hold under the condition (41). To some extent, relation (46) does not seem unexpected, since both one-particle clothed and in(out) states are the H eigenvectors. Of course, it does not mean that $a_{\text{in(out)}}(\mathbf{k}) = a_c(\mathbf{k})$!

Furthermore, one should keep in mind the following recipe of practical calculations with

$$\Omega_c^{(\pm)} |E; c\rangle = \Omega_c^{(\pm)}(E) |E; c\rangle = \pm i \lim_{\epsilon \rightarrow +0} \epsilon G(E \pm i\epsilon) |E; c\rangle, \quad (50)$$

where we have introduced the notation $G(z) = (z - K)^{-1}$ for the Hamiltonian resolvent. In its turn, the resolvent can be expressed through the corresponding T or R operator to make a path to known methods. Just such an approach has been used in papers [34,54] when studying the properties of two-nucleon systems. Evidently, the appearance of these operators reflects the inclusion of the initial (final) state interaction effects into entrance (exit) reaction channels.

D. Deuteron and np -pair states in the CPR

Working in the two-nucleon sector of the Fock space, the initial and final states in the rest frame can be represented as

$$\begin{aligned} |\Psi_{M_d}\rangle &= \int d\mathbf{p}' |\mathbf{p}' S' M'_S\rangle \langle \mathbf{p}' S' M'_S | \Psi_{M_d}\rangle \\ &\equiv \int d\mathbf{p}' \Psi_{M_d}(\mathbf{p}' S' M'_S) |\mathbf{p}' S' M'_S\rangle, \end{aligned} \quad (51)$$

$$\begin{aligned} |\Psi_{\mathbf{0}, \mathbf{p} S M_S}^{(-)}\rangle &\equiv |\Psi_{\mathbf{p} S M_S}^{(-)}\rangle = \int d\mathbf{p}' |\mathbf{p}' S' M'_S\rangle \langle \mathbf{p}' S' M'_S | \Psi_{\mathbf{p} S M_S}^{(-)}\rangle \\ &\equiv \int d\mathbf{p}' \Psi_{\mathbf{p} S M_S}^{(-)}(\mathbf{p}' S' M'_S) |\mathbf{p}' S' M'_S\rangle, \end{aligned} \quad (52)$$

where

$$|\mathbf{p} S M_S\rangle = \sum_{\mu_1 \mu_2} \left(\frac{1}{2} \mu_1 \frac{1}{2} \mu_2 \middle| S M_S \right) b_c^\dagger(\mathbf{p} \mu_1) b_c^\dagger(-\mathbf{p} \mu_2) |\Omega\rangle. \quad (53)$$

These states are the $P = (H, \mathbf{P})$ eigenvectors, viz.,

$$P |\Psi_{M_d}\rangle = (m_d, \mathbf{0}) |\Psi_{M_d}\rangle, \quad P |\Psi_{\mathbf{p} S M_S}^{(-)}\rangle = (2E_{\mathbf{p}}, \mathbf{0}) |\Psi_{\mathbf{p} S M_S}^{(-)}\rangle, \quad (54)$$

with $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$, where $\mathbf{p}$ is the relative momentum in the rest frame and m (m_d) is the physical nucleon (deuteron) mass.

Further, according to Appendix B of Ref. [34] the basis vectors (53) are the common eigenstates of the operators S_{ferm}^2 and S_{ferm}^z :

$$\begin{aligned} S_{\text{ferm}}^2 |\mathbf{p} S M_S\rangle &= S(S+1) |\mathbf{p} S M_S\rangle, \\ S_{\text{ferm}}^z |\mathbf{p} S M_S\rangle &= M_S |\mathbf{p} S M_S\rangle. \end{aligned} \quad (55)$$

Then the np -pair states are also S_{ferm}^2 eigenvectors

$$\begin{aligned} S_{\text{ferm}}^2 |\Psi_{\mathbf{p} S M_S}^{(-)}\rangle &= \sum_{M'_S} \int d\mathbf{p}' \Psi_{\mathbf{p} S M_S}^{(-)}(\mathbf{p}' S' M'_S) S_{\text{ferm}}^2 |\mathbf{p}' S' M'_S\rangle \\ &= S(S+1) |\Psi_{\mathbf{p} S M_S}^{(-)}\rangle. \end{aligned} \quad (56)$$

The latter follows from the fact that our Hamiltonian is diagonal with respect to the spin quantum numbers

$$\langle \mathbf{p}' S' M'_S | H | \mathbf{p} S M_S \rangle = \delta_{S'S} \langle \mathbf{p}' S' M'_S | H | \mathbf{p} S M_S \rangle. \quad (57)$$

The $\mathbb{C}$ -number coefficient $\Psi_{M_d}(\mathbf{p} S M_S)$ can be written in terms of the S and D components of the deuteron WF in momentum space

$$\Psi_{M_d}(\mathbf{p} 1 M_S) = \sum_{l=0,2} (l m_l 1 M_S | 1 M_d) u_l(|\mathbf{p}|) Y_{l m_l}(\hat{\mathbf{p}}). \quad (58)$$

They have been already calculated in Refs. [38,56].

In a moving frame the corresponding np -pair state can be found via the link

$$|\Psi_{\mathbf{q}, \mathbf{p}_0 S M_S}^{(-)}\rangle = e^{-i\boldsymbol{\beta} \cdot \mathbf{B}} |\Psi_{\mathbf{p} S M_S}^{(-)}\rangle. \quad (59)$$

This relation is realized with the boost operator $\mathbf{B} = \mathbf{B}_F + \mathbf{B}_I$, where its free part $\mathbf{B}_F$ is determined via the Belinfante ansatz $\mathbf{B}_F = -\int \mathbf{x} K_F(\mathbf{x}) d\mathbf{x}$ and $K_F(\mathbf{x})$ is the free Hamiltonian density. Perhaps, one should note that the required $P|\Psi_{\mathbf{q}, \mathbf{p}_0 S M_S}^{(-)}\rangle = (E_{np}, \mathbf{q})|\Psi_{\mathbf{q}, \mathbf{p}_0 S M_S}^{(-)}\rangle$ follows from the property of the energy-momentum operator to be four-vector, viz.,

$$e^{i\boldsymbol{\beta} \cdot \mathbf{B}} P^\mu e^{-i\boldsymbol{\beta} \cdot \mathbf{B}} = P^\nu L_\nu^\mu(\boldsymbol{\beta}), \quad (60)$$

$$L_\nu^\mu(\boldsymbol{\beta}) = \begin{pmatrix} u_0 & \vdots & -u_j \\ \cdots & \cdots & \cdots \\ u^i & \vdots & \delta_j^i - \frac{u^i u_j}{1+u_0} \end{pmatrix}, \quad (61)$$

with the Lorentz transformation $(2E_p, \mathbf{0}) \Rightarrow (E_{np} = E_{\mathbf{k}_p} + E_{\mathbf{k}_n}, \mathbf{q}) = P$ and 4-velocity vector

$$u^\mu = (u_0, \mathbf{u}) = (\text{ch}\beta, \mathbf{n} \text{sh}\beta) = \frac{P^\mu}{\sqrt{E_{np}^2 - \mathbf{q}^2}}. \quad (62)$$

Relative momentum in the rest frame $\mathbf{p}$ is expressed through $\mathbf{q}$ and $\mathbf{p}_0$ in laboratory system as follows:

$$\mathbf{p}(\mathbf{p}_0, \mathbf{q}) = \mathbf{p}_0 + \frac{\mathbf{q}}{2} + \left(\frac{\mathbf{u} \cdot (\mathbf{p}_0 + \frac{\mathbf{q}}{2})}{1 + u_0} - E_{\mathbf{p}_0 + \frac{\mathbf{q}}{2}} \right) \mathbf{u}. \quad (63)$$

To meet Eq. (60) it is sufficient to meet the Lie-Poincaré commutation relations

$$[H, \mathbf{B}] = i\mathbf{P}, \quad [P^i, B^j] = i\delta^{ij}H. \quad (64)$$

To apply Eq. (59), one can employ the following operator relation:

$$e^{-i\boldsymbol{\beta} \cdot (\mathbf{B}_F + \mathbf{B}_I)} = Z(e^{-i\boldsymbol{\beta} \cdot \mathbf{B}_F} \mathbf{B}_I e^{i\boldsymbol{\beta} \cdot \mathbf{B}_F}) e^{-i\boldsymbol{\beta} \cdot \mathbf{B}_F}, \quad (65)$$

where the operator Z is a measure of noncommutativity of the operators $\mathbf{B}_F$ and $\mathbf{B}_I$. In this context, let us consider the relation

$$e^{\tau(a+b)} = Z(\tau) e^{\tau a} \quad (66)$$

for arbitrary operators a and b (in our case $a = -i\boldsymbol{\beta} \cdot \mathbf{B}_F$ and $b = -i\boldsymbol{\beta} \cdot \mathbf{B}_I$), where the operator $Z(\tau)$ meets the equation

$$\frac{\partial Z(\tau)}{\partial \tau} = Z(\tau) e^{\tau a} b e^{-\tau a}. \quad (67)$$

Recall that the similarity transformation in this equation is the Campbell-Hausdorff series

$$e^{\tau a} b e^{-\tau a} = \sum_{n=0}^{\infty} \frac{\tau^n}{n!} \underbrace{[a, \cdots [a, b]]}_{n} \cdots. \quad (68)$$

There are different ways on solving the equations similar to Eq. (67). First of all we keep in mind those based on the Baker-Hausdorff and Magnus theorems [57,58].

Taking into account $U_F(L) = \exp(-i\boldsymbol{\beta} \mathbf{B}_F)$, the separation (65) allows us to use the property

$$e^{-i\boldsymbol{\beta} \mathbf{B}_F} b^\dagger(\mathbf{p}\mu) e^{i\boldsymbol{\beta} \mathbf{B}_F} = D_{\mu\bar{\mu}}^{[1/2]}(W(L, p)) b^\dagger(L\mathbf{p}\bar{\mu}), \quad (69)$$

to get

$$\begin{aligned} |\Psi_{\mathbf{q}, \mathbf{p}_0 S M_S}^{(-)}\rangle &= \int d\mathbf{p}' \Psi_{\mathbf{p}' S M_S}^{(-)}(\mathbf{p}' \bar{\mu}_1 \bar{\mu}_2) D_{\bar{\mu}_1 \bar{\mu}_1'}^{[1/2]}(W(L, p')) \\ &\times D_{\bar{\mu}_2 \bar{\mu}_2'}^{[1/2]}(W(L, p')) Z(e^{-i\boldsymbol{\beta} \mathbf{B}_F} \mathbf{B}_I e^{i\boldsymbol{\beta} \mathbf{B}_F}) \\ &\times b_c^\dagger(L\mathbf{p}' \bar{\mu}_1') b_c^\dagger(L\mathbf{p}' \bar{\mu}_2') |\Omega\rangle, \end{aligned} \quad (70)$$

where the D function depends on the Wigner rotation $W(L, p)$ and we introduced three-vectors

$$(L\mathbf{p}')^i \equiv L_\mu^i p'^\mu, \quad (L\mathbf{p}_-')^i \equiv L_\mu^i p_-'^\mu, \quad (71)$$

with $p' = (E_{p'}, \mathbf{p}')$ and $p_-' = (E_{p'}, -\mathbf{p}')$. We will confine ourselves to calculations with $Z = 1$, i.e., we neglect the interaction term in the total boost operator.

Now, let us address the partial wave decomposition of the np -pair final state in the rest frame

$$|\Psi_{\mathbf{p} S M_S T 0}^{(-)}\rangle \equiv |\Psi_{\mathbf{p} S M_S}^{(-)}| \langle T 0 |, \quad (72)$$

i.e.,

$$\begin{aligned} &|\Psi_{\mathbf{p} S M_S T 0}^{(-)}| \mathbf{p}' S' M'_S T' 0 \rangle \\ &= \delta_{S'S} \delta_{T'T} \sum_{J=0}^{J_{\max}} \sum_{M_J=-J}^J Y_{lm}(\hat{\mathbf{p}}) (l m S M_S | J M_J) \\ &\times \Psi_{pl'l'}^{\alpha(-)}(p') (l' m' S M'_S | J M_J) Y_{l'm'}^*(\hat{\mathbf{p}}'), \end{aligned} \quad (73)$$

where we restore the indices for the total isospin T and its projection $M_T = 0$, $J_{\max}$ is the maximum value of the total angular momentum in the np -pair partial-wave scattering states $\alpha = \{J, S, T\}$ and summation over l, l' as well as m and m' is implied. Here we use the basis vectors

$$|pJ(lS)M_J T M_T\rangle = \int d\hat{\mathbf{p}} Y_{lm_l}(\hat{\mathbf{p}}) |\mathbf{p} S M_S T M_T\rangle (l m_l S M_S | J M_J) \quad (74)$$

and vice versa

$$|\mathbf{p} S M_S T M_T\rangle = (l m_l S M_S | J M_J) Y_{lm_l}^*(\hat{\mathbf{p}}) |pJ(lS)M_J T M_T\rangle, \quad (75)$$

with the unit vector $\hat{\mathbf{p}} = \mathbf{p}/p$. Details of the construction and physical properties of this basis can be found in Appendix B of Ref. [34].

The partial waves $\Psi_{pl'l'}^{\alpha(\pm)}(p')$ are related by

$$\Psi_{pl'l'}^{\alpha(\pm)}(p') = \sum_{l''} O_{ll''}^{\alpha(\pm)}(p) \varphi_{pl''l'}^{\alpha}(p'), \quad (76)$$

with the WFs $\varphi_{pl'l'}^\alpha(p')$, which have the asymptotics of standing waves. The latter meet the equation

$$\varphi_{pl'l'}^\alpha(p') = \frac{1}{p'^2} \delta(p' - p) \delta_{l'l} + P \frac{1}{E_p - E_{p'}} R_{l'l}^\alpha(p', p), \quad (77)$$

where ε_α is the mixing parameter, while for uncoupled partial waves ($l' = l = J$) we have $O^{\alpha(\pm)}(p) = \Delta^{\alpha(\pm)}$. The quantities $\Delta_\pm^{\alpha(\pm)}$ ($\Delta^{\alpha(\pm)}$) can be expressed through the corresponding phase shifts $\delta_\pm^\alpha = \delta_{J\pm 1}^\alpha$ (δ_J^α): $\Delta^{\alpha(\pm)} = \exp(\pm i\delta^\alpha) \cos \delta^\alpha$.

To get expressions for the partial states of standing waves, we will employ the so-called matrix inversion method [60]. Let us consider the typical integral of the generalized functions

$$I = \int_0^\infty \varphi(q) h(q) q^2 dq, \quad (79)$$

with $h(q)$ from the class of functions on which the quadrature calculation scheme works. Using Eq. (77) one can show

$$I = h(N+1) - \sum_{j=1}^{N+1} \Omega_j p_j^2 R(j, N+1) h(j) \\ = \sum_{j=1}^{N+1} B(j) h(j), \quad (80)$$

$$B(j) \equiv \delta_{j, N+1} - \Omega_j p_j^2 R(j, N+1), \quad (81)$$

so

$$\varphi(p') = \sum_{j=1}^{N+1} B(j) \frac{\delta(p' - p_j)}{p'^2}. \quad (82)$$

This idea can be applied to all partial standing waves:

$$\varphi_{pl'l'}^\alpha(p') = \sum_{j=1}^{N+1} B_{pl'l'}^\alpha(j) \frac{\delta(p' - p_j)}{p_j^2}, \quad (83)$$

where coefficients $B_{pl'l'}^\alpha(j)$ are the solutions of the set of linear algebraic equations approximately equivalent to the underlying Lippman-Schwinger type equation for the n - p scattering problem in c.m. system. Here N is the number of Gaussian nodes on the interval $[-1, 1]$, p_j are the corresponding grid points and $p_{N+1} = p$ (see details in Ref. [61]). It is important to stress that the representation (83) should be meant in an integral sense. Previously this method was successfully applied for the treatment of the final-state interaction in Refs. [27,62,63]. In Appendix C one can find an additional test of our np -pair WF calculations.

E. Current density operator in the CPR without partial wave expansions

Our matrix elements (4) with the states (51) and (70) are reduced to the expectation value with respect to the physical vacuum $|\Omega\rangle$,

$$\langle L\mathbf{p}'\mu'_1, L\mathbf{p}'_-\mu'_2 | J^\mu(0) | \mathbf{p}\mu_1, -\mathbf{p}\mu_2 \rangle, \quad (84)$$

where elements of the R -matrix satisfies its own Lippman-Schwinger type equations. For coupled partial waves ($l', l = J \pm 1$) the matrix $O^{\alpha(\pm)}(p)$ is parametrized as in Ref. [59],

$$\hat{O}^{\alpha(\pm)}(p) = \begin{pmatrix} \Delta_-^{\alpha(\pm)} \cos^2 \varepsilon_\alpha + \Delta_+^{\alpha(\pm)} \sin^2 \varepsilon_\alpha & \frac{1}{2} (\Delta_-^{\alpha(\pm)} - \Delta_+^{\alpha(\pm)}) \sin 2\varepsilon_\alpha \\ \frac{1}{2} (\Delta_-^{\alpha(\pm)} - \Delta_+^{\alpha(\pm)}) \sin 2\varepsilon_\alpha & \Delta_-^{\alpha(\pm)} \sin^2 \varepsilon_\alpha + \Delta_+^{\alpha(\pm)} \cos^2 \varepsilon_\alpha \end{pmatrix}, \quad (78)$$

where $|\mathbf{p}_1\mu_1, \mathbf{p}_2\mu_2\rangle \equiv b_c^\dagger(\mathbf{p}_1\mu_1)b_c^\dagger(\mathbf{p}_2\mu_2)|\Omega\rangle$.

The Noether current operator $J(0)$ in the CPR can be found by using the Campbell-Hausdorff formula [31,34,40]

$$J^\mu(0) = e^R J_c^\mu(0) e^{-R} \\ = J_c^\mu(0) + [R, J_c^\mu(0)] + \frac{1}{2} [R, [R, J_c^\mu(0)]] + \dots, \quad (85)$$

where $J_c^\mu(0)$ is the primary Noether current in which “bare” operators $\{\alpha\}$ are replaced by their clothed counterparts $\{\alpha_c\}$ and R is the generator of the clothing transformation $W = \exp(R)$. Remind that the Noether current density $J^\mu(\mathbf{x}) = J_N^\mu(\mathbf{x}) + J_M^\mu(\mathbf{x})$ in the Schrödinger (S) picture consists of the nucleon part

$$J_N^\mu(\mathbf{x}) = e : \bar{\psi}(\mathbf{x}) \gamma^\mu \frac{1 + \tau^z}{2} \psi(\mathbf{x}) :, \quad \text{where } \psi = \begin{pmatrix} \psi_{1/2} \\ \psi_{-1/2} \end{pmatrix}, \quad (86)$$

and mesonic one. For the latter, we have the following definitions:

$$J_{ps}^\alpha(\mathbf{x}) = e : [\varphi_{ps}(\mathbf{x}) \times \partial^\alpha \varphi_{ps}(\mathbf{x})]^z :, \quad (87)$$

$$J_s^\alpha(\mathbf{x}) = e : [\varphi_s(\mathbf{x}) \times \partial^\alpha \varphi_s(\mathbf{x})]^z :, \quad (88)$$

$$J_v^\alpha(\mathbf{x}) = e : [\varphi_v^{\alpha\beta}(\mathbf{x}) \times \varphi_{v\beta}(\mathbf{x})]^z :, \quad (89)$$

with the isovectors $\boldsymbol{\varphi}$.

Decomposition (85) involves one-body, two-body and more complicated interaction currents that are often called the MECs, if one uses the terminology which is adopted in the theory of e.m. interactions with nuclei.

In Eq. (84) the current density operator $J^\mu(0)$ is sandwiched between the clothed two-nucleon states. As shown in Ref. [40] for such expectation values the current $J^\mu(0)$ will contribute only with its one-body $J_{[1]}$ and two-body $J_{[2]}$ parts

$$\langle \mathbf{p}'_1\mu'_1, \mathbf{p}'_2\mu'_2 | J^\mu(0) | \mathbf{p}_1\mu_1, \mathbf{p}_2\mu_2 \rangle \\ = \langle \mathbf{p}'_1\mu'_1, \mathbf{p}'_2\mu'_2 | (J_{[1]}^\mu + J_{[2]}^\mu) | \mathbf{p}_1\mu_1, \mathbf{p}_2\mu_2 \rangle, \quad (90)$$

which, in general, can be written as

$$J_{[1]}^\mu = \sum_{\mu'_1\mu'_2} \int d\mathbf{p}' d\mathbf{p} F^\mu(\mathbf{p}'\mu'_1, \mathbf{p}\mu) b_c^\dagger(\mathbf{p}'\mu'_1) b_c(\mathbf{p}\mu), \quad (91)$$

$$J_{[2]}^\mu = \sum_{\mu'_1\mu'_2, \mu_1\mu_2} \int d\mathbf{p}'_1 d\mathbf{p}'_2 d\mathbf{p}_1 d\mathbf{p}_2 F_{\text{MEC}}^\mu \\ \times (\mathbf{p}'_1\mu'_1, \mathbf{p}'_2\mu'_2; \mathbf{p}_1\mu_1, \mathbf{p}_2\mu_2) b_c^\dagger(\mathbf{p}'_1\mu'_1) \\ \times b_c^\dagger(\mathbf{p}'_2\mu'_2) b_c(\mathbf{p}_1\mu_1) b_c(\mathbf{p}_2\mu_2). \quad (92)$$

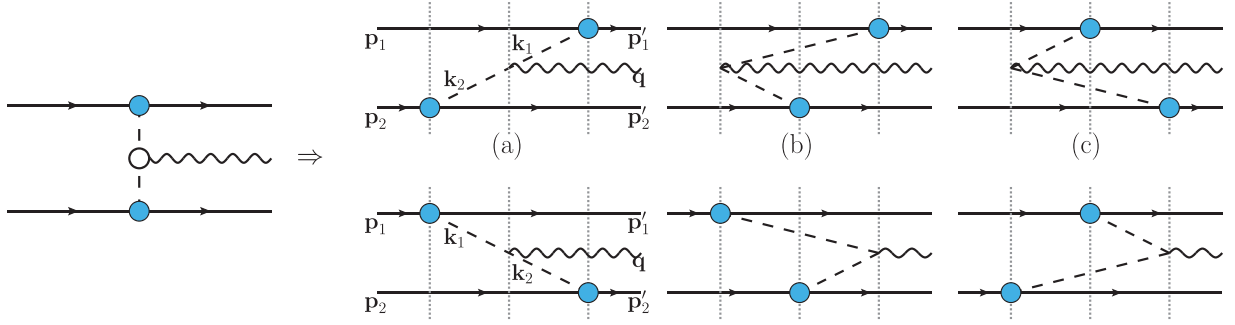


FIG. 3. Illustration of the mechanisms that contribute to the mesonic current J_{MCC} . The blue dot corresponds to the cutoff function g_M at the nucleon-meson vertex (see Appendix B).

The $\mathbb{C}$ -number functions F^μ and F_{MEC}^μ can be found from the series (85) by separating the respective $b_c^\dagger b_c$ and $b_c^\dagger b_c^\dagger b_c b_c$ contributions

$$\begin{aligned} J_{[1]} &= J_c(0)_{b_c^\dagger b_c} + [R, J_c(0)]_{b_c^\dagger b_c} + \frac{1}{2}[R, [R, J_c(0)]]_{b_c^\dagger b_c} + \dots, \\ J_{[2]} &= J_c(0)_{b_c^\dagger b_c^\dagger b_c b_c} + [R, J_c(0)]_{b_c^\dagger b_c^\dagger b_c b_c} \\ &\quad + \frac{1}{2}[R, [R, J_c(0)]]_{b_c^\dagger b_c^\dagger b_c b_c} + \dots \end{aligned} \quad (93)$$

It is important to stress that the many-nucleon currents introduced in such a way do not depend on the choice of states between which we calculate the amplitude. In particular, it means that the one-nucleon matrix elements

$$\langle \mathbf{p}'\mu' | J_h(0) | \mathbf{p}\mu \rangle = \langle \mathbf{p}'\mu' | J_{[1]} | \mathbf{p}\mu \rangle = F(\mathbf{p}'\mu', \mathbf{p}\mu), \quad (94)$$

where $|\mathbf{p}\mu\rangle = b_c^\dagger(\mathbf{p}\mu)|\Omega\rangle$ and the operator $J_{[1]}$ is the same as in Eq. (90).

For the one-nucleon current we have an alternative way to find the function $F(\mathbf{p}'\mu', \mathbf{p}\mu)$ than from the series (93). It follows from the observation that the primary Noether current density, being sandwiched between the one-nucleon states, yields the usual on-mass-shell expression

$$\begin{aligned} F^\mu(\mathbf{p}'\mu', \mathbf{p}\mu) &= \bar{u}(\mathbf{p}'\mu') \left[F_1[(p' - p)^2] \gamma^\mu \right. \\ &\quad \left. + i\sigma^{\mu\nu} \frac{(p' - p)_\nu}{2m} F_2[(p' - p)^2] \right] u(\mathbf{p}\mu), \end{aligned} \quad (95)$$

in terms of the Dirac (F_1) and Pauli (F_2) nucleon FFs (see, for example, p. 242 in Ref. [64]). That fact allows us to use the form (95) to determine the $\mathbb{C}$ -number coefficient in Eq. (91) instead of evaluating the infinite number of contributions in the series (93). It is impossible for the two-nucleon contribution that can be interpreted as a meson exchange current, so the determination of F_{MEC}^μ requires sequential evaluation of the $[R, \dots [R, J_c(0)] \dots]$ commutators. The total hadronic current density J_c composed of the meson $J_{M,c}$ and nucleon $J_{N,c}$ currents so the two-nucleon meson exchange current density also composed of the two parts,

$$J_{[2]} = e^R(J_{M,c}(0) + J_{N,c}(0))e^{-R}|_{b_c^\dagger b_c^\dagger b_c b_c} \equiv J_{MCC} + J_{MNN}, \quad (96)$$

with the so-called mesonic J_{MCC} and seagull J_{MNN} currents.

It is important to note that the single-nucleon form factors in Eq. (95) depend on the four-momentum transfer to the individual nucleon $Q_N^2 \equiv -(p' - p)^2$, rather than on the four-momentum transfer to the deuteron as a whole. The latter is fixed by the electron kinematics

$$Q^2 = \mathbf{q}^2 - \omega^2. \quad (97)$$

In general $Q_N^2 \neq Q^2$, even when the final-state interaction and meson-exchange currents contributions are neglected. Indeed, in the plane-wave impulse approximation, the form factors depend on

$$Q_N^2 = \mathbf{q}^2 - (E_p - E_n)^2 = Q^2 + (2E_p - m_d)(2E_n - m_d) > Q^2. \quad (98)$$

Hence, the squared spacelike momentum transfer experienced by an individual nucleon is larger than that transferred to the deuteron as a whole. Similar treatment of one-body currents, with the nucleon form factors depending on the momentum transferred to the struck nucleon rather than to the nucleus as a whole, have been employed, e.g., in Refs. [65,66].

In Refs. [40,42] using the recursive technique for computing the multiple commutators from Ref. [32] we considered the first nonvanishing contribution to the two-nucleon current from the series (93) that stem from the commutator $\frac{1}{2}[R, [R, J_c^\mu(0)]]$. Thus, we have found F_{MEC}^μ for the model of interacting clothed nucleons and π , ρ , ω , η , δ , σ mesons. Explicit expressions for the MEC operators can be found in Appendix B. The interaction mechanisms embodied in these operators are depicted in Figs. 3 and 4. In the right-hand side of Fig. 3 (Fig. 4) one can see all the separate contributions to the mesonic (seagull) current that are wired to the simple graph in the left-hand side of the corresponding figure. Being topologically equivalent to the time-ordered Feynman diagrams, these graphs have a different interpretation: they are related to one and the same instant $t = 0$, so each of them is associated with a pair of the noncovariant energy denominators, the three-momentum being conserved in every vertex with all the particles on their mass shells.

At this point, let us discuss the distinctive features of our MECs in comparison with several other models proposed in the literature. In Sec. IV our numerical results are compared with those of Ref. [27], where the MECs of Maize and Kim [67] were used. Their mesonic current and seagull

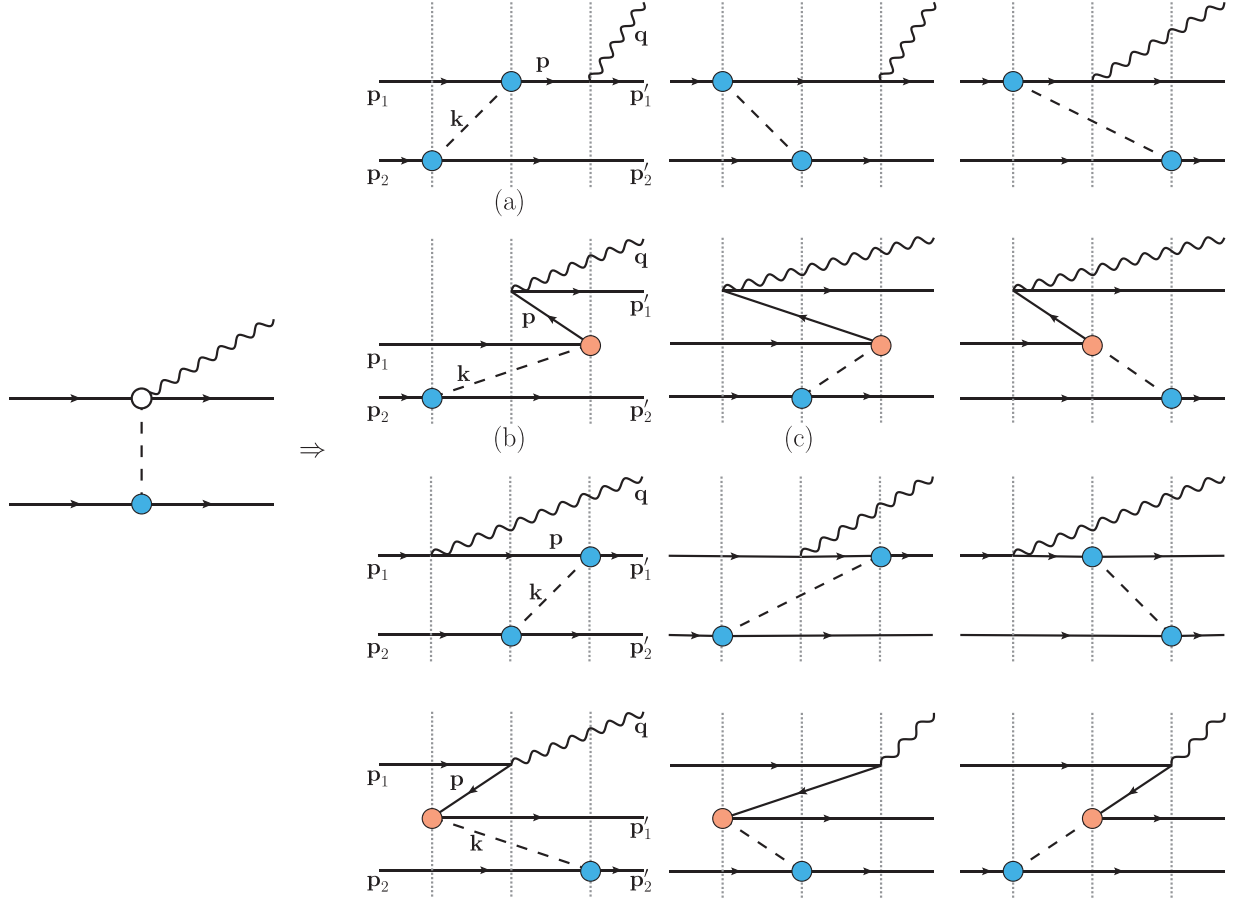


FIG. 4. Illustration of the mechanisms that contribute to the seagull current $J_{\mathcal{M}NN}$: the “retarded” ones are shown in the two top rows and the “advanced” ones in the two bottom rows. The blue and orange dots correspond to the cutoff functions $g_{\mathcal{M}}$ and $\bar{g}_{\mathcal{M}}$, respectively (see Appendix B).

current (called the “pair current” in Ref. [67]) are given by

$$\mathbf{j}_{\pi CC} = \frac{ig_\pi^2}{2m^2(2\pi)^3} F_\pi(Q^2) [\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]_z (\boldsymbol{\sigma}_1 \cdot \mathbf{k}_1) (\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2) \frac{\mathbf{k}_2 - \mathbf{k}_1}{\mathbf{k}_2^2 - \mathbf{k}_1^2} \left[\frac{f_{\pi NN}^2(\mathbf{k}_1)}{\mathbf{k}_1^2 + m_\pi^2} - \frac{f_{\pi NN}^2(\mathbf{k}_2)}{\mathbf{k}_2^2 + m_\pi^2} \right], \quad (99)$$

and

$$\mathbf{j}_{\pi NN} = \frac{ig_\pi^2}{4m^2(2\pi)^3} F_1^V(Q^2) [\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]_z \left[\frac{(\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2) \boldsymbol{\sigma}_1}{\mathbf{k}_2^2 + m_\pi^2} f_{\pi NN}^2(\mathbf{k}_2^2) - \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{k}_1) \boldsymbol{\sigma}_2}{\mathbf{k}_1^2 + m_\pi^2} f_{\pi NN}^2(\mathbf{k}_1^2) \right], \quad (100)$$

with momenta $\mathbf{k}_1 = \mathbf{p}_1 - \mathbf{p}'_1$, $\mathbf{k}_2 = \mathbf{p}_2 - \mathbf{p}'_2$, πNN vertex cutoff function

$$f_{\pi NN}(\mathbf{k}^2) = \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 + \mathbf{k}^2}, \quad (101)$$

e.m. pion FF F_π and isovector nucleon FF $F_1^V = F_1^p - F_1^n$. Being derived from Feynman diagrams, these expressions represent the matrix elements defined only on the energy shell. In contrast, in our approach we have no need to “invent” any analytical extensions to the off-energy-shell region. For comparison with Ref. [67], we consider the nonrelativistic reduction of the two-nucleon matrix elements of the pion-induced currents (B9) and (B13):

$$\langle 1', 2' | \mathbf{J}_{\pi CC} | 1, 2 \rangle \approx \frac{ig_\pi^2}{2m^2(2\pi)^6} [\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]_z (\boldsymbol{\sigma}_1 \cdot \mathbf{k}_1) (\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2) \frac{\mathbf{k}_2 - \mathbf{k}_1}{\mathbf{k}_2^2 - \mathbf{k}_1^2} \left[\frac{f_{\pi NN}(\mathbf{k}_1^2) f_{\pi NN}(\mathbf{k}_2^2)}{\mathbf{k}_1^2 + m_\pi^2} - \frac{f_{\pi NN}(\mathbf{k}_1^2) f_{\pi NN}(\mathbf{k}_2^2)}{\mathbf{k}_2^2 + m_\pi^2} \right], \quad (102)$$

$$\langle 1', 2' | \mathbf{J}_{\pi NN} | 1, 2 \rangle \approx \frac{ig_\pi^2}{4m^2(2\pi)^6} [\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]_z \left[\frac{(\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2) \boldsymbol{\sigma}_1}{\mathbf{k}_2^2 + m_\pi^2} f_{\pi NN}^2(\mathbf{k}_2^2) - \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{k}_1) \boldsymbol{\sigma}_2}{\mathbf{k}_1^2 + m_\pi^2} f_{\pi NN}^2(\mathbf{k}_1^2) \right]. \quad (103)$$

Note that the cutoff function (B7) reduces, in the nonrelativistic limit, to Eq. (101), whereas the time components $\langle 1', 2' | J_{\pi CC, \pi NN}^0 | 1, 2 \rangle$ are smaller by two orders in $|\mathbf{p}|/m$ than the spatial ones. The expressions (102) and (103) differ from the Maize–Kim currents in two aspects. First, they do not contain the phenomenological e.m. pion F_π and isovector nucleon F_1^V FFs. Second, the cutoff dependence in the mesonic current is different: Eq. (102) contains the product $f_{\pi NN}(\mathbf{k}_1^2)f_{\pi NN}(\mathbf{k}_2^2)$, whereas Eq. (99) contains $f_{\pi NN}^2(\mathbf{k}_1^2)$ and $f_{\pi NN}^2(\mathbf{k}_2^2)$. Therefore, in the nonrelativistic limit our pion-exchange MECs reduce, up to an overall normalization factor, to the Maize–Kim currents if one sets $F_\pi = F_1^V = 1$ and takes the cutoff parameter $\Lambda_\pi \rightarrow \infty$.

One may also compare currents (102) and (103) with those from Ref. [19] (see Eqs. (A2) and (A7) therein). The only difference is the presence of the isovector FF F_1^V in the latter. The authors of Ref. [19] also provide relativistic corrections

to these currents, but a detailed comparison with those terms is less straightforward. In addition to the mesonic and seagull two-body currents (called exchange and contact currents, respectively, in Ref. [19]), they include Δ -isobar currents and the so-called dissociation currents associated with the $\gamma\pi\rho$ and $\gamma\pi\omega$ vertices. Such mechanisms are introduced there in a nonminimal way and are absent from our primary Lagrangian. We plan to consider such extensions of the considered field model in future research.

It is also important to emphasize that the currents (99) and (100) are purely isovector, whereas our MECs contain both isovector and isoscalar contributions. For the pseudoscalar-meson exchange, the isoscalar currents are suppressed in the nonrelativistic limit $\langle 1', 2' | \mathbf{J}_{\pi NN}^s | 1, 2 \rangle \approx 0$, while the scalar- and vector-meson exchange generate nonvanishing isoscalar currents. From Eq. (B14) we get

$$\langle 1', 2' | \mathbf{J}_{\sigma NN}^{is} | 1, 2 \rangle \approx \frac{eg_\sigma^2}{4m^2(2\pi)^6} (1 + \tau_1^z) \frac{f_{\sigma NN}^2(\mathbf{k}_2^2)}{\mathbf{k}_2^2 + m_\sigma^2} \{(\mathbf{p}' + \mathbf{p}_1) - i[\boldsymbol{\sigma}_1 \times (\mathbf{k}_1 + \mathbf{k}_2)]\} + (1 \leftrightarrow 2). \quad (104)$$

The corresponding result for the vector boson (ω meson) exchange presents as

$$\langle 1', 2' | \mathbf{J}_{\omega NN}^{is} | 1, 2 \rangle \approx \frac{eg_\omega^2}{4m^2(2\pi)^6} (1 + \tau_1^z) \frac{f_{\omega NN}^2(\mathbf{k}_2^2)}{\mathbf{k}_2^2 + m_\omega^2} \left\{ (\mathbf{p}' + \mathbf{p}_2) - \left(1 + \frac{f_\omega}{g_\omega}\right) i[(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \times \mathbf{k}_2] \right\} + (1 \leftrightarrow 2). \quad (105)$$

Here we have also assumed $4m^2 \gg m_{\sigma,\omega}^2$. For the mesons with isospin $T = 1$ (δ , $\sigma(1)$ and ρ), one should replace $1 + \tau_1^z \mapsto \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \tau_2^z$. The expressions (104) and (105) have much in common with the isoscalar currents derived in Refs. [68,69]. For example, compare them with exchange currents in Eqs. (A.1a) and (A.1b) of Ref. [68]:

$$\mathbf{J}_{\sigma NN}^{is} = \frac{eg_\sigma^2}{4m^2} (1 + \tau_1^z) \frac{1}{\mathbf{k}_2^2 + m_\sigma^2} \{(\mathbf{p}' + \mathbf{p}_1) - i[\boldsymbol{\sigma}_1 \times (\mathbf{k}_1 + \mathbf{k}_2)]\} + (1 \leftrightarrow 2), \quad (106)$$

$$\mathbf{J}_{\omega NN}^{is} = \frac{eg_\omega^2}{4m^2} (1 + \tau_1^z) \frac{1}{\mathbf{k}_2^2 + m_\omega^2} \left\{ (\mathbf{p}' + \mathbf{p}_2) - \left(1 + \frac{f_\omega}{g_\omega}\right) i[(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \times \mathbf{k}_2] \right\} + (1 \leftrightarrow 2). \quad (107)$$

Once again, these expressions coincide with Eqs. (104) and (105), up to an overall normalization factor, in the limit $\Lambda_\pi \rightarrow \infty$.

To summarize, the main features of our MECs are:

- (1) They are consistent with the nuclear Hamiltonian in the CPR.
- (2) Instead of a common procedure of extracting MECs from the Feynman diagrams and extrapolating them to the off-shell region, we derive our MECs just from the given conserved nucleon and meson Noether currents.
- (3) The MEC operators do not depend on the choice of states between which we calculate the amplitude that considerably expands the range of applications.
- (4) They contain both isovector and isoscalar contributions. The latter are often omitted in other approaches, although they are essential, for example, in elastic ed scattering, where isovector currents do not contribute.
- (5) Our approach produces relativistic expressions for the MECs, without any nonrelativistic reduction. This allows us to use them in a broad range of energies and momentum transfers.

F. Details of computations

Using the commutation relations (B5) and the antisymmetric property of the deuteron WF, we get

$$\begin{aligned} \mathcal{F}^\mu(\mathbf{p}_0, \mathbf{q}) = & 2 \sum \int d\mathbf{p}' d\mathbf{p}'' \Psi_{\mathbf{p}_{SM_S}}^{(-)*}(\mathbf{p}' \bar{\mu}_1 \bar{\mu}_2) D_{\bar{\mu}_1 \mu'_1}^{[1/2]*}(W(L, p')) D_{\bar{\mu}_2 \mu'_2}^{[1/2]*}(W(L, p'_-)) \\ & \times \{J^\mu(L\mathbf{p}' \mu'_1, L\mathbf{p}' \mu'_2, \mathbf{p}'' \mu_1, -\mathbf{p}'' \mu_2) + J^\mu(L\mathbf{p}' \mu'_2, L\mathbf{p}' \mu'_1, -\mathbf{p}'' \mu_2, \mathbf{p}'' \mu_1)\} \Psi_{M_d}(\mathbf{p}'' \mu_1 \mu_2), \end{aligned} \quad (108)$$

where the C-number function $J^\mu(\mathbf{p}' \mu'_1, \mathbf{p}' \mu'_2; \mathbf{p}_1 \mu_1, \mathbf{p}_2 \mu_2)$ consists of the one-body and two-body terms

$$J^\mu(\mathbf{p}' \mu'_1, \mathbf{p}' \mu'_2; \mathbf{p}_1 \mu_1, \mathbf{p}_2 \mu_2) = J_{[1]}^\mu(\mathbf{p}' \mu'_1, \mathbf{p}' \mu'_2; \mathbf{p}_1 \mu_1, \mathbf{p}_2 \mu_2) + J_{[2]}^\mu(\mathbf{p}' \mu'_1, \mathbf{p}' \mu'_2; \mathbf{p}_1 \mu_1, \mathbf{p}_2 \mu_2), \quad (109)$$

with

$$J_{[1]}^\mu(\mathbf{p}'_1\mu'_1, \mathbf{p}'_2\mu'_2; \mathbf{p}_1\mu_1, \mathbf{p}_2\mu_2) = \delta(\mathbf{p}'_2 - \mathbf{p}_2)\delta_{\mu'_2\mu_2}F^\mu(\mathbf{p}'_1\mu'_1, \mathbf{p}_1\mu_1), \quad (110)$$

$$J_{[2]}^\mu(\mathbf{p}'_1\mu'_1, \mathbf{p}'_2\mu'_2; \mathbf{p}_1\mu_1, \mathbf{p}_2\mu_2) = -F_{\text{MEC}}^\mu(\mathbf{p}'_1\mu'_1, \mathbf{p}'_2\mu'_2; \mathbf{p}_1\mu_1, \mathbf{p}_2\mu_2). \quad (111)$$

Recall that in Eq. (108), $\mathbf{p}$ is the np -pair relative momentum in the rest frame that is given by relation (63).

Further, it would be helpful to show the spin dependence of the quantities (109). Relying upon previous experience [27] it can be done as follows:

$$\begin{aligned} J^\mu(Lp'\mu'_1, Lp'_-\mu'_2, \mathbf{p}''\mu_1, -\mathbf{p}''\mu_2) &\equiv \langle \mu'_1\mu'_2 | J(\mathbf{p}', \mathbf{p}'', \mathbf{q}) | \mu_1\mu_2 \rangle \\ &= \sum_{k_1k_2, k\kappa} \langle \mu'_1\mu'_2 | \{\sigma_{k_1}(1) \otimes \sigma_{k_2}(2)\}_{k\kappa} | \mu_1\mu_2 \rangle J_{k\kappa}^{k_1k_2, \mu}(\mathbf{p}', \mathbf{p}'', \mathbf{q}), \end{aligned} \quad (112)$$

with the irreducible tensor product $\{\sigma_{k_1} \otimes \sigma_{k_2}\}_k$ of rank k and the spin-tensors $\sigma_{00} = \mathbb{I}$, $\sigma_{1\kappa} = (\sigma)_\kappa$. Henceforth, $(\mathbf{a})_{k=0, \pm 1} = a_{0, \pm 1}$ denotes the cyclic components for any three-vector $\mathbf{a}$:

$$a_{\pm 1} = \mp \frac{\sqrt{2}}{2}(a_x \pm ia_y), \quad a_0 = a_z, \quad a^\kappa = (-1)^\kappa a_{-\kappa}. \quad (113)$$

The components of the spatial tensor $J_{k\kappa}^{k_1k_2, \mu}$ can be easily found by taking the traces

$$\begin{aligned} J_{00}^{00, \mu}(\mathbf{p}', \mathbf{p}'', \mathbf{q}) &= \frac{1}{4} \text{Tr}(J(\mathbf{p}', \mathbf{p}'', \mathbf{q})), \\ J_{1\kappa}^{10, \mu}(\mathbf{p}', \mathbf{p}'', \mathbf{q}) &= \frac{1}{4} \text{Tr}(J(\mathbf{p}', \mathbf{p}'', \mathbf{q})\sigma^\kappa(1)), \\ J_{1\kappa}^{01, \mu}(\mathbf{p}', \mathbf{p}'', \mathbf{q}) &= \frac{1}{4} \text{Tr}(J(\mathbf{p}', \mathbf{p}'', \mathbf{q})\sigma^\kappa(2)), \\ J_{1\kappa}^{11, \mu}(\mathbf{p}', \mathbf{p}'', \mathbf{q}) &= \frac{1}{4} \text{Tr} \left(J(\mathbf{p}', \mathbf{p}'', \mathbf{q}) \sum_{\kappa_1\kappa_2} \sigma^{\kappa_1}(1)\sigma^{\kappa_2}(2)(1\kappa_1\ 1\kappa_2 | k\kappa) \right), \end{aligned} \quad (114)$$

where $\text{Tr}(O) \equiv \sum_{\mu_1\mu_2} \langle \mu_1\mu_2 | O | \mu_1\mu_2 \rangle$.

Substituting Eq. (112) into Eq. (108) we obtain the six-dimensional overlapping integrals

$$S_{ll'm'LM_L}^\alpha(\mathbf{p}_0, \mathbf{q}) = \int d\mathbf{p}' d\mathbf{p}'' \psi_{pl'l'}^\alpha(p') Y_{l'm'}^*(\hat{\mathbf{p}}') J(\mathbf{p}', \mathbf{p}'', \mathbf{q}) u_L(p'') Y_{LM_L}(\hat{\mathbf{p}}''), \quad (115)$$

so the one-body current contributes via the 3D integrals

$$S_{ll'm'LM_L}^{[1]\alpha}(\mathbf{p}_0, \mathbf{q}) = \int d\mathbf{p}' \psi_{pl'l'}^\alpha(p') Y_{l'm'}^*(\hat{\mathbf{p}}') J\left(\mathbf{p}' - \frac{1}{2}\mathbf{q}, \mathbf{q}\right) u_L\left(\left|\mathbf{p}' - \frac{1}{2}\mathbf{q}\right|\right) Y_{LM_L}\left(\widehat{\mathbf{p}' - \frac{1}{2}\mathbf{q}}\right). \quad (116)$$

Their measure is decreasing owing to the delta functions in the formula (83), while the remaining angular integrals in Eq. (116) can be expressed through the quantities

$$I_{\lambda m, LM}(p', \mathbf{q}) = \int d\hat{\mathbf{p}}' Y_{\lambda m}(\hat{\mathbf{p}}') Y_{LM}\left(\widehat{\mathbf{p}' - \frac{1}{2}\mathbf{q}}\right) u_L\left(\left|\mathbf{p}' - \frac{1}{2}\mathbf{q}\right|\right). \quad (117)$$

In its turn, the Wigner-Eckart theorem allows us to express them through the reduced quantities $a_L^{\lambda\lambda'}(p', q)$,

$$I_{\lambda m, LM}(p', \mathbf{q}) = \sum_{\lambda'} \sqrt{\frac{(2L+1)(2\lambda+1)}{4\pi(2\lambda'+1)}} (\lambda m LM | \lambda' m + M) (\lambda 0 L 0 | \lambda' 0) Y_{\lambda' m + M}(\hat{\mathbf{q}}) a_L^{\lambda\lambda'}(p', q) \quad (118)$$

(see details in Ref. [27]). That gives us analytic results for the integrals (116) for a given deuteron WFs model.

We could reduce the integrals (115) to the two-dimensional ones via the same technique, but here we prefer to proceed more straightforwardly by addressing the Korobov integration method [70]. This efficient method was successfully applied in studying the two-body ^3He breakup by linearly polarized photons [11].

G. Toward gauge-independent description

Following a remarkable research [71], the properties of gauge invariance for a quantum-mechanical system that interacts with an e.m. field can be formulated by considering the Schrödinger equation

$$i \frac{\partial \Psi}{\partial t} = H\{A_\mu\} \Psi \quad (119)$$

and the gradient displacement (the gauge transformation after Weyl [72])

$$A_\mu(t, \mathbf{x}) \rightarrow A'_\mu(t, \mathbf{x}) = A_\mu(t, \mathbf{x}) + \partial_\mu G(t, \mathbf{x}) \quad (120)$$

of the e.m. potential $A^\mu(t, \mathbf{x}) = (A^0(t, \mathbf{x}), \mathbf{A}(t, \mathbf{x}))$ at the space-time point $x = (t, \mathbf{x})$ with an arbitrary function $G(x)$. This point is sufficiently delicate [71]. In particular, one has to consider that the function $G(x)$ has well-defined limits at $t = \pm\infty$.

According to the fundamental idea by Fock [73], the theory is gauge invariant if there exists a unitary transformation $\Psi \rightarrow \Psi' = U\Psi$ such that Eq. (119) remains unchanged in its form, viz.,

$$i\frac{\partial\Psi'}{\partial t} = H\{A'_\mu\}\Psi'. \quad (121)$$

This physical requirement is equivalent to the constraint

$$H\{A'_\mu\} = UH\{A_\mu\}U^\dagger + i\frac{\partial U}{\partial t}U^\dagger = UH\{A_\mu\}U^\dagger - iU\frac{\partial U^\dagger}{\partial t} \quad (122)$$

imposed upon the Hamiltonian $H\{A_\mu\}$ of the system in the e.m. field. Relations (121) and (122) give a constructive way for the description of e.m. interactions of quantum systems. We allow ourselves to call such a formulation the Fock-Weyl criterion (see also the surveys [74,75] addressing the historical roots of the gauge invariance principle).

Let us show how this criterion works for the reaction amplitudes of interest. Using Eq. (22) the amplitude of the process $e + i \rightarrow e' + f$ within the OPEA, is expressed as

$$T_{if} = \varepsilon_\mu(e', e)\langle f; \mathbf{P}_f | J_h^\mu(0) | i; \mathbf{P}_i \rangle, \quad (123)$$

with the virtual photon polarization vector $\varepsilon_\mu(e', e) = \bar{u}_e(k')\gamma_\mu u_e(k)$. The initial $|h\rangle = |i; \mathbf{P}_i\rangle$ and final $|h'\rangle = |f; \mathbf{P}_f\rangle$ hadronic states are the eigenvectors of the operator $P^\mu = (H_h, \mathbf{P})$

$$\begin{aligned} H_h|i; \mathbf{P}_i\rangle &= E_i|i; \mathbf{P}_i\rangle, & \mathbf{P}|i; \mathbf{P}_i\rangle &= \mathbf{P}_i|i; \mathbf{P}_i\rangle, \\ H_h|f; \mathbf{P}_f\rangle &= E_f|f; \mathbf{P}_f\rangle, & \mathbf{P}|f; \mathbf{P}_f\rangle &= \mathbf{P}_f|f; \mathbf{P}_f\rangle, \end{aligned} \quad (124)$$

where H_h is a model Hamiltonian for the system of interacting meson and nucleons.

The current density operator $J_h^\mu(\mathbf{x}) \equiv J^\mu(t=0, \mathbf{x})$ meets the continuity equation (CE)

$$i \operatorname{div} \mathbf{J}(\mathbf{x}) = [H_h, \rho_h(\mathbf{x})] \quad (125)$$

or in more compact form

$$[P_\mu, J^\mu(0)] = 0. \quad (126)$$

It is the e^1 -order consequence of the gauge invariance principle that has been formulated above (for details see Ref. [76]). Of great interest is the GI condition (not to be confused with the gauge invariance principle)

$$q_\mu \langle f; \mathbf{P}_f + \mathbf{q} | J^\mu(0) | i; \mathbf{P}_i \rangle = 0 \quad (127)$$

that follows directly from the CE being sandwiched between the exact P^μ eigenstates.

It becomes clear that the amplitude (123) is GI being equal to zero after the replacement $\varepsilon_\mu(e', e) \rightarrow q_\mu$, only if, first, Eq. (126) fulfills and, second, the states $|f\rangle, |i\rangle$ are the exact H_h eigenstates. The latter should be kept in mind in

practical calculations, when one has to handle approximate states of a hadronic system. In this connection, let us stress that the explicitly gauge-independent approach described in the remainder of this section is not employed in the numerical calculations of Sec. IV yet. Since all the components of the current operator used there are generated by one and the same unitary clothing transformation, the residual violation of the condition (127) due to the adopted approximations is of higher order in the coupling constants than the retained contributions.

In our consideration, to provide the GI of calculations, we would like to make use of the extension of the Siegert theorem (cf. Ref. [77]) proposed in Refs. [78,79]. This way allows us to express the amplitude of interest in explicitly GI way through the Fourier transforms of electric $[\mathbf{E}(\mathbf{q})]$ and magnetic $[\mathbf{H}(\mathbf{q})]$ field strengths

$$T_{if} = \mathbf{E}(\mathbf{q})\mathbf{D}_{if}(\mathbf{q}) + \mathbf{H}(\mathbf{q})\mathbf{M}_{if}(\mathbf{q}),$$

$$\mathbf{E}(\mathbf{q}) = i(\omega\boldsymbol{\varepsilon}(\mathbf{q}) - \mathbf{q}\varepsilon_0(\mathbf{q})), \quad \mathbf{H}(\mathbf{q}) = i\mathbf{q} \times \boldsymbol{\varepsilon}(\mathbf{q}), \quad (128)$$

with $\mathbf{D}_{if}(\mathbf{q})$ and $\mathbf{M}_{if}(\mathbf{q})$ being matrix elements of generalized electric and magnetic dipole moments of the hadronic system containing the information on the nuclear dynamics

$$\begin{aligned} \mathbf{D}_{if}(\mathbf{q}) &= i\omega^{-1} \int_0^1 d\lambda \nabla_{\lambda\mathbf{q}} [(E_f(\lambda\mathbf{q}) - E_i) \\ &\quad \times \langle f; \mathbf{P}_i + \lambda\mathbf{q} | \rho(0) | i; \mathbf{P}_i \rangle], \end{aligned} \quad (129)$$

$$\mathbf{M}_{if}(\mathbf{q}) = i \int_0^1 \lambda d\lambda [\nabla_{\lambda\mathbf{q}} \times \langle f; \mathbf{P}_i + \lambda\mathbf{q} | \mathbf{J}(0) | i; \mathbf{P}_i \rangle]. \quad (130)$$

These equations have been derived using the GI condition (126), the property of translational invariance

$$J^\mu(\mathbf{x}) = e^{-i\mathbf{x}\cdot\mathbf{P}} J^\mu(0) e^{i\mathbf{x}\cdot\mathbf{P}} \quad (131)$$

and the formula [80]

$$\mathbf{a}e^{i\mathbf{b}\cdot\mathbf{c}} = \int_0^1 d\lambda (\nabla_{\mathbf{c}}((\mathbf{a}\cdot\mathbf{c})e^{i\lambda\mathbf{b}\cdot\mathbf{c}}) + i\lambda[\mathbf{c} \times [\mathbf{a} \times \mathbf{b}]]e^{i\lambda\mathbf{b}\cdot\mathbf{c}}), \quad (132)$$

with arbitrary vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$.

Within this approach the matrix element (4) now expressed in terms of $\mathbf{D}_{if}(\mathbf{q})$ and $\mathbf{M}_{if}(\mathbf{q})$,

$$\mathcal{F}_{if}^0 = -i\mathbf{q} \cdot \mathbf{D}_{if}(\mathbf{q}), \quad \mathcal{F}_{if} = -i\omega\mathbf{D} + i\mathbf{q} \times \mathbf{M}_{if}(\mathbf{q}), \quad (133)$$

and satisfy the continuity equation (8) by construction. From the practical point of view, Eq. (133) gives an opportunity to study separately the influence of the electric and magnetic components on the respective process. In our opinion, this is an attractive feature of such approach.

Up to now, we have not paid attention to another important property of the current density operator, viz., to be four-vector

$$e^{i\beta\cdot\mathbf{B}} J^\mu(x) e^{-i\beta\cdot\mathbf{B}} = J^\nu(Lx) L_\nu^\mu. \quad (134)$$

In its turn, the latter leads to the so-called local analog of the Siegert theorem

$$i\mathbf{J}(x=0) = [\mathbf{B}, J^0(0)], \quad (135)$$

that we encounter in Ref. [81] instead of Siegert's relation

$$\mathbf{J}(\mathbf{q}=0) = i[\mathcal{H}, \mathcal{D}] \quad (136)$$

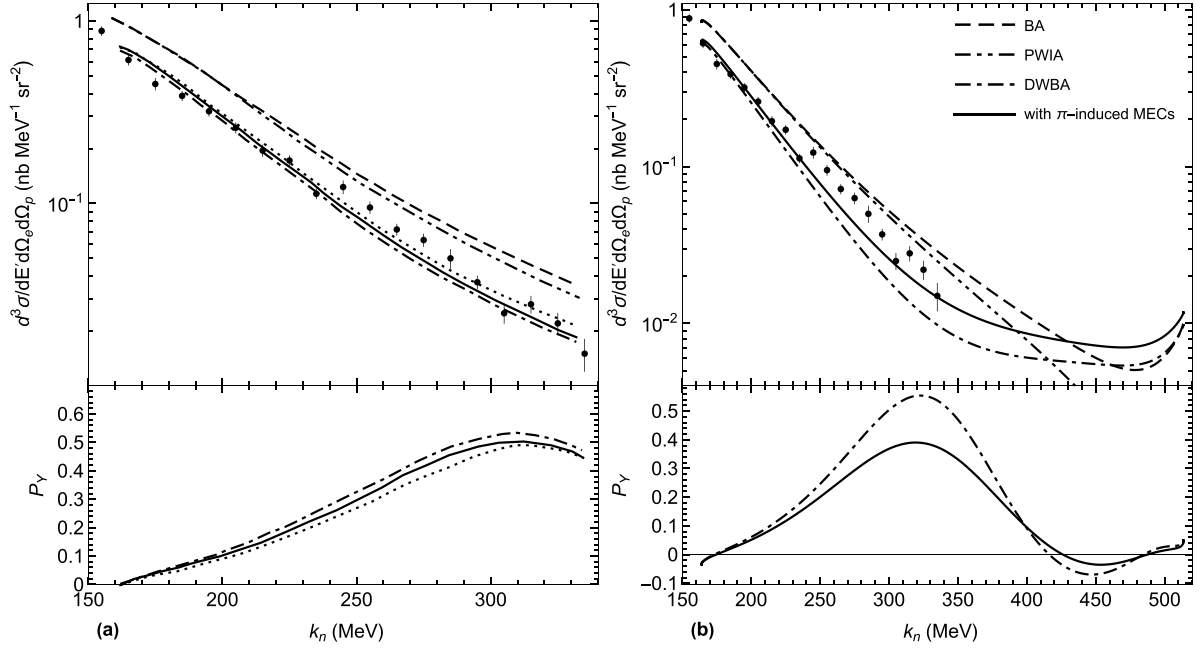


FIG. 5. Differential cross section (upper part) and induced polarization (lower part) of outgoing protons versus the neutron momentum for the Saclay kinematics [4]. The azimuth angle (reaction plane angle) is fixed $\varphi = 180^\circ$. (a) The results of Ref. [27] obtained within the nonrelativistic approach with the Paris potential and pion-induced MECs by Maize and Kim [67]. Here the following calculations are compared: PWIA (dash-double dotted curve), BA (dashed curve), DWBA (dash-dotted curve) and with inclusion of FSI effects and the MECs with $\Lambda = 4m_\pi$ ($\Lambda = \infty$) (solid (dotted) curve). (b) Calculations made within the formalism of the present paper. Here, the solid curve corresponds to the calculation with the deuteron and np -pair WFs obtained within the CPR, as well as, pion-induced MECs.

for integral quantities

$$\begin{aligned} \mathbf{J}(\mathbf{q}) &\equiv \int \mathbf{J}(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}} d\mathbf{x}, \\ \mathcal{D} &\equiv \int \mathbf{x} J^0(\mathbf{x}) d\mathbf{x}, \\ \mathcal{H} &\equiv \int H(\mathbf{x}) d\mathbf{x}. \end{aligned} \quad (137)$$

This result justifies the title of the article [81] and allows us to restore the spatial part of the current $\mathbf{J}(0)$ from the time component $J^0(0)$. In this context, the recipe for constructing the boost operators $\mathbf{B}$ in the CPR has been given in Ref. [50].

The numerical implementation of such explicitly GI and covariant calculations is underway.

IV. RESULTS AND DISCUSSION

This research extends the results [27,85] obtained in the 1990s. Therefore, let us compare our new calculations with those pioneering experimental data and theoretical predictions (see Figs. 5 and 6). In both calculations, the nucleon form factors are taken from Ref. [84]. The FSI effects were included by computing n - p partial-wave scattering states with $J \leq J_{\max} = 3$ while in Ref. [27] $J_{\max} = 2$. The authors of Ref. [27] used the one-body current of Ref. [86] and pion-induced MECs by Maize and Kim [67] with the Paris potential and deuteron WF [83]. Despite the nonrelativistic nature of these ingredients, qualitatively these results have much in common with the relativistic calculations of this paper.

The experimental data used for comparison are taken from Refs. [4,5], corresponding to the kinematical ranges $155 \text{ MeV} \leq k_n \leq 335 \text{ MeV}$ with $Q^2 = 0.10 \text{ GeV}^2$ and Bjorken variable $x_B = Q^2/2m\omega = 0.36$, and $294 \text{ MeV} \leq k_n \leq 500 \text{ MeV}$ with $Q^2 = 0.038 \text{ GeV}^2$ and $x_B = 0.10$, respectively. As the neutron momentum increases, the Born approximation (BA) and plane-wave impulse approximation (PWIA) curves diverge because the recoil mechanism becomes more important. For these kinematics, including FSI generally reduces the cross section over most of the neutron-momentum range. However, in our new calculations with an extended momentum range, we find that this trend does not hold at the highest momenta: in the far-right region, the distorted-wave Born approximation (DWBA) cross section exceeds the BA result. This behavior has been discussed extensively (see, e.g., Refs. [25,87]) and is related to the underlying structure of the respective amplitudes. In this kinematic region, the BA amplitude is dominated by the PWIA, which is directly proportional to the deuteron wave function evaluated at the specific value of the missing momentum, which drops off sharply as k_n increases. In contrast, the FSI contribution involves an integral over momentum space. This integral is dominated by contributions from lower internal momenta where the wave function is significantly larger, thereby sustaining the cross section at higher values of k_n .

It is important to note that even in these kinematics, relativistic effects can be significant. This becomes evident already at the level of the one-body current. In Figs. 5 and 6 we see that the nonrelativistic curves with only one-body current contribution are higher than the respective relativistic ones.

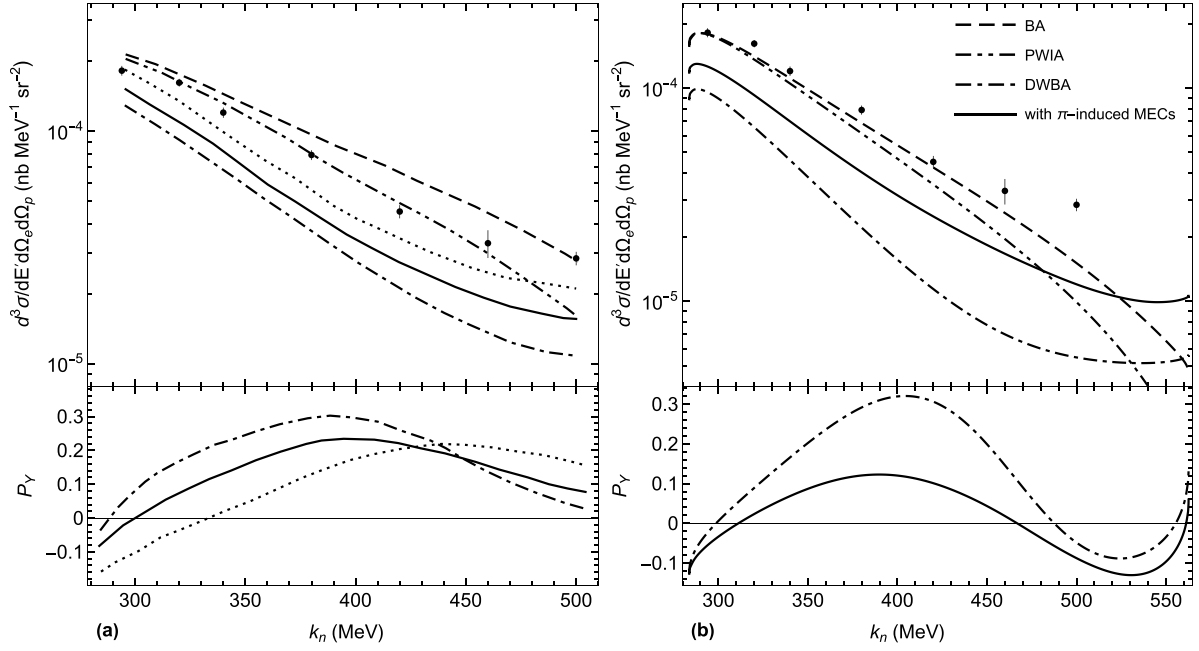


FIG. 6. Same as in Fig. 5 but for the Saclay kinematics [5].

That is demonstrated clearly in Fig. 7(a), where three BA calculations with different one-body currents are shown: the relativistic current (91) (purple solid curve), its nonrelativistic reduction

$$F^0(\mathbf{p}'\mu', \mathbf{p}\mu) = F_1[(\mathbf{p}' - \mathbf{p})^2]\delta_{\mu'\mu},$$

$$\mathbf{F}(\mathbf{p}'\mu', \mathbf{p}\mu) = F_1[(\mathbf{p}' - \mathbf{p})^2]\frac{\mathbf{p}' + \mathbf{p}}{2m}\delta_{\mu'\mu} + (F_1[(\mathbf{p}' - \mathbf{p})^2] + F_2[(\mathbf{p}' - \mathbf{p})^2])\frac{i\boldsymbol{\sigma}_{\mu'\mu} \times (\mathbf{p}' - \mathbf{p})}{2m} \quad (138)$$

(purple dashed curve), and the current by McVoy and Van Hove [86] (orange dashed curve). In Fig. 5(a) we see that the DWBA result is already in good agreement with the data and the contribution of the MECs is not of great importance, while the relativistic DWBA calculation in Fig. 5(b) underestimates the cross section for $k_n \gtrsim 200$ MeV. The role of MECs is more important in the relativistic calculation, and only after their inclusion a good agreement with the data is obtained. The kinematic regime of Fig. 6 lies far to the right of the quasifree peak (QFP) and therefore the role of FSI becomes even more significant. And again we see the

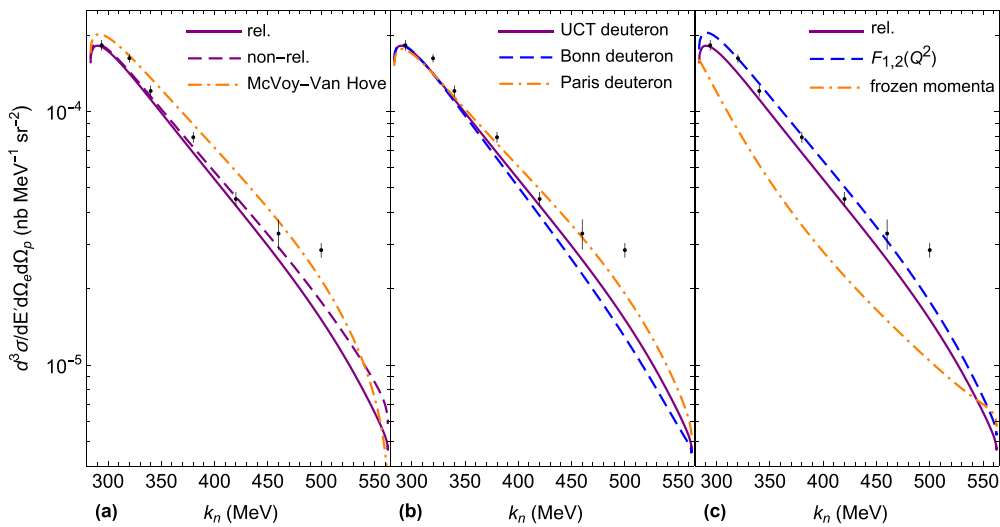


FIG. 7. Plots of different BA calculations of the differential cross section versus the missing momentum for the Saclay kinematics [5]. (a) Calculations with different one-body currents. We use the deuteron WF from Ref. [56] for all calculations here. (b) Calculations with relativistic one-body current but with different deuteron WFs. The purple solid curve—with UCT deuteron WF components [56], the blue dashed curve—with the Bonn deuteron WFs [82] and the orange dash-dotted one—with the Paris deuteron WFs [83]. (c) Demonstration of the role of the Fermi motion effects.

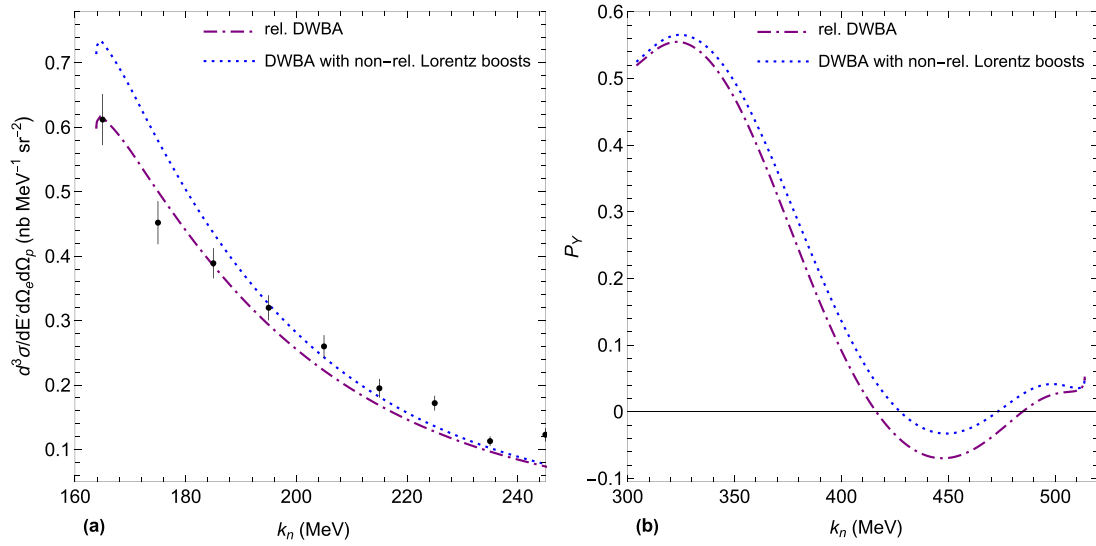


FIG. 8. Demonstration of the role of the Lorentz boosts in the deuteron electrodisintegration observables for the Saclay kinematics [4]. The dash-dotted curves corresponds to the relativistic DWBA calculation, while the dotted curves—with a nonrelativistic approximation to the Lorentz boosts. (a) Differential cross section, (b) induced polarization of knocked-out protons.

destructive interference between MEC and FSI contributions to the cross section that confirms the conclusion drawn in Ref. [88].

The cross section is also sensitive to the choice of the deuteron WF. Figure 7(b) demonstrates the role of the deuteron WF in the BA calculations with the relativistic

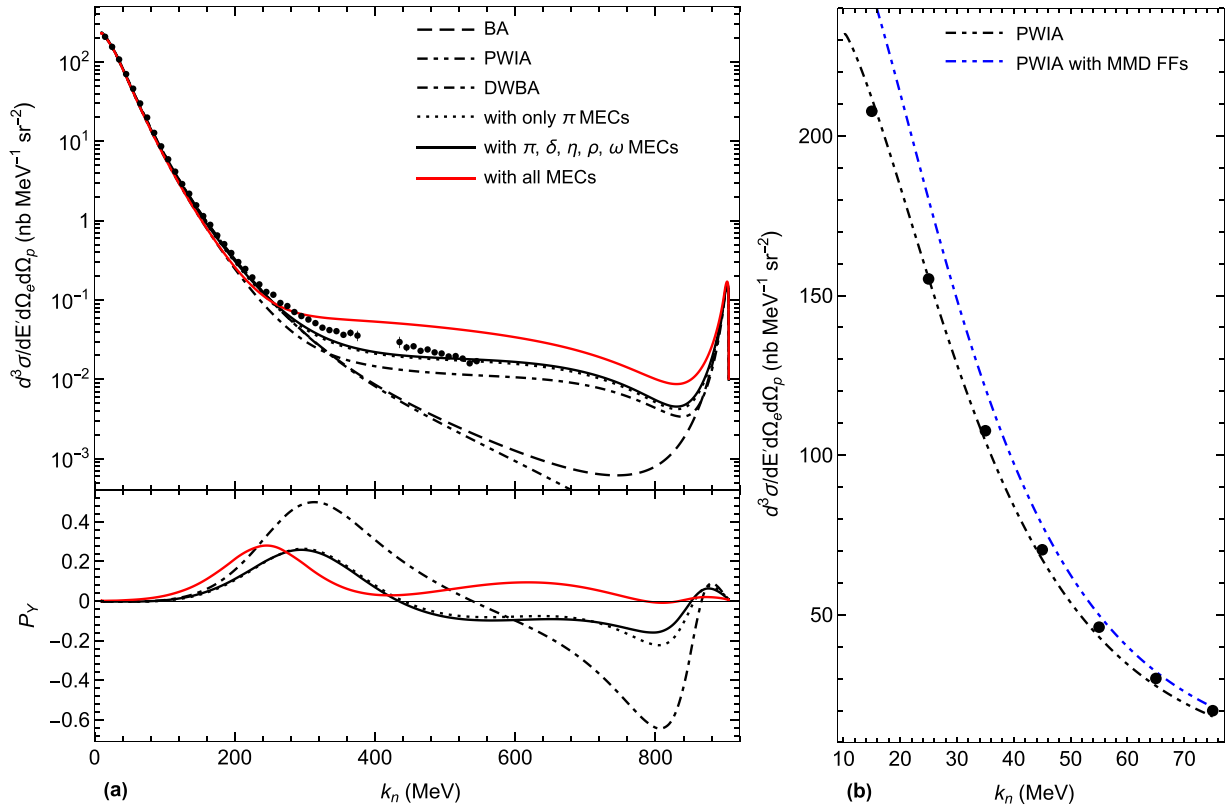


FIG. 9. (a) Differential cross section (upper part) and induced polarization (lower part) of knocked-out protons versus the neutron momentum for the kinematics of Ref. [12]. The following calculations without MECs are compared: PWIA (dash-double dotted curve), BA (dashed curve), DWBA (dash-dotted curve). As well as the calculations with inclusion of the FSI effects and the MECs: with the pion-induced MECs only (dotted), with MECs induced by the π , δ , η , ρ , ω mesons (solid), and with full set of MECs (red solid). (b) Linear plot of the differential cross section for the low missing momentum data. In this region, the cross section is well described by PWIA.

one-body current. For recoil momenta $k_n \lesssim 350$ MeV, these curves are almost indistinguishable, but for higher momenta they begin to differ significantly.

Now let us discuss the effects of Fermi motion, i.e., the relative motion of the nucleons in the nucleus. Note that, after substituting Eq. (95) into Eq. (108), the nucleon FFs depend not only on the transferred momentum q , but also on the relative nucleon momentum in the deuteron $\mathbf{p}$. This dependence is integrated out in the matrix elements (108) but as shown in Fig. 7(c) its effect is still present in the resulting calculations. Models in which the form factors depend only on Q^2 are widely used. The figure shows that the calculation with such “frozen” FFs (blue dotted curve) produces bigger cross section values for the entire range of recoil momenta. In such a model Fermi motion effects are still present, because the bispinors and the prefactor of F_2 in Eq. (95) depend on the relative momentum $\mathbf{p}$. If we “freeze” these dependencies as well, for example, putting $\mathbf{p}' = \mathbf{q}/2$ and $\mathbf{p} = -\mathbf{q}/2$ in Eq. (95) (orange dash-dotted curve) then the cross section drops by considerable amount for all k_n except for the lowest and highest values. This clearly demonstrates the importance of the Fermi motion effects in the deuteron electrodisintegration.

Other relativistic ingredients are the boosted momenta $L\mathbf{p}'$, $L\mathbf{p}'$ and D functions (Wigner rotations) that appear in the matrix elements (108). To demonstrate the role of these effects, we performed the following DWBA calculations of the cross section and induced polarization for the Saclay kinematics [4] (see Fig. 8): with all relativistic effects; with a nonrelativistic approximation to the Lorentz boosts, i.e., with the replacement $(Lp)^i \rightarrow (\mathbf{p} + \mathbf{q}/2)^i$; with the D functions replaced by the Kronecker deltas $D_{\bar{\mu}\mu}^{[1/2]}(W(L(q), p)) \rightarrow \delta_{\bar{\mu}\mu}$. It turns out that the role of the D functions is negligible for this and other kinematics we have considered (so we do not include the corresponding curve in Fig. 8), but the Lorentz boosts have a substantial impact. It is noteworthy that the influence of the latter is most noticeable at smaller values of recoil momentum ($k_n \lesssim 240$ MeV) for the cross section, but for the induced polarization, this influence is most pronounced at larger momenta ($k_n \gtrsim 300$ MeV). The relativistic treatment of the boosts is crucial for the correct description of the data in Fig. 8(a), and the nonrelativistic approximation leads to a significant overestimation of the cross section. The FSI and MEC contributions do not interfere destructively under all kinematic conditions. In Fig. 9 we present the results of calculations for the kinematic of Ref. [12] ($E = 3.11$ GeV, $E' = 2.74$ GeV, $\theta_e = 16.1^\circ$), where both FSI and MEC contributions lead to an increase in the cross section. This behavior was also observed for the Saclay kinematics in Figs. 5 and 6 but for the highest values of neutron momenta in both cases. Since the kinematic used in Fig. 9 allows much higher k_n up to 550 MeV this behavior becomes more obvious. Moreover, the contribution from the FSI effects becomes dominant in comparison with the plane-wave contribution for $400 \text{ MeV} \lesssim k_n \lesssim 800 \text{ MeV}$. Since the FSI contribution becomes so prominent, it is natural to examine the convergence of our calculations with respect to $J_{\max}$, the maximum total angular momentum retained in the partial-wave decomposition of the np scattering states (see Sec. III D). In Fig. 10 we compare the cross sections and

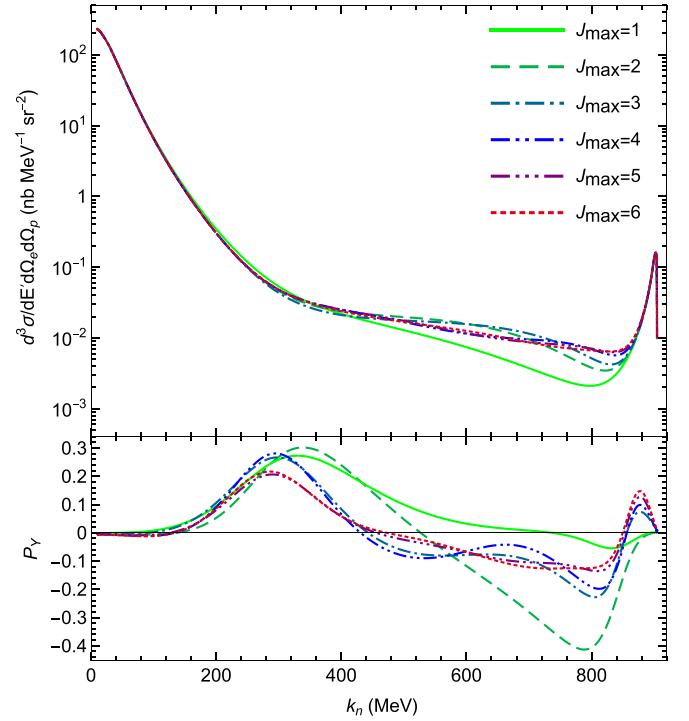


FIG. 10. Same calculations of the differential cross section (upper panel) and induced proton polarization (lower panel) as in Fig. 9(a) but with the np partial-wave scattering states restricted by different values of $J_{\max}$. All the curves are obtained with the inclusion of the FSI effects and the pion-induced MECs only.

induced polarizations evaluated as in Fig. 9(a), i.e., with the FSI effects and the pion-induced MECs included, for $J_{\max} = 1, \dots, 6$. For the cross sections, all the curves are practically indistinguishable for $k_n \lesssim 400$ MeV, where the plane-wave mechanism dominates. Beyond this region, only the $J_{\max} = 1$ curve deviates appreciably, while the others stay fairly close to each other. The induced polarization turns out to be more sensitive to the peripheral waves: the transition from $J_{\max} = 2$ to $J_{\max} = 3$ still changes P_γ considerably at $k_n \gtrsim 500$ MeV, whereas the difference between the $J_{\max} = 3$ and $J_{\max} = 4$ results is much smaller and shows up only at the highest recoil momenta. In fact, the full convergence of P_γ in this region is reached only around $J_{\max} = 5-6$, so this should be kept in mind when comparing with the data, if they are available. One should also keep in mind that the computations with larger $J_{\max}$ require a denser grid of integration nodes in the overlap integrals (115), becoming increasingly expensive. In this respect, the choice $J_{\max} = 3$ adopted throughout this paper provides a reasonable compromise between the accuracy and the computational cost. Calculations with MECs induced by the π , δ , η , ρ , ω mesons and only the π meson show only minor differences in Fig. 9(a). Contributions from different meson exchanges are not negligible, but they interfere destructively, so the final result with π , δ , η , ρ , ω meson exchanges could be well described with only pion-induced MECs. The contribution of the σ mesons is the largest among all MECs, and it leads to a significant increase of the cross section. This may appear unexpected because the role of the δ

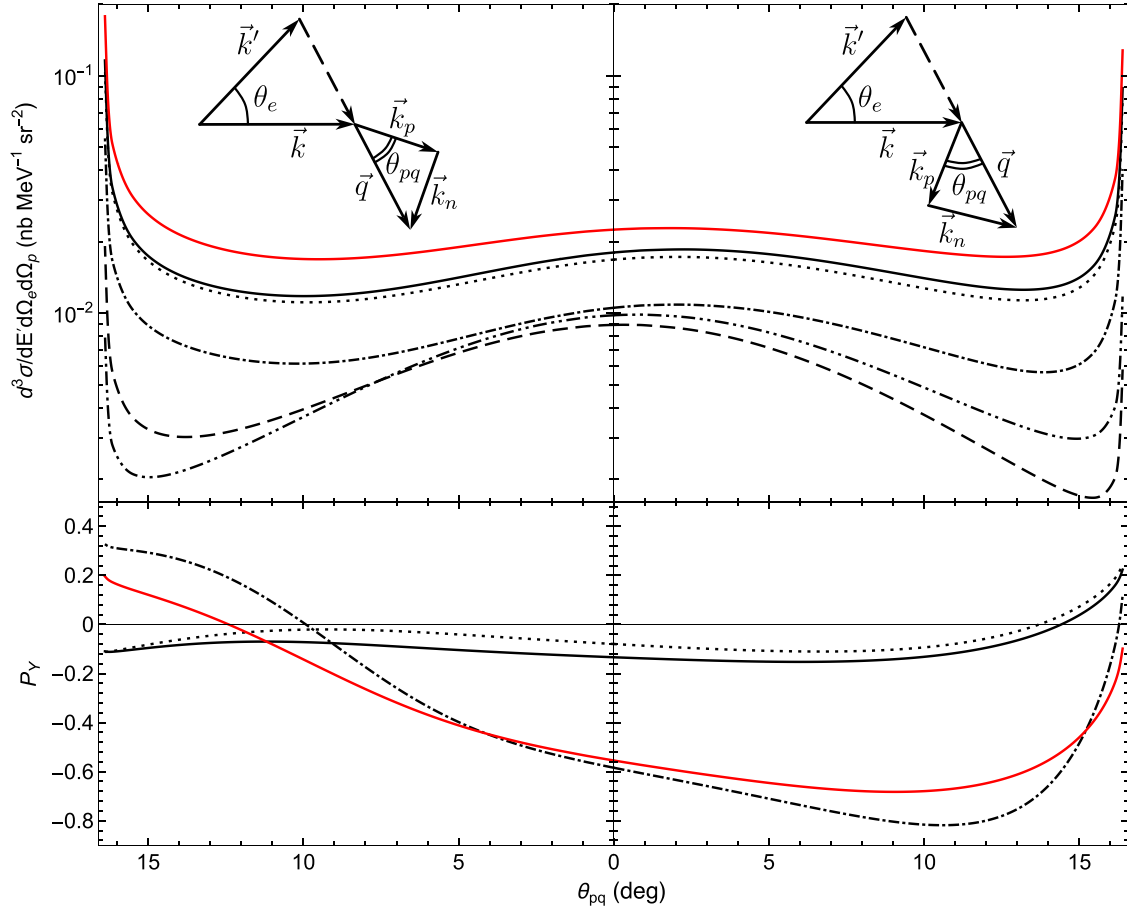


FIG. 11. Same as in Fig. 9 but versus the proton emission angle θ_{pq} for the kinematic conditions in the left wing of the QFP ($E = 1$ GeV, $E' = 827$ MeV, $\theta_e = 50^\circ$) from Ref. [27]. The left part of each plot corresponds to the azimuth angle (reaction plane angle) $\varphi = 0^\circ$ and the right part corresponds to $\varphi = 180^\circ$.

meson (which is also a scalar meson) is not that important, but the coupling constant for the σ ($T = 1$) meson (see UCT GS parameters from Table 1 in Ref. [63]) is much higher than for the δ meson ($g_\sigma^2/g_\delta^2 \approx 5.5$), so the contribution of the former is more significant. Recall that the introduction of sigma mesons aims to effectively account for two-pion exchanges. In this context, their role in electromagnetic processes with nuclei requires further investigation.

In general, our results are in good agreement with the experimental data. For values above $k_n \leq 335$ MeV MECs begin to play a significant role, the expected behavior at the modest Q^2 , where these experiments were performed. The cross-section calculations are quite sensitive to the corresponding two-body current model, for instance, the inclusion of sigma mesons leads to noticeable changes and overestimates the cross-section. An improved description can be achieved by introducing the Δ -isobar and dissociation currents. In this regard, the implicit inclusion of two-body currents via the generalized Siegert theorem from Sec. III G may prove useful. We will address these issues in our future works.

Figure 9(b) displays a linear plot for the low missing momentum data. In this region, the cross section is well described by PWIA. A reasonably accurate description of the

experimental data cannot be achieved using a nonrelativistic one-body current and only a fully relativistic form yields good agreement. Furthermore, the results are quite sensitive to the choice of form factor model. In addition to the calculations using form factors from Ref. [84], Fig. 9(b) includes results obtained with MMD form factors [89]. The latter tends to overestimate the cross section at low k_n , a result consistent with findings in Ref. [22] where these FFs were also employed. As k_n increases, both curves start getting closer to each other.

As seen in Figs. 11 and 12, the $e'p$ coincidence cross section and proton polarization exhibit azimuthal (left-right) asymmetry relative to the geometry with $\theta_{pq} = 0^\circ$ (parallel kinematics). This behavior is observed for both kinematic settings in the left (Fig. 11) and right (Fig. 12) wings of the QFP. As the momentum $\mathbf{k}_p$ lies on the right (left) of the momentum $\mathbf{q}$, the azimuthal angle φ equals $180^\circ(0^\circ)$. This asymmetry can be explained by using the transformation properties

$$R_z(180^\circ)J_{x,y}(\mathbf{q})R_z^{-1}(180^\circ) = -J_{x,y}(\mathbf{q}), \quad (139)$$

$$R_z(180^\circ)\sigma_y R_z^{-1}(180^\circ) = -\sigma_y, \quad (140)$$

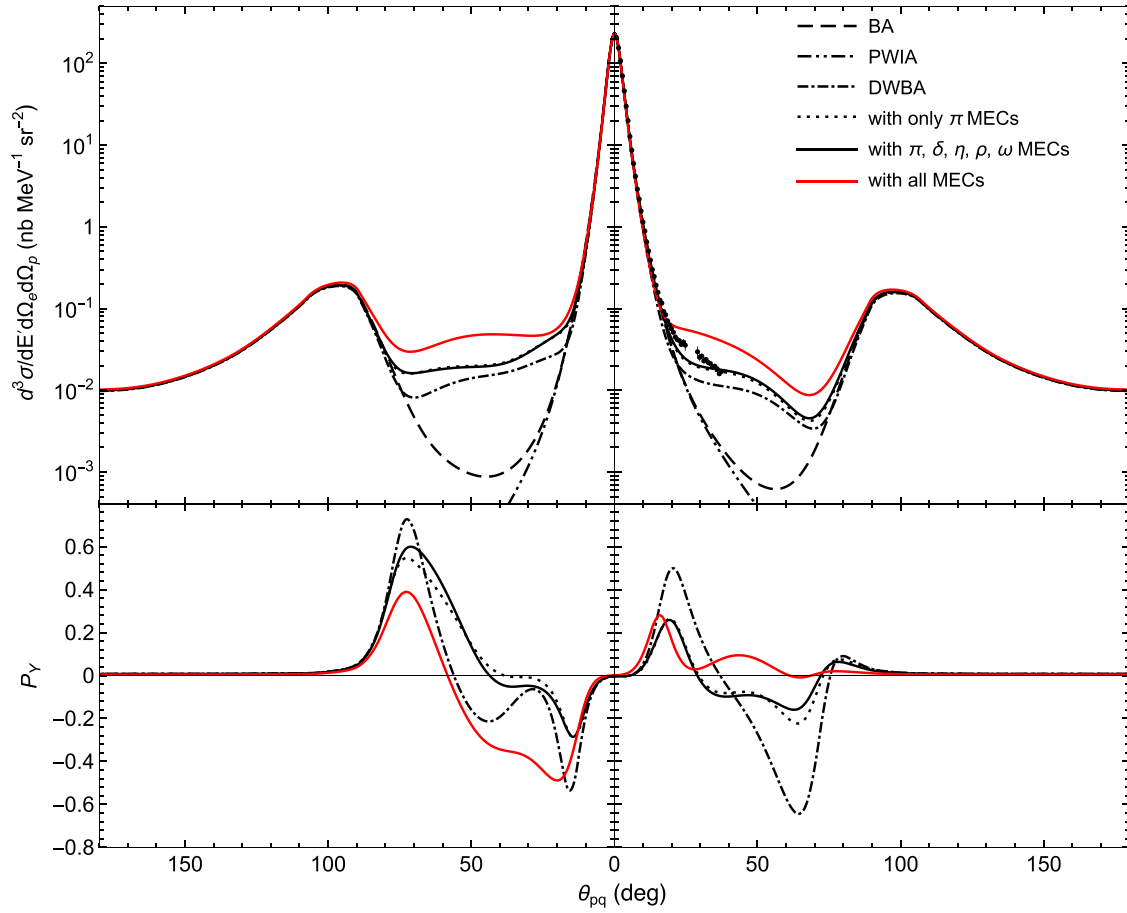


FIG. 12. Same as in Fig. 11 but for the kinematic conditions in the right wing of the QFP [12].

with respect to the frame rotation $R_z(180^\circ)$ by 180° about the vector $\hat{\mathbf{n}}_z$ ($\mathbf{q}$ direction). It follows from Eqs. (139) and (140) that for a given θ_{pq} value

$$W_{C,T,S}(\varphi = 0) = W_{C,T,S}(\varphi = \pi),$$

$$W_I(\varphi = 0) = -W_I(\varphi = \pi),$$

$$\Sigma_{C,T,S}(\varphi = 0) = -\Sigma_{C,T,S}(\varphi = \pi),$$

$$\Sigma_I(\varphi = 0) = \Sigma_I(\varphi = \pi). \quad (141)$$

The deuteron electrodisintegration on the left wing of the QFP [Figs. 11 and 13(a)] is characterized by a limiting proton

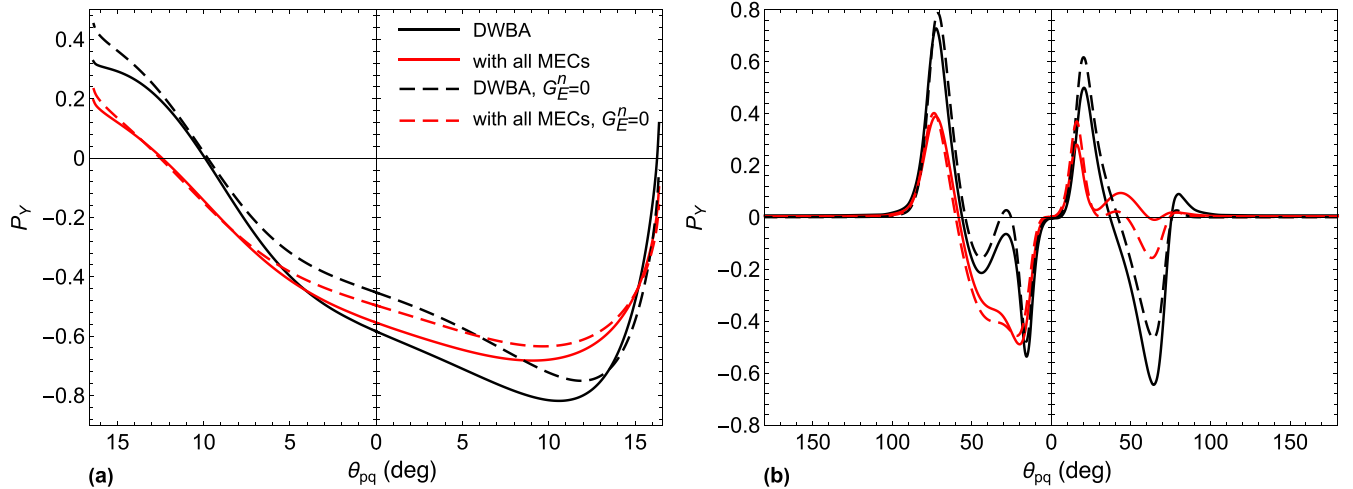


FIG. 13. Dependence of the induced proton polarization on the choice of the G_E^n model in the left wing of the QFP [27] (a) and in the right wing of the QFP [12] (b). The dashed curves were calculated with $G_E^n = 0$; solid curves with G_E^n from Ref. [84]. The black curves for DWBA results and the red ones for computations with all MECs contributions.

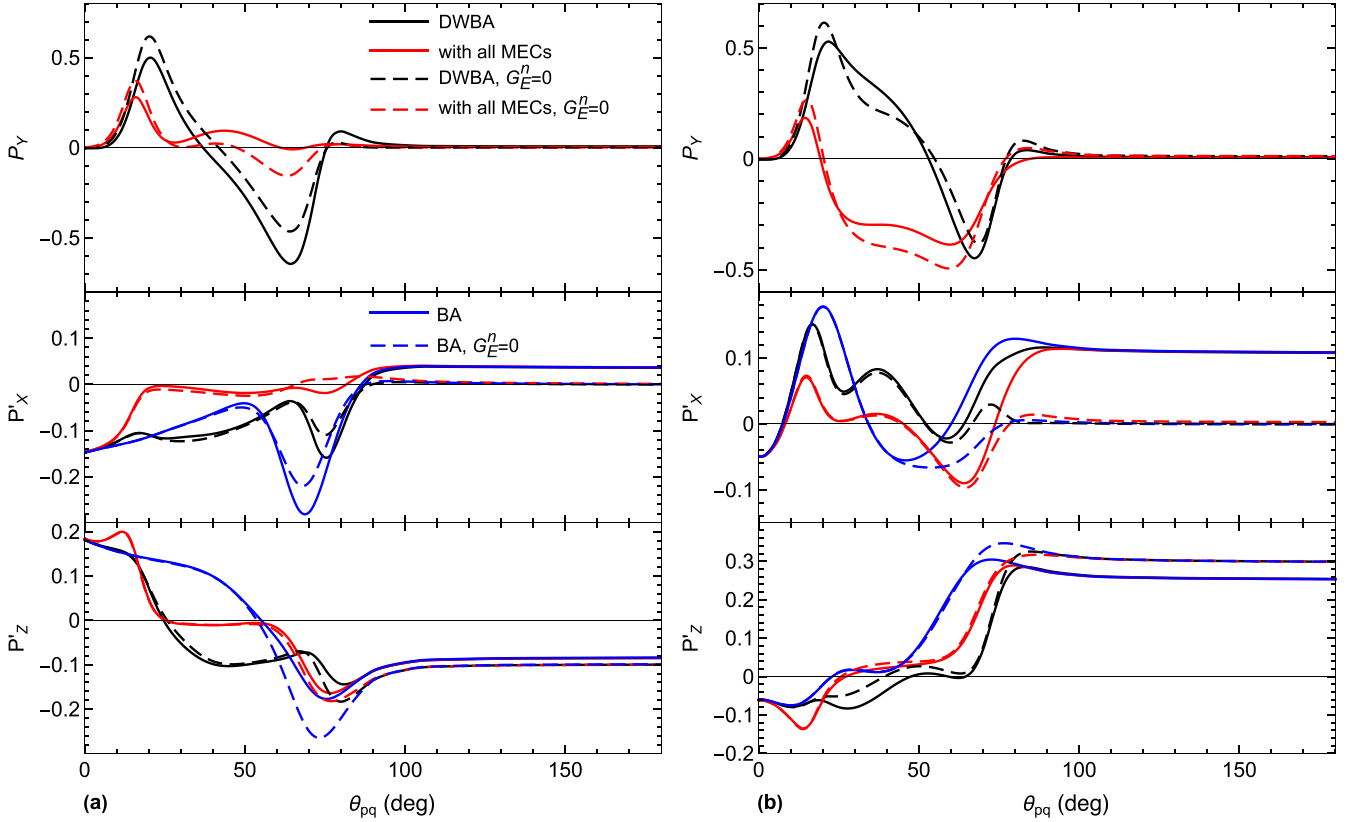


FIG. 14. Proton (a) and neutron (b) polarization components versus the proton emission angle θ_{pq} for the kinematics of Ref. [12]. The curves have the same meaning as in Fig. 13, with the addition of the blue ones representing the BA results. The BA curves are absent in the P_Y panels, since the induced polarization vanishes when the FSI is neglected.

emission angle in the laboratory frame,

$$\cos \theta_{\text{lim}} = \frac{1}{2qm_p} \sqrt{[\omega + m_d]^2 - [(\omega + m_d)^2 + m_p^2 - m_n^2 - \mathbf{q}^2]^2}. \quad (142)$$

As $\theta_{pq} \rightarrow \theta_{\text{lim}}$ (for the present case $\theta_{\text{lim}} \simeq 16.6^\circ$), the differential cross section increases sharply due to the kinematic factor R , which diverges in this limit. Nevertheless, this singular behavior is mild, and for any finite angular acceptance, the cross section remains finite.

As seen in Fig. 13 and noted in Ref. [27], the neutron electric form factor G_E^n plays a more pronounced role in the induced proton polarization on the left wing of the QFP than on the right wing. For parallel kinematics ($\theta_{pq} = 0^\circ$), the MEC contribution nearly vanishes when G_E^n is included, whereas setting $G_E^n = 0$ leads to a noticeable reduction of the polarization due to MEC effects. Thus, measurements of the induced polarization may help discriminate between different models of G_E^n , although one should keep in mind that P_Y in parallel kinematics remains sensitive to MEC contributions.

Measurements of the polarization transfer in $d(\bar{e}, e'\bar{p})n$ and $d(\bar{e}, e'\bar{n})p$ are a great source of information on the nucleon form factors, especially for the neutron. Figure 14 shows the results of calculations for the proton and neutron polarization components in the kinematics of Ref. [12]. In contrast to the left wing of the QFP, for parallel kinematics ($\theta_{pq} = 0^\circ$) in the right wing, MEC and FSI effects are negligible, as well

as the influence of G_E^n . However, for $\theta_{pq} \gtrsim 90^\circ$, the neutron electric form factor is of great importance for the polarization transfer (P'_X and P'_Z), while both MEC and FSI contributions remain insignificant so the BA is already sufficient to describe the observables in this region. It is the feature of the kinematic regimes near the QFP with the Bjorken variable $x_B \simeq 1$ ($x_B = 0.96$ in Ref. [12]) that makes them promising for the extraction of G_E^n from the polarization transfer measurements.

For the kinematics with $\mathbf{k}_n \parallel \mathbf{q}$ ($\theta_{pq} \simeq \theta_{\text{lim}}$) the following expressions can be derived in the BA under the assumption $u_{0,2}(q)/u_0(0) \ll 1$:

$$\begin{aligned} I_0 P'_X &\approx -\frac{1}{3} \xi \sqrt{\eta} \frac{q}{E_q} G_E^n G_M^n, \\ I_0 P'_Z &\approx -\frac{1}{3} \sqrt{\eta(\eta + \xi)} \frac{q^2}{E_q(E_q + m)} [G_M^n]^2, \end{aligned} \quad (143)$$

for proton polarization transfer and

$$\begin{aligned} I_0 P'_X &\approx -\xi \sqrt{\eta} \frac{q}{E_q} G_E^n G_M^n, \\ I_0 P'_Z &\approx \sqrt{\eta(\eta + \xi)} \frac{q^2}{E_q(E_q + m)} [G_M^n]^2, \end{aligned} \quad (144)$$

for neutron polarization transfer, with

$$I_0 = \frac{\sigma_0}{\sigma_M R} \left(\frac{e^2}{4\pi} u_0^2(0) \right)^{-1}. \quad (145)$$

Assumption $u_{0,2}(q)/u_0(0) \ll 1$ is justified for high q values. The above formulas show that the ratio of the transverse and longitudinal polarization transfer components is directly proportional to the ratio of the neutron electric and magnetic form factors, i.e., $P'_X/P'_Z \propto G_E^n/G_M^n$, being independent of the deuteron WF. This is a well-known result for the quasielastic scattering on the nucleon, but it can also be obtained for the deuteron electrodisintegration in the BA for parallel kinematics. Of course, contributions from MEC and FSI introduce a WF dependence that may distort the extraction of the form-factor ratio, but as we see in Fig. 14, we predict them to be negligible at $\theta_{pq} \simeq \theta_{\text{lim}} = 180^\circ$ ($\mathbf{k}_n \parallel \mathbf{q}$). Let us emphasize that this conclusion refers to the kinematic regimes near the QFP and to the present model, and should be reexamined for other kinematics.

V. CONCLUSIONS

To conclude:

- (i) A fully relativistic description of the $d(e, e'p)n$ reaction has been developed within the clothed particle representation. The nucleon-nucleon interaction (the Kharkiv potential), electromagnetic current operators (including one- and two-body terms), boost operators and other Poincaré group generators are constructed on a common field-theoretical footing.
- (ii) Comparison with the Saclay and Jefferson Lab data shows that relativistic effects are essential. In particular, the relativistic one-body current and the consistent treatment of Lorentz boosts significantly influence both the differential cross section and polarization observables.
- (iii) The role of FSI and MEC contributions depends on the kinematics. Contrary to the results in the considered Saclay kinematic regimes, for the higher-energy kinematics the FSI and MEC contributions may act constructively and increase the cross section at large recoil momenta. The contributions of different meson exchanges partially compensate each other, while the effective σ -meson mechanism gives a sizable effect that deserves further investigation.
- (iv) The observables exhibit a pronounced left-right asymmetry with respect to the reaction plane. In the left wing of the QFP, the induced proton polarization is especially sensitive to the neutron electric form factor G_E^n , in agreement with earlier findings. At the same time, for parallel kinematics the polarization transfer components satisfy simple analytical relations in the BA and become directly proportional to the ratio G_E^n/G_M^n , while being independent of the deuteron WF. Within our consistent calculation, where the deuteron WF, the FSI and the one- and two-body currents follow from one and the same Hamiltonian, the corrections to these relations near the QFP at $\theta_{pq} \simeq 180^\circ$ turn out to be small, which makes the corresponding

kinematic region particularly promising for the extraction of neutron form factors.

The present study demonstrates that a consistent field-theoretical construction of interactions and currents within the CPR framework provides a viable alternative to conventional nonrelativistic models. Further developments may include a fully gauge-independent description, more detailed analysis of subnucleonic degrees of freedom and applications to other electroweak processes involving few-nucleon systems.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: LINKS BETWEEN THE CLOTHED ONE- AND TWO-PARTICLE STATES AND (IN/OUT) STATES

In what follows we will explicitly derive the links (46)–(48) between in(out) states and the clothed particle states. The derivation will be done for the complex scalar field $\varphi(x)$, but it can be easily generalized to other cases. The scalar field $\varphi(x)$ satisfies the following Klein-Gordon (KG) equation:

$$(\square_x + \mu^2)\varphi(x) = j(x), \quad (A1)$$

where μ is the physical mass and current $j(x)$ describes a possible interaction. The field $\varphi(x)$ is in the Heisenberg picture

$$\varphi(x) = \varphi(\mathbf{x}, t) \equiv e^{iHt} \varphi(\mathbf{x}) e^{-iHt}, \quad (A2)$$

where H is the corresponding total Hamiltonian operator. Further, one introduces the quantities

$$A(\mathbf{k}, t) = (f_k^*, \varphi), \quad B^\dagger(\mathbf{k}, t) = (\varphi, f_k) = -(f_k, \varphi), \quad (A3)$$

with respect to the product

$$(F_1, F_2) = i \int d\mathbf{x} [F_1(\mathbf{x}, t) \partial_t F_2(\mathbf{x}, t) - F_2(\mathbf{x}, t) \partial_t F_1(\mathbf{x}, t)]. \quad (A4)$$

Here $f_k(x)$ denotes the plane waves

$$f_k(x) = f(\mathbf{x}, t) = [(2\pi)^3 2k_0]^{-1/2} \exp(-ikx), \quad (\text{A5})$$

with the four-vector $k = (k^0, \mathbf{k})$ is determined on the mass shell $k^0 = \sqrt{\mathbf{k}^2 + \mu^2}$, which enter in the expansion

$$\varphi(\mathbf{x}) = \int d\mathbf{k} [A(\mathbf{k}) f_k(\mathbf{x}, 0) + B^\dagger(\mathbf{k}) f_k^*(\mathbf{x}, 0)], \quad (\text{A6})$$

where the destruction and creation operators $A(\mathbf{k})$, $B^\dagger(\mathbf{k})$ correspond to the representation of the bare particles with physical masses. The clothed particle counterparts for these operators introduced via the unitary transformation W ,

$$A(\mathbf{k}) = W A_c(\mathbf{k}) W^\dagger, \quad B^\dagger(\mathbf{k}) = W B_c^\dagger(\mathbf{k}) W^\dagger, \quad (\text{A7})$$

which allows to express the operators (A3) through the clothed particle operators.

However, we shall employ the LSZ asymptotic condition,

$$\lim_{t \rightarrow -\infty} \langle \Phi | A^\dagger(\mathbf{k}, t) | \Psi \rangle = \langle \Phi | A_{\text{in}}^\dagger(\mathbf{k}) | \Psi \rangle, \quad (\text{A8})$$

that determines the *in* operator $A_{\text{in}}(\mathbf{k})$ for any normalizable states Φ and Ψ .

Equations (A3), (A7), and (A8) are our working formulas. In the context, the one-particle *in* state is given by

$$|\mathbf{k}; \text{in}\rangle = A_{\text{in}}^\dagger(\mathbf{k}) |\Omega\rangle = \lim_{t \rightarrow -\infty} A^\dagger(\mathbf{k}, t) |\Omega\rangle. \quad (\text{A9})$$

Explicit calculations give

$$A(\mathbf{k}, t) = i \int d\mathbf{x} \{ f_k^*(\mathbf{x}, t) e^{iHt} [H, \varphi(\mathbf{x}, 0)] e^{-iHt} - ik^0 \varphi(\mathbf{x}, t) f_k^*(\mathbf{k}, t) \} \equiv \bar{A}(\mathbf{k}, t) + A_I(\mathbf{k}, t), \quad (\text{A10})$$

where

$$\bar{A}(\mathbf{k}, t) = -e^{iHt} \int d\mathbf{x} f_k^*(\mathbf{x}, t) ([H_0, \varphi(\mathbf{x})] - k^0 \varphi(\mathbf{x})) e^{-iHt}, \quad (\text{A11})$$

and

$$A_I(\mathbf{k}, t) = -e^{iHt} \int d\mathbf{x} f_k^*(\mathbf{x}, t) [H_I, \varphi(\mathbf{x})] e^{-iHt}. \quad (\text{A12})$$

Here we assume the standard division of the total Hamiltonian operator into free and interaction parts $H = H_0(a) + H_I(a)$, where both terms are written in terms of the BPR operators $a = \{A(\mathbf{k}), B(\mathbf{k}), A^\dagger(\mathbf{k}), B^\dagger(\mathbf{k})\}$. In particular, the free part $H_0(a)$ is given by the usual expression

$$H_0(a) = \int d\mathbf{k} \sqrt{\mathbf{k}^2 + \mu^2} [A^\dagger(\mathbf{k}) A(\mathbf{k}) + B^\dagger(\mathbf{k}) B(\mathbf{k})]. \quad (\text{A13})$$

Taking into account the commutation relations $[H_0, A(\mathbf{k})] = -k^0 A(\mathbf{k})$ and $[H_0, B^\dagger(\mathbf{k})] = k^0 B^\dagger(\mathbf{k})$, Eq. (A11) can be put in the form

$$\bar{A}(\mathbf{k}, t) = e^{iHt} A(\mathbf{k}) e^{-iHt} e^{ik^0 t}. \quad (\text{A14})$$

Similarly, after integration over $d\mathbf{x}$ in Eq. (A12) one gets

$$A_I(\mathbf{k}, t) = -\frac{e^{iHt}}{2k_0} \{ [H, A(\mathbf{k}) + B^\dagger(-\mathbf{k})] + k_0 (A(\mathbf{k}) - B^\dagger(-\mathbf{k})) \} e^{ik^0 t} e^{-iHt}. \quad (\text{A15})$$

Now, with help of Eq. (A7) one can express $\bar{A}(\mathbf{k}, t)$ in terms of the clothed particle operators $a_c = \{A_c, B_c, A_c^\dagger, B_c^\dagger\}$,

$$\bar{A}(\mathbf{k}, t) = e^{iHt} W A_c(\mathbf{k}) W^\dagger e^{-Ht} e^{ik^0 t} = e^{iHt} e^{-iK_F(a_c)t} W_D(t) A_c(\mathbf{k}) W_D^\dagger(t) e^{iK_F(a_c)t} e^{-iHt}, \quad (\text{A16})$$

where we used that $e^{iK_F(a_c)t} A_c(\mathbf{k}) e^{-iK_F(a_c)t} = A_c(\mathbf{k}) e^{ik^0 t}$ with a new decomposition of the original Hamiltonian H into free and interaction parts $H = K_F(a_c) + K_I(a_c)$, where the free part is now given by

$$K_F(a_c) = H_0(a_c) = \int d\mathbf{k} \sqrt{\mathbf{k}^2 + \mu^2} [A_c^\dagger(\mathbf{k}) A_c(\mathbf{k}) + B_c^\dagger(\mathbf{k}) B_c(\mathbf{k})]. \quad (\text{A17})$$

Under asymptotic condition (41) one can get step by step the limits of interest

$$\begin{aligned} \lim_{t \rightarrow -\infty} \bar{A}^\dagger(\mathbf{k}, t) |\Omega\rangle &= \lim_{t \rightarrow -\infty} e^{iHt} e^{-iH_F(a_c)t} W_D(t) A_c^\dagger(\mathbf{k}) W_D^\dagger(t) e^{iH_F(a_c)t} e^{-iHt} |\Omega\rangle \\ &= \lim_{t \rightarrow -\infty} e^{iHt} e^{-iH_F(a_c)t} A_c^\dagger(\mathbf{k}) |\Omega\rangle = \lim_{t \rightarrow -\infty} e^{ik^0 t} e^{-ik^0 t} A_c^\dagger(\mathbf{k}) |\Omega\rangle = A_c^\dagger(\mathbf{k}) |\Omega\rangle, \end{aligned} \quad (\text{A18})$$

where along with the asymptotic condition we have used the following properties of the clothed particle states and the physical vacuum

$$K_F(\alpha_c)|\Omega\rangle = H|\Omega\rangle = 0 \quad (\text{A19})$$

and

$$K_F(\alpha_c)A_c^\dagger(\mathbf{k})|\Omega\rangle = HA_c^\dagger(\mathbf{k})|\Omega\rangle = k^0 A_c^\dagger(\mathbf{k})|\Omega\rangle. \quad (\text{A20})$$

In the same way one gets

$$\begin{aligned} \lim_{t \rightarrow -\infty} A_I^\dagger(\mathbf{k}, t)|\Omega\rangle &= \lim_{t \rightarrow -\infty} e^{iHt} \left(\frac{H - k_0}{2k_0} W A_c^\dagger(\mathbf{k}) W^\dagger + \frac{H + k_0}{2k_0} W B_c(-\mathbf{k}) W^\dagger \right) e^{-ik_0 t} e^{-iHt} |\Omega\rangle \\ &= \lim_{t \rightarrow -\infty} \frac{e^{iHt}}{2k_0} \left((H - k_0) e^{-iH_F(\alpha_c)t} W_D(t) A_c^\dagger(\mathbf{k}) W_D^\dagger(t) + (H + k_0) e^{-iH_F(\alpha_c)t} e^{-2ik_0 t} W_D(t) B_c(-\mathbf{k}) W_D^\dagger(t) \right) |\Omega\rangle \\ &= \lim_{t \rightarrow -\infty} e^{iHt} \frac{H - k_0}{2k_0} e^{-iH_F(\alpha_c)t} A_c^\dagger(\mathbf{k}) |\Omega\rangle = \lim_{t \rightarrow -\infty} e^{iHt} e^{-ik_0 t} \frac{k_0 - k_0}{2k_0} A_c^\dagger(\mathbf{k}) |\Omega\rangle = 0, \end{aligned} \quad (\text{A21})$$

where we used important condition that clothed particle destruction operators destroys the physical vacuum $B_c(-\mathbf{k})|\Omega\rangle = 0$. Therefore, $\lim_{t \rightarrow -\infty} A_I(\mathbf{k}, t)|\Omega\rangle = 0$ and

$$|\mathbf{k}; \text{in}\rangle = A_{\text{in}}^\dagger(\mathbf{k})|\Omega\rangle = \lim_{t \rightarrow -\infty} \bar{A}^\dagger(\mathbf{k}, t)|\Omega\rangle = A_c^\dagger(\mathbf{k})|\Omega\rangle = |\mathbf{k}; c\rangle. \quad (\text{A22})$$

Analogously, one can obtain links between *out* states $A_{\text{out}}^\dagger(\mathbf{k})|\Omega\rangle$, *in(out)* states $B_{\text{in(out)}}^\dagger(\mathbf{k})|\Omega\rangle$ and the corresponding one-particle clothed states.

Now, let's consider the two-particle *in* state

$$|\mathbf{k}_1 \mathbf{k}_2; \text{in}\rangle = A_{\text{in}}^\dagger(\mathbf{k}_1) A_{\text{in}}^\dagger(\mathbf{k}_2)|\Omega\rangle. \quad (\text{A23})$$

First, using the link between one-particle states $A_{\text{in}}^\dagger(\mathbf{k})|\Omega\rangle = A_c^\dagger(\mathbf{k})|\Omega\rangle$ one can write

$$|\mathbf{k}_1 \mathbf{k}_2; \text{in}\rangle = A_{\text{in}}^\dagger(\mathbf{k}_1) A_c^\dagger(\mathbf{k}_2)|\Omega\rangle = \lim_{t \rightarrow -\infty} A^\dagger(\mathbf{k}_1, t) A_c^\dagger(\mathbf{k}_2)|\Omega\rangle. \quad (\text{A24})$$

Similar to Eqs. (A18) and (A21) we obtain that

$$\lim_{t \rightarrow -\infty} \bar{A}^\dagger(\mathbf{k}_1, t) A_c^\dagger(\mathbf{k}_2)|\Omega\rangle = \lim_{t \rightarrow -\infty} e^{i(H - k_1^0 - k_2^0)t} A_c^\dagger(\mathbf{k}_1) A_c^\dagger(\mathbf{k}_2)|\Omega\rangle \quad (\text{A25})$$

and

$$\lim_{t \rightarrow -\infty} A_I^\dagger(\mathbf{k}_1, t) A_c^\dagger(\mathbf{k}_2)|\Omega\rangle = \frac{H - k_1^0 - k_2^0}{2k_1^0} \lim_{t \rightarrow -\infty} e^{i(H - k_1^0 - k_2^0)t} A_c^\dagger(\mathbf{k}_1) A_c^\dagger(\mathbf{k}_2)|\Omega\rangle. \quad (\text{A26})$$

According to definition (49) of the Møller operators $\Omega_c^{(\pm)}$ in the CPR, limits in Eqs. (A25) and (A26) can be put in the form

$$\lim_{t \rightarrow -\infty} e^{i(H - k_1^0 - k_2^0)t} A_c^\dagger(\mathbf{k}_1) A_c^\dagger(\mathbf{k}_2)|\Omega\rangle = \lim_{t \rightarrow -\infty} e^{iKt} e^{-iK_F t} A_c^\dagger(\mathbf{k}_1) A_c^\dagger(\mathbf{k}_2)|\Omega\rangle = \Omega_c^{(-)} A_c^\dagger(\mathbf{k}_1) A_c^\dagger(\mathbf{k}_2)|\Omega\rangle. \quad (\text{A27})$$

Further, using the property $H\Omega_c^{(\pm)} = \Omega_c^{(\pm)} K_F$ we obtain that Eq. (A26) vanishes and we get the final result

$$|\mathbf{k}_1 \mathbf{k}_2; \text{in}\rangle = \lim_{t \rightarrow -\infty} e^{i(H - k_1^0 - k_2^0)t} A_c^\dagger(\mathbf{k}_1) A_c^\dagger(\mathbf{k}_2)|\Omega\rangle = \Omega_c^{(-)} A_c^\dagger(\mathbf{k}_1) A_c^\dagger(\mathbf{k}_2)|\Omega\rangle = \Omega_c^{(-)} |\mathbf{k}_1 \mathbf{k}_2; c\rangle. \quad (\text{A28})$$

Its significant consequence is

$$|\mathbf{k}_1 \mathbf{k}_2; \text{in}\rangle \neq A_c^\dagger(\mathbf{k}_1) A_c^\dagger(\mathbf{k}_2)|\Omega\rangle. \quad (\text{A29})$$

Similar relations with $t \rightarrow +\infty$ can be derived for the *out* states.

APPENDIX B: EXPRESSIONS FOR MEC OPERATORS

The two-nucleon meson exchange current consists of the two parts

$$J_{2N} = J_{MCC} + J_{MNN}, \quad (\text{B1})$$

with the so-called mesonic J_{MCC} and seagull J_{MNN} currents. Both of them have the same operator structure

$$J_{MCC, MNN}^\alpha = \sum d1' d2' d1 d2 F_{MCC, MNN}^\alpha(1', 2', 1, 2) b_c^\dagger(1') b_c^\dagger(2') b_c(1) b_c(2), \quad (\mathcal{M} = \delta, \sigma, \pi, \eta, \rho, \omega); \quad (\text{B2})$$

hereafter we denote $\sum d1' \equiv \sum_{\mu'_1 \eta'_1} \int d\mathbf{p}'_1$, $b_c^\dagger(1') \equiv b_c^\dagger(\mathbf{p}'_1 \mu'_1 \eta'_1)$, and so on, with spin indices μ and isospin indices η . The matrix elements of these operators sandwiched between the two-nucleon states

$$|1, 2\rangle = b_c^\dagger(1)b_c^\dagger(2)|\Omega\rangle \quad (\text{B3})$$

can be easily found:

$$\langle 1', 2' | J^\alpha | 1, 2 \rangle = -F^\alpha(1', 2', 1, 2) + F^\alpha(1', 2', 2, 1) - F^\alpha(2', 1', 2, 1) + F^\alpha(2', 1', 1, 2). \quad (\text{B4})$$

Here we have used the commutation relation

$$\{b_c(\mathbf{p}\mu\eta), b_c^\dagger(\mathbf{p}'\mu'\eta')\} = \delta_{\mu'\mu} \delta_{\eta'\eta} \delta(\mathbf{p}' - \mathbf{p}). \quad (\text{B5})$$

Before we proceed to the explicit expressions for the MECs, we would like to establish some notations. For the sake of brevity, we denote the following isospin factors:

$$T_1 = \boldsymbol{\tau}_{\eta'_1 \eta_1} \cdot \boldsymbol{\tau}_{\eta'_2 \eta_2} + \tau_{\eta'_2 \eta_2}^z, \quad T_2 = 1 + \tau_{\eta'_1 \eta_1}^z, \quad T_3 = i[\boldsymbol{\tau}_{\eta'_1 \eta_1} \times \boldsymbol{\tau}_{\eta'_2 \eta_2}]^z, \quad (\text{B6})$$

with $\boldsymbol{\tau}$ being the Pauli matrices in the isospin space. Following Ref. [34] we introduce cutoff functions that are defined as follows:

$$g_{\mathcal{M}}(n', n) = \left[\frac{\Lambda_{\mathcal{M}}^2 - m_{\mathcal{M}}^2}{\Lambda_{\mathcal{M}}^2 - (E'_n - E_n)^2 + (\mathbf{p}'_n - \mathbf{p}_n)^2} \right]^{n_{\mathcal{M}}}, \quad (\text{B7})$$

where $n_{\mathcal{M}} = 1$ except for $n_{\rho} = n_{\omega} = 2$ and all the parameters can be found in Table 1 from Ref. [63]. The cutoff factors are supposed to account for finite-size effects (see details in Refs. [34,50]). In Ref. [35] these cutoffs have been used in the evaluation of the radial deuteron functions $u_{0,2}(|\mathbf{p}|)$. They are present in the definition of the generator R of the unitary clothing transformation and therefore enter the expressions for the MECs that stem from the commutator $\frac{1}{2}[R, [R, J_c^\mu(0)]]$.

As discussed in Appendix C of Ref. [34] we actually have the possibility to introduce two independent cutoff functions for each meson, one corresponds to the nucleon-meson-nucleon vertex (B7) and another one corresponds to the nucleon-meson-antinucleon vertex. But at the time authors of the paper considered only nucleon-nucleon interaction in the CPR that in the first nonvanishing approximation embodies only the first type of vertex. In contrast, the seagull currents we are considering here may contain this second type of cutoff factor. Let us denote it analogously to Eq. (B7):

$$\bar{g}_{\mathcal{M}}(n', n) = \left[\frac{\bar{\Lambda}_{\mathcal{M}}^2 - m_{\mathcal{M}}^2}{\bar{\Lambda}_{\mathcal{M}}^2 - (E'_n - E_n)^2 + (\mathbf{p}'_n - \mathbf{p}_n)^2} \right]^{n_{\mathcal{M}}}, \quad (\text{B8})$$

with new parameters $\bar{\Lambda}_{\mathcal{M}}$ instead of $\Lambda_{\mathcal{M}}$. In our calculations presented in Sec. IV we have used the same values for $\bar{\Lambda}_{\mathcal{M}}$ as for $\Lambda_{\mathcal{M}}$, but in principle they can be different and should be fitted to the experimental data for $N\bar{N}$ scattering.

Now we present the explicit expressions for F_{MCC}^α and F_{MNN}^α functions that determine the MEC operators and have been derived by computing the $\frac{1}{2}[R, [R, J_c^\mu(0)]]$ commutator from the series (93). Mesonic MECs are given by the following expressions for π mesons (pseudoscalar with isospin $T = 1$):

$$F_{\pi CC}^\alpha(1', 2', 1, 2) = -\frac{eg_\pi^2 m^2}{2(2\pi)^6} T_3 \frac{g_\pi(1', 1)g_\pi(2', 2)}{\sqrt{E'_1 E'_2 E_1 E_2}} (p'_1 - p_1 + p_2 - p'_2)^\alpha \frac{\bar{u}(\mathbf{p}'_1 \mu'_1) \gamma_5 u(\mathbf{p}_1 \mu_1)}{(p'_1 - p_1)^2 - m_\pi^2} \frac{\bar{u}(\mathbf{p}'_2 \mu'_2) \gamma_5 u(\mathbf{p}_2 \mu_2)}{(p'_2 - p_2)^2 - m_\pi^2}, \quad (\text{B9})$$

δ mesons (scalar with isospin $T = 1$):

$$F_{\delta CC}^\alpha(1', 2', 1, 2) = \frac{eg_\delta^2 m^2}{2(2\pi)^6} T_3 \frac{g_\delta(1', 1)g_\delta(2', 2)}{\sqrt{E'_1 E'_2 E_1 E_2}} (p'_1 - p_1 + p_2 - p'_2)^\alpha \frac{\bar{u}(\mathbf{p}'_1 \mu'_1) u(\mathbf{p}_1 \mu_1)}{(p'_1 - p_1)^2 - m_\delta^2} \frac{\bar{u}(\mathbf{p}'_2 \mu'_2) u(\mathbf{p}_2 \mu_2)}{(p'_2 - p_2)^2 - m_\delta^2}, \quad (\text{B10})$$

ρ mesons (vector with isospin $T = 1$):

$$\begin{aligned} F_{\rho CC}^\alpha(1', 2', 1, 2) = & \frac{em^2}{8(2\pi)^6} \frac{T_3}{\omega_1 \omega_2} \frac{g_\rho(1', 1)g_\rho(2', 2)}{\sqrt{E'_1 E'_2 E_1 E_2}} \left\{ \left[g^{\alpha\delta} \left(k_1^\sigma - k_{2-}^\sigma \frac{(k_1 \cdot k_{2-})}{m_\rho^2} \right) - k_1^\alpha \left(g^{\delta\sigma} - \frac{k_{2-}^\delta k_{2-}^\sigma}{m_\rho^2} \right) \right. \right. \\ & + g^{\alpha\sigma} \left(k_{2-}^\delta - k_1^\delta \frac{(k_1 \cdot k_{2-})}{m_\rho^2} \right) - k_{2-}^\alpha \left(g^{\delta\sigma} - \frac{k_1^\delta k_1^\sigma}{m_\rho^2} \right) \left. \right] \frac{\bar{u}(1') [g_\rho \gamma_\delta - \frac{f_\rho}{2m} i k_1^\nu \sigma_{\nu\delta}] u(1)}{E'_1 - E_1 - \omega_1} \frac{\bar{u}(2') [g_\rho \gamma_\sigma + \frac{f_\rho}{2m} i k_{2-}^\nu \sigma_{\nu\sigma}] u(2)}{-E'_2 + E_2 - \omega_2} \\ & - \left[g^{\alpha\delta} \left(k_{1-}^\sigma - k_2^\sigma \frac{(k_{1-} \cdot k_2)}{m_\rho^2} \right) - k_{1-}^\alpha \left(g^{\delta\sigma} - \frac{k_2^\delta k_2^\sigma}{m_\rho^2} \right) + g^{\alpha\sigma} \left(k_2^\delta - k_{1-}^\delta \frac{(k_{1-} \cdot k_2)}{m_\rho^2} \right) \right. \\ & \left. \left. - k_2^\alpha \left(g^{\delta\sigma} - \frac{k_{1-}^\delta k_{1-}^\sigma}{m_\rho^2} \right) \right] \frac{\bar{u}(1') [g_\rho \gamma_\delta + \frac{f_\rho}{2m} i k_{1-}^\nu \sigma_{\nu\delta}] u(1)}{-E'_1 + E_1 - \omega_1} \frac{\bar{u}(2') [g_\rho \gamma_\sigma - \frac{f_\rho}{2m} i k_2^\nu \sigma_{\nu\sigma}] u(2)}{E'_2 - E_2 - \omega_2} \right\} \end{aligned}$$

$$\begin{aligned}
& + \left[g^{\alpha\delta} \left(k_1^\sigma - k_2^\sigma \frac{(k_1 \cdot k_2)}{m_\rho^2} \right) - k_1^\alpha \left(g^{\delta\sigma} - \frac{k_2^\delta k_2^\sigma}{m_\rho^2} \right) - g^{\alpha\sigma} \left(k_2^\delta - k_1^\delta \frac{(k_1 \cdot k_2)}{m_\rho^2} \right) \right. \\
& + k_2^\alpha \left(g^{\delta\sigma} - \frac{k_1^\delta k_1^\sigma}{m_\rho^2} \right) \left. \right] \frac{\bar{u}(1') [g_\rho \gamma_\delta - \frac{f_\rho}{2m} i k_1^\nu \sigma_{\nu\delta}] u(1)}{E'_1 - E_1 - \omega_1} \frac{\bar{u}(2') [g_\rho \gamma_\sigma - \frac{f_\rho}{2m} i k_2^\nu \sigma_{\nu\sigma}] u(2)}{E'_2 - E_2 - \omega_2} \\
& - \left[g^{\alpha\delta} \left(k_{1-}^\sigma - k_{2-}^\sigma \frac{(k_1 \cdot k_2)}{m_\rho^2} \right) - k_{1-}^\alpha \left(g^{\delta\sigma} - \frac{k_{2-}^\delta k_{2-}^\sigma}{m_\rho^2} \right) - g^{\alpha\sigma} \left(k_{2-}^\delta - k_{1-}^\delta \frac{(k_1 \cdot k_2)}{m_\rho^2} \right) \right. \\
& + k_{2-}^\alpha \left(g^{\delta\sigma} - \frac{k_{1-}^\delta k_{1-}^\sigma}{m_\rho^2} \right) \left. \right] \frac{\bar{u}(1') [g_\rho \gamma_\delta + \frac{f_\rho}{2m} i k_{1-}^\nu \sigma_{\nu\delta}] u(1)}{-E'_1 + E_1 - \omega_1} \frac{\bar{u}(2') [g_\rho \gamma_\sigma + \frac{f_\rho}{2m} i k_{2-}^\nu \sigma_{\nu\sigma}] u(2)}{-E'_2 + E_2 - \omega_2} \Bigg\}. \tag{B11}
\end{aligned}$$

Here

$$\begin{aligned}
\mathbf{k}_1 &= \mathbf{p}'_1 - \mathbf{p}_1, \quad \mathbf{k}_2 = \mathbf{p}'_2 - \mathbf{p}_2, \quad \mathbf{p}_3 = \mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}'_2, \quad \mathbf{p}'_3 = \mathbf{p}'_1 + \mathbf{p}'_2 - \mathbf{p}_2, \\
\omega_1 &= \sqrt{m_\rho^2 + \mathbf{k}_1^2}, \quad \omega_2 = \sqrt{m_\rho^2 + \mathbf{k}_2^2}, \quad E_n = \sqrt{m + \mathbf{p}_n^2}, \quad E'_n = \sqrt{m + \mathbf{p}'_n^2} \quad \text{for } n = 1, 2, 3, \\
k_1 &= (\omega_1, \mathbf{k}_1), \quad k_2 = (\omega_2, \mathbf{k}_2), \quad k_{1-} = (\omega_1, -\mathbf{k}_1), \quad k_{2-} = (\omega_2, -\mathbf{k}_2), \\
p'_{3-} &= (E'_3, -\mathbf{p}'_3), \quad p_{3-} = (E_3, -\mathbf{p}_3), \quad p'_n = (E'_n, \mathbf{p}'_n), \quad p_n = (E_n, \mathbf{p}_n) \quad \text{for } n = 1, 2, 3, \\
\not{p}_3 &= \gamma_\mu p_3^\mu, \quad \not{p}_{3-} = \gamma_\mu p_{3-}^\mu, \quad \not{p}'_3 = \gamma_\mu p_3'^\mu, \quad \not{p}'_{3-} = \gamma_\mu p_{3-}'^\mu, \quad g^{\delta\sigma} = \text{diag}(1, -1, -1, -1). \tag{B12}
\end{aligned}$$

Seagull MECs are given by the following expressions for π mesons:

$$\begin{aligned}
F_{\pi NN}^\alpha(1', 2', 1, 2) &= \frac{eg_\pi^2 m^2}{2(2\pi)^6} \frac{g_\pi(2', 2)}{\sqrt{E'_1 E'_2 E_1 E_2}} \frac{\bar{u}(2') \gamma_5 u(2)}{(p'_2 - p_2)^2 - m_\pi^2} \\
&\times \left[\frac{T_1 - T_3}{2} \bar{u}(1') \gamma^\alpha A(1', 2', 1, 2) \gamma_5 u(1) + \frac{T_1 + T_3}{2} \bar{u}(1') \gamma_5 A(1, 2, 1', 2') \gamma^\alpha u(1) \right], \tag{B13}
\end{aligned}$$

δ mesons:

$$\begin{aligned}
F_{\delta NN}^\alpha(1', 2', 1, 2) &= -\frac{eg_\delta^2 m^2}{2(2\pi)^6} \frac{g_\delta(2', 2)}{\sqrt{E'_1 E'_2 E_1 E_2}} \frac{\bar{u}(2') u(2)}{(p'_2 - p_2)^2 - m_\delta^2} \\
&\times \left[\frac{T_1 - T_3}{2} \bar{u}(1') \gamma^\alpha A(1', 2', 1, 2) u(1) + \frac{T_1 + T_3}{2} \bar{u}(1') A(1, 2, 1', 2') \gamma^\alpha u(1) \right], \tag{B14}
\end{aligned}$$

with

$$A(1', 2', 1, 2) = \frac{1}{2E_3} \left[g_{\pi,\delta}(1, 3) (\not{p}_3 + m) \frac{E_1 - E_2 + E'_2 - E_3}{(p_1 - p_3)^2 - m_{\mathcal{M}}^2} + \bar{g}_{\pi,\delta}(1, 3) (\not{p}_{3-} - m) \frac{E_1 - E_2 + E'_2 + E_3}{(p_1 + p_{3-})^2 - m_{\mathcal{M}}^2} \right], \tag{B15}$$

and for ρ mesons:

$$\begin{aligned}
F_{\rho NN}^\alpha(1', 2', 1, 2) &= \frac{em^2}{16(2\pi)^6} \frac{1}{\omega_2} \frac{g_\rho(2', 2)}{\sqrt{E'_1 E'_2 E_1 E_2}} \left\{ \frac{T_1 - T_3}{E_3} \left(\left[-g^{\delta\sigma} + \frac{k_{2-}^\delta k_{2-}^\sigma}{m_\rho^2} \right] \frac{\bar{u}(2') [g_\rho \gamma_\delta + \frac{f_\rho}{2m} i k_{2-}^\nu \sigma_{\nu\delta}] u(2)}{-E'_2 + E_2 - \omega_2} \bar{u}(1') \gamma^\alpha \right. \right. \\
&\times \left[g_\rho(3, 1) \frac{(\not{p}_3 + m)}{E_3 - E_1 - \omega_2} + \bar{g}_\rho(3, 1) \frac{(\not{p}_{3-} - m)}{-E_3 - E_1 - \omega_2} \right] \left[g_\rho \gamma_\sigma - \frac{f_\rho}{2m} i k_{2-}^\nu \sigma_{\nu\sigma} \right] u(1) \\
&- \left[-g^{\delta\sigma} + \frac{k_2^\delta k_2^\sigma}{m_\rho^2} \right] \frac{\bar{u}(2') [g_\rho \gamma_\delta - \frac{f_\rho}{2m} i k_2^\nu \sigma_{\nu\delta}] u(2)}{E'_2 - E_2 - \omega_2} \bar{u}(1') \gamma^\alpha \\
&\times \left[g_\rho(3, 1) \frac{(\not{p}_3 + m)}{-E_3 + E_1 - \omega_2} + \bar{g}_\rho(3, 1) \frac{(\not{p}_{3-} - m)}{E_3 + E_1 - \omega_2} \right] \left[g_\rho \gamma_\sigma + \frac{f_\rho}{2m} i k_2^\nu \sigma_{\nu\sigma} \right] u(1) \Bigg) \\
&+ \frac{T_1 + T_3}{E'_3} \left(\left[-g^{\delta\sigma} + \frac{k_2^\delta k_2^\sigma}{m_\rho^2} \right] \frac{\bar{u}(2') [g_\rho \gamma_\delta - \frac{f_\rho}{2m} i k_2^\nu \sigma_{\nu\delta}] u(2)}{E'_2 - E_2 - \omega_2} \bar{u}(1') \left[g_\rho \gamma_\sigma + \frac{f_\rho}{2m} i k_2^\nu \sigma_{\nu\sigma} \right] \right.
\end{aligned}$$

$$\begin{aligned}
& \times \left[g_\rho(1', 3') \frac{(\not{p}'_3 + m)}{-E'_1 + E'_3 - \omega_2} + \bar{g}_\rho(1', 3') \frac{(\not{p}'_{3-} - m)}{-E'_1 - E'_3 - \omega_2} \right] \gamma^\alpha u(1) \\
& - \left[-g^{\delta\sigma} + \frac{k_{2-}^\delta k_{2-}^\sigma}{m_\rho^2} \right] \frac{\bar{u}(2') [g_\rho \gamma_\delta + \frac{f_\rho}{2m} i k_{2-}^\nu \sigma_{\nu\delta}] u(2)}{-E'_2 + E_2 - \omega_2} \\
& \times \bar{u}(1') \left[g_\rho \gamma_\sigma - \frac{f_\rho}{2m} i k_{2-}^\nu \sigma_{\nu\sigma} \right] \left[g_\rho(1', 3') \frac{(\not{p}'_3 + m)}{E'_1 - E'_3 - \omega_2} + \bar{g}_\rho(1', 3') \frac{(\not{p}'_{3-} - m)}{E'_1 + E'_3 - \omega_2} \right] \gamma^\alpha u(1) \Bigg\}. \quad (\text{B16})
\end{aligned}$$

The σ mesons (scalar) with isospin $T = 1$ produces the same currents as the δ mesons, but with different coupling constant, cutoff parameter and mass. To get the expressions for the MECs induced by the mesons with isospin $T = 0$: $\sigma(0)$ (scalar), η (pseudoscalar) and ω (vector), one needs to replace $T_1 \rightarrow T_2$, $T_3 \rightarrow 0$ and substitute the corresponding parameters (coupling constants, cutoff parameters and masses) in the respective expressions above. It is clear that the isoscalar mesons that has zero charge do not produce mesonic MECs, but they do produce seagull ones.

Although the expressions (B9)–(B16) for MECs may appear rather complex, they can be efficiently calculated via the relations from Ref. [90]:

$$\bar{u}(\mathbf{p}'\mu') \Gamma u(\mathbf{p}\mu) = A(\mathbf{p}', \mathbf{p}) \delta_{\mu'\mu} + \mathbf{B}(\mathbf{p}', \mathbf{p}) \cdot \boldsymbol{\sigma}_{\mu'\mu}, \quad (\text{B17})$$

$$A(\mathbf{p}', \mathbf{p}) = \text{Tr}[(\not{p}' + m) \Gamma (\not{p} + m) (1 + \gamma_0)] / 8m \sqrt{(E_{\mathbf{p}'} + m)(E_{\mathbf{p}} + m)}, \quad (\text{B18})$$

$$\mathbf{B}(\mathbf{p}', \mathbf{p}) = \text{Tr}[(\not{p}' + m) \Gamma (\not{p} + m) (1 + \gamma_0) \boldsymbol{\gamma} \gamma_5] / 8m \sqrt{(E_{\mathbf{p}'} + m)(E_{\mathbf{p}} + m)}, \quad (\text{B19})$$

with $\not{p} = \gamma_\alpha p^\alpha$, $p' = (E_{\mathbf{p}'}, \mathbf{p}')$, $p = (E_{\mathbf{p}}, \mathbf{p})$, and $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$, for any combination of the γ matrices Γ . We have the same definitions for γ matrices and bispinors as in Ref. [64]

$$u(\mathbf{p}\mu) = \sqrt{\frac{E_{\mathbf{p}} + m}{2m}} \begin{pmatrix} |\mu\rangle \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E_{\mathbf{p}} + m} |\mu\rangle \end{pmatrix}, \quad v(\mathbf{p}\mu) = \sqrt{\frac{E_{\mathbf{p}} + m}{2m}} \begin{pmatrix} \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E_{\mathbf{p}} + m} |\mu\rangle^c \\ |\mu\rangle^c \end{pmatrix}, \quad (\text{B20})$$

$$\left| \frac{1}{2} \right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \left| -\frac{1}{2} \right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad |\mu\rangle^c = (-1)^{\frac{1}{2} - \mu} |-\mu\rangle. \quad (\text{B21})$$

APPENDIX C: ADDITIONAL TEST OF np -PAIR PARTIAL WAVE CALCULATIONS

In Sec. III D we have described our method of calculating the np -pair partial wave functions $\Psi_{pl'l'}^{\alpha(-)}(p')$ in the momentum space. The matrix inversion method allows us to find the representation (83) for the WFs $\varphi_{pl'l'}^\alpha(p')$ that have the asymptotics of standing waves. The links (76) give us an analogous representation for the np -pair partial WFs

$$\Psi_{pl'l'}^{\alpha(\pm)}(p') = \sum_{j=1}^{N+1} \Omega_{pl'l'}^{\alpha(\pm)}(j) \frac{\delta(p' - p_j)}{p_j^2}, \quad (\text{C1})$$

$$\Omega_{pl'l'}^{\alpha(\pm)}(j) \equiv \sum_{l''} O_{ll''}^{\alpha(\pm)}(p) B_{pl''l'}^\alpha(j), \quad (\text{C2})$$

that should be meant in an integral sense. Here $p_{N+1} = p$ is the relative momentum of the np -pair and for the uncoupled partial waves the matrix $O_{ll'}^{\alpha(\pm)}(p)$ reduces to the phase factor

$$O_{ll'}^{\alpha(\pm)}(p) = \exp(\pm i \delta_l^\alpha) \cos \delta_l^\alpha, \quad (l' = l). \quad (\text{C3})$$

In fact, Eq. (C1) allows us to find analytical expressions for the np -pair partial WFs in the coordinate space and verify their asymptotic behavior. From Eq. (80) it follows that the radial partial wave functions

$$\Psi_{pl'l'}^{\alpha(\pm)}(r) = \frac{1}{2\pi^2} i^{l'} \int_0^\infty p'^2 dp' \Psi_{pl'l'}^{\alpha(\pm)}(p') j_{l'}(p' r) \quad (\text{C4})$$

can be written as

$$\Psi_{pl'l'}^{\alpha(\pm)}(r) = \frac{1}{2\pi^2} i^{l'} \sum_{j=1}^{N+1} \Omega_{pl'l'}^{\alpha(\pm)}(j) j_{l'}(p_j r). \quad (\text{C5})$$

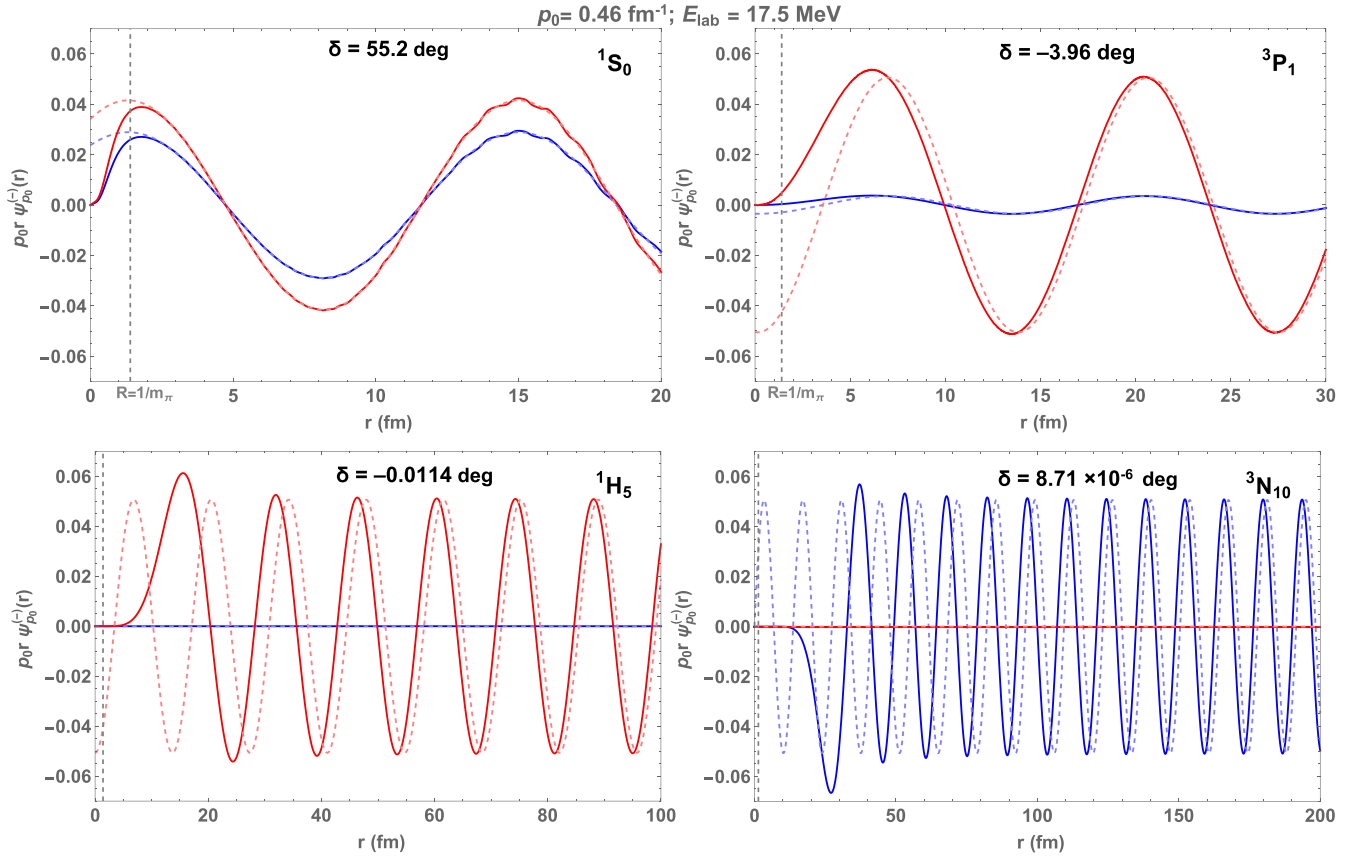


FIG. 15. Verification of the asymptotic behavior (C6) for the $^1S_0$, $^3P_1$, $^1H_5$, and $^3N_{10}$ uncoupled partial waves as examples for np -pair laboratory energy $E_{\text{lab}} = 17.5$ MeV ($p = 0.46$ fm $^{-1}$). Blue solid lines: real part of $pr\Psi_{pl}^{\alpha(-)}(r)$. Red solid lines: imaginary part of $pr\Psi_{pl}^{\alpha(-)}(r)$. Dotted lines are the respective real (blue) and imaginary (red) parts of the asymptotic functions (C6) multiplied by pr . The characteristic radius of the nuclear forces (vertical dotted line) $R = 1/m_\pi = 1.4$ fm is shown too.

Recall the known result of the scattering theory (see, e.g., Sec. 6.2 and Sec. 7.1 in Ref. [45]) for the asymptotic behavior of the partial wave functions of the continuous spectrum at large pr

$$\begin{aligned} \text{uncoupled waves: } \Psi_{pl}^{\alpha(\pm)}(r) &\asymp \frac{1}{2\pi^2} i^l e^{\pm i\delta_l^\alpha} \frac{\sin\left(pr - \frac{l\pi}{2} + \delta_l^\alpha\right)}{pr}, \quad pr \gg 1, \\ \text{coupled waves: } \Psi_{pl'}^{\alpha(\pm)}(r) &\asymp \frac{1}{2\pi^2} \frac{1}{pr} \begin{pmatrix} \cos \varepsilon_\alpha & -\sin \varepsilon_\alpha \\ \sin \varepsilon_\alpha & \cos \varepsilon_\alpha \end{pmatrix} \\ &\times \begin{pmatrix} i^{J-1} e^{\pm i\delta_-^\alpha} \sin\left(pr - \frac{(J-1)\pi}{2} + \delta_-^\alpha\right) & 0 \\ 0 & i^{J+1} e^{\pm i\delta_+^\alpha} \sin\left(pr - \frac{(J+1)\pi}{2} + \delta_+^\alpha\right) \end{pmatrix} \begin{pmatrix} \cos \varepsilon_\alpha & \sin \varepsilon_\alpha \\ -\sin \varepsilon_\alpha & \cos \varepsilon_\alpha \end{pmatrix}, \quad pr \gg 1, \end{aligned} \quad (\text{C6})$$

that gives us a possibility to verify once again the correctness of our calculations.

In Figs. 15 and 16 we show that the functions $pr\Psi_{pl'}^{\alpha(-)}(r)$ indeed have the expected asymptotic behavior. The numerical result for the state $^1S_0$ starts to follow the asymptotic one as soon as r exceeds the characteristic radius of nuclear forces $R = 1/m_\pi = 1.4$ fm, while for the states with higher orbital momenta the asymptotic behavior is observed at larger r .

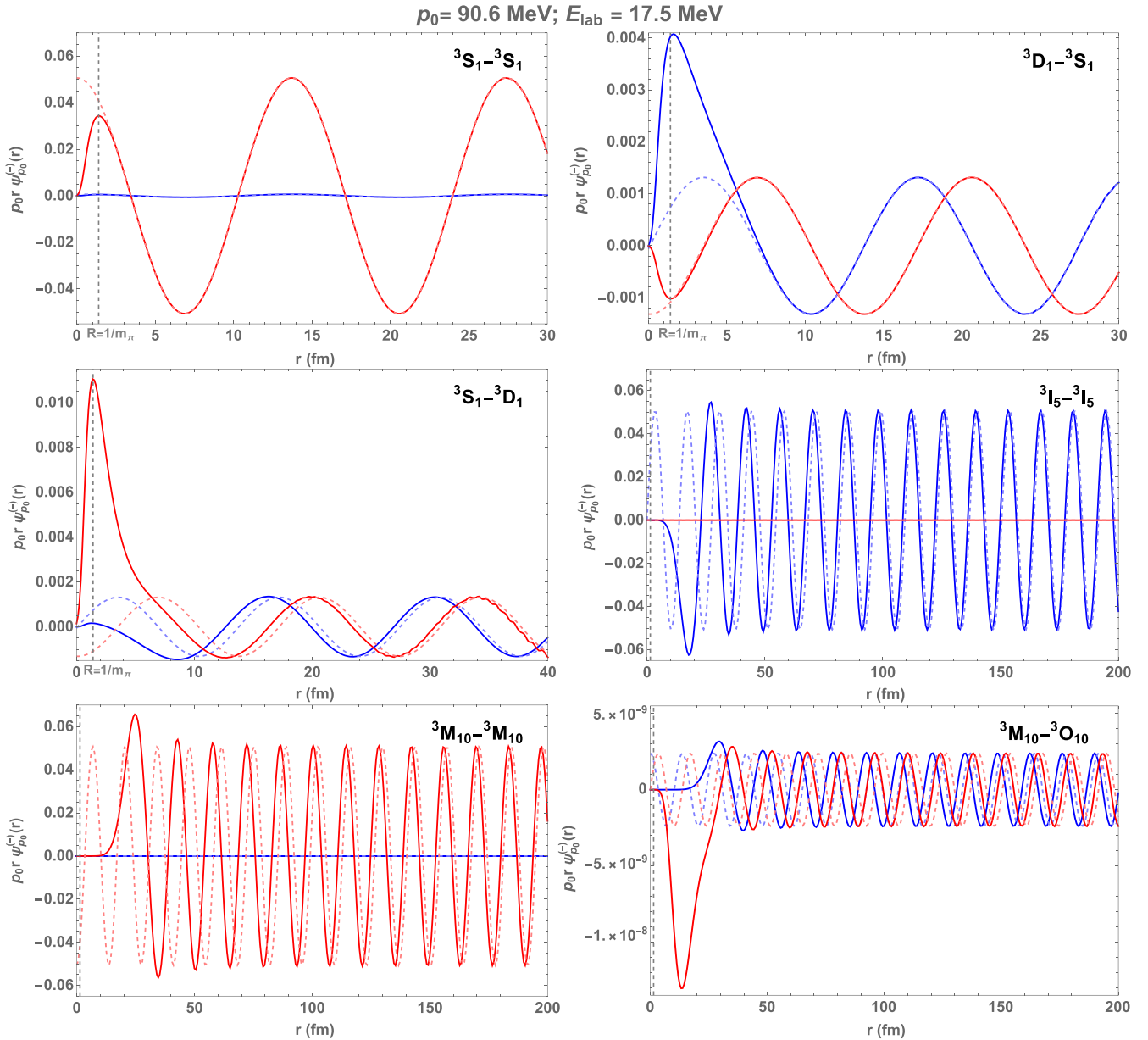


FIG. 16. Same as in Fig. 15 but for the coupled partial waves: $^3S_1 - ^3S_1$, $^3D_1 - ^3S_1$, $^3S_1 - ^3D_1$, $^3I_5 - ^3I_5$, $^3M_{10} - ^3M_{10}$, and $^3M_{10} - ^3O_{10}$.

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