

Relativistic spin hydrodynamics from novel relaxation time approximationSamapan Bhadury ^{*}*Institute of Theoretical Physics, Jagiellonian University, ul. St. Łojasiewicza 11, 30-348 Krakow, Poland*

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With the help of a semiclassical kinetic theory, a new collision kernel is proposed, which simultaneously conserves the energy-momentum tensor and the spin tensor of a relativistic fluid of spin-1/2 particles irrespective of the frame and matching conditions, even when relaxation time is momentum dependent. The relativistic Boltzmann's equation is solved using this new collision kernel that will lead to the transport coefficients with general definitions for the frame and matching conditions. The results indicate the expected existence of Barnett-like effect and the nonexistence of Einstein–de Haas-like effects. For the first time, a local collision kernel is being proposed that can be used to construct relativistic spin hydrodynamics along the line of Bemfica-Disconzi-Noronha-Kovtun (BDNK) formulation, which may lead to a first-order stable and causal theory.

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Introduction. During microscopic collisions, all fundamental interactions guarantee the conservation laws for the linear (p^μ) and angular momentum ($j^{\mu\nu} = \ell^{\mu\nu} + s^{\mu\nu}$), where $\ell^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu$ is the orbital angular momentum with x^μ being the space-time four-vector and $s^{\mu\nu}$ the internal angular momentum. The latter was introduced by Mathisson [1] in 1937 and is related to spin four-vector (s^μ) through the relation $s^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} p_\alpha s_\beta / m$, where m is the particle mass. Until recently, the study of relativistic kinetic theory focused only on the microscopic conservation of the momentum four-vector, leading to the formulation of relativistic hydrodynamics while the angular momentum part was mostly ignored. With the discovery of spin polarization phenomena in heavy-ion collision experiments [2–12], the development of spin hydrodynamics from various approaches has gained significant interest [13–67]. Specifically, the nontrivial dynamics due to local [68] and nonlocal collisions [60,69] causing exchange of spin in quantum kinetic theories are of particular interest [70,71]. In local collisions, the orbital and spin parts are conserved separately [21,72]. This simplification allows one to develop a theory of spin hydrodynamics for systems with conserved macroscopic spin tensor [28,73,74]. In general, such systems may be in an out-of-equilibrium state. The out-of-equilibrium dynamics is controlled by the collision kernel, whose complicated nature makes the Boltzmann equation an integro-differential equation. Consequently, some linearized approximations for collision kernel are used, which effectively preserve the properties of the original collision ker-

nel. While the relaxation time approximation (RTA) used in Refs. [28,73] fulfills the requirement of detailed balance of the collision kernel, it is not consistent with macroscopic conservation laws unless one imposes the specific Landau-Lifshitz (LL) frame and matching conditions [75]. Recently, it was shown in Ref. [74] that the newly proposed collision kernel of extended relaxation time approximation (ERTA) [76,77] can ensure all the conservation laws independent of the frame and matching conditions in an order-by-order manner in space-time gradients. While the simplistic nature of ERTA is very useful [78,79], it is desirable to construct a collision kernel that ensures the conservation laws at all orders. This can be helpful for the development of a first-order causal and stable theory of spin hydrodynamics [80–88] similar to the BDNK formalism of relativistic hydrodynamics for unpolarized systems. This is the purpose of this work. The implication of this new theory of spin hydrodynamics will be far reaching, as such a theory can be used to develop a (3 + 1)D hydrodynamic code for numerical simulation. The presently available numerical works consider a second-order theory of relativistic hydrodynamics and spin is evolved in a hydrodynamic background. These approaches have to consider the dissipative currents as independent degrees of freedom, which obey separate evolution equations. On the other hand, BDNK theories involve less number of degrees of freedom, which may lead to a simplified approach to study the evolution of spin.

Here, a new collision kernel is being proposed following the development of novel relaxation time approximation (NRTA) [89,90] for a system of massive spin-half particles with zero chemical potential that are spin polarizable. This collision kernel will be used to show that the macroscopic conservation laws of energy-momentum tensor ($T^{\mu\nu}$) and spin tensor ($S^{\lambda,\mu\nu}$) can be obtained at all orders as a direct consequence of the microscopic conservation laws, irrespective of any particular choice of frame and matching conditions. Furthermore, the Boltzmann equation will be solved and the transport coefficients associated with all the nonequilibrium

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processes will be obtained. The results indicate the gradients of the fluid variables affect the evolution of spin transport, resembling the Barnett effect. However, the converse effect of Einstein–de Haas is absent, i.e., the evolution of fluid variables is independent of the spin related gradients.

Notations and conventions. Throughout this Letter, natural units are used, i.e., $k_B = \hbar = c = 1$. For metric, the mostly negative convention has been chosen, i.e., $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$, whereas for the Levi-Civita tensor the choice is $\epsilon^{0123} = -\epsilon_{0123} = 1$. For both the ordinary phase-space distribution function, $f(x, p)$, and the extended phase-space distribution function, $f(x, p, s)$, the dependence on the microscopic variables are suppressed to denote them as f_p and f_s , respectively. Any macroscopic and microscopic variable at equilibrium will carry an additional subscript of "0," e.g., f_{0p} , whereas the nonequilibrium corrections will be denoted by adding a " δ " in front of the variable, e.g., δf_p . Bold letters will be used for three-vectors. The scalar product of four-vectors is denoted as $A \cdot B \equiv A_\mu B^\mu$ and the scalar product of rank-2 tensors is denoted as $A : B \equiv A_{\mu\nu} B^{\mu\nu}$.

Relativistic spin hydrodynamics. For a rotating relativistic fluid of massive spin-half particles with zero chemical potential the relevant conserved currents are the energy-momentum tensor ($T^{\mu\nu}$) and the total angular momentum tensor ($J^{\lambda,\mu\nu}$), which is the sum of an orbital part ($L^{\lambda,\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}$) and a spin part ($S^{\lambda,\mu\nu}$), i.e., $J^{\lambda,\mu\nu} = L^{\lambda,\mu\nu} + S^{\lambda,\mu\nu}$. The nonuniqueness of the division of the orbital and spin parts gives rise to the problem of pseudogauge transformation. In the absence of nonlocal collisions, under the de Groot–Leeuwen–Weert (GLW) pseudogauge, the energy-momentum tensor is symmetric [91], leading to a conserved spin tensor. The conserved currents for an out-of-equilibrium fluid can thus be expressed under GLW in terms of the single-particle distribution function in the extended phase space as

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - P \Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \pi^{\mu\nu} = \langle p^\mu p^\nu \rangle, \quad (1a)$$

$$S^{\lambda,\mu\nu} = u^\lambda S^{\mu\nu} + 2u^{[\lambda} \Delta^{\nu]\lambda} \Sigma + u^\mu \Sigma_{(s)}^{(\lambda\nu)} - u^\nu \Sigma_{(s)}^{(\lambda\mu)} + u^\mu \Sigma_{(a)}^{\lambda\nu} - u^\nu \Sigma_{(a)}^{\lambda\mu} + \Sigma^{\lambda,\mu\nu} = \langle p^\lambda s^{\mu\nu} \rangle, \quad (1b)$$

where $A^{[\mu} B^{\nu]} = \frac{1}{2}(A^\mu B^\nu - A^\nu B^\mu)$ is the antisymmetric combination, ε and P are the full energy density and pressure, respectively, which can be split into equilibrium and nonequilibrium parts as $\varepsilon = \varepsilon_0 + \delta\varepsilon$ and $P = P_0 + \delta P$, respectively, u^μ is the fluid four-velocity normalized to unity ($u_\mu u^\mu = 1$), and the projection operator orthogonal to u^μ is $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$. Additionally, the currents q^μ and $\pi^{\mu\nu}$ are the heat diffusion current and shear viscous pressure, respectively. While the components of the spin tensors lack an unanimous interpretation, in the present work, I will use the following terminologies: $S^{\mu\nu} \rightarrow$ spin density, $\Sigma \rightarrow$ spin pressure, $\Sigma_{(s)}^{(\mu\nu)} \rightarrow$ spin shear, $\Sigma_{(a)}^{\mu\nu} \rightarrow$ spin vorticity, and $\Sigma^{\lambda,\mu\nu} \rightarrow$ spin diffusion. Similar to ε , these variables can be split into equilibrium and nonequilibrium parts. Here the angular brackets enclosing the indices of the spin shear term are defined as $A^{[\mu} B^{\nu]} = \Delta_{\alpha\beta}^{\mu\nu} A^\alpha B^\beta$, where $\Delta_{\alpha\beta}^{\mu\nu} = \frac{1}{2}(\Delta_\alpha^\mu \Delta_\beta^\nu + \Delta_\beta^\mu \Delta_\alpha^\nu) - \frac{1}{3} \Delta^{\mu\nu} \Delta_{\alpha\beta}$ is the doubly symmetric traceless projection oper-

ator. The angular bracket appearing in Eqs. (1a) and (1b) is defined as

$$\langle (\cdots) \rangle = \int_\Gamma (\cdots) f_s, \quad (2)$$

where $\int_\Gamma \equiv \int dP dS$ with $dP = \frac{gd^3\mathbf{p}}{(2\pi)^3 E_p}$ being the momentum integral measure and $dS = \frac{m}{\pi s} d^4s \delta(s \cdot s + s^2) \delta(p \cdot s)$ being the spin integral measure. Here g is the degeneracy factor (spin degeneracy not included), $E_p = \sqrt{\mathbf{p}^2 + m^2}$ is the on-shell particle energy in the local rest frame, and s^2 is the eigenvalue of the Casimir operator. Using the fact that the distribution function can be split into equilibrium and nonequilibrium parts as $f_s = f_{0s} + \delta f_s$, one can also split the bracket in Eq. (2) as $\langle (\cdots) \rangle = \langle (\cdots) \rangle_0 + \langle (\cdots) \rangle_\delta$. One can intuitively understand the definition of these new brackets as $\langle (\cdots) \rangle_0 \equiv \int_\Gamma (\cdots) f_{0s}$ and $\langle (\cdots) \rangle_\delta \equiv \int_\Gamma (\cdots) \delta f_s$. One more bracket will be used frequently in this Letter that is defined as $\langle (\cdots) \rangle_{0p} \equiv \int dP (\cdots) f_{0p}$. The latter bracket will be used to replace $\langle (\cdots) \rangle_\delta$ once δf_s is defined in terms of f_{0p} .

The components of the conserved currents can now be expressed with the help of these brackets as

$$\varepsilon_0 = \langle E_p^2 \rangle_0, \quad \delta\varepsilon = \langle E_p^2 \rangle_\delta, \quad (3a)$$

$$P_0 = -\frac{1}{3} \langle (p \cdot \Delta \cdot p) \rangle_0, \quad \delta P = -\frac{1}{3} \langle (p \cdot \Delta \cdot p) \rangle_\delta, \quad (3b)$$

$$q^\mu = \langle E_p p^{(\mu)} \rangle_\delta, \quad \pi^{\mu\nu} = \langle p^{(\mu} p^{\nu)} \rangle_\delta, \quad (3c)$$

$$S^{\mu\nu} = u_\lambda S^{\lambda,\mu\nu} = \langle E_p s^{\mu\nu} \rangle, \quad (3d)$$

$$\Sigma = \frac{1}{3} u_\mu \Delta_{\lambda\nu} S^{\lambda,\mu\nu} = \langle (1/3) u_\mu p_{(\nu)} s^{\mu\nu} \rangle = 0, \quad (3e)$$

$$\Sigma_{(s)}^{(\lambda\nu)} = u_\mu \Delta_{\alpha\beta}^{\lambda\nu} S^{\alpha,\mu\beta} = \langle u_\mu s^{\mu(\nu} p^{\lambda)} \rangle, \quad (3f)$$

$$\Sigma_{(a)}^{\lambda\nu} = u_\mu \Delta_{\alpha\beta}^{[\lambda} \Delta^{\nu]} S^{\alpha,\mu\beta} = \langle u_\mu s^{\mu[\nu} p^{\lambda]} \rangle, \quad (3g)$$

$$\Sigma^{\lambda,\mu\nu} = \Delta_\gamma^\lambda \Delta_\alpha^\mu \Delta_\beta^\nu S^{\gamma,\alpha\beta} = \langle p^{(\lambda} s^{(\mu} p^{\nu)} \rangle, \quad (3h)$$

where the notation $A^{(\mu)} = \Delta_\alpha^\mu A^\alpha$ represents the part of the four-vector A^μ that is orthogonal to u_μ . The spin integral measure enforces the Frenkel condition, $p_\mu s^{\mu\nu} = 0$, leading to a vanishing Σ . The equilibrium parts can be expressed in terms of u^μ , temperature (T), and spin polarization tensor ($\omega^{\mu\nu}$), which is the Lagrange multiplier associated with conservation of angular momentum, by noting that the equilibrium distribution can be defined in the small polarization limit as [28]

$$f_{0s} = f_{0p} \exp[(s : \omega)/2] \approx f_{0p} [1 + (s : \omega)/2] + \mathcal{O}(\omega^2), \quad (4)$$

where $f_{0p} = e^{-\beta E_p}$. Here $\beta = 1/T$ is the inverse temperature. To determine the nonequilibrium quantities, one needs knowledge of $\delta f_s \equiv \phi_s f_{0s}$. In the next section, I obtain this by solving the Boltzmann equation.

Relativistic kinetic theory. The relativistic Boltzmann equation is given by

$$p^\mu \partial_\mu f_s = C[f_s], \quad (5)$$

¹It should be noted that this definition is required to fulfill the conservation law and does not lead to any inconsistency as the present work is restricted to linear order in $\omega_{\mu\nu}$.

where $C[f_s]$ is the collision kernel. To solve the Boltzmann equation, first the collision kernel is to be specified. In case of $2 \leftrightarrow 2$ local collisions, considering only the cases where both momentum and spin are exchanged, with the help of the weak equivalence principle [92], the collision kernel can be shown to be given by [72,93]

$$C[f_s] = \int_{\Gamma'} \int_{\Gamma_1} \int_{\Gamma'_1} W(f_{s_1} f_{s'_1} - f_s f_{s'}), \quad (6)$$

where W is the transition matrix. Using the fact that the distribution function can be written as $f_s = (1 + \phi_s) f_{0s}$ in Eq. (6) and keeping only the terms linear in ϕ_s , the Boltzmann equation can be rewritten under a linearized approximation of the collision kernel as

$$p^\mu \partial_\mu f_s = \hat{L} \phi_s, \quad (7)$$

where the linearized collision operator, $\hat{L}$ can be interpreted as a linear operator in Hilbert space, defined as

$$\hat{L} \phi_s \equiv \int_{\Gamma'} \int_{\Gamma_1} \int_{\Gamma'_1} W f_{0s} f_{0s'} (\phi_{s_1} + \phi_{s'_1} - \phi_s - \phi_{s'}). \quad (8)$$

Note that I have only considered terms linear in ϕ_s as I am constructing a linearized collision kernel. Taking advantage of the self-adjointness property of such a linearized collision kernel and the fact that its eigenfunctions with zero eigenvalue correspond to the microscopically conserved degrees of freedom of the particle, the authors in Ref. [90] proposed a

modified RTA. With such a collision kernel, one can obtain $\hat{L}1 = 0$ and $\hat{L}p^\mu = 0$, which correspond to the microscopic conservation of particle number and linear momentum four-vector, respectively. These properties of $\hat{L}$ allow one to build up a theory of relativistic hydrodynamics such that the number current and energy-momentum tensors are conserved by construction. Following this approach, in the present work, I propose a linearized collision kernel with the properties $\hat{L}p^\mu = 0$ and $\hat{L}s^{\mu\nu} = 0$, which will allow one to set up a theory of relativistic spin hydrodynamics.² Consequently, in this case, the number of degenerate eigenfunctions ($|\lambda_n\rangle$) of $\hat{L}$ with zero eigenvalue is 10. Thus the proposed abstract form of the linear collision operator in Hilbert space is

$$\hat{L} \sim -\mathbb{1} + \sum_{n=1}^{10} |\lambda_n\rangle \langle \lambda_n|, \quad (9)$$

where $\mathbb{1}$ corresponds to the Anderson-Witting RTA collision kernel. The new ten terms on the right-hand side correspond to counterterms that cancel out the homogeneous part of the solution of the Boltzmann equation. This cancellation allows one to build a theory of relativistic hydrodynamics independent of frame and matching conditions.³ The eigenfunctions, $|\lambda_n\rangle$ can be obtained explicitly with the help of the Gram-Schmidt orthonormalization process. Introducing the notation $\tilde{A}_\mu = u^\beta A_{(\mu)\beta}$ and $\tilde{A}_{\mu\nu} = A_{(\mu)(\nu)}$, the linear collision operator of Eq. (9) can be expressed in a less abstract form in terms of variables associated with spin hydrodynamics in the small polarization limit as

$$\begin{aligned} \hat{L} \phi_s = & - \left(\frac{E_{\mathbf{p}}}{\tau_R} \right) f_{0s} \left\{ \phi_s - \frac{\langle (E_{\mathbf{p}}^2 / \tau_R) \phi_s \rangle_0}{\langle (E_{\mathbf{p}}^3 / \tau_R) \rangle_0} E_{\mathbf{p}} - \frac{\langle (E_{\mathbf{p}} / \tau_R) p^{(\mu)} \phi_s \rangle_0}{\langle (1/3) (E_{\mathbf{p}} / \tau_R) p^{(\alpha)} p_{(\alpha)} \rangle_0} p_{(\mu)} \right. \\ & - \left[\langle (E_{\mathbf{p}} / \tau_R) \tilde{s}_\mu \phi_s \rangle_0 - \frac{\langle (E_{\mathbf{p}}^2 / \tau_R) \tilde{s}_\mu \rangle_0}{\langle (E_{\mathbf{p}}^3 / \tau_R) \rangle_0} \langle (E_{\mathbf{p}}^2 / \tau_R) \phi_s \rangle_0 - \frac{\langle (E_{\mathbf{p}} / \tau_R) p^{(\gamma)} \tilde{s}_\mu \rangle_0}{\langle (1/3) (E_{\mathbf{p}} / \tau_R) p^{(\alpha)} p_{(\alpha)} \rangle_0} \langle (E_{\mathbf{p}} / \tau_R) p_{(\gamma)} \phi_s \rangle_0 \right] \\ & \times \left[\tilde{s}^\mu - \frac{\langle (E_{\mathbf{p}}^2 / \tau_R) \tilde{s}^\mu \rangle_0}{\langle (E_{\mathbf{p}}^3 / \tau_R) \rangle_0} E_{\mathbf{p}} - \frac{\langle (E_{\mathbf{p}} / \tau_R) p^{(\rho)} \tilde{s}^\mu \rangle_0}{\langle (1/3) (E_{\mathbf{p}} / \tau_R) p^{(\beta)} p_{(\beta)} \rangle_0} p_{(\rho)} \right] \frac{1}{\langle (1/3) (E_{\mathbf{p}} / \tau_R) (\tilde{s} \cdot \tilde{s}) \rangle_0} \\ & - \left[\langle (E_{\mathbf{p}} / \tau_R) \tilde{s}_{\mu\nu} \phi_s \rangle_0 - \frac{\langle (E_{\mathbf{p}}^2 / \tau_R) \tilde{s}_{\mu\nu} \rangle_0}{\langle (E_{\mathbf{p}}^3 / \tau_R) \rangle_0} \langle (E_{\mathbf{p}}^2 / \tau_R) \phi_s \rangle_0 - \frac{\langle (E_{\mathbf{p}} / \tau_R) p^{(\gamma)} \tilde{s}_{\mu\nu} \rangle_0}{\langle (1/3) (E_{\mathbf{p}} / \tau_R) p^{(\alpha)} p_{(\alpha)} \rangle_0} \langle (E_{\mathbf{p}} / \tau_R) p_{(\gamma)} \phi_s \rangle_0 \right] \\ & \times \left[\tilde{s}^{\mu\nu} - \frac{\langle (E_{\mathbf{p}}^2 / \tau_R) \tilde{s}^{\mu\nu} \rangle_0}{\langle (E_{\mathbf{p}}^3 / \tau_R) \rangle_0} E_{\mathbf{p}} - \frac{\langle (E_{\mathbf{p}} / \tau_R) p^{(\rho)} \tilde{s}^{\mu\nu} \rangle_0}{\langle (1/3) (E_{\mathbf{p}} / \tau_R) p^{(\beta)} p_{(\beta)} \rangle_0} p_{(\rho)} \right] \frac{1}{\langle (1/3) (E_{\mathbf{p}} / \tau_R) (\tilde{s} \cdot \tilde{s}) \rangle_0} \Big\}. \end{aligned} \quad (10)$$

Here, the relaxation time could depend on particle momenta as well as spin. However, for the sake of simplicity, while determining the transport coefficients, the spin dependence on τ_R has been ignored. Note that all the terms appearing in Eq. (10) are essential to have $\hat{L}E_{\mathbf{p}} = \hat{L}p^{(\mu)} = \hat{L}s^{\mu\nu} = 0$ and hence the conservation laws, i.e., $\partial_\mu T^{\mu\nu} = \partial_\mu \langle p^\mu p^\nu \rangle_0 = 0$, $\partial_\lambda S^{\lambda, \mu\nu} = \partial_\lambda \langle p^\lambda s^{\mu\nu} \rangle_0 = 0$, hold true for any order. Another crucial point is the distribution function appearing outside the curly braces in the first line of Eq. (10) is f_{0s} and not $f_{0\mathbf{p}}$. It ensures that the conservation laws are satisfied and can be understood by noting that the brackets, $\langle (\cdots) \rangle_0$ appearing in

Eq. (10) are defined with the former. This complicates the solution process, but it is unavoidable.

²Note that $\hat{L}1 = 0$ has not been demanded. This is because such a constraint conserves the particle number separately and not the net particle number. In a relativistic system, where a particle and antiparticle pair can be produced, only the conservation of net particle number can be demanded. However, this is a highly nontrivial task and is left for a future study.

³In Refs. [74,77], similar cancellations of the homogeneous part occur through the counterterms denoted by δf^* .

Having introduced the new collision kernel, the next step is to solve ϕ_s . The solution may be split into a spin-independent part and a spin-dependent part as [43]

$$\phi_s = \phi_{\mathbf{p}} + s : \psi_{\mathbf{p}}. \quad (11)$$

Proceeding as in Refs. [43,90,94,95] one may expand the solutions in terms of irreducible tensors as

$$\phi_{\mathbf{p}} = \sum_{n \in \mathbb{S}_0^{(\ell)}} \sum_{\ell=0}^{\infty} \Phi_n^{(\mu_1 \dots \mu_\ell)} p_{(\mu_1} \dots p_{\mu_\ell)} P_n^{(0, \ell)}, \quad (12a)$$

$$\psi_{\mathbf{p}}^{\mu\nu} = \sum_{n \in \mathbb{S}_1^{(\ell)}} \sum_{\ell=0}^{\infty} \Psi_n^{\mu\nu, (\mu_1 \dots \mu_\ell)} p_{(\mu_1} \dots p_{\mu_\ell)} P_n^{(1, \ell)}, \quad (12b)$$

where $p_{(\mu_1} \dots p_{\mu_\ell)} = \Delta_{v_1 \dots v_\ell}^{\mu_1 \dots \mu_\ell} p^{(v_1} \dots p^{v_\ell)}$ form the irreducible orthogonal momentum basis with the rank- 2ℓ tensor $\Delta_{v_1 \dots v_\ell}^{\mu_1 \dots \mu_\ell}$ being defined in Refs. [91,94]. Here, $\mathbb{S}_j^{(\ell)}$ represents the set of values that the index n takes for a given value of j and ℓ . In principle, $\mathbb{S}_j^{(\ell)}$ should be the set $\{0, 1, 2, \dots, \infty\}$. For analytical calculations, however, it often becomes necessary to truncate this set to finite values.⁴ In this work, I have chosen the minimal set necessary to construct the theory of spin hydrodynamics with general frame and matching conditions. These are $\mathbb{S}_0^{(0)} = \{0, 1, 2\}$, $\mathbb{S}_0^{(1)} = \{0, 1\}$, $\mathbb{S}_0^{(2)} = \{0\}$ and $\mathbb{S}_1^{(0)} = \{0, 1\}$, $\mathbb{S}_1^{(1)} = \{0\}$, while all other $\mathbb{S}_j^{(\ell)}$ are null sets. This implies there are $14 + 30 = 44$ degrees of freedom. However, by imposing the frame and matching conditions, the degrees of freedom are further reduced by $4 + 6 = 10$ leaving $10 + 24 = 34$ degrees of freedom in total. However, of these, four spin degrees of freedom are redundant due to $p_\mu s^{\mu\nu} = 0$. The expansions in Eqs. (12a) and (12b) require one to use the set of orthogonal polynomials of energy $P_n^{(j, \ell)}$ satisfying the orthogonality condition

$$\frac{\ell!}{(2\ell + 1)!!} \left\langle \left(\frac{E_{\mathbf{p}}}{\tau_R} \right) (p \cdot \Delta \cdot p)^\ell P_r^{(j, \ell)} P_n^{(j, \ell)} \right\rangle_{0\mathbf{p}} = A_n^{(j, \ell)} \delta_{rn}, \quad (13)$$

where $A_n^{(j, \ell)}$ is obtained from Eq. (13) by setting $r = n$ on both sides. The polynomials $P_n^{(j, \ell)}$ can be determined through the Gram-Schmidt process. However, one has to still define one element of the set to initiate the process. In the present work, I impose $P_0^{(0, 0)} = \beta E_{\mathbf{p}}$ and, $P_0^{(0, \ell \geq 1)} = P_0^{(1, \ell \geq 0)} = 1$. The expansion coefficients, $\Phi_n^{(\mu_1 \dots \mu_\ell)}$ and $\Psi_n^{\mu\nu, (\mu_1 \dots \mu_\ell)}$, are functions of macroscopic variables only. Furthermore, $\Psi_n^{\mu\nu, (\mu_1 \dots \mu_\ell)} \sim \mathcal{O}(\omega)$ and hence I will ignore terms that go like $\psi_{\mathbf{p}} \omega^n$ for $n \geq 1$. Note that here the index ℓ corresponds to the tensorial rank of the coefficient and n represents the n th coefficient of the ℓ th rank coefficient. In the present work, $j = 0$ and $j = 1$ correspond to the spin-independent and spin-dependent parts, respectively.

Therefore, all that remains is to determine coefficients $\Phi_n^{(\mu_1 \dots \mu_\ell)}$ and $\Psi_n^{\mu\nu, (\mu_1 \dots \mu_\ell)}$. To obtain these coefficients up to

first order in space-time gradients, I first consider the left-hand side of Eq. (7). Following the Chapman-Enskog-like expansion, only the first-order gradient terms are kept so that the linearized Boltzmann equation is

$$(A_{\mathbf{p}} + B_{\mathbf{p}}^{(\mu)} p_\mu + C_{\mathbf{p}}^{(\mu\nu)} p_\mu p_\nu) \left[1 + \frac{1}{2} (s : \omega) \right] f_{0\mathbf{p}} + D_{\mathbf{p}}^{\mu, \alpha\beta} p_\mu s_{\alpha\beta} f_{0\mathbf{p}} = \hat{L} \phi_s, \quad (14)$$

where I have defined

$$A_{\mathbf{p}} = -E_{\mathbf{p}}^2 \dot{\beta} - \frac{\beta}{3} (p \cdot \Delta \cdot p) \theta, \quad (15a)$$

$$B_{\mathbf{p}}^{(\mu)} = -E_{\mathbf{p}} [(\nabla^\mu \beta) - \beta \dot{u}^\mu], \quad (15b)$$

$$C_{\mathbf{p}}^{(\mu\nu)} = -\beta \sigma^{\mu\nu}, \quad D_{\mathbf{p}}^{\mu, \alpha\beta} = \frac{1}{2} (u^\mu \dot{\omega}^{\alpha\beta} + \nabla^\mu \omega^{\alpha\beta}). \quad (15c)$$

Here, $\theta \equiv \partial \cdot u$ is the expansion scalar and $\sigma^{\mu\nu} = \partial^{(\mu} u^{\nu)}$ is the velocity stress tensor, $\hat{A} \equiv (u \cdot \partial) A$ denotes the comoving derivative, and $\nabla^\mu \equiv \Delta_{\alpha}^{\mu} \partial^\alpha$ denotes the spacelike derivative. On the right-hand side of Eq. (7), one may use Eq. (10) to substitute the linear collision kernel and ϕ_s is substituted from Eqs. (11), (12a), and (12b). However, not all of the coefficients can be determined from the Boltzmann equation. The ones corresponding to a homogeneous solution, i.e., Φ_0 , $\Phi_1^{(\mu)}$, and $\Psi_0^{\mu\nu}$, are obtained through the frame and matching conditions, which in the present case are taken to be

$$\langle q_1 \phi_s \rangle_{0\mathbf{p}} = 0, \quad \langle q_2 p^{(\mu)} \phi_s \rangle_{0\mathbf{p}} = 0, \quad \langle q_3 s^{\mu\nu} \phi_s \rangle_{0\mathbf{p}} = 0, \quad (16)$$

where $q_{1/2/3}$ are not specified at the moment. The choices $q_1 = E_{\mathbf{p}}^2$ and $q_2 = q_3 = E_{\mathbf{p}}$ lead to the ones used in Refs. [28,73]. The solutions can be now expressed as

$$\phi_{\mathbf{p}} = \sum_{n \in \mathbb{S}_0^{(0)}} \Phi_n P_n^{(0, 0)} + \sum_{n \in \mathbb{S}_0^{(1)}} \Phi_n^{(\mu_1)} P_n^{(0, 1)} p_{(\mu_1)} + \sum_{n \in \mathbb{S}_0^{(2)}} \Phi_n^{(\mu_1 \mu_2)} P_n^{(0, 2)} p_{(\mu_1} p_{\mu_2)}, \quad (17a)$$

$$\psi_{\mathbf{p}}^{\mu\nu} = \sum_{n \in \mathbb{S}_1^{(0)}} \Psi_n^{\mu\nu} P_n^{(1, 0)} + \sum_{n \in \mathbb{S}_1^{(1)}} \Phi_n^{\mu\nu, (\mu_1)} P_n^{(1, 1)} p_{(\mu_1)}, \quad (17b)$$

where

$$\Phi_0 = - \sum_{n \in \mathbb{S}_0^{(0)} - \{0\}} \frac{\langle q_1 P_n^{(0, 0)} \rangle_{0\mathbf{p}}}{\langle q_1 P_0^{(0, 0)} \rangle_{0\mathbf{p}}} \Phi_n, \quad (18a)$$

$$\Phi_n = - \frac{\langle A_{\mathbf{p}} P_n^{(0, 0)} \rangle_{0\mathbf{p}}}{A_n^{(0, 0)}}, \quad \text{for } n = 1, 2, \quad (18b)$$

$$\Phi_0^{(\mu)} = - \frac{\langle q_2 (p \cdot \Delta \cdot p) P_1^{(0, 1)} \rangle_{0\mathbf{p}}}{\langle q_2 (p \cdot \Delta \cdot p) P_0^{(0, 1)} \rangle_{0\mathbf{p}}} \Phi_1^{(\mu)}, \quad (18c)$$

$$\Phi_1^{(\mu)} = - \frac{1}{A_{(0, 1)}} \langle B_{\mathbf{p}}^{(\mu)} (p \cdot \Delta \cdot p) P_1^{(0, 1)} \rangle_{0\mathbf{p}}, \quad (18d)$$

$$\Phi_0^{(\mu\nu)} = - \frac{1}{A_{(0, 2)}} \langle C_{\mathbf{p}}^{(\mu\nu)} (p \cdot \Delta \cdot p)^2 P_0^{(0, 2)} \rangle_{0\mathbf{p}}, \quad (18e)$$

⁴In the present work, it is still possible to solve for $\phi_{\mathbf{p}}$ without truncating the set $\mathbb{S}_0^{(\ell)}$ following the Appendix A of Ref. [96]. However, this will make it impossible to obtain an analytical solution for $\psi_{\mathbf{p}}^{\mu\nu}$.

$$\Psi_0^{\mu\nu} = - \left[\frac{\langle q_3 P_1^{(1,0)} \rangle_{0\mathbf{p}}}{\langle q_3 P_0^{(1,0)} \rangle_{0\mathbf{p}}} \right] \Psi_1^{\mu\nu}, \quad \Psi_1^{\mu\nu} = C_1^{\mu\nu}, \quad (18f)$$

$$\Psi_0^{\mu\nu, \langle \mu_1 \rangle} = C_0^{\mu\nu, \langle \mu_1 \rangle}, \quad (18g)$$

with $C_1^{\mu\nu}$ being the solution of the equation $C_1^{\mu\nu} + A_1^{\mu\nu\alpha\beta} C_{1,\alpha\beta} = B_1^{\mu\nu}$. The more detailed expressions of the coefficients $\Phi_n^{\langle \mu_1 \dots \mu_\ell \rangle}$ and $\Psi_n^{\mu\nu, \langle \mu_1 \dots \mu_\ell \rangle}$ (in terms of thermodynamic integrals) will be provided in a future publication which will contain a more detailed description including the BDNK-like formulation of spin hydrodynamics. In the Supplemental Material [97] (also see [98,99] for details), I provide expressions of $A_1^{\mu\nu\alpha\beta}$, $B_1^{\mu\nu}$, and $C_0^{\mu\nu, \langle \mu_1 \rangle}$ and give a short description about how $C_1^{\mu\nu}$ can be obtained. It may be noted that, in Eq. (10), while all of the 21 terms are necessary for conservation laws, only five of them contribute in the determination of the coefficients $\Phi_n^{\langle \mu_1 \dots \mu_\ell \rangle}$ and $\Psi_n^{\mu\nu, \langle \mu_1 \dots \mu_\ell \rangle}$. Another important point is that the coefficients of $\psi_{\mathbf{p}}^{\mu\nu}$ were obtained using the fact that, due to the presence of the term $s : \psi_{\mathbf{p}}$ in Eq. (11), any part of $\psi_{\mathbf{p}}^{\mu\nu}$ that is parallel to momentum four-vector can be ignored, implying $p_\mu \psi_{\mathbf{p}}^{\mu\nu} = 0 = p_\mu \psi_{\mathbf{p}}^{\nu\mu}$.

Transport properties. In the previous section, the nonequilibrium corrections to the distribution function have been determined. Using this information, the transport properties can now be evaluated. Substituting Eq. (11) with the help of Eqs. (17a), (17b), and (18b)–(18g) into Eqs. (3a)–(3h) one finds

$$\delta\varepsilon = \sum_{n \in S_0^{(0)}} \Phi_n \langle E_{\mathbf{p}}^2 P_n^{(0,0)} \rangle_{0\mathbf{p}}, \quad (19a)$$

$$\delta P = - \sum_{n \in S_0^{(0)}} \Phi_n \langle (1/3)(p \cdot \Delta \cdot p) P_n^{(0,0)} \rangle_{0\mathbf{p}}, \quad (19b)$$

$$q^\mu = \sum_{n \in S_0^{(1)}} \Phi_n^{\langle \mu_1 \rangle} \langle (1/3) E_{\mathbf{p}} (p \cdot \Delta \cdot p) P_n^{(0,1)} \rangle_{0\mathbf{p}}, \quad (19c)$$

$$\pi^{\mu\nu} = \Phi_0^{\langle \mu\nu \rangle} \langle (2/15)(p \cdot \Delta \cdot p)^2 \rangle_{0\mathbf{p}}, \quad (19d)$$

$$\delta S^{\mu\nu} = \sum_{n \in S_1^{(0)}} \Psi_n^{\mu\nu} \langle E_{\mathbf{p}} P_n^{(1,0)} \rangle_{0\mathbf{p}}, \quad (19e)$$

$$\delta \Sigma_{(s)}^{\langle \mu\nu \rangle} = u_\alpha \Delta_{\rho\gamma}^{\mu\nu} \Psi_0^{\alpha\rho, \langle \gamma \rangle} \langle (1/3)(p \cdot \Delta \cdot p) \rangle_{0\mathbf{p}}, \quad (19f)$$

$$\delta \Sigma_{(a)}^{\mu\nu} = \sum_{n \in S_1^{(0)}} u_\alpha \Psi_n^{\alpha[\mu} u^{\nu]} \langle E_{\mathbf{p}} P_n^{(1,0)} \rangle_{0\mathbf{p}} + u_\alpha \Psi_0^{\alpha[\mu, \langle \nu \rangle]} \langle (1/3)(p \cdot \Delta \cdot p) \rangle_{0\mathbf{p}}, \quad (19g)$$

$$\delta \Sigma^{\lambda, \mu\nu} = \Psi_0^{\langle \mu \rangle \langle \nu \rangle, \langle \lambda \rangle} \langle (p \cdot \Delta \cdot p) \rangle_{0\mathbf{p}}. \quad (19h)$$

If one assumes the relaxation time to be independent of particle momenta and uses $q_1 = E_{\mathbf{p}}^2$, $q_2 = q_3 = E_{\mathbf{p}}$ in Eq. (16), then the collision kernel reduces back to the standard RTA [68] and the transport coefficients coincide with those obtained in Ref. [28]. The expressions of Eqs. (19a)–(19d) show that the hydrodynamic variables remain impervious to the spin dynamics, whereas the spin transport properties are influenced by the evolution of hydrodynamic variables. This is the expected behavior of relativistic fluids at small polarization in the absence of any external field's influence, as observed in Refs. [28,73,74]. Such a phenomenon is analogous to the Barnett effect [100], where a rotating magnetizable fluid can become magnetized under the influence of the rotation.

Conclusions and outlook. In this work, I proposed a new collision kernel that consistently guarantees macroscopic conservation laws of energy-momentum tensor and the spin tensor of a relativistic fluid whose evolution is dictated by local collisions. I solved the Boltzmann equation with this collision kernel in the limit of small polarization such that the relaxation time can be a function of particle four-momenta. The transport coefficients were determined under a general frame and matching conditions. Following the BDNK formulation of first-order relativistic hydrodynamics, the approach described here does not replace the comoving derivatives. One may therefore expect this theory of first-order spin hydrodynamics to be causal. Hence, in the future, I would like to perform the linear mode analysis of this theory [87,88,101,102].

I would like to extend this formulation to include the effect of the background magnetic field and obtain the transport coefficients for spin magnetohydrodynamics [73]. However, this can be achieved only after deriving a collision kernel that can describe particle and antiparticle pair production and annihilation. In the present work, the part of the collision kernel that exchanges only spin has been ignored for the sake of simplicity. It will be interesting to explore the implications of such a term in future studies.

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