Quark spin correlation inside hyperons

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The global spin polarization of hyperons in heavy-ion collisions have been investigated by including spin-correlation effects among their constituent quarks. The available data on global spin polarizations of hyperons and spin alignments of vector mesons provide constraints on phase-space functions of the spin polarization and correlation. These constraints can lead to inequalities under some approximations, which might provide possible clues for the presence of quark spin correlation inside hyperons at lower collision energies.

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I. INTRODUCTION

Spin physics in heavy-ion collisions has attracted a lot of interest since the first measurement of the global polarization of Λ hyperons by the STAR Collaboration at the Relativistic Heavy Ion Collider (RHIC) [1] followed by a series of later experiments at different energies or under different conditions [2–6]. This confirms the theoretical predictions made almost two decades ago [7–12].

Recently, the STAR Collaboration measured nonvanishing global spin alignment of the ϕ vector meson [13], implying a strong spin correlation between the strange and antistrange quark that combine to form the ϕ meson [14–16], for recent reviews, see, e.g., Refs. [17,18]. The experimental observations and corresponding theoretical studies provide an opportunity to study the spin correlations [19,20] in high-energy heavy-ion collisions for the first time. Very recently, the STAR Collaboration also measured the spin correlation between Λ and $\bar{\Lambda}$ in proton-proton collisions [21], which are inherited from spin-correlated strange quark-antiquark virtual pairs [22–26], which could serve as probes into QCD confinement.

A systematic description of quark (antiquark) spin correlations in quark matter can be provided by the multiquark spin-density matrix (SDM) in spin space [27,28]. From the multiquark SDM, one can obtain the SDM for a vector meson/hyperon through the transition operator to convert quark-antiquark/three quarks to the vector meson/hyperon. The hadron's spin polarization can then be determined by its SDM and depends not only on those of its constituent quarks but also on the spin correlations among them. The spin correlations in previous calculations of global and local spin polarizations for hyperons were all neglected: it seems that the data could be well described without the need for spin correlations [29–40]. However, there is an indication for a possible disagreement between the data and model calculations for the global polarization of Ω hyperons [41]. A natural question arises if the possible disagreement has to do with neglecting the spin correlation among strange quarks in Ω. In this paper, we try to examine this problem by including the spin-correlation effects into the spin polarizations of hyperons. Based on density operators for quarks (antiquarks) in spin-momentum space, we can derive spin-Wigner functions as the SDM elements in phase space for vector mesons and baryons which are expressed as phase-space integrals of their constituent quarks' SDM weighted by their coordinate wave functions, constituent quark distributions, and Clebsch-Gordan coefficients. In this way we obtain spin polarizations of vector mesons and hyperons depending on spin polarization and correlation functions for quarks. In comparison with data for global spin polarizations of hyperons and spin alignments of vector mesons, one can derive constraints for quark spin correlations under some approximations.

By convention, the components of a three-vector can be expressed as $\boldsymbol{a} = (a_x, a_y, a_z) = (a^x, a^y, a^z)$, where we do not distinguish superscripts and subscripts for its spatial components.

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II. SPIN-WIGNER FUNCTIONS FOR VECTOR MESONS AND BARYONS

Relativistic kinetic theory in the Wigner-function formalism [42–49] can describe the spin polarization of spin-1/2 massless [50–64] and massive fermions [65–73] determined by the axial-vector or tensor component of the Wigner function, see, e.g., Refs. [74,75] for recent reviews on this topic.

The spin-Wigner functions (SWF) are actually the spin-density matrix elements in phase space. The SWF for vector mesons and baryons can be obtained from those for quark-antiquark and three quarks through the spin-flavor or SU(6) wave functions [14,27,28,76]. In this section, we will derive the SWF in phase space for vector mesons and baryons in nonrelativistic quantum theory.

A. Spin-density operators for one-particle, two-particle, and three-particle systems

We start from the density operator for a nonrelativistic spin-1/2 particle on mass shell defined as

$$\begin{aligned}\rho^{\rm op}_{(1)} &= \sum_{r,s}\int [d^3\boldsymbol{p}_1][d^3\boldsymbol{p}_2] W(r,s;\boldsymbol{p}_1,\boldsymbol{p}_2)|r,\boldsymbol{p}_1\rangle\langle s,\boldsymbol{p}_2| \\ &= \sum_{r,s}\int [d^3\boldsymbol{p}][d^3\boldsymbol{q}] d^3\boldsymbol{x} e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} W(r,s;\boldsymbol{x},\boldsymbol{p}) \\ &\quad\times\Big|r,\boldsymbol{p}+\frac{\boldsymbol{q}}{2}\Big\rangle\Big\langle s,\boldsymbol{p}-\frac{\boldsymbol{q}}{2}\Big| \\ &= \sum_{r,s}\int d^3\boldsymbol{x}\int [d^3\boldsymbol{p}] W(r,s;\boldsymbol{x},\boldsymbol{p})\int [d^3\boldsymbol{q}] e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \\ &\quad\times\Big|r,\boldsymbol{p}+\frac{\boldsymbol{q}}{2}\Big\rangle\Big\langle s,\boldsymbol{p}-\frac{\boldsymbol{q}}{2}\Big|. \end{aligned} \tag{1}$$

Here, $r,s=\pm 1/2$ denote the spin states along the spin-quantization direction. We have used the shorthand notation $[d^3\boldsymbol{q}]\equiv d^3\boldsymbol{q}/(2\pi)^3$, and changed in second line the momenta $(\boldsymbol{p}_1,\boldsymbol{p}_2)$ to $(\boldsymbol{p},\boldsymbol{q})$ with $\boldsymbol{p}_{1,2}=\boldsymbol{p}\pm\boldsymbol{q}/2$; we also defined the Wigner function $W(r,s;\boldsymbol{x},\boldsymbol{p})$ as

$$\begin{aligned} W(r,s;\boldsymbol{q},\boldsymbol{p}) &\equiv W(r,s;\boldsymbol{p}+\boldsymbol{q}/2,\boldsymbol{p}-\boldsymbol{q}/2) \\ &= \int d^3\boldsymbol{x} e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} W(r,s;\boldsymbol{x},\boldsymbol{p}), \\ W(r,s;\boldsymbol{x},\boldsymbol{p}) &= \int [d^3\boldsymbol{q}] e^{i\boldsymbol{q}\cdot\boldsymbol{x}} W(r,s;\boldsymbol{q},\boldsymbol{p}) \\ &= \int [d^3\boldsymbol{q}] e^{i\boldsymbol{q}\cdot\boldsymbol{x}}\Big\langle r,\boldsymbol{p}+\frac{\boldsymbol{q}}{2}\Big|\rho^{\rm op}_{(1)}\Big|s,\boldsymbol{p}-\frac{\boldsymbol{q}}{2}\Big\rangle, \end{aligned} \tag{2}$$

where $\boldsymbol{x}$ is the particle's position conjugate to the relative momentum $\boldsymbol{q}$, and we have used the orthogonality relation for momentum states

$$\langle r,\boldsymbol{p}|s,\boldsymbol{k}\rangle = (2\pi)^3\delta_{rs}\delta^{(3)}(\boldsymbol{p}-\boldsymbol{k}). \tag{3}$$

We assume that the Wigner function of a nonrelativistic particle can be factorized into the polarized and unpolarized parts

$$\begin{aligned} W(r,s;\boldsymbol{x},\boldsymbol{p}) &= f(\boldsymbol{x},\boldsymbol{p})\langle r|\hat{\rho}_{(1)}(\boldsymbol{x},\boldsymbol{p})|s\rangle \\ &\equiv f(\boldsymbol{x},\boldsymbol{p})\tfrac{1}{2}[\delta_{rs}+\boldsymbol{\sigma}_{rs}\cdot\boldsymbol{P}(\boldsymbol{x},\boldsymbol{p})], \end{aligned} \tag{4}$$

where $\boldsymbol{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ denote the Pauli matrices, $f(\boldsymbol{x},\boldsymbol{p})$ and $\boldsymbol{P}(\boldsymbol{x},\boldsymbol{p})$ are the unpolarized and polarized distribution functions, respectively, and the second line defines the SDM for one particle $\hat{\rho}_{(1)}(\boldsymbol{x},\boldsymbol{p})$.

The density operator for two spin-1/2 particles can be defined as

$$\begin{aligned}\rho^{\rm op}_{(12)} &= \sum_{r_1,s_1,r_2,s_2}\int\prod_{n=1,2}[d^3\boldsymbol{p}_n][d^3\boldsymbol{q}_n]d^3\boldsymbol{x}_n e^{-i\boldsymbol{q}_n\cdot\boldsymbol{x}_n} \\ &\quad\times W(\{r_n,s_n;\boldsymbol{x}_n,\boldsymbol{p}_n\},n=1,2) \\ &\quad\times\prod_{n=1,2}^{\otimes}\Big|r_n,\boldsymbol{p}_n+\frac{\boldsymbol{q}_n}{2}\Big\rangle\Big\langle s_n,\boldsymbol{p}_n-\frac{\boldsymbol{q}_n}{2}\Big|, \end{aligned} \tag{5}$$

where the joint Wigner function for two particles can be put into the form

$$\begin{aligned} &W(\{r_n,s_n;\boldsymbol{x}_n,\boldsymbol{p}_n\},n=1,2) \\ &\quad= f_1(\boldsymbol{x}_1,\boldsymbol{p}_1)f_2(\boldsymbol{x}_2,\boldsymbol{p}_2)\langle r_1r_2|\hat{\rho}_{(12)}|s_1s_2\rangle, \end{aligned} \tag{6}$$

where $f_n(\boldsymbol{x}_n,\boldsymbol{p}_n)$ with $n=1,2$ are one-particle distribution functions, and the SDM for two particles $\hat{\rho}_{(12)}$ can be parametrized as

$$\begin{aligned}\hat{\rho}_{(12)} &= \hat{\rho}_{(1)}(\boldsymbol{x}_1,\boldsymbol{p}_1)\otimes\hat{\rho}_{(2)}(\boldsymbol{x}_2,\boldsymbol{p}_2) \\ &\quad+\frac{1}{2^2}c^{(12)}_{ij}(\boldsymbol{x}_1,\boldsymbol{p}_1,\boldsymbol{x}_2,\boldsymbol{p}_2)\sigma^{(1)}_i\otimes\sigma^{(2)}_j, \end{aligned} \tag{7}$$

with $i,j=x,y,z$ being the indices of Pauli matrices. Note that $\hat{\rho}_{(n)}(\boldsymbol{x}_n,\boldsymbol{p}_n)$ with $n=1,2$ are one-particle SDM defined in Eq. (4) and $c^{(12)}_{ij}(\boldsymbol{x}_1,\boldsymbol{p}_1,\boldsymbol{x}_2,\boldsymbol{p}_2)$ is the spin-correlation function between two particles.

Similar to Eq. (5), we can define the density operator $\rho^{\rm op}_{(123)}$ for three particles with the joint Wigner function similar to Eq. (6),

$$\begin{aligned} &W(\{r_n,s_n;\boldsymbol{x}_n,\boldsymbol{p}_n\},n=1,2,3) \\ &\quad=\prod_{n=1,2,3}f_n(\boldsymbol{x}_n,\boldsymbol{p}_n)\langle r_1r_2r_3|\hat{\rho}_{(123)}|s_1s_2s_3\rangle, \end{aligned} \tag{8}$$

where the SDM for three particles is given as

$$\begin{aligned}\hat{\rho}_{(123)} &= \hat{\rho}^{(1)}(\boldsymbol{x}_1,\boldsymbol{p}_1)\otimes\hat{\rho}^{(2)}(\boldsymbol{x}_2,\boldsymbol{p}_2)\otimes\hat{\rho}^{(3)}(\boldsymbol{x}_3,\boldsymbol{p}_3) \\ &\quad+\frac{1}{2^3}c^{(123)}_{ijk}(\boldsymbol{x}_1,\boldsymbol{p}_1;\boldsymbol{x}_2,\boldsymbol{p}_2;\boldsymbol{x}_3,\boldsymbol{p}_3)\sigma^{(1)}_i\otimes\sigma^{(2)}_j\otimes\sigma^{(3)}_k \\ &\quad+\frac{1}{2^2}\big[c^{(12)}_{ij}(\boldsymbol{x}_1,\boldsymbol{p}_1;\boldsymbol{x}_2,\boldsymbol{p}_2)\sigma^{(1)}_i\otimes\sigma^{(2)}_j\otimes\hat{\rho}^{(3)}(\boldsymbol{x}_3,\boldsymbol{p}_3) \\ &\quad+c^{(23)}_{jk}(\boldsymbol{x}_2,\boldsymbol{p}_2;\boldsymbol{x}_3,\boldsymbol{p}_3)\hat{\rho}^{(1)}(\boldsymbol{x}_1,\boldsymbol{p}_1)\otimes\sigma^{(2)}_j\otimes\sigma^{(3)}_k \\ &\quad+c^{(13)}_{ik}(\boldsymbol{x}_1,\boldsymbol{p}_1;\boldsymbol{x}_3,\boldsymbol{p}_3)\sigma^{(1)}_i\otimes\hat{\rho}^{(2)}(\boldsymbol{x}_2,\boldsymbol{p}_2)\otimes\sigma^{(3)}_k\big]. \end{aligned} \tag{9}$$

Here $i,j,k=x,y,z$ are the indices of Pauli matrices, and $c^{(123)}_{ijk}$ is the spin-correlation function for three particles.

B. Spin-Wigner function for vector mesons and baryons

Now we can apply the density operators $\rho^{\rm op}_{(12)}$ and $\rho^{\rm op}_{(123)}$ to the quark-antiquark and three-quark system, respectively,

in which we used the shorthand notations $(12) \equiv (q_1\bar{q}_2)$ and $(123) \equiv (q_1q_2q_3)$. We consider hadronization processes $q_1\bar{q}_2 \to V$ and $q_1q_2q_3 \to B$. The spin-density operators of the vector meson and baryon are given by

$$\rho_V^{\rm op} = \mathcal{M}_V \rho^{\rm op}_{(q_1\bar{q}_2)} \mathcal{M}_V^\dagger,$$
$$\rho_B^{\rm op} = \mathcal{M}_B \rho^{\rm op}_{(q_1q_2q_3)} \mathcal{M}_B^\dagger, \tag{10}$$

where $\mathcal{M}_{V/B}$ denote the transition operators for $q_1\bar{q}_2$ and $q_1q_2q_3$ to form the vector meson V and baryon B respectively. The SWF for the vector meson and baryon in phase space can be obtained by

$$\rho^{V/B}_{m'm}(\boldsymbol{x},\boldsymbol{p}) = \int [d^3\boldsymbol{q}] e^{i\boldsymbol{q}\cdot\boldsymbol{x}} \left\langle jm'; \boldsymbol{p}+\frac{\boldsymbol{q}}{2}\middle|\rho^{\rm op}_{V/B}\middle| jm; \boldsymbol{p}-\frac{\boldsymbol{q}}{2}\right\rangle, \tag{11}$$

where j and $m(m')$ denote the spin and magnetic quantum numbers for the vector meson and baryon.

Evaluation of $\rho^{V/B}_{m'm}$ by Eq. (11) involves the amplitudes of $\mathcal{M}$ which map the quarks to hadrons in the forms

$$\left\langle s_1s_2; \boldsymbol{p}_1-\frac{\boldsymbol{q}_1}{2}, \boldsymbol{p}_2-\frac{\boldsymbol{q}_2}{2}\middle|\mathcal{M}_V^\dagger\middle| jm; \boldsymbol{p}-\frac{\boldsymbol{q}}{2}\right\rangle$$
$$\approx \langle s_1s_2|\mathcal{M}_V^{\dagger\rm spin}|jm\rangle \left\langle \boldsymbol{p}_1-\frac{\boldsymbol{q}_1}{2}, \boldsymbol{p}_2-\frac{\boldsymbol{q}_2}{2}\middle|\mathcal{M}_V^{\dagger\rm mom}\middle|\boldsymbol{p}-\frac{\boldsymbol{q}}{2}\right\rangle, \tag{12}$$

$$\left\langle s_1s_2s_3; \boldsymbol{p}_1-\frac{\boldsymbol{q}_1}{2}, \boldsymbol{p}_2-\frac{\boldsymbol{q}_2}{2}, \boldsymbol{p}_3-\frac{\boldsymbol{q}_3}{2}\middle|\mathcal{M}_B^\dagger\middle| jm; \boldsymbol{p}-\frac{\boldsymbol{q}}{2}\right\rangle$$
$$\approx \langle s_1s_2s_3|\mathcal{M}_B^{\dagger\rm spin}|jm\rangle \times \left\langle \boldsymbol{p}_1-\frac{\boldsymbol{q}_1}{2}, \boldsymbol{p}_2-\frac{\boldsymbol{q}_2}{2}, \boldsymbol{p}_3-\frac{\boldsymbol{q}_3}{2}\middle|\mathcal{M}_B^{\dagger\rm mom}\middle|\boldsymbol{p}-\frac{\boldsymbol{q}}{2}\right\rangle. \tag{13}$$

Here, we have used the factorization ansatz for the transition operators $\mathcal{M}_{V/B} = \mathcal{M}^{\rm spin}_{V/B} \otimes \mathcal{M}^{\rm mom}_{V/B}$, where $\mathcal{M}^{\rm spin}_{V/B}$ and $\mathcal{M}^{\rm mom}_{V/B}$ are the spin and momentum parts of $\mathcal{M}_{V/B}$, respectively. The spin parts of the amplitudes are assumed to be proportional to the Clebsch-Gordan coefficients

$$\langle s_1s_2|\mathcal{M}_V^{\dagger\rm spin}|jm\rangle = N_V \langle s_1s_2|\, jm\rangle = N_V C(jm; s_1s_2),$$
$$\langle s_1s_2s_3|\mathcal{M}_B^{\dagger\rm spin}|jm\rangle = N_B \langle s_1s_2s_3|\, jm\rangle = N_B C(jm; s_1s_2s_3). \tag{14}$$

Here, N_V and N_B are two proportionality constants. The momentum parts of the amplitudes are assumed to be proportional to the vector meson's and baryon's nonrelativistic wave functions

$$\langle \boldsymbol{p}_1, \boldsymbol{p}_2|\mathcal{M}_V^{\dagger\rm mom}|\boldsymbol{p}\rangle = (2\pi)^3 \psi_V(\boldsymbol{p}_b)\delta^{(3)}(\boldsymbol{p}-\boldsymbol{p}_1-\boldsymbol{p}_2),$$
$$\langle \boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3|\mathcal{M}_B^{\dagger\rm mom}|\boldsymbol{p}\rangle = (2\pi)^3 \psi_B(\boldsymbol{p}_b, \boldsymbol{p}_c) \times \delta^{(3)}(\boldsymbol{p}-\boldsymbol{p}_1-\boldsymbol{p}_2-\boldsymbol{p}_3), \tag{15}$$

where $\boldsymbol{p}_b \equiv (\boldsymbol{p}_1-\boldsymbol{p}_2)/2$, $\boldsymbol{p}_c \equiv (\boldsymbol{p}_1+\boldsymbol{p}_2-2\boldsymbol{p}_3)/3$, and $\psi_{V,B}$ are the vector meson's and baryon's wave functions in momentum space.

Using Eqs. (7) and (12) in Eq. (11), we finally obtain the SWF for vector mesons as

$$\rho^V_{m'm}(\boldsymbol{x},\boldsymbol{p}) \approx N_V^2 \int d^3\boldsymbol{x}_b [d^3\boldsymbol{p}_b][d^3\boldsymbol{q}_b] \exp(-i\boldsymbol{q}_b\cdot\boldsymbol{x}_b) \times f_q\left(\boldsymbol{x}+\frac{1}{2}\boldsymbol{x}_b, \frac{1}{2}\boldsymbol{p}+\boldsymbol{p}_b\right) f_{\bar{q}}\left(\boldsymbol{x}-\frac{1}{2}\boldsymbol{x}_b, \frac{1}{2}\boldsymbol{p}-\boldsymbol{p}_b\right) \times \psi_V^*\left(\boldsymbol{p}_b+\frac{1}{2}\boldsymbol{q}_b\right)\psi_V\left(\boldsymbol{p}_b-\frac{1}{2}\boldsymbol{q}_b\right) \times \sum_{r_1,s_1,r_2,s_2} C(jm; s_1s_2)C^*(jm'; r_1r_2)\langle r_1r_2|\hat{\rho}_{(q_1\bar{q}_2)}|s_1s_2\rangle. \tag{16}$$

In the elements of SDM $\langle r_1r_2|\hat{\rho}_{(q_1\bar{q}_2)}|s_1s_2\rangle$, we have set $\boldsymbol{x}_{1,2} = \boldsymbol{x} \pm \boldsymbol{x}_b/2$ and $\boldsymbol{p}_{1,2} = \boldsymbol{p}/2 \pm \boldsymbol{p}_b$, where $\boldsymbol{x}_{1,2}$ and $\boldsymbol{p}_{1,2}$ are the positions and momenta of the quark and antiquark defined in Eq. (5). Note that the meson's wave function in momentum space is used in Eq. (16). We can rewrite Eq. (16) in terms of the meson's wave function in coordinate space as

$$\rho^V_{m'm}(\boldsymbol{x},\boldsymbol{p}) \approx N_V^2 \int d^3\boldsymbol{y}_1 d^3\boldsymbol{y}_2 [d^3\boldsymbol{p}_b] \exp[i\boldsymbol{p}_b\cdot(\boldsymbol{y}_1-\boldsymbol{y}_2)]\psi_V^*(\boldsymbol{y}_1)\psi_V(\boldsymbol{y}_2) \times f_q\left(\boldsymbol{x}+\frac{1}{4}(\boldsymbol{y}_1+\boldsymbol{y}_2), \frac{1}{2}\boldsymbol{p}+\boldsymbol{p}_b\right) f_{\bar{q}}\left(\boldsymbol{x}-\frac{1}{4}(\boldsymbol{y}_1+\boldsymbol{y}_2), \frac{1}{2}\boldsymbol{p}-\boldsymbol{p}_b\right) \times \sum_{r_1,s_1,r_2,s_2} C(jm; s_1s_2)C^*(jm'; r_1r_2)\langle r_1r_2|\hat{\rho}_{(q_1\bar{q}_2)}|s_1s_2\rangle, \tag{17}$$

where we have used the Fourier transformation for the meson's wave functions (the upper/lower sign corresponds to $\boldsymbol{y}_1/\boldsymbol{y}_2$)

$$\psi_V\left(\boldsymbol{p}_b \pm \frac{1}{2}\boldsymbol{q}_b\right) = \int d^3\boldsymbol{y}_{1,2}\psi_V(\boldsymbol{y}_{1,2}) \exp\left[-i\left(\boldsymbol{p}_b \pm \frac{1}{2}\boldsymbol{q}_b\right)\cdot\boldsymbol{y}_{1,2}\right], \tag{18}$$

and set $\boldsymbol{x}_{1,2} = \boldsymbol{x} \pm (\boldsymbol{y}_1+\boldsymbol{y}_2)/4$ and $\boldsymbol{p}_{1,2} = \boldsymbol{p}/2 \pm \boldsymbol{p}_b$ in all phase-space functions in $\langle r_1r_2|\hat{\rho}_{(q_1\bar{q}_2)}|s_1s_2\rangle$.

Similarly, using Eqs. (9) and (13) in Eq. (11), we can derive the SWF for baryons as

$$\rho^B_{m'm}(\boldsymbol{x},\boldsymbol{p}) \approx N_B^2 \int \prod_{n=b,c} [d^3\boldsymbol{p}_n][d^3\boldsymbol{q}_n] d^3\boldsymbol{x}_n \exp\left(-i\boldsymbol{q}_b\cdot\boldsymbol{x}_b - i\boldsymbol{q}_c\cdot\boldsymbol{x}_c\right) f_{q_1}(\boldsymbol{x}_1,\boldsymbol{p}_1) f_{q_2}(\boldsymbol{x}_2,\boldsymbol{p}_2) f_{q_3}(\boldsymbol{x}_3,\boldsymbol{p}_3) \psi_B^*\left(\boldsymbol{p}_b+\frac{1}{2}\boldsymbol{q}_b, \boldsymbol{p}_c+\frac{1}{2}\boldsymbol{q}_c\right)$$
$$\times \psi_B\left(\boldsymbol{p}_b-\frac{1}{2}\boldsymbol{q}_b, \boldsymbol{p}_c-\frac{1}{2}\boldsymbol{q}_c\right) \sum_{\{r_n,s_n,n=1,2,3\}} C(jm;s_1s_2s_3) C^*(jm';r_1r_2r_3) \langle r_1r_2r_3|\hat{\rho}_{(q_1q_2q_3)}|s_1s_2s_3\rangle, \tag{19}$$

where we have set the positions and momenta of three quarks in the distribution functions and in $\langle r_1r_2r_3|\hat{\rho}_{(q_1q_2q_3)}|s_1s_2s_3\rangle$ as

$$\begin{aligned}
\boldsymbol{x}_1 &= \boldsymbol{x} + \frac{1}{2}\boldsymbol{x}_b + \frac{1}{3}\boldsymbol{x}_c,\\
\boldsymbol{x}_2 &= \boldsymbol{x} - \frac{1}{2}\boldsymbol{x}_b + \frac{1}{3}\boldsymbol{x}_c,\\
\boldsymbol{x}_3 &= \boldsymbol{x} - \frac{2}{3}\boldsymbol{x}_c,
\end{aligned} \tag{20}$$

$$\begin{aligned}
\boldsymbol{p}_1 &= \frac{1}{3}\boldsymbol{p} + \boldsymbol{p}_b + \frac{1}{2}\boldsymbol{p}_c,\\
\boldsymbol{p}_2 &= \frac{1}{3}\boldsymbol{p} - \boldsymbol{p}_b + \frac{1}{2}\boldsymbol{p}_c,\\
\boldsymbol{p}_3 &= \frac{1}{3}\boldsymbol{p} - \boldsymbol{p}_c.
\end{aligned} \tag{21}$$

We can also rewrite Eq. (19) in terms of the baryon's wave functions in coordiante space as

$$\begin{aligned}
&\rho^B_{m'm}(\boldsymbol{x},\boldsymbol{p})\\
&\approx N_B^2 \int d^3\boldsymbol{y}_1 d^3\boldsymbol{y}_2 d^3\boldsymbol{z}_1 d^3\boldsymbol{z}_2 [d^3\boldsymbol{p}_b][d^3\boldsymbol{p}_c]\\
&\quad\times \exp\left[i\boldsymbol{p}_b\cdot(\boldsymbol{y}_1-\boldsymbol{y}_2) + i\boldsymbol{p}_c\cdot(\boldsymbol{z}_1-\boldsymbol{z}_2)\right]\\
&\quad\times f_{q_1}(\boldsymbol{x}_1,\boldsymbol{p}_1) f_{q_2}(\boldsymbol{x}_2,\boldsymbol{p}_2) f_{q_3}(\boldsymbol{x}_3,\boldsymbol{p}_3) \psi_B^*(\boldsymbol{y}_1,\boldsymbol{z}_1)\psi_B(\boldsymbol{y}_2,\boldsymbol{z}_2)\\
&\quad\times \sum_{\{r_n,s_n,n=1,2,3\}} C(jm;s_1s_2s_3) C^*(jm';r_1r_2r_3)\\
&\quad\times \langle r_1r_2r_3|\hat{\rho}_{(q_1q_2q_3)}|s_1s_2s_3\rangle,
\end{aligned} \tag{22}$$

where we have set the positions of three quarks in the distribution functions and in $\langle r_1r_2r_3|\hat{\rho}_{(q_1q_2q_3)}|s_1s_2s_3\rangle$ as

$$\begin{aligned}
\boldsymbol{x}_1 &= \boldsymbol{x} + \frac{1}{4}(\boldsymbol{y}_1+\boldsymbol{y}_2) + \frac{1}{6}(\boldsymbol{z}_1+\boldsymbol{z}_2),\\
\boldsymbol{x}_2 &= \boldsymbol{x} - \frac{1}{4}(\boldsymbol{y}_1+\boldsymbol{y}_2) + \frac{1}{6}(\boldsymbol{z}_1+\boldsymbol{z}_2),\\
\boldsymbol{x}_3 &= \boldsymbol{x} - \frac{1}{3}(\boldsymbol{z}_1+\boldsymbol{z}_2),
\end{aligned} \tag{23}$$

while the momenta of three quarks are the same as in Eq. (21). We also used the Fourier transformation to convert the baryon's wave functions from momentum space to coordinate space

$$\begin{aligned}
&\psi_B\left(\boldsymbol{p}_b \pm \frac{1}{2}\boldsymbol{q}_b, \boldsymbol{p}_c \pm \frac{1}{2}\boldsymbol{q}_c\right)\\
&= \int d^3\boldsymbol{y}_{1,2} d^3\boldsymbol{z}_{1,2} \psi_B(\boldsymbol{y}_{1,2},\boldsymbol{z}_{1,2})\\
&\quad\times \exp\left[-i\left(\boldsymbol{p}_b \pm \frac{1}{2}\boldsymbol{q}_b\right)\cdot\boldsymbol{y}_{1,2} - i\left(\boldsymbol{p}_c \pm \frac{1}{2}\boldsymbol{q}_c\right)\cdot\boldsymbol{z}_{1,2}\right],
\end{aligned} \tag{24}$$

where the upper/lower sign corresponds to $(\boldsymbol{y}_1,\boldsymbol{z}_1)/(\boldsymbol{y}_2,\boldsymbol{z}_2)$.

C. Short- and long-range correlations

We have finally derived in Eqs. (17) and (22) the SWF for the vector meson and baryon in phase space, which are the main results of this section. In the SWF for the vector meson given by Eq. (17), we see integrals over coordinates $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ in the vector meson's wave functions $\psi_V^*(\boldsymbol{y}_1)\psi_V(\boldsymbol{y}_2)$ and also an integral over $\boldsymbol{p}_b$ (the relative momentum of q and $\bar{q}$) which is conjugate to $\boldsymbol{y}_1-\boldsymbol{y}_2$. In the integrand, there are phase- space distributions for q and $\bar{q}$ which depend on $\boldsymbol{y}_1+\boldsymbol{y}_2$ and $\boldsymbol{p}_b$. As part of the integrand, the polarization functions $\boldsymbol{P}_q(\boldsymbol{x}_1,\boldsymbol{p}_1)$, $\boldsymbol{P}_{\bar{q}}(\boldsymbol{x}_2,\boldsymbol{p}_2)$, and correlation function $c_{ij}^{(12)}(\boldsymbol{x}_1,\boldsymbol{p}_1,\boldsymbol{x}_2,\boldsymbol{p}_2)$, or any product of them, appear in the SDM. In other words, the SWF for the vector meson involves an average of the polarization and correlation functions for q and $\bar{q}$ over the phase-space volume defined by the meson's wave function and weighted by the distribution functions of q and $\bar{q}$. So the averages (or integrals) over $\boldsymbol{y}_1$, $\boldsymbol{y}_2$, and $\boldsymbol{p}_b$ is restricted by the meson's wave function and correspond to the short-range correlation for q and $\bar{q}$. In contrast, an average over the phase-space variables $(\boldsymbol{x},\boldsymbol{p})$ of the vector meson corresponds to the long-range correlation outside the meson's internal space.

Similar to the case of the vector meson, there are also short- and long-range correlations of three quarks in the baryon. The only difference from the meson's case is that the short-range correlation in the baryon involves the integrals over $(\boldsymbol{y}_1,\boldsymbol{z}_1)$, $(\boldsymbol{y}_2,\boldsymbol{z}_2)$, $\boldsymbol{p}_b$ (conjugate to $\boldsymbol{y}_1-\boldsymbol{y}_2$), and $\boldsymbol{p}_c$ (conjugate to $\boldsymbol{z}_1-\boldsymbol{z}_2$) in the baryon's SWF. This is because the baryon's wave function depends on two relative coordinates $\boldsymbol{y}$ and $\boldsymbol{z}$ of three quarks as shown in Eq. (23).

We can rewrite Eqs. (17) and (22) into compact forms:

$$\rho^V_{m'm}(\boldsymbol{x},\boldsymbol{p}) \approx N_V^2 \sum_{r_1,s_1,r_2,s_2} C(jm;s_1s_2) C^*(jm';r_1r_2) \times \langle r_1r_2|\langle\hat{\rho}_{(q_1\bar{q}_2)}(\boldsymbol{x}_1,\boldsymbol{p}_1,\boldsymbol{x}_2,\boldsymbol{p}_2)\rangle_V|s_1s_2\rangle, \tag{25}$$

$$\rho^B_{m'm}(\boldsymbol{x},\boldsymbol{p}) \approx N_B^2 \sum_{\{r_n,s_n,n=1,2,3\}} C(jm;s_1s_2s_3) C^*(jm';r_1r_2r_3) \times \langle r_1r_2r_3|\langle\hat{\rho}_{(q_1q_2q_3)}(\boldsymbol{x}_1,\boldsymbol{p}_1,\boldsymbol{x}_2,\boldsymbol{p}_2,\boldsymbol{x}_3,\boldsymbol{p}_3)\rangle_B \times |s_1s_2s_3\rangle, \tag{26}$$

where $\langle\hat{\rho}_{(q_1\bar{q}_2)}\rangle_V$ and $\langle\hat{\rho}_{(q_1q_2q_3)}\rangle_B$ represent the averages (integrals) in Eqs. (17) and (22) over phase-space volumes inside the meson and baryon, respectively, that give the short-range correlations.

D. Hadronization process as a quantum measurement

In quantum information science (QIS), a quantum measurement process is, in general, made through a collection of measurement operators satisfying a completeness relation, which define a generalized measurement [77]. Projective (von Neumann) measurements arise as a special case in which the measurement operators are orthogonal projectors. In Ref. [78], the decays of hyperons are described as effective generalized measurements acting on the spin degrees of freedom. In the following we argue that the hadronization process can also be interpreted as a type of generalized measurement.

The key observation is that the transition operators $\mathcal{M}_{V/B} = \mathcal{M}^{\rm spin}_{V/B} \otimes \mathcal{M}^{\rm mom}_{V/B}$ in Eq. (10) play the role of measurement operators. The effects of $\mathcal{M}_{V/B}$ are shown in Eqs. (17) and (22) or in compact forms in Eqs. (25) and (26). We can look at a very simple case in which the space and momentum dependence of all variables is neglected. Then the spin-density matrices for vector mesons and baryons in Eqs. (25) and (26) can be put into simple forms (we neglected the normalization of spin-density matrices which will be done later):

$$\begin{aligned}\rho^V_{m'm} &= \sum_{r_1,s_1,r_2,s_2} C(jm;s_1s_2)C^*(jm';r_1r_2)\langle r_1r_2|\hat\rho_{(q_1\bar q_2)}|s_1s_2\rangle,\\ \rho^B_{m'm} &= \sum_{\{r_n,s_n,n=1,2,3\}} C(jm;s_1s_2s_3)C^*(jm';r_1r_2r_3)\\ &\quad\times\langle r_1r_2r_3|\hat\rho_{(q_1q_2q_3)}|s_1s_2s_3\rangle,\end{aligned} \tag{27}$$

where $j = 1$ and $m = \pm1, 0$ in $\rho^V_{m'm}$ for the vector meson, and $j = 1/2$ or $3/2$ and $m = \pm1/2$ or $m = \pm1/2, \pm3/2$ in $\rho^B_{m'm}$ for the baryon. We see that the Clebsch-Gordan coefficients are actually measurement operators (matrices) in spin space

$$\begin{aligned}\mathcal{M}^{(j)}_M(m;r_1r_2) &\equiv C^*(jm;r_1r_2),\\ \mathcal{M}^{(j)}_B(m;r_1r_2r_3) &\equiv C^*(jm;r_1r_2r_3),\end{aligned} \tag{28}$$

where the subscript "M" in $\mathcal{M}^{(j)}_M$ stands for mesons with $j = 0, 1$ correpsonding to the scalar and vector meson, respectively. Note that the measurement operator for the vector meson is denoted as $\mathcal{M}^{(j)}_V \equiv \mathcal{M}^{(j=1)}_M$. So $\mathcal{M}^{(j)}_M$ is a $(2j+1)\times 4$ matrix and $\mathcal{M}^{(j)}_B$ is a $(2j+1)\times 8$ matrix, while $\mathcal{M}^{(j)\dagger}_M$ is a $4\times(2j+1)$ matrix and $\mathcal{M}^{(j)\dagger}_B$ is a $8\times(2j+1)$ matrix.

With the measurement operators (28) and their Hermitian conjugates, Eq. (27) can be put into the compact form

$$\begin{aligned}\rho_V &= \mathcal{M}^{(j)}_V\hat\rho_{(q_1\bar q_2)}\mathcal{M}^{(j)\dagger}_V,\\ \rho_B &= \mathcal{M}^{(j)}_B\hat\rho_{(q_1q_2q_3)}\mathcal{M}^{(j)\dagger}_B,\end{aligned} \tag{29}$$

where we have put the superscripts "V" and "B" in Eq. (27) to subscripts for notational convenience. The probabilities of obtaining the vector meson and the spin-j baryon are

$$\begin{aligned}\mathcal{P}_M(j=1) &= {\rm Tr}\rho_V = {\rm Tr}\big[\mathcal{M}^{(j)}_V\hat\rho_{(q_1\bar q_2)}\mathcal{M}^{(j)\dagger}_V\big],\\ \mathcal{P}_B(j) &= {\rm Tr}\rho_B = {\rm Tr}\big[\mathcal{M}^{(j)}_B\hat\rho_{(q_1q_2q_3)}\mathcal{M}^{(j)\dagger}_B\big],\end{aligned} \tag{30}$$

where the traces are taken in spin space. Note that $\{\mathcal{P}_M(j)\}$ and $\{\mathcal{P}_B(j)\}$ are probabilities for all possible j for mesons and baryons with the normalization conditions

$$\begin{aligned}\sum_{j=0,1}\mathcal{P}_M(j) &= 1,\\ \sum_{j=1/2,1/2,3/2}\mathcal{P}_B(j) &= 1.\end{aligned} \tag{31}$$

Here we have included two baryon doublets ($j = 1/2$) in the sum of $\mathcal{P}_B(j)$ with mixed-symmetric (MS) and antisymmetric (MA) spin states of quarks. For the measurement operators, we have the properties of the positive semidefiniteness

$$\mathcal{M}^{(j)\dagger}_{M/B}\mathcal{M}^{(j)}_{M/B} \geqslant 0, \tag{32}$$

and the completeness

$$\begin{aligned}\sum_{j=0,1}\mathcal{M}^{(j)\dagger}_M\mathcal{M}^{(j)}_M &= 1_4,\\ \sum_{j=1/2,1/2,3/2}\mathcal{M}^{(j)\dagger}_B\mathcal{M}^{(j)}_B &= 1_8,\end{aligned} \tag{33}$$

where 1_n denotes the $n\times n$ unit matrix, and there are two spin doublets (corresponding to the spin triplet and singlet of two quarks) in forming the baryon from three quarks. Based on the measurement postulate in quantum information science, by the generalized measurement, the initial states $\hat\rho_{(q_1\bar q_2)}$ and $\hat\rho_{(q_1q_2q_3)}$ instantaneously transform to the states $\hat\rho^{(j)}_M$ and $\hat\rho^{(j)}_B$, respectively:

$$\begin{aligned}\hat\rho_{(q_1\bar q_2)} &\mapsto \hat\rho^{(j)}_M \equiv \frac{1}{\mathcal{P}_M(j)}\mathcal{M}^{(j)}_M\hat\rho_{(q_1\bar q_2)}\mathcal{M}^{(j)\dagger}_M,\\ \hat\rho_{(q_1q_2q_3)} &\mapsto \hat\rho^{(j)}_B \equiv \frac{1}{\mathcal{P}_B(j)}\mathcal{M}^{(j)}_B\hat\rho_{(q_1q_2q_3)}\mathcal{M}^{(j)\dagger}_B.\end{aligned} \tag{34}$$

Note that $\hat\rho^{(j)}_M$ and $\hat\rho^{(j)}_B$ are normalized so that we have ${\rm Tr}\hat\rho^{(j)}_M = 1$ and ${\rm Tr}\hat\rho^{(j)}_B = 1$.

We can calculate the von Neumann entropy for $q_1\bar q_2$ and $q_1q_2q_3$ as well as for the meson and baryon through their normalized SDMs as

$$S(\hat\rho_i) = -{\rm Tr}(\hat\rho_i\ln\hat\rho_i), \tag{35}$$

where $i = q_1\bar q_2, q_1q_2q_3, M, B$. We can rewrite $\hat\rho$ ($\hat\rho \equiv \hat\rho_i$, we suppress the index i) as $\hat\rho = \hat\rho_0 + \delta\hat\rho$, where $\hat\rho_0 = 1_D/D$ with 1_D being the unit matrix of dimension D. When the matrix elements $\delta\hat\rho_{mn} \ll 1$ we can approximate the von Neumann entropy to the quadratic order $(\delta\hat\rho)^2$:

$$S(\hat\rho) \approx \ln D - \frac{D}{2}{\rm Tr}[(\delta\hat\rho)^2]. \tag{36}$$

For a maximal-mixed state we have ${\rm Tr}(\hat\rho^2) = {\rm Tr}(\hat\rho_0^2) = 1/D$, while for a pure state we have ${\rm Tr}(\hat\rho^2) = 1$. Then we obtain the inequality

$$0 \leqslant {\rm Tr}[(\delta\hat\rho)^2] \leqslant 1 - 1/D. \tag{37}$$

So we see that the second term of the von Neumann entropy (36) is always negative.

Using Eq. (36) we can calculate $S(\hat\rho_{(q_1\bar q_2)})$ and $S(\hat\rho_V)$ to the quadratic order of the spin density's deviation from $\hat\rho_0$. By assuming $P_{q_1/\bar q_2} \equiv \sqrt{\boldsymbol{P}_{q_1/\bar q_2}\cdot\boldsymbol{P}_{q_1/\bar q_2}} \ll 1$, $c^{(q_1\bar q_2)}_{ij} \ll 1$, and

$P_{q_1/\bar{q}_2} \sim c_{ij}^{(q_1\bar{q}_2)}$ in Eq. (7), we obtain to the quadratic order in $P_{q_1/\bar{q}_2}$ and $c_{ij}^{(q_1\bar{q}_2)}$

$$S\big(\hat{\rho}_{(q_1\bar{q}_2)}\big) \approx 2\ln 2 - \frac{1}{2}\big[P_{q_1}^2 + P_{\bar{q}_2}^2 + \mathrm{Tr}\big(c^{(q_1\bar{q}_2)}c^{(q_1\bar{q}_2)T}\big)\big],$$
$$S(\hat{\rho}_V) \approx \ln 3 - \frac{1}{3}\big[\big(\boldsymbol{P}_{q_1} + \boldsymbol{P}_{\bar{q}_2}\big)^2 + \mathrm{Tr}(c^{(q_1\bar{q}_2)}c^{(q_1\bar{q}_2)}) + \mathrm{Tr}(c^{(q_1\bar{q}_2)}c^{(q_1\bar{q}_2)T}) - \frac{2}{3}(\mathrm{Tr}c^{(q_1\bar{q}_2)})^2\big], \quad (38)$$

where the superscript "T" denotes the transpose of a matrix, so we have $c_{ij}^{(q_1\bar{q}_2)T} = c_{ji}^{(q_1\bar{q}_2)}$. By the inequality (37), one can show

$$\mathrm{Tr}\big(c^{(q_1\bar{q}_2)}c^{(q_1\bar{q}_2)T}\big) \geqslant 0,$$
$$\mathrm{Tr}\big(c^{(q_1\bar{q}_2)}c^{(q_1\bar{q}_2)}\big) + \mathrm{Tr}\big(c^{(q_1\bar{q}_2)}c^{(q_1\bar{q}_2)T}\big) - \frac{2}{3}\big(\mathrm{Tr}c^{(q_1\bar{q}_2)}\big)^2 \geqslant 0. \quad (39)$$

So we see from Eq. (38) that nonzero quark polarizations or correlations always lower the entropy. It is easy to check the inequalities in Eq. (39) by rewriting $c^{(12)} \equiv c^{(q_1\bar{q}_2)}$ as a sum over symmetric (S) and antisymmetric (A) part

$$c^{(12)} = c_S^{(12)} + c_A^{(12)}, \quad (40)$$

where $c_{S,ij}^{(12)} = c_{S,ji}^{(12)}$ and $c_{A,ij}^{(12)} = -c_{A,ji}^{(12)}$. Then the inequalities in Eq. (39) now become

$$\mathrm{Tr}\big(c_S^{(12)}\big)^2 - \mathrm{Tr}\big(c_A^{(12)}\big)^2 \geqslant 0,$$
$$F\big[c^{(12)}\big] \equiv \mathrm{Tr}\big(c_S^{(12)}\big)^2 - \frac{1}{3}\big(\mathrm{Tr}c_S^{(12)}\big)^2 \geqslant 0, \quad (41)$$

which hold, obviously.

In a similar way, we can also calculate with Eq. (36) $S(\hat{\rho}_{(q_1q_2q_3)})$ and $S(\hat{\rho}_B)$ to the quadratic order of the spin density's deviation from $\hat{\rho}_0$. The results are

$$S\big(\hat{\rho}_{(q_1q_2q_3)}\big) \approx 3\ln 2 - \frac{1}{2}\big[P_{q_1}^2 + P_{q_2}^2 + P_{q_3}^2 + \mathrm{Tr}\big(c^{(12)}c^{(12)T}\big) + \mathrm{Tr}\big(c^{(23)}c^{(23)T}\big) + \mathrm{Tr}\big(c^{(13)}c^{(13)T}\big)\big],$$
$$S\big(\hat{\rho}_{(3/2)B}\big) \approx \ln 4 - \frac{5}{18}\big(\boldsymbol{P}_{q_1} + \boldsymbol{P}_{q_2} + \boldsymbol{P}_{q_3}\big)^2 - \frac{1}{3}F\big[c^{(12)} + c^{(23)} + c^{(13)}\big],$$
$$S\big(\hat{\rho}_{(1/2)B}^{MS}\big) \approx \ln 2 - \frac{1}{18}\big(2\boldsymbol{P}_{q_1} + 2\boldsymbol{P}_{q_2} - \boldsymbol{P}_{q_3}\big)^2,$$
$$S\big(\hat{\rho}_{(1/2)B}^{MA}\big) \approx \ln 2 - \frac{1}{2}\boldsymbol{P}_{q_3}^2. \quad (42)$$

Among ground states of octet baryons with spin-partity $(1/2)^+$, the proton, neutron, $\Sigma^{\pm}$, and $\Xi^{-,0}$ are in mixed-symmetric states with the flavor configuration $q_1q_2q_3 = aab$ (flavor $a \neq b$); Σ^0 is also in the mixed-symmetric state with the flavor configuration $q_1q_2q_3 = uds$, while the Λ hyperon is in the mixed antisymmetric state with the flavor configuration $q_1q_2q_3 = uds$. We observe in Eq. (42) that there are no spin-correlation terms in the entropy for octet baryons at the quadratic order since they are of cubic or higher order, while there are spin-correlation terms for decuplet baryons. This means that decuplet baryons are more correlated than octet baryons in case of no quark polarization.

The comparison of the entropies of $q_1\bar{q}_2$ and $q_1q_2q_3$ with those of the corresponding meson and baryon states can provide insights into the role of polarization and spin correlations in the entropy evolution associated with quark recombination [79–84] in the hadronization process of relativistic heavy-ion collisions. It can also shed light on the entropy puzzle in the quark-combination model [85]. Note that the discussion in this subsection is based on the simplified SDM for the vector meson and baryon in which we neglected the space and momentum dependence of all variables. By using the SWFs for the vector meson and baryon in Eqs. (25) and (26), the von Neumann entropies for the quark and hadron systems should have more sophisticated features which will be reserved for a future study.

III. SPIN POLARIZATIONS OF VECTOR MESONS AND BARYONS

From the SWF for the vector meson and baryon we can obtain their spin polarizations [86–89]

$$\boldsymbol{P} = \frac{1}{S}\frac{\mathrm{Tr}(\rho \boldsymbol{S})}{\mathrm{Tr}\rho}, \quad (43)$$

where $\boldsymbol{S}$ is the spin operator (matrix) for the vector meson or baryon, S is the spin quantum number with the eigenvalue of $\boldsymbol{S}^2$ being $S(S+1)$, and the polarization has an upper bound $P \equiv \sqrt{\boldsymbol{P}\cdot\boldsymbol{P}} \leqslant 1$. Here ρ denotes the SWF as a spin-density matrix given in Eq. (17) for the vector meson and Eq. (22) for the baryon. If we choose the z direction as the spin-quantization direction, the spin polarizations for the vector meson, spin-1/2, and -3/2 baryons are given by

$$P_V^z = \frac{\rho_{11} - \rho_{-1,-1}}{\rho_{11} + \rho_{-1,-1} + \rho_{00}},$$
$$P_{B_{1/2}}^z = \frac{\rho_{1/2,1/2} - \rho_{-1/2,-1/2}}{\rho_{1/2,1/2} + \rho_{-1/2,-1/2}},$$
$$P_{B_{3/2}}^z = \frac{2}{3}\frac{\sum_{s_z=\pm 3/2,\pm 1/2} S_z\rho_{S_z,S_z}}{\sum_{s_z=\pm 3/2,\pm 1/2}\rho_{S_z,S_z}} = \frac{\rho_{3/2,3/2} - \rho_{-3/2,-3/2} + \frac{1}{3}(\rho_{1/2,1/2} - \rho_{-1/2,-1/2})}{\rho_{3/2,3/2} + \rho_{-3/2,-3/2} + \rho_{1/2,1/2} + \rho_{-1/2,-1/2}}, \quad (44)$$

where we have suppressed the indices V, $B_{1/2}$, and $B_{3/2}$ in ρ for the vector meson, spin-1/2, and spin-3/2 baryons, respectively. Note that the spin polarizations in Eq. (44) are functions of phase-space variables of mesons and baryons before taking long-range average. The spin polarizations of hyperons can be measured through their weak decays, but vector mesons only decay through strong interaction, so their polarizations cannot be measured in a conventional way.

Using the spin-flavor or SU(6) wave functions of mesons and baryons [90], we obtain the spin polarizations for the spin-1/2 baryons $\Xi^-(ssd)$, $\Xi^0(ssu)$, and $\Lambda(usd)$ [27] and the spin-3/2 baryon $\Omega^-(sss)$ [28] including spin correlations among

their constituent quarks

$$
\begin{aligned}
P^z_{\Xi^-} &= \frac{1}{\langle C_{\Xi^-}\rangle_B}\left\langle \frac{1}{3}\left(4P^z_s - P^z_d\right)C_{\Xi^-} + \delta\rho_{\Xi^-}\right\rangle_B,\\
P^z_{\Xi^0} &= P_{\Xi^-}(d\to u),\\
P^z_{\Lambda} &= \frac{1}{\langle C_{\Lambda}\rangle_B}\langle P^z_s C_{\Lambda} - \delta\rho_{\Lambda}\rangle_B,\\
P^z_{\Omega} &= \frac{1}{\langle C_{\Omega}\rangle_B}\left\langle \frac{5}{3}P^z_s C_{\Omega} + \delta\rho_{\Omega}\right\rangle_B,
\end{aligned}
\tag{45}
$$

where

$$
\begin{aligned}
\delta\rho_{\Xi^-} &= -\frac{4}{3}\left(\boldsymbol{P}_s\cdot\boldsymbol{P}_s + c^{(ss)}_{ii} - \boldsymbol{P}_s\cdot\boldsymbol{P}_d - c^{(sd)}_{ii}\right)\left(P^z_s - P^z_d\right)\\
&\quad - 4c^{(ss)}_{iz}P^i_d + 2\left(c^{(sd)}_{iz} - 2c^{(sd)}_{zi}\right)P^i_s + c^{(ssd)}_{iiz} - 4c^{(ssd)}_{zii},\\
C_{\Xi^-} &= 3 + \boldsymbol{P}_s\cdot\boldsymbol{P}_s + c^{(ss)}_{ii} - 4\left(\boldsymbol{P}_s\cdot\boldsymbol{P}_d + c^{(sd)}_{ii}\right),\\
\delta\rho_{\Xi^0} &= \delta\rho_{\Xi^-}(d\to u),\\
C_{\Xi^0} &= C_{\Xi^-}(d\to u),\\
\delta\rho_{\Lambda} &= c^{(us)}_{iz}P^i_d + c^{(ds)}_{iz}P^i_u + c^{(uds)}_{iiz},\\
C_{\Lambda} &= 1 - \left(\boldsymbol{P}_u\cdot\boldsymbol{P}_d + c^{(ud)}_{ii}\right),\\
\delta\rho_{\Omega} &= c^{(sss)}_{zii} + 2c^{(ss)}_{zi}P^i_s - 4\left(\boldsymbol{P}_s\cdot\boldsymbol{P}_s + c^{(ss)}_{ii}\right)P^z_s,\\
C_{\Omega} &= 3 + 3\left(\boldsymbol{P}_s\cdot\boldsymbol{P}_s + c^{(ss)}_{ii}\right).
\end{aligned}
\tag{46}
$$

In Eq. (45) the short-range average is taken inside a phase-space volume defined by a specific baryon's wave function and weighted by the distribution functions of the baryon's constituent quarks as described in Eq. (26) and Sec. II C.

For the following discussion, we assume that all baryon wave functions in coordinate or momentum space have a similar form at the hadronic scale, which provides a good approximation for the present problem. In other words, we adopt the same wave-function form for all baryon species.

The 00 element of the spin-density matrix for the ϕ meson is usually called spin alignment and given by

$$
\rho^{\phi}_{00} = \frac{1}{\langle C_{\phi}\rangle_V}\left[1 + \left\langle \boldsymbol{P}_s\cdot\boldsymbol{P}_{\bar{s}} + c^{(s\bar{s})}_{ii} - 2\left(P^z_s P^z_{\bar{s}} + c^{(s\bar{s})}_{zz}\right)\right\rangle_V\right], \tag{47}
$$

where

$$
C_{\phi} = 3 + \boldsymbol{P}_s\cdot\boldsymbol{P}_{\bar{s}} + c^{(s\bar{s})}_{ii}. \tag{48}
$$

In Eq. (47) the short-range average is taken inside the ϕ meson with its spatial or momentum wave function.

IV. INEQUALITIES FOR SPIN CORRELATIONS

The spin polarizations of hyperons and spin alignment of vector mesons can be measured in experiments. From Eqs. (45) and (47), we see that these observables depend on the spin polarizations and correlation functions of constituent quarks which have a huge number of unknown components or variables. In order to identify the relevant features of quark polarizations and correlations, we have to reduce the number of variables by making appropriate approximations. In this section we will derive a few inequalities for spin correlations implied by experimental data under these approximations.

A. Mean-field approximation

We make a mean-field approximation for all quark flavors in the short-range averages in hyperon polarizations given by Eq. (45). This means we do not distinguish flavor dependences in $\boldsymbol{P}_q$, $c^{(q_1q_2)}_{ij}$, and $c^{(q_1q_2q_3)}_{ijk}$. By this way, we imply that $c^{(q_1q_2)}_{ij}$ and $c^{(q_1q_2q_3)}_{ijk}$ are symmetric for the interchange of any two spatial indices. There are four types of terms:

$$
\begin{aligned}
B_1 &= \left\langle c^{(q_1q_2)}_{zi}P^i_{q_3}\right\rangle_B \equiv \left\langle c^{(qq)}_{zi}P^i_q\right\rangle_B,\\
B_2 &= \left\langle \left(c^{(q_1q_2)}_{ii} + \boldsymbol{P}_{q_1}\cdot\boldsymbol{P}_{q_2}\right)P^z_{q_3}\right\rangle_B \equiv \left\langle \left(c^{(qq)}_{ii} + \boldsymbol{P}_q\cdot\boldsymbol{P}_q\right)P^z_q\right\rangle_B,\\
B_3 &= \left\langle c^{(q_1q_2q_3)}_{zii}\right\rangle_B \equiv \left\langle c^{(qqq)}_{zii}\right\rangle_B,\\
B_4 &= \left\langle c^{(q_1q_2)}_{ii} + \boldsymbol{P}_{q_1}\cdot\boldsymbol{P}_{q_2}\right\rangle_B\left\langle P^z_{q_3}\right\rangle_B \equiv \left\langle c^{(qq)}_{ii} + \boldsymbol{P}_q\cdot\boldsymbol{P}_q\right\rangle_B\left\langle P^z_q\right\rangle_B,
\end{aligned}
\tag{49}
$$

and we assume $B_i \ll 1$ with $i = 1, 2, 3, 4$. Then we obtain from Eq. (45)

$$
\begin{aligned}
P^z_{\Lambda} &\approx \left\langle P^z_q\right\rangle_B - 2B_1 - B_2 - B_3 + B_4,\\
P^z_{\Xi^-} &\approx P^z_{\Lambda},\\
P^z_{\Omega} &\approx \frac{5}{3}\left\langle P^z_q\right\rangle_B + \frac{1}{3}(2B_1 + B_2 + B_3) - \frac{5}{3}B_4\\
&\approx \frac{5}{3}P^z_{\Lambda} + 2(2B_1 + B_2 + B_3) - \frac{10}{3}B_4.
\end{aligned}
\tag{50}
$$

Therefore, within the mean-field approximation we clearly see that $P_{\Xi^-} \approx P_{\Lambda}$ and $P_{\Omega} \neq (5/3)P_{\Lambda}$ with

$$
P^z_{\Omega} - \frac{5}{3}P^z_{\Lambda} \approx 2(2B_1 + B_2 + B_3) - \frac{10}{3}B_4, \tag{51}
$$

which should be larger than zero in heavy-ion collisions at energies lower than 20 GeV, as implied by data on Ω baryon's global polarization in Au + Au collisions [41]. This leads to an inequality

$$
2B_1 + B_2 + B_3 - \frac{5}{3}B_4 > 0, \tag{52}
$$

when the collisional energy is less than 20 GeV in Au + Au collisions.

The inequality (52) clearly shows the presence of the spin correlation in heavy-ion collisions at energies lower than 20 GeV in Au + Au collisions. If quarks are not polarized the inequality (52) is reduced to $\langle c^{(qqq)}_{zii}\rangle_B > 0$, i.e., the three-quark correlation inside hyperons is nonvanishing. On the other hand, if spin-correlation functions are all zero, the inequality (52) is reduced to $\langle(\boldsymbol{P}_q\cdot\boldsymbol{P}_q)P^z_q\rangle_B > (5/3)\langle\boldsymbol{P}_q\cdot\boldsymbol{P}_q\rangle_B\langle P^z_q\rangle_B$, i.e., the three quarks' polarizations are correlated inside hyperons.

B. With only *s*-quark correlations inside hyperons

In this subsection we only consider short-range correlations among strange quarks and antiquarks inside hyperons, neglecting mixed correlations between quarks of different flavors. We can then relate the average polarization of Ξ and Ω hyperons with the spin alignment of the ϕ meson.

1. Short- and long-range components of spin polarizations and correlations

The spin polarizations for quarks and antiquarks $\boldsymbol{P}_{q/\bar{q}}$ can be decomposed into the long- and short-range components. The long-range component $\overline{\boldsymbol{P}}_{q/\bar{q}}$ is induced by local vorticity and has the same sign for the quark and antiquark. The short range component $\widetilde{\boldsymbol{P}}_{q/\bar{q}}$ is from vector fields and has the opposite sign for the quark and antiquark. Namely, the polarization vectors can be written as

$$\begin{aligned}\boldsymbol{P}_q &= \overline{\boldsymbol{P}}_q + \widetilde{\boldsymbol{P}}_q,\\ \boldsymbol{P}_{\bar{q}} &= \overline{\boldsymbol{P}}_{\bar{q}} - \widetilde{\boldsymbol{P}}_{\bar{q}},\end{aligned} \tag{53}$$

for $q = u, d, s$. We assume that the averages of short-range components inside hyperons are restricted by their wave functions and satisfy

$$\begin{aligned}\langle\widetilde{\boldsymbol{P}}_q\rangle_B &= 0,\\ \langle(\widetilde{\boldsymbol{P}}_{q_1}\cdot\widetilde{\boldsymbol{P}}_{q_2})\widetilde{\boldsymbol{P}}_{q_3}\rangle_B &= 0,\\ \langle\widetilde{\boldsymbol{P}}_q\cdot\widetilde{\boldsymbol{P}}_q\rangle_B &\neq 0,\end{aligned} \tag{54}$$

where B represents the baryon wave function. So we have $\langle\boldsymbol{P}_q\rangle_B = \overline{\boldsymbol{P}}_q$. We can further assume that long-range components do not distinguish the quark, antiquark, and flavor, $\overline{\boldsymbol{P}}_{q_i} = \overline{\boldsymbol{P}}_{\bar{q}_j}$, with $i, j = u, d, s$. All spin-correlation matrices $c_{ij}^{(q_1q_2)}$ and $c_{ijk}^{(q_1q_2q_3)}$ are assumed to be of short range.

When taking the short-range average of $\delta\rho_B$ over the hadron volume we can approximate

$$\begin{aligned}\left\langle c_{ij}^{(q_1q_2)}P_{q_3}^k\right\rangle_B &\approx \left\langle c_{ij}^{(q_1q_2)}\right\rangle_B\overline{P}_{q_3}^k,\\ \left\langle(\boldsymbol{P}_{q_1}\cdot\boldsymbol{P}_{q_2})P_{q_3}^z\right\rangle_B &\approx \left\langle\widetilde{\boldsymbol{P}}_{q_1}\cdot\widetilde{\boldsymbol{P}}_{q_2}\right\rangle_B\overline{P}_{q_3}^z + \left\langle\widetilde{P}_{q_1}^i\widetilde{P}_{q_3}^z\right\rangle_B\overline{P}_{q_2}^i\\ &\quad + \left\langle\widetilde{P}_{q_2}^i\widetilde{P}_{q_3}^z\right\rangle_B\overline{P}_{q_1}^i,\\ \left\langle\widetilde{\boldsymbol{P}}_{q_1}\cdot\widetilde{\boldsymbol{P}}_{q_2}\right\rangle_B &\propto \delta_{q_1q_2},\ \left\langle c_{ij}^{(q_1q_2)}\right\rangle_B \propto \delta_{q_1q_2}\\ \overline{\boldsymbol{P}}_q\cdot\overline{\boldsymbol{P}}_q &\ll \overline{\boldsymbol{P}}_q \sim \left\langle\widetilde{\boldsymbol{P}}_q\cdot\widetilde{\boldsymbol{P}}_q\right\rangle_B \sim \left\langle c_{ij}^{(qq)}\right\rangle_B \ll 1.\end{aligned} \tag{55}$$

In this way we can relate the average polarization of Ξ and Ω hyperons with the spin alignment of the ϕ meson.

2. Spin polarizations of hyperons and spin alignment of ϕ meson

With the decomposition (53) and the approximations in short-range averages in Eqs. (54) and (55), we obtain

$$\begin{aligned}P_\Lambda^z &\approx \overline{P}_s^z = \overline{P}_q^z,\\ P_{\Xi^-}^z &\approx \overline{P}_q^z(1+\epsilon_\Xi) - \frac{4}{3}\delta,\\ P_\Omega^z &\approx \frac{5}{3}\overline{P}_q^z(1+\epsilon_\Omega) + \frac{2}{3}\delta + \frac{1}{3}\delta',\end{aligned} \tag{56}$$

where we have defined

$$\begin{aligned}\epsilon_\Xi &\equiv -\frac{4}{3}\left\langle\left(\widetilde{P}_s^z\right)^2 + c_{zz}^{(ss)}\right\rangle_B,\\ \epsilon_\Omega &\equiv -\frac{4}{5}\left\langle\widetilde{\boldsymbol{P}}_s\cdot\widetilde{\boldsymbol{P}}_s + c_{ii}^{(ss)}\right\rangle_B + \frac{2}{5}\left\langle(P_s^z)^2 + c_{zz}^{(ss)}\right\rangle_B,\\ \delta &\equiv \left\langle c_{zx}^{(ss)}\right\rangle_B\overline{P}_q^x + \left\langle c_{zy}^{(ss)}\right\rangle_B\overline{P}_q^y,\\ \delta' &\equiv \left\langle c_{zii}^{(sss)}\right\rangle_B.\end{aligned} \tag{57}$$

In the derivation of Eq. (56) we have kept $c^{(ss)}$ and $c^{(sss)}$ only and neglected all other spin-correlation functions.

For the spin alignment of the ϕ meson, we obtain from Eq. (47)

$$\rho_{00}^\phi \approx \frac{1}{3}\left[1 + \frac{2}{3}\left\langle\widetilde{\boldsymbol{P}}_s\cdot\widetilde{\boldsymbol{P}}_{\bar{s}} + c_{ii}^{(s\bar{s})}\right\rangle_V - 2\left\langle\widetilde{P}_s^z\widetilde{P}_{\bar{s}}^z + c_{zz}^{(s\bar{s})}\right\rangle_V\right]. \tag{58}$$

If the short-range spin polarization and correlation are caused by vector fields which distinguish particles and antiparticles, we can make an approximation on the short-range effective correlations

$$\begin{aligned}\left\langle\widetilde{\boldsymbol{P}}_s\cdot\widetilde{\boldsymbol{P}}_{\bar{s}} + c_{ii}^{(s\bar{s})}\right\rangle_V &\approx -\eta\left\langle\widetilde{\boldsymbol{P}}_s\cdot\widetilde{\boldsymbol{P}}_s + c_{ii}^{(ss)}\right\rangle_B,\\ \left\langle\widetilde{P}_s^z\widetilde{P}_{\bar{s}}^z + c_{zz}^{(s\bar{s})}\right\rangle_V &\approx -\eta\left\langle\widetilde{P}_s^z\widetilde{P}_s^z + c_{zz}^{(ss)}\right\rangle_B,\end{aligned} \tag{59}$$

where $\eta > 0$ is a proportionality constant. Then Eq. (58) becomes

$$\rho_{00}^\phi \approx \frac{1}{3}(1+\epsilon_\phi), \tag{60}$$

where we defined

$$\epsilon_\phi \equiv -\frac{2}{3}\eta\left\langle\widetilde{\boldsymbol{P}}_s\cdot\widetilde{\boldsymbol{P}}_s + c_{ii}^{(ss)}\right\rangle_B + 2\eta\left\langle\widetilde{P}_s^z\widetilde{P}_s^z + c_{zz}^{(ss)}\right\rangle_B. \tag{61}$$

The quantities ϵ_Ξ, ϵ_Ω, and ϵ_ϕ depend on spin correlations in the spin-quantization direction and in transverse directions perpendicular to it. Therefore, we can define longitudinal and transverse effective correlations as

$$\begin{aligned}A_L &= \left\langle\left(\widetilde{P}_s^z\right)^2 + c_{zz}^{(ss)}\right\rangle_B,\\ A_T &= \left\langle\left(\widetilde{P}_s^x\right)^2 + \left(\widetilde{P}_s^y\right)^2 + c_{xx}^{(ss)} + c_{yy}^{(ss)}\right\rangle_B,\end{aligned} \tag{62}$$

in terms of which the ϵ quantities in Eqs. (57) and (60) can be rewritten as

$$\begin{aligned}\epsilon_\Xi &= -\frac{4}{3}A_L,\\ \epsilon_\Omega &= -\frac{2}{5}(2A_T + A_L),\\ \epsilon_\phi &= -\frac{2}{3}\eta(A_T - 2A_L).\end{aligned} \tag{63}$$

3. Inequalities for quark spin correlations

By comparing Eqs. (56) and (60) with the experimental data, we obtain the following constraints:

$$\begin{aligned}P_{\Xi^-} - P_\Lambda &\approx -\frac{4}{3}\overline{P}_q^z A_L - \frac{4}{3}\delta \approx 0,\\ P_\Omega - \frac{5}{3}P_\Lambda &\approx -\frac{2}{3}\overline{P}_q^z(2A_T + A_L) + \frac{2}{3}\delta + \frac{1}{3}\delta' > 0,\\ \rho_{00}^\phi - \frac{1}{3} &= \frac{2}{9}\eta(2A_L - A_T) > 0,\end{aligned} \tag{64}$$

which leads to one equality and two inequalities for quark spin correlations,

$$\begin{aligned}\delta &\approx -\overline{P}_q^z A_L,\\ \delta' - 4\overline{P}_q^z(A_T + A_L) &> 0,\\ 2A_L - A_T &> 0.\end{aligned} \tag{65}$$

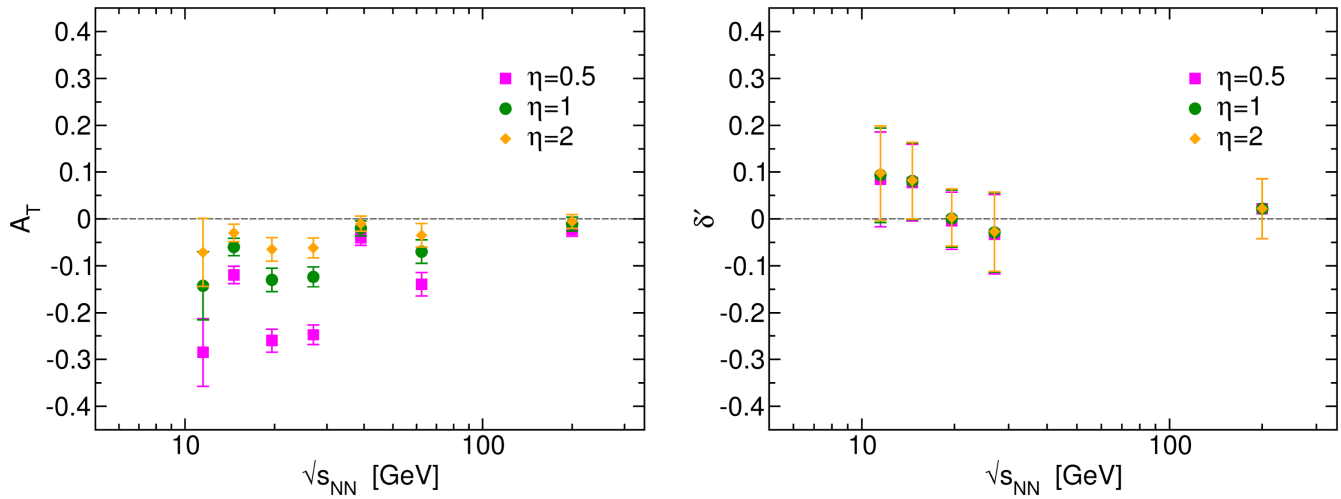

FIG. 1. Estimate of two-quark transverse correlations A_T (left panel) and the three-quark correlations δ' (right panel) as a function of collision energy for $\eta = 0.5, 1, 2$.

The explicit expressions of above equality and inequalities are

$$\begin{aligned}
&\langle c_{zx}^{(ss)}\rangle_B \overline{P}_q^x + \langle c_{zy}^{(ss)}\rangle_B \overline{P}_q^y \approx -\overline{P}_q^z \left\langle \left(\widetilde{P}_s^z\right)^2 + c_{zz}^{(ss)} \right\rangle_B, \\
&4\overline{P}_q^z \langle \widetilde{\boldsymbol{P}}_s \cdot \widetilde{\boldsymbol{P}}_s + c_{ii}^{(ss)}\rangle_B < \langle c_{zii}^{(sss)}\rangle_B, \\
&\langle \widetilde{\boldsymbol{P}}_s \cdot \widetilde{\boldsymbol{P}}_s + c_{ii}^{(ss)}\rangle_B < 3\left\langle \left(\widetilde{P}_s^z\right)^2 + c_{zz}^{(ss)} \right\rangle_B.
\end{aligned} \tag{66}$$

The results in Eq. (66) clearly show the presence of spin correlations between s quarks in hyperons. If we neglect the three-quark spin correlation, i.e., $\langle c_{zii}^{(sss)}\rangle_B = 0$, from the first inequality we obtain nonvanishing spin correlation between two s quarks

$$c_{xx}^{(ss)} + c_{yy}^{(ss)} + c_{zz}^{(ss)} < 0. \tag{67}$$

The second inequality in Eq. (66) looks like a quadrupole structure in spin correlations.

4. Estimates of quark spin correlations

In order to extract some quantitative estimates of quark spin correlations inside hyperons, in this subsection we make a further assumption that long-range correlations are only along the z direction, which is assumed to be the direction of the global orbital angular momentum in noncentral heavy-ion collisions. So we have

$$\overline{P}_q^z \approx P_\Lambda, \ \overline{P}_q^x \approx \overline{P}_q^y \approx 0. \tag{68}$$

This leads to $\delta \approx 0$ from the first equation of Eq. (64). Then, the experimental constraints on P_{Ξ^-}, P_Ω, and ρ_{00}^ϕ in Eq. (64) become

$$\begin{aligned}
&P_{\Xi^-} - P_\Lambda \approx -\frac{4}{3} P_\Lambda A_L \approx 0, \\
&P_\Omega - \frac{5}{3} P_\Lambda \approx -\frac{2}{3} P_\Lambda (2A_T + A_L) + \frac{1}{3}\delta' > 0, \\
&\rho_{00}^\phi - \frac{1}{3} \approx \frac{2}{9}\eta(2A_L - A_T).
\end{aligned} \tag{69}$$

This is a system of three equations in four unknown variables: A_L, A_T, δ', and η. From the first equation we can extract A_L: the fact that there is no significant difference between Λ and Ξ global polarization implies that $A_L \approx 0$, i.e., longitudinal effective correlations between s quarks are negligible. We can consider η as a fixed parameter, so that the other two variables, A_T and δ', can be determined from the second and third equations in Eq. (69):

$$\begin{aligned}
&A_T \approx \frac{3}{2\eta}(1 - 3\rho_{00}^\phi), \\
&\delta' \approx 3P_\Omega - P_\Lambda \left[5 - \frac{6}{\eta}(1 - 3\rho_{00}^\phi)\right].
\end{aligned} \tag{70}$$

We estimate the transverse two-particle correlations A_T and the three-particle correlations δ' as a function of collision energy from available experimental data on P_Λ, P_Ω, and ρ_{00}^ϕ. We also estimate the uncertainties on A_T and δ' in the extraction, by considering the experimental errors on P_Ω and ρ_{00}^ϕ, while neglecting those on P_Λ (them being small). In Fig. 1 we show results for the two-particle transverse correlations A_T (left panel) and the three-quark correlations δ' (right panel) as a function of collision energy for different values of the parameter η. For the extraction of A_T we consider all the collision energies where there are data on ρ_{00}^ϕ: 11.5, 14.6, 19.6, 27.0, 39.0, 62.4, 200 GeV. Regarding δ', we can extract it only at the common collision energies of the different spin-observable measurements: 11.5, 14.6, 19.6, 27.0, 200 GeV. From Fig. 1 we see that transverse effective correlation is negative:

$$\left\langle \left(\widetilde{P}_s^x\right)^2 + \left(\widetilde{P}_s^y\right)^2 + c_{xx}^{(ss)} + c_{yy}^{(ss)} \right\rangle_B < 0. \tag{71}$$

This implies that transverse genuine correlations between strange quarks are nonvanishing and negative, i.e., $\langle c_{xx}^{(ss)} + c_{yy}^{(ss)}\rangle_B < 0$.

We make some remarks about experimental constraints on spin-correlation parameters. In order to reduce the number of unknown variables in short-range correlations and to close the system of equations such as Eq. (69), the only way to our knowledge is through inputs from theoretical models. We can build models for strong force fields

produced by source particles and calculate their short-range correlations and fluctuations. We can also calculate the spin-correlation matrices among quarks in scattering processes which are purely quantum effects. Under the approximation in Sec. IV B, for short-range spin correlations among s quarks, the spin-alignment data of K^{*0} does not lead to additional constraints.

V. SUMMARY AND CONCLUSION

The global spin polarization of hyperons have been investigated by including spin-correlation effects among their constituent quarks. Based on density operators for multi-quarks (including antiquarks) in spin-momentum space, we derived the spin-Wigner functions (SWF) for vector mesons or baryons by computing elements of density operators between two spin-momentum states of the vector meson or baryon. The SWF are actually spin-density matrices in phase space for vector mesons and baryons which are expressed as phase-space integrals of their constituent quarks' reduced spin-density matrices weighted by their coordinate wave functions, constituent quark distributions, and CG coefficients. The short-range correlation is defined by the average over phase space of quarks (antiquarks) inside the hadron, while the long-range correlation is by the average over phase space of the hadron. The available data in global spin polarizations of hyperons and spin alignments of vector mesons provide constraints on phase-space functions of the spin polarization and correlation appearing in the reduced spin-density matrices for constituent quarks, which consist of large number of unknown variables. In order to reduce the number of variables and derive meaningful and simplified constraints, we make two approximations. In the first one, which is called the mean-field approximation, we do not distinguish different flavors in $\boldsymbol{P}_q$, $c_{ij}^{(q_1q_2)}$, and $c_{ijk}^{(q_1q_2q_3)}$. In the second approximation, we only consider spin correlation among s quarks and neglect others. In each approximation, we derived a set of inequalities for spin polarization and correlation functions. These inequalities might provide possible clues for the presence of spin correlation inside hyperons at lower collision energies.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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