

Meson spin alignment and baryon polarization from coalescence with spin-vorticity nonequilibrium

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We estimate both the polarization of spin 1/2 baryons and the spin density matrix coefficients of spin 1 mesons using a thermal model that incorporates vorticity and polarization to describe quarks, along with a coalescence formalism where spin and vorticity are not in equilibrium. We find that while our model is not predictive due to its considerable number of free parameters, it has the potential to explain several seemingly puzzling features of experimental spin measurements, such as the absence of baryon polarization alongside significant vector meson spin alignment. We suggest measuring meson spin off-diagonal matrix elements, and examining the dependence of polarization observables on azimuthal angles, as methods to falsify this model and gain insights into the freeze-out details of baryon and meson spin structure.

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I. INTRODUCTION AND MOTIVATION

The measurements of baryon polarization [1] and vector meson spin alignment [2,3] have heralded a fertile era of experimental and theoretical study of relativistic hydrodynamics with vorticity [4–6].

On one hand, it is encouraging that hydrodynamics together with a Cooper-Frye formula extended to spinors [7] can describe global polarization data [6] based on freeze-out from a vortical fluid. On the other hand, some phenomenological issues remain:

- (i) *Local* polarization appears to have the wrong sign [6].
- (ii) Vector meson spin alignment is not described by theoretical models [2], which has been interpreted as suggesting new physics such as classical fields [8–11] (suggesting the ϕ is coherent) and spin distillation [12] (which would mean the ϕ is more akin to a quarkonium state), as well as hadronic dynamics [13].

In addition, while [4,6] treat and vorticity as quantities related by an equilibrium relation (as in [14,15]), it is clear that causality forbids spin vorticity from being in full equilibrium [16,17]. There is now a consensus among theorists that a hydrodynamics framework incorporating both vorticity and spin is necessary. Several versions of such hydrodynamics have been proposed [15,18–22] making it imperative to establish a connection with phenomenology, specifically regarding the issues discussed in the previous paragraph.

As previously argued [12,23] vector mesons are a promising probe in this regard, as their 3×3 density matrix contains quantifiable traces of quantum coherence that can be directly connected to experimental observables [24,25].

Specifically, the spin 1/2 baryon angular distribution in its rest frame is governed by a single coefficient,¹ where ϕ^* is the daughter particle emission angle measured with respect to parent particle spin axis in its rest frame [26]:

$$W(\phi^*) \propto 1 + \alpha_H \chi_{1/2} \cos \phi^*,$$

$$\chi_{1/2}^L = \frac{(\rho^B)_{\frac{1}{2}, \frac{1}{2}}^L - (\rho^B)_{\frac{1}{2}, -\frac{1}{2}}^L}{(\rho^B)_{\frac{1}{2}, \frac{1}{2}}^L + (\rho^B)_{\frac{1}{2}, -\frac{1}{2}}^L}, \quad (1)$$

where α_H is the baryon decay parameter, and $\chi_{1/2}^L$ is the baryon polarization. On the other hand, for vector mesons decaying via p -wave processes (for instance $\phi \rightarrow KK$), provided we can reliably determine a common axis of the θ^* angle perpendicular to that of ϕ^* for all events (in other words if the rapidity distribution of the parent vector meson is understood), one can obtain the θ^*, ϕ^* distribution in the rest frame of the parent particle and relate it to the density matrix,

$$W(\theta^*, \phi^*) \propto \cos^2 \phi^* \rho_{00} + \sin^2 \phi^* \left(\frac{1 - \rho_{00}}{2} + r_{1,-1} \cos(2\theta^*) \right. \\ \left. + \alpha_{1,-1} \sin(2\theta^*) \right) \\ + \sin(2\phi^*) (r_{10} \cos \theta^* + \alpha_{10} \sin \theta^*). \quad (2)$$

The coefficients of the trigonometric expressions in the above formula are related to the density matrix elements of the

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¹ $W(\dots)$ refers to the probability of the daughter to decay. As we explain later it needs to be convoluted with the parent distribution.

meson ensemble through a normalization factor $3/(4\pi)$

Variable	Element	coefficient $\times \frac{3}{4\pi}$
ρ_{00}^M	ρ_{00}^M	$\cos^2 \phi^*$
$\frac{1-\rho_{00}^M}{2}$	$\frac{\rho_{11}^M + \rho_{-1-1}^M}{2}$	$\sin^2 \phi^*$
r_{10}	$\frac{1}{\sqrt{2}} \text{Re}[\rho_{-10}^M - \rho_{10}^M]$	$\sin(2\phi^*) \cos(\theta^*)$
α_{10}	$\frac{1}{\sqrt{2}} \text{Im}[\rho_{-10}^M + \rho_{10}^M]$	$\sin(2\phi^*) \sin(\theta^*)$
$r_{1,-1}$	$-\text{Re}[\rho_{1,-1}^M]$	$\sin^2 \phi^* \cos(2\theta^*)$
$\alpha_{1,-1}$	$\text{Im}[\rho_{1,-1}^M]$	$\sin^2 \phi^* \sin(2\theta^*)$

Notably, it has long been observed [23,27–29] that these off-diagonal matrix elements can provide insights into the hadronization of vector mesons.

In this work we develop this idea by combining a previously used blast wave model augmented with vorticity [30] with the coalescence ansatz of [23] to attempt to describe both vector meson spin alignment, including nondiagonal terms, and baryon polarization. We note that our aim is not quantitative precision as there are considerable phenomenological uncertainties. This is because

- (i) The model in [30] has not been tuned for the coalescence dynamics of [23].
- (ii) The model in [23] relies on parameters that we do not know how to estimate from first principles, such as the spin projection of the coalescence Wigner function and the projector P_L , which regulates the angular momentum transfer from vorticity to spin. This parameter could in principle be calculated from an open quantum systems formalism [12].

Rather, the goal of this exercise is to provide a proof of principle, showing that hydrodynamics with spin in nonequilibrium combined with a dynamical hadronization process such as coalescence, has the potential to account for the phenomenological puzzles described earlier. Specifically

- (i) Vector mesons coalescing in a vortical medium combine two spin 1/2 and an integer angular momentum into a spin 1 state, whereas spin 1/2 baryons coalescing in a vortical medium combine three spin 1/2 and an integer angular momentum into a spin 1/2 state. The latter requires a lot more cancellations (some exact due to spin-statistics theorem), which suppresses polarization.
- (ii) The concrete phenomenological picture implied by spin-vorticity nonequilibrium assumes [12] a global (perpendicular to reaction plane) vorticity formed in the initial state and a local vorticity formed on the timescale of elliptic flow. It is reasonable to expect the latter to be less equilibrated with spin than the former. We note that if angular momentum vorticity is used [31] (vorticity of enthalpy $\times$ flow), it is nonzero also initially due to density gradients, but it is out of phase with the final vorticity (for the same reason as elliptic flow is out of phase with the initial density ellipticity).

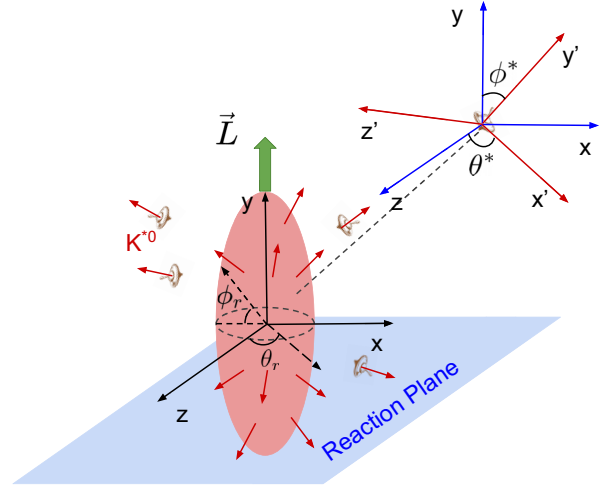


FIG. 1. We show a representation of two frames: decay frame, parametrized by the angles θ^* and ϕ^* , and the laboratory frame, defined by the angles θ_r and ϕ_r . Neglecting relativistic corrections, θ_r follows the reaction plane (global polarization) and ϕ_r follows the longitudinal direction in accordance with Sec. II A.

Thus the interaction between the different vorticities and polarizations alters the local dependence on the azimuthal angle.

Given the large number of parameters used here (and the fact that [30] was not tuned to our model in mind), we do not aim to prove that our model is the *ultimate* explanation.

Rather, we aim to show how spin hydrodynamics can be phenomenologically probed, and how the parameters above can be quantitatively determined, through a particular observable, namely the azimuthal dependence of both the baryon polarization and the experimentally accessible spin density matrix elements of vector mesons.

We understand that this observable is problematic as it is prone to systematic errors, and that so far only a preliminary measurement exists [32]. However, we hope to demonstrate that the phenomenological potential of such studies motivates their further development. A confirmation that off-equilibrium density matrix elements for vector mesons are nonzero and strongly depend on the azimuthal angle would provide evidence supporting our model.

II. THE MODEL

A. The coordinate systems

We begin by describing the coordinate system, shown in Fig. 1. θ^* , ϕ^* refer to the position of the daughter particle within the rest frame of the polarized parent, while ϕ_r and θ_r (where spacetime rapidity is typically used instead) refer to the position of the parent particle with respect to the laboratory frame. Any ensemble average must therefore be obtained in terms of both the daughter decay and the parent distribution.

The first angle ϕ^* is measured with respect to the polarization axis of the parent ($\phi^* = 0$ in the polarization direction), while ϕ_r is measured with respect to the impact parameter

($\phi_r = 0$ corresponds to the maximum of the elliptic flow, with $\pi/2$ relative to $\phi^* = 0$). The second angle, event by event, is provided by pseudorapidity η (Fig. 1).² The θ^* axis is defined relative to the beam, with $\theta^* = 0$ corresponding to the pseudorapidity of the mother particle and θ_r corresponding to the pseudorapidity of the daughter particle.

Therefore, for a quantitative estimate, the coalescence equations governing hadron production need to be convoluted with a thermal model distribution (described in Sec. II B) to obtain the full momentum rapidity, p_T , and angle dependence, considering that

$$\frac{dN}{d\theta^* d\phi^*} = \int p_T dp_T dy d\phi^* \left(\frac{dN_{\text{parent}}}{y dy dp_T d\phi} \right)_{\text{parent}} \times \frac{\partial(y, p_T, \phi)}{\partial(\theta_r, \phi_r, p)} W(\theta^* - \theta_r, \phi^* - \phi_r), \quad (4)$$

where $dN_{\text{parent}}/y dy dp_T d\phi$ is the parent distribution in momentum space (incorporating longitudinal and elliptic flow), and the Jacobian $\partial(\dots)/\partial(\dots)$ is the transformation from the parent rest frame (in the usual y - p_T - ϕ coordinates) to the rest frame (defined by the harmonic angles θ^* , ϕ^* and eventually the momentum p) where (2) is defined.

In the fully relativistic limit in transverse space, the Jacobian is a very complicated object. However, we consider the following limit:

- (i) The coalescence is local in rapidity (this is reasonable in most coalescence models).
- (ii) The longitudinal dynamics is Bjorken-like, in that spacetime and momentum rapidity are identical (this is true for higher energies, but we expect it to be violated at $\sqrt{s} \sim O(1)$ GeV).
- (iii) The transverse dynamics is nonrelativistic (which can be implemented by focusing on $p_T \ll m$).

In this limit, dynamics simplifies considerably, since angles are not affected by boosts and the azimuthal dependence of particle abundances remains constant. Therefore

$$\frac{\partial(\eta, p_T, \phi^*)}{\partial(\theta_r, \phi_r, p)} \simeq \underbrace{\frac{d\theta^*}{d\theta_r}}_{\sim \text{const.}} \times \frac{d\theta_r}{d\eta} \times \frac{\partial p_T}{\partial p}, \quad \eta \simeq \tanh^{-1} \cos\left(\frac{\theta_r}{2}\right), \quad (5)$$

where the last term $\frac{\partial p_T}{\partial p} m$ when integrated in p_T is just a normalization constant as long as the assumptions listed above hold.

Thus, for an approximately boost invariant system where one observes a small slice in rapidity, it is sufficient to multiply the previous expressions by a weight factor, usually $\sim O(1)$:

$$\frac{d\eta}{d\theta_r} = -\frac{\sqrt{1-\eta^2}}{2(1-\eta^2)}.$$

²This notation is opposite to what is usually used in the polarization literature. For consistency with the thermal model used later in this paper, θ and ϕ are also exchanged compared to [23].

Since the model in [30,33] was not tuned for coalescence and the relevant observables are not yet published, we believe such a more precise quantitative comparison is premature.

B. The blast wave model

As a simplified quantitative model for our medium we use the blast wave model [30], which we briefly review here. The blast wave model applied in heavy-ion collisions assumes the creation of a medium where the particles are formed in local thermal equilibrium, and expand in both longitudinal and transverse directions [34]. The longitudinal expansion is defined as boost invariant, and the transverse expansion is described using elliptic fluid flow parametrization, allowing for the inclusion of the elliptical flow effects observed in the experimental transverse momentum spectra [30]. Following Ref. [30], employing anisotropic fluid flow parametrization incorporated in the blast wave model, we determine the fluid vorticity required for our study. This provides approximate longitudinal component of the vortical structures resulting from elliptical fireball shape in the x - y plane as represented in Fig. 1. The x and y components of the freeze-out hypersurface points are given by (see Sec. II A and Fig. 1)

$$\begin{aligned} x &= r_{\text{max}} \sqrt{1-\epsilon} \cos \phi_r, \\ y &= r_{\text{max}} \sqrt{1+\epsilon} \sin \phi_r, \end{aligned} \quad (6)$$

where r_{max} and ϵ are model parameters, and the asymmetric flow velocity is given by

$$u^\mu = \left(\frac{t}{N}, \frac{x}{N} \sqrt{1+\delta}, \frac{y}{N} \sqrt{1-\delta}, \frac{z}{N} \right). \quad (7)$$

The normalization parameter in the above equation is

$$N = \sqrt{\tau^2 - (x^2 - y^2)\delta} \quad (8)$$

with the proper time expressed as

$$\tau^2 = t^2 - x^2 - y^2 - z^2. \quad (9)$$

We define z as (η is related to θ_r ; see Sec. II A)

$$z = \sqrt{\tau_f^2 + x^2 + y^2} \sinh \eta. \quad (10)$$

In Fig. 1, all the angles used in this work are clearly depicted. The angle ϕ_r is the angle in the x - y plane from the blast wave model, Eq. (6), and the angles ϕ^* and θ^* represent the angles between the parent particle's spin and the momentum of the daughter particle.

The δ and ϵ parameters describe the nontrivial effects in the x - y plane in noncentral collisions, extracted from a fit to 130 GeV spectra and elliptic flow data in [30,33]. Now, we can determine the thermal vorticity in the following form:

$$\Omega_{\mu,\nu} = -\frac{1}{2T}(\partial_\mu u_\nu - \partial_\nu u_\mu) - \frac{1}{2T^2}(u_\mu \partial_\nu T - u_\nu \partial_\mu T). \quad (11)$$

The spatial vorticity component is defined as

$$\Omega_i = \frac{1}{2} \epsilon_{ijk} \Omega_{jk}, \quad (12)$$

where $i, j, k = 1, 2, 3$. To obtain the explicit value of vorticity, we need to utilize the fireball parametrization Eqs. (6) and

(10) together with the flow velocity (7) in the thermal vorticity definition (11). Thus, we obtain the following expressions:

$$\begin{aligned}\Omega_1 &= -\frac{yz}{TN^3}(\sqrt{1+\delta} - 1 + \delta), \\ \Omega_2 &= \frac{xz}{TN^3}(1 + \delta - \sqrt{1+\delta}), \\ \Omega_3 &= \frac{xy\sqrt{1-\delta^2}}{TN^3}(\sqrt{1+\delta} - \sqrt{1-\delta}).\end{aligned}\quad (13)$$

In this way, we can connect the thermal vorticity developed in [30] and the coalescence model for vector mesons [12], and

extend this formalism to include baryons. This is important because it allows us to examine the influence of vorticity on the alignment factor and particle polarization within the same framework.

C. Production of baryons and vector mesons

Using the coalescence model, we can study the effects of vortices in the quark-gluon plasma (QGP) on vector mesons with spin 1, such as ϕ and K^* , and compare these effects to those on baryons with spin 1/2. We calculate the density matrix coefficients for vector mesons,

$$\begin{aligned}(\hat{\rho}^M)_{m',m''}^L &= \sum_{m'',m} \sum_{m'_1,m'_2,m'_L} \sum_{m_1,m_2,m_L} \int_0^{2\pi} P_W(v-v_0) P_L(\omega) d_{m'm''}^j(v) C_{S_1+S_2,m'_1+m'_2,L,m'_L}^{J,m''} \\ &\times C_{S_1,m'_1,S_2,m'_2}^{S_1+S_2,m'_1+m'_2} C_{S_1+S_2,m_1+m_2,L,m_L}^{J,m} C_{S_1,m_1,S_2,m_2}^{S_1+S_2,m_1+m_2} \rho_{m_1,m'_1}^1(\Omega) \rho_{m_2,m'_2}^2(\Omega) [d_{mm''}^j(v)]^{-1} dv,\end{aligned}\quad (14)$$

and for baryons,

$$\begin{aligned}(\hat{\rho}^M)_{m',m''}^L &= \sum_{m'',m} \sum_{m'_1,m'_2,m'_3,m'_L} \sum_{m_1,m_2,m_3,m_L} \int_0^{2\pi} P_W(v-v_0) P_L(\omega) d_{m'm''}^j(v) \\ &\times C_{S_1+S_2+S_3,m'_1+m'_2+m'_3,L,m'_L}^{J,m''} C_{S_1+S_2,m'_1+m'_2,S_3,m'_3}^{S_1+S_2,m'_1+m'_2} C_{S_1,m'_1,S_2,m'_2}^{S_1+S_2,m'_1+m'_2} C_{S_1+S_2+S_3,m_1+m_2+m_3,L,m_L}^{J,m} \\ &\times C_{S_1+m_1,S_2+m_2,S_3,m_3}^{S_1+S_2+S_3,m} C_{S_1,m_1,S_2,m_2}^{S_1+S_2,m_1+m_2} \rho_{m_1,m'_1}^1(\Omega) \rho_{m_2,m'_2}^2(\Omega) \rho_{m_3,m'_3}^3(\Omega) [d_{mm''}^j(v)]^{-1} dv,\end{aligned}\quad (15)$$

where Ω represents the vorticity created early in the collision, while ω is the vorticity formed during the hadronization phase, associated with the angular momentum L , as described in detail in [23]:

- (i) $d_{m'm''}^j$ rotate the spin direction of quarks to align with the vorticity ω .
- (ii) $C_{S_1,m'_1,S_2,m'_2}^{S_1+S_2,m'_1+m'_2}$ are Clebsch-Gordan coefficients that couple the spin of quarks, which make up the mesons and baryons, with the angular momentum arising from vorticity.
- (iii) $P_L(\omega)$ is the probability of a thermalized fluid with vorticity ω to transfer the absolute angular momentum quantum number L to the hadron spin structure (though this is something not yet well understood, it is in principle calculable from an open quantum system approach). Note that the only vorticity that can be consistently described this way is the one that carries angular momentum, i.e., thermal vorticity.
- (iv) $P_W(v-v_0)$ is the coalescence Wigner function in spin space, centered around a specific angle, similar to that in previous coalescence work [35] but with just spin degrees of freedom included. As explained in [23] we assume it to be a δ function as in [29] (although it can also be smeared with a Gaussian [23]):

$$P_W(v-v_0) = \delta(v-v_0). \quad (16)$$

- (v) $\rho^{1,2,3}$ is given by

$$\hat{\rho}_{kl}^{1,2,3} = \frac{1}{Z} \exp \left[\frac{\hat{\Omega} \cdot \hat{\sigma}}{2} \right]. \quad (17)$$

It is important to note that, in the case presented here, we do not consider the rotation $U(\phi_r, \theta_r)$. This is because, assuming longitudinal Bjorken flow (where spacetime and momentum rapidities coincide) and in the nonrelativistic limit, the effect of the longitudinal expansion of the QGP is already accounted for in the thermal model [30]. To illustrate this, we refer to [29], where the local spin alignment is considered using the longitudinal expansion of the QGP. We can reproduce the results of [29] by setting $\theta_r = \Delta\psi$ and using the numerical values $\phi_r = -\pi/2$ and $\Omega/T = 1/5$ for $L = 0$. For more details, consult Appendix C in [23].

We can obtain the alignment factor as a function of rapidity, η , using the blast wave model to determine the vorticity, while accounting for the effects of the elliptic flow. In Eqs. (A1)–(A3), ρ_{00} are given for $L = 0, 1, 2$ respectively in Appendix 1, while the off-diagonal matrix elements will be in Appendix 2.

Finally, from Eq. (15), we can determine the polarization of baryons for different values of angular momentum $L = 0$ and $L = 1$. The longitudinal polarization $\chi_{1/2}^L$ [Eq. (1)] is given in Appendix 3 [Eqs. (A16) and (A17)].

III. RESULTS AND DISCUSSION

From Eqs. (A1)–(A3) and vorticity equations defined in (13), we can derive the alignment factor in relation to the

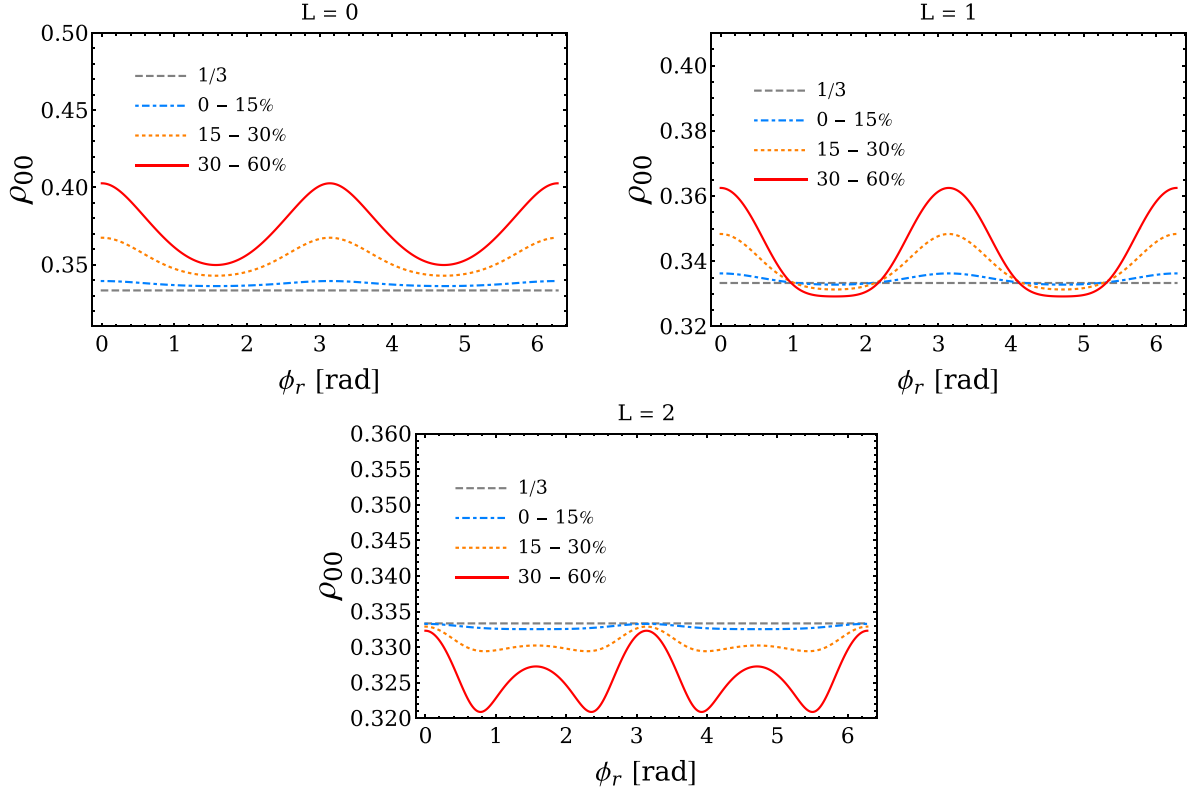


FIG. 2. The alignment factor obtained from the coalescence model combined with the blast wave model at $\sqrt{s_{NN}} = 130$ GeV, relative to azimuthal angle ϕ_r .

azimuthal angle ϕ_r for the spacetime rapidity range $|\eta| < 4$. In Fig. 2 all angular momentum contributions ($L = 0, 1, 2$) of alignment factor exhibit an oscillatory behavior, with the amplitude increasing as centrality increases.

In Figure 3, we observe the alignment factor as function of spacetime rapidity for different centralities. Here ρ_{00} increases with spacetime rapidity for $L = 0$ and $L = 1$ as expected experimentally [36]. In the case of $L = 2$, however, there is a decrease in ρ_{00} . Since the experimental value is expected to be a mixture of these components, we anticipate a relatively small influence from this component. It is important to note that we performed this analysis for *spacetime* rapidity. Assuming the convolution factor in (4) follows a Bjorken like scenario (where spacetime and momentum rapidities coincide, and the parent particle rapidity distribution is approximately flat), we expect that the observed momentum rapidity will follow the same behavior.

Figures 4–7 show the potential of measuring the off-diagonal elements as function of the azimuthal angle ϕ_r . As can be seen, while these elements mostly average to zero, they are expected to vary considerably with the azimuthal angle in the cases where the angle between spin and vorticity is nontrivial and also a considerable amount of angular momentum is transferred from vorticity to spin. This reinforces our previous conclusions [12,23] that these off-diagonal matrix elements are crucial in understanding hadronization in a vortical environment, for they directly probe how different

components of the angular momentum projections to the spin space density matrix. We also notice that the parity with respect to ϕ_r ($\phi_r \rightarrow \phi_r + \pi$) is broken for off-diagonal observables such as $\alpha_{1,-1}$, as expected because the axes of rotation do not commute, something also present in models such as [29]. It will be interesting therefore to see if such a breaking appears in experiment.

The only measurement so far of these observables is in quarkonium [3], where only a reduced set of coefficients of (3) is obtainable since it is not a p -wave decay. The result is compatible with Cooper-Frye style maximal mixing, but as Figs. 4–7 show it is the azimuthal dependence (not as yet measured) that is necessary to establish this.

From Eqs. (A16) and (A17), we can observe the effects on baryon polarization for particles with spin 1/2 under the influence of angular momentum $L = 0$ and $L = 1$ with $v_0 = 0$ and $v_0 = \pi$ in Fig. 8.

In Fig. 8 one can also see that the baryon polarization oscillates with the azimuthal angle, but averages to zero, in agreement with the expectation of zero local baryon polarization. However, Fig. 8 further shows that the azimuthal dependence of the baryon polarization coefficient is a sensitive probe for disentangling the coefficient that regulates the transfer of angular momentum from vorticity to polarization. In particular, we observe that varying angle between vorticity and polarization shifts the phase of the longitudinal polarization distribution with respect to the reaction plane. This

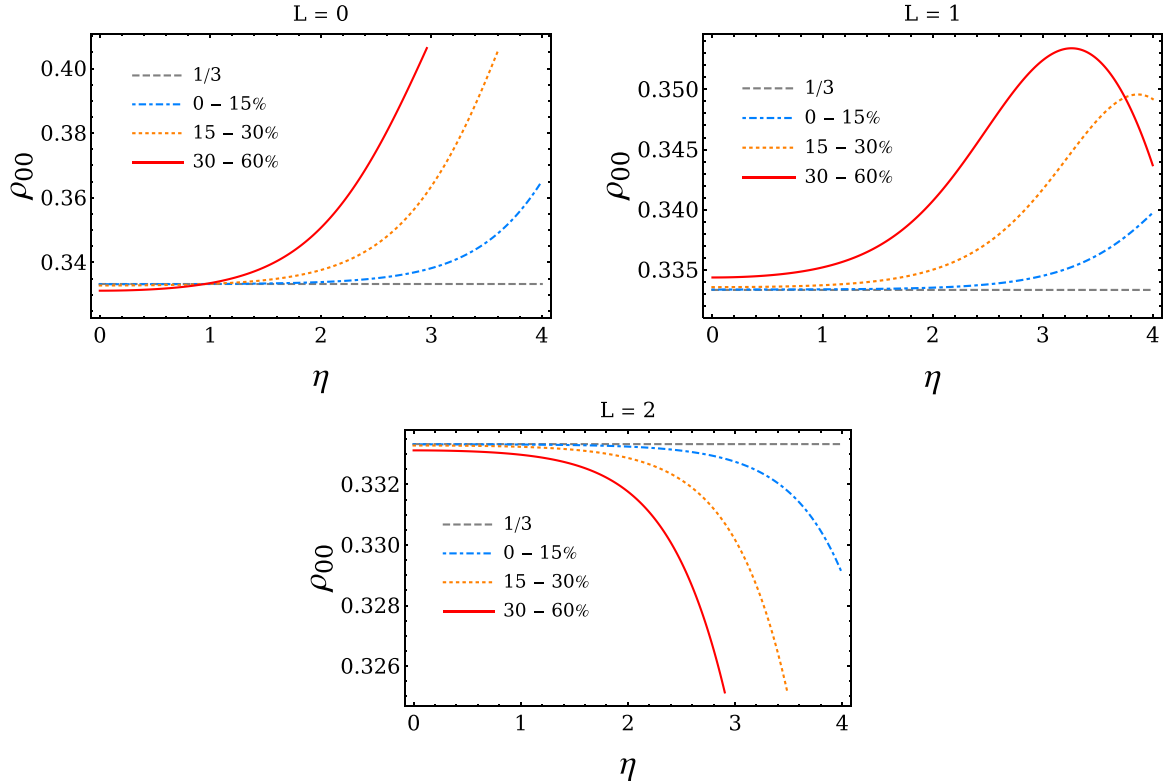


FIG. 3. The alignment factor obtained from the coalescence model together with the blast wave model at $\sqrt{s_{NN}} = 130$ GeV, as a function of spacetime rapidity for different values of angular momentum transferred from the vorticity to the hadron. Note that as the convolution in (4) approaches the Bjorken limit the rapidity distribution of ρ_{00} should follow η .

suggests that nonequilibrium between spin and vorticity could be a potential solution to the local polarization puzzle [6] (a similar result, with a detailed hydrodynamic simulation, was recently shown by [37]).

Moreover, since symmetric shear can arise from spin-vorticity nonequilibrium [17], this solution might be fundamentally related to the already established symmetric shear model [5]

Figure 9 confirms that at ultrarelativistic energies where the thermal model applies the transverse polarization is small. However, when binned in ϕ_r it shows considerable local variation. This observation appears consistent with preliminary data [32].

Comparing meson to baryon plots we can conclude that spin-vorticity nonequilibrium has the potential to resolve all “puzzles” of spin in heavy ion data. However, our model for now is not predictive as it is nonrelativistic and contains a lot of *ad hoc* parameters. That said, if $r_{1,0}$, $\alpha_{1,0}$, $r_{1,-1}$, and $\alpha_{1,-1}$ are found to be significant in mesons³ and strong evidence is found for a nontrivial modulation of ρ_{00} and $\rho_{1/2,1/2}$ of mesons and baryons respectively modulation in ϕ_r , it will be worth to develop this model into a predictive one. This would

create a direct link between experimental observables and the interaction between spin and vorticity at hadronization. In this context, it is worth reiterating that preliminary experimental evidence [32] suggests that the “zero” baryon polarization measured at top energies available at the Relativistic Heavy Ion Collider may in fact conceal significant modulation, which is compatible with our rather large theoretical estimates. However, this evidence is still in the preliminary stage.

Our conclusion is that relaxing the assumption of equilibrium between vorticity and spin, well motivated by theory [15,16,19–22], and augmenting it with a reasonable hadronization dynamics (such as coalescence [23]) could have profound phenomenological consequences. This approach might help solve current “puzzles” in interpreting experimental data on hadronic spin, including the out-of-phase dependence of local polarization and the comparison of vector meson spin alignment with baryon polarization.

However, this model inevitably introduces additional parameters, related specifically to spin physics [such as Wigner function P_W and the vorticity projection to spin space P_L in (14)]. This makes detailed quantitative fits premature at this stage. Nevertheless, our work encourages further experimental work. The most promising observable, advocated previously, is the azimuthal dependence of spin-sensitive observables. Specifically, significant deviations in the off diagonal elements of the spin 1 density matrix from zero would

³As argued in [23] and calculated here in Figs. 4–7, the Cooper-Frye formalism would imply such off-diagonal matrix elements should be zero because of maximal mixing, unlike coalescence.

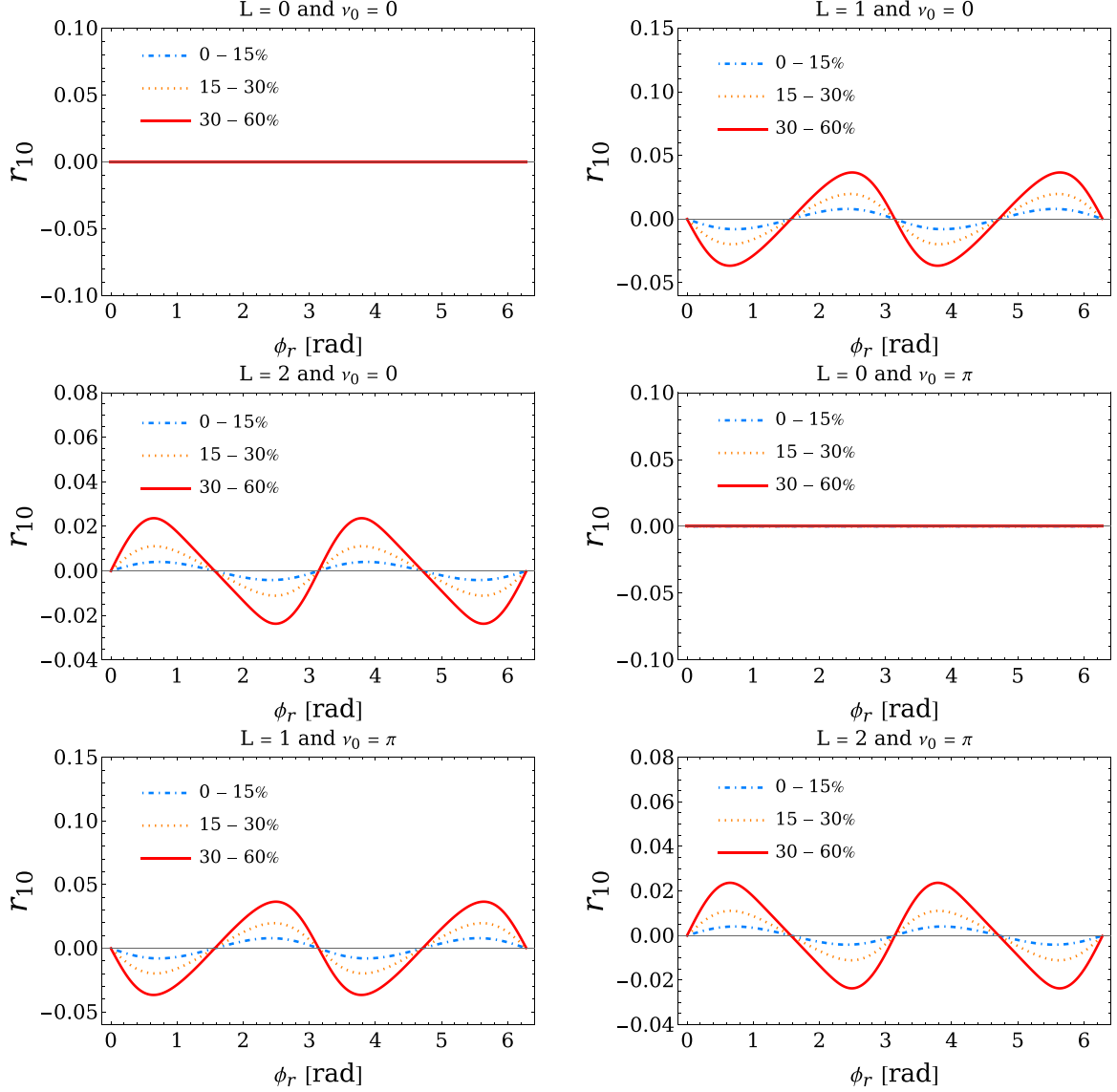


FIG. 4. The density matrix coefficient r_{10} with respect to the reaction plane azimuthal angle ϕ_r for the angles between spin and vorticity $v_0 = 0$ and $v_0 = \pi$.

indicate that the Cooper-Frye freeze-out dynamics is insufficient.

Another important prediction of this model is that spin $3/2$ baryons, such as Δ, Ω , will exhibit behaviors different from their spin $1/2$ counterparts. In particular, the decay chain $\Omega \rightarrow \Lambda K \rightarrow p\pi K$, with its double weak decay, is promising. Its sequential azimuthal analysis could offer full measurement of its very large 4×4 density matrix. The fact that Ω polarization appears to be significantly higher than Λ [38] is encouraging because it is consistent with the general idea of this work that quark polarization reflects initial state values which are higher than Cooper-Frye estimates, but, in the coalescence process in lower hadronic

spin states, spin and angular momentum cancel. The detailed analysis of the Ω will be the focus of a future project.

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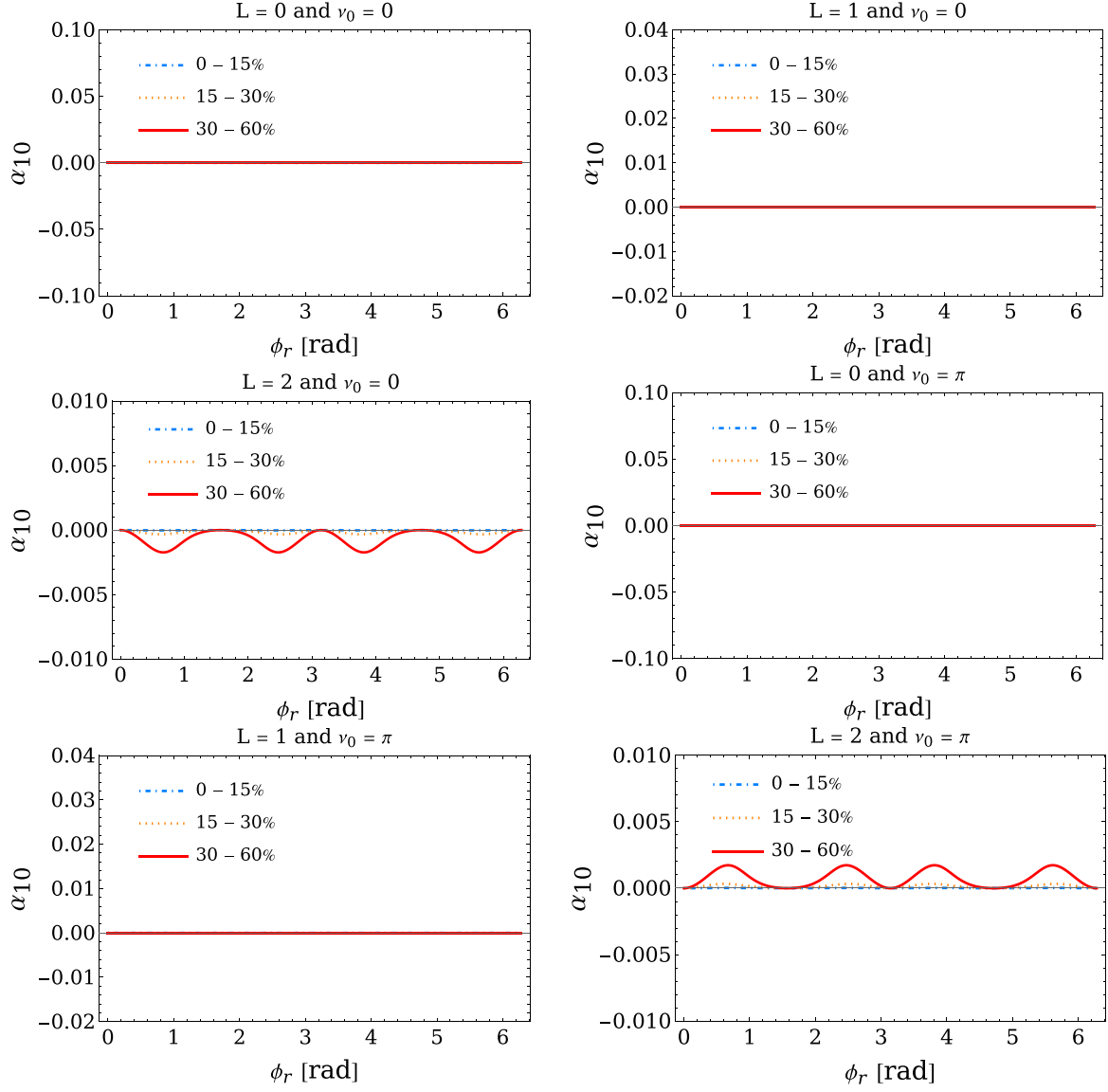


FIG. 5. The off-diagonal density matrix coefficient α_{10} in relation to azimuthal angle ϕ_r for the angles between spin and vorticity $\nu_0 = 0$ and $\nu_0 = \pi$.

APPENDIX: EXPLICIT EXPRESSIONS FOR SPIN DENSITY MATRIX COEFFICIENTS

1. Vector meson diagonal matrix elements

The coefficient $\langle(\rho^M)_{00}^{L=0}(\Omega_1, \Omega_2, \Omega_3, T)\rangle$:

$$\begin{aligned}
 \langle(\rho^M)_{00}^{L=0}(\Omega_1, \Omega_2, \Omega_3, T)\rangle = & \frac{1}{N_0[\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)]} \left\{ \Omega_3^2 \cos^2(\nu_0) + 2\sqrt{2}\Omega_1\Omega_3 \sin(2\nu_0) \right. \\
 & \times \sinh^2\left(\frac{1}{2}\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) + \cosh\left(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) [\Omega_3^2 \sin^2(\nu_0) \\
 & \left. + 2\Omega_2^2] + 2\Omega_1^2 [\cos^2(\nu_0) \cosh\left(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) + \sin^2(\nu_0)] \right\}, \quad (A1)
 \end{aligned}$$

$$N_0 = 1 + 2 \cosh\left(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right).$$

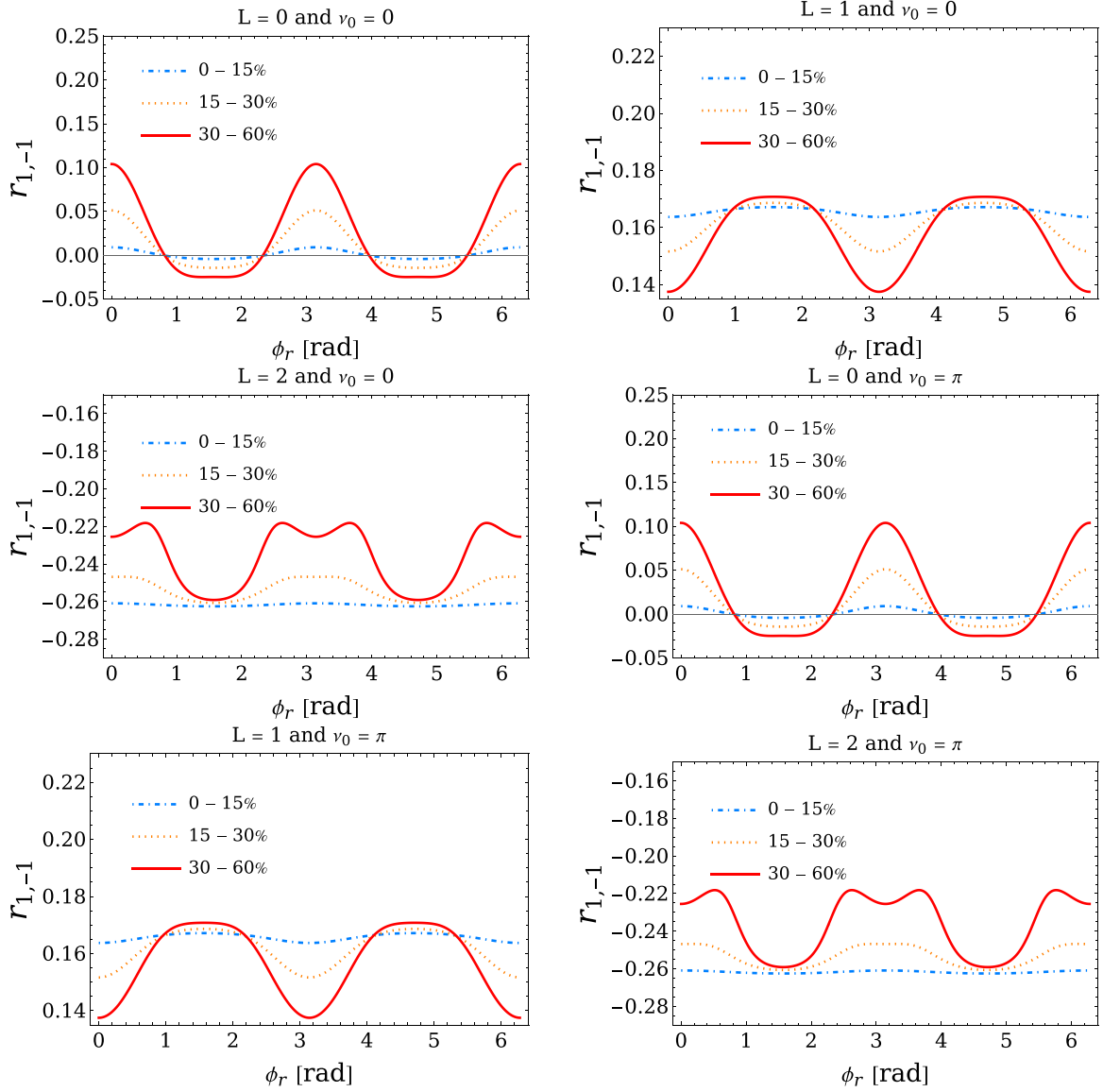


FIG. 6. The off-diagonal density matrix coefficient $r_{1,-1}$ for the cases where the angle between the spin and vorticity $v_0 = 0$ and $v = \pi$, in relation to the azimuthal angle ϕ_r for various centralities.

The coefficient $\langle (\rho^M)_{00}^{L=1}(\Omega_1, \Omega_2, \Omega_3, T) \rangle$:

$$\begin{aligned}
 \langle (\rho^M)_{00}^{L=1}(\Omega_1, \Omega_2, \Omega_3, T) \rangle &= \frac{1}{N_1 [4\Omega_3^2 + 8(\Omega_1^2 + \Omega_2^2)]} 4(\Omega_2^2 + \Omega_3^2) \sin^2(v_0) - 8\Omega_1 \\
 &\times \left[\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)} \sin^2(v_0) \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) + \sqrt{2}\Omega_3 \sin(2v_0) \right. \\
 &\times \left. \sinh^2\left(\frac{1}{2}\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) \right] + 2\Omega_3 \sqrt{2\Omega_3^2 + 4(\Omega_1^2 + \Omega_2^2)} \sin(2v_0) \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) \\
 &+ \cosh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) \Omega_3^2 [\cos(2v_0) + 3] + 8\Omega_2^2 + 4\Omega_1^2 [3 \sin^2(v_0) \cosh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) \\
 &+ 2 \cos^2(v_0)], N_1 = \frac{1}{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)} \left[-2\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)} \Omega_1 \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) \right. \\
 &\left. + (3\Omega_1^2 + 5\Omega_2^2 + 2\Omega_3^2) \cosh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) + 3\Omega_1^2 + \Omega_2^2 + \Omega_3^2 \right]. \quad (A2)
 \end{aligned}$$

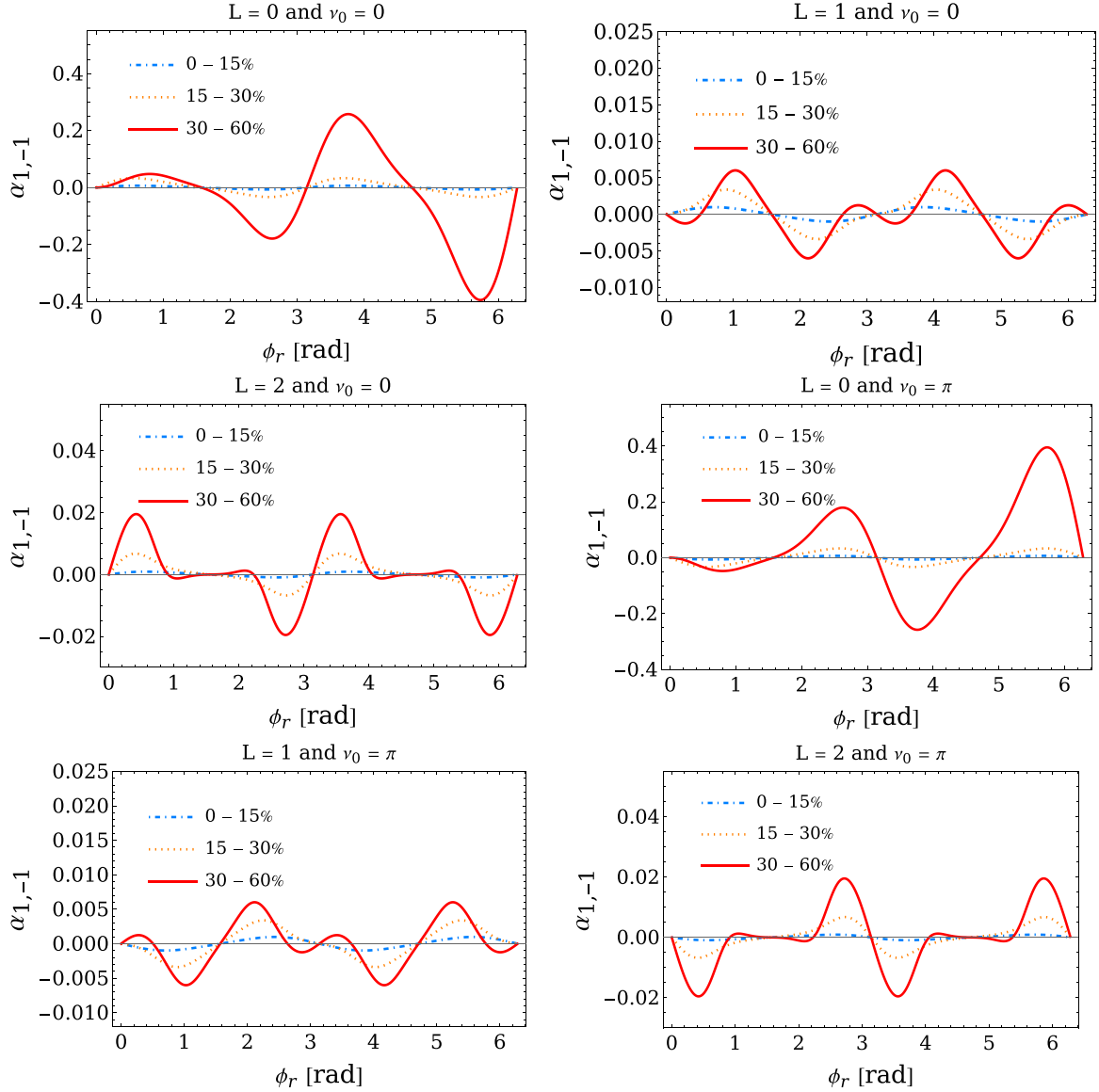
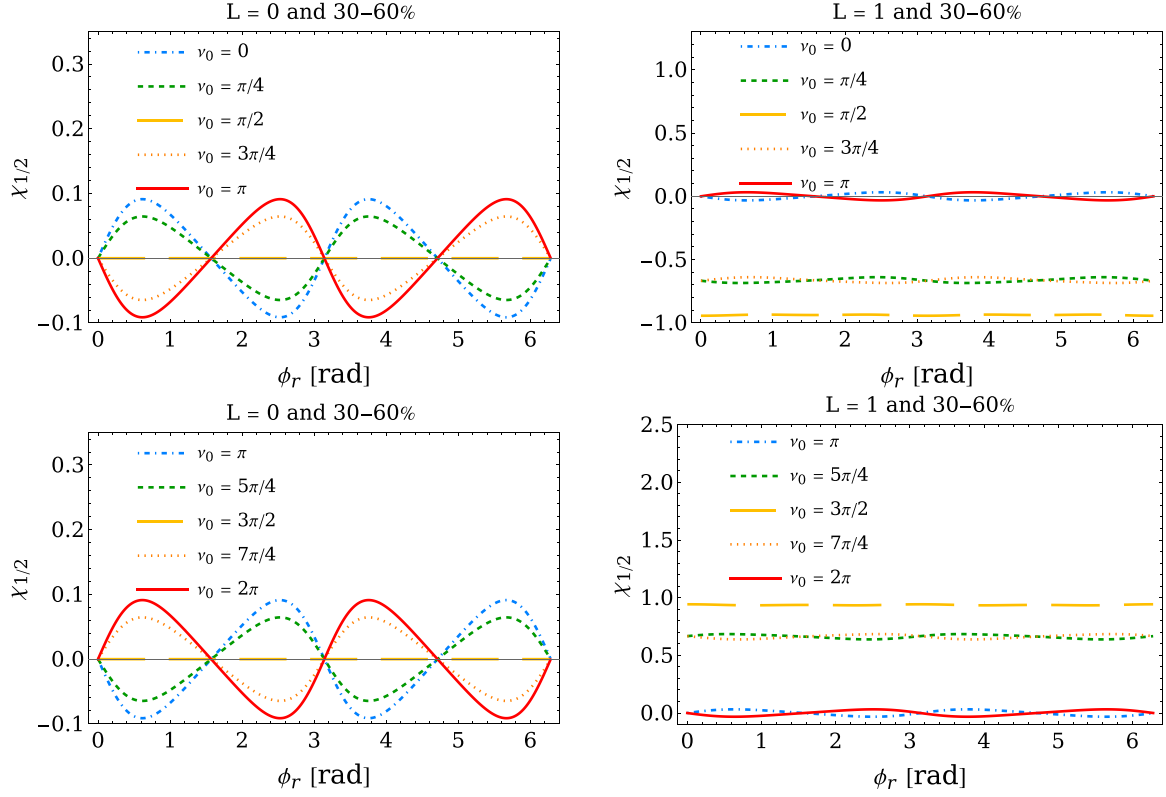


FIG. 7. The off-diagonal density matrix coefficient $\alpha_{1,-1}$ in relation to the azimuthal angle ϕ_r is given for angles between spin and vorticity $\nu_0 = 0$ and $\nu_0 = \pi$ for various centrality values.

The coefficient $\langle (\rho^M)_{00}^{L=2}(\Omega_1, \Omega_2, \Omega_3, T) \rangle$:

$$\begin{aligned}
 \langle (\rho^M)_{00}^{L=2}(\Omega_1, \Omega_2, \Omega_3, T) \rangle = & \frac{1}{10N_2[\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)]} (4\Omega_1((2\sqrt{3} - \sqrt{2})\Omega_3 \sin(2\nu_0) \\
 & \times \sinh^2\left(\frac{1}{2}\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) - 2\sqrt{3\Omega_3^2 + 6(\Omega_1^2 + \Omega_2^2)} \cos^2(\nu_0) \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)})) \\
 & + 2\Omega_1^2(10 \cos^2(\nu_0) \cosh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) + (7 - 2\sqrt{6}) \sin^2(\nu_0)) + 2\Omega_2^2 \\
 & \times (\cosh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)})((7 - 2\sqrt{6}) \sin^2(\nu_0) + 4 \cos^2(\nu_0)) + 6 \cos^2(\nu_0)) \\
 & + \Omega_3((\sqrt{6} - 6)\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)} \sin(2\nu_0) \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) \\
 & + \Omega_3(\cosh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)})((7 - 2\sqrt{6}) \sin^2(\nu_0) + 6 \cos^2(\nu_0)) + 4 \cos^2(\nu_0))).
 \end{aligned}$$

FIG. 8. The longitudinal baryon polarization at various values of ν_0 as a function of reaction azimuthal ϕ_r .

$$\begin{aligned}
 N_2 = & \frac{1}{5[\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)]} \left\{ -6(\sqrt{2} + \sqrt{3})\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\Omega_1 \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) \right. \\
 & + \Omega_1^2[(23 + 2\sqrt{6})\cosh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) - 2\sqrt{6} + 7] + [(17 - 2\sqrt{6})\Omega_2^2 + 10\Omega_3^2] \\
 & \times \cosh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) + (13 + 2\sqrt{6})\Omega_2^2 + 5\Omega_3^2 \left. \right\}. \quad (A3)
 \end{aligned}$$

Note that some combinations of P_L and polarization are forbidden by symmetry (the Pauli-Lubanski decomposition ensures $L = 0$, $\chi_{1/2} = 0$). For example $L = 0$ cannot result in baryon polarization. Nevertheless, we have calculated these cases as a test of our numerical model.

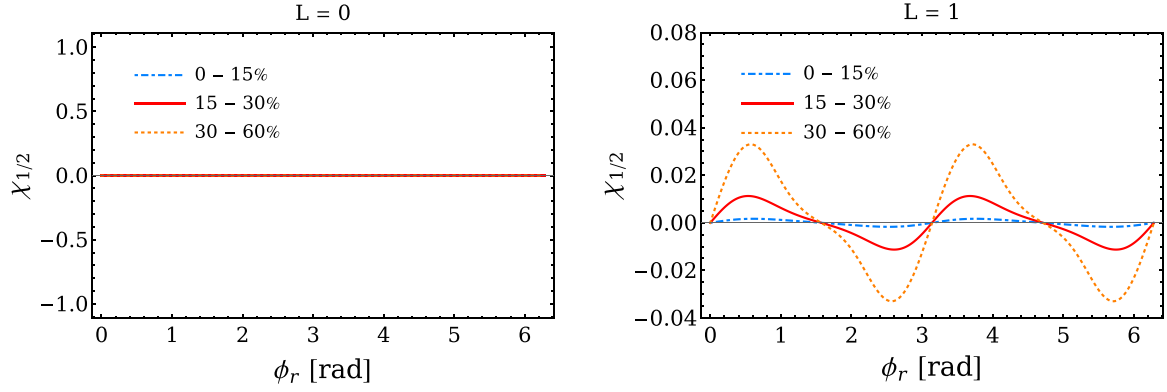
2. Vector meson off-diagonal matrix elements

The coefficient $\langle r_{1,0}^{L=0}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle$:

$$\begin{aligned}
 \langle r_{1,0}^{L=0}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle = & \frac{1}{N_0(2\Omega_1^2 + 2\Omega_2^2 + \Omega_3^2)} \left(\sinh^2\left(\frac{1}{2}\sqrt{2\Omega_1^2 + 2\Omega_2^2 + \Omega_3^2}\right)(2\Omega_1^2 \sin(2\nu_0) \right. \\
 & \left. - \Omega_3^2 \sin(2\nu_0) - 2\sqrt{2}\Omega_3\Omega_1 \cos(2\nu_0)) \right). \quad (A4)
 \end{aligned}$$

The coefficient $\langle r_{1,0}^{L=1}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle$:

$$\begin{aligned}
 \langle r_{1,0}^{L=1}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle = & \frac{1}{2N_1[\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)]^{3/2}} (2\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\Omega_1^2 \sin(2\nu_0) \\
 & - \Omega_2^2\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)} \sin(2\nu_0) - \Omega_3^2\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)} \sin(2\nu_0) + 4\Omega_3\sqrt{2\Omega_3^2 + 4(\Omega_1^2 + \Omega_2^2)} \\
 & \times \Omega_1 \cos(2\nu_0) \sinh^2\left(\frac{1}{2}\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) + 8\Omega_1^3 \sin(\nu_0) \cos(\nu_0) \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)})
 \end{aligned}$$

FIG. 9. The transverse baryon polarization as a function of azimuthal angle ϕ_r .

$$\begin{aligned}
& + \sinh\left(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) \left(2\Omega_1(2\Omega_2^2 + \Omega_3^2) \sin(2\nu_0) - \sqrt{2}\Omega_3(\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)) \cos(2\nu_0)\right) \\
& - 6\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\Omega_1^2 \sin(\nu_0) \cos(\nu_0) \cosh\left(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) + \Omega_3^2\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)} \\
& \times \sin(\nu_0) \cos(\nu_0) \cosh\left(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right). \tag{A5}
\end{aligned}$$

The coefficient $\langle r_{1,0}^{L=2}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle$:

$$\begin{aligned}
\langle r_{1,0}^{L=2}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle &= \frac{1}{20N_2[\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)]} \left(2\Omega_1^2 \sin(2\nu_0)\right. \\
& \times (10 \cosh\left(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) + 2\sqrt{6} - 7) + 2\Omega_2^2 \sin(2\nu_0)((2\sqrt{6} - 3) \\
& \times \cosh\left(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) + 6) + 8\Omega_1((\sqrt{2} - 2\sqrt{3})\Omega_3 \cos(2\nu_0) \\
& \times \sinh^2\left(\frac{1}{2}\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) - \sqrt{3}\Omega_3^2 + 6(\Omega_1^2 + \Omega_2^2) \sin(2\nu_0) \sinh\left(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right)) \\
& + \Omega_3(\Omega_3 \sin(2\nu_0)((2\sqrt{6} - 1) \cosh\left(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) + 4) - 2(\sqrt{6} - 6) \\
& \times \sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)} \cos(2\nu_0) \sinh\left(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right)) \left. \right). \tag{A6}
\end{aligned}$$

The coefficient $\langle \alpha_{1,0}^{L=0}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle$:

$$\langle \alpha_{1,0}^{L=0}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle = \frac{2\Omega_2 \sinh^2\left(\frac{1}{2}\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) (2\Omega_1 \sin(\nu_0) - \sqrt{2}\Omega_3 \cos(\nu_0))}{N_0[\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)]}. \tag{A7}$$

The coefficient $\langle \alpha_{1,0}^{L=1}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle$:

$$\langle \alpha_{1,0}^{L=1}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle = \frac{\Omega_2 \sin(\nu_0)}{N_1} \left(\frac{\sinh\left(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right)}{\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}} - \frac{\Omega_1 [\cosh\left(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) - 1]}{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)} \right). \tag{A8}$$

The coefficient $\langle \alpha_{1,0}^{L=2}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle$:

$$\begin{aligned} \langle \alpha_{1,0}^{L=2}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle = & \frac{1}{5N_2[\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)]} \left(\Omega_2(\sqrt{21 - 6\sqrt{6}}\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)} \right. \\ & \times \sin(\nu_0) \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) - 2 \sinh^2\left(\frac{1}{2}\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) \\ & \left. \times (5\Omega_1 \sin(\nu_0) + 2(\sqrt{2} - \sqrt{3})\Omega_3 \cos(\nu_0)) \right). \end{aligned} \quad (A9)$$

The coefficient $\langle r_{1,-1}^{L=0}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle$:

$$\begin{aligned} \langle r_{1,-1}^{L=0}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle = & -\frac{1}{N_0[\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)]} \left[\sinh^2\left(\frac{1}{2}\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) \right. \\ & \left. \times (\Omega_3^2 \sin^2(\nu_0) + \sqrt{2}\Omega_3\Omega_1 \sin(2\nu_0) + 2\Omega_1^2 \cos^2(\nu_0) - 2\Omega_2^2) \right]. \end{aligned} \quad (A10)$$

The coefficient $\langle r_{1,-1}^{L=1}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle$:

$$\begin{aligned} \langle r_{1,-1}^{L=1}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle = & -\frac{1}{4N_1[\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)]^{3/2}} (8\Omega_1^3 \cos^2(\nu_0) \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) \\ & + \Omega_3^2(\sqrt{2}\Omega_3 \sin(2\nu_0) \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) \\ & - \sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}(\sin^2(\nu_0) \cosh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) + 2\cos^2(\nu_0))) \\ & + 2\Omega_1^2(\sqrt{2}\Omega_3 \sin(2\nu_0) \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) + \sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}(\cos(2\nu_0) \\ & - 3\cos^2(\nu_0) \cosh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}))) + 2\Omega_1(\Omega_3\sqrt{2\Omega_3^2 + 4(\Omega_1^2 + \Omega_2^2)} \sin(2\nu_0) \\ & + 2(2\Omega_2^2 + \Omega_3^2) \cos^2(\nu_0) \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) - 2\Omega_3\sqrt{2\Omega_3^2 + 4(\Omega_1^2 + \Omega_2^2)} \sin(\nu_0) \cos(\nu_0) \\ & \times \cosh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)})) - \Omega_2^2(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) \\ & \times (\cos(2\nu_0) + 2\cosh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) + 1) \\ & - 4\sqrt{2}\Omega_3 \sin(\nu_0) \cos(\nu_0) \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)})). \end{aligned} \quad (A11)$$

The coefficient $\langle r_{1,-1}^{L=2}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle$:

$$\begin{aligned} \langle r_{1,-1}^{L=2}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r) \rangle = & -\frac{1}{20N_2[\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)]^{3/2}} (-8\Omega_1^3(\sqrt{3}\cos(2\nu_0) + 3\sqrt{2}) \\ & \times \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) - 4\Omega_1(2\Omega_2^2(\sqrt{3}\cos(2\nu_0) + 3\sqrt{2}) \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) \\ & + \Omega_3((\sqrt{2} - 2\sqrt{3})\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)} \sin(2\nu_0) \sinh^2\left(\frac{1}{2}\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) \\ & + \Omega_3(\sqrt{3}\cos(2\nu_0) + 3\sqrt{2}) \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}))) + \Omega_1^2(2(\sqrt{6} - 6)\Omega_3 \sin(2\nu_0) \\ & \times \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) + 2\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}((5\cos(2\nu_0) + 2\sqrt{6} + 8) \\ & \times \cosh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) + (2\sqrt{6} - 7)\cos^2(\nu_0))) + \Omega_2^2(2(\sqrt{6} - 6)\Omega_3 \sin(2\nu_0) \\ & \times \sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) + \sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}((2\sqrt{6} - 3)\cos(2\nu_0) + 2\sqrt{6} + 1) \end{aligned}$$

$$\begin{aligned}
& \times \cosh\left(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) + 6\cos(2\nu_0) + 4\sqrt{6} + 8) + \frac{1}{2}\Omega_3^2(2(\sqrt{6} - 6)\Omega_3\sin(2\nu_0) \\
& \times \sinh\left(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) + \sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}((2\sqrt{6} - 1)\cos(2\nu_0) + 6\sqrt{6} + 1) \\
& \times \cosh\left(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) + 4(\cos(2\nu_0) + 2))). \tag{A12}
\end{aligned}$$

The coefficient $\langle\alpha_{1,-1}^{L=0}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r)\rangle$:

$$\langle\alpha_{1,-1}^{L=0}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r)\rangle = -\frac{2\Omega_2 \sinh^2\left(\frac{1}{2}\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right)(\sqrt{2}\Omega_3\sin(\nu_0) + 2\Omega_1\cos(\nu_0))}{N_0(\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2))} \tag{A13}$$

The coefficient $\langle\alpha_{1,-1}^{L=1}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r)\rangle$:

$$\langle\alpha_{1,-1}^{L=1}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r)\rangle = \frac{\Omega_2 \cos(\nu_0)}{N_1} \left(\frac{\Omega_1(\cosh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}) - 1)}{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)} - \frac{\sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)})}{\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}} \right). \tag{A14}$$

The coefficient $\langle\alpha_{1,-1}^{L=2}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r)\rangle$:

$$\begin{aligned}
\langle\alpha_{1,-1}^{L=2}(\Omega_1, \Omega_2, \Omega_3, T, \phi_r)\rangle &= \frac{1}{N_2\{5[\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)]\}} \left(\Omega_2(2\sinh^2\left(\frac{1}{2}\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)}\right) \right. \\
&\times (2\sqrt{5 - 2\sqrt{6}}\Omega_3\sin(\nu_0) + 5\Omega_1\cos(\nu_0)) - \sqrt{21 - 6\sqrt{6}}\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)} \\
&\times \cos(\nu_0)\sinh(\sqrt{\Omega_3^2 + 2(\Omega_1^2 + \Omega_2^2)})) \Big). \tag{A15}
\end{aligned}$$

3. $S = 1/2$ baryon coefficients

The coefficient $\langle\chi_{1/2}^{L=0}(\Omega_1, \Omega_2, \Omega_3, T, \nu_0)\rangle$:

$$\begin{aligned}
\langle\chi_{1/2}^{L=0}(\Omega_1, \Omega_2, \Omega_3, T, \nu_0)\rangle &= \frac{\tanh\left(\frac{|\mathbf{\Omega}|}{2}\right)[\Omega_3\cos(\nu_0) - \Omega_1\sin(\nu_0)]}{|\mathbf{\Omega}|}, \\
|\mathbf{\Omega}| &= \sqrt{\Omega_1^2 + \Omega_2^2 + \Omega_3^2}. \tag{A16}
\end{aligned}$$

The coefficient $\langle\chi_{1/2}^{L=1}(\Omega_1, \Omega_2, \Omega_3, T, \nu_0)\rangle$:

$$\begin{aligned}
\langle\chi_{1/2}^{L=1}(\Omega_1, \Omega_2, \Omega_3, T, \nu_0)\rangle &= \frac{1}{D} \left[[\Omega_3\cos(\nu_0) - 3\Omega_1\sin(\nu_0)] \sinh\left(\frac{|\mathbf{\Omega}|}{2}\right) + 2\sqrt{2}|\mathbf{\Omega}|\sin(\nu_0) \cosh\left(\frac{|\mathbf{\Omega}|}{2}\right) \right], \\
D &= 2\sqrt{2}\Omega_1 \sinh\left(\frac{|\mathbf{\Omega}|}{2}\right) - 3|\mathbf{\Omega}| \cosh\left(\frac{|\mathbf{\Omega}|}{2}\right). \tag{A17}
\end{aligned}$$

The transverse polarization is defined as $\chi_{1/2}^{L=1}(\Omega_1, \Omega_2, \Omega_3, T, \nu_0) = \text{Tr}[\hat{\sigma}_y \cdot \hat{\rho}^B]$, and thus

The coefficient $\langle\chi_{1/2}^{L=0}(\Omega_1, \Omega_2, \Omega_3, T, \nu_0)\rangle$:

$$\langle\chi_{1/2}^{L=0}(\Omega_1, \Omega_2, \Omega_3, T, \nu_0)\rangle = -\frac{\Omega_2 \tanh\left(\frac{1}{2}|\mathbf{\Omega}|\right)}{|\mathbf{\Omega}|}. \tag{A18}$$

The coefficient $\langle\chi_{1/2}^{L=1}(\Omega_1, \Omega_2, \Omega_3, T, \nu_0)\rangle$:

$$\langle\chi_{1/2}^{L=1}(\Omega_1, \Omega_2, \Omega_3, T, \nu_0)\rangle = \frac{\Omega_2}{2\sqrt{2}\Omega_1 - 3|\mathbf{\Omega}| \coth\left(\frac{1}{2}|\mathbf{\Omega}|\right)}. \tag{A19}$$

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