

Semileptonic decays of $\Lambda \rightarrow p\ell^-\bar{\nu}_\ell$ in light-front dynamics

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We investigate the exclusive semileptonic decays of $\Lambda \rightarrow p\ell^-\bar{\nu}_\ell$ ($\ell = e, \mu$) within the standard model using the light-front quark model. The transition form factor behaviors of $\Lambda \rightarrow p$ are obtained from the effective treatment of nonvalence contributions in addition to the valence ones in the Drell-Yan-West frame due to the Bethe-Salpeter formalism. Based on these form factors, we obtain that the branching ratios of $\Lambda \rightarrow pe^-\bar{\nu}_e$ and $p\mu^-\bar{\nu}_\mu$, including nonvalence contributions, are around 8.32×10^{-4} and 1.31×10^{-4} , which are consistent with the latest measurements from the BESIII Collaboration, respectively. Our results indicate that nonvalence contributions can play a non-negligible role in the semileptonic baryon decays within the light-front framework.

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I. INTRODUCTION

Semileptonic decays of mesons and baryons have long provided valuable insights for testing the standard model (SM) and play a crucial role in understanding the interplay between strong and weak interactions. In particular, they can be used to conduct rigorous tests of the Cabibbo-Kobayashi-Maskawa (CKM) mixing elements within the SM [1–3]. The ratio of the semileptonic decay rates, $R_{\mu e}^{B_1 B_2} = \Gamma(B_1 \rightarrow B_2 \mu^-\bar{\nu}_\mu)/\Gamma(B_1 \rightarrow B_2 e^-\bar{\nu}_e)$, between the muon and electron decay channels has been extensively studied in the literature [4–13]. Recently, the BESIII [14,15] and LHCb [16] Collaborations have reported updated results on the $\Lambda \rightarrow p\ell^-\bar{\nu}_\ell$ decays given by

$$\begin{aligned} \mathcal{B}(\Lambda \rightarrow p\mu^-\bar{\nu}_\mu) &= (1.48 \pm 0.21 \pm 0.08) \times 10^{-4}, \\ \mathcal{B}(\Lambda \rightarrow pe^-\bar{\nu}_e) &= (8.16 \pm 0.22 \pm 0.14) \times 10^{-4}, \\ R_{\mu e}^{\Lambda p}|_{\text{expt}} &= 0.181 \pm 0.028, \end{aligned} \quad (1)$$

and

$$\begin{aligned} \mathcal{B}(\Lambda \rightarrow p\mu^-\bar{\nu}_\mu) &= (1.462 \pm 0.1) \times 10^{-4} \\ R_{\mu e}^{\Lambda p}|_{\text{exp}} &= 0.175 \pm 0.012, \end{aligned} \quad (2)$$

respectively. As pointed out in Ref. [12], the ratio of $R_{\mu e}^{\Lambda p}$ is not only independent of V_{us} and the lifetime of Λ , but is free of hadronic form-factor uncertainties at the next-to-leading order in the SM. Explicitly, the authors [12] predicted that $R_{\mu e}^{\Lambda p} = 0.153 \pm 0.008$, which was recently updated to be

0.162 ± 0.004 in Ref. [13] based on the data in the PDG [17]. From a theoretical perspective, this type of decay process involves an $s \rightarrow u$ transition mediated by a virtual W boson, making it a useful process to study weak interactions and hadron structures.

The transition form factor is a fundamental property of hyperons that describes the dynamic behavior of the transition between two states. The hadron part of the weak decay amplitude of a hyperon into a proton is described by the transition matrix element $\langle p|\bar{u}\gamma^\mu(1-\gamma_5)s|\Lambda\rangle$, which is parametrized by six form factors of $f_i(q^2)$ and $g_i(q^2)$ ($i = 1, 2, 3$), given by

$$\begin{aligned} &\left\langle \mathcal{B}_p(P', S') \left| \bar{u}\gamma^\mu(1-\gamma_5)s \right| \mathcal{B}_\Lambda(P, S = \frac{1}{2}, S_z) \right\rangle \\ &= \bar{u}_\alpha(P', S'_z) \left[\gamma^\mu f_1(q^2) + i \frac{f_2(q^2)}{M_\Lambda} \sigma^{\mu\nu} q_\nu + \frac{f_3(q^2)}{M_\Lambda} q^\mu \right] \\ &\quad \times u(P, S_z) - \bar{u}_\alpha(P', S'_z) \left[\gamma^\mu g_1(q^2) + i \frac{g_2(q^2)}{M_\Lambda} \sigma^{\mu\nu} q_\nu \right. \\ &\quad \left. + \frac{g_3(q^2)}{M_\Lambda} q^\mu \right] \gamma_5 u(P, S_z), \end{aligned} \quad (3)$$

where $q = P - P'$, $P^{(\prime)}$ and $S_z^{(\prime)}$ represent the momentum and spin of Λ (proton), respectively. It follows that these form factors encode the nonperturbative strong-interaction dynamics of the baryonic transition.

In this work, we employ the light-front quark model (LFQM) based on the light-front (LF) quantization [18–23] to calculate the hadronic form factors in Eq. (3) and evaluate the decay rates of $\Lambda \rightarrow p\ell^-\bar{\nu}_\ell$ ($\ell = e, \mu$) within the SM. From a phenomenological perspective, the LF formalism provides a simple, nonperturbative, and relativistic framework for calculating hadronic form factors. In this paper, the LFQM incorporates a diquark model [24,25], in which a baryon consists of an active quark and a spectator diquark, thereby allowing the nonperturbative interaction between the two light quarks to be effectively incorporated into the diquark mass. This approximation effectively reduces the baryonic system to

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a two-body problem, significantly reducing the computational burden. The LFQM [23] framework adopted in this work has several noteworthy features compared to other LFQM [26,27] analyses: (1) We evaluate the six form factors in the spin $1/2 \rightarrow 1/2$ transition matrix element in LF coordinates. (2) We analyze the form factors in a timelike space and extend them to the entire physical region, thereby obtaining the form factors of the baryon semileptonic decays. (3) We use the recent methods proposed in Refs. [20–23] with an effective treatment based on the Bethe-Salpeter (B-S) formalism (see [20–22,28]) for addressing the higher-order Fock contributions to the form factors in the $q^+ > 0$ coordinate system.

In the LF framework, the μ index in the V-A current takes the $+$ component, while the calculation is performed in the range with $q^+ \neq 0$, i.e., in the timelike domain ($q^2 > 0$). From a physical perspective, since semileptonic decays probe the in physical timelike region, it is preferable to evaluate the form factors in the $q^2 > 0$ domain without relying solely on analytic continuation. Similarly, when conducting studies in the timelike domain, contributions from nonvalence Fock states may also appear due to the inclusion of higher-order Fock states in the wave function, in addition to valence states [20]. In other words, we will inevitably encounter nonvalence diagrams arising from the creation of quark-antiquark pairs. Note that, effective methods for handling nonvalence contributions in meson-meson scattering processes have already been established in the literature [20–22,28].

This paper is organized as follows. In Sec. II, we present the framework for the baryonic semileptonic decays of $\Lambda \rightarrow p \ell^- \bar{\nu}_\ell$. Our numerical results and discussions are given in Sec. III. We conclude in Sec. IV.

II. FRAMEWORK FOR THE SEMILEPTONIC DECAYS OF $\Lambda \rightarrow p \ell^- \bar{\nu}_\ell$

In this section, we present the theoretical framework for the semileptonic decay of $\Lambda \rightarrow p \ell^- \bar{\nu}_\ell$ within the LFQM. The decay amplitudes are governed by the effective weak Hamiltonian describing the transition of $s \rightarrow u$ at quark-level,

$$\begin{aligned} H_{\text{eff}}(\Lambda \rightarrow p \ell^- \bar{\nu}_\ell) &= \frac{G_F}{\sqrt{2}} V_{us} \left\langle \mathcal{B}_p \left(P', S' = \frac{1}{2} \right) \right| \bar{u} \gamma^\mu (1 - \gamma_5) s \\ &\times \left| \mathcal{B}_\Lambda \left(P, S = \frac{1}{2} \right) \right\rangle \bar{\nu}_\ell \gamma^\mu (1 - \gamma_5) \ell. \end{aligned} \quad (4)$$

We note that the discussion in this section can be viewed as a generic study of $\mathcal{B}'(P', S') \rightarrow \mathcal{B}(P, S = \frac{1}{2}) \ell^- \bar{\nu}_\ell$ for any $\mathcal{B}'(1/2) \rightarrow \mathcal{B}(1/2)$ transition.

A. Helicity amplitudes

The transition matrix elements in Eq. (3) can be applied to the helicity amplitudes, given by [29–31]

$$H_{\lambda_p \lambda_W}^{V(A)} \equiv \langle p | (\bar{u}s)_{V(A)} | \Lambda \rangle \varepsilon_W^\mu, \quad (5)$$

where ε_W^μ is the polarization of the W boson, and $\lambda_p = \pm 1/2$ represent the helicity states of the proton. Based on the helicity conservation, $\lambda_\Lambda = \lambda_p - \lambda_W$ is held.

The helicity amplitudes are related to the form factors through the following expressions:

$$\begin{aligned} H_{\frac{1}{2},t}^V &= -i \frac{\sqrt{Q_+}}{\sqrt{q^2}} \left((M_\Lambda - M_p) f_1 + \frac{q^2}{M_\Lambda} f_3 \right), \\ H_{\frac{1}{2},0}^V &= -i \frac{\sqrt{Q_-}}{\sqrt{q^2}} \left((M_\Lambda + M_p) f_1 - \frac{q^2}{M_\Lambda} f_2 \right), \\ H_{\frac{1}{2},1}^V &= -i \sqrt{2Q_-} \left(f_1 - \frac{M_\Lambda + M_p}{M_\Lambda} f_2 \right), \\ H_{\frac{1}{2},t}^A &= -i \frac{\sqrt{Q_-}}{\sqrt{q^2}} \left((M_\Lambda + M_p) g_1 - \frac{q^2}{M_\Lambda} g_3 \right), \\ H_{\frac{1}{2},0}^A &= -i \frac{\sqrt{Q_+}}{\sqrt{q^2}} \left((M_\Lambda - M_p) g_1 + \frac{q^2}{M_\Lambda} g_2 \right), \\ H_{\frac{1}{2},1}^A &= -i \sqrt{2Q_+} \left(g_1 + \frac{M_\Lambda - M_p}{M_\Lambda} g_2 \right), \end{aligned} \quad (6)$$

where the subscript “ t ” denotes $H_{\frac{1}{2},t}^{V(A)}$ from the temporal component of the current of $(\bar{u}s)_{V(A)}$, q^2 is the lepton pair invariant mass, $Q_\pm = (M_\Lambda \pm M_p)^2 - q^2$, and M_Λ (M_p) is the mass of the parent (daughter) baryon. The negative helicity amplitudes are defined by

$$H_{-\lambda_p, -\lambda_W}^V = H_{\lambda_p, \lambda_W}^V \quad \text{and} \quad H_{-\lambda_p, -\lambda_W}^A = -H_{\lambda_p, \lambda_W}^A, \quad (7)$$

while the helicity ones for the left-handed current are obtained as

$$H_{\lambda_p, \lambda_W} = H_{\lambda_p, \lambda_W}^V - H_{\lambda_p, \lambda_W}^A. \quad (8)$$

Consequently, the differential decay widths of the semileptonic processes read

$$\begin{aligned} \frac{d\Gamma_L}{dq^2} &= \frac{G_F^2 |V_{us}|^2}{(2\pi)^3} \frac{(q^2 - m_\ell^2)^2}{24M_\Lambda^2 q^2} \text{p}_{\text{cm}} \\ &\times \left\{ \left(1 + \frac{m_\ell^2}{2q^2} \right) \left[|H_{\frac{1}{2},0}^V|^2 + |H_{-\frac{1}{2},0}^V|^2 + |H_{\frac{1}{2},1}^V|^2 \right. \right. \\ &\left. \left. + |H_{-\frac{1}{2},-1}^V|^2 \right] + \frac{3m_\ell^2}{2q^2} (|H_{\frac{1}{2},t}^V|^2 + |H_{-\frac{1}{2},t}^V|^2) \right\}, \end{aligned} \quad (9)$$

where $\text{p}_{\text{cm}} = \sqrt{Q_+ Q_-} / 2M_\Lambda$.

B. Form factors in light-front quark model

In order to calculate the form factors in the LFQM, we treat the baryon as a bound state described in the quark-diquark picture of q_1 and $q_{2,3}$, where $q_{2,3}$ are combined into a single diquark, expressed as $q_{[2,3]}$. Explicitly, the baryon bound state with the total momentum P and spin S can be written as [23,25,26]

$$\begin{aligned} |\mathcal{B}(P, S, S_z)\rangle &= \int \{d^3 p_1\} \{d^3 p_2\} 2(2\pi)^3 \delta^3(P - p_1 - p_2) \\ &\times \sum_{\lambda_1, \lambda_2} \Psi^{SS_z}(p_1, p_2, \lambda_1, \lambda_2) \\ &\times |q_1(p_1, \lambda_1) [q_2, q_3](p_2, \lambda_2)\rangle, \end{aligned} \quad (10)$$

where $q_1 = s$ or u denotes the active quark corresponding to Λ or p , $[q_2, q_3]$ represents the diquark, Ψ^{SS_z} corresponds to the momentum-space wave function and $p_{1,2}$ are the on-mass-shell LF momenta, and

$$\begin{aligned} p_1^+ &= x_1 P^+, & p_2^+ &= x_2 P^+, & x_1 + x_2 &= 1, \\ p_{1\perp} &= x_1 P_\perp + k_\perp, & p_{2\perp} &= x_2 P_\perp - k_\perp \end{aligned} \quad (11)$$

with $(x, k_\perp)$ being the longitudinal momentum fractions and transverse momenta of the constituent particles. By the Melosh transformation [32], it is more convenient to work with the following representation of the wave function:

$$\begin{aligned} \Psi^{SS_z}(p_1, p_2, \lambda_1, \lambda_2) &= \frac{1}{\sqrt{2(p_1 \cdot P + m_1 M_0)}} \bar{u}(p_1, \lambda_1) \\ &\times \Gamma_{l,m} u(P, S_z) \phi(x, k_\perp), \end{aligned} \quad (12)$$

where $\Gamma_{l,m}$ is the coupling vertex function of the decaying quark q_1 and the diquark in the baryon state. For the scalar

diquark, the coupling vertex is $\Gamma_{l,m} = 1$. For the distribution amplitude function of $\phi(x, k_\perp)$ in Eq. (12), we use the Gaussian-type function in this work, given by

$$\phi(x, k_\perp) = 4 \left(\frac{\pi}{\beta^2} \right)^{3/4} \sqrt{\frac{dk_z}{dx}} \exp \left(\frac{-\vec{k}^2}{2\beta^2} \right), \quad (13)$$

where β is the baryon shape parameter and k_z is defined by

$$k_z = \frac{xM_0}{2} - \frac{m_2^2 + k_\perp^2}{2xM_0}, \quad M_0^2 = \frac{m_1^2 + k_\perp^2}{1-x} + \frac{m_2^2 + k_\perp^2}{x}. \quad (14)$$

Using the bound states of $|\Lambda(P, S, S_z)\rangle$ and $|p(P', S', S'_z)\rangle$ in Eq. (10) and the above identities, we derive the matrix elements of the baryonic transition in the LF frame. By considering the $\mu = +$ component, the transition matrix elements are given by

$$\langle \mathcal{B}_p(P', S', S'_z) | \bar{q}' \gamma^+ (1 - \gamma_5) q | \mathcal{B}_\Lambda(P, S, S_z) \rangle = N_{fs} \int \{d^4 p_2\} \frac{\chi_{B_\Lambda}(x, \mathbf{k}_\perp) I^+ \chi'_{B_p}(x', \mathbf{k}'_\perp)}{(p_1^2 - m_1^2 + i\epsilon)(p_1'^2 - m_1'^2 + i\epsilon)}, \quad (15)$$

where $I^+ = \sum_{\lambda_2} \bar{u}(\bar{P}', S'_z) [\bar{\Gamma}'_{S(A)} (\not{p}'_1 + m'_1) \gamma^+ (1 - \gamma_5) (\not{p}_1 + m_1) \Gamma_{S(A)}] u(\bar{P}, S_z)$, $\bar{\Gamma} = \gamma^0 \Gamma^\dagger \gamma^0$, $\chi_{B_\Lambda}(\chi'_{B_p})$ corresponds to the vertex function of the baryon $\Lambda(p)$, and the flavor spin factor N_{fs} is the overlap factor of particle transformations, given by the specific process. We refer to the literature [33–36] to compute $f_3(g_3)$, consider the case with “ $\mu = +$ ” and $q^+ \neq 0$, and derive the following relationships:

$$\begin{aligned} \bar{u}(P', S'_z) q^+ u(P, S_z) &= \sqrt{P^+ P'^+} \left[\left(\frac{M_\Lambda}{P^+} + \frac{M_p}{P'^+} \right) \delta_{S'_z S_z} + \left(\frac{P'_\perp}{P'^+} - \frac{P_\perp}{P^+} \right) \sigma^3 \delta_{S'_z, -S_z} \right] q^+, \\ \bar{u}(P', S'_z) q^+ \gamma_5 u(P, S_z) &= -\sqrt{P^+ P'^+} \left[\left(\frac{M_\Lambda}{P^+} - \frac{M_p}{P'^+} \right) \sigma^3 \delta_{S'_z S_z} - \left(\frac{P'_\perp}{P'^+} - \frac{P_\perp}{P^+} \right) \delta_{S'_z, -S_z} \right] q^+. \end{aligned} \quad (16)$$

Subsequently, in the LF framework, Eq. (3) can be rewritten as

$$\begin{aligned} &\langle \mathcal{B}_p(P', S', S'_z) | V^+ | \mathcal{B}_\Lambda(P, S, S_z) \rangle \\ &= -2\sqrt{P^+ P'^+} \left\{ f_1(q^2) \delta_{S'_z S_z} + \frac{f_2(q^2)}{M_\Lambda} (\sigma \cdot q_\perp) \sigma^3 \delta_{S'_z S_z} + \frac{f_3(q^2)}{2M_\Lambda} \left[\left(\frac{M_\Lambda}{P^+} + \frac{M_p}{P'^+} \right) \delta_{S'_z S_z} + \left(\frac{P'_\perp}{P'^+} - \frac{P_\perp}{P^+} \right) \sigma^3 \delta_{S'_z, -S_z} \right] q^+ \right\}, \\ &\langle \mathcal{B}_p(P', S', S'_z) | A^+ | \mathcal{B}_\Lambda(P, S, S_z) \rangle \\ &= 2\sqrt{P^+ P'^+} \left\{ g_1(q^2) \sigma^3 \delta_{S'_z S_z} + \frac{g_2(q^2)}{M_\Lambda} (\sigma \cdot q_\perp) \delta_{S'_z S_z} - \frac{g_3(q^2)}{2M_\Lambda} \left[\left(\frac{M_\Lambda}{P^+} - \frac{M_p}{P'^+} \right) \sigma^3 \delta_{S'_z S_z} - \left(\frac{P'_\perp}{P'^+} - \frac{P_\perp}{P^+} \right) \delta_{S'_z, -S_z} \right] q^+ \right\}. \end{aligned} \quad (17)$$

Using the orthogonality relations among the matrices $\delta_{S'_z S_z}$, $\sigma^3 \delta_{S'_z S_z}$, $\sigma_\perp^i \sigma^3 \delta_{S'_z S_z}$, and $\sigma_\perp^i \delta_{S'_z S_z}$, the form factors can be extracted through trace operations. We note that $f_1(q^2)(g_1(q^2))$ and $f_3(q^2)(g_3(q^2))$ are orthogonal to $f_2(q^2)(g_2(q^2))$, which can be derived by using the following identities:

$$\begin{aligned} \frac{1}{2} \sum_{S'_z S_z} u(P, S_z) \delta_{S'_z S_z} \bar{u}(P', S'_z) &= \frac{1}{4\sqrt{P^+ P'^+}} (\not{P} + M_0) \gamma^+ (\not{P}' + M'_0), \\ \frac{1}{2} \sum_{S'_z S_z} u(P, S_z) \sigma^3 \delta_{S'_z S_z} \bar{u}(P', S'_z) &= \frac{1}{4\sqrt{P^+ P'^+}} (\not{P} + M_0) \gamma^+ \gamma_5 (\not{P}' + M'_0), \\ \frac{1}{2} \sum_{S'_z S_z} u(P, S_z) \sigma_\perp^i \delta_{S'_z S_z} \bar{u}(P', S'_z) &= \frac{-i}{4\sqrt{P^+ P'^+}} (\not{P} + M_0) \sigma^{i+} (\not{P}' + M'_0), \\ \frac{1}{2} \sum_{S'_z S_z} u(P, S_z) \sigma^i \delta_{S'_z S_z} \bar{u}(P', S'_z) &= \frac{i}{4\sqrt{P^+ P'^+}} (\not{P} + M_0) \sigma^{i+} \gamma_5 (\not{P}' + M'_0). \end{aligned} \quad (18)$$

Through Eqs. (15), (17), and (18), we obtain the expressions of the form factors as follows:

$$f_1(q^2)\delta_{S'_z S_z} + \frac{f_3(q^2)}{2M_\Lambda} \left(\frac{M_\Lambda}{P^+} + \frac{M_p}{P'^+} \right) q^+ \delta_{S'_z S_z} = \frac{N_{fs}}{8P^+P'^+} \int \{d^4 p_2\} \frac{\chi_{B_\Lambda}(x, \mathbf{k}_\perp) \chi'_{B_p}(x', \mathbf{k}'_\perp)}{(p_1^2 - m_1^2 + i\epsilon)(p_1'^2 - m_1'^2 + i\epsilon)} \\ \times \text{Tr}[(\not{P} + M_0)\gamma^+(\not{P}' + M'_0)(\not{p}'_1 + m'_1)\gamma^+(\not{p}_1 + m_1)], \quad (19)$$

$$g_1(q^2)\sigma^3\delta_{S'_z S_z} - \frac{g_3(q^2)}{2M_\Lambda} \left(\frac{M_\Lambda}{P^+} - \frac{M_p}{P'^+} \right) \sigma^3 q^+ \delta_{S'_z S_z} = \frac{N_{fs}}{8P^+P'^+} \int \{d^4 p_2\} \frac{\chi_{B_\Lambda}(x, \mathbf{k}_\perp) \chi'_{B_p}(x', \mathbf{k}'_\perp)}{(p_1^2 - m_1^2 + i\epsilon)(p_1'^2 - m_1'^2 + i\epsilon)} \\ \times \text{Tr}[(\not{P} + M_0)\gamma^+\gamma_5(\not{P}' + M'_0)(\not{p}'_1 + m'_1)\gamma^+\gamma_5(\not{p}_1 + m_1)], \quad (20)$$

$$\frac{f_2(q^2)}{M_\Lambda} (\sigma \cdot q_\perp \sigma^3) \delta_{S'_z S_z} = \frac{N_{fs}}{8P^+P'^+} \int \{d^4 p_2\} \frac{\chi_{B_\Lambda}(x, \mathbf{k}_\perp) \chi'_{B_p}(x', \mathbf{k}'_\perp)}{(p_1^2 - m_1^2 + i\epsilon)(p_1'^2 - m_1'^2 + i\epsilon)} \\ \times \text{Tr}[(\not{P} + M_0)\sigma^{v+}(\not{P}' + M'_0)(\not{p}'_1 + m'_1)\gamma^+(\not{p}_1 + m_1)],$$

$$\frac{g_2(q^2)}{M_\Lambda} (\sigma \cdot q_\perp) \delta_{S'_z S_z} = \frac{N_{fs}}{8P^+P'^+} \int \{d^4 p_2\} \frac{\chi_{B_\Lambda}(x, \mathbf{k}_\perp) \chi'_{B_p}(x', \mathbf{k}'_\perp)}{(p_1^2 - m_1^2 + i\epsilon)(p_1'^2 - m_1'^2 + i\epsilon)} \\ \times \text{Tr}[(\not{P} + M_0)\sigma^{v+}\gamma_5(\not{P}' + M'_0)(\not{p}'_1 + m'_1)\gamma^+\gamma_5(\not{p}_1 + m_1)], \quad (21)$$

where $v = 1, 2$ and the variables of $(x', \mathbf{k}'_\perp)$ are the LF relative momentum variables of $\mathcal{B}_f(P', S', S'_z)$ with the definitions given by replacing $x \rightarrow x'$ in Eq. (11). The extraction of f_2 and g_2 is performed before taking the $q_\perp \rightarrow 0$ limit. Furthermore, since the system is analyzed in the timelike domain (i.e., $q^2 = q^+q^- - q_\perp^2 \geq 0$), momentum is transmitted only in the longitudinal direction, i.e., $q_\perp = 0$. Therefore, we can choose a reference frame such that $P(P')_\perp = 0$ which causes the term containing $\delta_{S'_z, -S_z}$ in Eqs. (16) and (17) to vanish. To decouple $f_1(g_1)$ and $f_3(g_3)$, we rewrite Eqs. (19) and (20) as

$$f_1(q^2) + Af_3(q^2) = Hv(q^2) = \frac{N_{fs}}{8P^+P'^+} \int \{d^4 p_2\} \frac{\chi_{B_\Lambda}(x, \mathbf{k}_\perp) \chi'_{B_p}(x', \mathbf{k}'_\perp)}{(p_1^2 - m_1^2 + i\epsilon)(p_1'^2 - m_1'^2 + i\epsilon)} \\ \times \text{Tr}[(\not{P} + M_0)\gamma^+(\not{P}' + M'_0)(\not{p}'_1 + m'_1)\gamma^+(\not{p}_1 + m_1)], \\ g_1(q^2) + Bg_3(q^2) = Ha(q^2) = \frac{N_{fs}}{8P^+P'^+} \int \{d^4 p_2\} \frac{\chi_{B_\Lambda}(x, \mathbf{k}_\perp) \chi'_{B_p}(x', \mathbf{k}'_\perp)}{(p_1^2 - m_1^2 + i\epsilon)(p_1'^2 - m_1'^2 + i\epsilon)} \\ \times \text{Tr}[(\not{P} + M_0)\gamma^+\gamma_5(\not{P}' + M'_0)(\not{p}'_1 + m'_1)\gamma^+\gamma_5(\not{p}_1 + m_1)], \quad (22)$$

where

$$A = \frac{1}{2M_\Lambda} \left(\frac{M_\Lambda}{P^+} + \frac{M_p}{P'^+} \right) q^+ = \frac{1}{2M_\Lambda} \frac{1 - \alpha}{\alpha} (\alpha M_\Lambda + M_p), \\ B = -\frac{1}{2M_\Lambda} \left(\frac{M_\Lambda}{P^+} - \frac{M_p}{P'^+} \right) q^+ = -\frac{1}{2M_\Lambda} \frac{1 - \alpha}{\alpha} (\alpha M_\Lambda - M_p), \quad (23)$$

where the minus sign in the axial contribution has been absorbed into the definition of B and $\alpha = P'^+/P^+$. There are two solutions for α , namely,

$$\alpha_\pm = \frac{M_\Lambda^2 + M_p^2 - q^2 \pm \sqrt{(M_\Lambda^2 + M_p^2 - q^2)^2 - 4M_\Lambda^2 M_p^2}}{2M_\Lambda^2} \quad (24)$$

Based on the definition of Eq. (24), the $+$ ($-$) signs correspond to the outgoing baryon recoiling in the positive (negative) z direction relative to incoming baryon. The form factors are independent of the reference frame chosen for the moving direction. Therefore, α can be decomposed into two parts, α_+ and α_- , to solve for $f_1(g_1)$ and $f_3(g_3)$ where the consistency between α_+ and α_- serve as a self-consistency check:

$$f_1(q^2) + A_+ f_3(q^2) = Hv(q^2)|_{\alpha=\alpha_+}, \\ f_1(q^2) + A_- f_3(q^2) = Hv(q^2)|_{\alpha=\alpha_-}, \quad (25)$$

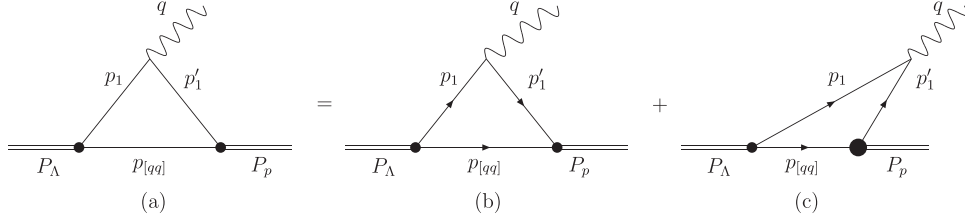


FIG. 1. The effective treatment of the LF amplitude (a) can be decomposed into the LF valence part (b) in $0 < x < \alpha$ and the nonvalence one (c) in $\alpha < x < 1$, where the small and large solid circles of the mediator-quark vertices in (b) and (c) represent the LF ordinary and nonvalence wave function vertices, respectively.

where $A_+ = A|_{\alpha=\alpha_+}$ and $A_- = A|_{\alpha=\alpha_-}$. One obtains

$$\begin{aligned} f_1(q^2) &= \frac{A_- H v(q^2)|_{\alpha=\alpha_+} - A_+ H v(q^2)|_{\alpha=\alpha_-}}{A_- - A_+}, \\ f_3(q^2) &= \frac{H v(q^2)|_{\alpha=\alpha_-} - H v(q^2)|_{\alpha=\alpha_+}}{A_- - A_+}. \end{aligned} \quad (26)$$

By repeating the steps from Eqs. (22) to (25), we obtain

$$\begin{aligned} g_1(q^2) &= \frac{B_- H a(q^2)|_{\alpha=\alpha_+} - B_+ H a(q^2)|_{\alpha=\alpha_-}}{B_- - B_+}, \\ g_3(q^2) &= \frac{H a(q^2)|_{\alpha=\alpha_-} - H a(q^2)|_{\alpha=\alpha_+}}{B_- - B_+}, \end{aligned} \quad (27)$$

where $B_+ = B|_{\alpha=\alpha_+}$ and $B_- = B|_{\alpha=\alpha_-}$.

C. Valence contribution

The trace term I^+ in Eq. (15) can be written as the sum of the valence I_V^+ and nonvalence I_{NV}^+ parts, as shown in Fig. 1(a). In the region of $0 < x < \alpha$ with $0 < p^+ < P'^+$ and $p_{[qq]}^- = p_{[qq]on}^- = (m_{[qq]}^2 + k_\perp^2)/p_{[qq]}^+$, in which the subscript “on” indicates on the mass shell, called the valence region as seen in Fig. 1(b), the effective contribution of the LF valence amplitude is given by

$$\begin{aligned} \mathcal{M}_{\text{val}} &= \frac{N_{fs}}{16\pi^3} \int_0^\alpha dx \int d^2\mathbf{k}_\perp \Phi_\Lambda(x, \mathbf{k}_\perp) I_V^+ \Phi_p(x', \mathbf{k}'_\perp), \\ I_V^+ &= \bar{u}(\bar{P}', S'_z) \bar{\Gamma}'(\not{p}'_1 + m'_1) \gamma^+ (1 - \gamma_5) (\not{p}_1 + m_1) \Gamma u(\bar{P}, S_z). \end{aligned} \quad (28)$$

Following Refs. [33–36], we drive the transition form factors

$$\begin{aligned} H v_{\text{val}}(q^2) &= \frac{N_{fs}}{16\pi^3} \int_0^\alpha dx \int d^2\mathbf{k}_\perp \Phi_\Lambda(x, \mathbf{k}_\perp) \Phi_p(x', \mathbf{k}'_\perp) [k_\perp \cdot k'_\perp + ((1-x)M_0 + m_1)((1-x')M'_0 + m'_1)], \\ H a_{\text{val}}(q^2) &= \frac{N_{fs}}{16\pi^3} \int_0^\alpha dx \int d^2\mathbf{k}_\perp \Phi_\Lambda(x, \mathbf{k}_\perp) \Phi_p(x', \mathbf{k}'_\perp) [-k_\perp \cdot k'_\perp + ((1-x)M_0 + m_1)((1-x')M'_0 + m'_1)], \\ \frac{f_{2\text{val}}(q^2)}{M} &= \frac{N_{fs}}{16\pi^3 q_\perp^2} \int_0^\alpha dx \int d^2\mathbf{k}_\perp \Phi_\Lambda(x, \mathbf{k}_\perp) \Phi_p(x', \mathbf{k}'_\perp) [-(m_1 + (1-x)M_0)k'_\perp \cdot q_\perp + (m'_1 + (1-x')M'_0)k_\perp \cdot q_\perp], \\ \frac{g_{2\text{val}}(q^2)}{M} &= \frac{N_{fs}}{16\pi^3 q_\perp^2} \int_0^\alpha dx \int d^2\mathbf{k}_\perp \Phi_\Lambda(x, \mathbf{k}_\perp) \Phi_p(x', \mathbf{k}'_\perp) [-(m_1 + (1-x)M_0)k'_\perp \cdot q_\perp - (m'_1 + (1-x')M'_0)k_\perp \cdot q_\perp] \end{aligned} \quad (29)$$

with

$$\Phi_B(x, \mathbf{k}_\perp) = \frac{\phi(x, k_\perp)}{\sqrt{2x_1(p_1 \cdot P + m_1 M_0)}}. \quad (31)$$

D. Nonvalence contribution

In the nonvalence region of $\alpha < x < 1$, $P'^+ < p^+ < P^+$, and $p_{1on}^- = p_{1on}^- = (m_1^2 + k_{1\perp}^2)/p_1^+$, as shown in Fig. 1(c), the trace term in Eq. (15) can be separated into the on-shell propagating and instantaneous parts of I_{on}^μ and I_{inst}^μ via

$$\not{p} + m = (\not{p}_{on} + m) + \frac{1}{2} \gamma^+ (p^- - p_{on}^-), \quad (32)$$

respectively. The effective contribution can be found as

$$\mathcal{M}_{\text{non-val}} = \frac{N_{fs}}{16\pi^3} \int_\alpha^1 dx \int d^2\mathbf{k}_\perp \Gamma_g(x, \mathbf{k}_\perp) I_{NV}^+ \Phi_\Lambda(x, \mathbf{k}_\perp) \int \frac{dy}{y(1-y)} \int d^2\mathbf{l}_\perp \mathcal{K}(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) \Phi_p(y, \mathbf{l}_\perp), \quad (33)$$

where I_{NV}^+ is the trace term in the nonvalence region. Substituting Eq. (32) into I^+ , we get $I_{NV}^+ = I_V^+(p_i^- = p_{1on}^- = (m_i^2 + k_{i\perp}^2)/p_i^+) + I_{\text{inst}}^+$. The LF vertex function of a gauge boson Γ_g in Eq. (33) corresponds to the LF energy denominator with its

explicit form given by [20,22,28]

$$\Gamma_g^{-1}(x, \mathbf{k}_\perp) = \alpha \left[\frac{q^2}{1-\alpha} - \left(\frac{\mathbf{k}_\perp^2 + m_1^2}{1-x} + \frac{\mathbf{k}_\perp'^2 + m_1'^2}{\alpha-x} \right) \right]. \quad (34)$$

The trace terms in Eqs. (28) and (33), both corresponding to the products of the initial and final LF spin wave functions, can be obtained by off-shell Melosh transformations. The form factors related to the nonvalence diagram I_{NV}^+ are given by

$$\begin{aligned} H_{NV}(q^2) &= \frac{N_{fs}}{16\pi^3} \int_\alpha^1 dx \int d^2\mathbf{k}_\perp \Gamma_g(x, \mathbf{k}_\perp) \Phi_\Lambda(x, \mathbf{k}_\perp) \{ [k_\perp \cdot k'_\perp + ((1-x)M_0 + m_1) \\ &\quad \times ((1-x')M_p + m'_1)] + I_{\text{inst}}^+ \} \int \frac{dy}{y(1-y)} \int d^2\mathbf{l}_\perp \mathcal{K}(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) \Phi_p(y, \mathbf{l}_\perp), \\ H_{aNV}(q^2) &= \frac{N_{fs}}{16\pi^3} \int_\alpha^1 dx \int d^2\mathbf{k}_\perp \Gamma_g(x, \mathbf{k}_\perp) \Phi_\Lambda(x, \mathbf{k}_\perp) \{ [-k_\perp \cdot k'_\perp + ((1-x)M_0 + m_1) \\ &\quad \times ((1-x')M_p + m'_1)] + I_{\text{inst}}^+ \} \int \frac{dy}{y(1-y)} \int d^2\mathbf{l}_\perp \mathcal{K}(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) \Phi_p(y, \mathbf{l}_\perp), \end{aligned} \quad (35)$$

$$\begin{aligned} \frac{f_{2NV}(q^2)}{M} &= \frac{N_{fs}}{16\pi^3 q_\perp^2} \int_\alpha^1 dx \int d^2\mathbf{k}_\perp \Gamma_g(x, \mathbf{k}_\perp) \Phi_\Lambda(x, \mathbf{k}_\perp) \{ [(m_1 + (1-x)M_0)k'_\perp \cdot q_\perp \\ &\quad - (m'_1 + (1-x')M_p)k_\perp \cdot q_\perp] + I_{\text{inst}}^+ \} \int \frac{dy}{y(1-y)} \int d^2\mathbf{l}_\perp \mathcal{K}(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) \Phi_p(y, \mathbf{l}_\perp), \\ \frac{g_{2NV}(q^2)}{M} &= \frac{N_{fs}}{16\pi^3 q_\perp^2} \int_\alpha^1 dx \int d^2\mathbf{k}_\perp \Gamma_g(x, \mathbf{k}_\perp) \Phi_\Lambda(x, \mathbf{k}_\perp) \{ [-(m_1 + (1-x)M_0)k'_\perp \cdot q_\perp - (m'_1 \\ &\quad + (1-x')M_p)k_\perp \cdot q_\perp] + I_{\text{inst}}^+ \} \int \frac{dy}{y(1-y)} \int d^2\mathbf{l}_\perp \mathcal{K}(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) \Phi_p(y, \mathbf{l}_\perp), \end{aligned} \quad (36)$$

where $I_{\text{inst}}^+ = I_{\text{inst}}^- = 0$. The complete form factors are $f(g)_j = f(g)_{jV} + f(g)_{jNV}$ ($j = 1, 2$). Notice that the instantaneous contribution exists in the nonvalence diagram only when the “+” component is used. The vertex of the nonvalence wave function is usually obtainable from the Bethe-Salpeter (B-S) amplitude in the B-S theory [20,37]. The corresponding light-front bound-state equation can be written as [20,38,39]

$$\begin{aligned} (M^2 - M_0^2) \Phi'(x_i, k_{i\perp}) \\ = \int [dy] [d^2\mathbf{l}_\perp] \mathcal{K}(x_i, \mathbf{k}_{i\perp}; y, \mathbf{l}_\perp) \Phi(y, \mathbf{l}_\perp). \end{aligned} \quad (37)$$

Both valence and nonvalence B-S amplitudes can be regarded as solutions of Eq. (37). The normal and nonvalence B-S amplitudes correspond to $x < \alpha$ and $x > \alpha$, respectively. In Fig. 1(c), the nonvalence B-S amplitude can be phenomenologically effectively related through an analytic continuation procedure of the valence B-S amplitude. In the LFQM, the relationship between the B-S amplitudes of two regions is given in Refs. [20–22]. However, for the integral equation of Eq. (37) it is necessary to use the nonperturbative quantum chromodynamics (QCD) method to obtain the kernel.

The relevant operator $\mathcal{K}$ in Eq. (37) is the B-S core, which in principle contains contributions from high Fock states. As shown in Fig. 2, the kernel $\mathcal{K}$ provides the dynamical connection between the higher-Fock and ordinary valence configurations. We define that

$$G_{B_\Lambda B_p} \equiv \int [dy] [d^2\mathbf{l}_\perp] \mathcal{K}(x_i, \mathbf{k}_{i\perp}; y, \mathbf{l}_\perp) \Phi_p(y, \mathbf{l}_\perp), \quad (38)$$

which depends only on x and $\mathbf{k}_\perp$. The range of the momentum fraction x relies on the external momenta for the embedded states. In this study, we approximate $G_{B_\Lambda B_p}$ as a constant because the initial wave function $\Psi_\Lambda(x, \mathbf{k}_\perp)$ acts as a weighting factor in the nonvalence state contribution, which causes the initial Gaussian wave function to act as a suppressor, with the main contribution originating from regions with narrow x values and low transverse momentum $k_\perp$. Consequently, the momentum dependence of $G_{B_\Lambda B_p}$ becomes effectively weak, allowing us to approximate it as a constant parameter [20–22]. Although this approximation has mainly been tested in mesonic systems, the same suppression mechanism from the Gaussian wave function is expected to persist in the quark-diquark description of baryons.

III. NUMERICAL RESULTS AND DISCUSSIONS

A. Form factors

To numerically evaluate the exclusive transition form factors in the LFQM, we work directly in the timelike region.

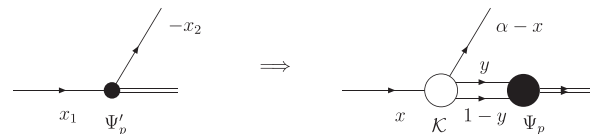


FIG. 2. Relationship between the ordinary LF wave function (large solid vertex) and the nonvalence vertex (small solid vertex).

TABLE I. Input parameters for the $\Lambda \rightarrow p$ transitions.

$m_{u,d}$	m_s	$m_{[qq']}$	$\beta_{u[qq]}$	$\beta_{s[qq]}$
0.25	0.48	0.4–0.7	0.34 ± 0.02	0.38 ± 0.02

In the previous calculations, we assumed $q_\perp \neq 0$ and only set $q_\perp = 0$ in the final numerical calculation. In the numerical analysis, the quark masses of m_s are taken from Ref. [27]; m_u is set to be 0.25 in GeV; the mass of diquark $m_{[ud]}$, parameters $\beta_{s,[ud]}$ and $\beta_{u,[ud]}$ are chosen from Refs. [25–27]. The input parameters are shown in Table I. In Eq. (33), $G_{B_\Lambda B_p}$ is treated as a constant in the range of 1.0–6.0, which was previously tested in some exclusively semileptonic mesonic decay processes and shown to be a good approximation for processes with a small momentum transfer [20–22]. Explicitly, in our numerical evaluation we take $G_{B_\Lambda B_p} = 2$, which is generally determined by comparison with the experimental results. Furthermore, we use $N_{fs} = \frac{1}{\sqrt{6}}$ with detailed calculation given in the Appendix of Ref. [27].

To describe the momentum q^2 behaviors, we parametrize the form factor using the double-pole forms of

$$F(q^2) = \frac{F(0)}{1 + a(q^2/M_{\text{pole}}^2) + b(q^4/M_{\text{pole}}^4)}, \quad (39)$$

where $M_{\text{pole}} = 1.115$ GeV is treated as an effective hadronic scale characterizing the momentum dependence of the form factors, and $(F(0), a, b)$ can be determined in the numerical analysis. The sign convention in Eq. (39) follows the fitting definition adopted in the present work. Another form of parametrizing the form factors is

$$\begin{aligned} \mathcal{F}(q^2) &= \frac{1}{1 - \frac{q^2}{M_{\text{pole}}^2}} [F(0) + C(z(q^2) - z(0))], \\ z(q^2) &= \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \end{aligned} \quad (40)$$

where $t_\pm = (M_\Lambda \pm M_p)^2$ and $t_0 = t_+(1 - \sqrt{t_-/t_+})$. For Eqs. (39) and (40), describing the q^2 dependence of the form factors, the results of the parametrization are shown in Table II. From Table II, $g_3(0)$ is numerically large. This is mainly because, in Eq. (23), the value of B is mathematically about one order of magnitude smaller than that of A . Physically, $g_3(0)$ contributes only through the scalar helicity amplitude and is further suppressed by the lepton mass. Additionally, in Eq. (40), the value of the fitting parameter associated with $g_3(q^2)$ is large because the pseudoscalar contribution exhibits strong momentum dependence in the timelike domain. All

fitting parameters are derived primarily from the momentum dependence of the form factor rather than the physical observables themselves. Although the fitted coefficients associated with g_3 become numerically large, their impact on the physical decay observables remains moderate due to the lepton-mass suppression of the corresponding helicity amplitudes.

In Fig. 3, we present the numerical results of the form factors as functions of q^2 for the $\Lambda \rightarrow p$ transition. In Table III, we compare the obtained form factors, expressed as constant ratios, with experimental and predicted values from several phenomenological models, where $f_1^{SU(3)} = \sqrt{3/2}$ [40]. We observe that the results for g_1/f_1 are in good agreement with the latest results from the BESIII experiments and QCD sum rule calculations, except that f_2/f_1 is slightly larger. Overall, the largest deviations from the results of lattice QCD [11] are f_2/f_1 and g_2/f_1 , mainly because our results of f_3 and g_2 are significantly larger, especially g_2 , while the other ratios are basically consistent within the error range. We also note that the sign of f_2/f_1 differs from that of some existing literature. This sign depends on the definition of the form factor, and different methods in the literature define f_2 differently.

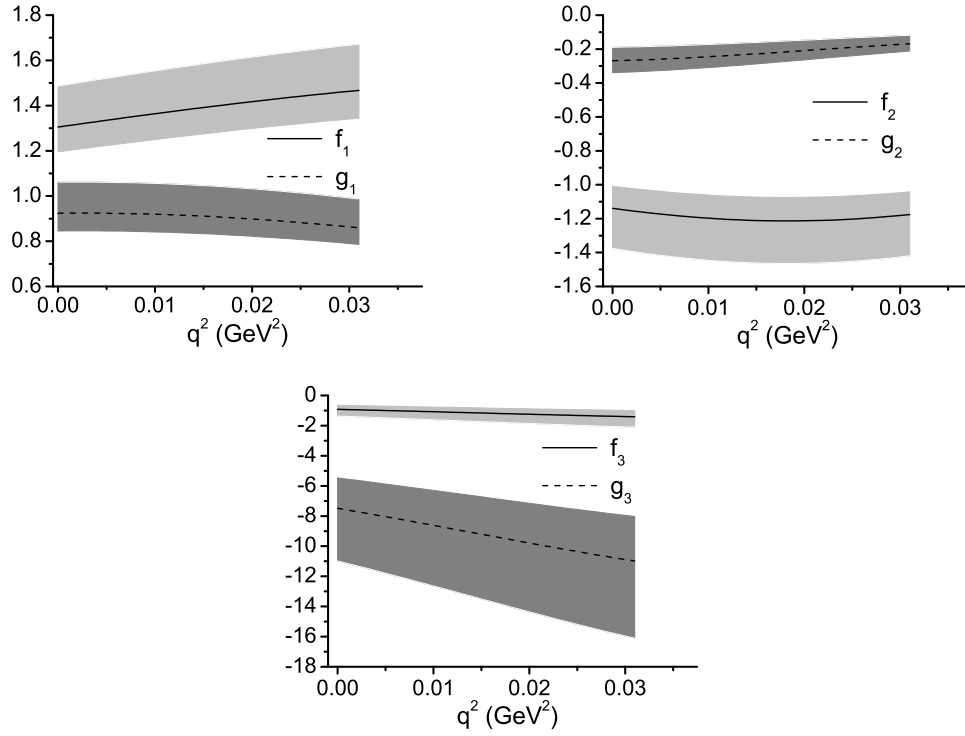
B. Decay branching ratios

The amplitudes of $\Lambda \rightarrow p \ell^- \bar{\nu}_\ell$ ($\ell = e, \mu$) contain some independent mixtures of helicity components, described by $h_{\lambda, \lambda_p}^{V(A)}$, where λ and λ_p represent the helicity components of the final baryon and W propagator, respectively. From Eq. (9), we can easily separate the integrals of the longitudinal and transverse polarization asymmetries. In Table IV, we show the decay branching ratios of $\Lambda \rightarrow p \ell^- \bar{\nu}_\ell$ ($\ell = e, \mu$) in various LFQMs along with the BESIII data, where I and II represent our predictions without and with $f_3(g_3)$ contributions, respectively. In particular, $f_3(g_3)$ appears only in the time component of the helicity amplitude $H_{\frac{1}{2}, t}^{V/A}$, specifically in the second term of Eq. (9). Therefore, $f_3(g_3)$ gives a negligible contribution to the e-mode due to the tiny electron mass but does affect the μ mode. It can be seen that the contribution with the inclusion of $f_3(g_3)$ is about 6% less than that without it for the muon mode. In any case, our results are consistent with the BESIII data. A specific observable is the ratio of muonic to electronic decay rates, which is shown in Table IV. As shown in the table, our results are lower than those of BESIII, primarily due to our lower muon decay rate.

Finally, we discuss the sensitivities of the input parameters to our results. In Table IV, the central values of our results for the decay branching ratios correspond to the central ones of $\beta_{s[qq]}$, $\beta_{u[qq]}$, and $m_{[qq]}$ in Table I, while those of upper (lower)

TABLE II. Form factors of the $\Lambda \rightarrow p$ transitions.

$\Lambda \rightarrow p$	f_1	f_2	f_3	g_1	g_2	g_3
$F(0)$	$1.305^{+0.18}_{-0.1}$	$-1.139^{+0.123}_{-0.235}$	$-0.917^{+0.212}_{-0.450}$	$0.924^{+0.137}_{-0.062}$	$-0.269^{+0.079}_{-0.022}$	$-7.487^{+1.537}_{-3.491}$
a	-5.976	-8.562	-20.152	-0.464	4.975	-17.325
b	55.964	303.263	257.322	143.982	833.088	197.214
C	-80.119	96.730	21.09	61.492	8.846	16152.9

FIG. 3. Transition form factors for $\Lambda \rightarrow p$. The gray bands denote the uncertainties associated with the shape parameters.TABLE III. Ratios of form factors from the $\Lambda \rightarrow p$ transition.

$\Lambda \rightarrow p$	$f_1/f_1^{SU(3)}$	g_1/f_1	f_2/f_1	g_2/f_1
Our results	$1.065^{+0.18}_{-0.1}$	$0.708^{+0.144}_{-0.072}$	$-0.872^{+0.153}_{-0.192}$	$-0.206^{+0.067}_{-0.023}$
BESIII [15]	—	$0.706^{+0.088}_{-0.086}$	$0.77^{+0.53}_{-0.49}$	$-0.19^{+0.65}_{-0.63}$
QCD sum rules [4,5]	0.963 ± 0.061	0.708 ± 0.047	0.752 ± 0.074	—
Cabibbo's model fit [7]	—	0.718 ± 0.015	1.066	—
Soliton model [41,42]	—	0.68	0.71	—
Quark model [8,9]	—	0.724	1	—
$1/N_c$ expansion [10]	1.02 ± 0.02	0.729	0.9	—
Lattice QCD [11]	0.9674 ± 0.0047	0.6902 ± 0.0044	-0.693 ± 0.017	-0.069 ± 0.016
Lattice QCD [13]	—	0.6902 ± 0.0044	—	—

TABLE IV. Branching fractions for the semileptonic decays $\Lambda \rightarrow p \ell^- \bar{\nu}_\ell$ ($\ell = e, \mu$).

Result	$\mathcal{B}(\Lambda \rightarrow p e^- \bar{\nu}_e) \times 10^{-4}$	$\mathcal{B}(\Lambda \rightarrow p \mu^- \bar{\nu}_\mu) \times 10^{-4}$	$R^{e\mu} = \frac{\mathcal{B}(\Lambda \rightarrow p \mu^- \bar{\nu}_\mu)}{\mathcal{B}(\Lambda \rightarrow p e^- \bar{\nu}_e)}$
I	$(8.32^{+2.63}_{-1.35})$	$(1.38^{+0.42}_{-0.21})$	$(0.166^{+0.073}_{-0.037})$
II	$(8.32^{+2.63}_{-1.35})$	$(1.31^{+0.36}_{-0.20})$	$(0.158^{+0.066}_{-0.035})$
BESIII [14,15]	8.16 ± 0.26	1.48 ± 0.22	0.181 ± 0.028
LHCb [16]	—	1.462 ± 0.1	0.175 ± 0.012
PDG [17]	8.34 ± 0.14	1.51 ± 0.19	0.181 ± 0.022
QCD sum rules [4,5]	7.72 ± 0.64	1.35 ± 0.11	0.174 ± 0.016
QCD polynomial [6]	$7.64^{+3.56}_{-0.94}$	$1.5^{+0.80}_{-0.32}$	$0.196^{+0.009}_{-0.012}$
QCD z-expansion [6]	$7.23^{+2.87}_{-0.94}$	$1.26^{+0.45}_{-0.14}$	$0.174^{+0.002}_{-0.005}$
Cabibbo's model fit [7]	8.43 ± 0.17	—	—
quark model [8,9]	8.58	1.49	0.173
Lattice QCD [11]	7.68 ± 0.48	1.33 ± 0.16	0.174 ± 0.010
NLO-QCD [12]	—	—	0.153 ± 0.008
(Updated [13])	—	—	0.162 ± 0.004
Lattice QCD [13]	—	—	0.163 ± 0.004

uncertainties are calculated by taking the upper (lower) values of β and $m_{[qq]}$, respectively. Note that our results are based on fixed values of m_q in Table I. We note that, different choices of these parameters will also affect the results. For instance, for $\Lambda \rightarrow p\ell^-\bar{\nu}_\ell$, if the diquark mass $m_{[qq]}$, which ranges from 0.4 to 0.7 GeV [25], changes without altering the baryon's β value, a variation of diquark mass $m_{[qq]}$ by ± 0.1 GeV changes the branching ratios by approximately $\pm 11\%$. Since this study assumes that the uncertainty of the β factor is small, the uncertainty of the results is also small. Therefore, $m_{[qq]}$ is the main source of uncertainty.

IV. CONCLUSION

We have calculated all six form factors of the $\Lambda \rightarrow p$ transition matrix element in the timelike region under the LF coordinate system and extended them to the entire physical region. These form factors are obtained based on the B-S formalism for the case $q^+ > 0$ and include both valence and nonvalence contributions. In our calculations, by analyzing the form factors, as shown in Table II, the ratios g_1/f_1 , f_2/f_1 , and g_2/f_1 are given to be $0.708^{+0.012}_{-0.027}$, $-0.872^{+0.014}_{-0.035}$, and $-0.206^{+0.089}_{-0.077}$, respectively. These form factors include nonvalence contributions and agree well with the existing

data from the BESIII experiments as well as other theoretical estimates. In addition, for the results that include the contribution from $f_3(g_3)$, we have found the branching ratios for the semileptonic decays of $\Lambda \rightarrow p e^- \bar{\nu}_e$ and $\Lambda \rightarrow p \mu^- \bar{\nu}_\mu$ being approximately 8.32×10^{-4} and 1.31×10^{-4} , which are close to the BESIII experimental results, respectively. However, we have obtained that $R^{e\mu} = \frac{\mathcal{B}(\Lambda \rightarrow p \mu^- \bar{\nu}_\mu)}{\mathcal{B}(\Lambda \rightarrow p e^- \bar{\nu}_e)} = 0.158$, which is slightly smaller than the BESIII experimental value of 0.181. The present analysis indicates that nonvalence contributions play an important role for a consistent light-front description of semileptonic baryon decays in the physical timelike region. Our framework may also be extended to other semileptonic baryonic transitions where nonvalence contributions become important in the physical timelike region.

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DATA AVAILABILITY

The data that support the findings of this article are publicly available [4–17].

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- [1] J. F. Donoghue and S. Pakvasa, Signals of CP nonconservation in hyperon decay, *Phys. Rev. Lett.* **55**, 162 (1985).
 - [2] J. Fu, H.-B. Li, J.-P. Wang, F.-S. Yu, and J. Zhang, Probing hyperon electric dipole moments with a full angular analysis, *Phys. Rev. D* **108**, L091301 (2023).
 - [3] J. M. Gaillard and G. Sauvage, Hyperon beta decays, *Annu. Rev. Nucl. Part. Sci.* **34**, 351 (1984).
 - [4] S. Q. Zhang, X. H. Zhang, and C. F. Qiao, Hyperon semileptonic decays in QCD sum rules, *J. High Energy Phys.* **06** (2024) 122.
 - [5] S. Q. Zhang and C. F. Qiao, Baryons and baryoniums in the perspective of QCD sum rules, *AAPPS Bull.* **36**, 12 (2026).
 - [6] M. Ahmadi, Z. Rajabi Najjar, and K. Azizi, Study of the semileptonic decay $\Lambda \rightarrow p\ell^-\bar{\nu}_\ell$ in QCD, *Phys. Rev. D* **112**, 094035 (2025).
 - [7] N. Cabibbo, E. C. Swallow, and R. Winston, Semileptonic hyperon decays, *Annu. Rev. Nucl. Part. Sci.* **53**, 39 (2003).
 - [8] F. Schlumpf, β decay of hyperons in a relativistic quark model, *Phys. Rev. D* **51**, 2262 (1995).
 - [9] A. Faessler, T. Gutsche, B. R. Holstein, M. A. Ivanov, J. G. Körner, and V. E. Lyubovitskij, Semileptonic decays of the light $J^P = 1/2^+$ ground state baryon octet, *Phys. Rev. D* **78**, 094005 (2008).
 - [10] R. Flores-Mendieta, E. E. Jenkins, and A. V. Manohar, $SU(3)$ symmetry breaking in hyperon semileptonic decays, *Phys. Rev. D* **58**, 094028 (1998).
 - [11] S. Bacchio and A. Konstantinou, Study of the $\Lambda \rightarrow p\ell^-\bar{\nu}_\ell$ semileptonic decay in lattice QCD, *Phys. Rev. Lett.* **135**, 231901 (2025).
 - [12] H.-M. Chang, M. González-Alonso, and J. Martin Camalich, Nonstandard semileptonic hyperon decays, *Phys. Rev. Lett.* **114**, 161802 (2015).
 - [13] C. Alexandrou, S. Bacchio, A. Konstantinou, and E. Vakana, Scalar and tensor form factors for $\Lambda \rightarrow p\ell^-\bar{\nu}_\ell$ from lattice QCD, *Phys. Rev. D* **114**, 014507 (2026).
 - [14] M. Ablikim, M. N. Achasov, P. Adlarson, S. Ahmed, M. Albrecht, R. Aliberti, A. Amoroso, M. R. An, Q. An, X. H. Bai, Y. Bai, O. Bakina, R. Baldini Ferroli, I. Balossino, Y. Ban, K. Begzsuren, N. Berger, M. Bertani, D. Bettoni, F. Bianchi, *et al.* (BESIII Collaboration), First measurement of the absolute branching fraction of $\Lambda \rightarrow p\mu^-\bar{\nu}_\mu$, *Phys. Rev. Lett.* **127**, 121802 (2021).
 - [15] M. Ablikim, M. N. Achasov, P. Adlarson, X. C. Ai, R. Aliberti, A. Amoroso, Q. An, Y. Bai, O. Bakina, Y. Ban, H.-R. Bao, V. Batzskaya, K. Begzsuren, N. Berger, M. Berlowski, M. Bertani, D. Bettoni, F. Bianchi, E. Bianco, A. Bortone, *et al.* (BESIII Collaboration), Exploring baryon semi-leptonic decays through polarization and entanglement, *Nature (London)* **657**, 92 (2026).
 - [16] R. Aaij *et al.* (LHCb Collaboration), Branching fraction measurement of the $\Lambda \rightarrow p\mu^-\bar{\nu}_\mu$ decay, *J. High Energy Phys.* **04** (2026) 096.
 - [17] S. Navas, C. Amsler, T. Gutsche, C. Hanhart, J. J. Hernández-Rey, C. Lourenço, A. Masoni, M. Mikhasenko, R. E. Mitchell, C. Patrignani, C. Schwanda, S. Spanier, G. Venanzoni, C. Z. Yuan, K. Agashe, G. Aielli, B. C. Allanach, J. Alvarez-Muñiz, M. Antonelli, E. C. Aschenauer, *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
 - [18] H.-M. Choi and C.-R. Ji, Mixing angles and electromagnetic properties of ground state pseudoscalar and vector meson nonets in the light-cone quark model, *Phys. Rev. D* **59**, 074015 (1999).
 - [19] H.-M. Choi and C.-R. Ji, Light-front quark model analysis of exclusive $0^- \rightarrow 0^-$ semileptonic heavy meson decays,

- Phys. Lett. B* **460**, 461 (1999); Kaon electroweak form factors in the light-front quark model, *Phys. Rev. D* **59**, 034001 (1998).
- [20] S. J. Brodsky and D. S. Hwang, Exact light-cone wavefunction representation of matrix elements of electroweak currents, *Nucl. Phys. B* **543**, 239 (1999); C.-R. Ji and H.-M. Choi, New effective treatment of the light-front nonvalence contribution in timelike exclusive processes, *Phys. Lett. B* **513**, 330 (2001).
- [21] C.-R. Ji and H.-M. Choi, New progress in time-like exclusive processes, [arXiv:hep-ph/0105248v3](https://arxiv.org/abs/hep-ph/0105248v3).
- [22] H.-M. Choi, C.-R. Ji, and L. S. Kisslinger, Skewed quark distribution of the pion in the light-front quark model, *Phys. Rev. D* **64**, 093006 (2001).
- [23] C.-C. Lih and C.-Q. Geng, Semileptonic decays of Λ_c^+ in light-front quark model with nonvalence contributions, *Phys. Rev. D* **112**, 076023 (2025).
- [24] M. Anselmino, E. Predazzi, S. Ekelin, S. Fredriksson, and D. B. Lichtenberg, Diquarks, *Rev. Mod. Phys.* **65**, 1199 (1993).
- [25] H. G. Dosch, M. Jamin, and B. Stech, Diquarks, QCD sum rules, and weak decays, *Z. Phys. C - Particles and Fields* **42**, 167 (1989).
- [26] H.-W. Ke, X.-Q. Li, and Z.-T. Wei, Diquarks and $\Lambda_b \rightarrow \Lambda_c$ weak decays, *Phys. Rev. D* **77**, 014020 (2008).
- [27] Z. X. Zhao, Weak decays of heavy baryons in the light-front approach, *Chin. Phys. C* **42**, 093101 (2018); Z.-T. Wei, H.-W. Ke, and X.-Q. Li, Evaluating decay rates and asymmetries of Λ_b into light baryons in the light-front quark model, *Phys. Rev. D* **80**, 094016 (2009).
- [28] H.-M. Choi, C.-R. Ji, and L. S. Kisslinger, Light-front quark model analysis of rare $B \rightarrow Kl^+l^-$ decays, *Phys. Rev. D* **65**, 074032 (2002).
- [29] A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner, and V. E. Lyubovitskij, Semileptonic decays of double heavy baryons in a relativistic constituent three-quark model, *Phys. Rev. D* **80**, 034025 (2009).
- [30] J. G. Körner and M. Krämer, Exclusive non-leptonic charm baryon decays, *Z. Phys. C - Particles and Fields* **55**, 659 (1992).
- [31] J. G. Körner, Helicity amplitudes and angular decay distributions, [arXiv:hep-ph/1402.2787v1](https://arxiv.org/abs/hep-ph/1402.2787v1).
- [32] H. J. Melosh, Quarks: Currents and constituents, *Phys. Rev. D* **9**, 1095 (1974).
- [33] S. Tawfiq, P. J. O'Donnell, and J. G. Körner, Charmed baryon strong coupling constants in a light-front quark model, *Phys. Rev. D* **58**, 054010 (1998).
- [34] H.-C. Pauli, A compendium of light-cone quantization, *Nucl. Phys. B Proc. Suppl.* **90**, 259 (2000).
- [35] G. P. Lepage and S. J. Brodsky, Exclusive processes in perturbative quantum chromodynamics, *Phys. Rev. D* **22**, 2157 (1980).
- [36] F. Schlumpf, Relativistic constituent quark model of electroweak properties of baryons, *Phys. Rev. D* **47**, 4114 (1993); **49**, 6246(E) (1994).
- [37] B. L. G. Bakker and C.-R. Ji, Disentangling intertwined embedded states and spin effects in light-front quantization, *Phys. Rev. D* **62**, 074014 (2000); B. L. G. Bakker, H.-M. Choi, and C.-R. Ji, Regularizing the divergent structure of light-front currents, *ibid.* **63**, 074014 (2001).
- [38] S. J. Brodsky, C.-R. Ji, and M. Sawicki, Evolution equation and relativistic bound-state wave functions for scalar-field models in four and six dimensions, *Phys. Rev. D* **32**, 1530 (1985).
- [39] J. H. O. Sales, T. Frederico, B. V. Carlson, and P. U. Sauer, Light-front Bethe-Salpeter equation, *Phys. Rev. C* **61**, 044003 (2000).
- [40] L. S. Geng, J. Martin Camalich, and M. J. Vicente Vacas, SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$ in covariant baryon chiral perturbation theory, *Phys. Rev. D* **79**, 094022 (2009).
- [41] G. S. Yang and H. C. Kim, Hyperon semileptonic decay constants with flavor SU(3) symmetry breaking, *Phys. Rev. C* **92**, 035206 (2015).
- [42] T. Ledwig, A. Silva, H. C. Kim, and K. Goeke, Semileptonic hyperon decays in the self-consistent SU(3) chiral quark-soliton model, *J. High Energy Phys.* **07** (2008) 132.