

Meson-exchange currents in quasielastic charged-current neutrino reactions with single-nucleon knockout

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The effect of meson-exchange currents on charged-current quasielastic neutrino scattering with single-nucleon emission is computed and analyzed within the relativistic Fermi gas model. This contribution arises primarily from the interference between one-body and two-body currents, where the two-body operator excites a 1p1h state in the presence of a second, spectator nucleon. The results obtained show a reduction of the vector, axial, and vector-axial transverse response functions and, consequently, a decrease in the total neutrino cross section. In addition to a comparison with the nonrelativistic limit, other models are also explored, such as the relativistic mean field model for nuclear matter and superscaling analysis with relativistic effective mass, both of which yield qualitatively similar results.

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I. INTRODUCTION

The increasing precision of current and future neutrino experiments demands accurate modeling of neutrino-nucleus interactions to extract reliable information about fundamental neutrino properties, such as masses and CP violation [1–3]. Providing robust theoretical predictions for cross sections across a wide energy range and for realistic nuclear targets is a major challenge [4–9]. This requires combining nuclear physics tools—effective theories, many-body methods, and phenomenological models—to capture the complex nuclear dynamics [10–17]. These models are implemented in neutrino event generators, which are essential for simulating and analyzing experimental data [18,19].

In particular, a precise understanding of quasielastic (QE) neutrino-nucleus scattering is crucial for interpreting data from accelerator-based experiments such as Mini-BooNE, T2K, MicroBooNE, MINERvA, and the forthcoming Hyper-Kamiokande and DUNE projects [1,20–22]. The

charged-current quasielastic (CCQE) cross section is largely dominated by the one-particle–one-hole (1p1h) channel, where a single nucleon is emitted and the residual nucleus remains in a bound state. However, it is now well established that additional reaction mechanisms, including multinucleon emission and meson-exchange currents (MECs), contribute significantly to the total cross section and must be properly accounted for in theoretical models to ensure a comprehensive and accurate description [9,23–25].

In fact, an important outcome of modern theoretical developments is the identification of an enhancement in the quasielasticlike cross section—defined here as excluding pion production and inelastic channels—arising from processes involving the emission of two nucleons. These so-called two-particle–two-hole (2p2h) contributions are associated with nuclear correlations and meson-exchange currents. In models based on a sum over exclusive final states, the 2p2h channel corresponds to a distinct class of configurations that do not interfere with the standard one-particle–one-hole (1p1h) contribution, as they lead to orthogonal final states. This mechanism has been studied using a variety of nuclear approaches, including relativistic and nonrelativistic frameworks [9,23,25,26], each relying on different treatments of the nuclear dynamics and current operators. Despite methodological differences, the predicted enhancement is relatively consistent, typically accounting for about 20% of the total QE cross section. While the precise impact of this contribution on experimental observables is still under investigation, its inclusion is necessary for a more accurate and complete theoretical description.

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The interference between MEC and the one-body current has received limited attention in the context of neutrino QE scattering, and remains an open issue. While the MEC contribution has been extensively studied in the 2p2h sector, to our knowledge, the only prior work addressing MEC in quasielastic CC neutrino-nucleus scattering in the 1p1h channel is Ref. [27], which employed a soft-pion MEC dominance model without including the Δ current. Apart from that, the present study represents one of the first fully dedicated analyses of CCQE scattering in the 1p1h channel including MEC interference effects. In the case of inclusive electron scattering, many investigations have highlighted the role of the interference term, yet recent studies have raised confusion regarding its sign [28–31].

Calculations based on independent-particle models—such as the relativistic Fermi gas, mean-field approaches, or even spectral function models—typically yield a negative interference in the transverse response [32–40]. This behavior results from a partial cancellation between the seagull current (which interferes positively with the one-body current) and the Δ -current and pion-in-flight diagrams (which contribute negatively). Models such as the Valencia approach do not include the one-body–two-body (1b-2b) interference explicitly, likely under the assumption that its net effect is small compared to other mechanisms [41], such as final-state interactions (FSIs) or long-range correlations treated within the random phase approximation (RPA). However, this assumption remains untested.

Interestingly, a variational calculation [42] showed that the sign of the interference can be reversed depending on the type of correlations included in the nuclear wave function: it becomes positive when tensor correlations are incorporated via a correlated basis function approach, while it remains negative with purely Jastrow-type correlations. This contrasts with Green’s function Monte Carlo (GFMC) results for ^4He [43], where the net effect of MEC appears to be positive even when using simplified wave functions without tensor correlations. Nevertheless, such comparisons must be made with caution, as GFMC calculations cannot isolate the 1p1h channel; the total MEC contribution includes both interference and 2p2h mechanisms, which may partially cancel each other. Moreover, an additional layer of complexity arises from the interference between one-body and two-body operators within the 2p2h channel itself [44], further complicating the disentanglement of different reaction mechanisms and their impact on the total cross section.

Given the complexity of the problem, in this work we report the results of a calculation performed within the simplest possible framework, with the aim of gaining insight into the fundamental features of the interference between one-body and two-body currents in the 1p1h channel for CCQE neutrino scattering. This serves as a first step towards more elaborate approaches. Specifically, we consider the relativistic Fermi gas (RFG) model without nucleon-nucleon correlations, and evaluate the contribution of MEC to the QE response arising from their interference with the one-body current, restricted to genuine 1p1h excitations, meaning final states involving only one emitted nucleon and one hole in the residual system. The two-body current operators are taken from a relativistic model

that has been widely used in previous studies [25,45,46]. For comparison, we also analyze the nonrelativistic limit, in which the vector and axial MEC reduce to the standard expressions found, for instance, in Refs [47,48]. These currents have been extensively applied in the context of inclusive (e, e') and exclusive $(e, e'p)$ scattering.

We compute the interference responses in both the relativistic and nonrelativistic schemes, and verify that the former consistently reduce to the latter through two independent calculations, providing a valuable internal check. Furthermore, we explore the impact of nuclear dynamics beyond the RFG by implementing the relativistic mean field (RMF) model in nuclear matter [49,50] and the super scaling analysis with relativistic effective mass (SuSAM*) model [40], which incorporates effects associated with nuclear mean fields. We present results for neutrino-induced interference responses and for the flux-averaged differential cross sections relevant to accelerator-based experiments, allowing for a preliminary comparison with available measurements.

This paper is a natural extension to CC neutrino scattering of Ref. [31], where the transverse interference response was studied in electron scattering; while avoiding unnecessary overlap by referring to that work for the vector current formalism, we focus here on the new ingredients, namely the axial MEC and the weak response functions, with special attention to the axial Δ current.

In Sec. II we introduce the formalism within the RFG model, along with the relativistic MEC operators employed in this work. In Sec. III, we examine the nonrelativistic limit and compare the results with the relativistic calculation in for low momentum and energy transfer, where both approaches are expected to converge. Section IV presents numerical results for the neutrino response functions and the flux-averaged cross section, including the interference between one-body and two-body currents. Finally, in Sec. V we summarize our conclusions.

II. FORMALISM

In this section we outline the general formalism for charged-current quasielastic (CCQE) neutrino scattering off nuclei. The framework is based on the relativistic Fermi gas model, which allows a fully covariant treatment of the kinematics and current operators.

A. Cross section

We follow the formalism of Ref. [16]. We consider the process $\nu_\mu + A \rightarrow \mu^- + X$, where a muon neutrino of four-momentum $k^\mu = (\epsilon, \mathbf{k})$ scatters off a nucleus at rest, transferring four-momentum $Q^\mu = k^\mu - k'^\mu = (\omega, \mathbf{q})$ to the nuclear system. Here, $k'^\mu = (\epsilon', \mathbf{k}')$ is the four-momentum of the outgoing muon, and $Q^2 = \omega^2 - q^2 < 0$ is the squared four-momentum transfer.

The energy transfer is denoted by $\omega = \epsilon - \epsilon'$, and the three-momentum transfer $\mathbf{q}$ is taken along the z axis. The scattering angle between the incoming neutrino and outgoing muon is θ_μ , so that the muon energy is $\epsilon' = \sqrt{|\mathbf{k}'|^2 + m_\mu^2}$ and its kinetic energy is $T_\mu = \epsilon' - m_\mu$.

In the laboratory frame, assuming the target nucleus is initially at rest, the inclusive cross section reads

$$\frac{d^2\sigma}{d\epsilon' d\Omega_\mu} = \frac{G_F^2 \cos^2 \theta_C}{4\pi^2} \frac{|\mathbf{k}'|}{\epsilon} L_{\mu\nu} W^{\mu\nu}, \quad (1)$$

where $G_F = 1.666 \times 10^{-11} \text{ MeV}^{-2}$ is the Fermi constant, θ_C is the Cabibbo angle, $L_{\mu\nu}$ is the leptonic tensor, and $W^{\mu\nu}$ is the hadronic tensor that encodes the five nuclear response functions. Performing the tensor contraction, and integrating over the muon azimuthal angle, the differential cross section with respect to T_μ , $\cos \theta_\mu$, can be written as

$$\frac{d^2\sigma}{dT_\mu d\cos \theta_\mu} = \frac{G_F^2 \cos^2 \theta_C}{2\pi} \frac{k'}{\epsilon} \frac{v_0}{2} (V_{CC} R_{CC} + 2V_{CL} R_{CL} + V_{LL} R_{LL} + V_T R_T \pm 2V_{T'} R_{T'}), \quad (2)$$

where the minus sign is for antineutrino scattering, $\bar{\nu}_\mu + A \rightarrow \mu^+ + X$. In this equation we have defined the factor $v_0 = (\epsilon + \epsilon')^2 - q^2$, and the coefficients V_K are obtained from the components of the leptonic tensor:

$$\begin{aligned} V_{CC} &= 1 + \delta^2 \frac{Q^2}{v_0}, \\ V_{CL} &= \frac{\omega}{q} - \frac{\delta^2 Q^2}{\rho' v_0}, \\ V_{LL} &= \frac{\omega^2}{q^2} + \left(1 + \frac{2\omega}{q\rho'} + \rho\delta^2\right) \delta^2 \frac{Q^2}{v_0}, \\ V_T &= \frac{Q^2}{v_0} + \frac{\rho}{2} - \frac{\delta^2}{\rho'} \left(\frac{\omega}{q} + \frac{1}{2}\rho\rho'\delta^2\right) \frac{Q^2}{v_0}, \\ V_{T'} &= \frac{1}{\rho'} \left(1 - \frac{\omega\rho'}{q}\delta^2\right) \frac{Q^2}{v_0}, \end{aligned} \quad (3)$$

with the dimensionless factors

$$\delta = \frac{m_\mu}{\sqrt{|Q^2|}}, \quad \rho = \frac{|Q^2|}{q^2}, \quad \rho' = \frac{q}{\epsilon + \epsilon'}. \quad (4)$$

The five nuclear response functions only depend on (q, ω) , and are the following components of the hadronic tensor:

$$\begin{aligned} R_{CC} &= W^{00}, \\ R_{CL} &= -\frac{1}{2}(W^{03} + W^{30}), \\ R_{LL} &= W^{33}, \\ R_T &= W^{11} + W^{22}, \\ R_{T'} &= -\frac{i}{2}(W^{12} - W^{21}). \end{aligned} \quad (5)$$

In the case of charged-current weak interactions, the nuclear current operator is the sum of a vector component and an axial-vector component. As a result, the response functions CC , CL , LL , and T , can each be written as the sum of two separate contributions: one arising from the vector-vector (VV) part of the current, and the other from the axial-axial (AA) part:

$$R_K = R_K^{VV} + R_K^{AA}, \quad K = CC, CL, LL, T. \quad (6)$$

On the other hand, the T' response originates from the interference between the vector and axial components of the current, and can be written as

$$R_{T'} = R_{T'}^{VA} + R_{T'}^{AV}. \quad (7)$$

B. Hadronic tensor

We now describe the construction of the hadronic tensor in the $1p1h$ sector within the RFG model, which provides a transparent and analytically tractable framework for studying MEC effects. In this case, the hadronic tensor is constructed from the matrix elements of the nuclear current between the ground state of the RFG and excited $1p1h$ states, which are represented by Slater determinants built from plane-wave single-particle states within a volume V

$$\begin{aligned} W_{1p1h}^{\mu\nu} &= \sum_{ph} \langle ph^{-1} | \hat{J}^\mu(\mathbf{q}) | F \rangle^* \langle ph^{-1} | \hat{J}^\nu(\mathbf{q}) | F \rangle \\ &\times \delta(E_p - E_h - \omega) \theta(p - k_F) \theta(k_F - h), \end{aligned} \quad (8)$$

where $E_p = \sqrt{p^2 + m_N^2}$ and $E_h = \sqrt{h^2 + m_N^2}$ are the on-shell energies of the nucleons involved in the $1p1h$ excitation, with momenta $\mathbf{p}$ and $\mathbf{h}$, respectively. The step functions ensure that the initial nucleons have a momentum below the Fermi momentum $h < k_F$ and the final nucleons have a momentum $p > k_F$. The sums in Eq. (8) include sums over the spin projections s_p, s_h and isospin states t_p, t_h of the particle and hole, respectively. However, in the case of charged-current neutrino scattering, the weak interaction selects specific isospin transitions: the initial hole must be a neutron and the final particle a proton (the opposite holds for antineutrino scattering).

The nuclear current is taken as the sum of one-body (1b) and two-body (2b) operators,

$$\hat{J}^\mu = \hat{J}_{1b}^\mu + \hat{J}_{2b}^\mu. \quad (9)$$

The $1p1h$ matrix element of these operators in the RFG is given by

$$\langle ph^{-1} | \hat{J}_{1b}^\mu | F \rangle = \langle p | \hat{J}_{1b}^\mu | h \rangle, \quad (10)$$

$$\langle ph^{-1} | \hat{J}_{2b}^\mu | F \rangle = \sum_{k < k_F} [\langle pk | \hat{J}_{2b}^\mu | hk \rangle - \langle pk | \hat{J}_{2b}^\mu | kh \rangle]. \quad (11)$$

Note that the matrix element of the MEC, being a two-body operator, involves a transition between pairs of nucleons; however, in a $1p1h$ excitation, one of the nucleons, $|k\rangle = |k, s_k, t_k\rangle$, remains in its initial state and acts merely as a spectator.

The next step is to write the elementary matrix elements of the one-body and two-body current operators between plane-wave states, using momentum conservation:

$$\langle p | \hat{J}_{1b}^\mu | h \rangle = \frac{(2\pi)^3}{V} \delta^3(\mathbf{q} + \mathbf{h} - \mathbf{p}) j_{1b}^\mu(\mathbf{p}, \mathbf{h}), \quad (12)$$

$$\begin{aligned} \langle p'_1 p'_2 | \hat{J}_{2b}^\mu | p_1 p_2 \rangle &= \frac{(2\pi)^3}{V^2} \delta^3(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{q} - \mathbf{p}'_1 - \mathbf{p}'_2) \\ &\times j_{2b}^\mu(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{p}_1, \mathbf{p}_2). \end{aligned} \quad (13)$$

The current functions $j_{1b}^\mu(\mathbf{p}, \mathbf{h})$ and $j_{2b}^\mu(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{p}_1, \mathbf{p}_2)$ implicitly depend on spin and isospin indices. Inserting Eqs. (12) and

(13) in Eqs. (10) and (11), respectively, we can write the total current matrix element in the form

$$\langle ph^{-1} | \hat{j}^\mu | F \rangle = \frac{(2\pi)^3}{V} \delta^3(\mathbf{q} + \mathbf{h} - \mathbf{p}) j^\mu(\mathbf{p}, \mathbf{h}), \quad (14)$$

where

$$j^\mu(\mathbf{p}, \mathbf{h}) \equiv j_{1b}^\mu(\mathbf{p}, \mathbf{h}) + j_{2b}^\mu(\mathbf{p}, \mathbf{h}) \quad (15)$$

and

$$j_{2b}^\mu(\mathbf{p}, \mathbf{h}) \equiv \frac{1}{V} \sum_{k < k_F} [j_{2b}^\mu(\mathbf{p}, \mathbf{k}, \mathbf{h}, \mathbf{k}) - j_{2b}^\mu(\mathbf{p}, \mathbf{k}, \mathbf{k}, \mathbf{h})]. \quad (16)$$

This effective current, $j_{2b}^\mu(\mathbf{p}, \mathbf{h})$, accounts for the fact that the two-body current operator, when acting on a nucleon pair in the Fermi sea, can lead to a 1p1h excitation if one of the nucleons remains a spectator. This mechanism is responsible for the interference contribution in the 1p1h channel. Then the total current function $j^\mu(\mathbf{p}, \mathbf{h})$ incorporates the contribution of the 1b and 2b currents.

To evaluate the hadronic tensor (8) in the RFG we insert the matrix element (14), and take the thermodynamic limit by replacing the discrete sum over hole states with an integral over momentum space,

$$\sum_h \rightarrow \frac{V}{(2\pi)^3} \int d^3h \sum_{s_h t_h}. \quad (17)$$

The integration over the final particle states can then be performed using the momentum-conserving delta function, which fixes the particle momentum to $\mathbf{p} = \mathbf{h} + \mathbf{q}$:

$$W^{\mu\nu} = \frac{V}{(2\pi)^3} \int d^3h \delta(E_p - E_h - \omega) 2w^{\mu\nu}(\mathbf{p}, \mathbf{h}) \times \theta(p - k_F) \theta(k_F - h), \quad (18)$$

where

$$w^{\mu\nu}(\mathbf{p}, \mathbf{h}) \equiv \frac{1}{2} \sum_{s_p s_h} j^\mu(\mathbf{p}, \mathbf{h})^* j^\nu(\mathbf{p}, \mathbf{h}) \quad (19)$$

is the single-nucleon hadronic tensor. The sums over isospin t_p, t_h no longer appear because we have already imposed the condition that, in the case of neutrino scattering, the hole state h corresponds to a neutron and the particle p to a proton, while the opposite holds for antineutrino scattering.

The single-nucleon tensor contains the square of the sum of the one-body and two-body currents. By expanding this square, one obtains

$$w^{\mu\nu} = w_{1b}^{\mu\nu} + w_{2b}^{\mu\nu} + w_{1b2b}^{\mu\nu}, \quad (20)$$

where $w_{1b}^{\mu\nu}$ and $w_{2b}^{\mu\nu}$ are the pure one-body and two-body contributions, respectively, and $w_{1b2b}^{\mu\nu}$ is the interference term:

$$w_{1b}^{\mu\nu} = \frac{1}{2} \sum (j_{1b}^\mu)^* j_{1b}^\nu, \quad (21)$$

$$w_{2b}^{\mu\nu} = \frac{1}{2} \sum (j_{2b}^\mu)^* j_{2b}^\nu, \quad (22)$$

$$w_{1b2b}^{\mu\nu} = \frac{1}{2} \sum [(j_{1b}^\mu)^* j_{2b}^\nu + (j_{2b}^\mu)^* j_{1b}^\nu]. \quad (23)$$

The response functions also decompose accordingly:

$$R^K = R_{1b}^K + R_{2b}^K + R_{1b2b}^K. \quad (24)$$

From calculations in electron scattering, it has been observed that the pure two-body MEC contribution, R_{2b}^K , is generally small and can often be neglected when compared to the interference term R_{1b2b}^K , which tends to dominate the MEC effects in the 1p1h channel [39,40]. This justifies our focus on the interference term, which captures the leading MEC effect in the 1p1h sector.

The one-body current operator consists of two terms: $j_{1b}^\mu(\mathbf{p}, \mathbf{h}) = j_{1bV}^\mu(\mathbf{p}, \mathbf{h}) + j_{1bA}^\mu(\mathbf{p}, \mathbf{h})$. The vector current is

$$j_{1bV}^\mu(\mathbf{p}, \mathbf{h}) = \bar{u}(\mathbf{p}) \left(2F_1^V \gamma^\mu + i \frac{2F_2^V}{2m_N} \sigma^{\mu\nu} Q_\nu \right) u(\mathbf{h}), \quad (25)$$

where the isovector nucleon form factors are defined as $F_{1,2}^V = (F_{1,2}^p - F_{1,2}^n)/2$. The axial current is

$$j_{1bA}^\mu(\mathbf{p}, \mathbf{h}) = -\bar{u}(\mathbf{p}) \left(G_A \gamma^\mu \gamma_5 + G_P \frac{Q^\mu}{2m_N} \gamma_5 \right) u(\mathbf{h}), \quad (26)$$

where G_A is the nucleon axial-vector form factor and G_P is the pseudoscalar form factor, given by

$$G_A = \frac{g_A}{(1 - \frac{Q^2}{M_A^2})^2}, \quad (27)$$

$$G_P = \frac{4m_N^2}{m_\pi^2 - Q^2} G_A \quad (28)$$

with $g_A = 1.26$ and $M_A = 1032$ MeV.

Note that the minus sign in the axial one-body current (26) arises from our convention of defining the total current as the sum of the vector and axial parts, $J^\mu = J_V^\mu + J_A^\mu$, whereas it is often written in the literature as $V - A$. We adopt this convention for consistency with the meson-exchange currents, which are also defined as the sum $V + A$. Of course, physical results are independent of this choice.

C. Meson exchange currents

In this section we present the MECs for CC neutrino scattering considered in this work, corresponding to the Feynman diagrams shown in Fig. 1. They were derived in [51] from the pion weak production model of Ref. [52]. Similarly to the 1b current, the weak MECs are the sum of vector and axial currents. The different current operators are the seagull current [diagrams (a) and (b)], the pion-in-flight current [diagram (c)], the pion-pole current [diagrams (d) and (e)], and the Δ forward [(f), (g)] and backward currents [(h), (i)]:

$$j_2^\mu(\mathbf{p}_1', \mathbf{p}_2', \mathbf{p}_1, \mathbf{p}_2) = j_{\text{sea}}^\mu + j_\pi^\mu + j_{\text{pole}}^\mu + j_\Delta^\mu. \quad (29)$$

All these currents share a common structure involving the following spin matrix element in each πNN vertex:

$$V_{s_1' s_1}(p_1', p_1) \equiv F_{\pi NN}(k_1^2) \frac{\bar{u}_{s_1'}(\mathbf{p}_1') \gamma^5 k_1 u_{s_1}(\mathbf{p}_1)}{k_1^2 - m_\pi^2}, \quad (30)$$

where $k_1 = p_1' - p_1$ is the momentum transfer to the individual nucleon (and the momentum carried by the pion), the spinors $u_s(\mathbf{p})$ are the solutions of the Dirac equation with

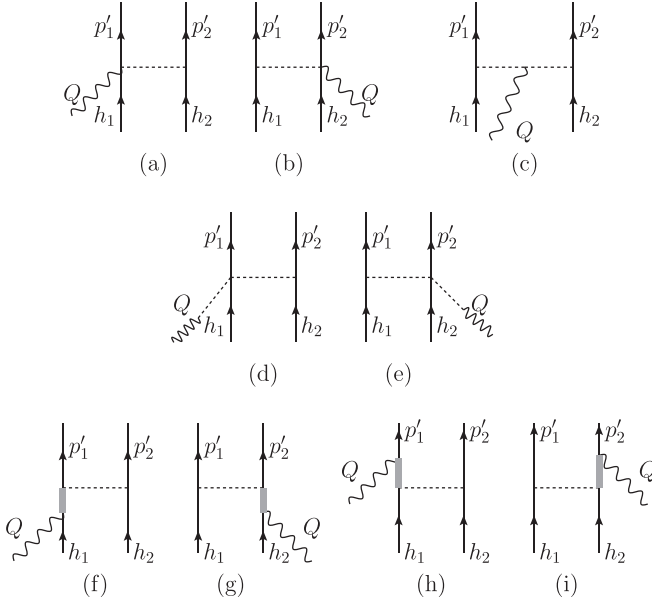


FIG. 1. Feynman diagrams for the electroweak MEC model used in this work.

momentum $\mathbf{p}$, m_π is the pion mass, and $F_{\pi NN}(k^2)$ is a strong form factor [53–55]

$$F_{\pi NN}(k^2) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - k^2}, \quad \Lambda = 1300 \text{ MeV} \quad (31)$$

Seagull. The seagull current is given as the sum of the vector and axial operators,

$$j_{\text{sea}}^\mu = (j_{\text{sea}}^\mu)_V + (j_{\text{sea}}^\mu)_A, \quad (32)$$

$$(j_{\text{sea}}^\mu)_V = i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_\pm \frac{f^2}{m_\pi^2} F_1^V F_{\pi NN}(k_1^2) V_{s'_1 s_1}(p'_1, p_1) \times \bar{u}_{s'_2}(p'_2) \gamma^5 \gamma^\mu u_{s_2}(p_2) + (1 \leftrightarrow 2), \quad (33)$$

$$(j_{\text{sea}}^\mu)_A = i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_\pm \frac{f^2}{m_\pi^2} \frac{F_\rho(k_1^2)}{g_A} F_{\pi NN}(k_1^2) V_{s'_1 s_1}(p'_1, p_1) \times \bar{u}_{s'_2}(p'_2) \gamma^\mu u_{s_2}(p_2) + (1 \leftrightarrow 2), \quad (34)$$

where $f^2 = 1$ is the πNN coupling constant, $\boldsymbol{\tau}^{(i)}$ is the isospin operator of nucleon i , $F_1^V(Q^2) = F_1^p - F_1^n$ is the isovector form factor of the nucleon, and F_ρ is the ρ meson form factor. Note that the current has been written as proportional to an isospin raising operator. The isospin matrix elements are computed in Appendix A.

Pion-in-flight. The pion-in-flight (or pionic) current has only a vector part:

$$(j_\pi^\mu)_V = i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_\pm F_1^V \frac{f^2}{m_\pi^2} V_{s'_1 s_1}(p'_1, p_1) \times V_{s'_2 s_2}(p'_2, p_2) (k_1^\mu - k_2^\mu), \quad (35)$$

$$(j_\pi^\mu)_A = 0. \quad (36)$$

Pion-pole. The pion-pole current is purely axial (this current could be considered as the axial part of the pionic one):

$$(j_{\text{pole}}^\mu)_V = 0, \quad (37)$$

$$(j_{\text{pole}}^\mu)_A = i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_\pm \frac{f^2}{m_\pi^2} \frac{F_\rho(k_1^2)}{g_A} F_{\pi NN}(k_2^2) \times \frac{Q^\mu \bar{u}_{s'_1}(p'_1)/Q u_{s_1}(p_1)}{Q^2 - m_\pi^2} V_{s'_2 s_2}(p'_2, p_2) + (1 \leftrightarrow 2). \quad (38)$$

Note that this current is proportional to the four-momentum transfer, Q^μ , and it only contributes to the longitudinal and time components of the hadronic tensor.

Delta (Δ). The Δ excitation current operator has both vector and axial parts,

$$j_\Delta^\mu = (j_\Delta^\mu)_V + (j_\Delta^\mu)_A, \quad (39)$$

corresponding to the vertices $\Gamma_V^{\beta\mu}$ and $\Gamma_A^{\beta\mu}$, respectively, in the $N \rightarrow \Delta$ transition, given in Eq. (43) below. The Δ current is further divided into forward and backward operators

$$j_{\Delta F}^\mu = [U_F(1, 2)_\pm] \frac{f^* f}{m_\pi^2} F_{\pi N \Delta}(k_2^2) V_{s'_2 s_2}(p'_2, p_2) \times \bar{u}_{s'_1}(p'_1) k_2^\alpha G_{\alpha\beta}(p_1 + Q) \Gamma^{\beta\mu}(Q) u_{s_1}(p_1) + (1 \leftrightarrow 2), \quad (40)$$

$$j_{\Delta B}^\mu = [U_B(1, 2)_\pm] \frac{f^* f}{m_\pi^2} F_{\pi N \Delta}(k_2^2) V_{s'_2 s_2}(p'_2, p_2) \times \bar{u}_{s'_1}(p'_1) k_2^\beta \hat{\Gamma}^{\mu\alpha}(Q) G_{\alpha\beta}(p'_1 - Q) u_{s_1}(p_1) + (1 \leftrightarrow 2), \quad (41)$$

where the $\pi N \Delta$ coupling constant is $f^* = 2.13f$. The $\gamma N \Delta$ vertices are

$$\Gamma^{\beta\mu}(Q) = \Gamma_V^{\beta\mu}(Q) + \Gamma_A^{\beta\mu}(Q), \quad (42)$$

$$\Gamma_V^{\beta\mu}(Q) = \frac{C_3^V}{m_N} (g^{\beta\mu} \not{Q} - Q^\beta \gamma^\mu) \gamma_5, \quad (43)$$

$$\Gamma_A^{\beta\mu}(Q) = C_5^A g^{\beta\mu}, \quad (44)$$

$$\hat{\Gamma}^{\beta\mu}(Q) = \hat{\Gamma}_V^{\beta\mu}(Q) + \hat{\Gamma}_A^{\beta\mu}(Q), \quad (45)$$

$$\hat{\Gamma}_V^{\beta\mu}(Q) = -\Gamma_V^{\beta\mu}(Q),$$

$$\hat{\Gamma}_A^{\beta\mu}(Q) = \Gamma_A^{\beta\mu}(Q). \quad (46)$$

The vector and axial form factors are taken from [52]:

$$C_3^V(Q^2) = \frac{2.13}{(1 - \frac{Q^2}{M_V^2})^2} \frac{1}{1 - \frac{Q^2}{4M_V^2}}, \quad (47)$$

$$C_5^A(Q^2) = \frac{1.2}{(1 - \frac{Q^2}{M_{A\Delta}^2})^2} \frac{1}{1 - \frac{Q^2}{4M_{A\Delta}^2}}, \quad (48)$$

with $M_V = 0.84 \text{ GeV}$ and $M_{A\Delta} = 1.05 \text{ GeV}$. The $\pi N \Delta$ form factor is taken as [53,54]

$$F_{\pi NN}(k) = F_{\pi N \Delta}(k). \quad (49)$$

The forward Δ current corresponds to processes where the Δ resonance is produced and then decays back to a nucleon, while the backward Δ current involves the exchange of a pion, leading to the creation of a Δ resonance in the intermediate state. The charge dependence of these processes is embedded in the isospin operators $U_F(1, 2)_\pm = U_F(1, 2)_x \pm iU_F(1, 2)_y$ for the forward term and $U_B(1, 2)_\pm = U_B(1, 2)_x \pm iU_B(1, 2)_y$ for the backward term, where

$$U_F(1, 2)_i = \sqrt{\frac{3}{2}} \sum_{j=1}^3 T_j^{(1)} T_i^{(1)\dagger} \tau_j^{(2)}, \quad (50)$$

$$U_B(1, 2)_i = \sqrt{\frac{3}{2}} \sum_{j=1}^3 T_i^{(1)} T_j^{(1)\dagger} \tau_j^{(2)}. \quad (51)$$

The operator $T_i^\dagger$ is the isospin raising operator that connects isospin-1/2 states to isospin-3/2 states and satisfies the condition $T_i T_j^\dagger = \frac{2}{3} \delta_{ij} - \frac{i}{3} \epsilon_{ijk} \tau_k$. Using this property one can write

$$U_F(1, 2)_\pm = \frac{2}{\sqrt{6}} \tau_\pm^{(2)} - \frac{i}{\sqrt{6}} [\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_\pm, \quad (52)$$

$$U_B(1, 2)_\pm = \frac{2}{\sqrt{6}} \tau_\pm^{(2)} + \frac{i}{\sqrt{6}} [\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_\pm. \quad (53)$$

Finally, the Δ propagator is

$$G_{\alpha\beta}(P) = \frac{\mathcal{P}_{\alpha\beta}(P)}{P^2 - m_\Delta^2 + im_\Delta \Gamma(P^2) + \frac{\Gamma(P^2)^2}{4}}, \quad (54)$$

where m_Δ and Γ are the Δ mass and width respectively. The projector $\mathcal{P}_{\alpha\beta}(P)$ over spin 3/2 is

$$\begin{aligned} \mathcal{P}_{\alpha\beta}(P) = & -(P + M_\Delta) \left[g_{\alpha\beta} - \frac{\gamma_\alpha \gamma_\beta}{3} \right. \\ & \left. - \frac{2P_\alpha P_\beta}{3m_\Delta^2} + \frac{P_\alpha \gamma_\beta - P_\beta \gamma_\alpha}{3m_\Delta} \right]. \end{aligned} \quad (55)$$

The Δ width $\Gamma(P^2)$ is given by

$$\Gamma(P^2) = \Gamma_0 \frac{m_\Delta}{\sqrt{P^2}} \left(\frac{p_\pi}{p_\pi^{\text{res}}} \right)^3, \quad (56)$$

where $\Gamma_0 = 120$ MeV is the Δ width at rest, p_π is the momentum of the final pion in the Δ decay, and p_π^{res} is its value at resonance ($P^2 = m_\Delta^2$).

The sign $\pm$ in the isospin matrix elements of Eqs. (33)–(41) refers to neutrino (+) or antineutrino (−) scattering.

The vector part of the weak meson-exchange currents reduces to the electromagnetic MEC when the isospin-raising operators are replaced by their third components [40].

D. Calculation of the responses in the RFG

In this section we outline the numerical procedure used to compute the response functions in the relativistic Fermi gas model, including both one-body and meson exchange currents. First we perform a change of variables to (E_h, E_p, ϕ) in Eq. (18), where E_h and E_p are the energies of the hole and particle states, and ϕ is the azimuthal angle. Then the volume element in spherical coordinates becomes $h^2 dh d\cos\theta d\phi =$

$(E_h E_p / q) dE_h dE_p d\phi$. The integration over E_p can then be carried out using the energy-conserving delta function, which fixes the polar angle θ between $\mathbf{q}$ and $\mathbf{h}$:

$$\cos\theta = \frac{2E_h \omega + Q^2}{2hq}. \quad (57)$$

Since the z axis is chosen along the direction of the momentum transfer $\mathbf{q}$, the integrand is independent of the azimuthal angle ϕ . We choose $\phi = 0$ and the integration over ϕ yields a factor of 2π . The remaining integral is then performed numerically over the hole energy E_h :

$$R_K(q, \omega) = \frac{V}{(2\pi)^3} \frac{2\pi m_N^3}{q} \int_{\epsilon_0}^{\infty} d\epsilon n(\epsilon) 2w_K(\epsilon, q, \omega), \quad (58)$$

where we have defined the adimensional energies $\epsilon = E_h/m_N$ and $\epsilon_F = E_F/m_N$. Moreover we have introduced the energy distribution of the Fermi gas $n(\epsilon) = \theta(\epsilon_F - \epsilon)$. The lower limit of the integral (58), denoted ϵ_0 , corresponds to the minimum energy of an on-shell nucleon that can absorb the energy and momentum transfer $(\omega, \mathbf{q})$. It is given by [16]

$$\epsilon_0 = \text{Max} \left\{ \kappa \sqrt{1 + \frac{1}{\tau}} - \lambda, \epsilon_F - 2\lambda \right\}, \quad (59)$$

where we use the dimensionless variables

$$\lambda = \frac{\omega}{2m_N}, \quad \kappa = \frac{q}{2m_N}, \quad \tau = \kappa^2 - \lambda^2. \quad (60)$$

It can be seen that the minimum energy ϵ_0 is always greater than or equal to 1, which corresponds to a nucleon at rest. This situation defines the center of the QE peak, $\omega = Q^2/(2m_N)$. It is also easy to verify that this occurs when $\lambda = \tau$, in which case $\kappa^2 = \tau(\tau + 1)$.

The integrand in Eq. (58) is the single-nucleon response $w_K(\epsilon, q, \omega)$ corresponding to a $1p1h$ excitation, obtained from the hadronic tensor of a single nucleon using the same definition as in Eqs. (5). This function depends in general on the momenta $\mathbf{h}$ and $\mathbf{p}$, but in practice it only depends on three variables. Once ϵ is fixed, the hole energy is given by $E_h = m_N \epsilon$, from which the magnitude of the momentum $h = |\mathbf{h}|$ can be determined. For fixed q and ω , the scattering angle $\cos\theta$ can then be calculated using (57), which determines the direction of $\mathbf{h}$ (we take $\phi = 0$ without loss of generality). By adding the momentum transfer $\mathbf{q}$, the final momentum $\mathbf{p} = \mathbf{h} + \mathbf{q}$ is fully specified.

In the RFG model, the total current $j^\mu(\mathbf{p}, \mathbf{h})$, including the $1b$ and $2b$ currents, and the single-nucleon responses w_K are computed numerically. This requires performing a numerical integration over the three-momentum $\mathbf{k}$ of the intermediate nucleon, as well as sums over the spin projections s_k , s_p , and s_h .

The isospin sums over t_k that appear in the two-body current are provided in Appendix A. The resulting effective one-body current consists of a direct term minus an exchange term,

$$j_{2b}^\mu(\mathbf{p}, \mathbf{h}) = j_{2b}^\mu(\mathbf{p}, \mathbf{h})_{\text{dir}} - j_{2b}^\mu(\mathbf{p}, \mathbf{h})_{\text{exch}} \quad (61)$$

The exchange contribution corresponds to the diagrams shown in Fig. 2. In symmetric nuclear matter, the direct term of the

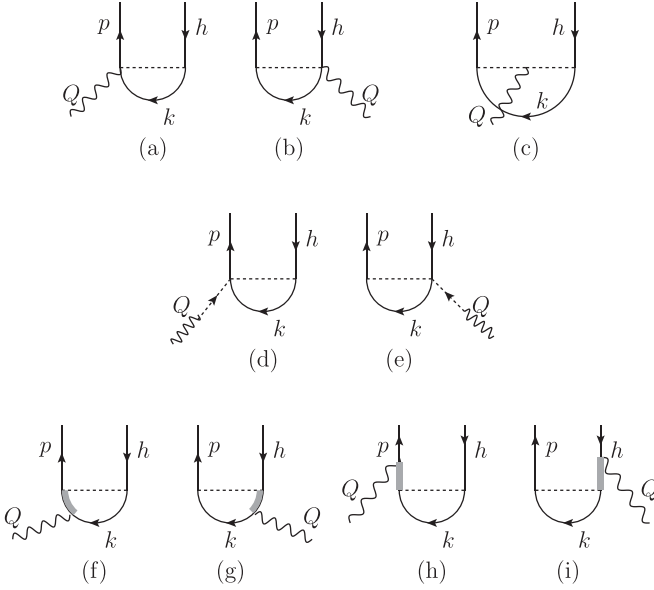


FIG. 2. Exchange many-body diagrams of the MEC matrix elements contributing to the 1p1h excitation channel considered in this work.

vector current vanishes—similarly to the case of the electromagnetic exchange current—because it arises solely from the Δ current, which is transverse, and therefore vanishes upon contraction with Q^μ . When summing over spin in the direct matrix element, this contraction appears explicitly, leading to a vanishing contribution.

However, for the axial Δ current, the direct term does not vanish, and in principle should be included when computing the neutrino response. Nevertheless, we have checked that this term contributes significantly only to the longitudinal response R_{LL} , which is known to be small. This behavior can be seen explicitly in the nonrelativistic limit, where the structure of the current becomes more transparent. In the results section we show through explicit calculations that the LL response gives a negligible contribution to the cross section for quasielastic neutrino scattering. Since the direct axial MEC term contributes significantly only to the LL channel, and this channel plays a minor role in the kinematics of interest, it is not included in the present work.

III. NONRELATIVISTIC LIMIT

In this section we present the expressions for the interference responses in the nonrelativistic (NR) limit, which will be compared with the corresponding relativistic results in Sec. IV. This comparison serves several purposes: (i) to verify that relativistic and nonrelativistic results agree at low momentum and energy transfer, as expected; (ii) to compare our results with known expressions for axial MEC currents in the NR literature [47]; and (iii) to test the accuracy of our numerical implementation, since the NR Fermi gas allows partial analytical evaluation of the response functions [37,38]. Furthermore, this limit helps identify the dominant responses and current components at moderate momentum transfer.

A. Nonrelativistic one-body current

The vector part of the 1b current is the sum of magnetization and convection currents:

$$\mathbf{j}_{1b}(\mathbf{p}, \mathbf{h})_V = \mathbf{j}_{mV}(\mathbf{p}, \mathbf{h}) + \mathbf{j}_{cV}(\mathbf{p}, \mathbf{h}), \quad (62)$$

$$\mathbf{j}_{mV}(\mathbf{p}, \mathbf{h}) = -\frac{2G_M^V}{2m_N} i\mathbf{q} \times \boldsymbol{\sigma}_{spsh}, \quad (63)$$

$$\mathbf{j}_{cV}(\mathbf{p}, \mathbf{h}) = \delta_{spsh} \frac{2G_E^V}{m_N} \left(\mathbf{h} + \frac{\mathbf{q}}{2} \right), \quad (64)$$

with $\mathbf{q} = \mathbf{p} - \mathbf{h}$ by momentum conservation. Here G_M^V (G_E^V) is the isovector magnetic (electric) form factor of the nucleon, $G_E^V = (G_E^p - G_E^n)/2$.

The transverse (perpendicular to $\mathbf{q}$) one-body axial current at leading order is [56]

$$\mathbf{j}_{1b}^\perp(\mathbf{p}, \mathbf{h})_A = -G_A \boldsymbol{\sigma}_{spsh}^\perp. \quad (65)$$

The time component of the axial current, j_{1b}^0 , or axial charge-density is typically not included in nonrelativistic calculations because it is small at leading order; however, this suppression is not a strict consequence of the non-relativistic expansion itself. To obtain its static limit, we have considered the semi-relativistic expansion of the electroweak current introduced in Refs. [56,57]. It can be written as the sum of a convective term and a magnetization term, in analogy with the structure of the transverse vector current [56]:

$$j_{1b}^0(\mathbf{p}, \mathbf{h})_A = j_{mA}^0(\mathbf{p}, \mathbf{h}) + j_{cA}^0(\mathbf{p}, \mathbf{h}), \quad (66)$$

$$j_{mA}^0(\mathbf{p}, \mathbf{h}) = -\frac{G_A'}{2m_N} \mathbf{q} \cdot \boldsymbol{\sigma}_{spsh}, \quad (67)$$

$$j_{cA}^0(\mathbf{p}, \mathbf{h}) = -G_A \frac{\mathbf{h}^\perp}{m_N} \cdot \boldsymbol{\sigma}_{spsh}, \quad (68)$$

where $\mathbf{h}^\perp$ is the transverse component of $\mathbf{h}$, perpendicular to the momentum transfer. The auxiliary form factor G_A' is defined by

$$G_A' = G_A - \tau G_P = \left(1 - \frac{Q^2}{Q^2 - m_\pi^2} \right) G_A. \quad (69)$$

B. Nonrelativistic expansion of the weak MECs

The weak MEC operators are the sum of vector plus axial components. The vector MECs are also isovectors and closely related to the electromagnetic MECs, which are also isovectors. The key difference lies in the isospin structure: while the electromagnetic current corresponds to the third component of the isospin operator, the charged weak current involves the raising and lowering $\pm$ operators. Consequently, the nonrelativistic expressions for electromagnetic MECs derived in Ref. [31] can be directly adapted to the weak case by substituting $\tau_z \rightarrow \tau_\pm$ and $[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_z \rightarrow [\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_\pm$, depending on the specific charge-changing process.

The nonrelativistic expansion of the MEC is obtained by applying standard reduction rules to matrix elements involving products of gamma matrices between Dirac spinors,

retaining only leading-order terms in $1/m_N$:

$$\gamma^0 \longrightarrow 1, \quad \gamma^i \longrightarrow 0, \quad \gamma_5 \gamma^0 \longrightarrow 0, \quad (70)$$

$$\gamma_5 \gamma^i \longrightarrow -\sigma_i, \quad \gamma^i \gamma^j \longrightarrow -\sigma_i \sigma_j, \quad \gamma^0 \gamma^j \longrightarrow 0. \quad (71)$$

For a nucleon momentum,

$$p^\mu \longrightarrow (m_N, p^i), \quad \not{p} \longrightarrow p_0. \quad (72)$$

For the momentum transfer to nucleon i ,

$$k^\mu \longrightarrow (0, k^i), \quad \gamma_5 \not{k} \longrightarrow \mathbf{k} \cdot \boldsymbol{\sigma}. \quad (73)$$

Finally, the V function of Eq. (30) becomes

$$V(1', 1) \longrightarrow -\frac{\mathbf{k}_1 \cdot \boldsymbol{\sigma}^{(1)}}{\mathbf{k}_1^2 + m_\pi^2}. \quad (74)$$

As a result, only the spatial components of the vector MEC survive at leading order in the nonrelativistic expansion. This feature is also confirmed with the fully relativistic calculation.

Then

$$j_{sV}^\mu \xrightarrow{\text{nr}} (0, \mathbf{j}_{sV}),$$

$$j_{\pi V}^\mu \xrightarrow{\text{nr}} (0, \mathbf{j}_{\pi V}),$$

$$j_{\Delta V}^\mu \xrightarrow{\text{nr}} (0, \mathbf{j}_{\Delta V}).$$

The axial seagull current is proportional to the matrix element of γ^μ , and, in the nonrelativistic limit, only its time component (proportional to γ^0) survives. On the other hand, only the spatial components of the axial Δ current remain nonzero, as shown in Appendix B. The pion-pole current, being proportional to $q_\mu \gamma^\mu$, vanishes at leading order. The surviving axial currents are

$$j_{sA}^\mu \xrightarrow{\text{nr}} (j_{sA}^0, \vec{0}),$$

$$j_{\Delta A}^\mu \xrightarrow{\text{nr}} (0, \mathbf{j}_{\Delta A}).$$

The corresponding nonrelativistic operators are

$$\mathbf{j}_{sV}(p'_1, p'_2, p_1, p_2) = i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_+ \frac{f^2}{m_\pi^2} F_1^V \left(\frac{\mathbf{k}_1 \cdot \boldsymbol{\sigma}^{(1)}}{\mathbf{k}_1^2 + m_\pi^2} \boldsymbol{\sigma}^{(2)} - \frac{\mathbf{k}_2 \cdot \boldsymbol{\sigma}^{(2)}}{\mathbf{k}_2^2 + m_\pi^2} \boldsymbol{\sigma}^{(1)} \right), \quad (75)$$

$$\mathbf{j}_{\pi V}(p'_1, p'_2, p_1, p_2) = i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_+ \frac{f^2}{m_\pi^2} F_1^V \frac{\mathbf{k}_1 \cdot \boldsymbol{\sigma}^{(1)}}{\mathbf{k}_1^2 + m_\pi^2} \frac{\mathbf{k}_2 \cdot \boldsymbol{\sigma}^{(2)}}{\mathbf{k}_2^2 + m_\pi^2} (\mathbf{k}_1 - \mathbf{k}_2), \quad (76)$$

$$\begin{aligned} \mathbf{j}_{\Delta V}(p'_1, p'_2, p_1, p_2) = & i\sqrt{\frac{3}{2}} \frac{2}{9} \frac{ff^*}{m_\pi^2} \frac{C_3^V}{m_N} \frac{1}{m_\Delta - m_N} \left\{ \frac{\mathbf{k}_2 \cdot \boldsymbol{\sigma}^{(2)}}{\mathbf{k}_2^2 + m_\pi^2} [4\tau_+^{(2)} \mathbf{k}_2 + [\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_+ \mathbf{k}_2 \times \boldsymbol{\sigma}^{(1)}] \right. \\ & \left. + \frac{\mathbf{k}_1 \cdot \boldsymbol{\sigma}^{(1)}}{\mathbf{k}_1^2 + m_\pi^2} [4\tau_+^{(1)} \mathbf{k}_1 - [\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_+ \mathbf{k}_1 \times \boldsymbol{\sigma}^{(2)}] \right\} \times \mathbf{q}, \end{aligned} \quad (77)$$

$$j_{sA}^0(p'_1, p'_2, p_1, p_2) = -i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_+ \frac{f_{\pi NN}^2}{m_\pi^2} \frac{1}{g_A} \left[\frac{\mathbf{k}_1 \cdot \boldsymbol{\sigma}^{(1)}}{\mathbf{k}_1^2 + m_\pi^2} - \frac{\mathbf{k}_2 \cdot \boldsymbol{\sigma}^{(2)}}{\mathbf{k}_2^2 + m_\pi^2} \right], \quad (78)$$

$$\begin{aligned} \mathbf{j}_{\Delta A}(p'_1, p'_2, p_1, p_2) = & -\sqrt{\frac{3}{2}} \frac{2}{9} \frac{ff^*}{m_\pi^2} C_5^A \frac{1}{m_\Delta - m_N} \left\{ 4\tau_+^{(1)} \frac{(\mathbf{k}_1 \cdot \boldsymbol{\sigma}^{(1)}) \mathbf{k}_1}{\mathbf{k}_1^2 + m_\pi^2} + 4\tau_+^{(2)} \frac{(\mathbf{k}_2 \cdot \boldsymbol{\sigma}^{(2)}) \mathbf{k}_2}{\mathbf{k}_2^2 + m_\pi^2} \right. \\ & \left. + [\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_+ \left[\frac{(\mathbf{k}_2 \cdot \boldsymbol{\sigma}^{(2)}) (\mathbf{k}_2 \times \boldsymbol{\sigma}^{(1)})}{\mathbf{k}_2^2 + m_\pi^2} - \frac{(\mathbf{k}_1 \cdot \boldsymbol{\sigma}^{(1)}) (\mathbf{k}_1 \times \boldsymbol{\sigma}^{(2)})}{\mathbf{k}_1^2 + m_\pi^2} \right] \right\}. \end{aligned} \quad (79)$$

These operators match the standard nonrelativistic MECs in the literature [47] modulo differences in coupling constants and form factors.

C. MEC effective one-body currents

The 1p1h matrix elements of the vector and axial MECs are

$$j_{2b}^\mu(p, h) = -\int \frac{d^3k}{(2\pi)^3} \sum_{t_k, s_k} j_{2b}^\mu(p, k, k, h) = j_s^\mu(p, h) + j_\pi^\mu(p, h) + j_\Delta^\mu(p, h), \quad (80)$$

where we have neglected the direct part in the axial Δ current, as previously mentioned. After performing the sums over t_k, s_k , the results are the following for the three MEC, seagull, pionic and Δ currents:

$$\mathbf{j}_s(p, h)_V = 4 \frac{f^2}{m_\pi^2} F_1^V \int \frac{d^3k}{(2\pi)^3} \left(\frac{\delta_{s_p s_h} \mathbf{k}_1 + i\boldsymbol{\sigma}_{ph} \times \mathbf{k}_1}{\mathbf{k}_1^2 + m_\pi^2} - \frac{\delta_{s_p s_h} \mathbf{k}_2 + i\mathbf{k}_2 \times \boldsymbol{\sigma}_{ph}}{\mathbf{k}_2^2 + m_\pi^2} \right), \quad (81)$$

$$\mathbf{j}_\pi(p, h)_V = 4 \frac{f^2}{m_\pi^2} F_1^V \int \frac{d^3k}{(2\pi)^3} \frac{\delta_{s_p s_h} \mathbf{k}_1 \cdot \mathbf{k}_2 + i(\mathbf{k}_1 \times \mathbf{k}_2) \cdot \boldsymbol{\sigma}_{ph}}{(\mathbf{k}_1^2 + m_\pi^2)(\mathbf{k}_2^2 + m_\pi^2)} (\mathbf{k}_1 - \mathbf{k}_2), \quad (82)$$

$$\mathbf{j}_\Delta(p, h)_V = i\sqrt{\frac{3}{2}} \frac{8}{9} \frac{f f^*}{m_\pi^2} \frac{C_3^V}{m_N} \frac{1}{m_\Delta - m_N} \mathbf{q} \times \int \frac{d^3k}{(2\pi)^3} \left(\frac{\mathbf{k}_1^2 \sigma_{ph} + (\sigma_{ph} \cdot \mathbf{k}_1) \mathbf{k}_1}{\mathbf{k}_1^2 + m_\pi^2} + \frac{\mathbf{k}_2^2 \sigma_{ph} + (\sigma_{ph} \cdot \mathbf{k}_2) \mathbf{k}_2}{\mathbf{k}_2^2 + m_\pi^2} \right), \quad (83)$$

$$j_s^0(p, h)_A = -4 \frac{f^2}{m_\pi^2} \frac{1}{g_A} \int \frac{d^3k}{(2\pi)^3} \left[\frac{\mathbf{k}_1 \cdot \sigma_{ph}}{\mathbf{k}_1^2 + m_\pi^2} - \frac{\mathbf{k}_2 \cdot \sigma_{ph}}{\mathbf{k}_2^2 + m_\pi^2} \right], \quad (84)$$

$$\mathbf{j}_\Delta(p, h)_A = \sqrt{\frac{3}{2}} \frac{8}{9} \frac{f f^*}{m_\pi^2} C_5^A \frac{1}{m_\Delta - m_N} \int \frac{d^3k}{(2\pi)^3} \left(\frac{\mathbf{k}_1^2 \sigma_{ph} + (\sigma_{ph} \cdot \mathbf{k}_1) \mathbf{k}_1}{\mathbf{k}_1^2 + m_\pi^2} + \frac{\mathbf{k}_2^2 \sigma_{ph} + (\sigma_{ph} \cdot \mathbf{k}_2) \mathbf{k}_2}{\mathbf{k}_2^2 + m_\pi^2} \right), \quad (85)$$

with $\mathbf{k}_1 = \mathbf{p} - \mathbf{k}$, $\mathbf{k}_2 = \mathbf{k} - \mathbf{h}$, and $\sigma_{ph} = \sigma_{sp} s_h$.

D. Non relativistic 1b-2b interference single-nucleon responses

Here we provide separate expressions for the different contributions to the interference terms, which arise from the cross products between the various components of the one-body current and the meson exchange currents:

$$w_{CC,1b2b}^{AA} = w_{CC,ms}^{AA} + w_{CC,cs}^{AA}, \quad (86)$$

$$w_{CC,ms}^{AA} = \text{Re} \sum_{spsh} j_{mA}^0(p, h)^* j_{sA}^0(p, h), \quad (87)$$

$$w_{CC,cs}^{AA} = \text{Re} \sum_{spsh} j_{cA}^0(p, h)^* j_{sA}^0(p, h), \quad (88)$$

$$w_{T,1b2b}^{VV} = w_{T,ms}^{VV} + w_{T,cs}^{VV} + w_{T,m\pi}^{VV} + w_{T,c\pi}^{VV} + w_{T,m\Delta}^{VV}, \quad (89)$$

$$w_{T,ms}^{VV} = \text{Re} \sum_{spsh} \mathbf{j}_{mV}(p, h)^* \cdot \mathbf{j}_{sV}(p, h), \quad (90)$$

$$w_{T,cs}^{VV} = \text{Re} \sum_{spsh} \mathbf{j}_{cV}^T(p, h)^* \cdot \mathbf{j}_{sV}(p, h), \quad (91)$$

$$w_{T,m\pi}^{VV} = \text{Re} \sum_{spsh} \mathbf{j}_{mV}(p, h)^* \cdot \mathbf{j}_\pi(p, h), \quad (92)$$

$$w_{T,c\pi}^{VV} = \text{Re} \sum_{spsh} \mathbf{j}_{cV}^T(p, h)^* \cdot \mathbf{j}_\pi(p, h), \quad (93)$$

$$w_{T,m\Delta}^{VV} = \text{Re} \sum_{spsh} \mathbf{j}_{mV}(p, h)^* \cdot \mathbf{j}_{\Delta V}(p, h), \quad (94)$$

$$w_{T,1b2b}^{AA} = w_{T,1b\Delta}^{AA} = \text{Re} \sum_{spsh} \mathbf{j}_{1bA}^\perp(p, h)^* \cdot \mathbf{j}_{\Delta A}(p, h). \quad (95)$$

In the case of the T' response we separate the VA and AV contributions:

$$w_{T',1b2b} = w_{T',1b2b}^{VA} + w_{T',1b2b}^{AV}, \quad (96)$$

$$w_{T',1b2b}^{VA} = \frac{1}{2} \text{Im} \sum_{spsh} [j_{1bV}^{1*}(p, h) j_{2bA}^2(p, h) + j_{2bA}^{1*}(p, h) j_{1bV}^2(p, h)], \quad (97)$$

$$w_{T',1b2b}^{AV} = \frac{1}{2} \text{Im} \sum_{spsh} [j_{1bA}^{1*}(p, h) j_{2bV}^2(p, h) + j_{2bV}^{1*}(p, h) j_{1bA}^2(p, h)]. \quad (98)$$

Then

$$w_{T',1b2b}^{VA} = w_{T',m\Delta}^{VA}, \quad (99)$$

$$w_{T',1b2b}^{AV} = w_{T',1bs}^{AV} + w_{T',1b\pi}^{AV} + w_{T',1b\Delta}^{AV}. \quad (100)$$

Note that, in the case of the Δ current, both the vector and axial parts contribute to the response T' , whereas, in the case of the seagull and pionic current, only the vector part is considered, since the axial part is longitudinal in the nonrelativistic limit.

The explicit expressions for the different contributions to the single nucleon interference responses are the following (with $\mathbf{k}_1 = \mathbf{p} - \mathbf{k}$ and $\mathbf{k}_2 = \mathbf{k} - \mathbf{h}$):

$$w_{T,ms}^{VV} = 2 \frac{f^2}{m_\pi^2} F_1^V \frac{G_M^V}{m_N} \int \frac{d^3k}{(2\pi)^3} \left(\frac{4\mathbf{q} \cdot \mathbf{k}_1}{\mathbf{k}_1^2 + m_\pi^2} + \frac{4\mathbf{q} \cdot \mathbf{k}_2}{\mathbf{k}_2^2 + m_\pi^2} \right), \quad (101)$$

$$w_{T,cs}^{VV} = 4 \frac{f^2}{m_\pi^2} F_1^V \frac{G_E^V}{m_N} \int \frac{d^3k}{(2\pi)^3} \left(\frac{2\mathbf{h}_T \cdot \mathbf{k}_1}{\mathbf{k}_1^2 + m_\pi^2} - \frac{2\mathbf{h}_T \cdot \mathbf{k}_2}{\mathbf{k}_2^2 + m_\pi^2} \right), \quad (102)$$

$$w_{T,m\pi}^{VV} = -2 \frac{f^2}{m_\pi^2} F_1^V \frac{G_M^V}{m_N} \int \frac{d^3k}{(2\pi)^3} \frac{4(\mathbf{q} \times \mathbf{k}_2)^2}{(\mathbf{k}_1^2 + m_\pi^2)(\mathbf{k}_2^2 + m_\pi^2)}, \quad (103)$$

$$w_{T,c\pi}^{VV} = -4 \frac{f^2}{m_\pi^2} F_1^V \frac{G_E^V}{m_N} \int \frac{d^3k}{(2\pi)^3} \frac{4(\mathbf{q} \cdot \mathbf{k}_2 - \mathbf{k}_2^2) \mathbf{h}_T \cdot \mathbf{k}_2}{(\mathbf{k}_1^2 + m_\pi^2)(\mathbf{k}_2^2 + m_\pi^2)}, \quad (104)$$

$$w_{T,m\Delta}^{VV} = -2 \sqrt{\frac{3}{2}} \frac{f f^*}{m_\pi^2} \frac{C_3^V}{m_N} \frac{1}{m_\Delta - m_N} \frac{G_M^V}{m_N} \int \frac{d^3k}{(2\pi)^3} 2 \left(\frac{3q^2 k_1^2 - (\mathbf{q} \cdot \mathbf{k}_1)^2}{\mathbf{k}_1^2 + m_\pi^2} + \frac{3q^2 k_2^2 - (\mathbf{q} \cdot \mathbf{k}_2)^2}{\mathbf{k}_2^2 + m_\pi^2} \right), \quad (105)$$

$$w_{CC,ms}^{AA} = \frac{f^2}{m_\pi^2} \frac{1}{g_A} \frac{G'_A}{m_N} \int \frac{d^3k}{(2\pi)^3} \left(\frac{4\mathbf{q} \cdot \mathbf{k}_1}{\mathbf{k}_1^2 + m_\pi^2} + \frac{4\mathbf{q} \cdot \mathbf{k}_2}{\mathbf{k}_2^2 + m_\pi^2} \right), \quad (106)$$

$$w_{CC,cs}^{AA} = 4 \frac{f^2}{m_\pi^2} \frac{1}{g_A} \frac{G_A}{m_N} \int \frac{d^3k}{(2\pi)^3} \left(\frac{2\mathbf{h}_T \cdot \mathbf{k}_1}{\mathbf{k}_1^2 + m_\pi^2} - \frac{2\mathbf{h}_T \cdot \mathbf{k}_2}{\mathbf{k}_2^2 + m_\pi^2} \right), \quad (107)$$

$$w_{T',lb\Delta}^{AA} = -\sqrt{\frac{3}{2}} \frac{16}{9} \frac{ff^*}{m_\pi^2} \frac{1}{m_\Delta - m_N} \frac{C_5^A G_A}{q^2} \int \frac{d^3k}{(2\pi)^3} 2 \left(\frac{3q^2 k_1^2 - (\mathbf{q} \cdot \mathbf{k}_1)^2}{\mathbf{k}_1^2 + m_\pi^2} + \frac{3q^2 k_2^2 - (\mathbf{q} \cdot \mathbf{k}_2)^2}{\mathbf{k}_2^2 + m_\pi^2} \right), \quad (108)$$

$$w_{T',lbs}^{AV} = 2 \frac{f^2}{m_\pi^2} \frac{G_A F_1^V}{q} \int \frac{d^3k}{(2\pi)^3} \left(\frac{4\mathbf{q} \cdot \mathbf{k}_1}{\mathbf{k}_1^2 + m_\pi^2} + \frac{4\mathbf{q} \cdot \mathbf{k}_2}{\mathbf{k}_2^2 + m_\pi^2} \right), \quad (109)$$

$$w_{T',lb\pi}^{AV} = -4 \frac{f^2}{m_\pi^2} F_1^V \frac{G_A}{q} \int \frac{d^3k}{(2\pi)^3} \frac{4(\mathbf{q} \times \mathbf{k}_2)^2}{(\mathbf{k}_1^2 + m_\pi^2)(\mathbf{k}_2^2 + m_\pi^2)}, \quad (110)$$

$$w_{T',lb\Delta}^{AV} = -\sqrt{\frac{3}{2}} \frac{4}{9} \frac{ff^*}{m_\pi^2} \frac{G_A C_3^V}{m_N q} \frac{1}{m_\Delta - m_N} \int \frac{d^3k}{(2\pi)^3} 2 \left(\frac{3q^2 k_1^2 - (\mathbf{q} \cdot \mathbf{k}_1)^2}{\mathbf{k}_1^2 + m_\pi^2} + \frac{3q^2 k_2^2 - (\mathbf{q} \cdot \mathbf{k}_2)^2}{\mathbf{k}_2^2 + m_\pi^2} \right), \quad (111)$$

$$w_{T',m\Delta}^{VA} = -\sqrt{\frac{3}{2}} \frac{4}{9} \frac{ff^*}{m_\pi^2} \frac{G_M^V C_5^A}{m_N q} \frac{1}{m_\Delta - m_N} \int \frac{d^3k}{(2\pi)^3} 2 \left(\frac{3q^2 k_1^2 - (\mathbf{q} \cdot \mathbf{k}_1)^2}{\mathbf{k}_1^2 + m_\pi^2} + \frac{3q^2 k_2^2 - (\mathbf{q} \cdot \mathbf{k}_2)^2}{\mathbf{k}_2^2 + m_\pi^2} \right). \quad (112)$$

IV. RESULTS

In this section, we present numerical results for the effect of two-body MECs on the charged-current (CC) neutrino response functions in the 1p1h channel. We consider three nuclear models: the RFG, the RMF in nuclear matter, and the SuSAM* approach. These results allow us to assess the model dependence of the MEC contributions. Finally, we illustrate the impact of these effects on neutrino cross sections with

selected examples. To this end, we will first examine in detail the interference terms between the one-body and two-body currents, which provide the dominant MEC contribution, and later compare them with the pure one-body responses.

A. Nonrelativistic Fermi gas results

We begin by analyzing the dominant response functions to leading order in the non-relativistic Fermi gas (NRFG),

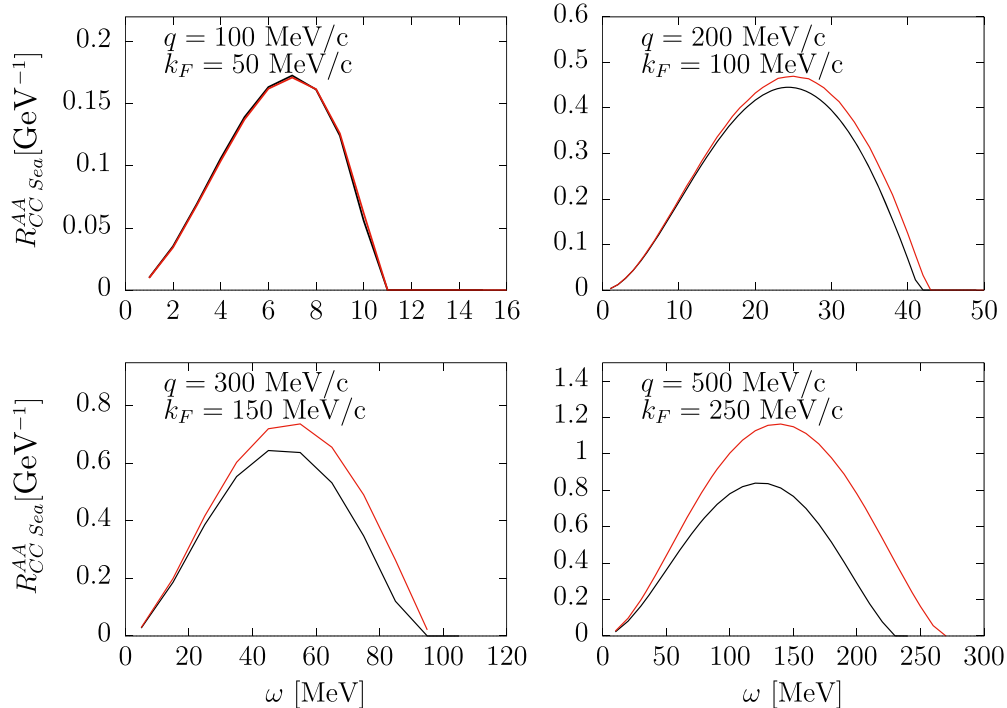


FIG. 3. Interference response R_{CC}^{AA} between the one-body axial current and the seagull current, for increasing values of the momentum transfer q , with $k_F = q/2$. In each panel, the nonrelativistic results (red lines) are compared with the relativistic ones (black lines).

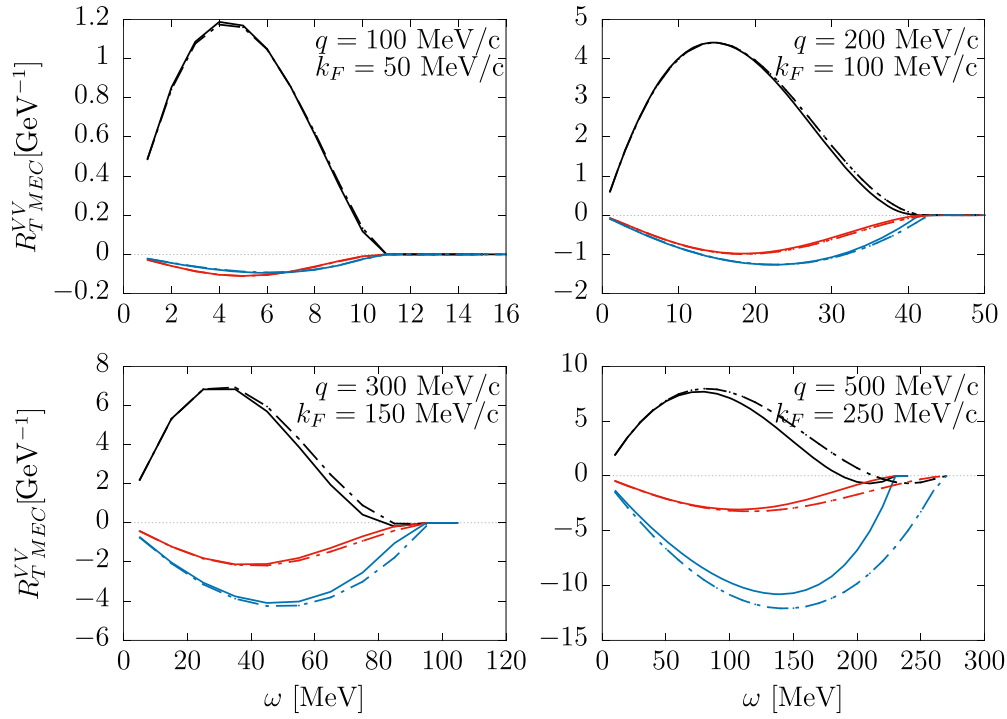


FIG. 4. Nonrelativistic (dot-dashed lines) versus relativistic (solid lines) transverse interference response R_T^{VV} between the 1b current and the seagull (black), pion-in-flight (red), and Δ -excitation (blue) two-body currents, for increasing values of q and with $k_F = q/2$.

and comparing them with the fully relativistic results obtained within the RFG model. As previously discussed, this comparison serves as a consistency test of the calculation, since the NRFG and RFG models are implemented in different ways. In the non-relativistic case, the integrals over the momentum

of the intermediate nucleon are performed analytically following the method of Refs. [37,38], except for the pion-in-flight current, where they reduce to one-dimensional integrals. In addition, the spin traces have been computed explicitly in the NR case. On the other hand, the RFG responses are evaluated

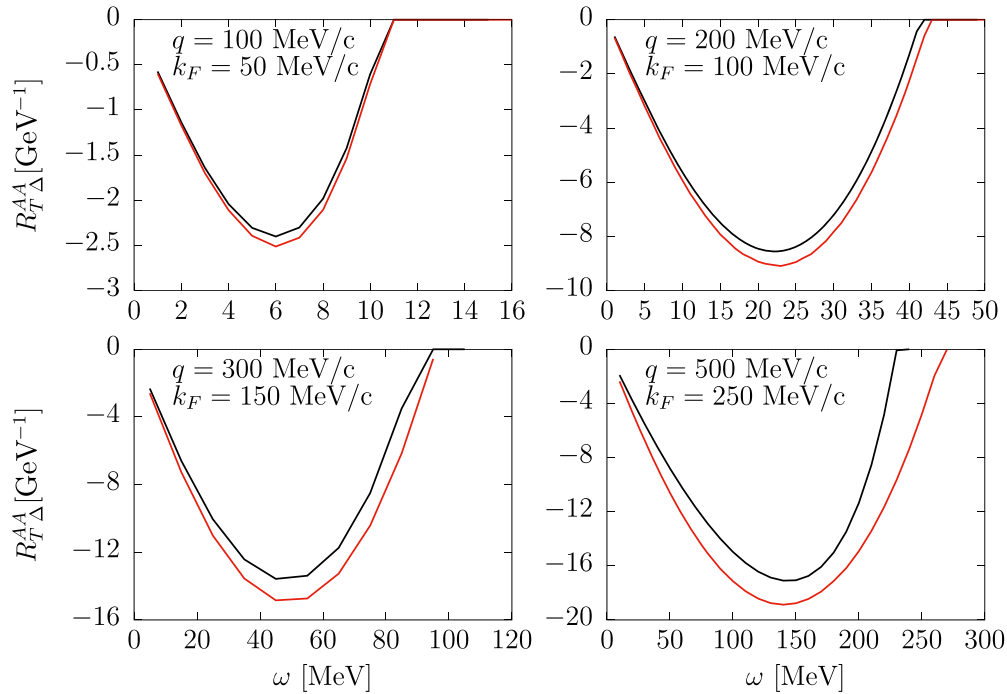


FIG. 5. The same as Fig. 3 but for the interference response R_T^{AA} between the axial 1b and Δ currents.

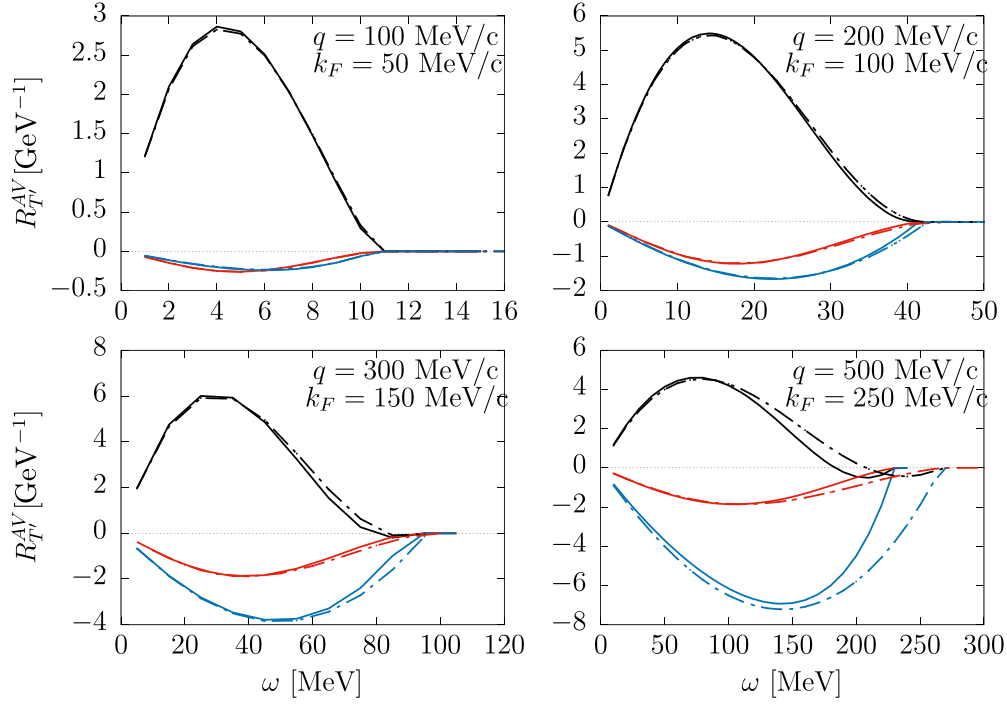


FIG. 6. The same as Fig. 4 but for the interference response $R_{T'1b2b}^{AV}$ between the axial 1b current and the vector seagull (black), pionic (red), and delta (blue) currents.

fully numerically. As expected, the RFG results approach those of the NRFG in the low- q , low- ω region, as we explicitly demonstrate below.

In Fig. 3 we show the interference response $R_{CC,1bs}^{AA}$, which corresponds to the interference between the axial one-body

current and the axial seagull two-body current in the CC channel. This is the dominant MEC contribution to the CC responses at low momentum transfer. To examine how the relativistic response approaches the nonrelativistic one as q becomes small, we present results for several values of the

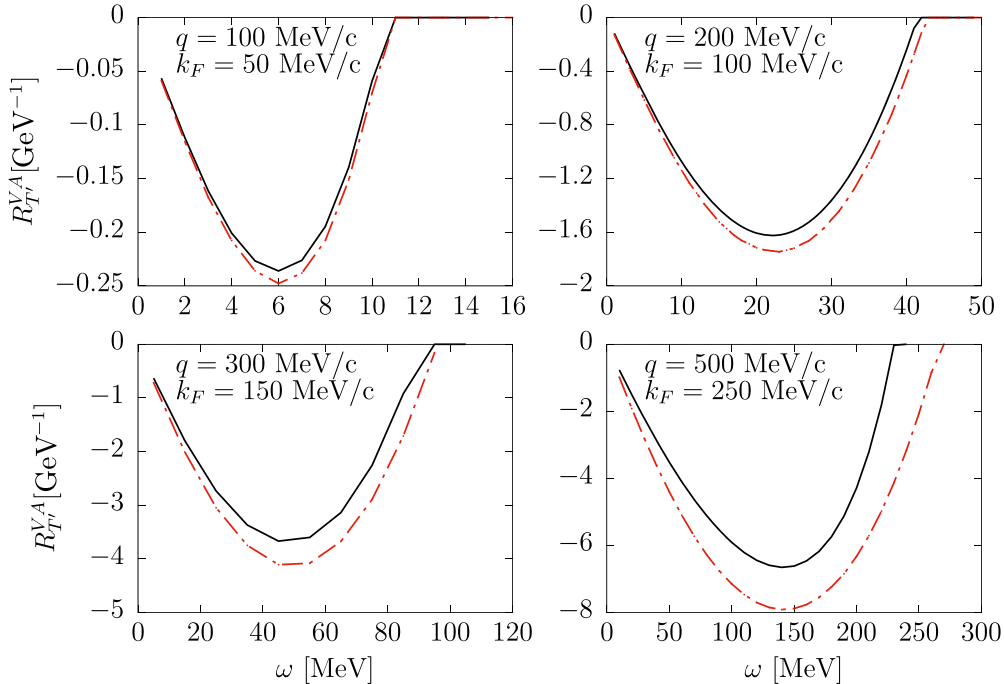


FIG. 7. The same as Fig. 3 but for the $R_{T'1b\Delta}^{VA}$ interference response.

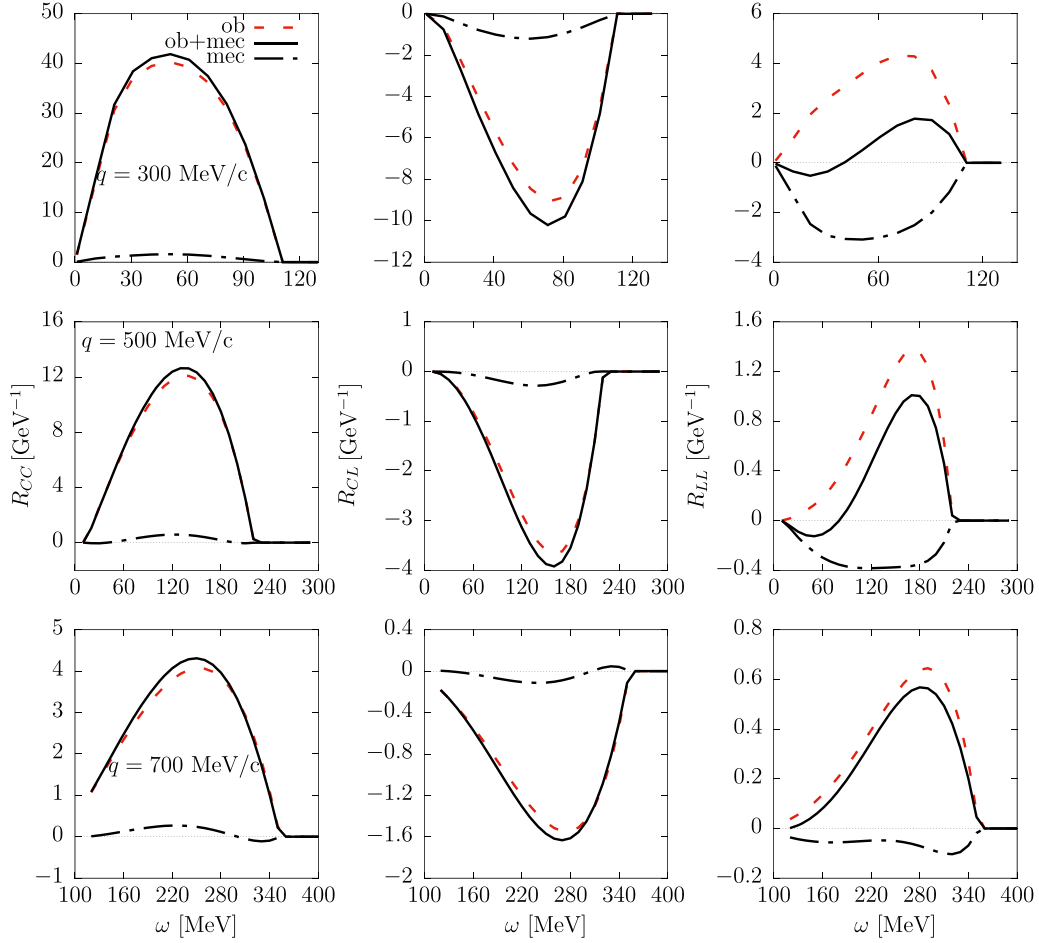


FIG. 8. Response functions R_{CC} , R_{CL} , and R_{LL} computed in the RFG model for different values of q . We compare one-body responses with the MEC contribution and the total results.

momentum transfer: $q = 100, 200, 300$, and 500 MeV/ c . In each panel we set the Fermi momentum to $k_F = q/2$, so that the momenta of the initial nucleons are also small when q is small. This choice also minimizes the effects of Pauli blocking in the comparison. As expected, we observe that the relativistic and nonrelativistic responses are nearly identical for $q = 100$ MeV/ c , and begin to differ progressively as q increases.

In Fig. 4 we show the interference contributions to the R_T^{VV} response, separating the effects of the seagull, pion-in-flight, and Δ currents. These vector-vector responses are exactly twice as large as the corresponding electromagnetic responses, as proved in Appendix B. We observe that the seagull contribution is positive, while the pion-in-flight and Δ contributions are negative, leading to a partial cancellation. The Δ term increases with q more rapidly than the seagull term, and becomes the dominant contribution at $q = 500$ MeV/ c . This behavior is consistent with the electromagnetic response results reported in Ref. [31].

In Fig. 5 we present the axial transverse interference response R_T^{AA} . At $q = 100$ MeV/ c , the relativistic and nonrelativistic results are nearly identical, serving as a triple consistency check: first, of the nonrelativistic reduction of the axial Δ current performed in Appendix C; second, of

the numerical implementation of the RFG and NRFG frameworks; and third, of the analytic and numerical procedures used in each calculation. The response is negative and its absolute value increases with q , becoming slightly larger than the corresponding vector Δ contribution.

In Fig. 6 we show the interference responses $R_{T',1b2b}^{AV}$ between the axial one-body current and the vector MECs, separating the contributions of the vector seagull, pionic, and Δ currents. These responses exhibit a behavior very similar to the corresponding $R_{T',1b2b}^{VV}$ ones. In fact, by inspecting the expressions for the nonrelativistic single-nucleon responses, one finds that $w_{T',m2b}^{VV}$ (interference with magnetization current) and $w_{T',1b2b}^{AV}$ involve exactly the same integrals over the intermediate nucleon momentum $\mathbf{k}$. The difference lies in the coupling factors between the axial current and the magnetization current. Since the convection current has little impact on the transverse responses, the resulting behavior is nearly identical in both cases.

Finally, in Fig. 7 we show the $R_{T',1b\Delta}^{VA}$ interference response. Once again, we observe a close similarity between this response and both $R_{T',1b\Delta}^{AV}$ and $R_{T',1b\Delta}^{VV}$, for the same reasons discussed previously. This similarity originates from the fact that the structure of the Δ current and the magnetization current is nearly the same in the axial and vector sectors.

The main difference lies in a vector product with $\vec{q}$, while the spin operator is identical in both cases, except for different coupling constants associated with the axial and vector currents. This can also be seen by inspecting the corresponding single-nucleon tensors, which involve the same integrals in all three cases.

In all previous results, the sign of the one-body- Δ interference responses is always found to be negative. This result can be clearly understood by inspecting the expressions of the single-nucleon response functions, Eqs. (105), (108), (111), and (112). All of them carry an overall minus sign and are proportional to the same integral, which is positive since the integrand contains terms of the form $3q^2k_1^2 - (\mathbf{q} \cdot \mathbf{k}_1)^2 > 0$. This result for the sign of the $1b$ - Δ interference was already pointed out in Ref. [31] for the electromagnetic transverse response. A similar argument applies to the $R_{T,m\pi}^{VV}$ and $R_{T',1b\pi}^{AV}$ responses, which are also negative. Although these sign arguments are strictly valid in the nonrelativistic limit, numerical calculations show that they remain valid in the relativistic case, except at very large q , where sign changes can occur depending on the value of ω .

B. Results in the RFG model

We now introduce the results obtained within the RFG model. In this section, we apply the formalism to the case of ^{12}C , using a Fermi momentum $k_F = 225 \text{ MeV}/c$.

In Fig. 8 we show the longitudinal responses R_{CC} , R_{CL} , and R_{LL} , while in Fig. 9 we display the transverse responses R_T and $R_{T'}$. For each response, we present three curves: the pure one-body (1b) contribution, the full result including MECs (1b+mec), and the interference term between one-body and two-body currents (mec). It is worth noting that the full results comprise both the interference term and the (small) standalone contribution from MECs.

It is observed that the effect of MECs in the R_{CC} response is a very small increase. In the R_{CL} response, MECs introduce a visible modification, and in the R_{LL} response they are of the same order of magnitude as the one-body part for small values of q . Nevertheless, this will have little or no impact on neutrino cross sections, since R_{LL} is very small and longitudinal responses in general contribute less significantly than the transverse ones to the total cross section.

Figure 9 shows the transverse responses R_T and $R_{T'}$ in the relativistic Fermi gas model. The interference is negative and leads to a significant reduction of both responses. This reduction was also found in the electromagnetic transverse response [31]. This behavior is due to the interference with the Δ current that gives a dominant contribution in the relevant kinematics.

The MEC-induced suppression of the transverse neutrino responses is of similar magnitude in the vector and axial sectors. To illustrate this point, in Fig. 10 we show separately the vector-vector (VV) and axial-axial (AA) contributions to the transverse response R_T . It can be seen that the reduction induced by MECs, relative to the one-body current, is similar in the two channels. In both cases, the dominant effect is the interference between the one-body current and the Δ current, which leads to a negative contribution.

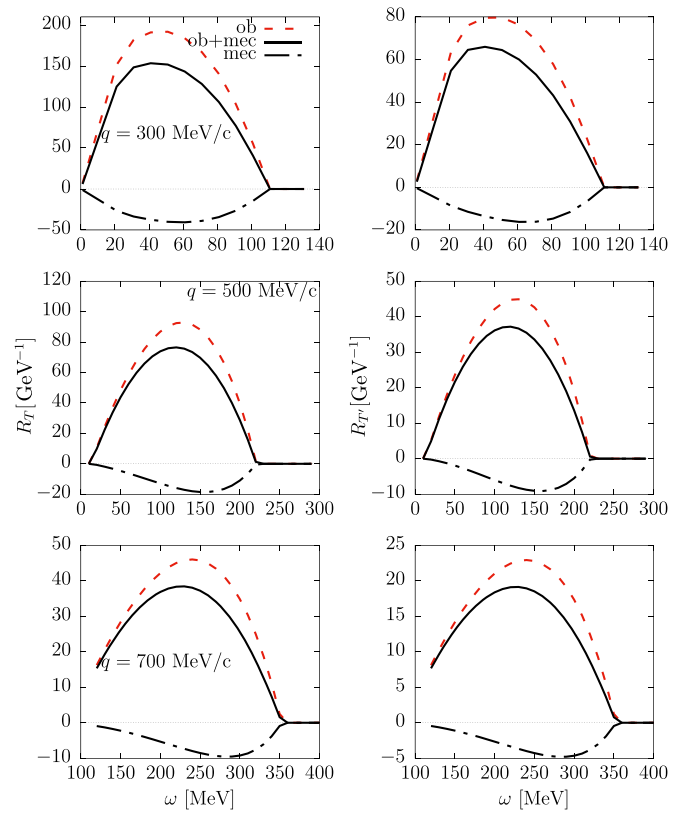
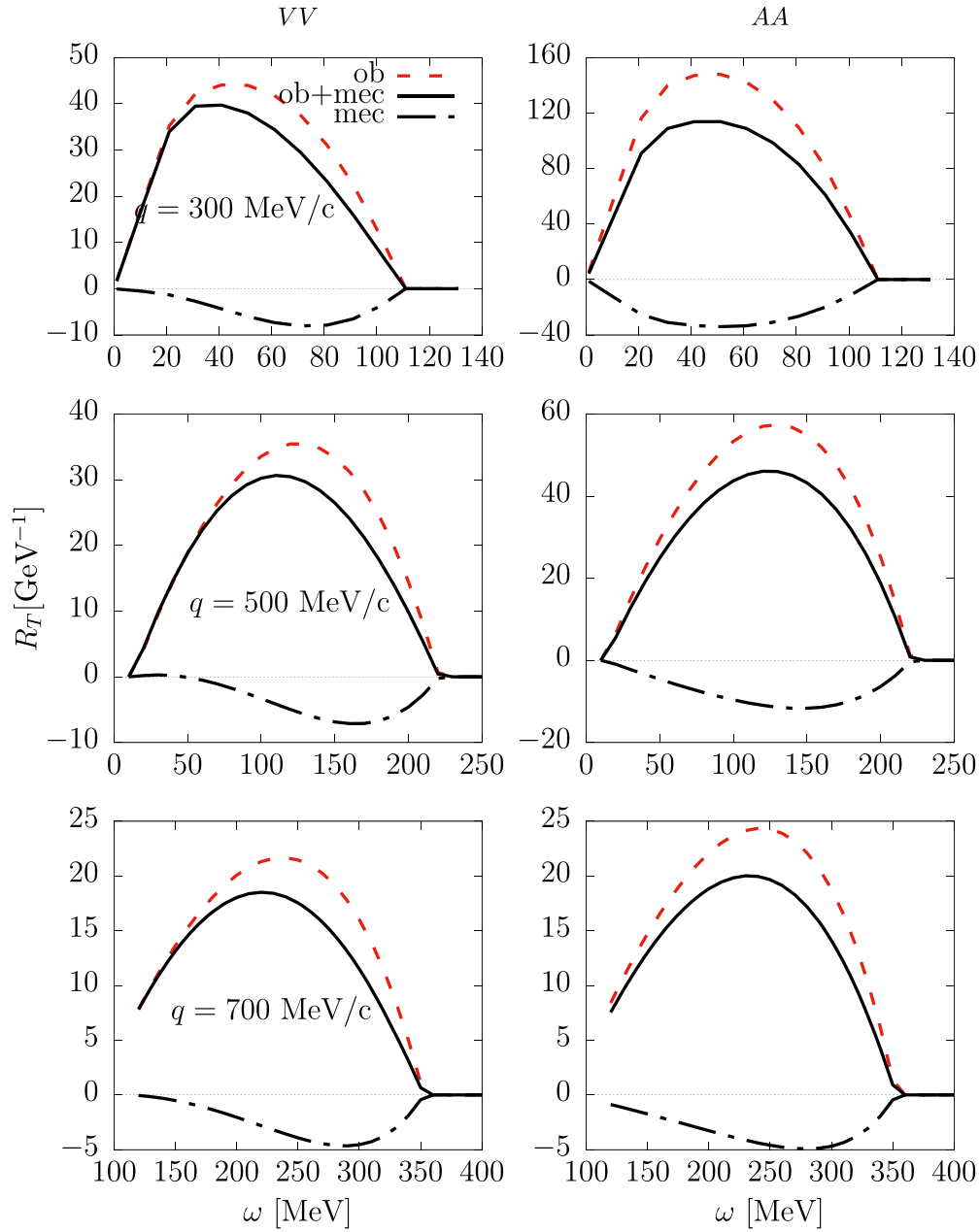


FIG. 9. The same as in Fig. 8 but for the transverse responses R_T and $R_{T'}$.

A more detailed scrutiny of the relevance of each MEC contribution to the different response functions is provided in Figs. 11 and 12, where all the interference terms are shown separately for the seagull, pionic, pion-pole, and Δ currents. The effect of MECs on the longitudinal responses is diverse (Fig. 11), although these interferences have a limited impact on the total responses and an even smaller one on the neutrino cross section. For example, the pionic and seagull contributions tend to cancel each other in the VV -type R_{CC} , R_{CL} , and R_{LL} responses. In the axial R_{CC} , the seagull clearly dominates, as anticipated in the nonrelativistic development, while in the axial R_{CL} and R_{LL} responses a non-negligible contribution from the Δ current is observed, since the axial Δ is no longer purely transverse. The pion pole is negligible. Nevertheless, we reiterate that the overall impact of these longitudinal interferences on observables is minimal. The MEC effect that plays a significant role appears in the transverse responses of Fig. 12, where the Δ current is clearly dominant and produces a sizable reduction of the response, partially compensated by the seagull contribution.

An important outcome of this analysis is the identification of a sizeable negative interference between the one-body and Δ -current contributions in the axial channel, particularly in the transverse and axial-transverse responses. This effect, which to our knowledge has not been previously highlighted in the literature on quasielastic neutrino scattering, is a novel result of this work. It is especially relevant for neutrino

FIG. 10. The same as Fig. 9 but for the R_T^{VV} and R_T^{AA} response functions.

interactions, where the axial current plays a central role. Since current neutrino event generators and models often neglect such interference terms and treat MEC contributions only as a 2p2h additive term, this finding suggests that existing models may require revision in order to properly account for interference effects, especially those involving the Δ current in axial channels.

C. Relativistic mean field and superscaling

In this subsection we present results obtained with two additional nuclear models beyond the RFG: the RMF model of nuclear matter and the superscaling analysis with relativistic effective mass.

1. RMF

In this model, nucleons interact via a relativistic constant mean field composed of scalar and vector potentials, as described in the Walecka model [49]. As a result, nucleons acquire an effective mass m_N^* due to their coupling to the scalar field, along with a constant vector potential energy E_v . The single-particle energy of nucleons in the RMF is thus given by

$$E_{\text{RMF}} = \sqrt{p^2 + (m_N^*)^2} + E_v. \quad (113)$$

The formalism for computing the nuclear response functions is modified accordingly: the on-shell energy $E = \sqrt{p^2 + m_N^{*2}}$

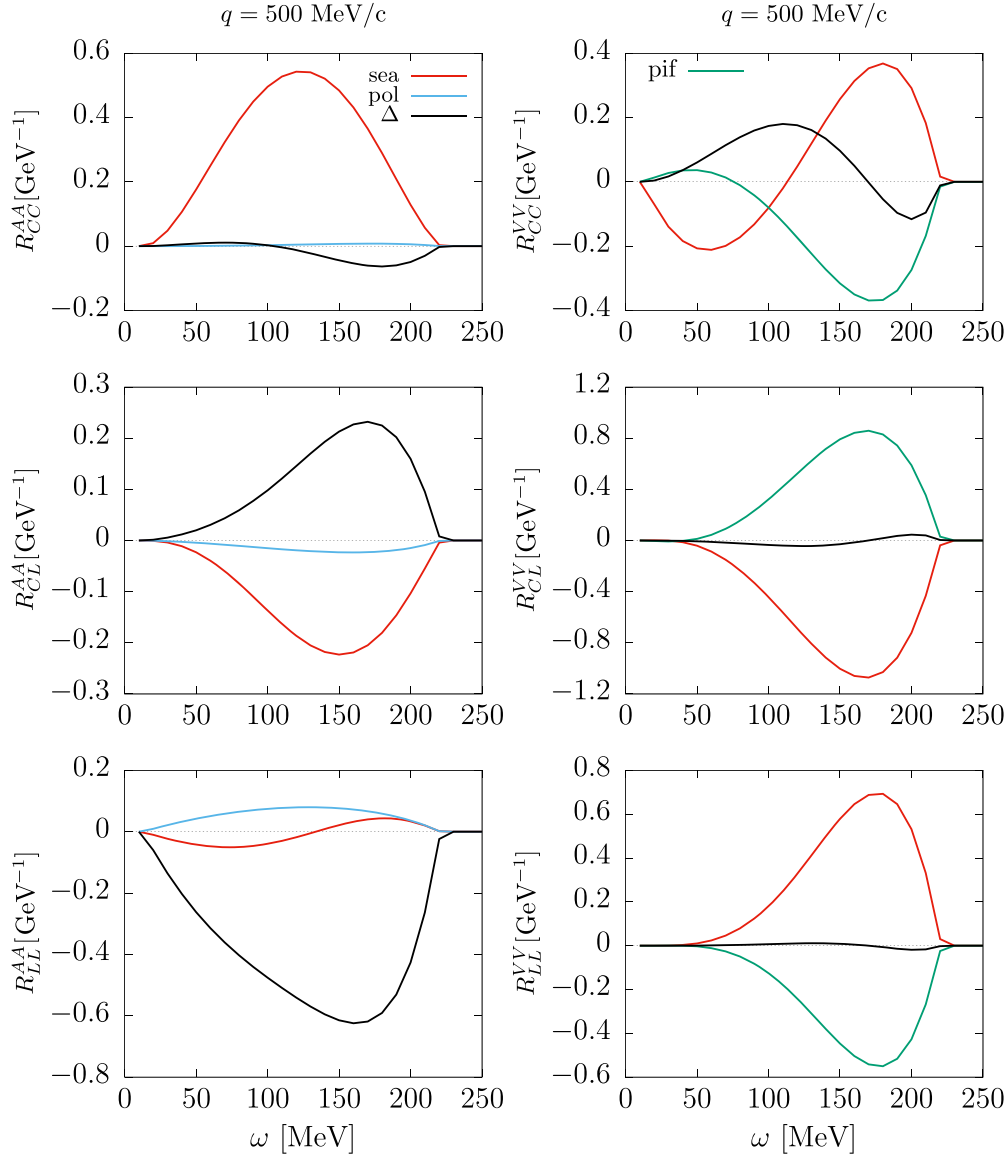


FIG. 11. Interference longitudinal responses between the 1b current and the different MECs seagull (sea), pionic (pif), pion pole (pol), and Δ , separated in vector and axial contributions, for $q = 500$ MeV/c.

is used in the kinematics, and the free Dirac spinors $u(p)$ are replaced by spinors constructed with the effective mass m_N^* . Therefore all the formulas from the previous section can be applied but replacing m_N by m_N^* everywhere except in the current operators, which remain unmodified in this model. Additionally, the vector potential energy E_v must be included in the Δ propagators to compute the four momenta $p + q$ and $p' - q$. With these prescriptions, the electromagnetic response functions including MEC in the 1p1h channel were evaluated in Ref. [40]. We now extend this framework to the weak sector, focusing on the role of MEC and interference effects in neutrino-nucleus reactions.

In Fig. 13, we compare the total five response functions, including 1b plus MEC contributions, calculated in the RFG and RMF models for three values of the momentum transfer. The main effect of the relativistic mean field is to produce a shift of the strength towards higher values of ω due to the use

of an effective mass for the nucleons. This shift effectively incorporates, through the mean field, part of the dynamical effects related to the binding and interaction energy of the nucleon in the final state.

2. SuSAM*

The SuSAM* model combines the superscaling approach with elements of the RMF model through the introduction of an effective mass. This approach is based on the assumption that the response functions can be factorized as the product of an averaged single-nucleon response and a phenomenological superscaling function, extracted from electron scattering data [40]. The SuSAM* model provides an alternative to the SuSA approach, using a single universal scaling function for all channels, and it allows for the inclusion of MEC effects both in the generalized single-nucleon responses and in the

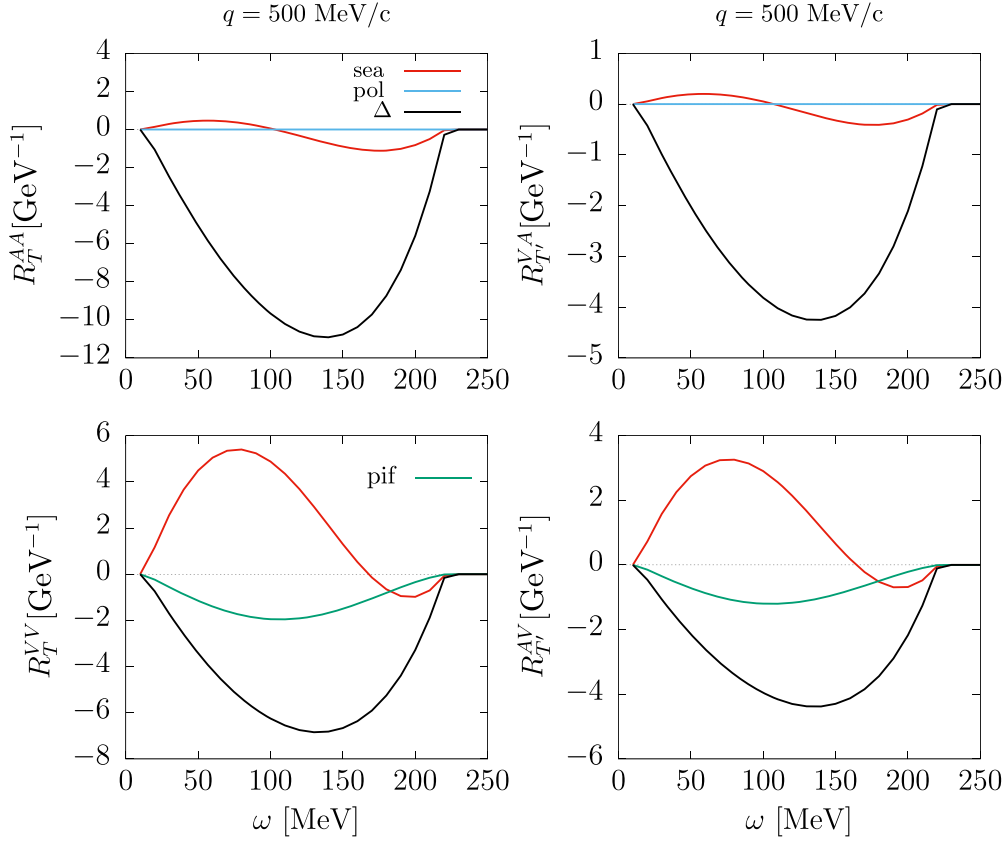


FIG. 12. The same as Fig. 11 but for the transverse responses.

scaling function itself. The model was originally developed to describe electromagnetic responses, and it is now extended to the weak sector including MEC contributions.

The starting point is to rewrite the RMF response functions, Eq. (58), in a factorized form by defining

- (1) an averaged single-nucleon response $\overline{w_K}$

$$\overline{w_K}(q, \omega) = \frac{\int_{\epsilon_0}^{\infty} d\epsilon n(\epsilon) w_K(\epsilon, q, \omega)}{\int_{\epsilon_0}^{\infty} d\epsilon n(\epsilon)}, \quad (114)$$

- (2) a scaling function

$$f^*(\psi^*) = \frac{3}{4} \frac{1}{\epsilon_F - 1} \int_{\epsilon_0}^{\infty} d\epsilon n(\epsilon), \quad (115)$$

- (3) and a scaling variable

$$\psi^* = \sqrt{\frac{\epsilon_0 - 1}{\epsilon_F - 1}} \text{sgn}(\lambda - \tau). \quad (116)$$

Using $V/(2\pi^3) = N/(\frac{8}{3}\pi k_F^3)$, where N is the number of neutrons, the responses can be written in the factorized form

$$R_K(q, \omega) = \frac{\epsilon_F - 1}{m_N^* \eta_F^3 \kappa} N \overline{w_K}(q, \omega) f^*(\psi^*), \quad (117)$$

where $\eta_F = k_F/m_N^*$. In the RMF (and also the RFG in the particular case $m_N^* = m_N$) the condition $\epsilon_0 < \epsilon_F$ restricts the kinematical region in ω where the response is nonzero, corresponding to the interval $-1 < \psi^* < 1$ in terms of the scaling

variable. In the SuSAM* model, this region is extended beyond the RFG boundaries using a phenomenological scaling function, $f_{\text{ph}}^*(\psi^*)$, extracted from inclusive electron scattering data.

However, there is the problem that the averaged single nucleon response in Eq. (114) is not defined for $\epsilon_0 > \epsilon_F$ (or $|\psi^*| > 1$) because the denominator is zero. To apply the model beyond the RFG limits, one must extrapolate the averaged single-nucleon responses outside the physically allowed region (as in the SuSA approach), which is not feasible when MEC are included, unless an explicit analytical expression is available. The solution adopted in Ref. [40] was to slightly modify the momentum distribution of the Fermi gas by introducing a smooth smearing function of Fermi type, which allows all the values of ϵ_0 :

$$n(p) = \frac{1}{1 + \exp[(p - k_F)/a]}, \quad (118)$$

where k_F is the Fermi momentum and $a \approx 25$ MeV is a diffuseness parameter that controls the smoothness of the falloff. This modification allows one to compute the averaged single-nucleon responses $\overline{w_K}$ in a broader kinematic domain and to include the contribution of MECs in a consistent way (see [40,58] for details).

In the SuSAM* approach, we use Eq. (117) together with the phenomenological scaling function extracted in Ref. [40]. The results for the interference $1b$ -MEC response functions are shown in Figs. 14 and 15 for $q = 500$ MeV/ c . As we can

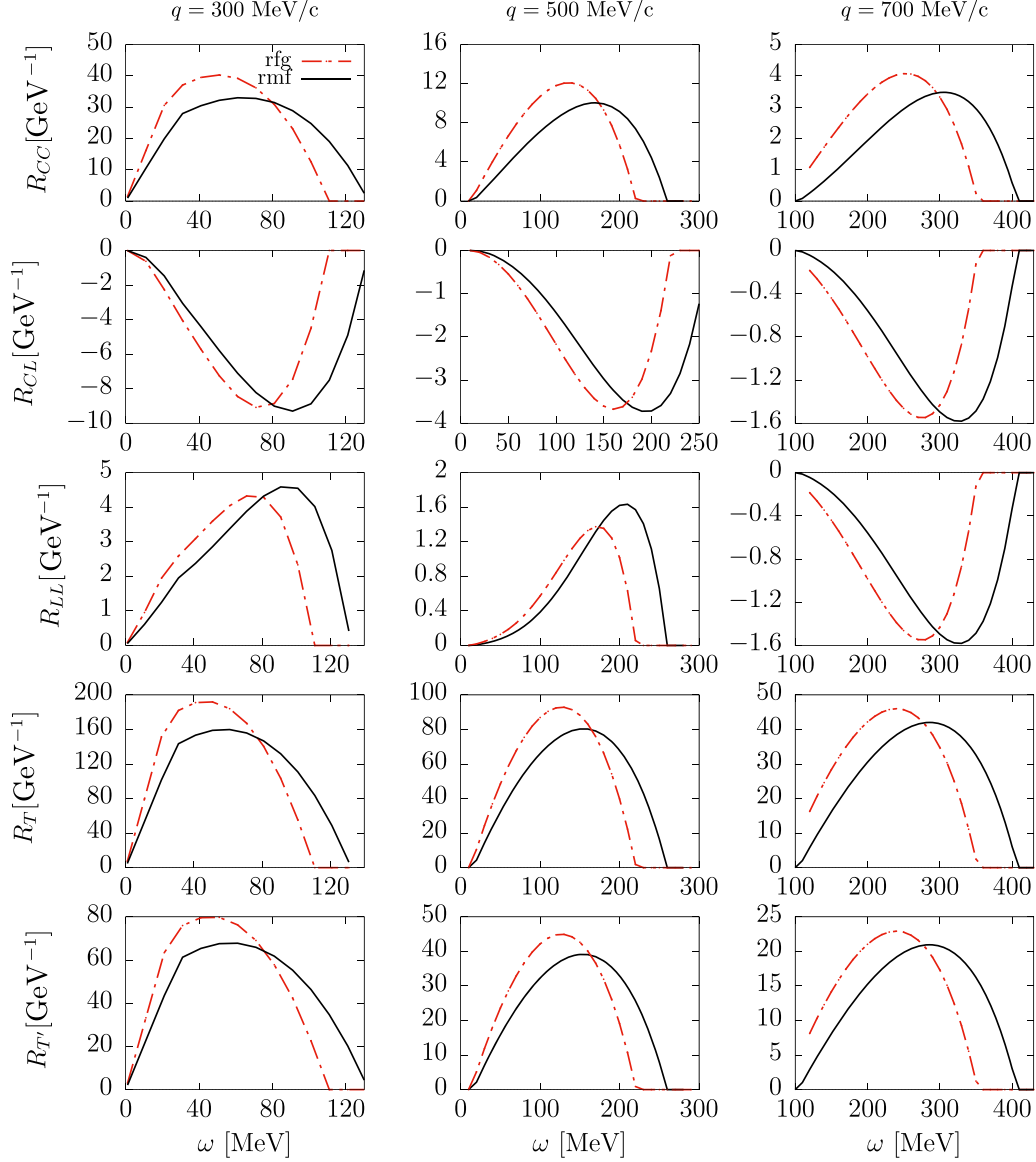


FIG. 13. Comparison of the five total (1b+MEC) response functions R_{CC} , R_{CL} , R_{LL} , R_T , and $R_{T'}$ computed within the RFG and the RMF models for momentum transfers $q = 300, 500$, and 700 MeV/ c .

see, the interference responses extend well beyond the allowed region of the RFG, enabling the estimation of MEC effects at large ω values. The phenomenological scaling function was parametrized as a sum of two Gaussians, which explains why some of the responses display two peaks. It is also apparent that in the transverse responses the interference remains negative due to the Δ current.

In Fig. 16 we compare the total response functions computed in RMF and SuSAM*. The longitudinal responses CL and LL are small and contribute little to the neutrino cross section. For the dominant responses—the transverse (T , T') and charge-charge (CC) ones—the effect of the SuSAM* model is to introduce a high-energy tail in the responses. The relative size of the responses can be clearly seen in this figure for $q = 500$ MeV/ c : the T response reaches a maximum of ≈ 60 GeV $^{-1}$, T' is about half of that (≈ 30 GeV $^{-1}$), CC peaks at ≈ 10 GeV $^{-1}$, CL at ≈ -4 GeV $^{-1}$, and LL at ≈ 1.5 GeV $^{-1}$.

This shows that the CC contribution is relatively small, that of CL even smaller, and the LL contribution is almost negligible. Note, however, that each response is weighted by a different kinematic factor v_K in the cross section, Eq. (2). In particular, the LL response is so small that it becomes extremely sensitive to fine details of the model, but such differences are likely to be unobservable in the total cross section, which is largely dominated by the transverse responses and, to a lesser extent, by the CC component.

D. Cross section

In this final section we present results for the neutrino (ν_μ, μ^-) and antineutrino ($\bar{\nu}_\mu, \mu^+$) inclusive cross sections. To compare theoretical predictions with experimental data, the double differential cross section, expressed as a function of the muon kinetic energy and the scattering angle, must

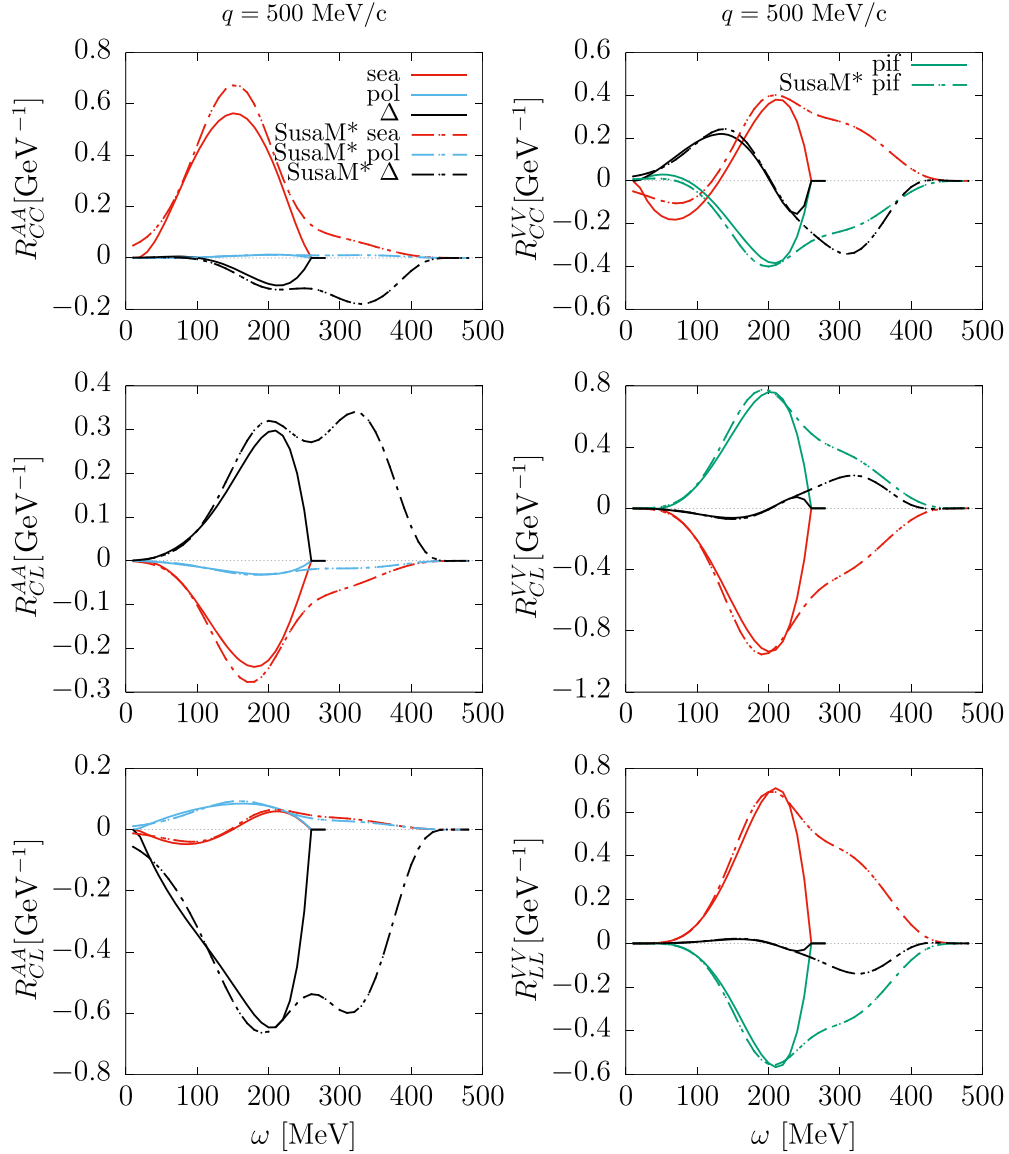


FIG. 14. Longitudinal 1b-MEC interference responses in the SuSAM* approach compared to the RMF model for $q = 500$ MeV/ c .

be integrated over the neutrino flux. The flux-averaged cross section is defined as:

$$\frac{d^2\sigma}{dT_\mu d\cos\theta} = \frac{1}{\Phi_{\text{tot}}} \int dE_\nu \Phi(E_\nu) \frac{d^2\sigma}{dT_\mu d\cos\theta}(E_\nu), \quad (119)$$

where $\Phi(E_\nu)$ is the neutrino flux, $\frac{d^2\sigma}{dT_\mu d\cos\theta}(E_\nu)$ is the cross section evaluated at a fixed neutrino energy E_ν and Φ_{tot} is the total integrated flux,

$$\Phi_{\text{tot}} = \int dE_\nu \phi(E_\nu). \quad (120)$$

The experimental data are typically provided in bins of $\cos\theta$, where θ is the scattering angle of the outgoing muon. For each bin, what is actually given is the cross section averaged over

the bin width, which implies an integration over $\cos\theta$.

$$\begin{aligned} & \left\langle \frac{d^2\sigma}{dT_\mu d\cos\theta} \right\rangle_{\text{Bin}} \\ &= \frac{1}{\Delta \cos\theta} \int_{\cos\theta_i}^{\cos\theta_f} \frac{d^2\sigma}{dT_\mu d\cos\theta}(\cos\theta) d\cos\theta. \end{aligned} \quad (121)$$

In Figs. 17 and 18 we present results for the double-differential charged-current neutrino and antineutrino cross sections, respectively, corresponding to the kinematics and flux of the MiniBooNE experiment [20,59]. The experimental data correspond to the fully inclusive CCQE measurements, without any kinematical cuts, where the $\text{CC}1\pi^+$ interaction with intranuclear pion absorption has been subtracted. Each panel shows the cross section for a given $\cos\theta$ bin, with bin width $\Delta \cos\theta = 0.1$, as a function of the kinetic energy of the outgoing muon. A broad peak is observed, which arises from

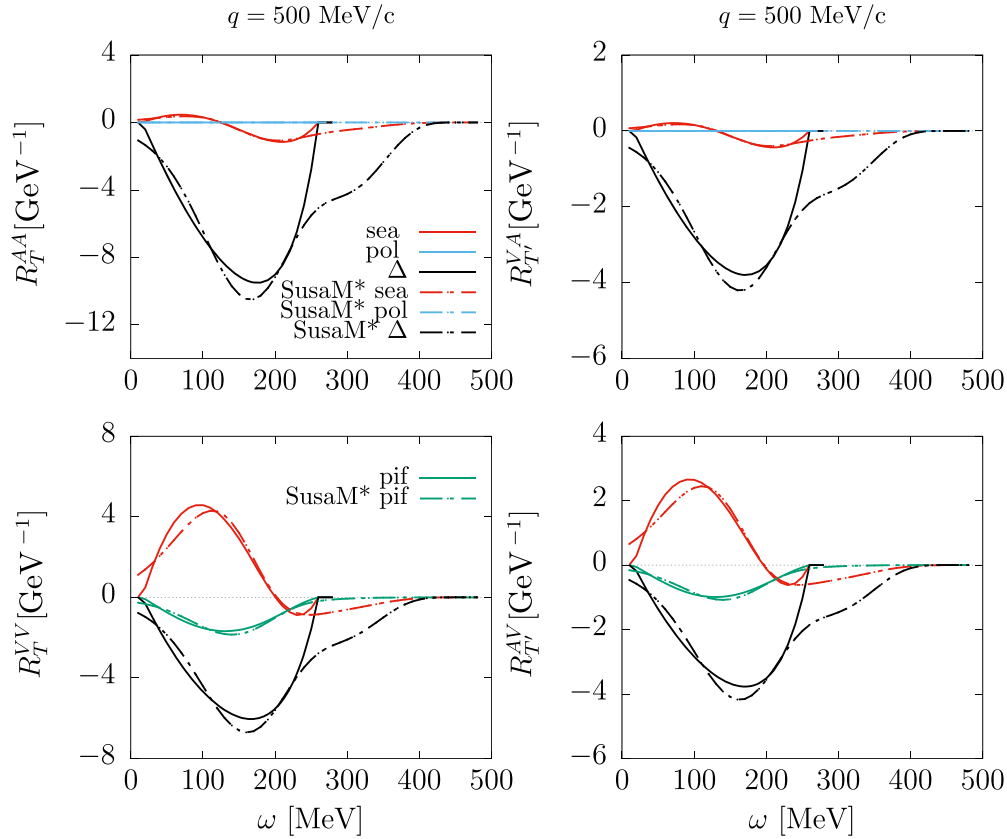


FIG. 15. The same as Fig. 14 but for the transverse interference responses.

an average over many cross sections corresponding to different values of the incident neutrino energy E_ν , weighted with the flux. This averaging produces a much broader shape than what would be expected from a quasielastic cross section at fixed E_ν .

The theoretical calculations have been performed using the RMF model with effective mass $M^* = 0.8$ and Fermi momentum $k_F = 225$ MeV/c, corresponding to ^{12}C .

In Figs. 17 and 18 we show the one-body (1b) results together with the sum of 1b and the 1b–2b interference. One observes that the interference with MEC produces a reduction of the cross section of about 10%. For antineutrinos the effect is smaller due to the partial cancellation between the T and T' responses. In the same figures we also show separately the 1b–2b interference and the 2p2h contributions for comparison. The 2p2h responses, computed in Ref. [60], are positive, somewhat larger than the interference, and display their maximum at lower energy transfer. As a consequence, the two contributions partially cancel each other, although the 2p2h still dominates. Therefore, one expects that when both are summed the net effect would be an enhancement smaller than that produced by the 2p2h contribution alone. We have not included this sum in the figure, since the focus of the present work is on the interference, but it is clear that both effects, 2p2h and 1b–2b interference, are of comparable size.

Other models in the literature often neglect the interference between one-body and two-body currents, and their predictions vary significantly among them, contributing to the theoretical systematic uncertainties in neutrino oscillation

experiments. Our results indicate that the 1b–2b interference constitutes an additional source of uncertainty that should also be taken into account.

Overall, the agreement with experimental data is good at very forward angle, while the theoretical calculation tends to underestimate the data as θ increases. This is expected given that the 2p2h contributions have not been included yet.

In order to gain deeper insight into the flux-folded results of Figs. 17 and 18, in Fig. 19 we show the impact of the interference term on double-differential cross sections at fixed neutrino energies and different scattering angles. Specifically, results are presented for $\cos\theta = 0.95, 0.75, 0.55$ and 0.05 . Because the cross section varies rapidly at forward angles, the case $\cos\theta = 0.95$ is averaged over the bin $\cos\theta \in [0.9, 1]$, corresponding to scattering angles between 0° and 25° . For larger angles the cross section varies smoothly within the bin, and it is sufficient to show the result at the midpoint, corresponding to $41^\circ, 57^\circ$ and 87° , respectively. Each panel displays results for incident neutrino energies of 0.5 and 1 GeV, which approximately span the region where the flux is most intense. For each kinematics we show the cross section obtained with the 1b current and after including the interference (1b+2b). The results indicate that the interference effect depends only weakly on kinematics, consistent with the behavior observed in Figs. 17 and 18. One might expect that the transverse response becomes more relevant at larger angles, thereby increasing the relative effect of MEC. This is also illustrated in Fig. 19, where the separated contributions of the CC , T , and T' responses are shown (the

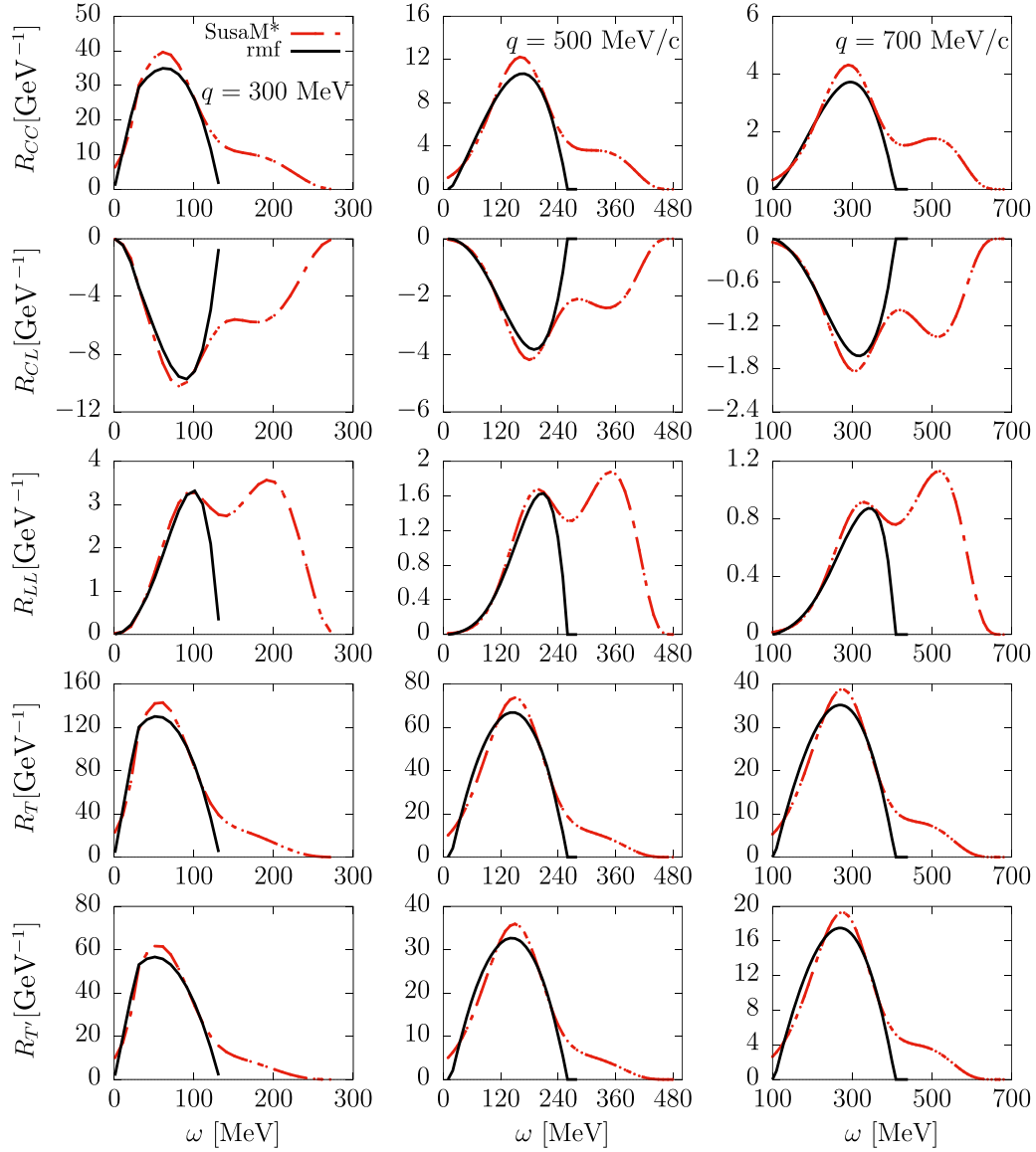


FIG. 16. Total responses in the RMF and SuSAM* models for momentum transfers $q = 300, 500, 700$ MeV/ c .

CL and LL components are not displayed because they are smaller). For forward angles the CC response is larger than T' , but the T response dominates. This arises because in neutrino scattering the axial current contributes significantly to T , while its role in CC is minor. With increasing angle the CC contribution decreases relative to T and T' , and the T' component becomes more important. In summary, except for very small angles, the neutrino cross section is dominated by the transverse response, which explains why the relative impact of MEC does not vary strongly with kinematics. A more detailed study would be required to quantify in depth the kinematic dependence of MEC effects.

V. DISCUSSION AND CONCLUDING REMARKS

In this work we have presented a detailed study of meson-exchange currents in the $1p1h$ channel of CC neutrino and antineutrino scattering on nuclei. Our analysis focused on

the interference between one-body and two-body currents, an effect often neglected in neutrino event generators but that, as we have shown, can be sizable and may contribute to the systematic uncertainties in oscillation experiments.

The formalism employed was developed starting from the relativistic Fermi gas model, including $1p1h$ excitations produced by one-body and two-body current operators. In this context, we computed the five nuclear response functions (CC , CL , LL , T , and T') including the interference between the $1b$ current and the seagull, pion-in-flight, pion-pole, and Δ currents. The nonrelativistic limit of the response functions has been carefully examined as a preliminary step to validate the relativistic model; in this limit, the spin sums can be calculated analytically, allowing us to understand the signs and relative importance of the dominant terms. In particular, we found that the Δ current gives rise to a strong negative interference in the transverse responses T and T' . This is a new and important result, which is absent in existing neutrino

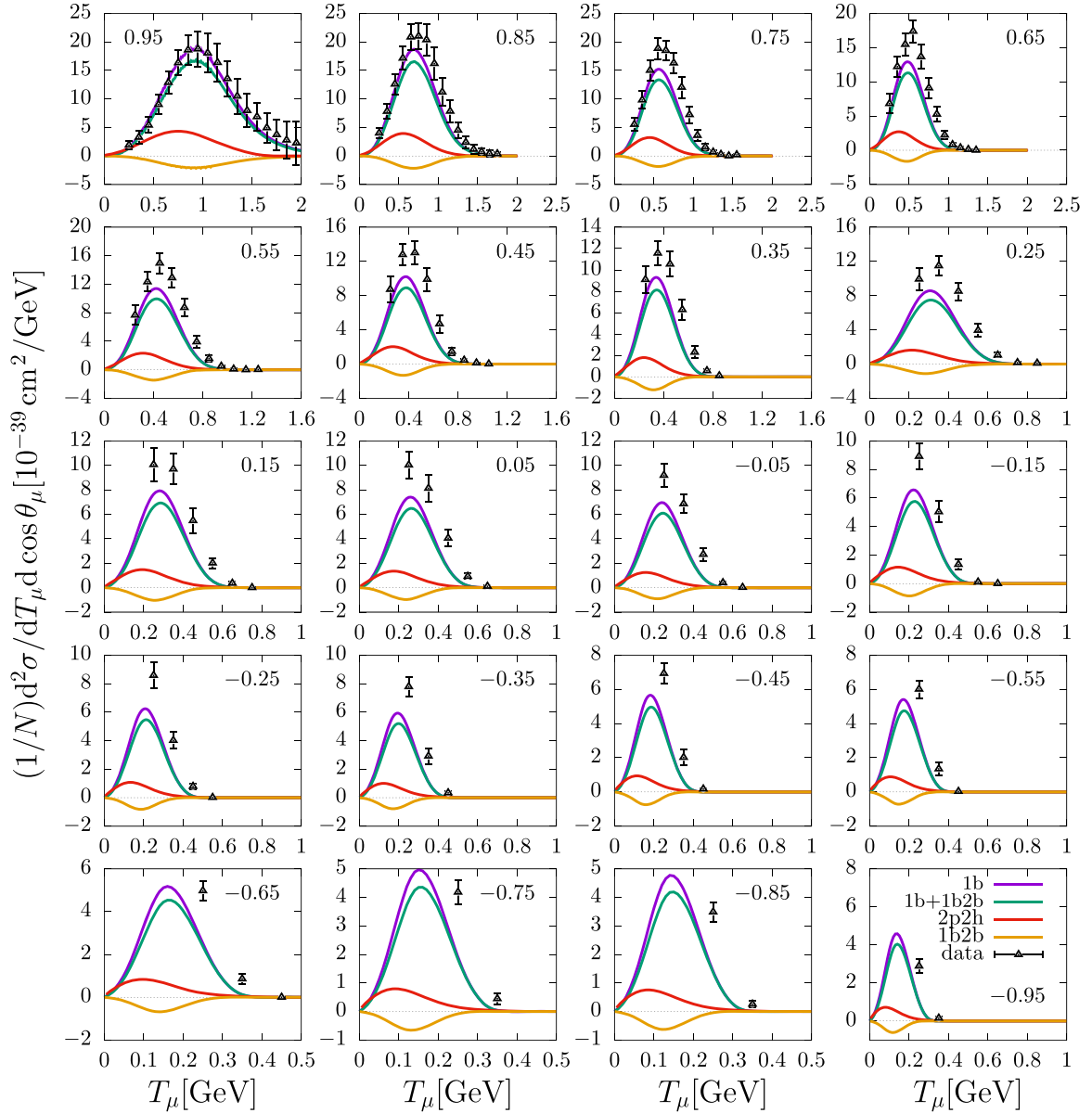


FIG. 17. Flux-integrated double-differential cross section per neutron for CC neutrino scattering on ^{12}C in the RMF model. The experimental points are the inclusive CCQE measurements from MiniBooNE [20]. Shown are the one-body (1b) results, the 1b-2b interference (1b2b), their sum (1b+1b2b), and the 2p2h contribution from Ref. [60] computed within the same RMF model.

models and calls for a careful reevaluation of the role of 1b-2b interference terms in neutrino-nucleus scattering.

To assess the model dependence of the interference effects, we extended the analysis beyond the RFG by employing two additional nuclear models: the relativistic mean field in nuclear matter, and the superscaling approach with effective mass, SuSAM*. In the RMF, the interaction of the nucleons with scalar and vector mean fields modifies their dispersion relation through an effective mass m_N^* and a vector energy shift. These modifications affect the response functions by shifting their peak positions and redistributing strength, thereby mimicking to some extent the effects of binding and final-state interactions in a more realistic manner.

The SuSAM* model, on the other hand, incorporates the relativistic effective mass from RMF into the superscaling

framework. It assumes a factorization of the nuclear response into a single-nucleon response multiplied by a phenomenological scaling function, extracted from inclusive (e, e') data. One of the advantages of the SuSAM* approach is that it effectively incorporates nuclear effects beyond the Fermi gas model, such as final-state interactions (FSIs) and other complex mechanisms, which are phenomenologically embedded on average in the experimental scaling function. The SuSAM* model also enables a consistent inclusion of MEC effects in both the scaling function and the averaged single-nucleon responses.

Our results show that the 1b-2b interference contributions, though relatively small compared to the full cross section, are not negligible and may be comparable in size to other nuclear effects usually included in more sophisticated models.

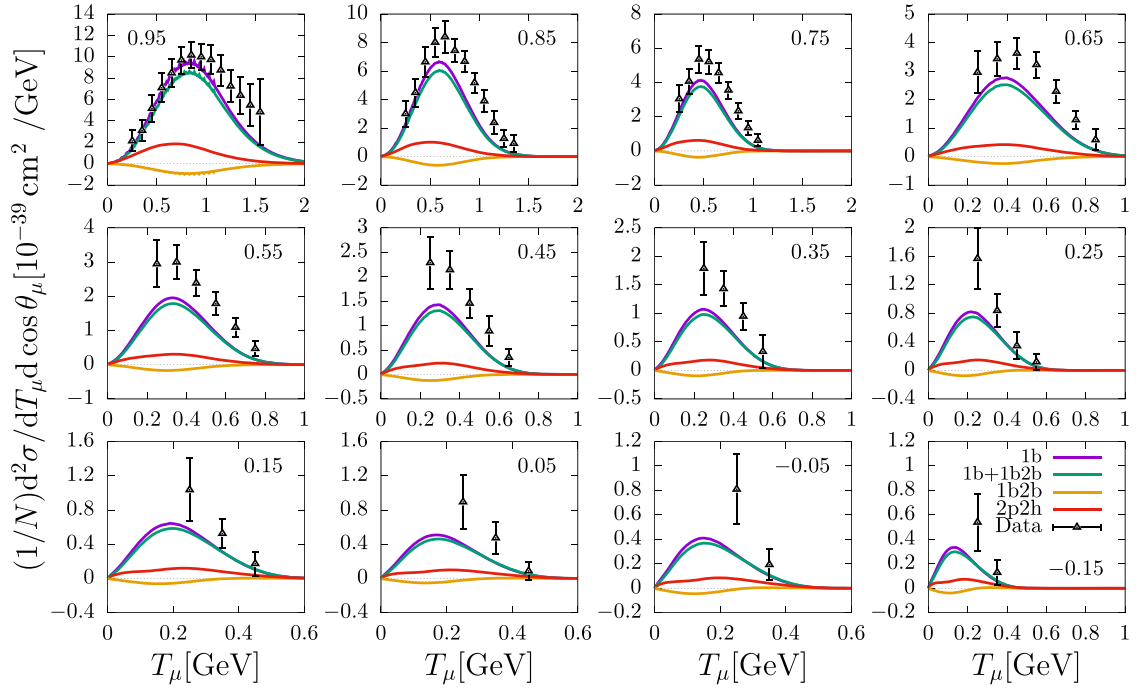


FIG. 18. The same as Fig. 17 but for the antineutrino cross section. Experimental data are the CCQE antineutrino measurements from MiniBoone [59].

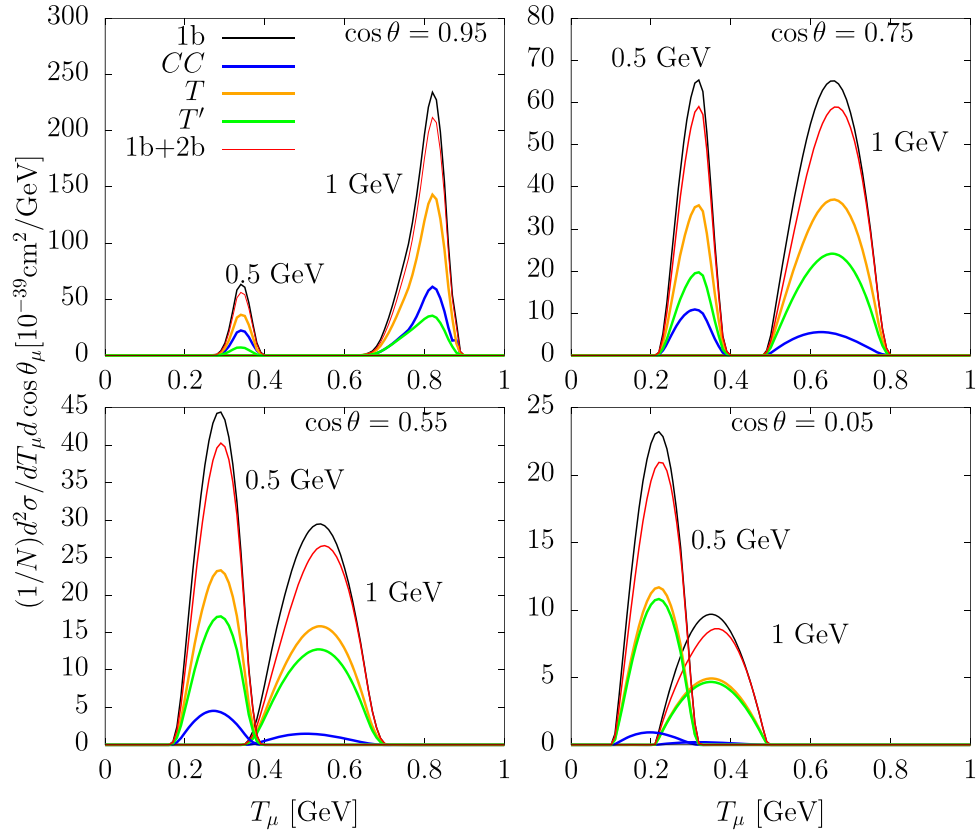


FIG. 19. Double-differential cross sections for CC neutrino scattering on ^{12}C in the RMF model at fixed neutrino energies $E_\nu = 0.5$ and 1 GeV. Results are shown for different scattering angles corresponding to $\cos \theta = 0.95, 0.75, 0.55$, and 0.05 . For the most forward bin, $\cos \theta = 0.95$, the cross section is averaged over $\cos \theta \in [0.9, 1]$, corresponding to angles between 0° and 25° . In each panel we display the results with the one-body current alone (1b) and including the interference contribution (1b+1b2b). The 1b contributions of the CC, T, and T' responses are also shown separately.

For instance, effects such as long-range correlations modeled via the random phase approximation [9,61], final-state interactions (FSIs) in energy-dependent RMF approaches [62], finite-size shell effects in Hartree-Fock based models [63], or GiBUU transport theory [6], all modify the shape and size of the nuclear response. The presence of a non-negligible 1b-2b interference implies that those models may need to be revisited or extended to estimate at least the order of magnitude of this contribution and its possible interplay with the aforementioned mechanisms.

The transverse enhancement observed in calculations using correlated wave functions [42] or Green's function Monte Carlo [43] may point to nuclear dynamics beyond the mean-field approximation, possibly involving spin-entangled two-body structures that interact with the MEC operator but are averaged out in mean-field models. This may help explain the enhancement seen in electron scattering data [64,65], although in GFMC calculations it is not possible to separate 1p1h and 2p2h final states. This problem requires a shift in perspective that is currently under investigation.

In view of the present study, it is pertinent to comment on recent results reported in Refs. [28–30], where a positive T interference between one- and two-body currents in electron scattering has been obtained in the RMF and spectral function models. This outcome contrasts with our comparative analyses across several models of Ref. [31], where the interference has consistently been found to be negative. In particular, the positive result of Refs. [28,29] is due to a Δ current with a sign opposite to the standard operator, as was explicitly demonstrated in the Appendix G of Ref. [31]. A similar feature can be identified in Ref. [30], where the propagator of the Δ resonance is taken with opposite sign. On the other hand, Ref. [43] employs a fully correlated model with the standard operator, in which case the enhancement plausibly originates from the role of correlations, consistent with the findings of Ref. [42].

Although in this paper we have worked with some of the most elementary nuclear models, this choice was necessary in order to formulate the problem rigorously and within a fully reproducible framework. We believe that this strategy provides a solid baseline that will allow more realistic models to incorporate and test the effects we have identified. Our hope is that this study serves as a starting point for a deeper investigation and helps indicate the direction in which further theoretical efforts should proceed.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and

hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: ISOSPIN SUMMATIONS IN THE 1P1H MEC MATRIX ELEMENT

Here we provide the sums over the isospin index t_k of the terms appearing in the 1p1h MEC matrix element. We first note that the MEC can be written as a linear combination of $\tau_{\pm}^{(1)}$ and $\tau_{\pm}^{(2)}$ and $i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_{\pm}$,

$$j_{2b}^{\mu} = \tau_{\pm}^{(1)} j_1^{\mu} + \tau_{\pm}^{(2)} j_2^{\mu} + i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_{\pm} j_3^{\mu}, \quad (\text{A1})$$

where the 2b currents j_1^{μ} , j_2^{μ} , and j_3^{μ} are isospin independent. To be specific, we will consider the case of the (+) component of the isospin current, corresponding to $N \rightarrow P$ transitions:

$$\tau_{+} = \tau_1 + i\tau_2 = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}, \quad (\text{A2})$$

$$i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_{+} = \tau_{+}^{(1)} \tau_3^{(2)} - \tau_3^{(1)} \tau_{+}^{(2)}, \quad (\text{A3})$$

from which we obtain

$$\tau_{+}|P\rangle = 0, \quad \tau_{+}|N\rangle = 2|P\rangle, \quad (\text{A4})$$

$$i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_{+}|NP\rangle = 2|PP\rangle,$$

$$i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_{+}|NN\rangle = -2|PN\rangle + 2|NP\rangle,$$

$$i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_{+}|PP\rangle = 0,$$

$$i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_{+}|PN\rangle = -2|PP\rangle. \quad (\text{A5})$$

From these elementary results, it is straightforward to compute the isospin sums. The results are the following.

a. Direct terms:

$$\sum_{t_k=\pm 1/2} \langle Pt_k | \tau_{+}^{(1)} | Nt_k \rangle = 4, \quad (\text{A6})$$

$$\sum_{t_k} \langle Pt_k | \tau_z^{(2)} | Pt_k \rangle = 0, \quad (\text{A7})$$

$$\sum_{t_k} \langle Pt_k | i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_{+} | Nt_k \rangle = 0. \quad (\text{A8})$$

b. Exchange terms:

$$\sum_{t_k} \langle Pt_k | \tau_{+}^{(1)} | t_k N \rangle = 2, \quad (\text{A9})$$

$$\sum_{t_k} \langle Pt_k | \tau_{+}^{(2)} | t_k N \rangle = 2, \quad (\text{A10})$$

$$\sum_{t_k} \langle Pt_k | i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_{+} | t_k N \rangle = -4. \quad (\text{A11})$$

Similar results are obtained for the $P \rightarrow N$ transition with the (−) isospin components.

After performing these sums the effective one-body matrix element $j_{2b}^{\mu}(\mathbf{p}, \mathbf{h})$ induced by the two-body current, Eq. (16), can be written as sum of direct minus exchange currents:

$$j_{2b}^{\mu}(\mathbf{p}, \mathbf{h}) = j_{2b}^{\mu}(\mathbf{p}, \mathbf{h})_{\text{dir}} - j_{2b}^{\mu}(\mathbf{p}, \mathbf{h})_{\text{exch}}, \quad (\text{A12})$$

where

$$j_{2b}^\mu(\mathbf{p}, \mathbf{h})_{\text{dir}} = \frac{1}{V} \sum_{\mathbf{k}} \sum_{s_k} 4j_1^\mu(\mathbf{p}, \mathbf{k}, \mathbf{h}, \mathbf{k}), \quad (\text{A13})$$

$$j_{2b}^\mu(\mathbf{p}, \mathbf{h})_{\text{exch}} = \frac{1}{V} \sum_{\mathbf{k}} \sum_{s_k} [2j_1^\mu(\mathbf{p}, \mathbf{k}, \mathbf{k}, \mathbf{h}) + 2j_2^\mu(\mathbf{p}, \mathbf{k}, \mathbf{k}, \mathbf{h}) - 4j_3^\mu(\mathbf{p}, \mathbf{k}, \mathbf{k}, \mathbf{h})]. \quad (\text{A14})$$

APPENDIX B: RELATION BETWEEN CC AND EM INTERFERENCE RESPONSES IN THE VECTOR SECTOR

In this Appendix, we demonstrate that the interference responses between the one-body and two-body currents in CC neutrino scattering in the vector sector are exactly twice the corresponding electromagnetic responses when both proton and neutron emission are summed. This relation holds in symmetric nuclear matter and follows from the isovector nature of both the CC and the electromagnetic two-body currents.

Indeed, the one-body electromagnetic and CC vector currents written as isospin operators are

$$j_{1b,e}^\mu = s^\mu + f^\mu \tau_z, \quad (\text{B1})$$

$$j_{1b,v}^\mu = f^\mu \tau_+, \quad (\text{B2})$$

where s^μ and f^μ are the isoscalar and isovector 1b currents, respectively.

Therefore the electromagnetic current in a 1p1h excitation between isospin states $|t_h\rangle$ and $|t_p\rangle$ is

$$\langle t_p | j_{1b,e}^\mu(\mathbf{p}, \mathbf{h}) | t_h \rangle = \delta_{t_p t_h} [s^\mu(\mathbf{p}, \mathbf{h}) + 2t_h f^\mu(\mathbf{p}, \mathbf{h})], \quad (\text{B3})$$

and the CC vector current in a neutron-to-proton transition is

$$\langle P | j_{1b,v}^\mu(\mathbf{p}, \mathbf{h}) | N \rangle = 2f^\mu(\mathbf{p}, \mathbf{h}). \quad (\text{B4})$$

The two-body MEC currents are written in a similar way:

$$j_{2b,v}^\mu = \tau_\pm^{(1)} j_1^\mu + \tau_\pm^{(2)} j_2^\mu + i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_\pm j_3^\mu, \quad (\text{B5})$$

$$j_{2b,e}^\mu = \tau_z^{(1)} j_1^\mu + \tau_z^{(2)} j_2^\mu + i[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_z j_3^\mu, \quad (\text{B6})$$

where the 2b vector currents j_1^μ , j_2^μ , and j_3^μ are isospin independent. Since the two-body current $(j_{2b})_V$ has no axial component, the corresponding effective one-body current contains only the exchange term, as given in Eq. (A14). Thus in a transition where h is a neutron and p is a proton, we have

$$\langle P | j_{2b,v}^\mu(\mathbf{p}, \mathbf{h}) | N \rangle = -j_{2b}^\mu(\mathbf{p}, \mathbf{h})_{\text{ex}}, \quad (\text{B7})$$

where $j_{2b}^\mu(\mathbf{p}, \mathbf{h})_{\text{ex}}$ is given by Eq. (A14) with only the vector part. On the other hand the electromagnetic current for a transition from isospin t_h to t_p was computed in Ref. [31], and is given by

$$\langle t_p | (j_{2b,e}^\mu)(\mathbf{p}, \mathbf{h}) | t_h \rangle = -\delta_{t_p t_h} t_h j_{2b}^\mu(\mathbf{p}, \mathbf{h})_{\text{ex}}. \quad (\text{B8})$$

Let us compute a generic contribution to the interference tensor $w_{1b2b}^{\mu\nu}$ of Eq. (23), such as $j_{1b}^{\mu*} j_{2b}^\nu$, with the one-body and two-body currents. In the case of neutrino scattering the vector part is

$$\langle P | j_{1b,v}^\mu | N \rangle^* \langle P | j_{2b,v}^\nu | N \rangle = -2f^{\mu*}(j_{2b}^\nu)_{\text{ex}}. \quad (\text{B9})$$

In the EM case it is

$$\langle t_p | j_{1b,e}^\mu | t_h \rangle^* \langle t_h | j_{2b,e}^\nu | t_h \rangle = -\delta_{t_p t_h} (s^\mu + 2t_h f^\mu)^* t_h (j_{2b}^\nu)_{\text{ex}}. \quad (\text{B10})$$

Performing the sum over isospin in the EM case,

$$\sum_{t_p t_h = \pm 1} \langle t_p | j_{1b,e}^\mu | t_h \rangle^* \langle t_h | j_{2b,e}^\nu | t_h \rangle = -f^{\mu*}(j_{2b}^\nu)_{\text{ex}}. \quad (\text{B11})$$

This factor of 2 between the CC (B9) and EM (B11) expressions thus applies to all interference response functions considered in this work.

APPENDIX C: NONRELATIVISTIC REDUCTION OF THE AXIAL Δ CURRENT

Here we compute the non relativistic axial Δ current to leading order in the $1/m_N$ expansion. The nonrelativistic reduction follows the same procedure as that used for the vector current in Ref. [31]. We start by writing the forward and backward relativistic currents (40), (41) in the form

$$j_{\Delta F}^\mu = U_F(1, 2) K_F^\mu + (1 \leftrightarrow 2), \quad (\text{C1})$$

$$j_{\Delta B}^\mu = U_B(1, 2) K_B^\mu + (1 \leftrightarrow 2), \quad (\text{C2})$$

where

$$K_F^\mu = \frac{f f^*}{m_\pi^2} C_5^A V(2'2) A^\mu, \quad (\text{C3})$$

$$K_B^\mu = \frac{f f^*}{m_\pi^2} C_5^A V(2', 2) B^\mu, \quad (\text{C4})$$

and we have defined

$$A^\mu = \bar{u}(1') k_2^\alpha G_{\alpha\beta}(p_1 + Q) g^{\beta\mu} u(1), \quad (\text{C5})$$

$$B^\mu = \bar{u}(1') k_2^\beta g^{\mu\alpha} G_{\alpha\beta}(p'_1 - Q) u(1). \quad (\text{C6})$$

In the following, we omit the explicit spinors, $u(1)$, $u(1')$, and reduce the expressions to leading order. Using $k_2^\alpha \rightarrow (0, \mathbf{k}_2)$, we have

$$A^\mu = k_2^\alpha G_{\alpha\beta} g^{\beta\mu} \rightarrow k_2^k G_{k\beta} g^{\beta\mu}, \quad (\text{C7})$$

$$B^\mu = k_2^\beta g^{\mu\alpha} G_{\alpha\beta} \rightarrow k_2^k g^{\mu\alpha} G_{\alpha k}. \quad (\text{C8})$$

1. Time components

We first show that the time component of the axial Δ current vanishes in the static limit. In fact the forward and backward parts are, respectively, proportional to

$$A^0 \rightarrow k_2^k G_{k\beta} g^{\beta 0} = k_2^k G_{k0} g^{00}, \quad (\text{C9})$$

$$B^0 \rightarrow k_2^k g^{\alpha 0} G_{\alpha k} = k_2^k g^{00} G_{0k}. \quad (\text{C10})$$

Writing the Δ propagator in the static limit in the form [31]

$$G_{\alpha\beta} \sim -\frac{\frac{\gamma_\alpha \gamma_\beta}{3} - \frac{2P_\alpha P_\beta}{3m_\Delta^2} + \frac{P_\alpha \gamma_\beta - P_\beta \gamma_\alpha}{3m_\Delta}}{m_N - m_\Delta}, \quad (\text{C11})$$

we have

$$G_{k0} \sim -\frac{g_{k0} - \frac{\gamma_k \gamma_0}{3} - \frac{2P_k P_0}{3m_\Delta^2} + \frac{P_k \gamma_0 - P_0 \gamma_k}{3m_\Delta}}{m_N - m_\Delta}. \quad (\text{C12})$$

In the static limit $P_k \rightarrow 0$ and $\gamma_k \rightarrow 0$. Then $G_{k0} \rightarrow 0$ and consequently $A^0 \rightarrow 0$. Similarly we find $G_{0k} \rightarrow 0$ and $B^0 \rightarrow 0$.

2. Space components

Using (71), the spatial components G_{kj} of the Δ propagator to leading order are written as [31]

$$G_{kj} \sim \frac{-1}{m_N - m_\Delta} \left(g_{kj} - \frac{\gamma_k \gamma_j}{3} \right) \rightarrow \frac{1}{m_N - m_\Delta} \left(\frac{2}{3} \delta_{kj} - \frac{i}{3} \epsilon_{kjl} \sigma_l \right). \quad (\text{C13})$$

Substituting into Eq. (C7) we obtain

$$\begin{aligned} A^i &\sim k_2^k G_{k\beta} g^{\beta i} = k_2^k G_{kj} g^{ji} \\ &\rightarrow k_2^k \frac{1}{m_N - m_\Delta} \left(\frac{2}{3} \delta_{kj} - \frac{i}{3} \epsilon_{kjl} \sigma_l \right) g^{ji} \\ &= \frac{1}{m_\Delta - m_N} \left(\frac{2}{3} k_2^i - \frac{i}{3} \epsilon_{kil} k_2^k \sigma_l \right) \end{aligned} \quad (\text{C14})$$

Or, in vector form

$$\mathbf{A} \rightarrow \frac{1}{m_\Delta - m_N} \left(\frac{2}{3} \mathbf{k}_2 + \frac{i}{3} \mathbf{k}_2 \times \boldsymbol{\sigma}_l^{(1)} \right). \quad (\text{C15})$$

(recall that the operator A^μ was acting on the spinor $u(1)$). Similarly, from Eq. (C8),

$$\begin{aligned} B^i &\sim k_2^k g^{\alpha i} G_{\alpha k} = k_2^k g^{ji} G_{jk} \\ &\rightarrow k_2^k g^{ji} \frac{1}{m_N - m_\Delta} \left(\frac{2}{3} \delta_{jk} - \frac{i}{3} \epsilon_{jkl} \sigma_l \right) \\ &= \frac{1}{m_\Delta - m_N} \left(\frac{2}{3} k_2^i - \frac{i}{3} \epsilon_{ikl} k_2^k \sigma_l \right). \end{aligned} \quad (\text{C16})$$

Moreover, in vector notation, $\mathbf{B}$ can be expressed as

$$\mathbf{B} \rightarrow \frac{1}{m_\Delta - m_N} \left(\frac{2}{3} \mathbf{k}_2 - \frac{i}{3} \mathbf{k}_2 \times \boldsymbol{\sigma}_l^{(1)} \right). \quad (\text{C17})$$

Once we have the nonrelativistic expressions of $\mathbf{A}$ and $\mathbf{B}$, we can insert them into the definitions of $\mathbf{K}_F$ and $\mathbf{K}_B$, given in

equations (C3) and (C4). Applying also the nonrelativistic limit of the function V from Eq. (74), we have

$$\mathbf{K}_F \rightarrow -\frac{ff^*}{m_\pi^2} C_5^A \frac{\mathbf{k}_2 \cdot \boldsymbol{\sigma}^{(2)}}{\mathbf{k}_2^2 + m_\pi^2} \mathbf{A}, \quad (\text{C18})$$

$$\mathbf{K}_B \rightarrow -\frac{ff^*}{m_\pi^2} C_5^A \frac{\mathbf{k}_2 \cdot \boldsymbol{\sigma}^{(2)}}{\mathbf{k}_2^2 + m_\pi^2} \mathbf{B}. \quad (\text{C19})$$

3. Total axial Δ current

Using Eqs. (52) and (53) for the isospin operators U_F and U_B we can write the axial Δ current in the form

$$\mathbf{j}_{\Delta A} = \frac{2}{\sqrt{6}} \tau_+^{(2)} (\mathbf{k}_F + \mathbf{K}_B) + \frac{1}{\sqrt{6}} i [\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_+ (\mathbf{k}_B - \mathbf{K}_F). \quad (\text{C20})$$

To leading order we have

$$\mathbf{A} + \mathbf{B} = \frac{1}{m_\Delta - m_N} \frac{4}{3} \mathbf{k}_2, \quad (\text{C21})$$

$$\mathbf{B} - \mathbf{A} = -\frac{1}{m_\Delta - m_N} \frac{2i}{3} \mathbf{k}_2 \times \boldsymbol{\sigma}^{(1)}. \quad (\text{C22})$$

Thus, we obtain

$$\mathbf{K}_F + \mathbf{K}_B = -\frac{ff^*}{m_\pi^2} \frac{C_5^A}{m_\Delta - m_N} \frac{\mathbf{k}_2 \cdot \boldsymbol{\sigma}^{(2)}}{\mathbf{k}_2^2 + m_\pi^2} \frac{4}{3} \mathbf{k}_2, \quad (\text{C23})$$

$$\mathbf{K}_B - \mathbf{K}_F = \frac{ff^*}{m_\pi^2} \frac{C_5^A}{m_\Delta - m_N} \frac{\mathbf{k}_2 \cdot \boldsymbol{\sigma}^{(2)}}{\mathbf{k}_2^2 + m_\pi^2} \frac{2i}{3} \mathbf{k}_2 \times \boldsymbol{\sigma}^{(1)}. \quad (\text{C24})$$

Finally, the nonrelativistic axial Δ current takes the form

$$\begin{aligned} \mathbf{j}_{\Delta A} &= -\sqrt{\frac{3}{2}} \frac{ff^*}{m_\pi^2} \frac{C_5^A}{m_\Delta - m_N} \left[4 \tau_+^{(2)} \frac{\mathbf{k}_2 \cdot \boldsymbol{\sigma}^{(2)}}{\mathbf{k}_2^2 + m_\pi^2} \mathbf{k}_2 \right. \\ &\quad \left. + [\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}]_+ \frac{\mathbf{k}_2 \cdot \boldsymbol{\sigma}^{(2)}}{\mathbf{k}_2^2 + m_\pi^2} \mathbf{k}_2 \times \boldsymbol{\sigma}^{(1)} \right] \\ &\quad + (1 \leftrightarrow 2), \end{aligned} \quad (\text{C25})$$

By writing explicitly the $(1 \leftrightarrow 2)$ term, we directly arrive at Eq. (79), which matches the form given in Ref. [47].

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- [1] K. Abe *et al.* (T2K Collaboration), *Phys. Rev. Lett.* **121**, 171802 (2018).
 - [2] K. Abe *et al.* (T2K Collaboration), *Nature (London)* **580**, 339 (2020).
 - [3] B. Abi *et al.* (DUNE Collaboration), *Eur. Phys. J. C* **80**, 978 (2020).
 - [4] A. M. Ankowski, A. Ashkenazi, S. Bacca, J. L. Barrow, M. Betancourt, A. Bodek, M. E. Christy, L. D. S. Dytman, A. Friedland, O. Hen *et al.*, *J. Phys. G* **50**, 120501 (2023).
 - [5] J. E. Sobczyk and J. Nieves, *Phys. Rev. C* **111**, 025502 (2025).
 - [6] U. Mosel, *Annu. Rev. Nucl. Part. Sci.* **66**, 171 (2016).
 - [7] T. Katori and M. Martini, *J. Phys. G* **45**, 013001 (2018).
 - [8] L. Alvarez-Ruso *et al.* (NuSTEC Collaboration), *Prog. Part. Nucl. Phys.* **100**, 1 (2018).
 - [9] M. Martini, M. Ericson, G. Chanfray, and J. Marteau, *Phys. Rev. C* **80**, 065501 (2009).
 - [10] V. Pandey, *Prog. Part. Nucl. Phys.* **134**, 104078 (2024).
 - [11] M. Sajjad Athar, A. Fatima, and S. K. Singh, *Prog. Part. Nucl. Phys.* **129**, 104019 (2023).
 - [12] J. G. Morfin, J. Nieves, and J. T. Sobczyk, *Adv. High Energy Phys.* **2012**, 934597 (2012).
 - [13] L. Alvarez-Ruso, A. M. Ankowski, A. Ashkenazi, J. Barrow, M. Betancourt, K. Borah, M. Sajjad Athar, E. Catano-Mur, P. Coloma, P. Dunne *et al.*, *arXiv:2503.23556*.
 - [14] A. M. Ankowski and C. Mariani, *J. Phys. G* **44**, 054001 (2017).
 - [15] O. Benhar, P. Huber, C. Mariani, and D. Meloni, *Phys. Rep.* **700**, 1 (2017).
 - [16] J. E. Amaro, M. B. Barbaro, J. A. Caballero, R. González-Jiménez, G. D. Megias, and I. Ruiz Simo, *J. Phys. G* **47**, 124001 (2020).

- [17] L. Alvarez-Ruso, A. M. Ankowski, S. Bacca, A. B. Balantekin, J. Carlson, S. Gardiner, R. González-Jiménez, R. Gupta, T. J. Hobbs, and M. Hoferichter *et al.*, *J. Phys. G* **52**, 043001 (2025).
- [18] C. Andreopoulos *et al.*, *Nucl. Instrum. Methods Phys. Res., Sect. A* **614**, 87 (2010).
- [19] H. Gallagher, G. Garvey, and G. P. Zeller, *Annu. Rev. Nucl. Part. Sci.* **61**, 355 (2011).
- [20] A. A. Aguilar-Arevalo *et al.* (MiniBooNE Collaboration), *Phys. Rev. D* **81**, 092005 (2010).
- [21] L. Aliaga *et al.* (MINERvA Collaboration), *Phys. Rev. D* **94**, 092005 (2016).
- [22] B. Abi *et al.* (DUNE Collaboration), [arXiv:2002.03005](https://arxiv.org/abs/2002.03005).
- [23] J. Nieves, I. Ruiz Simo, and M. J. Vicente Vacas, *Phys. Rev. C* **83**, 045501 (2011).
- [24] J. Nieves, F. Sanchez, I. Ruiz Simo, and M. J. Vicente Vacas, *Phys. Rev. D* **85**, 113008 (2012).
- [25] G. D. Megias *et al.*, *Phys. Rev. D* **94**, 093004 (2016).
- [26] J. E. Amaro, M. B. Barbaro, J. A. Caballero, T. W. Donnelly, and C. F. Williamson, *Phys. Lett. B* **696**, 151 (2011).
- [27] Y. Umino, J. M. Udías, and P. J. Mulders, *Phys. Rev. C* **52**, 3399 (1995).
- [28] T. Franco-Munoz, J. García-Marcos, R. González-Jiménez, and J. M. Udías, *Phys. Rev. C* **108**, 064608 (2023).
- [29] T. Franco-Munoz, R. González-Jiménez, and J. M. Udías, *J. Phys. G* **52**, 025103 (2025).
- [30] A. Lovato, N. Rocco, and N. Steinberg, *Phys. Rev. C* **112**, 045501 (2025).
- [31] P. R. Casale, J. E. Amaro, V. Belocchi, M. B. Barbaro, A. De Pace, and M. Martini, *Phys. Rev. C* **112**, 024603 (2025).
- [32] J. E. Amaro, M. B. Barbaro, J. A. Caballero, T. W. Donnelly, and A. Molinari, *Nucl. Phys. A* **643**, 349 (1998).
- [33] J. E. Amaro, M. B. Barbaro, J. A. Caballero, T. W. Donnelly, and A. Molinari, *Nucl. Phys. A* **697**, 388 (2002).
- [34] J. E. Amaro, M. B. Barbaro, J. A. Caballero, T. W. Donnelly, and A. Molinari, *Phys. Rep.* **368**, 317 (2002).
- [35] M. Kohno and N. Ohtsuka, *Phys. Lett. B* **98**, 335 (1981).
- [36] W. M. Alberico, T. W. Donnelly, and A. Molinari, *Nucl. Phys. A* **512**, 541 (1990).
- [37] J. E. Amaro, A. M. Lallena, and G. Co, *Int. J. Mod. Phys. E* **3**, 735 (1994).
- [38] J. E. Amaro, A. M. Lallena, and G. Co, *Nucl. Phys. A* **578**, 365 (1994).
- [39] J. E. Amaro, M. B. Barbaro, J. A. Caballero, T. W. Donnelly, and A. Molinari, *Nucl. Phys. A* **723**, 181 (2003).
- [40] P. R. Casale, J. E. Amaro, and M. B. Barbaro, *Symmetry* **15**, 1709 (2023).
- [41] J. M. Nieves (private communication).
- [42] A. Fabrocini, *Phys. Rev. C* **55**, 338 (1997).
- [43] J. Carlson, J. Jourdan, R. Schiavilla, and I. Sick, *Phys. Rev. C* **65**, 024002 (2002).
- [44] O. Benhar, A. Lovato, and N. Rocco, *Phys. Rev. C* **92**, 024602 (2015).
- [45] A. De Pace, M. Nardi, W. M. Alberico, T. W. Donnelly, and A. Molinari, *Nucl. Phys. A* **726**, 303 (2003).
- [46] J. E. Amaro, M. B. Barbaro, J. A. Caballero, T. W. Donnelly, and J. M. Udías, *Phys. Rev. C* **82**, 044601 (2010).
- [47] D. O. Riska, *Phys. Rep.* **181**, 207 (1989).
- [48] M. Ericson and W. Weise, *Pions and Nuclei* (Clarendon, Oxford, 1988).
- [49] H. c. Kim, J. Piekarewicz, and C. J. Horowitz, *Phys. Rev. C* **51**, 2739 (1995).
- [50] H. C. Kim, S. Schramm, and C. J. Horowitz, *Phys. Rev. C* **53**, 2468 (1996).
- [51] I. Ruiz Simo, J. E. Amaro, M. B. Barbaro, A. De Pace, J. A. Caballero, and T. W. Donnelly, *J. Phys. G* **44**, 065105 (2017).
- [52] E. Hernandez, J. Nieves, and M. Valverde, *Phys. Rev. D* **76**, 033005 (2007).
- [53] W. Alberico, M. Ericson, and A. Molinari, *Ann. Phys.* **154**, 356 (1984).
- [54] B. Sommer, *Nucl. Phys. A* **308**, 263 (1978).
- [55] R. Machleidt, K. Holinde, and Ch Elster, *Phys. Rep.* **149**, 1 (1987).
- [56] J. E. Amaro, M. B. Barbaro, J. A. Caballero, T. W. Donnelly, and C. Maieron, *Phys. Rev. C* **71**, 065501 (2005).
- [57] J. E. Amaro, J. A. Caballero, T. W. Donnelly, A. M. Lallena, E. Moya de Guerra, and J. M. Udías, *Nucl. Phys. A* **602**, 263 (1996).
- [58] P. R. Casale, J. E. Amaro, V. L. Martinez-Consentino, and I. Ruiz Simo, *Universe*, **9**, 158 (2023).
- [59] A. A. Aguilar-Arevalo *et al.* (MiniBooNE Collaboration), *Phys. Rev. D* **88**, 032001 (2013).
- [60] V. L. Martinez-Consentino and J. E. Amaro, *Phys. Rev. D* **108**, 113006 (2023).
- [61] J. Nieves, J. E. Amaro, and M. Valverde, *Phys. Rev. C* **70**, 055503 (2004).
- [62] R. González-Jiménez, N. Jachowicz, K. Niewczas, J. Nieves, and M. B. Barbaro, *Phys. Rev. C* **95**, 065501 (2017).
- [63] N. Jachowicz, K. Heyde, J. Ryckebusch, and S. Rombouts, *Phys. Rev. C* **65**, 025501 (2002).
- [64] A. Bodek, H. Budd, and M. E. Christy, *Eur. Phys. J. C* **71**, 1726 (2011).
- [65] A. Bodek and M. E. Christy, *Phys. Rev. C*, **106**, L061305 (2022).