

Benchmark study of the relativistic plane wave impulse approximation for polarized $(\vec{p}, 2\vec{p})$ reactions on the $3s_{1/2}$ state in ^{208}Pb at 392 MeV

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The relativistic plane wave impulse approximation (RPWIA) is benchmarked against the established relativistic distorted wave impulse approximation (RDWIA) for exclusive proton-induced proton knockout reactions $(\vec{p}, 2\vec{p})$ from the $3s_{1/2}$ state in ^{208}Pb at 392 MeV. While the RDWIA provides a proper description of the reaction, the RPWIA—which neglects nuclear distortion and absorption effects—fails dramatically for absolute cross sections, requiring a scaling factor of $1/26$ to match the data. However, within ± 10 MeV of the recoilless condition, the RPWIA quantitatively describes the analyzing power (A_y) data and yields similar results as the RDWIA predictions for all polarization transfer observables (D_{Tj}). These polarization transfer observables are insensitive to the choice of the Relativistic Mean Field model used. The study establishes clear boundaries for the RPWIA's utility: Computationally simple predictions for polarization observables near zero recoil, but essential reliance on the RDWIA for spectroscopic factor extraction.

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I. INTRODUCTION

The study of nuclear reactions with polarized beams, particularly exclusive $(\vec{p}, 2\vec{p})$ reactions, provides insights into nuclear structure and reaction mechanisms. These experiments have been instrumental in determining nucleon momentum distributions, spectroscopic factors, and the medium-modification of the nucleon-nucleon (NN) interaction [1–4]. The most sophisticated theoretical descriptions of these reactions are provided by distorted wave models which incorporate the effects of the nuclear medium through optical potentials that describe the distortion of the incident and outgoing proton waves. These potentials can be derived phenomenologically by fitting elastic scattering data [5,6] or microscopically through folding models [7,8].

The nonrelativistic distorted wave impulse approximation (NRDWIA) has achieved broad success in spectroscopic analyses of knockout reactions for a wider range of nuclei and single-particle states [3,9]. For lighter nuclei such as ^{12}C , ^{16}O , and ^{40}Ca , the NRDWIA model provides spectroscopic factors which are consistent with those extracted from the electron-induced knockout ($e, e'p$) measurements [3]. However, for heavier nuclei such as ^{208}Pb , NRDWIA was found to overestimate the spectroscopic factors as compared to the $(e, e'p)$, which suggests that additional effects may become important in heavier nuclei [3]. The relativistic distorted wave impulse

approximation (RDWIA) is a fully relativistic framework that treats the scattering and the bound-state wave functions within the Dirac theory, and its applications have been focused largely on medium modifications of the NN interaction and spin-dependent observables where the relativistic effects are expected to be significant [2,6,10].

A known challenge in distorted-wave approaches, whether nonrelativistic or relativistic, is their sensitivity to the choice of optical potentials, which can introduce ambiguity in the extraction of absolute spectroscopic factors [3,11]. This has motivated investigations into simpler models to identify observables and kinematics that are not sensitive to optical potentials. We therefore consider investigating whether a simpler Relativistic Plane Wave Impulse Approximation (RPWIA) model which treats all proton wave functions as Dirac plane waves that neglect the distortion and absorption effects [12,13] can describe analyzing powers for $(\vec{p}, 2\vec{p})$ reactions under specific kinematic regimes such as near the recoilless condition.

This paper presents a benchmark of the RPWIA model against the established RDWIA standard. We do not propose RPWIA as a replacement for sophisticated distorted-wave models. Instead, we seek to quantitatively map its performance across a complete set of observables—the unpolarized triple differential cross section (σ), the analyzing power (A_y), and the full set of polarization transfer observables (D_{Tj})—for proton knockout from the $3s_{1/2}$ state in ^{208}Pb . This paper aims to

- (1) Quantify the failure of the RPWIA to describe the absolute cross section.
- (2) Identify the specific kinematic region where its predictions for A_y and D_{Tj} coincide with the RDWIA and experimental data.

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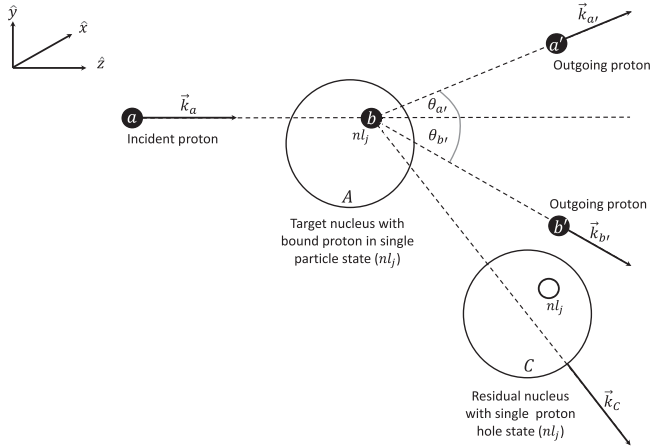


FIG. 1. Schematic diagram of the exclusive $(\bar{p}, 2\bar{p})$ knockout reaction.

- (3) Investigate the sensitivity of these findings to the underlying nuclear structure model.
- (4) Compare the spectroscopic factor for the $(\bar{p}, 2\bar{p})$ reaction extracted from the RDWIA for the $3s_{1/2}$ state in ^{208}Pb with NRDWIA analyses and $(e, e'p)$ measurements.

This analysis provides a clear reference for the utility and limitations of the RPWIA model, which may be of value for exploratory calculations, and for informing future $(\bar{p}, 2\bar{p})$ experiments on stable and unstable nuclei, particularly those focusing on polarization transfer observables.

This manuscript is organized as follows. Section II outlines the theoretical framework, including the RPWIA formalism (Sec. II A), the relativistic bound-state wave functions (Sec. II B), and the Nucleon-Nucleon (NN) interaction with the IA1 parametrization (Sec. II C). Section III presents the results and discussion for the triple differential cross section, analyzing power and polarization transfer observables for the $3s_{1/2}$ state of ^{208}Pb . Our conclusions with a discussion of the utility and limitations of the RPWIA framework are presented in Sec. IV.

II. THEORETICAL FRAMEWORK

A. Relativistic plane wave impulse approximation

The exclusive polarized proton-induced proton-knockout reaction $(\bar{p}, 2\bar{p})$, illustrated by Fig. 1, involves a polarized projectile proton (a) with momentum $\vec{k}_a$, knocking out a bound proton (b) from a single-particle state (nl_j) of a stationary target nucleus ($A = C + b$), leaving behind the residual nucleus

C , such that two outgoing protons a' and b' with momenta $\vec{k}_{a'}$ and $\vec{k}_{b'}$, respectively, are detected at coplanar scattering angles $\theta_{a'}$ and $\theta_{b'}$ in the exit channels [13,14]. The principal quantum number, the orbital angular momentum, and the total angular momentum are given as n , l and $j = l \pm \frac{1}{2}$, respectively. The transition amplitude for this particular reaction is given by

$$T_{ljm_j}(s_a, s_{a'}, s_{b'}) = \int d^3\vec{x} [\bar{\psi}^{(-)}(\vec{x}, \vec{k}_{a'}, s_{a'}) \otimes \bar{\psi}^{(-)}(\vec{x}, \vec{k}_{b'}, s_{b'})] \times \hat{F}[\psi^{(+)}(\vec{x}, \vec{k}_a, s_a) \otimes \phi_{ljm_j}(\vec{x})], \quad (1)$$

where the wave functions are treated as relativistic Dirac plane waves:

$$\begin{aligned} \psi^{(+)}(\vec{x}, \vec{k}_a, s_a) &= e^{i\vec{k}_a \cdot \vec{x}} U(\vec{k}_a, s_a), \\ \bar{\psi}^{(-)}(\vec{x}, \vec{k}_{a'}, s_{a'}) &= e^{-i\vec{k}_{a'} \cdot \vec{x}} U(\vec{k}_{a'}, s_{a'}), \\ \bar{\psi}^{(-)}(\vec{x}, \vec{k}_{b'}, s_{b'}) &= e^{-i\vec{k}_{b'} \cdot \vec{x}} U(\vec{k}_{b'}, s_{b'}). \end{aligned} \quad (2)$$

The four-component Dirac spinor is given as

$$U(\vec{k}_i, s_i) = \sqrt{\frac{E_i + m_i}{2m_i}} \begin{pmatrix} 1 \\ \frac{\vec{\sigma} \cdot \vec{k}_i}{E_i + m_i} \end{pmatrix} \chi_{s_i}, \quad (3)$$

with the Pauli spin matrices denoted by $\vec{\sigma}$, the energy and the rest mass of the nucleon by E_i and m_i , respectively. The two-component Pauli spinors with projection s are represented by χ_{s_i} . The wave function of the incident proton is represented as $\psi^{(+)}(\vec{x}, \vec{k}_a, s_a)$, while the wave functions of the two outgoing protons are denoted as $\bar{\psi}^{(-)}(\vec{x}, \vec{k}_{a'}, s_{a'})$ and $\bar{\psi}^{(-)}(\vec{x}, \vec{k}_{b'}, s_{b'})$, respectively. The four-component wave functions are solutions to the free Dirac equation. The Kronecker product is denoted by $\otimes$, the nucleon-nucleon (NN) interaction by $\hat{F}$, and the relativistic bound-state wave function by $\phi_{ljm_j}(\vec{x})$. The Relativistic Impulse Approximation, which assumes that the NN scattering matrix inside the nuclear medium is identical to that of free NN scattering, is also employed.

Within the RPWIA framework, the incident and the outgoing protons are treated as free Dirac plane waves given by Eqs. (2), neglecting both the strong nuclear distortion and Coulomb interactions between the protons and the target nucleus [12,13,15].

B. Relativistic bound-state wave functions

The bound-state proton radial wave functions $\phi_{ljm_j}(\vec{x})$ are obtained via self-consistent solutions of the Dirac-Hartree field equations from quantum hadrodynamics (QHD) within the relativistic mean field (RMF) approximation [16]. The wave function is expressed as

$$\phi_{ljm_j}(\vec{x}) = \frac{1}{x} \sum_{s_B} \begin{bmatrix} u_{lj}(x) \langle l, m_j - s_B, \frac{1}{2}, s_B | j, m_j \rangle Y_{l, m_j - s_B}(\hat{x}) \chi_{s_B} \\ i w_{lj}(x) \langle 2j - l, m_j - s_B, \frac{1}{2}, s_B | j, m_j \rangle Y_{2j-l, m_j - s_B}(\hat{x}) \chi_{s_B} \end{bmatrix}, \quad (4)$$

where u_{lj} and w_{lj} represent the upper and lower proton radial wave functions, respectively. The quantum numbers: l ,

j , and m_j denote the orbital angular momentum, total angular momentum, and its projection of the bound proton. The

summation runs over the spin projection $s_B = \pm 1/2$ of the bound proton. The brackets $\langle \dots \rangle$ are the Clebsch-Gordon coefficients, which couple the orbital and spin angular momenta to total angular momentum j . The functions Y_{lm} are the spherical harmonics, and χ_{s_B} are two-component Pauli spinors.

The different RMF models applied to extract the bound-state wave functions are QHDII, NL3, FSUGold, and FSUGold2. QHDII is based on quantum hadrodynamics and includes linear scalar and vector couplings, with its parameters fitted to reproduce bulk properties of nuclear matter [16]. In contrast, the NL3 model introduces self-interactions of the sigma meson, which improves its ability to describe the ground-state properties of spherical nuclei [17]. FSUGold is an RMF model with additional vector meson self-interactions which improve the description of the symmetry energy and neutron star properties [18]. An updated RMF version, FSUGold2, has refined parameters for improved properties of infinite nuclear matter [19]. Comprehensive model comparisons are beyond the present scope; interested readers should consult Refs. [16–19].

C. Nucleon-Nucleon interaction

For the NN interaction ($\hat{F}$), we adopt the IA1 representation [20,21]:

$$\hat{F} = \sum_{i=S}^T F_i(\lambda^{i(1)} \cdot \lambda_{i(2)}), \quad (5)$$

where the Dirac matrices λ_i correspond to scalar, vector, pseudoscalar, axial-vector, and tensor interactions.

We acknowledge the previous studies by van der Ventel and Hillhouse [13] where they compared the RPWIA calculations of the $(\vec{p}, 2\vec{p})$ reactions on ^{208}Pb at 392 MeV, using both IA1 and IA2 parametrizations of the NN interaction, which found that some observables discriminate between the two parametrizations. However, our primary goal is to benchmark RPWIA against the established RDWIA model and not to investigate the dependence on the NN interaction, and to the best of our knowledge, most RDWIA calculations for exclusive knockout reactions (including those in this paper) employ the IA1 representation. To ensure a consistent comparison between RPWIA and RDWIA, we employ IA1 in both frameworks. A systematic study of IA1 versus IA2, within both RPWIA and RDWIA, is beyond the scope of this current work but is planned for future publication.

D. Observables

The unpolarized triple differential cross section is given by [6]

$$\sigma = \frac{d^3\sigma}{dT_a d\Omega_{a'} d\Omega_b} = S_{lj} \sigma_{\text{calc}}, \quad (6)$$

with

$$\sigma_{\text{calc}} = \frac{F_{\text{kin}}}{(2s_a + 1)(2j + 1)} \sum_{m_j, s_b} \text{Tr}(TT^\dagger), \quad (7)$$

where S_{lj} represents the spectroscopic factor and F_{kin} is the kinematic factor. The polarization transfer observables, D_{ij} , are defined as

$$D_{ij} = \frac{\text{Tr}(T\sigma_j T^\dagger \sigma_i)}{\text{Tr}(TT^\dagger)}. \quad (8)$$

The transition amplitude in Eq. (1) is used to construct a 2×2 matrix T in the spin space of the incident and scattered protons which is used to calculate the cross section and the polarization transfer observables in Eqs. (7) and (8), respectively. The T matrix is expressed as

$$T = \begin{pmatrix} T_{lj}^{s_a=\frac{1}{2}, s_{a'}=\frac{1}{2}} & T_{lj}^{s_a=-\frac{1}{2}, s_{a'}=\frac{1}{2}} \\ T_{lj}^{s_a=-\frac{1}{2}, s_{a'}=-\frac{1}{2}} & T_{lj}^{s_a=\frac{1}{2}, s_{a'}=-\frac{1}{2}} \end{pmatrix}, \quad (9)$$

where $s_a = \pm \frac{1}{2}$ and $s_{a'} = \pm \frac{1}{2}$ denote the spin projections of both the incident (a) and the outgoing proton (a'). The matrix elements $T_{lj m_j}(s_a, s_{a'}, s_{b'})$ are related to $T_{lj}^{s_a, s_{a'}}$ through [6]

$$T_{lj}^{s_a, s_{a'}} = \sum_{m_j, s_{b'}} T_{lj m_j}(s_a, s_{a'}, s_{b'}).$$

III. RESULTS AND DISCUSSION

This section presents calculations for exclusive proton knockout from the $3s_{1/2}$ state in ^{208}Pb at an incident laboratory kinetic energy of 392 MeV. The scattering angles are fixed in coplanar geometry at 32.5° , -50.0° . In Figs. 2 and 3, the solid black line (RPWIA) and the dashed black line (RDWIA) both employ the FSUGold2 RMF model for the bound-state wave functions and the gray shaded band represents the spread in the RPWIA predictions from other RMF models (QHDII, NL3, and FSUGold).

A. Cross section and analyzing power: A benchmark against data

We begin by comparing the predictions of the RPWIA and RDWIA models with experimental data for the unpolarized triple differential cross section (σ) and the analyzing power (A_y) for proton knockout from the $3s_{1/2}$ state in ^{208}Pb from Ref. [3]. Figure 2(a) shows the energy-sharing distribution of σ . The RDWIA calculation provides a good qualitative description of the data's shape, requiring only a minor normalization value for a quantitative fit, $S = 1/1.7$. In stark contrast, the RPWIA model, while reproducing the symmetric shape, dramatically overestimates the magnitude of the cross section. To match the data, the RPWIA result must be scaled down by a factor of 1/26. This factor is a measure of the total suppression caused by nuclear distortion and absorption—effects that are completely absent within the RPWIA treatment. This result underscores the fundamental inadequacy of the RPWIA for predicting absolute cross sections.

The dramatic difference between the spectroscopic factor extracted from the RDWIA ($S = 1/1.7$) and the normalization factor required for the RPWIA (1/26) provides a quantitative measure of the importance of distortion effects. While the RDWIA achieves agreement with experimental data through a physically meaningful spectroscopic factor of 1/1.7, the

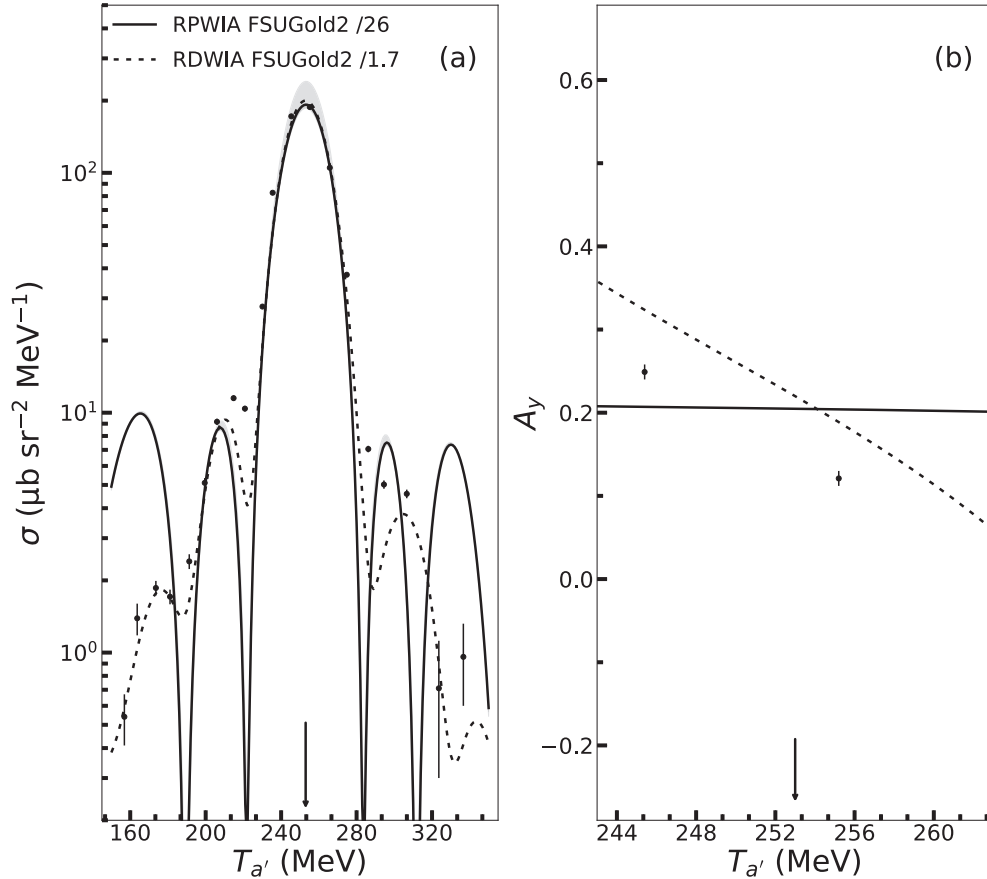


FIG. 2. (a) Triple differential cross section σ , (b) analyzing power A_y for the $^{208}\text{Pb}(\bar{p}, 2\bar{p})$ reaction on the $3s_{1/2}$ state at an incident energy of 392 MeV, and fixed coplanar scattering angles, 32.5° , -50.0° , shown as a function of the laboratory kinetic energy $T_{a'}$ of the outgoing proton. The solid (dashed) black line represents the RPWIA (RDWIA) results using the FSUGold2 RMF model. The gray shaded band shows the collective variations in the RPWIA predictions across different RMF models. Experimental data are from Ref. [3] and the arrows represent the minimum recoil position.

RPWIA requires a phenomenological factor of $1/26$ that has no direct physical interpretation. This difference underscores that RPWIA cannot be used for extracting nuclear structure information such as spectroscopic factors, even though it may approximate certain polarization transfer observables within specific kinematic regions. Figure 2(b) shows the analyzing power A_y within the window of ± 10 MeV around the peak of the cross section corresponding to zero recoil momentum, the full energy range is shown in Supplemental Material Fig. 1 [22]. Here, the performance of the RPWIA model is markedly different. Within a limited kinematic range (corresponding to low recoil momentum), the RPWIA calculation quantitatively describes the experimental data and agrees closely with the RDWIA prediction. This finding indicates that, while the overall reaction probability is severely suppressed, the spin-dependent aspects of the interaction captured by A_y can be modeled reasonably well through the RPWIA approach in this specific kinematic window.

B. Polarization transfer observables: A benchmark against RDWIA

As no experimental data currently exist for the complete set of polarization transfer observables (D_{ij}'), we benchmark the

RPWIA predictions against the established RDWIA model. The results are shown in Figs. 3(a)–3(f).

Strikingly, within the same ± 10 MeV window where A_y is well described, the RPWIA predictions for all D_{ij}' observables are in close agreement with the RDWIA results. This suggests that for these specific spin-transfer observables, the net effect of distortions on the spin dynamics is minimal near the recoilless condition. At these conditions, the momentum transfer to the residual nucleus is essentially zero, since the outgoing protons carry away nearly all the transferred momentum. The optical potentials, which depend on the local momentum of the nucleons, have an insignificant influence on the spin dynamics, since the phase shifts experienced by the incoming and the outgoing waves cancel in spin-dependent ratios. Away from the recoilless regime, however, the residual nucleus acquires non-zero momentum, and the distortion effects become significant, hence leading to noticeable deviations between RPWIA and RDWIA calculations. This behavior therefore highlights a useful, although limited, application of RPWIA: It can serve as a computationally inexpensive tool for generating initial predictions for polarization transfer observables in this kinematic regime.

We have demonstrated the polarization transfer observables in the limited range of the recoilless condition in Fig. 3

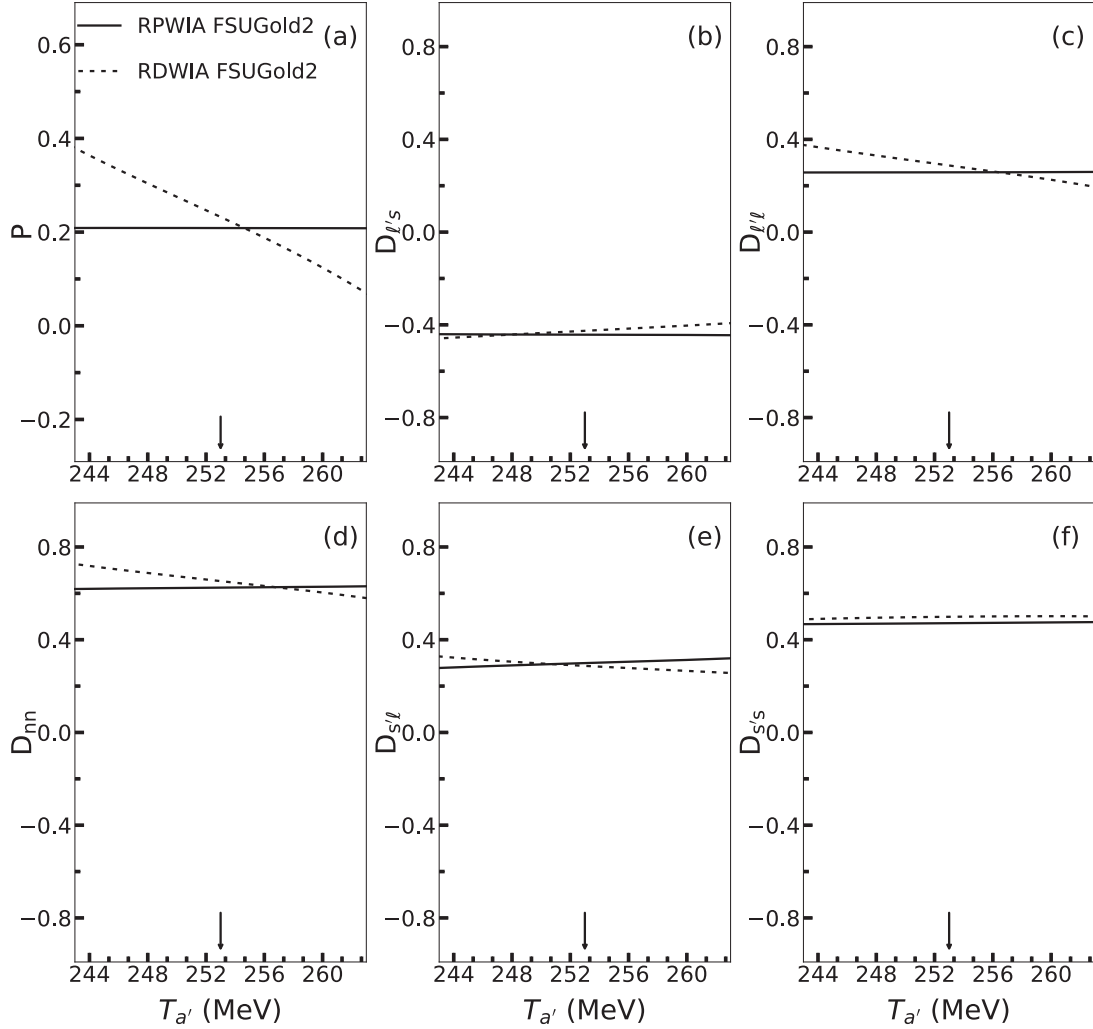


FIG. 3. The complete set of polarization transfer observables (D_{ij}) of the $3s_{1/2}$ state in ^{208}Pb with similar RMF model representation as Fig. 2.

(the ± 10 MeV window where RPWIA and RDWIA agree around the peak of the cross section). The results for the full energy range (145–355 MeV) are shown in the Supplemental Material (Fig. 2) [22], where RPWIA and RDWIA deviate outside the ± 10 MeV from the peak of the cross section.

C. Sensitivity to Nuclear Structure Inputs

We investigate the sensitivity of our results to the choice of RMF models (QHDII, NL3, FSUGold, FSUGold2) used to generate the bound-state wave functions. The unpolarized cross section σ shown in Fig. 2(a), exhibits noticeable sensitivity to the RMF model. At the recoilless energy ($T_{a'} = 253$ MeV), the scaled RPWIA cross sections range from 185.9 μb (FSUGold) to 240.6 μb (QHDII), with FSUGold2 and NL3 falling between them at 192.4 μb and 186.8 μb , respectively, giving a spread of approximately 29% between the highest and the lowest values. For comparison, the experimental peak cross section is 188.0 ± 1.3 μb . The NL3, FSUGold, and FSUGold2 models are within 2.4% of the experimental data, while QHDII overestimates the cross section by 28%. However, the spin-dependent observables—both A_y

[Fig. 2(b)] and the complete set of D_{ij} presented in Figs. 3(a)–3(f) show remarkable insensitivity to these changes. This indicates that while the overall probability of the reaction depends on the structural details of the bound state, the spin dynamics governing the polarization observables are predominantly dictated by the free NN interaction and are largely decoupled from the specific nuances of the RMF parametrization, at least for this s state. We note similar behavior of the polarization transfer observables for other single-particle states in ^{208}Pb within the same kinematical conditions; a detailed analysis of these cases will be presented in a future publication.

D. Spectroscopic factors and relativistic effects

The spectroscopic factor (S) for the $3s_{1/2}$ state in ^{208}Pb extracted from our fully relativistic RDWIA analysis is $S = 0.61(14)$, obtained by normalizing the theoretical cross section to experimental data following procedures in Ref. [3]. This value is presented alongside comparative results in Table I.

TABLE I. Spectroscopic factors for the $3s_{1/2}$ state in ^{208}Pb from various reaction mechanisms. The present RDWIA analysis yields $S = 0.61(14)$, which is lower than values obtained from other approaches.

Reaction	Model/reference	Spectroscopic factor S	Notes
$(\bar{p}, 2\bar{p})$	Present RDWIA work	0.61(14)	Fully relativistic treatment
$(p, 2p)$	EDAD1 [3]	1.64(32)	Semirelativistic
$(p, 2p)$	Timora [3]	1.77(41)	Semirelativistic
$(e, e'p)$	Various [3,9,23]	0.98(09)	Electron scattering
$(d, {}^3\text{He})$	Transfer reactions [9,23]	1.5	Nuclear transfer

The NRDWIA analyses in Ref. [3] yielded larger values ($S \approx 1.6\text{--}1.8$) for the same reaction on ^{208}Pb . As noted in Ref. [3], the tendency of the NRDWIA to overestimate spectroscopic factors is not universal since it has provided S factors which are consistent with those extracted from $(e, e'p)$ measurements for lighter nuclei, i.e., ^{12}C , ^{16}O , and ^{40}Ca . The discrepancy appears for a heavy nucleus like ^{208}Pb , where the relativistic and distortion effects play a significant role.

The reduction in spectroscopic strength extracted from the RDWIA framework compared to other analyses ($S = 0.61$ versus $S \approx 1.6\text{--}1.8$ from earlier $(\bar{p}, 2\bar{p})$ studies) may be influenced by several factors inherent to the consistent relativistic treatment:

- (1) Strong relativistic potentials: The RDWIA framework includes both scalar and vector potentials of substantial magnitude ($\approx 300\text{--}400\text{ MeV}$ in the nuclear interior [5]), which may strongly distort the Dirac spinors of the bound and scattering states.
- (2) Enhanced wave function suppression: These strong potentials may lead to additional suppression of the overlap between initial and final states beyond what is captured in nonrelativistic or semirelativistic frameworks.
- (3) Consistency with relativistic dynamics: The lower S value may reflect a more complete treatment of relativistic dynamics throughout the reaction process.

It is important to exercise caution when interpreting the differences in the S factors from different reaction models. The RDWIA model itself faces several challenges, including the proper treatment of the recoil effects, sensitivity to the choice of relativistic optical potentials, and limited validation for lighter nuclei where NRDWIA has been extensively tested. Consequently, while our RDWIA result for ^{208}Pb is physically plausible, a more comprehensive assessment for a wider range of nuclei and kinematics is required before making conclusions about the relative performance of relativistic (RDWIA) and nonrelativistic (NRDWIA) frameworks. Even though the results of a fully relativistic treatment may yield lower S factor for the $3s_{1/2}$ state of ^{208}Pb , this should not be interpreted as a claim of RDWIA's superiority over NRDWIA.

Our discussion reinforces the message of the paper regarding RPWIA's limitations. The large difference between the factor required to scale the RPWIA results to data ($1/26$) and that of the RDWIA (0.61) demonstrates that distortion effects are important for quantitative descriptions of the ab-

solute cross sections. Even for sophisticated distorted-wave models (RDWIA and NRDWIA), the extracted S factors vary and depend on the type of theoretical framework used. This highlights the need to establish the use of validated reaction models when extracting nuclear structure information from knockout reactions.

The discrepancy between our RDWIA result and previous $(p, 2p)$ analyses highlights the significant impact of theoretical framework choices on extracted spectroscopic factors. Notably, earlier $(p, 2p)$ analyses employed hybrid approaches (e.g., nonrelativistic distorted waves with relativistic bound states) [3], which may not fully account for the strong relativistic distortion effects present in heavy nuclei like ^{208}Pb .

The factor of ≈ 16 difference between the RDWIA normalization ($S = 0.61$) and the RPWIA normalization ($1/26$) is smaller than the ≈ 43 difference when comparing to NRDWIA analyses. This emphasizes that:

- (1) Even sophisticated models like RDWIA show significant variation in extracted spectroscopic factors depending on implementation details and relativistic consistency.
- (2) RPWIA's failure is extreme when compared to either relativistic or nonrelativistic treatments, requiring a normalization factor $1/26$ that has no relation to physically meaningful spectroscopic strengths.
- (3) The separation between what RPWIA can do (approximate certain polarization observables in specific kinematics) and what it cannot do (extract nuclear structure information) is now quantitatively established through this comparison.

The systematically lower spectroscopic factor extracted from the RDWIA framework ($S = 0.61$) compared to earlier semirelativistic $(\bar{p}, 2\bar{p})$ analyses ($S \approx 1.6\text{--}1.8$) demonstrates the critical importance of maintaining relativistic consistency throughout the reaction model. This consistency appears to yield spectroscopic strengths that align more closely with values from fundamentally different reaction mechanisms like $(e, e'p)$, potentially offering a more unified description of nuclear structure across diverse experimental probes. This finding underscores that the precise value of extracted spectroscopic factors remains strongly dependent on the completeness of the theoretical framework, particularly for heavy nuclei where relativistic effects are significant.

IV. CONCLUSION

We have conducted a benchmark of the RPWIA against the established RDWIA for exclusive $(\vec{p}, 2\vec{p})$ knockout reactions on the $3s_{1/2}$ state in ^{208}Pb at 392 MeV. Our key findings are:

- (1) RPWIA fails for absolute cross sections, requiring an unphysical normalization factor of $1/26$ compared to the spectroscopic factor $S = 0.61(14)$ extracted from RDWIA.
- (2) Within ± 10 MeV of the recoilless condition, RPWIA quantitatively describes both the analyzing power (A_y) data and RDWIA predictions for all polarization transfer observables D_{ij} .
- (3) Polarization observables in this kinematic window are insensitive to the choice of RMF model, governed primarily by the free NN interaction.
- (4) The spectroscopic factor from RDWIA ($S = 0.61$) is lower than values yielded by the NRDWIA analyses ($S \approx 1.6 - 1.8$). This highlights the sensitivity of the extracted S factors to the reaction model for heavy nuclei.

Therefore, RPWIA has a limited but specific utility: It provides computationally simple, accurate estimates for po-

larization observables near zero recoil, making it suitable for exploratory studies with radioactive beams where optical potentials are unknown. For spectroscopic factor extraction or full kinematic analyses, RDWIA and NRDWIA remain essential. This work establishes clear boundaries for applying simplified models in polarized knockout reactions.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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