

Locating the QCD critical point through contours of constant entropy density

Hitansh Shah¹, Mauricio Hippert^{2,3}, Jorge Noronha⁴, Claudia Ratti¹, and Volodymyr Vovchenko¹

¹Department of Physics, *University of Houston*, Houston, Texas 77204, USA

²Instituto de Física, *Universidade do Estado do Rio de Janeiro*, Rua São Francisco Xavier, 524, Rio de Janeiro, Rio de Janeiro 20550-013, Brazil

³Centro Brasileiro de Pesquisas Físicas, Rua Dr. Xavier Sigaud 150, Rio de Janeiro, Rio de Janeiro 22290-180, Brazil

⁴Illinois Center for Advanced Studies of the Universe and Department of Physics, *University of Illinois at Urbana-Champaign*, Urbana, Illinois 61801-3003, USA



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We propose a new method to investigate the existence and location of the conjectured high-temperature critical point of strongly interacting matter via contours of constant entropy density. By approximating these lines as a power series in the baryon chemical potential μ_B , one can extrapolate them from first-principle results at zero net-baryon density, and use them to locate the quantum chromodynamics (QCD) critical point, including the associated first-order and spinodal lines. As a proof of principle, we employ currently available continuum-extrapolated lattice data from the Wuppertal-Budapest collaboration to find a critical point at a temperature and a baryon chemical potential of $T_c = 114.3 \pm 6.9$ MeV and $\mu_{B,c} = 602.1 \pm 62.1$ MeV, respectively, at expansion order $\mathcal{O}(\mu_B^2)$. We advocate for a more precise determination of the required expansion coefficients via lattice QCD simulations as a means of pinpointing the location of the critical endpoint in the phase diagram of strongly interacting matter.

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Introduction. One of the most challenging questions in the field of quantum chromodynamics (QCD) concerns the existence of a first-order phase transition at finite baryon chemical potential μ_B and high temperature T , separating the hadronic and quark-gluon plasma phases. From state of the art lattice QCD calculations, we know that the transition from hadronic matter to deconfined quark gluon plasma at $\mu_B = 0$ is an analytic, smooth crossover [1], taking place at a pseudocritical temperature of 155–158 MeV [2,3]. Because direct lattice QCD simulations are not feasible at finite density due to the fermion sign problem, whether a first-order phase transition occurs at large chemical potential remains an open question. Extrapolations of lattice results to small baryon densities indicate that the analytic crossover persists at least up to $\mu_B/T \leq 3$ [2,4,5], although a hint of a narrowing of the crossover toward large μ_B was observed for the first time in [6]. Several effective QCD theories predict a first-order phase transition at large μ_B , originating from a critical point (CP) where the transition is of second order (for recent reviews, see [7,8]). While in the past the theoretical predictions for the location of the QCD critical point were scattered all over the QCD phase diagram, more recent ones seem to concentrate around a considerably narrower region, with chemical potentials in the range $400 \leq \mu_B \leq 650$ MeV [9–14].

The universal features and peculiar behavior expected near the QCD critical point make a compelling case for its experimental search in heavy-ion collisions, in particular utilizing event-by-event fluctuations in multiplicities of produced particles [15–17]. The experimental search for the QCD critical point is one of the primary goals of the Beam Energy Scan program at the BNL Relativistic Heavy Ion Collider (RHIC), which finished its runs in 2021 [7,18]. Proton cumulants are expected to show characteristic signatures of critical behavior, although interpreting experimental results in this context is challenging. The data from the STAR Collaboration [19,20], including the new preliminary data from BESII on proton number factorial cumulants [21], hint at an interesting behavior for energies $\sqrt{s_{NN}} \leq 20$ GeV. These features cannot be explained by noncritical baselines [22,23], even though no conclusive statement on the critical point existence/location has been made so far either using the proton cumulants, or other considerations such as finite-size scaling [24–26].

A common strategy to sidestep the obstacles for a direct lattice QCD determination of the equation of state at high baryon densities is to rely on extrapolations from zero or imaginary chemical potentials, such as analytic continuation and Taylor expansion [27–31]. However, these methods often fail at large densities. Especially, the Taylor expansion of thermodynamic pressure in powers of μ_B/T is restricted by nonanalyticities in the complex plane, including the CP itself [32], and hence cannot reach the CP. Recent approaches to extract information about the CP from lattice QCD results analyze the expected Lee-Yang edge singularities at complex baryon chemical potential and extrapolate down in temperature to locate the CP through their crossing of the real μ_B axis [13,14]. In this Letter, we present a new method to locate the

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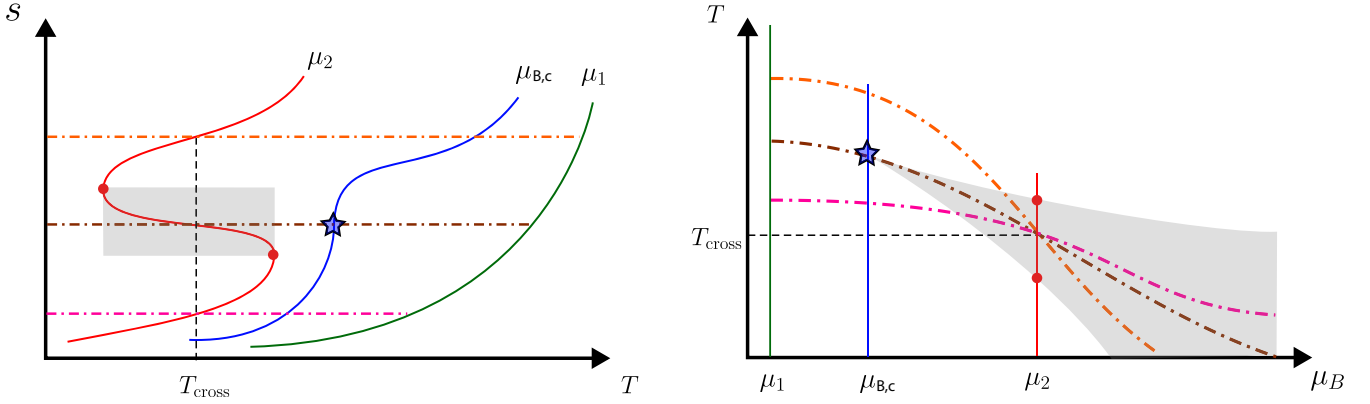


FIG. 1. Left: entropy as a function of the temperature, at three different chemical potentials. In this scheme, $\mu_1 < \mu_{B,c} < \mu_2$. Right: corresponding constant-entropy contours in the (T, μ_B) plane. The blue star indicates the critical point, while the shaded area corresponds to the spinodal region. The red dots indicate the spinodal points for $\mu_B = \mu_2$.

CP in the QCD phase diagram which utilizes first-principles lattice QCD input, exploiting the fact that the entropy density s is a multivalued function in the region of a first-order phase transition, reflecting phase coexistence. The existence of a CP thus implies crossings of constant entropy density contours. Since lattice QCD simulations can only be performed at vanishing baryon chemical potentials, we extrapolate the entropy density contours from $\mu_B = 0$ by approximating them with an expansion in powers of μ_B . In contrast to other expansions used in the literature, this scheme explicitly permits the crossings of these contours.

As we discuss in the following, it describes the CP and associated mixed-phase region in a mean-field approximation. Thus, the method allows for the determination of whether and where a CP is found in the phase diagram, as well as yielding spinodal and coexistence lines for the QCD phase transition.

To utilize the first-principles input, we extract expansion coefficients for the lines of constant entropy density from state-of-the-art lattice QCD results from the Wuppertal-Budapest collaboration. Our reconstructed entropy density agrees with the lattice QCD results from [33] up to $\mu_B/T \leq 3.5$. At larger chemical potentials, it develops the S-shape (Maxwell loops) of a first-order phase transition, indicating the emergence of the CP singularity. The results remain consistent with the HotQCD Collaboration's input for entropy density [34]. As a consistency check, we find that the method successfully locates the known CP in the holographic model developed in [35]. Our method also shows indications for a CP at imaginary μ_B close to where the endpoint of the Roberge-Weiss transition is expected.

Crossings of entropy contours. The intuitive picture behind our approach is grounded in the multi-valued behavior of the entropy density s around a first-order phase transition. Strictly speaking, entropy density is a single-valued function of T and μ_B in equilibrium, with the only exception being the phase coexistence line in the thermodynamic limit. However, the multivaluedness can appear by considering metastable states. The most prominent example is the van der Waals equation of state (and other analytic models), which neglects fluctuations of the order parameter and describes the first-order phase transition in the mean field approximation. The Maxwell

construction is then employed to eliminate the multivaluedness and determine the equilibrium state. Our method here is based on analytic extrapolation of constant entropy contours from crossover toward a CP and yields a similar structure, which we discuss in detail below.

The expected relationship between the temperature and entropy density in the presence of the QCD CP at finite μ_B , in the mean-field approximation, is illustrated in Fig. 1. The curves on the left panel show s vs T for different values of the chemical potential: the critical value $\mu_{B,c}$, $\mu_1 < \mu_{B,c}$, and $\mu_2 > \mu_{B,c}$. For $\mu_B < \mu_{B,c}$, the entropy is a continuous function of the temperature, with a steep rise around the crossover region. As μ_B increases, this steep rise becomes more pronounced, making the lines of constant entropy density closer in temperature, until a diverging slope is achieved at the critical point. For $\mu_B > \mu_{B,c}$, the entropy can have three values for the same (T, μ_B) pair. Thus, up to three lines of constant s can cross at the same point in the (T, μ_B) plane. This is shown by the horizontal dash-dotted lines depicting three different contours of entropy density. These contours are shown in the T - μ_B plane in the right panel of Fig. 1, where they cross at the same point. In fact, crossings between contours of constant s will spread through the region in the (T, μ_B) plane between the spinodal curves.

Precisely at the spinodal points, the temperature as a function of s displays a local maximum or minimum (see Fig. 1), so that $(\partial T / \partial s)_{\mu_B} = 0$. As the critical point is approached, the local maximum and minimum coalesce, leading to an inflection point with $(\partial^2 T / \partial s^2)_{\mu_B} = 0$, while still satisfying $(\partial T / \partial s)_{\mu_B} = 0$. The location of the CP is thus determined by a pair of equations

$$(\partial T / \partial s)_{\mu_B} = 0, \quad (\partial^2 T / \partial s^2)_{\mu_B} = 0. \quad (1)$$

Similar arguments hold for other quantities that become multivalued around the first-order phase transition and may serve as order parameters. For instance, the CP of a liquid-gas transition corresponds to the vanishing first and second derivatives of the pressure with respect to the density at constant temperature. We note that the relations (1) hold at the CP not only in the mean-field limit, but also in the three-dimensional (3D) Ising universality class expected in QCD [36].

New expansion. We describe the contours of constant entropy density in terms of the function $T_s(\mu_B; T_0)$, defined such that

$$s[T_s(\mu_B; T_0), \mu_B] = s(T_0, \mu_B = 0). \quad (2)$$

That is, for increasing values of μ_B , one can keep the entropy fixed at $s_0 = s(T_0, \mu_B = 0)$ by choosing the temperature $T = T_s(\mu_B; T_0)$. Anchoring the entropy density contour at $\mu_B = 0$ will allow us to utilize the lattice QCD input. The expansion of $T_s(\mu_B; T_0)$ in powers of μ_B reads

$$T_s(\mu_B; T_0) \approx T_0 + \sum_{n=1}^N \alpha_{2n}(T_0) \frac{\mu_B^{2n}}{(2n)!} + \mathcal{O}(\mu_B^{2(N+1)}), \quad (3)$$

where we truncate the series at order $\mathcal{O}(\mu_B^{2N})$. Of course, $T_s(\mu_B = 0; T_0) = T_0$ by definition. Note that the expansion (3) explicitly permits the existence of the crossings even at leading order ($N = 1$), namely $T_s(\mu_B; T_{0,1}) = T_s(\mu_B; T_{0,2})$ is possible for two different values of T_0 .

From the function $T_s(\mu_B; T_0)$, we can evaluate the conditions for the presence of a critical point and the shape of spinodal lines, following the discussion above. At the spinodal lines,

$$\left(\frac{\partial T_s}{\partial s} \right)_{\mu_B} = 0 \quad \Rightarrow \quad \left(\frac{\partial T_s}{\partial T_0} \right)_{\mu_B} = 0, \quad (4)$$

where the second equation follows from the first one due to the chain rule. At the critical point, Eq. (4) is complemented by the condition

$$\left(\frac{\partial^2 T_s}{\partial s^2} \right)_{\mu_B} = 0 \quad \Rightarrow \quad \left(\frac{\partial^2 T_s}{\partial T_0^2} \right)_{\mu_B} = 0. \quad (5)$$

The new expansion is different from the commonly used Taylor expansion method. The Taylor method expands the thermodynamic observable of interest, typically the pressure, in powers of μ_B directly. A truncated Taylor expansion does not permit the crossings of lines of constant observable and cannot satisfy the condition (1) for the CP. Our new expansion does not expand the observable directly, but rather provides a scheme to reconstruct the observable (here, the entropy density) via Eq. (2). As we show below, it can satisfy Eq. (1) even at leading $\mathcal{O}(\mu_B^2)$ expansion order.

Expansion coefficient α_2 . The expansion coefficients α_{2n} are evaluated at $\mu_B = 0$ and can be expressed in terms of other thermodynamic quantities (see Supplemental Material for technical details [37]). The expression for α_2 reads

$$\alpha_2(T_0) = -\frac{2T_0\chi_2^B(T_0) + T_0^2\chi_2^{B'}(T_0)}{s'(T_0)}. \quad (6)$$

Here, $\chi_2^B = [\frac{\partial^2(p/T)}{\partial(\mu_B/T)^2}]_T$ is the baryon number susceptibility, $s(T_0)$ is the entropy density, and $'$ indicates the temperature derivative at a constant μ_B . In Eq. (6), we can see that the expansion coefficient contains not only the second derivative of pressure with respect to μ_B , but also the second derivative with respect to T and a mixed derivative with respect to T and (twice) μ_B . This reflects the fact that the present expansion is not taken at constant temperature, but rather on contours of constant entropy density—which curve toward

lower temperatures at higher values of μ_B —thus covering an extended region of the phase diagram. High-order coefficients α_{2n} involve high-order baryon number susceptibilities and high-order temperature derivatives. We restrict the present analysis to α_2 and leave the inclusion of higher order terms for the future when more precise lattice results become available.

Lattice QCD input. As follows from Eq. (6), the input quantities consist of the temperature dependencies of the entropy density $s(T)$ and baryon number susceptibility $\chi_2^B(T)$ at $\mu_B = 0$. Continuum extrapolated results from the Wuppertal-Budapest collaboration have been presented in Ref. [38] for the former and in Ref. [33] for the latter. As our analysis relies on temperature derivatives of these two quantities, we parametrize the lattice input, based on Ref. [39]. We extract the values of the parameters and their full covariance matrix from the lattice results, considering strong correlations between lattice data errors of neighboring temperatures, and use this covariance matrix for Monte Carlo sampling to perform error analysis. Additionally, we used smoothing splines to describe $s(T)$ and $\chi_2^B(T)$ and their temperature derivatives, as a more agnostic approach to employing the lattice input. The obtained results are consistent within errors to those obtained using the parametrization.

Entropy density at finite baryochemical potential. The left panel of Fig. 2 depicts the temperature dependence of the scaled entropy density s/T^3 at different values of μ_B/T : 1 (red), 1.5 (orange), 2 (purple), 2.5 (black), 3 (brown), and 3.5 (blue). The results are obtained within the new expansion described above at order $\mathcal{O}(\mu_B^2)$. As an important cross-check, we compare our results with state-of-the-art lattice QCD data from Ref. [33]. One can see excellent agreement of our results with lattice QCD. In addition, the hazy lines show the different Monte Carlo samples of our parametrization, reflecting the propagation of the errors from the lattice QCD input. The resulting spread shows good agreement with the error bars of the lattice data of Ref. [33].

The right panel of Fig. 2 shows the behavior of s/T^3 at larger chemical potentials, $\mu_B = 450$ MeV (red), 602 MeV (blue), and 750 MeV (orange). One can see the development of a distinct S-shaped, multivalued behavior of the entropy density as μ_B is increased to $\mu_B \gtrsim 600$ MeV—a hallmark feature of a first-order phase transition.

Critical point location. The CP is determined by a pair of equations $(\frac{\partial T_s}{\partial T_0})_{\mu_B} = 0$ and $(\frac{\partial^2 T_s}{\partial T_0^2})_{\mu_B} = 0$. At order $\mathcal{O}(\mu_B^2)$, the function $T_s(\mu_B; T_0)$ reads

$$T_s(\mu_B; T_0) = T_0 + \alpha_2(T_0) \frac{\mu_B^2}{2}. \quad (7)$$

Denoting $\mu_{B,c}$ and $T_{0,c}$ as the values of μ_B and T_0 corresponding to the CP, the first equation, $(\frac{\partial T_s}{\partial T_0})_{\mu_B} = 0$, yields the relationship between $\mu_{B,c}$ and $T_{0,c}$:

$$1 + \alpha_2'(T_{0,c}) \frac{\mu_{B,c}^2}{2} = 0 \quad \Rightarrow \quad \mu_{B,c} = \sqrt{-\frac{2}{\alpha_2'(T_{0,c})}}. \quad (8)$$

This equation also determines the spinodal lines at $\mu_B > \mu_{B,c}$ where it has two solutions with respect to T_0 .

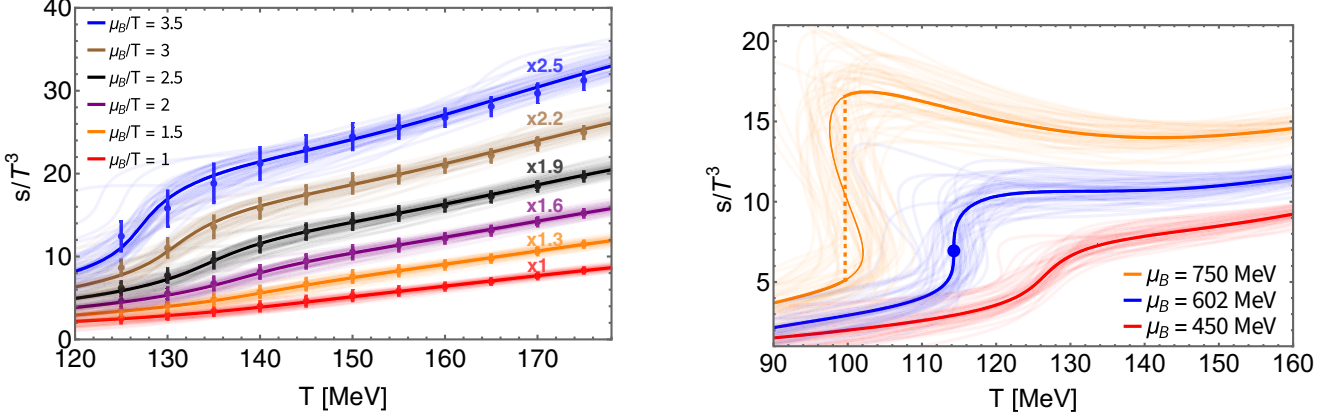


FIG. 2. Left panel: Scaled entropy density as a function of the temperature for different slices of constant μ_B/T . The curves are multiplied by different factors to avoid plotting overlap. Lattice QCD data points with error bars from Ref. [33] are also shown by the dots and vertical lines. Right panel: Scaled entropy density as a function of the temperature for different slices of constant baryochemical potential, $\mu_B = 450$ MeV (red lines, crossover regime), $\mu_B = 602$ MeV (blue lines, critical regime), $\mu_B = 750$ MeV (orange lines, mixed phase regime). The blue circle shows the critical point. In both panels, the solid curves correspond to the mean parametrization of lattice QCD results, while the translucent curves reflect the error propagation of the lattice QCD input via Monte Carlo sampling.

The second equation, $(\frac{\partial^2 T_c}{\partial \mu_B^2})_{\mu_B} = 0$, corresponds to

$$\alpha_2''(T_{0,c}) = 0, \quad (9)$$

which determines $T_{0,c}$. The determination of the CP location thus proceeds by solving Eq. (9) for $T_{0,c}$, plugging the result into Eq. (8) to determine $\mu_{B,c}$, and computing $T_c = T_s(\mu_{B,c}; T_{0,c})$ through Eq. (7). At $\mu_B > \mu_{B,c}$, the Maxwell construction of equal areas for entropy density can be utilized to determine the phase coexistence line.

Utilizing the above equations and a parametrized lattice QCD input, we obtain $T_{0,c} = 140.9 \pm 2.0$ MeV, and

$$(T_c, \mu_{B,c}) = (114.3 \pm 6.9, 602.1 \pm 62.1) \text{ MeV} \quad (10)$$

for the CP location in the T - μ_B plane. The uncertainties in T_c and $\mu_{B,c}$ are (anti)correlated, with a Pearson correlation coefficient of -0.91 . The uncertainties correspond exclusively to the Gaussian error propagation of the lattice QCD input and do not incorporate the truncation error due to terminating the expansion (7) at $\mathcal{O}(\mu_B^2)$. Furthermore, our expansion yields a CP in the mean-field universality class, as opposed to the 3D Ising universality class expected in QCD. Our estimate thus contains additional systematic error due to the mean-field approximation. One can expect the size of this uncertainty to reflect the size of the critical scaling region.

The obtained CP is shown in the T - μ_B plane in Fig. 3, together with the spinodals and the coexistence line. The shown covariance ellipse corresponds to the 68% confidence interval. We also crosscheck the error estimate by sampling the (Gaussian) log-likelihood of our lattice QCD input parametrization and depicting the resulting spread of critical points by hazy gray circles in Fig. 3.

The obtained CP location is consistent with the possible continuation of the chiral crossover line [2], shown in Fig. 3 by the green band at $\mu_B < 400$ MeV. In the context of heavy-ion collisions, the CP should lie above the chemical freeze-out line where the hadron resonance gas (HRG) model descrip-

tion of hadron yields applies. The chemical freeze-out bound on the lowest CP temperature obtained in Ref. [40] under $\mu_Q = \mu_S = 0$ conditions appropriate for our analysis is shown in Fig. 3 by the orange line, and it lies ≈ 10 MeV below our CP estimate. Based on the μ_B dependence of the chemical freeze-out curve [41], we find that the collision energy range of $\sqrt{s_{NN}} = 4$ –6 GeV is in the closest vicinity to our CP location estimate.

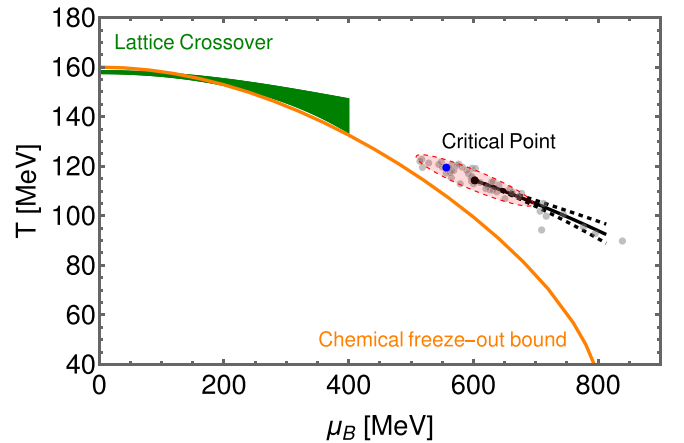


FIG. 3. The solid points depict the location of the QCD critical point extracted using the contours of constant entropy density based on lattice QCD data using either parametrization (black) or smoothing splines (blue). The dashed ellipse corresponds to the 68% confidence interval and reflects the error propagation of the lattice input. The hazy gray points show the scatter of the critical points obtained by sampling the lattice QCD input. The solid and dashed black lines depict the coexistence curve and the spinodals, respectively. The green band shows the chiral crossover line from [2]. The orange line corresponds to the heavy-ion chemical freeze-out bound on the CP from [40].

Our results are generally compatible with other CP estimates in recent literature based on different approaches, such as the Dyson-Schwinger equation, functional renormalization group, or holography [9–12]. Estimates from Lee-Yang edge singularity analyses yield somewhat smaller T or μ_B values [13,14], however they rely on high-order baryon susceptibilities that lack continuum-extrapolated data and have large systematic uncertainties and cut-off effects. Our value of T_c is considerably lower, however, than suggested by a recent finite-size scaling analysis of proton cumulants in heavy-ion collisions [26].

Consistency checks. Lattice QCD groups typically use splines to perform analyses on higher order fluctuations [5,42]. We performed a corresponding check by using a smoothed spline interpolation of the lattice data for $s(T)$ and $\chi_2^B(T)$ to calculate $\alpha_2(T)$ and determine the CP (see Supplemental Material) [37], which is found at $(T_c, \mu_{B,c}) = (119.5 \pm 5.4, 556.5 \pm 49.8)$ MeV. The mean value is inside the 68% confidence ellipse obtained with the parametrization, as shown by the blue point in Fig. 3.

As a different check, we utilized the parametrization of the $\mu_B = 0$ equation of state from the HotQCD Collaboration from Ref. [34] in place of our parametrization for the entropy density. Using the mean values of the parameters, we obtain a CP located at $(T_c, \mu_{B,c}) = (111.4, 634.8)$ MeV, inside our 68% confidence region.

We also checked the performance of our method against solvable models. The holographic model [35] constrained by lattice QCD data contains a CP at $(T_c, \mu_{B,c}) \approx (103, 599)$ MeV [12]. When we use the input for $s(T_0)$ and $\chi_2^B(T_0)$ to determine $\alpha_2(T_0)$ in the same holographic model, we obtain a CP at $(T_c, \mu_{B,c}) \approx (104.5, 637.6)$ MeV corresponding to a relative error of 1.4% and 6.4% in T_c and $\mu_{B,c}$, respectively.

Another useful test is provided by the ideal HRG model, which has no CP. Thus, this model can be used to test whether our method may produce a fake signature of a CP. Utilizing an open source HRG code [43], we tested that the CP equation $\alpha_2''(T_{0,c}) = 0$ [Eq. (9)] is not satisfied at any temperature T_0 in the range of $120 < T_0 < 200$ MeV in the ideal HRG model.

Roberge-Weiss endpoint. Equation (6) determining the value of T_0 corresponding to a CP permits a second solution at $\tilde{T}_{0,c} = 160.2 \pm 1.8$ MeV. The value of $\alpha_2'(\tilde{T}_{0,c})$ at this temperature is positive and, therefore, the critical chemical potential $\tilde{\mu}_{B,c}$ given by Eq. (8) is purely imaginary. By calculating the critical temperature through Eq. (7) we obtain

$$\tilde{T}_c = 197.1 \pm 7.1 \text{ MeV}, \quad \frac{\tilde{\mu}_{B,c}}{\tilde{T}_c} = i(3.50 \pm 0.30). \quad (11)$$

Remarkably, this value is close to where the Roberge-Weiss (RW) transition endpoint [44–46] is expected in the plane of temperature and imaginary chemical potential, namely $T_{RW} = 208 \pm 5$ MeV [47,48], and $\mu_{B,RW}/T_{RW} = i\pi$ [44]. One should note that our new expansion does not explicitly incorporate the RW periodicity $\mu_B \rightarrow \mu_B + i2\pi T$ of the partition function and the exact relation between the extracted CP at imaginary μ_B and the RW endpoint requires further investigations.

Conclusion. In this Letter, a novel method for determining the QCD critical point is proposed, based on the crossings of constant entropy density contours. The expansion (3) of the $s = \text{const.}$ line away from $\mu_B = 0$ is utilized, which overcomes the shortcomings of other expansion schemes by permitting the description of a CP and a first-order phase transition already at the leading expansion order. Utilizing this novel expansion at order $\mathcal{O}(\mu_B^2)$ and the continuum-extrapolated lattice QCD data from the Wuppertal-Budapest collaboration for entropy density $s(T)$ and baryon susceptibility $\chi_2^B(T)$ as input, we find evidence for the presence of the QCD critical point in the QCD phase diagram at $(T_c, \mu_{B,c}) = (114.3 \pm 6.9, 602.1 \pm 62.1)$ MeV with the quoted uncertainty corresponding exclusively to the propagation of lattice QCD errors for second order baryon susceptibility and entropy density. It remains to be seen whether the CP estimate obtained here at $\mathcal{O}(\mu_B^2)$ order reflects the true QCD CP or a breakdown of the expansion for unrelated reasons.

The analysis can be improved in the future by utilizing more precise lattice input. The CP is determined by the behavior of the coefficient α_2 and its temperature derivatives, which are expressed in terms of entropy density and baryon susceptibility. Therefore, a more direct determination of α_2 and its temperature derivatives on the lattice can significantly improve the statistical error. Analysis of the truncation error due to using the expansion (3) at order $\mathcal{O}(\mu_B^2)$ is also important, and how it may modify the result will depend on the convergence properties of the expansion. Such an analysis will require the accurate determination of higher-order susceptibilities such as χ_4^B , and high-order temperature derivatives, which are presently not all available in the continuum limit from lattice QCD. Alternatively, one could utilize direct extrapolations of entropy density contours from zero and imaginary μ_B toward positive, real μ_B , and analyze the possible emergence of their crossings. Given that the extracted values of $T_{0,c}$ are of order $T_{0,c} \approx 140$ MeV, and the analysis relies on temperature derivatives, it is suggestive to extend lattice QCD simulations to lower temperatures, down to $T \approx 120$ MeV.

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Data availability. The data supporting the findings of this study are publicly available in Zenodo at [49].

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