One-, two-, and three-dimensional photon femtoscopy

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Femtoscopy with photon pairs is a particularly attractive tool for studying high-energy nuclear collisions. Proposed and extensively discussed in several influential theory articles, it has seen only a few applications in experiment because of statistics limitations. With the progress of detectors and electronics, only now is it coming within reach. In this paper we discuss the choice of kinematic variables for two-photon correlation functions. In particular, we argue against $C(Q_{\mathrm{inv}})$ and in favor of $C(\Delta E, Q_{\mathrm{inv}})$.

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I. INTRODUCTION

Femtoscopy, the analysis of momentum correlations between pairs of particles, is a powerful and well-established technique for assessing the geometry of particle-emitting systems created in energetic hadronic and nuclear collisions [1]. It is most commonly applied to pions, for which the correlations are dominated by the Bose-Einstein (BE) enhancement of pairs of identical bosons with small relative momenta. The width of the BE peak located at zero relative momentum is inversely proportional to the width of the distribution of pions of a given momentum at the point of their last scattering (kinetic freeze-out). This, along with momentum spectra, gives access to the final stage of the collision.

Even more valuable information can be obtained from the analysis of direct photons, which are produced mostly at the beginning of the collision and do not scatter subsequently. Unfortunately, these photons are buried under a background of photons coming from the decays of hadrons, mostly π^0. The decay photons dilute the correlation function, reducing the amplitude of the BE peak by a factor of f^2, where f is the fraction of direct photons. One could argue that this does not spoil the measurement of the geometry (width of the peak), and the reduction of the peak height can even be used to measure the direct-photon yield, complementing the standard method based on the cocktail subtraction [2]. The price for this, however, is the necessity to deal with large statistical uncertainties coming from the background photons.

In this situation, the choice of the data representation is particularly important. In the experimental photon femtoscopy studies published so far the invariant momentum difference Q_{inv} (see below for definition), well established in identical-hadron femtoscopy, has been adopted. Our central claim is that the one-dimensional correlation function $C(Q_{\mathrm{inv}})$ is not optimal for photon femtoscopy and should be replaced by a two-dimensional analysis.

II. PHOTON FEMTOSCOPY IN THEORY

The BE correlations between photons have been the subject of several seminal theoretical articles. In Ref. [3] by Neuhauser, the two-photon correlation functions are considered in two dimensions, as a function of their opening angle and the energy difference. Sinyukov [4] discussed a longitudinally expanding source of photons and analyzed the correlation function in transverse (to the beam axis) and longitudinal momentum difference. Srivastava and Kapusta [5] studied the shape of the two-photon correlation functions vs the rapidity and transverse momentum differences as well as the azimuthal opening angle. In the subsequent papers [6,7] they switched to the Bertsch-Pratt [8,9] coordinates, which entered the scene around that time and became the de facto standard axes in pion femtoscopy. The relative momentum here is decomposed into

$$\mathbf{q} = \mathbf{p}_2 - \mathbf{p}_1 = (q_{\mathrm{out}}, q_{\mathrm{side}}, q_{\mathrm{long}}) \tag{1}$$

with the "out" axis pointing along the pair transverse momentum, the "side" axis perpendicular to it in the transverse plane, and the "long" axis along the beam. This three-dimensional coordinate system was also used by Bass, Muller, and Srivastava [10], Frodermann and Heinz [11], and Alam *et al.* [12]. The latter article also shows one-dimensional correlation functions $C(Q_{\mathrm{inv}})$,

$$Q_{\mathrm{inv}} = \sqrt{-\left(p_2^\mu - p_1^\mu\right)^2}, \tag{2}$$

albeit under the condition that $q_{\mathrm{out}} = q_{\mathrm{long}} = 0$.

While the two-particle correlation functions of identical and nonidentical hadrons exhibit structures coming from quantum statistics and Coulomb and strong interactions at small relative velocities (two particles "flying together" in the laboratory), for photon pairs the situation is different. The photon velocity in vacuum does not depend on the photon energy and thus any two photons emitted close to each other

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in space-time and in the same direction "fly together" even if they differ in energy. In this situation one might wonder if the BE enhancement occurs also for such pairs. Intuition suggests that this should not be the case, and indeed in all the above theory papers the BE peak is restricted to small $q_{\rm out}$.

III. PHOTON FEMTOSCOPY IN EXPERIMENT

The experimental data on two-photon Bose-Einstein correlations are scarce. The first analysis was performed on data collected in heavy-ion collisions at 40–60 MeV per nucleon (i.e., well below the π^0 production threshold) by TAPS at GANIL [13,14]. The correlation function was represented as $C(q, q_0)$ with q_0 defined as the energy difference $q_0 = \Delta E = E_2 - E_1$. In order to reduce the statistical uncertainties, the q_0 dependence was dropped, arguing that the energy spectrum of direct photons is exponential, hence q_0 is small, and thus the averaging over q_0 does not destroy the signal. A significant fraction of photons originated from decays of subthreshold π^0, as clearly seen in the invariant mass spectrum. Since for photons $Q_{\rm inv} = M_{\rm inv}$, for the sake of keeping the π^0 peak narrow the authors went one step further and switched from $C(q)$ to $C(Q_{\rm inv})$. The same representation was used in a subsequent centrality dependent analysis of the same collision system with increased statistics [15].

At the CERN Super Proton Synchrotron (SPS), the WA98 collaboration published a two-photon $C(Q_{\rm inv})$ measured in central Pb–Pb collisions [16,17]. The fitted correlation strength (Bose-Einstein peak height) λ was used to calculate the direct-photon fraction.

The STAR collaboration at the BNL Relativistic Heavy Ion Collider (RHIC) presented a preliminary $C(Q_{\rm inv})$ from Au-Au collisions in the QM2005 conference [18]. In QM2006, the PHENIX collaboration showed their preliminary results [19]. Also in this case $C(Q_{\rm inv})$ has been used.

IV. ABSCISSA: CHOICE OF THE KINEMATIC VARIABLES

As shown above, the theory predictions and the experimental measurements of two-photon correlation functions so far were performed in terms of $C(q_{\rm out}, q_{\rm side}, q_{\rm long})$ and $C(Q_{\rm inv})$, respectively. The measurements were plagued by statistical uncertainties resulting from small number of events at GANIL and from π^0 decay background at the SPS and above. The representation chosen by experimentalists, $C(Q_{\rm inv})$, keeps the π^0 peak narrow and exposes the flat part of the correlation function between the Bose-Einstein peak and π^0 peak, facilitating the normalization of the correlation function [14]. Let us look more closely at the meaning of $Q_{\rm inv}$. We start from the quantities measured in the experiment, the photon momenta (i.e., energies) p_1, p_2, and the pair opening angle α. The invariant momentum difference $Q_{\rm inv}$ coincides with the invariant mass and is equal to

$$Q_{\rm inv} = \sqrt{2\,p_1\,p_2\,(1-\cos\alpha)} \tag{3}$$

$$= 2\,\sqrt{p_1\,p_2}\,\sin\frac{\alpha}{2}. \tag{4}$$

In the limits of collinear and back-to-back pairs it becomes

$$Q_{\rm inv} = \begin{cases} \sqrt{p_1 p_2}\,\alpha, & \text{for } \alpha \to 0, \\ 2\sqrt{p_1 p_2} = 2p, & \text{for } \alpha \to \pi. \end{cases} \tag{5}$$

As one can see, $Q_{\rm inv}$ is not directly sensitive to the energy difference between the two photons. For pairs of massive identical particles a vanishing $Q_{\rm inv}$ implies $\mathbf{p}_1 = \mathbf{p}_2$. For photons it does not. $Q_{\rm inv}$ mostly reflects $q_{\rm side}$ and $q_{\rm long}$ and is only weakly related to $q_{\rm out}$. This means that the Bose-Einstein peak, which sits at $q_{\rm out} = q_{\rm side} = q_{\rm long} = 0$, in $C(Q_{\rm inv})$ is diluted by uncorrelated pairs with large $q_{\rm out}$. This deficiency of $C(Q_{\rm inv})$ has been noticed (Refs. [16,17,19,20]; see also Eq. (38) in Ref. [1]) and the resulting reduction of the peak amplitude λ was estimated and corrected for by performing a simulation accounting for the detector acceptance, the shape of the photon spectrum, and the assumed $R_{\rm out}$[20]. The direct photon yield published by WA98 [16] was presented for two cases, $R_{\rm out} = 0$ and $R_{\rm out} = 6$ fm. The magnitude of this effect increases with the collision energy (see Appendix).

Preserving the full height of the BE peak and at the same time keeping the π^0 peak narrow is possible in a two-dimensional analysis

$$C(\Delta E, Q_{\rm inv}). \tag{6}$$

The additional axis represents the energy difference between the two photons $\Delta E = p_2 - p_1$. It is easy to show that the length of the vector $(\Delta E, Q_{\rm inv})$ is equal to q:

$$\Delta E^2 + Q_{\rm inv}^2 = q_{\rm out}^2 + q_{\rm side}^2 + q_{\rm long}^2. \tag{7}$$

The representation (6) is similar to Ref. [3] except the opening angle is replaced by $Q_{\rm inv}$, and similar to Ref. [14] but with q replaced by $Q_{\rm inv}$. The suppression of the BE peak and the systematic uncertainty related to its correction are absent in this case.

Our representation (6) is closely related to

$$C\big(q_{\rm out}, \sqrt{q_{\rm side}^2 + q_{\rm long}^2}\big). \tag{8}$$

In order to demonstrate this, we first note that the components of the momentum difference $\mathbf{q}$ parallel and perpendicular to the pair momentum $\mathbf{K} := (\mathbf{p}_1 + \mathbf{p}_2)/2$ are

$$q_{||} = \frac{(\mathbf{p}_2 - \mathbf{p}_1)\cdot(\mathbf{p}_1 + \mathbf{p}_2)}{2\,K} = \frac{p_2^2 - p_1^2}{2\,K} \tag{9}$$

and

$$q_{\perp} = \frac{(\mathbf{p}_2 - \mathbf{p}_1)\times(\mathbf{p}_1 + \mathbf{p}_2)}{2\,K} = \frac{p_1\,p_2\,\sin\alpha}{K}. \tag{10}$$

Secondly, we eliminate p_1, p_2, α and get the direct relation between $(\Delta E, Q_{\rm inv})$ and $(q_{||}, q_{\perp})$:

$$Q_{\rm inv}^2 = \frac{1}{2}\left[q^2 - 4K^2 + \sqrt{(q^2 + 4K^2)^2 - (4Kq_{||})^2}\right],$$

$$\Delta E = \frac{2Kq_{||}}{\sqrt{Q_{\rm inv}^2 + 4K^2}} \tag{11}$$

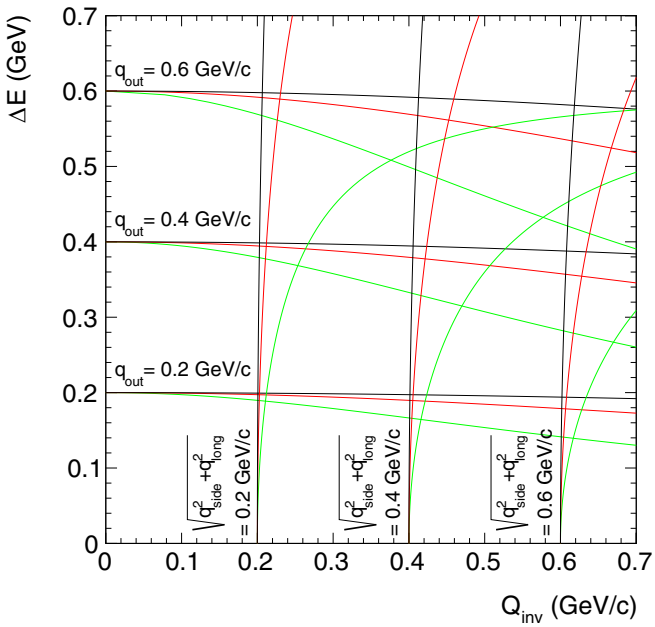

FIG. 1. Mapping of $(q_{\rm out}, \sqrt{q_{\rm side}^2 + q_{\rm long}^2})$ on the $(\Delta E, Q_{\rm inv})$ plane in LCMS. Lines of constant $q_{\rm out}$ and those of constant $\sqrt{q_{\rm side}^2 + q_{\rm long}^2}$ are shown in green, red, and black for the pair momentum K of 0.3, 0.6, and 1.2 GeV/c, respectively. In the region below 0.1 GeV/c, where the Bose-Einstein peak is expected to be located for direct photons from heavy-ion collisions, the two representations are very close to each other.

with $q^2 = q_{||}^2 + q_\perp^2 = q_{\rm out}^2 + q_{\rm side}^2 + q_{\rm long}^2$. The inverse transformation reads

$$q_{||} = \Delta E \sqrt{1 + \left(\frac{Q_{\rm inv}}{2K}\right)^2},$$
$$q_\perp = Q_{\rm inv} \sqrt{1 - \left(\frac{\Delta E}{2K}\right)^2}. \tag{12}$$

Third, in the longitudinally comoving coordinate system (LCMS), commonly used in femtoscopy [1], the pair momentum **K** is perpendicular to the beam direction, $K = K_{\rm T}$, and

$$q_{||} = q_{\rm out},$$
$$q_\perp = \sqrt{q_{\rm side}^2 + q_{\rm long}^2}. \tag{13}$$

The resulting relation between the $(\Delta E, Q_{\rm inv})$ coordinates and $(q_{\rm out}, \sqrt{q_{\rm side}^2 + q_{\rm long}^2})$ in LCMS is shown in Fig. 1. In the region below 0.1 GeV/c, where the Bose-Einstein peak is expected to be located for direct photons from heavy-ion collisions, the two sets of variables are very close to each other.

In order to make a more quantitative statement, let us look at the shape of the BE peak in both representations. We start from a Gaussian in $C(q_{\rm out}, q_{\rm side}, q_{\rm long})$ [11]:

$$C = 1 + \lambda \exp\left(-R_{\rm out}^2 q_{\rm out}^2 - R_{\rm side}^2 q_{\rm side}^2 - R_{\rm long}^2 q_{\rm long}^2\right). \tag{14}$$

Its parameters can be directly extracted from a fit to $C(\Delta E, Q_{\rm inv})$:

$$C = 1 + \lambda \exp\left(-R_{\rm out}^2 \Delta E^2 - R_\perp^2 Q_{\rm inv}^2\right) \times \exp\left[\left(R_\perp^2 - R_{\rm out}^2\right)\left(\frac{\Delta E\, Q_{\rm inv}}{2K_{\rm T}}\right)^2\right]. \tag{15}$$

The value of $R_\perp$ obtained from the fit represents a weighted mean of $R_{\rm side}$ and $R_{\rm long}$. The weights are determined by the experiment's acceptance and its two-photon resolution. For example, in an experiment with narrow pseudorapidity coverage and broad azimuthal acceptance, $R_\perp$ will be closer to $R_{\rm side}$ than to $R_{\rm long}$. The exponent in the second line is close to unity for $q \ll K_{\rm T}$ and thus fitting the BE peak in $C(\Delta E, Q_{\rm inv})$ is practically equivalent to fitting it in $C(q_{\rm out}, \sqrt{q_{\rm side}^2 + q_{\rm long}^2})$, as already suggested by Fig. 1.

Around midrapidity, an additional practical advantage of the representation (6) is that the experimental effects related to the determination of the photon momentum and the opening angle act nearly separately on the two dimensions of the correlation function. The finite opening-angle resolution affects only $Q_{\rm inv}$, and ΔE is mostly affected by the energy resolution.

Similarly, correlated background may manifest mostly in one of the two dimensions. Not only the π^0 decay but also anisotropic flow and, for calorimeters, photon conversion and cluster splitting will result in background pairs with small opening angles but not necessarily close in energy.

At this point we would like to mention an intermediate stage between $C(\Delta E, Q_{\rm inv})$ and $C(q_{\rm out}, \sqrt{q_{\rm side}^2 + q_{\rm long}^2})$, namely,

$$C(q_{\rm out}, Q_{\rm inv}). \tag{16}$$

This representation spares the transformation between ΔE and $q_{\rm out}$. It is, however, less elegant in that only at low ΔE and $Q_{\rm inv}$ the vector $(\Delta E, 0)$ is orthogonal to $(0, Q_{\rm inv})$ and the length is preserved as in Eq. (7).

V. ORDINATE: STATISTICAL UNCERTAINTIES IN VARIOUS REPRESENTATIONS

The tendency to reduce the number of dimensions of the correlation function is common in experiments encountering large statistical uncertainties. Reducing the point-to-point scatter of the correlation function, be it by reducing the number of dimensions or by increasing the bin width, has indeed a great positive impact on the visual appearance of the correlation. However, it does not lead to a more precise fit. (If anything, an overly coarse binning can distort the fit result.) A properly performed fit—one based on the maximum likelihood method, assuming the Poissonian distribution of the number of counts, and taking its standard deviation not from the numerator but from the denominator multiplied by the fit function—can perfectly well deal with low numbers of counts per bin, including empty bins [21,22]. One-dimensional plots of the correlation function appropriate for visual inspection can be constructed as usual by building a ratio between the projections of the numerator and denominator. The fit function will be visualized by plotting the projection of the denominator multiplied by the fit, divided by the projection of

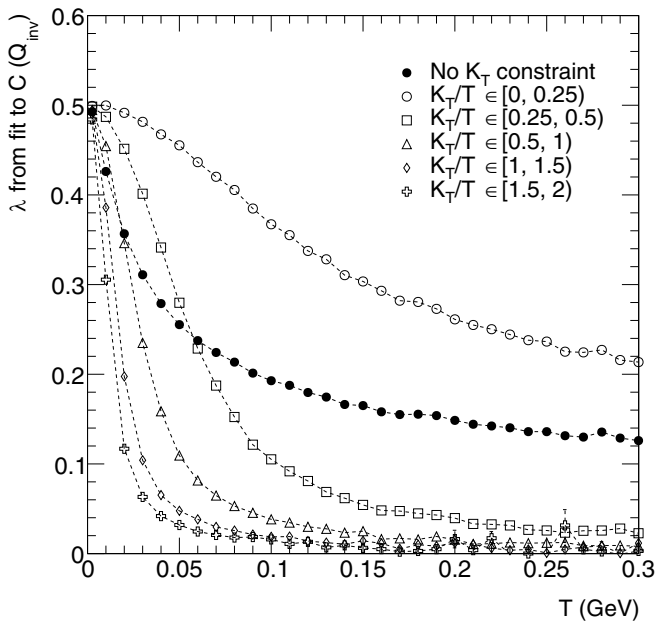

FIG. 2. The correlation strength λ obtained by fitting a Gaussian to $C(Q_{\rm inv})$ (Monte Carlo simulation). The reduction of λ is particularly strong at high collision energies and high pair momenta.

the denominator. Whenever such projections are made, cuts on the other dimensions will be applied such as to maximize the BE signal. Following these standard procedures, the additional dimension in $C(\Delta E, Q_{\rm inv})$, compared to $C(Q_{\rm inv})$, will come at no cost.

VI. CONCLUSION

We advocate using the two-dimensional correlation function $C(\Delta E, Q_{\rm inv})$ in LCMS for analysis of Bose-Einstein correlations of photons in heavy-ion collisions. This representation avoids the dilution of the BE peak happening when the one-dimensional $C(Q_{\rm inv})$ is used. At the same time it preserves the advantage of the latter, namely it keeps the π^0 peak narrow. The fit precision will not be affected by the additional dimension.

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DATA AVAILABILITY

The data represented in Figs. 1 and 2 are publicly available [23].

APPENDIX: DILUTION OF THE BOSE-EINSTEIN PEAK IN THE $C(Q_{\rm inv})$ REPRESENTATION

We estimated the reduction of the peak height λ in $C(Q_{\rm inv})$ by a Monte Carlo simulation. We sampled photons from

$$\frac{{\rm d}N}{p_{\rm T}\,{\rm d}p_{\rm T}\,{\rm d}y\,{\rm d}\varphi} \propto \frac{1}{\exp(p_{\rm T}/T)-1} \tag{A1}$$

within $|y|<1$ and $0\leqslant\varphi<2\pi$. To each photon pair we assigned a weight corresponding to the three-dimensional correlation function

$$C = 1 + 0.5\exp\left(-R^2_{\rm out}q^2_{\rm out} - R^2_{\rm side}q^2_{\rm side} - R^2_{\rm long}q^2_{\rm long}\right) \tag{A2}$$

with $R_{\rm out}=R_{\rm side}=R_{\rm long}=6$ fm. We histogrammed the pairs with and without the weight factor to obtain, respectively, the numerator and denominator of $C(Q_{\rm inv})$. As expected, the correlation strength λ (peak height) obtained by fitting a Gaussian to $C(Q_{\rm inv})$ is reduced and the effect is particularly strong at high T (high collision energy) and high pair transverse momentum $K_{\rm T}$ (Fig. 2).

[1] M. A. Lisa, S. Pratt, R. Soltz, and U. Wiedemann, Femtoscopy in relativistic heavy ion collisions: Two decades of progress, Annu. Rev. Nucl. Part. Sci. **55**, 357 (2005).
[2] M. M. Aggarwal *et al.* (WA98 Collaboration), Observation of direct photons in central 158A GeV $^{208}Pb+^{208}Pb$ collisions, Phys. Rev. Lett. **85**, 3595 (2000).
[3] D. Neuhauser, Photon interferometry for heavy ion collisions, Phys. Lett. B **182**, 289 (1986).
[4] Y. M. Sinyukov, Hanbury Brown/Twiss correlations for expanding hadron and quark-gluon matter, Nucl. Phys. A **498**, 151 (1989).
[5] D. K. Srivastava and J. I. Kapusta, Photon interferometry of quark-gluon dynamics, Phys. Lett. B **307**, 1 (1993).
[6] D. K. Srivastava and J. I. Kapusta, History of quark-gluon plasma evolution from photon interferometry, Phys. Rev. C **48**, 1335 (1993).
[7] D. K. Srivastava and J. I. Kapusta, Nature of the photon correlation function for quark-gluon plasma, Phys. Rev. C **50**, 505 (1994).
[8] G. F. Bertsch, Pion interferometry as a probe of the plasma, Nucl. Phys. A **498**, 173 (1989).
[9] S. Pratt, Pion interferometry of quark-gluon plasma, Phys. Rev. D **33**, 1314 (1986).
[10] S. A. Bass, B. Muller, and D. K. Srivastava, Photon interferometry of Au + Au collisions at the BNL Relativistic Heavy-Ion Collider, Phys. Rev. Lett. **93**, 162301 (2004).
[11] E. Frodermann and U. Heinz, Photon Hanbury-Brown–Twiss interferometry for noncentral heavy-ion collisions, Phys. Rev. C **80**, 044903 (2009).
[12] J. E. Alam, B. Mohanty, P. Roy, S. Sarkar, and B. Sinha, Photon interferometry and size of the hot zone in relativistic heavy ion collisions, Phys. Rev. C **67**, 054902 (2003).

[13] M. Marqués *et al.*, Hard photon intensity interferometry in heavy ion reactions, Phys. Rev. Lett. **73**, 34 (1994).
[14] F. M. Marques, G. Martinez, T. Matulewicz, R. W. Ostendorf, and Y. Schutz, Two-photon correlations: From Young's experiments to heavy-ion collision dynamics, Phys. Rep. **284**, 91 (1997).
[15] K. Piasecki and T. Matulewicz, Two-photon interferometry in Ta + Au collisions at 40*A* MeV, Acta Phys. Polon. B **37**, 175 (2006).
[16] M. M. Aggarwal *et al.* (WA98 Collaboration), Interferometry of direct photons in central ^{208}Pb + ^{208}Pb collisions at 158 *A* GeV, Phys. Rev. Lett. **93**, 022301 (2004).
[17] D. Peresunko (WA98 Collaboration), Interferometry of direct photons in ^{208}Pb + ^{208}Pb collisions at 158 *A* GeV, J. Phys. G: Nucl. Part. Phys. **30**, S1065 (2004).
[18] D. Das *et al.* (STAR Collaboration), Preliminary results on direct photon-photon HBT measurements in $\sqrt{s_{\rm NN}} = 62.4$ GeV and 200 GeV Au + Au collisions at RHIC, Nukleonika **51**, S55 (2006).
[19] D. Peressounko (PHENIX Collaboration), Bose–Einstein correlations of direct photons in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV, Int. J. Mod. Phys. E **16**, 2235 (2007).
[20] D. Peressounko, Direct photon interferometry, AIP Conf. Proc. **828**, 581 (2006).
[21] D. Miśkowiec *et al.* (E877 Collaboration), Two-particle correlations in Au + Au collisions at AGS energy, Nucl. Phys. A **610**, 227 (1996).
[22] L. Ahle *et al.* (E802 Collaboration), System, centrality, and transverse mass dependence of two-pion correlation radii in heavy ion collisions at 11.6*A* and 14.6*A* GeV/c, Phys. Rev. C **66**, 054906 (2002).
[23] D. Miśkowiec and K. Reygers, One-, two-, and three-dimensional photon femtoscopy, arXiv:2602.12184.