

Anisotropic flows of light-flavor and charmed hadrons in Pb+Pb collisions at energies available at the CERN Large Hadron Collider

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We apply a constituent quark equal-velocity combination (EVC) model to study the elliptic flow (v_2) and triangular flow (v_3) of light-flavor and single-charmed hadrons in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV. $v_{2,3}$ of hadrons in the EVC model can be expressed as a linear superposition of the $v_{2,3}$ of quarks at the same velocity as that of the hadrons. We find that available experimental data for v_2 and v_3 of p , Λ , Ξ , and Ω as a function of transverse momentum (p_T) can be consistently explained by the EVC formula using the v_2 of up/down quarks and the v_2 of strange quarks. In comparison with v_2 data of ϕ at $\sqrt{s_{NN}} = 2.76$ TeV, which can be naturally explained by the v_2 of strange quarks, explanation of the data of ϕ mesons at $\sqrt{s_{NN}} = 5.02$ TeV requires an additional contribution of two-kaon coalescence, which indicates approximately 20% influence of final-state hadronic rescattering on v_2 of ϕ at $\sqrt{s_{NN}} = 5.02$ TeV. Using v_2 and v_3 of light-flavor quarks obtained in studying light-flavor hadrons and v_2 and v_3 of charm quarks determined from D^0 meson data, we apply the EVC model to predict the anisotropic flows of D_s^+ , Λ_c^+ , Ξ_c^0 , and Ω_c^0 and compare them with the available experimental data. The preliminary data for v_2 of D^0 , D_s^+ , and Λ_c^+ at $\sqrt{s_{NN}} = 5.36$ TeV are found to be naturally described by the EVC model.

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I. INTRODUCTION

In ultrarelativistic heavy-ion collisions, a state of deconfined quarks and gluons is formed, known as the quark-gluon plasma (QGP) [1,2]. In noncentral collisions, the initial anisotropic collision geometry causes anisotropic expansion of the QGP in the transverse momentum plane, which in turn leads to anisotropic emission of particles in transverse momentum space [3,4]. This anisotropic property is measured in experiments by the harmonic flow coefficients $v_n = \langle \cos[n(\varphi - \Psi_n)] \rangle$ in the Fourier expansion of the transverse momentum distribution of hadrons [4,5].

There have already been many measurements of the anisotropic flow of particles produced in relativistic heavy-ion collisions at the Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC). These measurements provide rich and precise data for v_2 , v_3 , and higher-order flows of light-flavor and heavy-flavor hadrons. These data and the relevant theoretical studies reveal lots of important physical properties of the QGP created in ultrarelativistic

heavy-ion collisions [6–10]. An interesting phenomenon related to the hadronization of the QGP is that v_2 data of light-flavor hadrons (e.g., π , K , p , Λ) exhibit a remarkable number-of-constituent-quark (NCQ) scaling [10–14]. This scaling behavior is an indication of the quark recombination mechanism at QGP hadronization. The NCQ effect is particularly pronounced in the intermediate transverse momentum region ($2 \lesssim p_T \lesssim 6$ GeV/c), where the enhanced baryon-to-meson ratio is also observed and can be naturally explained by the quark combination models [15–21]. Recent measurements of charmed hadrons (such as D^0 , D_s^+ , and Λ_c^+) in Pb+Pb collisions by CMS and ALICE Collaborations have shown that their anisotropic flow exhibits similar behavior to that of light-flavor hadrons [22–28], suggesting that charm quarks may undergo partial thermalization and participate in the collective expansion of the QGP.

The ALICE and CMS Collaborations have reported the precise measurement of the v_2 and v_3 of light-flavor and single-charmed hadrons in Pb+Pb Collisions at the LHC energies [13,14,22–25,29,30]. These experimental data have been studied in various theoretical models and event generators such as the AMPT model [31,32], the viscous hydrodynamic model [33,34], the VISHNU hybrid model [35], the Boltzmann transport equation [36], the blast-wave-Tsallis-power model (BWTPM) [37], and the HYDJET++ model [38,39]. In this paper, we apply an equal-velocity quark combination model [40–44] to systematically study v_2 and v_3 of light-flavor and single-charmed hadrons in Pb+Pb Collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV, and we put particular emphasis on the self-consistency of the quark combination mechanism

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in explaining the experimental data of v_2 and v_3 of different hadrons measured by the CMS and ALICE collaborations. The equal-velocity combination of constituent quarks gives simple formulas for hadronic flow by the linear superposition of flow of quarks, which are convenient for testing by experimental data and meanwhile are also quite visible in explaining/exhibiting the correlations among flow of different hadrons.

The paper is organized as follows. In Sec. II, we present the analytical formulas of the anisotropic flow hadrons in a constituent quark equal-velocity combination model and show the v_2 and v_3 of hadrons as the linear superposition of quark flows. In Sec. III, we apply the EVC model to study the v_2 and v_3 of light-flavor hadrons ϕ , Λ , Ξ , Ω , and p in different centralities at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV. In Sec. IV, we study the v_2 and v_3 of single-charmed hadrons D^0 , D_s^+ , Λ_c^+ , Ξ_c^0 , and Ω_c^0 , and discuss the scaling property of $v_{2,3}$ of light-flavor quarks and charm quarks. Finally, the summary is given in Sec. V.

II. THE ANISOTROPIC FLOWS OF HADRONS IN QCM WITH EVC

In this section, we employ a constituent quark combination model (QCM) under the equal-velocity combination (EVC) approximation [16,40,44–46] to describe the production of single-charmed hadrons and derive their elliptic flow (v_2) and triangular flow (v_3) at midrapidity in heavy-ion collisions. In the EVC approximation, the hadron is formed by the combination of constituent quarks and antiquarks with the same velocity as that of the hadron. In this ideal case, the momentum vectors of quarks are parallel to those of hadrons they

form, and we have $\mathbf{p}_q = x_q \mathbf{p}_h$, where x_q is momentum fraction of quark q in the hadron h . For the production of hadrons at midrapidity, we define the distribution function $f(p_T, \varphi) \equiv dN/dp_T d\varphi$, where p_T is transverse momentum and φ is the azimuthal angle. EVC mechanism constrains $p_{T,q} = x_q p_{T,h}$ and $\varphi_q = \varphi_h$. Following the spirit of EVC model, the momentum distribution function of single-charmed hadrons can be expressed as

$$f_{M_j}(p_T, \varphi) = \kappa_{M_j} f_{q_1}(x_1 p_T, \varphi) f_{\bar{q}_2}(x_2 p_T, \varphi), \quad (1)$$

$$f_{B_j}(p_T, \varphi) = \kappa_{B_j} f_{q_1}(x_1 p_T, \varphi) f_{q_2}(x_2 p_T, \varphi) f_{q_3}(x_3 p_T, \varphi). \quad (2)$$

Here, the coefficients κ_{M_j} and κ_{B_j} are independent of the transverse momentum p_T . The momentum fractions are defined as $x_{1,2} = m_{1,2}/(m_1 + m_2)$ for mesons $M_j(q_1 \bar{q}_2)$, under the momentum conservation condition $x_1 + x_2 = 1$, and $x_{1,2,3} = m_{1,2,3}/(m_1 + m_2 + m_3)$ for baryons $B_j(q_1 q_2 q_3)$, under the momentum conservation condition $x_1 + x_2 + x_3 = 1$. The constituent masses of quarks are adopted as $m_u = m_d = 0.3$ GeV, $m_s = 0.5$ GeV, and $m_c = 1.5$ GeV.

The azimuthal dependence of particle distribution function is expressed through the Fourier series expansion

$$f(p_T, \varphi) = f(p_T) \left[1 + 2 \sum_{n=1}^{\infty} v_n(p_T) \cos(n\varphi) \right], \quad (3)$$

where $f_q(p_T)$ is independent of the azimuthal angle φ . The anisotropic coefficient v_n is calculated by

$$v_n(p_T) = \frac{\int f(p_T, \varphi) \cos(n\varphi) d\varphi}{\int f(p_T, \varphi) d\varphi}. \quad (4)$$

Using Eqs. (1)–(3), we can obtain the $v_n(p_T)$ for the meson $M_j(q_1 \bar{q}_2)$ and baryon $B_j(q_1 q_2 q_3)$, and the expressions up to order v_n^3 are

$$\begin{aligned} v_{n,M_j} = \frac{\int f_{M_j}(p_T, \varphi) \cos(n\varphi) d\varphi}{\int f_{M_j}(p_T, \varphi) d\varphi} = v_{n,q_1} \left[1 + \sum_{k=1}^{\infty} \frac{v_{k,q_1}}{v_{n,q_1}} v_{n+k,\bar{q}_2} - 2 \sum_{k=1}^{\infty} v_{k,q_1} v_{k,\bar{q}_2} \right] \\ + v_{n,\bar{q}_2} \left[1 + \sum_{k=1}^{\infty} \frac{v_{k,\bar{q}_2}}{v_{n,\bar{q}_2}} v_{n+k,q_1} - 2 \sum_{k=1}^{\infty} v_{k,q_1} v_{k,\bar{q}_2} \right] + \sum_{k=1}^{n-1} v_{k,q_1} v_{n-k,\bar{q}_2} + O(v^4) \end{aligned} \quad (5)$$

and

$$\begin{aligned} v_{n,B_j} = v_{n,q_1} \left\{ 1 + \sum_{k=1}^{\infty} \frac{v_{k,q_1}}{v_{n,q_1}} (v_{n+k,q_2} + v_{n+k,q_3}) - 2 \sum_{k=1}^{\infty} (v_{k,q_2} v_{k,q_1} + v_{k,q_2} v_{k,q_3} + v_{k,q_1} v_{k,q_3}) \right. \\ \left. + \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} \frac{v_{k,q_1}}{v_{n,q_1}} v_{m,q_2} (v_{m+k+n,q_3} + v_{m+k-n,q_3}) \right\} \\ + v_{n,q_2} \left\{ 1 + \sum_{k=1}^{\infty} \frac{v_{k,q_2}}{v_{n,q_2}} (v_{n+k,q_3} + v_{n+k,q_1}) - 2 \sum_{k=1}^{\infty} (v_{k,q_2} v_{k,q_1} + v_{k,q_2} v_{k,q_3} + v_{k,q_1} v_{k,q_3}) \right. \\ \left. + \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} \frac{v_{k,q_2}}{v_{n,q_2}} v_{m,q_3} (v_{m+k+n,q_1} + v_{m+k-n,q_1}) \right\} \\ + v_{n,q_3} \left\{ 1 + \sum_{k=1}^{\infty} \frac{v_{k,q_3}}{v_{n,q_3}} (v_{n+k,q_1} + v_{n+k,q_2}) - 2 \sum_{k=1}^{\infty} (v_{k,q_2} v_{k,q_1} + v_{k,q_2} v_{k,q_3} + v_{k,q_1} v_{k,q_3}) \right. \\ \left. + \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} \frac{v_{k,q_3}}{v_{n,q_3}} v_{m,q_1} (v_{m+k+n,q_2} + v_{m+k-n,q_2}) \right\} \end{aligned}$$

$$\begin{aligned}
& + \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} \frac{v_{k,q_3}}{v_{n,q_3}} v_{m,q_1} (v_{m+k+n,q_2} + v_{m+k-n,q_2}) \Big\} \\
& + \sum_{k=1}^{n-1} (v_{k,q_1} v_{n-k,q_2} + v_{k,q_2} v_{n-k,q_3} + v_{k,q_3} v_{n-k,q_1}) + \sum_{k,m=1}^{m+k \leq n-1} v_{k,q_1} v_{m,q_2} v_{n-m-k,q_3} + O(v^4). \quad (6)
\end{aligned}$$

In the above formulas, we use the abbreviation v_{n,q_i} for $v_{n,q_i}(x_i p_T)$ with $i = 1, 2, 3$ and $v_{n,\bar{q}_2}$ for $v_{n,\bar{q}_2}(x_2 p_T)$.

From Eqs. (5) and (6), we see that v_n of hadrons have a general structure of the summation of v_n of quarks and the product of lower-order quark flows. There is a modification factor before $v_{n,q}$ which contains complex contributions of flows of quarks at various orders, even for low order flows of hadrons. In order to obtain simple formulas for hadronic flow at midrapidity, we should consider some approximations based on the properties of hadronic flow measured in relativistic heavy-ion collisions.

First, the experimental measurements show a very small value of $v_{1,h} \leq 10^{-3}$ at midrapidity in heavy-ion collisions at both RHIC and LHC energies, [47,48], which indicates $v_{1,q} \approx (\frac{1}{2} - \frac{1}{3})v_{1,h} \leq 10^{-3}$ and can be neglected in our analysis. Second, the rough estimation for $v_{2-5,q} \approx (\frac{1}{2} - \frac{1}{3})v_{2-5,h}$ is approximately 10^{-2} according to experimental data of $v_{2-5,h}$ [10–14]. Higher-power terms such as $(v_{n,q})^{4,5}$ become negligible ($\leq 10^{-8}$) and can be safely omitted from the calculations. Furthermore, we adopt a scaling relation $v_{n,q} = a_n v_{2,q}^{n/2}$ at the quark level, which has been indicated by both experimental and theoretical studies [49–55]. Using this relation, we can estimate that the ratio $v_{n+1,q_1} v_{n+1,q_2} / v_{n,q_1} v_{n,q_2} \sim 10^{-2}$ is very small. Consequently, correction terms with harmonic indices $k, m > 2$ in the modification factor of v_{n,q_i} can be safely neglected. The detailed derivation of these approximations is given in Ref. [45].

Based on the above considerations, the v_2 and v_3 of hadrons can be significantly simplified as follows:

$$v_{2,M_j} \approx v_{2,q_1} [1 + (a_4 - 2)v_{2,q_1} v_{2,\bar{q}_2}] + v_{2,\bar{q}_2} [1 + (a_4 - 2)v_{2,q_1} v_{2,\bar{q}_2}], \quad (7)$$

$$\begin{aligned}
v_{2,B_j} \approx & v_{2,q_1} \{1 + (a_4 - 2)(v_{2,q_1} v_{2,q_2} + v_{2,q_1} v_{2,q_3}) - 2v_{2,q_2} v_{2,q_3}\} \\
& + v_{2,q_2} \{1 + (a_4 - 2)(v_{2,q_1} v_{2,q_2} + v_{2,q_2} v_{2,q_3}) - 2v_{2,q_1} v_{2,q_3}\} \\
& + v_{2,q_3} \{1 + (a_4 - 2)(v_{2,q_1} v_{2,q_3} + v_{2,q_2} v_{2,q_3}) - 2v_{2,q_2} v_{2,q_1}\}, \quad (8)
\end{aligned}$$

$$v_{3,M_j} \approx v_{3,q_1} \left[1 + \left(\frac{a_5}{a_3} - 2\right)v_{2,q_1} v_{2,\bar{q}_2}\right] + v_{3,\bar{q}_2} \left[1 + \left(\frac{a_5}{a_3} - 2\right)v_{2,q_1} v_{2,\bar{q}_2}\right], \quad (9)$$

$$\begin{aligned}
v_{3,B_j} \approx & v_{3,q_1} \left\{1 + \left(\frac{a_5}{a_3} - 2\right)(v_{2,q_1} v_{2,q_2} + v_{2,q_1} v_{2,q_3}) - 2v_{2,q_2} v_{2,q_3}\right\} \\
& + v_{3,q_2} \left\{1 + \left(\frac{a_5}{a_3} - 2\right)(v_{2,q_1} v_{2,q_2} + v_{2,q_2} v_{2,q_3}) - 2v_{2,q_1} v_{2,q_3}\right\} \\
& + v_{3,q_3} \left\{1 + \left(\frac{a_5}{a_3} - 2\right)(v_{2,q_1} v_{2,q_3} + v_{2,q_2} v_{2,q_3}) - 2v_{2,q_2} v_{2,q_1}\right\}. \quad (10)
\end{aligned}$$

Finally, we obtain formulas of hadronic v_2 and v_3 that have the property of a relatively simple sum rule, although there are some nonlinear correction terms. In the following text, we can properly treat them according to the practical situation in relativistic heavy-ion collisions and obtain more simple forms of hadronic v_2 and v_3 . These simple formulas for v_2 and v_3 of hadrons can enable us to test the relationship between flows of different hadrons in a direct manner. In order to test the self-consistency of the model, we will extract quark flows from data of several hadrons and use them to calculate flows of other hadrons and compare results with experimental data in relativistic heavy-ion collisions at LHC energies. In addition, because the EVC model in the current version is suitable for describing the production of light-flavor hadrons in the low and intermediate p_T range by the combination of soft light-flavor quarks and that of charm hadrons by the combination of charm quarks and soft light-flavor quarks, we will compare model results with experimental data for light-flavor hadrons

in the range $p_T \lesssim 7 \text{ GeV}/c$ and for single-charm hadrons in the range $p_T \lesssim 8 \text{ GeV}/c$ in Pb+Pb collisions at LHC energies according to our previous study of p_T spectra of these hadrons [16].

In popular combination models using the Wigner wave function method [18,19,56], v_2 and v_3 of hadrons are not of such simple form due to the superposition of contributions from quarks with relative momentum intervals. The current EVC model does not explicitly consider such a momentum smearing effect. With the given quark anisotropic flows, v_2 and v_3 of hadrons under EVC model will be slightly higher than those considering the momentum smearing effect. Taking ϕ as an example, given a strange quark v_2 such as that in 30%–40% centrality in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76 \text{ TeV}$ shown in Fig. 1, our numerical calculations show that v_2 of ϕ under the EVC model is higher than that under a Gaussian momentum smearing function by about 3%–8% in the range $2 < p_{T,\phi} < 5 \text{ GeV}/c$ if we adopt a Gaussian combination

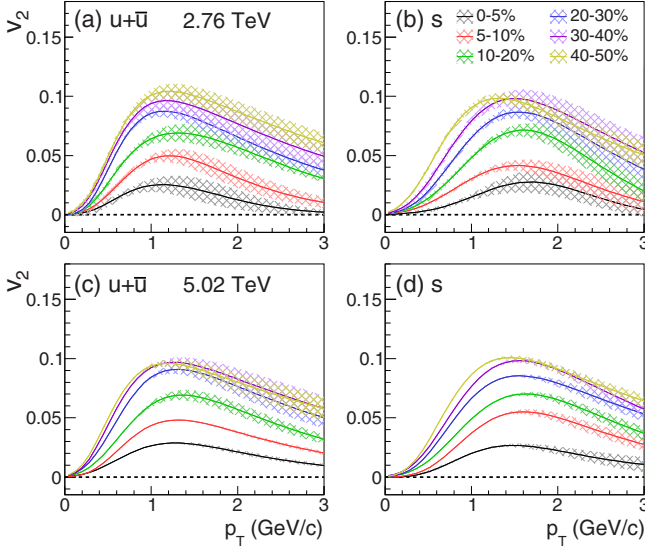


FIG. 1. v_2 of quarks as a function of p_T in different centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV.

radius of $0.3\text{--}0.4$ GeV/ c suggested in Ref. [56]. However, in our practical studies quark flows are extracted from experimental data of some hadrons and then are applied to other hadrons. This approach essentially simplifies the backward and forward propagation of the momentum smearing effect by effective v_2 and v_3 of quarks under the EVC model and brings the advantage that flows of hadrons can be written as simple analytic expressions and therefore can be conveniently and visibly tested by experimental data.

III. THE ANISOTROPY FLOW OF LIGHT-FLAVOR HADRONS

In this section, we employ the formulas of anisotropic flow of hadrons in EVC model presented above to study the p_T dependence of the v_2 and v_3 for light-flavor hadrons ϕ , Λ ($\Lambda + \bar{\Lambda}$), Ξ ($\Xi^- + \Xi^+$), Ω ($\Omega^- + \Omega^+$), and p ($p + \bar{p}$) at midrapidity in different centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV, and compare results with available experimental data of the ALICE Collaboration. Except for ϕ and Ω , experimental data for p , Λ , and Ξ usually contain certain decay contributions from heavier resonances. Considering that the main decay channel of baryon resonances is $B \rightarrow B' + \pi$, where pion mass is very small in comparison with that of the daughter and mother baryons, the decay kinematics leads to the result that v_2 and v_3 of the daughter baryon are very close to those of the mother baryon, and thus slight deviation of the flow information occurs during the decay process. Therefore, we neglect this decay influence and directly apply our formulas to these hadrons.

A. The elliptic flow of light-flavor hadrons

In the v_2 of light-flavor hadrons in Eqs. (7) and (8), there is the contribution from the quadratic term of quark v_2 . In a previous study [45], we estimated $a_4 \approx 2$ using the experimental data of v_2 of hadrons in Au+Au collisions at

$\sqrt{s_{NN}} = 200$ GeV [10–14]. The magnitude of $(a_4 - 2)v_{2,q}^2$ in the modification term is therefore very small, and its contribution can be safely neglected. Consequently, the expression for the v_2 of light-flavor hadrons can be simplified as follows:

$$v_{2,M_j}(p_T) \approx v_{2,q_1}(x_1 p_T) + v_{2,q_2}(x_2 p_T), \quad (11)$$

$$v_{2,B_j}(p_T) \approx v_{2,q_1}(x_1 p_T) + v_{2,q_2}(x_2 p_T) + v_{2,q_3}(x_3 p_T). \quad (12)$$

Using the simplified expressions from Eqs. (11) and (12), we obtain the following analytic formulas for the v_2 of the various hadrons:

$$v_{2,p}(p_T) = 3v_{2,u}(p_T/3), \quad (13)$$

$$v_{2,\Omega}(p_T) = 3v_{2,s}(p_T/3), \quad (14)$$

$$v_{2,\phi}(p_T) = 2v_{2,s}(p_T/2), \quad (15)$$

$$v_{2,\Lambda}(p_T) = 2v_{2,u}\left(\frac{1}{2+r_{su}}p_T\right) + v_{2,s}\left(\frac{r_{su}}{2+r_{su}}p_T\right), \quad (16)$$

$$v_{2,\Xi}(p_T) = 2v_{2,s}\left(\frac{r_{su}}{1+2r_{su}}p_T\right) + v_{2,u}\left(\frac{1}{1+2r_{su}}p_T\right). \quad (17)$$

Here, we assume approximate isospin symmetry between up and down quarks, i.e., $v_{2,u} = v_{2,d}$, and strangeness neutrality, i.e., $v_{2,s} = v_{2,\bar{s}}$, in the midrapidity region. The relative momentum ratio $r_{su} = m_s/m_u$ represents the constituent mass ratio between strange and up quarks. Considering uncertainties in constituent quark masses, e.g., $m_s = 0.5\text{--}0.55$ GeV and $m_u = 0.3\text{--}0.33$ GeV, the derived ratio r_{su} retains an approximate value of $5/3$, with an estimated uncertainty of approximately 10%. Our numerical results show that the v_2 of hadrons is not sensitive to such variations in r_{su} see the Appendix for an example calculation. Consequently, this study focuses exclusively on presenting calculated v_2 for hadrons using a fixed value of $r_{su} = 1.67$, which serves as a representative benchmark for our model predictions.

In Eqs. (13)–(17), we have only two inputs, $v_{2,u}$ and $v_{2,s}$, which can be fixed by fitting the experimental data for v_2 of Λ and Ξ [13,29] by Eqs. (16) and (17), respectively. If data of Ξ are unavailable, such as those in the 0%–5% and 5%–10% centralities at $\sqrt{s_{NN}} = 5.02$ TeV, the values of $v_{2,u}$ and $v_{2,s}$ are extracted using the available data for Λ and p in these centralities. v_2 of quarks is parametrized as

$$v_{2,q}(p_T) = a_q \exp\left[-\frac{p_T}{b_q} - c_q \exp\left(-\frac{p_T}{d_q}\right)\right], \quad (18)$$

where a_q , b_q , c_q , and d_q are parameters that control the shape of v_2 of quarks. The extracted results for the v_2 of quarks in central 0%–5% to semicentral 40%–50% centrality in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV are shown in Fig. 1. The shadow areas show the statistical uncertainties, which are passed from the statistical uncertainties of the experimental data for Λ , Ξ or Λ , p .

In Figs. 2 and 3, we show the results for the v_2 of light-flavor hadrons ϕ , Λ , Ξ , Ω , and p at midrapidity in different centralities (0%–5%, 5%–10%, 10%–20%, 20%–30%, 30%–40%, and 40%–50%) and compare them with the experimental data in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$

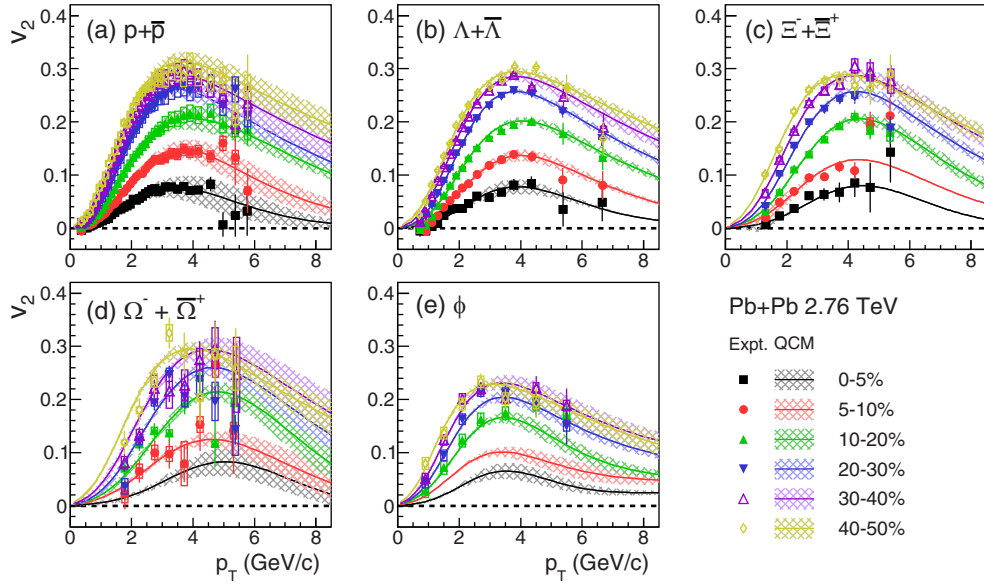


FIG. 2. The v_2 of (a) p , (b) Λ , (c) Ξ , (d) Ω , and (e) ϕ in different centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. Symbols are the experimental data [13] and lines are model results.

and 5.02 TeV. The QCM results are lines and experimental data [13,29,30] are symbols. The experimental data of Λ and Ξ in the 10%–20%, 20%–30%, 30%–40%, and 40%–50% centralities at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV are used to constrain the quark v_2 , while in 0%–5% and 5%–10% centralities at $\sqrt{s_{NN}} = 5.02$ TeV the data for Λ and p are used. The v_2 of other hadrons are theoretical predictions.

We see that the agreement between QCM results for light-flavor hadrons and experimental data is generally well. An exception is ϕ mesons at $\sqrt{s_{NN}} = 5.02$ TeV, where theoretical results are below the experimental data to a certain extent. This indicates other production channels may contribute to

ϕ meson production in heavy-ion collisions at $\sqrt{s_{NN}} = 5.02$ TeV. Here, we consider the coalescence of two kaons ($KK \rightarrow \phi$) at the hadronic rescattering stage [48,57,58] as a new channel of ϕ meson production besides the normal quark combination at hadronization. Accordingly, the final distribution of ϕ mesons is modeled by incorporating both production mechanisms:

$$f_{\phi}^{(\text{final})}(p_T, \varphi) = f_{\phi, s\bar{s}}(p_T, \varphi) + f_{\phi, KK}(p_T, \varphi).$$

The elliptic flow is

$$v_{2,\phi}^{(\text{final})}(p_T) = (1 - z)v_{2,s\bar{s}}(p_T) + zv_{2,KK}(p_T), \quad (19)$$

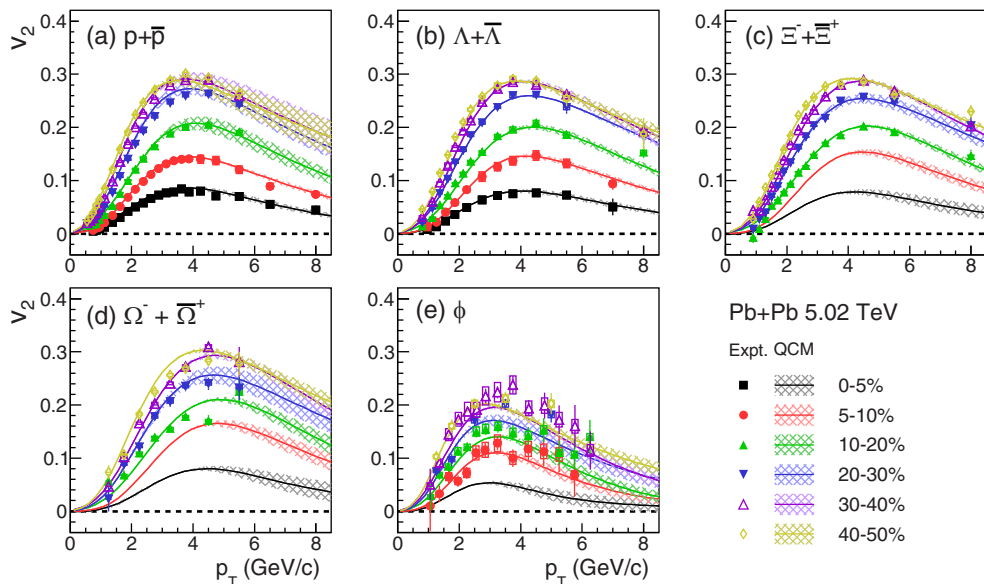


FIG. 3. The v_2 of (a) p , (b) Λ , (c) Ξ , (d) Ω , and (e) ϕ in different centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. Symbols are the experimental data [29,30] and lines are model results.

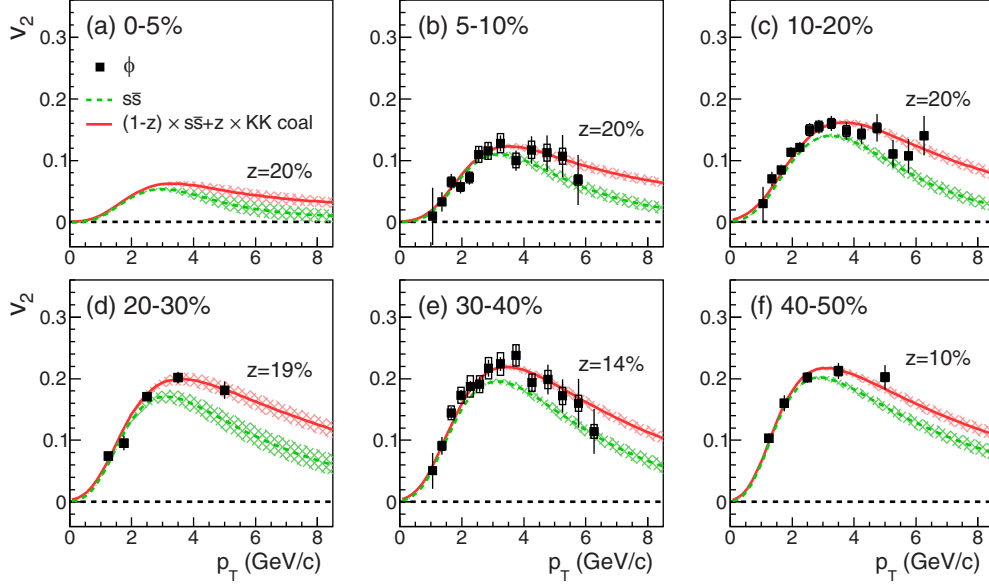


FIG. 4. The v_2 of ϕ mesons as a function of p_T in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. Experimental data (symbols) are from Refs. [29,30]. Theoretical results including only the $s\bar{s}$ combination are shown as dashed lines and those incorporating both $s\bar{s}$ combination and two-kaon coalescence are shown as solid lines.

where the parameter z denotes the fractional contribution of the two-kaon coalescence channel. Because v_2 data of kaon [30] are rich and precise, we carry out a functional fit to kaon v_2 and apply it in Eq. (19) to provide a smoothing evaluation of v_2 of ϕ .

In Fig. 4, we apply Eq. (19) to study v_2 of ϕ mesons and compare theoretical calculations with experimental data [29,30]. The results for the pure $s\bar{s}$ quark combination mechanism, labeled as $s\bar{s}$, are shown as dashed curves in Fig. 4. We see that this mechanism can describe the v_2 of the ϕ mesons in the low transverse momentum region ($p_T \lesssim 2.5$ GeV/c) but underestimates the data in the intermediate transverse momentum region ($p_T \gtrsim 2.5$ GeV/c). If we introduce the contribution of two-kaon coalescence ($KK \rightarrow \phi$), the final results of ϕ can be improved. For example, we consider a contribution fraction $z = 0.2$ and will find in Fig. 4(b) that the final results (the solid lines) can be in good agreement with experimental data for ϕ in 5%–10% centrality in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. This indicates that approximately 20% of ϕ meson come from the two-kaon coalescence at the hadronic rescattering state in this centrality. Results with fractions z about 0.20, 0.19, 0.14, and 0.10 in Figs. 4(c)–4(f) can well describe v_2 data of ϕ in 10%–20%, 20%–30%, 30%–40%, and 40%–50% centralities, respectively. The decrease of z with the increased centrality is related to the decrease of the hadronic rescattering effect from central collisions to semicentral collisions. The uncertainty of z can be estimated by Eq. (19) according to those of ϕ , kaon, and strange quarks. The relative uncertainty of z is about 20%, 19%, and 14% in 5%–10%, 10%–20%, and 30%–40% centralities with most abundant data points for ϕ , respectively. In Fig. 4(a), we predict v_2 of ϕ with $z = 0.20$ in most-central (0%–5% centrality) collisions for future testing.

B. The triangular flow of light-flavor hadrons

We further study the v_3 of light-flavor hadrons ϕ , Λ , Ξ , Ω , and p at midrapidity in different centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV [14,30]. v_3 of light-flavor hadrons in Eqs. (9) and (10) contains the contribution of the quadratic term of quark v_2 . According to the NCQ estimation of the second to fifth flow of hadrons [10–14], the a_5/a_3 ratio is about 2.5 [49–52,59]. Therefore, the magnitude of $(a_5/a_3 - 2)v_{2,q}^2$ is about 10^{-3} and its influence can be neglected in the formula, and we finally obtain

$$v_{3,M_j}(p_T) \approx v_{3,q_1}(x_1 p_T) + v_{3,q_2}(x_2 p_T), \quad (20)$$

$$v_{3,B_j}(p_T) \approx v_{3,q_1}(x_1 p_T) + v_{3,q_2}(x_2 p_T) + v_{3,q_3}(x_3 p_T). \quad (21)$$

Applying Eqs. (20) and (21), we obtain

$$v_{3,p}(p_T) = 3v_{3,u}(p_T/3), \quad (22)$$

$$v_{3,\Omega}(p_T) = 3v_{3,s}(p_T/3), \quad (23)$$

$$v_{3,\phi}(p_T) = v_{3,s}(p_T/2) + v_{3,\bar{s}}(p_T/2), \quad (24)$$

$$v_{3,\Lambda}(p_T) = 2v_{3,u}\left(\frac{1}{2 + r_{su}}p_T\right) + v_{3,s}\left(\frac{r_{su}}{2 + r_{su}}p_T\right), \quad (25)$$

$$v_{3,\Xi}(p_T) = 2v_{3,s}\left(\frac{r_{su}}{1 + 2r_{su}}p_T\right) + v_{3,u}\left(\frac{1}{1 + 2r_{su}}p_T\right). \quad (26)$$

Here, we also assume the approximate isospin symmetry ($v_{3,u} = v_{3,d}$) and strangeness neutrality ($v_{3,s} = v_{3,\bar{s}}$) at midrapidity in Pb+Pb collisions at LHC energies.

We adopt an approach similar to that used for the hadronic v_2 in the previous section to test EVC model. First, we adopt the formula in Eq. (18) to parametrize the v_3 of (anti)quarks, and then extract the up/down quark $v_{3,u}$ and strange quark $v_{3,s}$

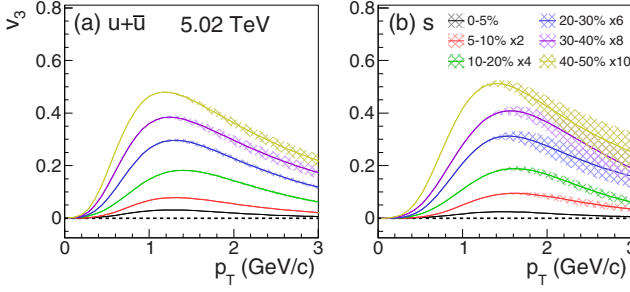


FIG. 5. v_3 of quarks as a function of p_T in different centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV.

by fitting experimental data of p and Λ measured in Pb+Pb collisions at LHC energies. Results are shown in Fig. 5, where data from different centralities are scaled by different factors for clarity. We further use the obtained $v_{3,q}$ to calculate the v_3 of other hadrons (such as Ξ , Ω , and ϕ) and compare them with corresponding experimental data. Figure 6 shows the results for the v_3 of ϕ , Λ , Ξ , Ω , and p in 0%–5%, 5%–10%, 10%–20%, 20%–30%, 30%–40%, 40%–50% centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. Experimental data of the v_3 of these hadrons are shown as symbols in Fig. 6 and are taken from Refs. [14,30]. In the calculation of ϕ mesons, we also consider the contribution of two-kaon coalescence to v_3 of ϕ and set $z = 0.2$ in the final prediction.

Compared with the data points of v_2 for hadrons, the data points of v_3 for hadrons are relatively fewer. By comparing between the theoretical results and those limited experimental data, we see that theoretical calculations of v_3 of Ξ and ϕ are generally in agreement with the available experimental data in the studied collision centralities.

IV. THE ANISOTROPY FLOW OF SINGLE-CHARMED HADRONS

In this section, we apply the EVC model in Sec. II to study the v_2 and v_3 of single-charmed hadrons D^0 , D_s^+ , Λ_c^+ , Ξ_c^0 , Ω_c^0 at midrapidity in different centralities (0%–10%, 10%–30%, 30%–50%) in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV. Here, the application of the same equal-velocity combination rule to charm quark does not imply the same thermalization and same flows for charm and light-flavor quarks. We determine v_2 and v_3 of charm quarks just before hadronization by data of the D^0 meson. Similarly to light-flavor hadrons, we also neglect the decay influence on v_2 and v_3 of these hadrons from heavier resonances such as D^* , D_s^* , Σ_c^* , Ξ_c^* , and Ω_c^* due to decay kinematics.

A. The elliptic flow of single-charmed hadrons

The v_2 of single-charmed hadrons is also given by Eqs. (7) and (8). For the light-flavor quarks, the scaling coefficient a_4 is approximated as 2 according to our previous analysis [45]. For charm quarks, although the exact value of a_4 is not determined, the magnitude of the $(a_4 - 2)v_{2,q}^2$ is estimated to be less than 0.4%. Therefore, we neglect the small contribution of quadratic terms of quark v_2 and simplify the v_2 of single-charmed hadrons as

$$v_{2,M_{cl}}(p_T) \approx v_{2,q_c}(x_1 p_T) + v_{2,q_l}(x_2 p_T), \quad (27)$$

$$v_{2,B_{cl'}}(p_T) \approx v_{2,q_c}(x_1 p_T) + v_{2,q_l}(x_2 p_T) + v_{2,q_l'}(x_3 p_T). \quad (28)$$

We observe a simple sum rule for the v_2 of charmed hadrons, which is convenient for experimental testing. Under the assumption of approximate isospin symmetry between up and down quarks (i.e., $v_{2,u} = v_{2,d}$), and strangeness neutrality

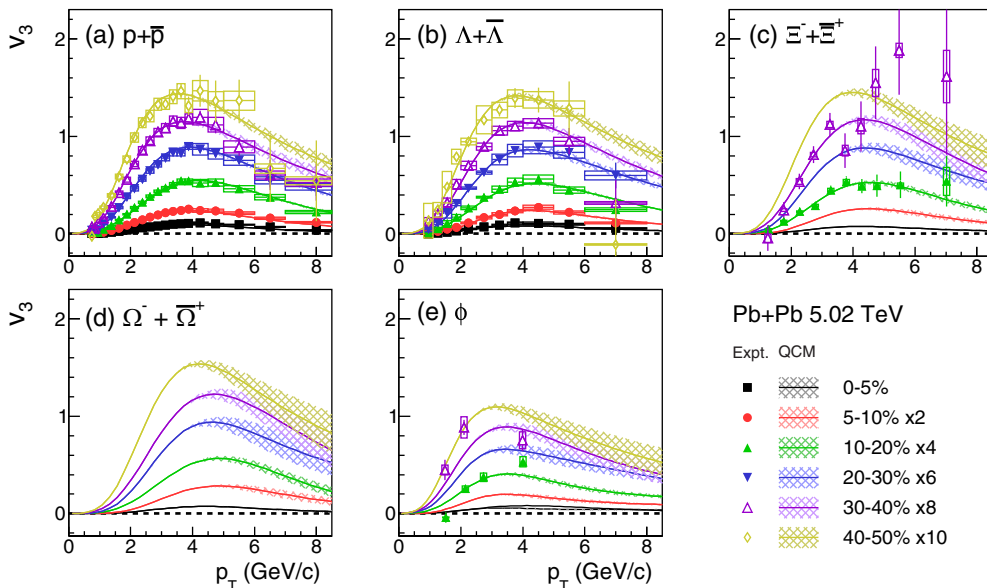


FIG. 6. The v_3 of (a) p , (b) Λ , (c) Ξ , (d) Ω , and (e) ϕ in different centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. Symbols are the experimental data [14,30] and lines are model results.

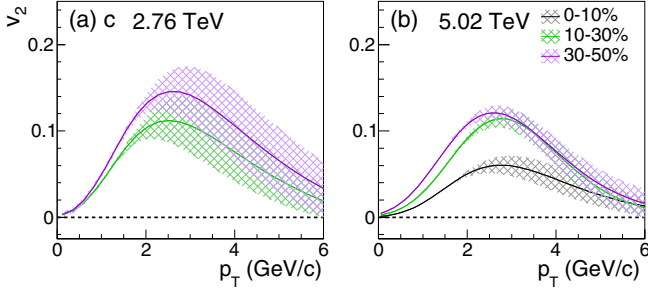


FIG. 7. v_2 of c quarks as a function of p_T in different centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV.

(i.e., $v_{2,s} = v_{2,\bar{s}}$), the formulas for D^0 , D_s^+ , Λ_c^+ , Ξ_c^0 , and Ω_c^0 can be further simplified using Eqs. (27) and (28) as follows:

$$v_{2,D}(p_T) = v_{2,c} \left(\frac{r_{cu}}{1 + r_{cu}} p_T \right) + v_{2,u} \left(\frac{1}{1 + r_{cu}} p_T \right), \quad (29)$$

$$v_{2,D_s}(p_T) = v_{2,c} \left(\frac{r_{cs}}{1 + r_{cs}} p_T \right) + v_{2,s} \left(\frac{1}{1 + r_{cs}} p_T \right), \quad (30)$$

$$v_{2,\Lambda_c}(p_T) = v_{2,c} \left(\frac{r_{cu}}{2 + r_{cu}} p_T \right) + 2v_{2,u} \left(\frac{1}{2 + r_{cu}} p_T \right), \quad (31)$$

$$v_{2,\Xi_c}(p_T) = v_{2,c} \left(\frac{r_{cu}}{1 + r_{su} + r_{cu}} p_T \right) + v_{2,u} \left(\frac{1}{1 + r_{su} + r_{cu}} p_T \right) + v_{2,s} \left(\frac{r_{su}}{1 + r_{su} + r_{cu}} p_T \right), \quad (32)$$

$$v_{2,\Omega_c}(p_T) = v_{2,c} \left(\frac{r_{cs}}{2 + r_{cs}} p_T \right) + 2v_{2,s} \left(\frac{1}{2 + r_{cs}} p_T \right). \quad (33)$$

Here, the relative momentum ratios are $r_{cu} = m_c/m_u = 5$ and $r_{cs} = m_c/m_s = 3$. It can be seen that the v_2 of charmed hadrons at p_T is the summation of the v_2 of the charm quark, whose p_T is close to that of the charmed hadron, and the v_2 of a light-flavor quark, whose p_T is much smaller. This composition pattern is different from that of light-flavor hadrons, where quarks have relatively close momentum.

Given the $v_{2,u}$ and $v_{2,s}$ obtained in the previous Sec. III A, the v_2 of charm quarks can be extracted from the experimental data of D^0 mesons,

$$v_{2,c}(p_T) = v_{2,D} \left(\frac{1 + r_{cu}}{r_{cu}} p_T \right) - v_{2,u} \left(\frac{1}{r_{cu}} p_T \right). \quad (34)$$

For the charm quark, the parametrization form also adopts Eq. (18). In Fig. 7, we show the extracted $v_{2,c}$ as a function of p_T in different centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV. The shadow areas show the statistical uncertainties, which are passed from the statistical uncertainties of the experimental data for D^0 [23,26,60]. It should be noted that this extraction method is only valid in the low and intermediate p_T region, where the combination mechanism dominates D meson production. In previous studies [16,41,61,62], we found that the experimental data for p_T spectra of single-charmed hadrons in the $p_T \lesssim 7.5$ GeV/c range from pp , p +Pb, and Pb+Pb collisions at LHC energies are well described by the EVC model. Therefore, the

experimental data of v_2 for D mesons with $p_T \lesssim 7.5$ GeV/c can be used to extract the v_2 of charm quarks with $p_T \lesssim 6$ GeV/c within the EVC model. In the higher p_T region ($p_T \gtrsim 7.5$ GeV/c), the fragmentation mechanism becomes significant [63,64], and Eq. (34) is no longer applicable. In addition, v_2 data of D^0 mesons may contain a certain contribution from the hadronic rescattering stage [65]. This hadronic rescattering effect will affect the extraction of charm quarks v_2 in Eq. (34) and will propagate to other charm hadrons via charm quark content.

In Figs. 8 and 9, we present the results for the v_2 of D^0 , D_s^+ , Λ_c^+ , Ξ_c^0 , and Ω_c^0 at 0%–10%, 10%–30%, and 30%–50% centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV. The symbols are the experimental data [22–26] from the CMS and ALICE Collaborations and lines are the calculated results. The results are in good agreement with the experimental data for D_s^+ in Fig. 8(b). Predictions from QCM for other charmed baryons, Λ_c^+ , Ξ_c^0 , Ω_c^0 , are presented in Figs. 8(c)–8(e) and 9(c)–9(e), which can be tested in future experimental measurements.

Recently, the ALICE collaboration report the preliminary data for v_2 of single-charmed hadrons D^0 , D_s^+ , and Λ_c^+ in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV. Because of quite close collision energy, these data can be used as the first test of our prediction. In Fig. 10 we show these data in 30%–50% centrality as well as v_2 of D^0 and D_s^+ at $\sqrt{s_{NN}} = 5.02$ TeV. We see that v_2 of D^0 at the two collision energies are quite close to each other, and v_2 of D_s^+ has a slight difference but the error bar at 5.02 TeV is large.

In order to clearly show the individual contributions of charm quarks and light-flavor quarks in the composition of v_2 of charmed hadrons, we plot the contribution of charm quarks as the dot-dashed lines and that of light-flavor quarks as the dotted lines. The solid lines represent v_2 of charmed hadrons in Figs. 10(a)–10(c). From Fig. 10(a), we see that v_2 of D^0 in the range $p_T \lesssim 2$ GeV/c is dominated by the contribution of charm quarks, and at large p_T the contribution of light-flavor quarks continuously strengthens and is even dominant at $p_T \gtrsim 5$ GeV/c. v_2 of D_s^+ and Λ_c^+ in Figs. 10(b)–10(c) also exhibit a similar property. A difference between Λ_c^+ and D^0 is that Λ_c^+ contains a double contribution from the v_2 of light-flavor quarks. Therefore, as shown in Fig. 10(d), the v_2 of Λ_c^+ is larger than that of D^0 in the range $3 < p_T < 8$ GeV/c. In addition, the peak position of the v_2 of Λ_c^+ is at about 5 GeV/c, while that of D^0 is at about 3.5 GeV/c. This is because the double contribution from light-flavor quarks in the Λ_c^+ formation will push the peak position for v_2 of Λ_c^+ to larger p_T in comparison with that of D^0 . Furthermore, v_2 of Λ_c^+ and D^0 do not follow the NCQ scaling exhibited in light-flavor hadrons because the v_2 of involved light-flavor quarks is quite different from the v_2 of charm quarks; see the crossing feature between the dash-dotted and dash-dot-dotted lines in Figs. 10(a) and 10(c). The model result for v_2 of D^0 has a certain difference from that of D_s^+ ; see the dashed, dash-dotted, and dash-dot-dotted lines in Figs. 10(a) and 10(b). We see that the v_2 result of D_s^+ is slightly smaller than that of D^0 in the low p_T range ($p_T \lesssim 3$ GeV/c) and is larger than the latter to a certain extent. Experimental data for v_2 of D_s^+ in Fig. 10(d) also show a smaller tendency in comparison with v_2 data of D^0 , although

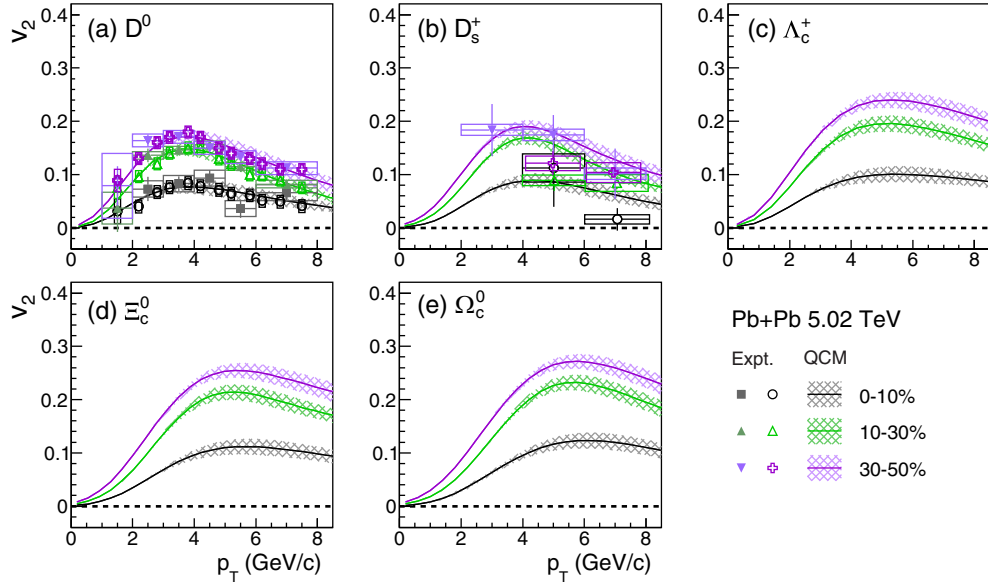


FIG. 8. The v_2 of (a) D^0 , (b) D_s^+ , (c) Λ_c^+ , (d) Ξ_c^0 , and (e) Ω_c^0 in different centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. Symbols are the experimental data [22–25] and lines are model results.

the experimental data have large uncertainties. We note that a sequential coalescence hadronization can generate a smaller v_2 of D_s^+ [66].

B. The triangular flow of single-charmed hadrons

In this subsection, we study the v_3 of single-charmed hadrons D^0 , D_s^+ , Λ_c^+ , Ξ_c^0 , and Ω_c^0 in different centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, and make comparisons with the available experimental data [25,60,67]. v_3 of single-charmed hadrons in Eqs. (9) and (10) includes the

small contribution from the quadratic terms of quark v_2 . By NCQ scaling estimation for the second to fifth harmonic flows of light-flavor hadrons [10–14], the relating coefficient $\frac{a_5}{a_3}$ is approximately 2.5 [49–52,59]. Consequently, the magnitude of $(\frac{a_5}{a_3} - 2)v_{2,q}^2$ is on the order of 10^{-3} , and its influence is negligible in the formula. We therefore finally obtain

$$v_{3,M_{cl}}(p_T) \approx v_{3,q_c}(x_1 p_T) + v_{3,q_l}(x_2 p_T), \quad (35)$$

$$v_{3,B_{cl'}}(p_T) \approx v_{3,q_c}(x_1 p_T) + v_{3,q_l}(x_2 p_T) + v_{3,q_l'}(x_3 p_T), \quad (36)$$

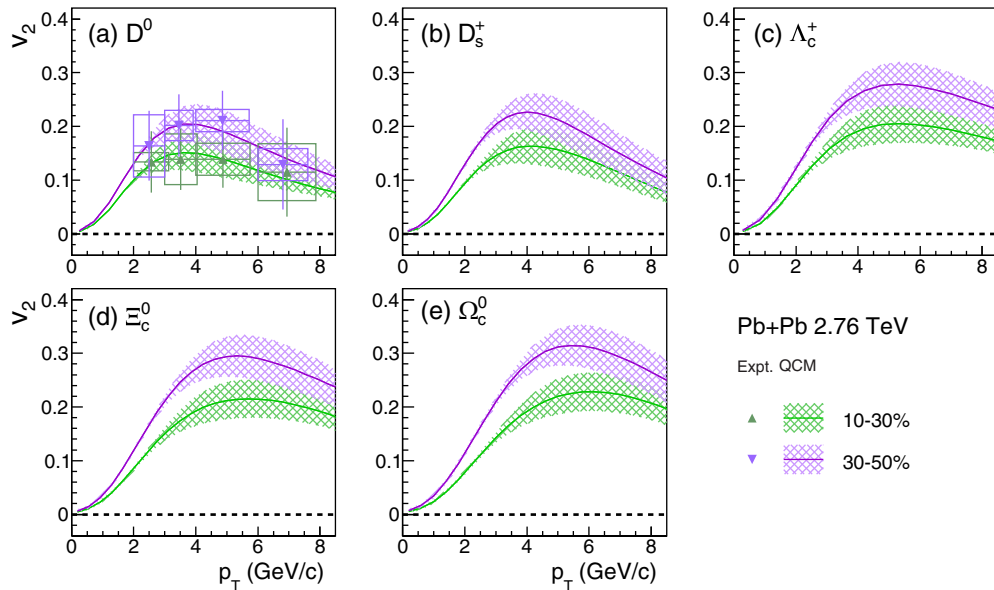


FIG. 9. The v_2 of (a) D^0 , (b) D_s^+ , (c) Λ_c^+ , (d) Ξ_c^0 , and (e) Ω_c^0 in different centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. Symbols are the experimental data [26] and lines are model results.

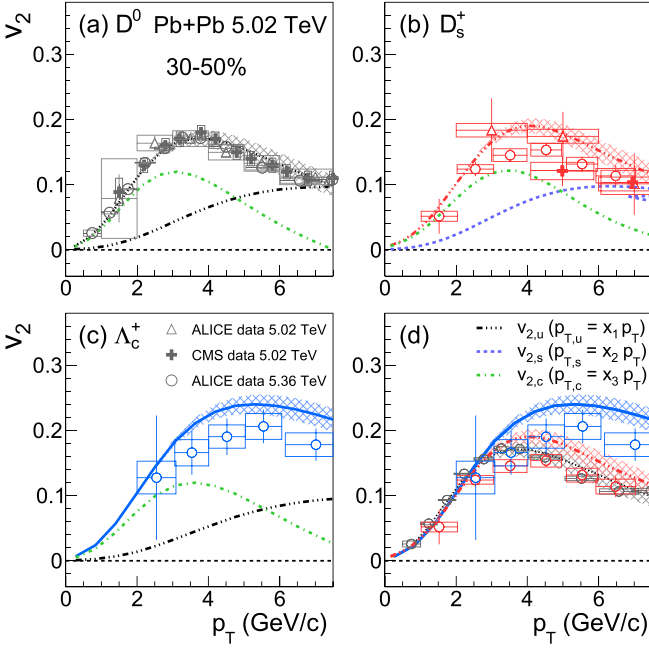


FIG. 10. The v_2 of (a) D^0 , (b) D_s^+ , and (c) Λ_c^+ in 30%–50% centrality in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ and 5.36 TeV. Panel (d) shows the comparison of v_2 of D^0 , D_s^+ , and Λ_c^+ at $\sqrt{s_{NN}} = 5.36$ TeV. Symbols are the experimental data [22–25] and lines are model results.

and

$$v_{3,D}(p_T) = v_{3,c} \left(\frac{r_{cu}}{1 + r_{cu}} p_T \right) + v_{3,u} \left(\frac{1}{1 + r_{cu}} p_T \right), \quad (37)$$

$$v_{3,D_s}(p_T) = v_{3,c} \left(\frac{r_{cs}}{1 + r_{cs}} p_T \right) + v_{3,s} \left(\frac{1}{1 + r_{cs}} p_T \right), \quad (38)$$

$$v_{3,\Lambda_c}(p_T) = v_{3,c} \left(\frac{r_{cu}}{2 + r_{cu}} p_T \right) + 2v_{3,u} \left(\frac{1}{2 + r_{cu}} p_T \right), \quad (39)$$

$$v_{3,\Xi_c}(p_T) = v_{3,c} \left(\frac{r_{cu}}{1 + r_{su} + r_{cu}} p_T \right) + v_{3,u} \left(\frac{1}{1 + r_{su} + r_{cu}} p_T \right) + v_{3,s} \left(\frac{r_{su}}{1 + r_{su} + r_{cu}} p_T \right), \quad (40)$$

$$v_{3,\Omega_c}(p_T) = v_{3,c} \left(\frac{r_{cs}}{2 + r_{cs}} p_T \right) + 2v_{3,s} \left(\frac{1}{2 + r_{cs}} p_T \right). \quad (41)$$

Here, we also adopt approximate isospin symmetry between up and down quarks, as well as strangeness neutrality between strange quarks and strange antiquarks at midrapidity in heavy-ion collisions at LHC energies.

We employ a procedure similar to that used in testing v_2 of the single-charmed hadrons in the previous section to examine the v_3 of the single-charmed hadrons. First, we again adopt Eq. (18) to parametrize the v_3 of charm quarks. By fitting experimental data from D^0 in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, the values of the parameters $v_{3,c}$ are extracted. Figure 11 shows the extracted $v_{3,c}$ in the three centralities, where data from different centralities are scaled by different factors for clarity. Second, we use the resulting $v_{3,c}$ and v_3 of light-flavor

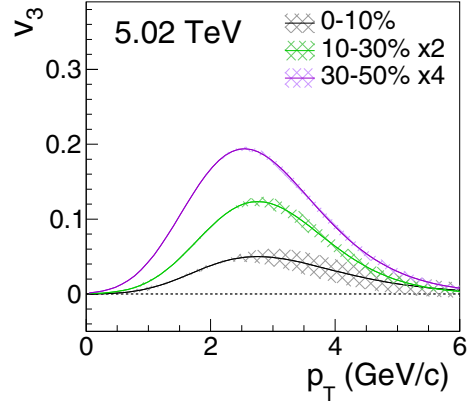


FIG. 11. v_3 of c quarks as a function of p_T in different centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV.

quarks (i.e., $v_{3,u}$, $v_{3,s}$) obtained in Sec. III B to calculate the v_3 of other single-charmed hadrons, D_s^+ , Λ_c^+ , Ξ_c^0 , and Ω_c^0 , by Eqs. (38)–(41), and compare them with experimental data.

Figure 12 shows the results of the v_3 of D^0 , D_s^+ , Λ_c^+ , Ξ_c^0 , and Ω_c^0 in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV in the 0%–10%, 10%–30%, and 30%–50% centralities. Comparing Eqs. (37)–(41) for v_3 and Eqs. (29)–(33) for v_2 of charmed hadrons, we see that the composition patterns of charm quarks and light-flavor quarks are very similar in v_2 and v_3 of charmed hadrons. Therefore, as shown in Fig. 10 for hadronic v_2 , v_3 of charmed hadrons in the range $p_T \lesssim 4$ GeV/c is dominated by v_3 of charm quarks and, in the intermediate p_T region ($4 \lesssim p_T \lesssim 7.5$ GeV/c), the contribution from light-flavor quarks gradually increases. Taking results in 30%–50% centrality as an example, we see that the peak position for v_3 of D^0 and D_s^+ is located at about 4.0 GeV/c, while that of baryons Λ_c^+ , Ξ_c^0 , and Ω_c^0 is located at about 5.5 GeV/c. The maximum v_3 value of Λ_c^+ at peak position is higher than that of D^0 by about 0.07 because of an extra contribution of light-flavor quark v_3 . The maximum v_3 of strange charmed hadrons D_s^+ , Ξ_c^0 , and Ω_c^0 are higher than those of D^0 and Λ_c^+ because the involved strange quarks have larger transverse momentum and thus larger v_3 at charm quark hadronization. These properties of v_3 of charmed hadrons are left for future test.

C. The properties of v_2 and v_3 of quarks

In this subsection, we utilize the previously obtained v_2 and v_3 of up/down quarks, strange quarks, and charm quarks just before hadronization to further investigate their properties, with a particular focus on the scaling behavior of anisotropic flow at different orders.

In Sec. II, we assumed a scaling relation $v_{n,q} = a_n v_{2,q}^{n/2}$ at quark level to estimate the contribution of higher-order quark flows to the lower-order anisotropic flow of single-charmed hadrons. Here, we test this relation using the obtained v_2 and v_3 of up/down, strange, and charm quarks. The results of $a_3 = v_3/v_2^{3/2}$ as a function of p_T are shown in Fig. 13. We observe that the values of a_3 for up, strange, and charm quarks all exhibit a weak dependence on p_T , clustering around

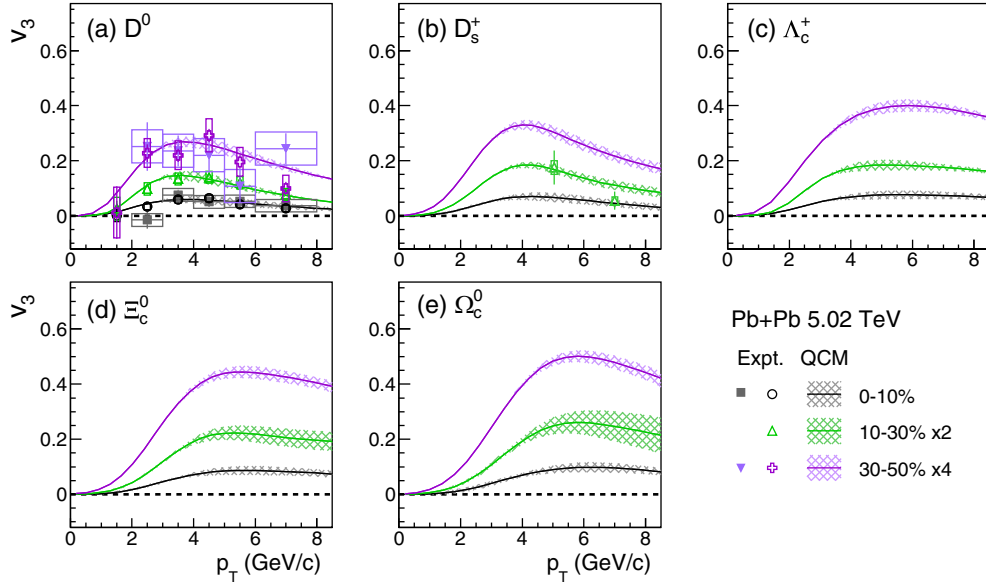


FIG. 12. The v_3 of (a) D^0 , (b) D_s^+ , (c) Λ_c^+ , (d) Ξ_c^0 , and (e) Ω_c^0 in different centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. Symbols are the experimental data [25,60,67] and lines are model results.

a value of approximately 2–4. The same behavior is also seen at RHIC. Moreover, the value of a_3 for charm quarks is observed to be significantly smaller than those for light-flavor quarks in the three centralities. This may be explained by the observation that heavier quarks exhibit a more pronounced centrality dependence in their elliptic flow, whereas triangular flow shows relatively little sensitivity to centrality variations. These results validate the consistency of the scaling relation $v_{n,q} = a_n v_{2,q}^{n/2}$ as used in the current model for analyzing the anisotropic flow v_n of single-charmed hadrons. The observed scaling behavior of quark flows can be qualitatively

understood within the framework of hydrodynamic evolution of hot quark matter [54,68].

V. SUMMARY

In this paper, we have applied the constituent quark equal-velocity combination model to study v_2 and v_3 of light-flavor hadrons and single-charmed hadrons in different centralities (0%–10%, 10%–30%, and 30%–50%) in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV. In the equal-velocity combination mechanism, the $v_{2,3}$ of hadrons can be finally expressed as the linear superposition of $v_{2,3}$ of constituent quarks and antiquarks after neglecting the higher order flows of quarks. These simplified analytical expressions for hadronic flows are conveniently tested by experimental data.

Using these analytical expressions for hadronic flows, we first systematically study the v_2 and v_3 of p , Λ , Ξ , Ω , and ϕ in different centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV. v_2 and v_3 of quarks are obtained by combination fitting to experimental data of Λ and Ξ , and we used them to calculate flows of other hadrons and compare them with the available experimental data. We found good agreement with data at $\sqrt{s_{NN}} = 2.76$. In Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, we found that v_2 data of ϕ are higher than our predictions using the $s\bar{s}$ quark combination. We therefore considered the possible contribution of final-state hadronic rescattering by including the production of ϕ via two-kaon coalescence. We found that the two-kaon coalescence channel contributes about 20% of ϕ production at $\sqrt{s_{NN}} = 5.02$ TeV. With this contribution estimation, we further predict v_3 of ϕ as a function of p_T for future testing.

Using the obtained v_2 and v_3 of light-flavor quarks in studying light-flavor hadrons, we further studied the v_2 and v_3 of single-charmed hadrons in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV. We used the experimental data of D^0 to

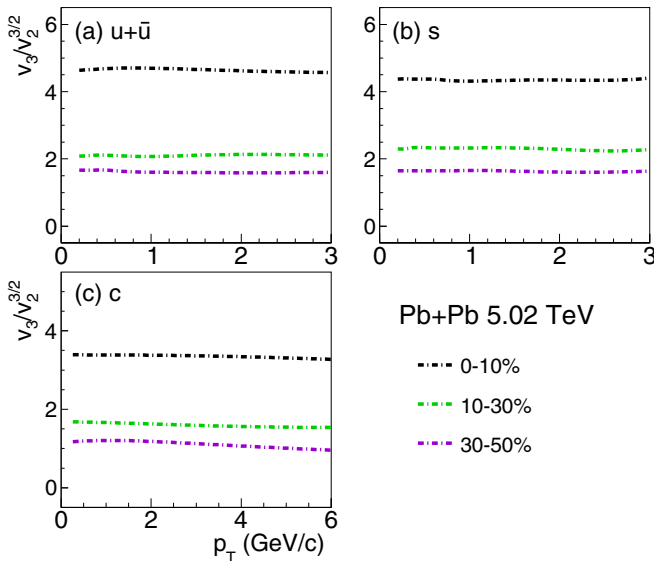


FIG. 13. The ratio $v_3/v_2^{3/2}$ of quarks as a function of p_T in different centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV.

fix the v_2 and v_3 of charm quarks and then applied the EVC model to predict flows of D_s^+ , Λ_c^+ , Ξ_c^0 , and Ω_c^0 at two collision energies. We also took the preliminary data of D^0 , D_s^+ , and Λ_c^+ in 30%–50% centrality in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV to compare with our predictions to show some features of v_2 of charmed hadrons; see Fig. 10. An interesting feature is that v_2 of Λ_c^+ is only slightly higher than those of D^0 and D_s^+ in small and intermediate p_T ranges, which is different from the NCQ scaling property for light-flavor hadrons. This is because in the EVC model charm quark hadronizes by capturing a light-flavor antiquark or two light-flavor quarks with much smaller momentum $p_{T,q} = m_q/m_c p_{T,c} \approx (0.2-0.3)p_{T,c}$. v_2 of charmed hadrons at p_T is therefore the summation of v_2 of charm quarks at a slightly smaller p_T and that of light-flavor quarks at a much smaller p_T , which has quite small value at low p_T . Another property is that the p_T position for the maximum v_2 of Λ_c^+ is larger than that of D mesons about 1 GeV/c. This is because in Λ_c^+ formation a charm quark will capture two light-flavor quarks and v_2 of light-flavor quarks involving Λ_c^+ formation is ascending function with p_T .

In deriving v_2 and v_3 of hadrons, we assumed a scaling relation $v_{n,q} = a_n(v_{2,q})^{n/2}$ for quark flows. Using the obtained v_2 and v_3 of light-flavor quarks and charm quarks, we found that this relation holds well for v_3 of light-flavor quarks and charm quarks at the two LHC energies. This was consistent with our previous results at RHIC energies [45]. The origin of this scaling relation for quark flows is related to hydrodynamic expansion evolution of QGP created in relativistic heavy-ion collisions.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this article. Requests for further information or data should be sent to the authors.

APPENDIX: INFLUENCE OF QUARK MASS UNCERTAINTIES ON HADRONIC FLOWS

In this appendix, we show the influence of the uncertainties of constituent quark masses on the anisotropic flows hadrons

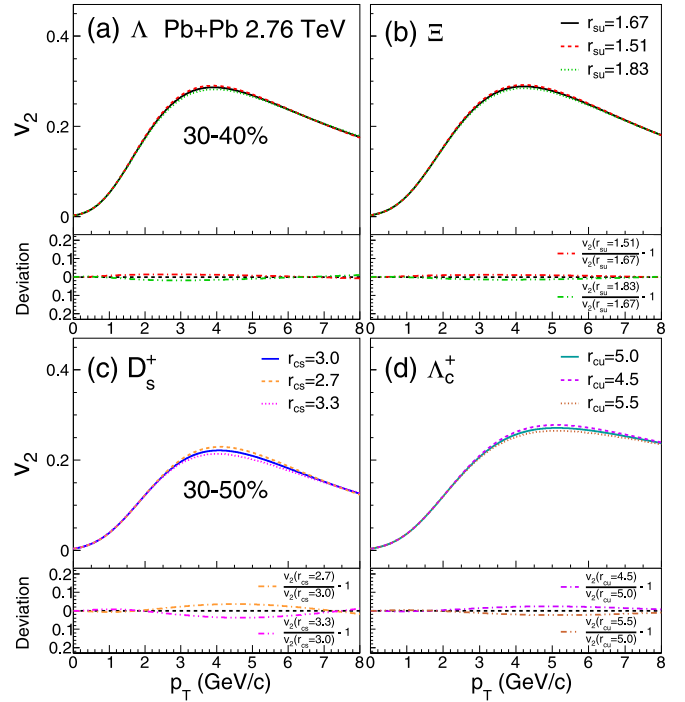


FIG. 14. v_2 of Λ (a), Ξ (b), D_s^+ (c), and Λ_c^+ (d) at different $r_{su} = m_s/m_u$, $r_{cu} = m_c/m_u$, and $r_{cs} = m_c/m_s$ in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV.

in our model. In the light-flavor sector, we take v_2 of hadrons in 30%–40% centrality in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV as an example. For simplicity, we extract v_2 of the up quark and that of the strange quark from data of the proton and ϕ [13], respectively. In Figs. 14(a) and 14(b), we show v_2 of Λ and Ξ with $r_{su} = m_s/m_u = 1.67, 1.51$, and 1.83 which correspond to 10% uncertainties of quark masses $m_s = 0.50-0.55$ GeV and $m_u = 0.30-0.33$ GeV. We see that v_2 of two hadrons are only changed about $\pm 2\%$ under $\pm 10\%$ change of r_{su} around 1.67. In Figs. 14(c) and 14(d), we further show v_2 of D_s^+ and Λ_c^+ at different constituent quark masses in 30%–50% centrality in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. Here, v_2 of charm quarks are extracted from data of D^0 [26]. We see that v_2 of Λ_c^+ at three values, $r_{cu} = m_c/m_u = 5.0, 4.5, 5.5$, changes only slightly. The maximum relative change is about 2% in the range $2.0 < p_T < 8.0$ GeV/c for $\pm 10\%$ change of r_{cu} around 5.0. v_2 of D_s^+ at three values, $r_{cs} = m_c/m_s = 3.0, 2.7, 3.3$, are also weakly changed, and the maximum relative change is about 4% in range $2.0 < p_T < 8.0$ GeV/c.

- [1] E. V. Shuryak, Quark-gluon plasma and hadronic production of leptons, photons and psions, *Phys. Lett. B* **78**, 150 (1978).
- [2] E. V. Shuryak, Quantum chromodynamics and the theory of superdense matter, *Phys. Rep.* **61**, 71 (1980).
- [3] J.-Y. Ollitrault, Anisotropy as a signature of transverse collective flow, *Phys. Rev. D* **46**, 229 (1992).

- [4] S. Voloshin and Y. Zhang, Flow study in relativistic nuclear collisions by Fourier expansion of azimuthal particle distributions, *Z. Phys. C* **70**, 665 (1996).
- [5] A. M. Poskanzer and S. A. Voloshin, Methods for analyzing anisotropic flow in relativistic nuclear collisions, *Phys. Rev. C* **58**, 1671 (1998).

- [6] P. F. Kolb, v_4 : A small, but sensitive observable for heavy ion collisions, *Phys. Rev. C* **68**, 031902 (2003).
- [7] P. F. Kolb, J. Sollfrank, and U. W. Heinz, Anisotropic transverse flow and the quark-hadron phase transition, *Phys. Rev. C* **62**, 054909 (2000).
- [8] P. Huovinen, P. F. Kolb, U. W. Heinz, P. V. Ruuskanen, and S. A. Voloshin, Radial and elliptic flow at RHIC: Further predictions, *Phys. Lett. B* **503**, 58 (2001).
- [9] D. Teaney, J. Lauret, and E. V. Shuryak, Flow at the SPS and RHIC as a quark-gluon plasma signature, *Phys. Rev. Lett.* **86**, 4783 (2001).
- [10] L. Adamczyk, J. K. Adkins, G. Agakishiev, M. M. Aggarwal, Z. Ahammed, I. Alekseev, J. Alford, C. D. Anson, A. Aparin, D. Arkhipkin, E. Aschenauer, G. S. Averichev, J. Balewski, A. Banerjee, Z. Barnovska, D. R. Beavis, R. Bellwied, M. J. Betancourt, R. R. Betts, A. Bhasin, *et al.* (STAR Collaboration), Elliptic flow of identified hadrons in Au+Au collisions at $\sqrt{s_{NN}} = 7.7\text{--}62.4$ GeV, *Phys. Rev. C* **88**, 014902 (2013).
- [11] A. Adare, S. Afanasiev, C. Aidala, N. N. Ajitanand, Y. Akiba, H. Al-Bataineh, J. Alexander, A. Al-Jamel, K. Aoki, L. Aphecetche, R. Armendariz, S. H. Aronson, J. Asai, E. T. Atomssa, R. Averbeck, T. C. Awes, B. Azmoun, V. Babintsev, G. Baksay, L. Baksay, *et al.* (PHENIX Collaboration), Scaling properties of azimuthal anisotropy in Au + Au and Cu + Cu collisions at $\sqrt{s_{NN}} = 200$ GeV, *Phys. Rev. Lett.* **98**, 162301 (2007).
- [12] L. Adamczyk, J. K. Adkins, G. Agakishiev, M. M. Aggarwal, Z. Ahammed, I. Alekseev, A. Aparin, D. Arkhipkin, E. C. Aschenauer, G. S. Averichev, V. Bairathi, A. Banerjee, R. Bellwied, A. Bhasin, A. K. Bhati, P. Bhattarai, J. Bielcik, J. Bielcikova, L. C. Bland, I. G. Bordyuzhin, *et al.* (STAR Collaboration), Centrality and transverse momentum dependence of elliptic flow of multistrange hadrons and ϕ meson in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV, *Phys. Rev. Lett.* **116**, 062301 (2016).
- [13] B. Abelev, J. Adam, D. Adamová, M. M. Aggarwal, M. Agnello, A. Agostinelli, N. Agrawal, Z. Ahammed, N. Ahmad, I. Ahmed, S. U. Ahn, S. A. Ahn, I. Aimo, S. Aiola, M. Ajaz, A. Akkindinov, S. N. Alam, D. Aleksandrov, B. Alessandro, D. Alexandre, *et al.*, (ALICE Collaboration), Elliptic flow of identified hadrons in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, *J. High Energy Phys.* **06** (2015) 190.
- [14] S. Acharya, F. T.-. Acosta, D. Adamová, J. Adolfsson, M. M. Aggarwal, G. Aglieri Rinella, M. Agnello, N. Agrawal, Z. Ahammed, S. U. Ahn, S. Aiola, A. Akkindinov, M. Al-Turany, S. N. Alam, D. S. D. Albuquerque, D. Aleksandrov, B. Alessandro, R. Alfaro Molina, Y. Ali, A. Alici, *et al.*, (ALICE Collaboration), Anisotropic flow of identified particles in Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, *J. High Energy Phys.* **09** (2018) 006.
- [15] V. Greco, C. M. Ko, and P. Lévai, Parton coalescence and the antiproton/pion anomaly at RHIC, *Phys. Rev. Lett.* **90**, 202302 (2003).
- [16] W.-B. Chang, R.-Q. Wang, J. Song, F.-L. Shao, Q. Wang, and Z.-T. Liang, Production of strange and charm hadrons in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, *Symmetry* **15**, 400 (2023).
- [17] R. J. Fries, B. Müller, C. Nonaka, and S. A. Bass, Hadronization in heavy-ion collisions: Recombination and fragmentation of partons, *Phys. Rev. Lett.* **90**, 202303 (2003).
- [18] V. Greco, C. M. Ko, and P. Lévai, Partonic coalescence in relativistic heavy ion collisions, *Phys. Rev. C* **68**, 034904 (2003).
- [19] R. J. Fries, B. Müller, C. Nonaka, and S. A. Bass, Hadron production in heavy ion collisions: Fragmentation and recombination from a dense parton phase, *Phys. Rev. C* **68**, 044902 (2003).
- [20] D. Molnár and S. A. Voloshin, Elliptic flow at large transverse momenta from quark coalescence, *Phys. Rev. Lett.* **91**, 092301 (2003).
- [21] V. Minissale, F. Scardina, and V. Greco, Hadrons from coalescence plus fragmentation in AA collisions at energies available at the BNL Relativistic Heavy Ion Collider to the CERN Large Hadron Collider, *Phys. Rev. C* **92**, 054904 (2015).
- [22] A. M. Sirunyan, A. Tumasyan, W. Adam, F. Ambroggi, T. Bergauer, M. Dragicevic, J. Erö, A. Escalante Del Valle, R. Frühwirth, M. Jeitler, N. Krammer, L. Lechner, D. Liko, T. Madlener, I. Mikulec, N. Rad, J. Schieck, R. Schöffbeck, M. Spanring, S. Templ, *et al.* (CMS Collaboration), Measurement of prompt D^0 and $\bar{D}^0$ meson azimuthal anisotropy and search for strong electric fields in PbPb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, *Phys. Lett. B* **816**, 136253 (2021).
- [23] S. Acharya, D. Adamová, A. Adler, J. Adolfsson, M. M. Aggarwal, G. Aglieri Rinella, M. Agnello, N. Agrawal, Z. Ahammed, S. Ahmad, S. U. Ahn, Z. Akbar, A. Akkindinov, M. Al-Turany, S. N. Alam, D. S. D. Albuquerque, D. Aleksandrov, B. Alessandro, H. M. Alfanda, R. Alfaro Molina, *et al.* (ALICE Collaboration), J/ψ elliptic and triangular flow in Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, *J. High Energy Phys.* **10** (2020) 141.
- [24] S. Acharya, D. Adamová, A. Adler, J. Adolfsson, G. Aglieri Rinella, M. Agnello, N. Agrawal, Z. Ahammed, S. Ahmad, S. U. Ahn, I. Ahuja, Z. Akbar, A. Akkindinov, M. Al-Turany, S. N. Alam, D. Aleksandrov, B. Alessandro, H. M. Alfanda, R. Alfaro Molina, B. Ali, *et al.* (ALICE Collaboration), Measurement of prompt D_s^+ meson production and azimuthal anisotropy in Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, *Phys. Lett. B* **827**, 136986 (2022).
- [25] R. Pradhan *et al.* (CMS Collaboration), Recent results from CMS, *J. Subatomic Part. Cosmol.* **6**, 100492 (2026).
- [26] R. Bailhache (ALICE Collaboration), Heavy-flavour elliptic flow measured in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV with ALICE, *Nucl. Phys. A* **931**, 530 (2014).
- [27] N. R. Saha *et al.* (CMS Collaboration), Measurement of azimuthal anisotropy of $f_0(980)$ and D^0 in heavy ion collisions at CMS, *Nuovo Cimento C* **48**, 20 (2024).
- [28] N. R. Saha (CMS Collaboration), Measurement of collective flow of D^0 mesons in heavy-ion collisions at CMS, *J. Subatomic Part. Cosmol.* **4**, 100095 (2025).
- [29] S. Acharya, D. Adamová, A. Adler, G. Aglieri Rinella, M. Agnello, N. Agrawal, Z. Ahammed, S. Ahmad, S. U. Ahn, I. Ahuja, A. Akkindinov, M. Al-Turany, D. Aleksandrov, B. Alessandro, H. M. Alfanda, R. Alfaro Molina, B. Ali, Y. Ali, A. Alici, N. Alizadehvandchali, *et al.* (ALICE Collaboration), Anisotropic flow and flow fluctuations of identified hadrons in Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, *J. High Energy Phys.* **05** (2023) 243.
- [30] Y. Zhu *et al.* (ALICE Collaboration), Anisotropic flow of identified particles in Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV with ALICE, *PoS ICHEP2018*, 441 (2019).
- [31] N. Mallick, S. Prasad, A. N. Mishra, R. Sahoo, and G. G. Barnaföldi, Deep learning predicted elliptic flow of identified particles in heavy-ion collisions at the RHIC and LHC energies, *Phys. Rev. D* **107**, 094001 (2023).

- [32] S. Tang, Z. Zhang, C. Zhang, L. Zheng, and R. Wan, Investigating the transverse-momentum- and pseudorapidity-dependent flow vector decorrelation in p -Pb collisions with a multi-phase transport model, *Chin. Phys. C* **49**, 124104 (2025).
- [33] M. Alqahtani, M. Nopoush, R. Ryblewski, and M. Strickland, Anisotropic hydrodynamic modeling of 2.76 TeV Pb-Pb collisions, *Phys. Rev. C* **96**, 044910 (2017).
- [34] H. Alalawi, M. Alqahtani, and M. Strickland, Resummed relativistic dissipative hydrodynamics, *Symmetry* **14**, 329 (2022).
- [35] X. Zhu, Spectra and elliptic flow of (multi)strange hadrons at RHIC and LHC within viscous hydrodynamics + hadron cascade hybrid model, *Adv. High Energy Phys.* **2016**, 4236492 (2016).
- [36] A. Akhil and S. K. Tiwari, Exploring anisotropic flow via the Boltzmann transport equation employing the Tsallis blast wave description at LHC energies, *J. Phys. G: Nucl. Part. Phys.* **51**, 035002 (2024).
- [37] S. Grigoryan, Blast-wave-Tsallis-power model for p_T spectra and elliptic flow v_2 of hadrons in collisions of identical nuclei at energies available at the CERN Large Hadron Collider, *Phys. Rev. C* **113**, 014903 (2026).
- [38] G. Devi, A. Singh, S. Pandey, and B. K. Singh, Higher flow harmonics of strange hadrons in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV with HYDJET++ model, *Eur. Phys. J. Plus* **138**, 921 (2023).
- [39] G. Devi and B. K. Singh, Production of multistrange hadrons in Pb + Pb collisions at energies available at the CERN Large Hadron Collider using the HYDJET++ model, *Phys. Rev. C* **112**, 034907 (2025).
- [40] J. Song, X.-r. Gou, F.-l. Shao, and Z.-T. Liang, Quark number scaling of hadronic p_T spectra and constituent quark degree of freedom in p -Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, *Phys. Lett. B* **774**, 516 (2017).
- [41] J. Song, H.-h. Li, and F.-l. Shao, New feature of low p_T charm quark hadronization in pp collisions at $\sqrt{s} = 7$ TeV, *Eur. Phys. J. C* **78**, 344 (2018).
- [42] J. Song, X.-f. Wang, H.-h. Li, R.-q. Wang, and F.-l. Shao, Strange hadron production in a quark combination model in Au + Au collisions at energies available at the BNL Relativistic Heavy Ion Collider, *Phys. Rev. C* **103**, 034907 (2021).
- [43] J. Song, H.-h. Li, and F.-l. Shao, Signals of quark combination at hadronization in pp collisions at $\sqrt{s} = 200$ GeV, *Phys. Rev. D* **105**, 074027 (2022).
- [44] J. Song, H.-h. Li, and F.-l. Shao, Elliptic flow of hadrons in equal-velocity quark combination mechanism in relativistic heavy-ion collisions, *Eur. Phys. J. C* **81**, 5 (2021).
- [45] Y.-t. Feng, R.-q. Wang, F.-l. Shao, and J. Song, Anisotropic flows of identified hadrons in the equal-velocity quark combination model at energies available at the BNL Relativistic Heavy Ion Collider, *Phys. Rev. C* **111**, 064902 (2025).
- [46] R.-Q. Wang, X.-L. Hou, Y.-H. Li, J. Song, and F.-L. Shao, Production characteristics of light nuclei, hypertritons, and Ω -hypernuclei in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, *Nucl. Sci. Tech.* **36**, 185 (2025).
- [47] B. I. Abelev, M. M. Aggarwal, Z. Ahammed, B. D. Anderson, D. Arkhipkin, G. S. Averichev, Y. Bai, J. Balewski, O. Barannikova, L. S. Barnby, J. Baudot, S. Baumgart, D. R. Beavis, R. Bellwied, F. Benedosso, R. R. Betts, S. Bhardwaj, A. Bhasin, A. K. Bhati, H. Bichsel, *et al.* (STAR Collaboration), System-size independence of directed flow measured at the BNL Relativistic Heavy-Ion Collider, *Phys. Rev. Lett.* **101**, 252301 (2008).
- [48] B. Abelev, J. Adam, D. Adamová, A. M. Adare, M. M. Aggarwal, G. Aglieri Rinella, M. Agnello, A. G. Agocs, A. Agostinelli, Z. Ahammed, N. Ahmad, A. Ahmad Masoodi, I. Ahmed, S. A. Ahn, S. U. Ahn, I. Aimo, M. Ajaz, A. Akkindinov, D. Aleksandrov, B. Alessandro, *et al.* (ALICE Collaboration), Directed flow of charged particles at midrapidity relative to the spectator plane in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, *Phys. Rev. Lett.* **111**, 232302 (2013).
- [49] J. Adams, M. M. Aggarwal, Z. Ahammed, J. Amonett, B. D. Anderson, D. Arkhipkin, G. S. Averichev, S. K. Badyal, Y. Bai, J. Balewski, O. Barannikova, L. S. Barnby, J. Baudot, S. Bekele, V. V. Belaga, R. Bellwied, J. Berger, B. I. Bezverkhny, S. Bharadwaj, A. Bhasin, *et al.* (STAR Collaboration), Azimuthal anisotropy in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, *Phys. Rev. C* **72**, 014904 (2005).
- [50] B. I. Abelev, M. M. Aggarwal, Z. Ahammed, B. D. Anderson, D. Arkhipkin, G. S. Averichev, Y. Bai, J. Balewski, O. Barannikova, L. S. Barnby, J. Baudot, S. Baumgart, S. Bekele, V. V. Belaga, A. Bellingeri-Laurikainen, R. Bellwied, F. Benedosso, R. R. Betts, S. Bhardwaj, A. Bhasin, *et al.* (STAR Collaboration), Mass, quark-number, and $\sqrt{s_{NN}}$ dependence of the second and fourth flow harmonics in ultrarelativistic nucleus-nucleus collisions, *Phys. Rev. C* **75**, 054906 (2007).
- [51] P. F. Kolb, L.-W. Chen, V. Greco, and C. M. Ko, Momentum anisotropies in the quark coalescence model, *Phys. Rev. C* **69**, 051901 (2004).
- [52] L.-W. Chen, C. M. Ko, and Z.-W. Lin, Partonic effects on higher-order anisotropic flows in relativistic heavy-ion collisions, *Phys. Rev. C* **69**, 031901 (2004).
- [53] F. G. Gardim, F. Grassi, M. Luzum, and J.-Y. Ollitrault, Anisotropic flow in event-by-event ideal hydrodynamic simulations of $\sqrt{s_{NN}} = 200$ GeV Au + Au collisions, *Phys. Rev. Lett.* **109**, 202302 (2012).
- [54] N. Borghini and J.-Y. Ollitrault, Momentum spectra, anisotropic flow, and ideal fluids, *Phys. Lett. B* **642**, 227 (2006).
- [55] C. Gombeaud and J.-Y. Ollitrault, Effects of flow fluctuations and partial thermalization on v_4 , *Phys. Rev. C* **81**, 014901 (2010).
- [56] L.-W. Chen and C. M. Ko, ϕ and Ω production in relativistic heavy-ion collisions in a dynamical quark coalescence model, *Phys. Rev. C* **73**, 044903 (2006).
- [57] C. Alt, T. Anticic, B. Baatar, D. Barna, J. Bartke, L. Betev, H. Bialkowska, C. Blume, B. Boimska, M. Botje, J. Bracinik, R. Bramm, P. Bunčić, V. Cerny, P. Christakoglou, P. Chung, O. Chvala, J. G. Cramer, P. Csátó, P. Dinkelaker, *et al.* (NA49 Collaboration), Energy dependence of ϕ meson production in central Pb + Pb collisions at $\sqrt{s_{NN}} = 6$ to 17 GeV, *Phys. Rev. C* **78**, 044907 (2008).
- [58] L.-X. Sun, R.-Q. Wang, J. Song, and F.-L. Shao, Hadronic rapidity spectra in heavy ion collisions at SPS and AGS energies in a quark combination model, *Chin. Phys. C* **36**, 55 (2012).
- [59] V. Bairathi (STAR Collaboration), Azimuthal anisotropy of ϕ meson in U+U and Au+Au collisions at RHIC, *J. Phys.: Conf. Ser.* **668**, 012039 (2016).
- [60] A. M. Sirunyan, A. Tumasyan, W. Adam, F. Ambrogio, E. Asilar, T. Bergauer, J. Brandstetter, E. Brondolin, M. Dragicevic, J. Erö, M. Flechl, M. Friedl, R. Frühwirth, V.M. Ghete, J. Grossmann, J. Hrubec, M. Jeitler, A. König, N. Krammer, I. Krätschmer, *et al.* (CMS Collaboration), Measurement

- of prompt D^0 meson azimuthal anisotropy in Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, *Phys. Rev. Lett.* **120**, 202301 (2018).
- [61] J. Song, H.-h. Li, and F.-l. Shao, Transverse momentum and multiplicity dependence of Λ_c^+/D^0 ratio in pp collisions at $\sqrt{s} = 13$ TeV, *Eur. Phys. J. C* **83**, 852 (2023).
- [62] H.-h. Li, F.-l. Shao, J. Song, and R.-q. Wang, Production of single-charm hadrons by the quark-combination mechanism in p -Pb collisions at $\sqrt{NN} = 5.02$ TeV, *Phys. Rev. C* **97**, 064915 (2018).
- [63] Y. Oh, C. M. Ko, S. H. Lee, and S. Yasui, Ratios of heavy baryons to heavy mesons in relativistic nucleus-nucleus collisions, *Phys. Rev. C* **79**, 044905 (2009).
- [64] S. Cao, G.-Y. Qin, and S. A. Bass, Energy loss, hadronization, and hadronic interactions of heavy flavors in relativistic heavy-ion collisions, *Phys. Rev. C* **92**, 024907 (2015).
- [65] S. K. Das, J. M. Torres-Rincon, and R. Rapp, Charm and bottom hadrons in hot hadronic matter, *Phys. Rep.* **1129-1131**, 1 (2025).
- [66] Z.-X. Xu, W. Dai, B.-W. Zhang, J. Zhao, and P. Zhuang, D -meson production via sequential hadronization in high-energy nuclear collisions, *Phys. Rev. Lett.* **136**, 232301 (2026).
- [67] S. Acharya, D. Adamová, A. Adler, J. Adolfsson, M. M. Aggarwal, G. Aglieri Rinella, M. Agnello, N. Agrawal, Z. Ahammed, S. Ahmad, S. U. Ahn, Z. Akbar, A. Akindinov, M. Al-Turany, S. N. Alam, D. S. D. Albuquerque, D. Aleksandrov, B. Alessandro, H. M. Alfanda, R. Alfaro Molina, *et al.* (ALICE Collaboration), Transverse-momentum and event-shape dependence of D -meson flow harmonics in Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, *Phys. Lett. B* **813**, 136054 (2021).
- [68] E. Retinskaya, M. Luzum, and J.-Y. Ollitrault, Constraining models of initial conditions with elliptic and triangular flow data, *Phys. Rev. C* **89**, 014902 (2014).