

Exploring nucleon-nucleon collisions and clusterization within the antisymmetrized molecular dynamics transport model by means of a Bayesian analysis on a dataset from the FAZIA setup

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The in-medium nucleon-nucleon cross section and the cluster production are key ingredients in transport models for heavy-ion collisions. They have been constrained by the comparison of predictions of the AMD transport model to experimental data of $^{20}\text{Ne} + ^{12}\text{C}$ at 50 MeV/nucleon [C. Frosin *et al.*, *Phys. Rev. C* **107**, 044614 (2023)], tuning the related parameters in the framework of a Bayesian inference. Experimental data have been collected with four blocks of the FAZIA setup. A large dataset of observables has been included in the analysis. The prior distribution of the examined parameters was chosen on the basis of the existing literature. The posterior distribution indicates a trend towards a larger number of collisions per event with respect to the previously used simulations.

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I. INTRODUCTION

Describing heavy-ion collisions in the intermediate energy regime (20–100 MeV/nucleon) is a challenging task for theoretical models. In fact, this energy region is a transition zone where both mean-field effects, prevailing in the low-energy region, and nucleon-nucleon collisions, dominating the relativistic regime, influence the reaction behavior. Indeed, from the experimental point of view, a rich variety of phenomena is observed in the collected data. In central collisions

multifragmentation and vaporization events can be detected; binary reactions dominate at larger impact parameters, but they are often accompanied by other processes, such as mid-velocity emission, quasiprojectile breakup. On top of that at all impact parameters also clustering phenomena take place. In the final stage of the reaction primary hot products decay by means of evaporative emission [1,2]. The description of these reactions has historically followed two main approaches: either a statistical description, which establishes the primary partitions on the basis of statistical weights, or a dynamical approach, adopted by transport models of both Boltzmann-Uehling-Uhlenbeck (BUU) and quantum molecular dynamics (QMD) type [3], in which the time evolution of the colliding system, described in terms of density functionals or wave packets, is followed up to several hundreds of fm/c (typically 300–500 fm/c). In both dynamical approaches the final ejectile distributions are obtained after applying to the hot primary products a secondary-decay afterburner based on a statistical model code.

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Concerning transport models, to our knowledge a simulation able to give a perfect description of nuclear reactions in every detail for all impact parameters does not exist [4].

Among the possible choices in the intermediate energy regime, the dynamical model AMD (antisymmetrized molecular dynamics) [5,6] has already demonstrated a good capability to describe the main properties of heavy-ion collisions for the whole impact parameter range [7–14], albeit with some limitations on the reproduction of the reaction details [9–11,13]. Other models, mainly of BUU type, proved to be able to reproduce exclusive observables [15–17], at the expenses of a global description of the events.

In this work we focus on AMD and on an experimental dataset collected with FAZIA [18–20] for the system $^{20}\text{Ne} + ^{12}\text{C}$ at 50 MeV/nucleon, which was found to be reasonably reproduced in its main features by the model [7], in order to see if it is possible to further improve the global agreement with a proper choice of the model parameters. AMD is based on the description of the nuclear many-body system in terms of a Slater determinant of Gaussian wave packets. Therefore, it naturally includes the antisymmetrization of the wave function and correctly treats the Pauli principle. Moreover, two nucleon collisions are taken into account and cluster formation during the time evolution of the system is explicitly encompassed. The introduction of clustering phenomena was proven to improve the agreement between simulations and experimental data [7]. However, there are still some parameters which should be tuned in order to better reproduce experimental data. First of all, the effective potential describing the mean-field interaction, including also the density dependence of the symmetry energy, the nucleon-nucleon cross section parametrization, the cluster formation probability, and their dissolution density (namely the Mott critical density). Concerning cluster formation, for example, the model takes into account nucleon-nucleon, nucleon-cluster, and intercluster correlations as described in Ref. [21], introducing phenomenological parameters to tune them. In Ref. [7] two extreme configurations, with and without cluster formation during the collision phase (see Sec. II B), have been compared, showing that the introduction of clusterization allows for a better reproduction of the experimental observables related to isotopic distributions, energy, and angular spectra for light ejectiles. This work aims at tuning two of the parameters of AMD, one related to the clustering and the other to the in-medium nucleon-nucleon cross section in order to obtain the best global reproduction of the experimental events coming from the same dataset of Ref. [7].

When a model depends on some parameters influencing observables for which experimental measurements are available, it is possible to estimate the probability distribution of the parameters better reproducing the experimental data by means of a Bayesian approach. Such an approach has recently gained a lot of interest since it allows to combine experimental results coming from different measurement techniques in order to constrain some model parameters, and mainly to get a better estimation of their uncertainty. Examples of such a method are shown in Refs. [22,23], where it was used to constrain the nuclear equation of state and the symmetry energy.

In the present work we present a Bayesian analysis performed on experimental data of $^{20}\text{Ne} + ^{12}\text{C}$ collisions at 50 MeV/nucleon and the AMD transport model coupled with the afterburner HFI introduced in Ref. [24] and presented with some details hereafter. Since we are interested in the best global reproduction of the experimental events, even if this can trigger a worsening of the reproduction of single observables, we have included in the Bayesian procedure as many as possible experimental observables, such as the elemental yields of all the ejectiles, the kinetic energy spectra and the polar angle distributions of light products (provided that the statistics is sufficient), their isotopic composition and the average multiplicities. Anyway, as shown in Sec. III C, in a first step we have separately tested each observable, in order to evidence possible conflicts in the best model parameter combination for the reproduction of single observables.

Among all the available experimental datasets of the INDRA-FAZIA Collaboration we have chosen this particular system mainly because it involves light nuclei in the entrance channel. Such a choice allows for the exploration of the parameters of interest with reasonable granularity and computational time. Indeed, running AMD simulations is extremely time consuming in particular for large systems. The drawback of the choice of this system is mainly related to the fact that data have been collected with limited geometrical acceptance (only four FAZIA [18–20] blocks were used), with respect to the current INDRA-FAZIA setup [12,14]. Anyway, as shown in Ref. [7], the angular coverage was sufficient to characterize the events in terms of centrality selection, multiplicities, kinetic energy and polar angle distributions. It should be noted that both target and projectile are α -conjugate nuclei; therefore there is a significant probability that α particles are preformed inside the nuclei, favoring a large amount of α particle emission, usually difficult to simulate. However, the AMD model includes the possibility of α particle preformation in the ground state, although not in a refined way; as a consequence this model remains reasonably suited to study α -conjugate systems as shown in Ref. [25].

The presentation of our work is organized as follows. In Sec. II, the technique used for the data analysis is presented, including a short presentation of the Bayesian inference, of the AMD model with a focus on the parameters under study, and of the experimental data. In Sec. III, the obtained results concerning the posterior distribution of the parameters coming from the Bayesian analysis are shown. Finally, in Sec. IV, a summary of our study and the obtained conclusions are presented, together with some perspectives of future developments of this analysis. The Appendix includes some tests demonstrating the robustness of the obtained results.

II. METHOD

A. Bayesian inference

In a Bayesian inference the *a priori* probability distribution of model parameters (the so-called prior distribution) is combined with the experimental data through the likelihood distribution, thus producing the so-called posterior distribution of model parameters. In order to be conclusive, the

prerequisite of such an approach is that within the framework of the model the considered observables are correlated enough to the investigated model parameters. The procedure can be applied iteratively, assuming the posterior distribution as the new prior when new experimental measurements of the same observables become available, or when new observables are taken into account. If no *a priori* information is available on the model parameters, as a first step, the prior distribution can be uniform between two reasonable boundaries.

According to the Bayes theorem, the posterior distribution $P_{\text{post}}(\{p_i\}|v_{\text{exp}})$ of parameters $\{p_i\}$ of the model reproducing the experimental data v_{exp} can be calculated as:

$$P_{\text{post}}(\{p_i\}|v_{\text{exp}}) = \frac{L(v_{\text{exp}}|\{p_i\})P_{\text{prior}}(\{p_i\})}{\int L(v_{\text{exp}}|\{p_i\})P_{\text{prior}}(\{p_i\})dp_i}, \quad (1)$$

where $P_{\text{prior}}(\{p_i\})$ is the prior distribution of the parameters, and $L(v_{\text{exp}}|\{p_i\})$ is the likelihood distribution obtained by comparing the model results as a function of the parameter values with the experimental measurement of the considered observables v_{exp} . Basically, the likelihood distribution, once normalized, gives the probability to obtain the experimental measurement of a given observable (or of many observables, combining the single likelihood distributions) for each value of the parameter set in the considered range.

In the present analysis the used observables are a selection of physical quantities among those that have been measured. We have focused our analysis on the yields of the ejectiles, indicated by the symbol Y , i.e., the experimental (or simulated) counts normalized to the number of events, and their dependence on the phase space, as follows:

- (i) the elemental (charge) yield $Y(Z)$ integrated over kinetic energy and polar angle, in the following also referred as the charge multiplicity;
- (ii) the yield $Y(A, Z)$ for a specific isotope characterized by its mass number A , neutron number N and proton number Z , integrated over kinetic energy and polar angle, in the following also referred as the isotopic multiplicity;
- (iii) the yields as a function of the kinetic energy $\frac{dY(Z, N)}{dE}$ for a specific isotope (Z, N) , in the following also referred as the kinetic energy distribution;
- (iv) the yields as a function of the polar angle $\frac{dY(Z, N)}{d\vartheta}$ for a specific isotope (Z, N) , in the following also referred as the polar angle distribution.

Following the standard approach when presenting the experimental results, the discrete observables $Y(Z)$ and $Y(A, Z)$ are usually aggregated into various combinations:

- (i) $Y(A, Z)$ is the isotopic yields for a given Z element $Y(A, Z) = \{Y(A, Z), A = A_{\text{min}}, \dots, A_{\text{max}}\}$;
- (ii) the average-per-event particle multiplicities $\langle M \rangle$ (shown in Fig. 5 of Ref. [7]) is a combined set of $Y(A, Z)$ for different Z in the range 1–5 (see the exact definition in the following Table I);
- (iii) $Y(Z)$ for $Z = (Z_{\text{min}}, Z_{\text{max}})$ is given by $\{Y(Z), Z = Z_{\text{min}}, \dots, Z_{\text{max}}\}$.

In order to circumvent the low statistics in simulations, instead of using the full spectra of continuous differential observables, such as $\frac{dY(Z, N)}{dE}$ and $\frac{dY(Z, N)}{d\vartheta}$, we used their average ($\Lambda_{k=1}$), standard deviation ($\Lambda_{k=2}$) and skewness ($\Lambda_{k=3}$).

We assumed a Gaussian likelihood $L_j(\{p_i\})$ for each considered observable j and simulation parameters $\{p_i\}$. In particular

$$L_j(\{p_i\}) = e^{-\frac{1}{2}\chi_j^2(\{p_i\})}, \quad (2)$$

where $\chi_j^2(\{p_i\})$ are the residuals between experimental data y_j^{exp} and model data y_j^{mod} calculated for given values of the parameters $\{p_i\}$

$$\chi_j^2(\{p_i\}) = \left[\frac{y_j^{\text{exp}} - y_j^{\text{mod}}(\{p_i\})}{\sigma_j} \right]^2. \quad (3)$$

The uncertainty σ_j is calculated as a combination of the uncertainty for the model σ_j^{mod} and for the experimental data σ_j^{exp} :

$$\sigma_j = \sqrt{(\sigma_j^{\text{mod}})^2 + (\sigma_j^{\text{exp}})^2}. \quad (4)$$

Therefore, for aggregated discrete observables, which are combinations of independent observables, in case of $Y(A, Z)$

$$\chi^2(\{p_i\}) = \sum_{A=A_{\text{min}}}^{A_{\text{max}}} \left[\frac{Y(A, Z)^{\text{exp}} - Y(A, Z)^{\text{mod}}(\{p_i\})}{\sigma(A, Z)} \right]^2, \quad (5)$$

while for $Y(Z)$ for $Z = (Z_{\text{min}}, Z_{\text{max}})$:

$$\chi^2(\{p_i\}) = \sum_{Z=Z_{\text{min}}}^{Z_{\text{max}}} \left[\frac{Y(Z)^{\text{exp}} - Y(Z)^{\text{mod}}(\{p_i\})}{\sigma(Z)} \right]^2. \quad (6)$$

The standard deviations are calculated assuming a Poissonian distribution of the counts and taking into account the normalization to the number of events for both model and experimental data.

For a continuous differential observable j , such as $\frac{dY(Z, N)}{dE}$ and $\frac{dY(Z, N)}{d\vartheta}$, $\chi_j^2(\{p_i\})$ is given by

$$\chi_j^2(\{p_i\}) = \sum_{k=1}^{N_m} \left[\frac{\Lambda_{j,k}^{\text{exp}} - \Lambda_{j,k}^{\text{mod}}(\{p_i\})}{\sigma_{j,k}} \right]^2. \quad (7)$$

In this formula $N_m = 3$ is the number of moments used to describe the spectrum of the observable j , $\Lambda_{j,k}^{\text{exp}}$ is the k th moment of the observable j for the experimental data, while $\Lambda_{j,k}^{\text{mod}}(\{p_i\})$ is the same moment for theoretical simulations, where the simulation parameters are set to the values $\{p_i\}$. The uncertainty $\sigma_{j,k}$ is calculated as

$$\sigma_{j,k} = \sqrt{(\sigma_{j,k}^{\text{mod}})^2 + (\sigma_{j,k}^{\text{exp}})^2}, \quad (8)$$

where $\sigma_{j,k}^{\text{exp(mod)}}$ is the error associated with $\Lambda_{j,k}^{\text{exp(mod)}}$.

It is worth citing that at variance with other authors (e.g., Eq. (25) of Ref. [22]) we do not introduce any scaling parameter in the calculation of χ^2 in Eqs. (5)–(7), which would broaden the likelihood distribution. Only statistical errors, on both experimental and simulated data, are considered in our approach. However, we point out that, when the observables

TABLE I. Results of the Bayesian analysis performed on experimental data of the colliding system $^{20}\text{Ne} + ^{12}\text{C}$ at 50 MeV/nucleon as reference data, in comparison with AMD+HFI simulations, using the prior distribution of η and Γ value shown in Fig. 3. Given here are the resulting coordinates on the (η, Γ) plane of the maximum of the likelihood ($\eta_{\text{like}}, \Gamma_{\text{like}}$), the values of the maximum of the $\ln(\text{likelihood})$, the coordinates on the (η, Γ) plane of the maximum of the posterior ($\eta_{\text{Post}}, \Gamma_{\text{Post}}$), being N_{dof} the number of degrees of freedom corresponding to the used observables. For continuous differential observables (i.e., polar angle distributions $\frac{dY}{d\vartheta}$ and kinetic energy spectra $\frac{dY}{dE}$), spectra are replaced by mean, standard deviation, and skewness ($\Lambda_{j,k}$, with $k = 1, 2, 3$), i.e., $N_{\text{dof}} = 3$. $\langle M \rangle_{Z=1-5}$ is the average-per-event multiplicity distribution for $Z < 6$ (as in Fig. 5 of Ref. [7]), while $\langle M \rangle_{\text{restricted}}$ is the same quantity limited to the light products with higher statistics. The observables have been ordered according to the maximum value of the $\ln(\text{likelihood})$; those in part B and C have a maximum of the $\ln(\text{likelihood})$ which differs from the expectation value for more than three times the standard deviation STD. The observables in part C (i.e., $Y(Z)$, $\langle M \rangle_{Z=1,5}$ and $\langle M \rangle_{\text{restricted}}$) are not included in the total likelihood (due to the redundancy of the information).

observable	η_{like}	Γ_{like}	$\text{Max}_{\ln(\text{like})}$	η_{Post}	Γ_{Post}	N_{dof}
Part A - $ \text{Max}_{\ln(\text{likelihood})} + \frac{N_{\text{dof}}}{2} < 3 \cdot \text{STD}$						
$dY(^3\text{He})/d\vartheta_{\text{cm}}$	0.85	100	-0.2	0.85	100	3
$dY(^{10}\text{Be})/d\vartheta_{\text{cm}}$	1.05	2	-0.5	1.15	∞	3
$dY(^9\text{Be})/d\vartheta_{\text{cm}}$	1.15	4	-0.6	0.35	10	3
$dY(^{10}\text{Be})/dE$	5	100	-1.0	0.95	15	3
$dY(^{10}\text{B})/d\vartheta_{\text{cm}}$	1.05	∞	-1.1	1.05	∞	3
$dY(^3\text{He})/dE$	1.5	50	-1.6	0.65	∞	3
$dY(d)/d\vartheta_{\text{cm}}$	1.5	2	-1.9	0.55	∞	3
$dY(p)/d\vartheta_{\text{cm}}$	1.15	15	-2.4	1.15	15	3
$dY(^6\text{Li})/d\vartheta_{\text{cm}}$	1.05	15	-2.7	1.05	∞	3
$dY(^{11}\text{B})/d\vartheta_{\text{cm}}$	2	∞	-3.3	2	∞	3
$dY(t)/d\vartheta_{\text{cm}}$	0.55	5	-3.5	0.55	5	3
$dY(^7\text{Li})/d\vartheta_{\text{cm}}$	0.55	2	-3.8	0.55	2	3
$Y(^A\text{B})$ for $A = 8, 10, 11, 12, 13$	5	3	-3.9	0.85	4	5
Part B - $ \text{Max}_{\ln(\text{likelihood})} + \frac{N_{\text{dof}}}{2} > 3 \cdot \text{STD}$						
$dY(^9\text{Be})/dE$	5	100	-12	0.15	5	3
$dY(d)/dE$	0.15	∞	-15	0.15	∞	3
$dY(^7\text{Li})/dE$	1.5	∞	-15	1.5	∞	3
$dY(t)/dE$	0.25	100	-19	0.25	100	3
$dY(^{10}\text{B})/dE$	2	15	-19	2	15	3
$dY(^6\text{Li})/dE$	5	100	-22	1.5	50	3
$dY(^{11}\text{B})/dE$	5	100	-27	2	∞	3
$Y(^A\text{Be})$ for $A = 7, 9, 10, 11, 12$	5	∞	-66	2	∞	5
$dY(p)/dE$	0.15	0	-148	0.15	0	3
$Y(^A\text{Li})$ for $A = 6, 7, 8, 9$	2.5	5	-934	2.5	5	4
$Y(^A\text{H})$ for $A = 1, 2, 3$	1.5	100	-1076	1.5	100	3
$dY(^4\text{He})/dE$	5	15	-1128	5	15	3
$Y(Z)$, $Z \geq 6$	5	0	-1565	5	0	7
$Y(^A\text{He})$ for $A=3, 4, 6$	0.95	10	-3444	0.95	10	3
$dY(^4\text{He})/d\vartheta_{\text{cm}}$	5	15	-7065	5	15	3
Part C - redundant observables						
$\langle M \rangle_{\text{restricted}}^a$	5	3	-1749	5	3	11
$Y(Z)$ for $Z = 1-13$	5	∞	-3430	5	∞	13
$\langle M \rangle_{Z=1-5}^b$	2.5	50	-5508	2.5	50	17

^a $\langle M \rangle_{\text{restricted}} = \{Y(^A\text{H}) A = 1, 2, 3; Y(^A\text{He}) A = 3, 4; Y(^A\text{Li}) A = 6, 7; Y(^A\text{Be}) A = 9, 10; Y(^A\text{B}) A = 10, 11\}$.

^b $\langle M \rangle_{Z=1-5} = \{Y(^A\text{H}) A = 1, 2, 3; Y(^A\text{He}) A = 3, 4, 6; Y(^A\text{Li}) A = 6, 7, 8; Y(^A\text{Be}) A = 7, 9, 10, 11; Y(^A\text{B}) A = 10, 11, 12, 13\}$.

are replaced by their moments, in the calculation of the errors, the possible correlations among the moments have not been taken into account. Tests on the effects of this assumption and of the introduction of a non statistical error in the model are shown in the Appendix.

The Gaussian function for the likelihood shown in Eq. (2) was adopted following some precedents found in the

literature [22,26]. However, in order to reduce the weight of badly performing simulations, or to apply the Bayesian analysis to observables beyond the model applicability (e.g., $Y(^8\text{B})$, which is known to be underestimated by AMD [25]), a Laplace distribution is preferable:

$$L_j(\{p_i\}) = e^{-x_j(\{p_i\})}, \quad (9)$$

where

$$x_j(\{p_i\}) = \sum_{k=N_{\min}}^{N_{\max}} \left| \frac{y_{j,k}^{\exp} - y_{j,k}^{\text{mod}}(\{p_i\})}{\sigma_{j,k}/\sqrt{2}} \right|. \quad (10)$$

For discrete aggregated observables, using the symbols of Eqs. (5) and (6):

if $j = Y(Z)$ for $Z = (Z_{\min}, Z_{\max})$,

$$\begin{aligned} \diamond k &= Z \\ \diamond N_{\min} &= Z_{\min}, N_{\max} = Z_{\max} \\ \diamond y_{j,k}^{\exp(\text{mod})} &= Y(Z)^{\exp(\text{mod})} \\ \diamond \sigma_{j,k} &= \sigma(Z) \end{aligned}$$

if $j = Y(AZ)$

$$\begin{aligned} \diamond k &= A \\ \diamond N_{\min} &= A_{\min}, N_{\max} = A_{\max} \\ \diamond y_{j,k}^{\exp(\text{mod})} &= Y(AZ)^{\exp(\text{mod})} \\ \diamond \sigma_{j,k} &= \sigma(AZ). \end{aligned}$$

For each continuous differential observable j :

$$\begin{aligned} N_{\min} &= 1, N_{\max} = 3 \\ y_{j,k}^{\exp(\text{mod})} &= \Lambda_{j,k}^{\exp(\text{mod})}, \end{aligned}$$

with the uncertainty $\sigma_{j,k}$ given by Eq. (8).

Independently of the adopted form for the likelihood distribution of the single observable [i.e., Eq. (2) or (9)], assuming that all the observables are independent (i.e., neglecting the residual correlations among them), the whole likelihood distribution for all the considered observables (total number N_{obs}) as a function of the simulation parameter values is simply given by:

$$L^T(\{p_i\}) = \prod_{j=1}^{N_{\text{obs}}} L_j(\{p_i\}), \quad (11)$$

and then it is normalized to its modal value [27].

B. AMD model and the investigated parameters

The AMD model [5,6] is a transport model belonging to the molecular dynamics class, widely used to describe the dynamical phase of heavy-ion collisions at intermediate and relativistic incident energies with satisfactory results, also when applied to experimental data collected by the FAZIA Collaboration [7,9–14] and specifically for the dataset already introduced in Ref. [7] and analyzed in the present work. In AMD the time evolution of a many-body system is calculated by means of an equation of motion (obtained from the time-dependent variational principle) whose Hamiltonian includes the mean-field contribution by means of an effective interaction. In the version used for the present work (like in Ref. [7]), the effective interaction is the Skyrme force SLy4 [28] with a soft symmetry energy density dependence (normal density value $S_0 = 32$ MeV, slope at normal density $L = 46$ MeV). At

each time step the system is represented by a Slater determinant of Gaussian wave packets. At the end of the dynamical calculation (typically 300–500 fm/c) a cluster recognition algorithm is applied. Such an algorithm recognizes the nucleons close in phase space as belonging to a fragment. Note that in this work cluster and fragment are used as synonym.

Nucleon-nucleon collisions are included in a stochastic way [29]. AMD encompasses also cluster correlations, including, among the possible final states of a collision, also cluster states. Each scattered nucleon N_i ($i = 1, 2$) in a nucleon-nucleon collision may form a light cluster C_i (deuterons, tritons or He isotopes) with the surrounding particles B_i , e.g., $N_1 + N_2 + B_1 + B_2 \rightarrow C_1 + C_2$, thus producing two clusters for each collision. Such light clusters C_i may bind with other nucleons or clusters forming heavier nuclei up to $A \sim 10$. Standard two nucleon collisions are a particular case with $N_1 + N_2 \rightarrow N_1 + N_2$ (no clusters produced); the scattering process may also end with only one cluster produced: $N_1 + N_2 + B_1 \rightarrow C_1 + N_2$.

The collision probability to a specific configuration (C_1, C_2) is given by

$$\frac{d\sigma_C(C_1, C_2)}{d\Omega} = P(C_1, C_2, p_f, \Omega) \frac{1}{v_i v_f} |M|^2 p_f. \quad (12)$$

In this formula v_i is the initial relative velocity between the colliding nucleons N_1 and N_2 , (p_f, Ω) is the relative momentum vector after the momentum transfer between the two nucleons. E is the energy of the system (including the effective interaction), conserved in the collision process, and therefore the final relative velocity $v_f = \partial E / \partial p_f$ depends on the effective interaction too. $P(C_1, C_2, p_f, \Omega)$ is the overlap probability factor between the wave functions of the colliding nucleons for the cluster formation, Ω being the solid angle. M is the matrix element associated to the two-nucleon scattering, which may be modified in the nuclear medium. The in-medium nucleon-nucleon cross section used in the simulation is the sum of Eq. (12) over all the (C_1, C_2) configurations. M is given by: $|M|^2 = (\frac{2}{m_N})^2 \frac{d\sigma}{d\Omega}$, where m_N is the nucleon mass.

In this work the formulation of Eq. (3) of Ref. [30] was used for σ :

$$\sigma = \sigma_0 \tanh\left(\frac{\sigma_{\text{free}}}{\sigma_0}\right) \quad (13)$$

where $\sigma_0 = \eta \rho^{-2/3}$. In this equation ρ is the nuclear density, η is the screening parameter. The smaller the η value the stronger the reduction of the in-medium cross section with respect to the free nucleon-nucleon cross section value σ_{free} , which depends on the nucleon-nucleon collision energy. The standard value for the screening parameter η is 0.85. It is important to note that even when σ approaches σ_{free} (for instance for large values of η) the AMD nucleon-nucleon cross section of Eq. (12) is always different from the free one, i.e., it still keeps into account some in-medium effects in the factors other than $|M|^2$.

When the cluster formation is turned-off only standard nucleon-nucleon collisions are allowed, i.e., $P(C_1, C_2, p_f, \Omega) = 1$ for $(C_1, C_2) = (N_1, N_2)$ and 0 for all the other kind of collisions. On the contrary, when cluster

formation is activated, the overlap probability factor for standard nucleon-nucleon collisions is less than 1, because the probability is shared among all the possible channels (including clusters).

Cluster correlations are tuned by means of a parameter Γ , which reduces the cluster formation probability when the momentum transfer $\mathbf{q}$ of a two-nucleon collision is small. To do so, $P(C_1, C_2)$ is replaced by [29]:

$$(1 - e^{-\frac{\Gamma q^2}{2\hbar\sqrt{\nu}}}) \cdot P(C_1, C_2) \quad (14)$$

for $(C_1, C_2) \neq (N_1, N_2)$ and by

$$e^{-\frac{\Gamma q^2}{2\hbar\sqrt{\nu}}} + (1 - e^{-\frac{\Gamma q^2}{2\hbar\sqrt{\nu}}}) \cdot P(N_1, N_2) \quad (15)$$

for $(C_1, C_2) = (N_1, N_2)$, where $\nu = 0.16 \text{ fm}^{-2}$ is the standard width parameter. The larger the Γ the higher the probability of cluster formation. $\Gamma = \infty$ indicates the largest cluster formation probability, given by the overlapping of the Gaussian wave functions of the involved nucleons. A detailed description of the AMD model and of the cluster treatment can be found in Ref. [25] and references therein.

The present Bayesian analysis was performed to tune the screening parameter η and the clustering parameter Γ . For such purposes, 165 AMD simulations with different parameter values for the system $^{20}\text{Ne} + ^{12}\text{C}$ at 50 MeV/nucleon, each of them including 6000 events in the whole impact parameter range according to a triangular distribution from 0 up to the grazing impact parameter (9 fm), were produced. For each simulation values of η and Γ chosen in a grid ranging for η from 0.15 up to 5, and for Γ from no cluster at all ($\Gamma = 0$) up to no reduction of the probability of cluster formation ($\Gamma = \infty$) were used. Note that grid steps are not regularly spaced. The investigated values of η start from 0.15 and increase by steps of 0.1 up to 1.15. After that, only the values 1.5, 2.0, 2.5, and 5.0 were used. It is worth mentioning that the upper limit $\eta = 5.0$ was imposed by the functional form of Eq. (13), which saturates at σ_{free} beyond that value, as shown in Fig. 1.

For Γ we investigated values of 0, 1, 2, 3, 4, 5, 10, 15, 50, 100, and ∞ ; we did not explore the region between $\Gamma = 0$ and $\Gamma = 1$ because we already know from the literature that a significant amount of clustering is mandatory to reproduce the experimental data [7,31]. The dynamical calculation was stopped at 500 fm/c, and the following secondary decay of excited primary fragments identified by the recognition algorithm was simulated by means of a statistical code used as an afterburner.

During the time evolution between 0 and 500 fm/c when $\Gamma > 0$ clusters may dynamically form and break up due to successive collisions. For the investigated system the effect of the simulation parameters η and Γ on the average number of collisions, and on the average number of clusters dynamically formed for each collision (i.e., a value ranging from 0 to 2 clusters) is shown in Fig. 2 top and bottom panels, respectively. A significant increase in the average number of collisions is observed when η increases, while Γ has much less influence. Concerning the number of clusters formed for each collision, it increases with Γ , though not strongly (therefore it would be useless doing the calculation also, for example, for $\Gamma = 500$ or $\Gamma = 200$). On the contrary, the η parameter slightly

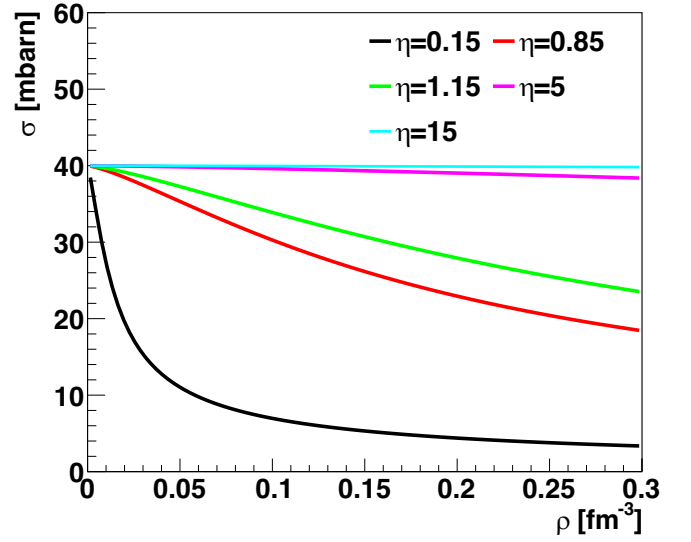


FIG. 1. Functional form of Eq. (13) as a function of the density for different values of the η parameter; the calculation was performed with $\sigma_{\text{free}} = 40$ mbarn.

affects the number of formed clusters, which increases as a function of η . We observe that, despite the nonuniform grid spacing for the investigated parameter values, both the average number of collisions per event and the average number of cluster per collision behave in a regular way, without sudden step, thus suggesting a smooth trend for the parameter values not included in the explored grid.

C. Afterburner of primary excited clusters

Although the AMD can be run up to very long times (e.g., in Ref. [8], where the dynamical calculation was tested up to 10^4 fm/c), it is customary to stop the dynamical calculation at 300–500 fm/c, since at that time the dynamical phase can be safely considered as concluded. The following evolution is simulated by means of a statistical code, generally implemented for the decay of a compound nucleus (with few exceptions, e.g., SIMON [32]) and applied to the hot primary fragments. The procedure can be repeated many times for each primary events in order to increase the statistics. While the most commonly used afterburner simulating secondary sequential decay [9–13,25,33] is the GEMINI++ code [34], in Ref. [7] the HFI code [24] was used. Both statistical codes are based on the Hauser-Feshbach formalism [35], but HFI is particularly optimized for the decay of light compound nuclei, since it includes all the tabulated discrete levels for particle decays up to $Z = 12$ [36–40]. However, unlike GEMINI, it does not include fission as possible decay channel.

In this work, on the basis of the experience gained in Ref. [7] for the same dataset, we used the HFI code and for each primary AMD event 100 secondary decays were simulated. This commonly used procedure introduces correlations among the events generated from the same primary cluster partition, whose contribution to the error of the model is not considered in our analysis. In this study we reduced both the

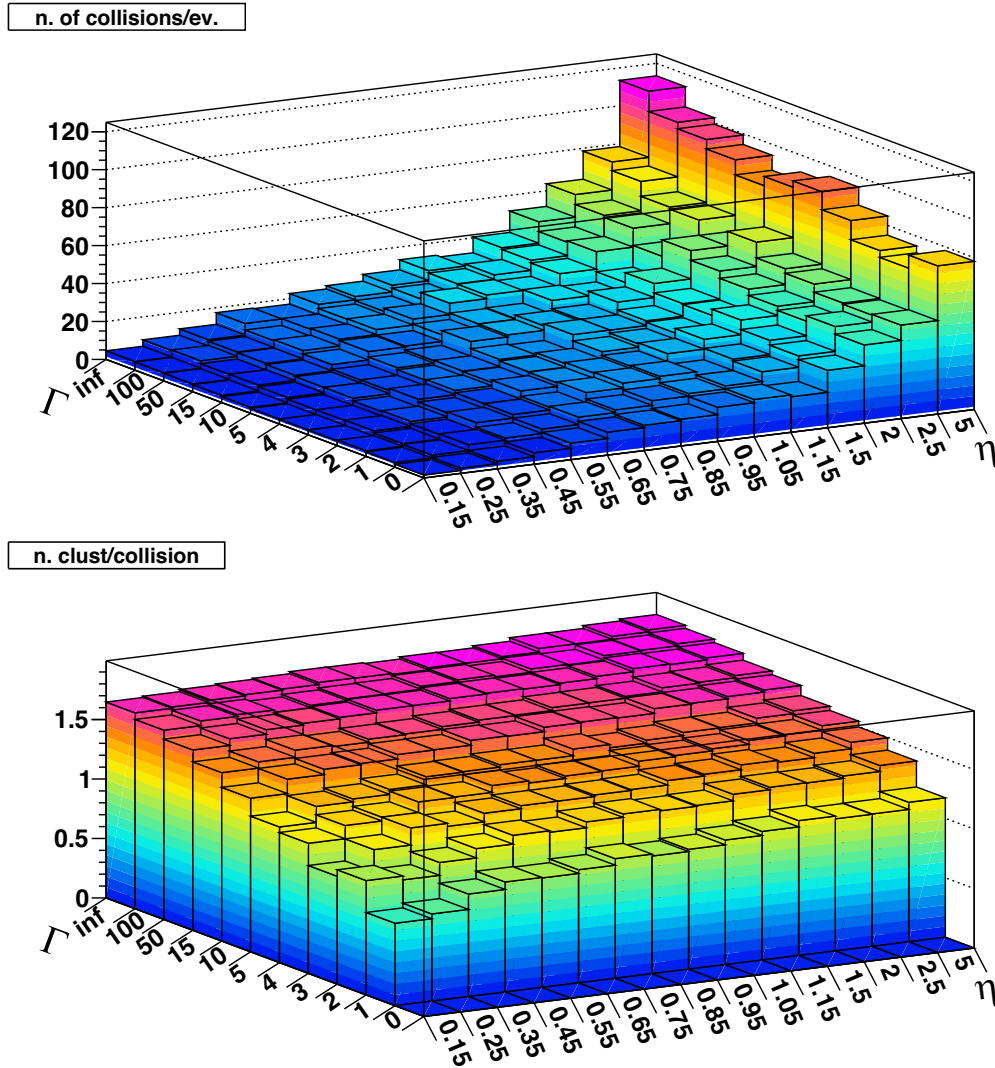


FIG. 2. Top: average number of collisions per event as a function of η and Γ parameters (see text for their description); bottom: average number of dynamically formed clusters per collision. Each nucleon-nucleon collision may end with 0 (standard nucleon-nucleon collision), one or two formed clusters. Both pictures refer to collisions of $^{20}\text{Ne} + ^{12}\text{C}$ at 50 MeV/nucleon incident energy simulated by the AMD transport model.

number of produced primary events and the number of secondary events produced for each primary by a factor 10 with respect to the simulations used for Ref. [7]. It is worth noting that, since in principle a different afterburner may produce different secondary events, the results of the Bayesian analysis shown in this paper must be interpreted as referred to the joint simulation AMD + HFI and not to AMD only. The effect of applying GEMINI++ is discussed in the Appendix.

Before applying the Bayesian inference, the simulated events were filtered via a software replica of the setup including geometry and identification thresholds, as in Ref. [7]. It is important to mention that the excitation energy of the primary fragments, necessary as input to the afterburner, was calculated using as ground state values not those coming from the tabulated values of binding energies but those produced by AMD by means of the procedure used to produce the ground states. In this way mismatches between the

tabulated values and those produced by AMD are strongly reduced.

D. Experimental data

The experimental dataset used for this work corresponds to the colliding system $^{20}\text{Ne} + ^{12}\text{C}$ at 50 MeV/nucleon incident energy (target thickness: $240 \mu\text{g}/\text{cm}^2$, beam current: 10^9 pps, global trigger rate: 1 kHz, with a maximum of 300 Hz for each detector). Data were collected with beam delivered at LNS-INFN in Catania, using four blocks of FAZIA [18–20] in wall configuration (see Fig. 1 of Ref. [7] for the layout of the setup), for a total of 64 telescopes located 80 cm from the target, covering polar angles between 2° and 8° . FAZIA three-layer telescopes [41,42] [Si 300 μm thick + Si 500 μm thick + CsI(Tl) 10 cm thick and read out by a photodiode], fully equipped with custom digital electronics,

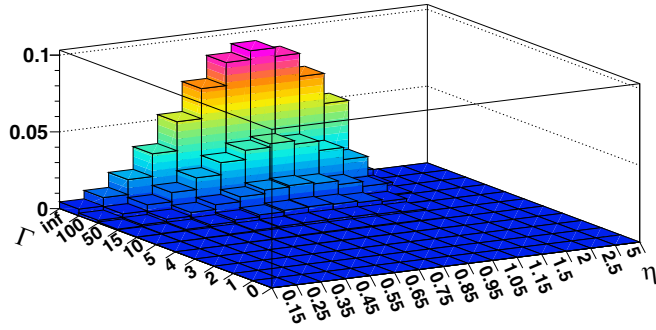


FIG. 3. Prior distribution of the AMD parameters η and Γ used in the Bayesian analysis [see Eq. (16)].

represent the state-of-the-art in terms of isotopic discrimination capabilities and identification thresholds, achieving results comparable to those of a spectrometer for medium-mass nuclei (isotopic identification up to $Z \sim 25$ for ions punching through the first silicon layer, up to $Z \sim 20$ for those stopped in the first layer thanks to the pulse shape analysis technique).

In Ref. [7] the experimental data concerning charge and velocity spectra, average multiplicities, angular distributions and energy spectra for light ejectiles have been compared with predictions of AMD + HFI. As already cited, two versions of the dynamical code were used: one version without cluster production (corresponding to Γ parameter value equal to 0) and one version without reduction of the probability for the cluster formation (corresponding to Γ parameter value equal to ∞). In both cases the screening parameter for the in-medium cross section was chosen as $\eta = 0.85$, which corresponds to a standard value used also in other comparisons of FAZIA data with AMD ([9–13]. Note that in all these references $\Gamma = \infty$ was used). As observed in Ref. [7], the model including clustering performed better in reproducing the light ejectile properties for all the investigated systems, in particular the angular distributions, the energy spectra, and the average multiplicities. We stress that even if clusters are not dynamically produced in the collision process (i.e., when $\Gamma = 0$), they are present in the output of AMD, as shown in Fig. 5 of Ref. [7] (green bars), since they are identified by the already-mentioned fragment recognition algorithm applied at the end of the dynamical calculation.

III. RESULTS

A. Prior distribution

Taking advantage of the existing literature, which suggests $\eta = 0.85$ and $\Gamma = \infty$ as standard model parameter values, the used prior distribution is (see Fig. 3):

$$\mathcal{F}(\eta, \Gamma) = K_{\text{norm}} \cdot e^{-\frac{(\eta - \eta_{\text{ref}})^2}{2\sigma_G^2}} \cdot e^{k_E \Gamma} \quad \text{with } \Gamma \in [0, \Gamma_{\text{ref}}], \quad (16)$$

where K_{norm} is a normalization constant. Namely, for the η parameter we used a Gaussian distribution centered around $\eta_{\text{ref}} = 0.85$, with a somewhat arbitrary standard deviation σ_G of 0.4, while for the Γ parameter an increasing exponential with constant k_E equal to 0.01 was chosen. In order to avoid a

divergence, a cut off on the Γ parameter is imposed at 1000 in the calculation of the prior distribution. Therefore Γ_{ref} , which should be equal to ∞ , is taken equal to 1000 in Eq. (16). Tests with different prior distributions, e.g., a broader Gaussian for the η parameter ($\sigma_G = 0.85$) and a milder-increasing exponential for the Γ distribution (with $k = 0.001$) have also been performed.

We remind that the effect of using a completely flat prior distribution can be obtained by looking directly at the resulting likelihood. The effects of the various choices will be discussed in Sec. III C 2.

B. Test of the method

Before performing the analysis with the experimental data, we tested the method, using dummy data generated by AMD (in particular with $\Gamma = 4$ and $\eta = 0.55$). The yields of all the particles obtained for these reference data were modified according to Gaussian distributions centered around the original yield values, with variances associated to Poissonian fluctuations. As an example, in Fig. 4 left panel the simulated kinetic energy distribution of protons in the laboratory reference frame, obtained with model parameters $\Gamma = 4$ and $\eta = 0.55$, is shown. The histogram corresponds to the original result of the simulation, while full dots correspond to the distribution obtained after applying the Gaussian perturbation. The right panel shows the logarithm of the posterior distribution obtained combining the likelihood distribution $L^T(\{p_i\})$ associated to a given set of observables (in this case, the kinetic energy of protons and ^4He in the laboratory reference frame and the polar angle distribution in the c.m. for ^7Li) with the prior of Fig. 3.

We observe that the position of the maximum of the posterior distribution corresponds to the values of η and Γ of the simulation used as reference data (black arrow in the picture), thus proving that the method is properly working. Similar results have been obtained both by choosing a simulation with different η and Γ values as reference data and by varying the set of investigated observables. Therefore, these outcomes support the validation of the method. We would like to stress that this was only a test to check whether the procedure is properly working. The choice of the observables employed for this test has no particular meaning for the constraining of η and Γ .

C. Choice of the observables and obtained results

Having verified the dependability of the adopted procedure, we now apply it to the experimental data using them as reference dataset for the Bayesian analysis.

1. Single observables

Table I lists all the observables considered in this study. In particular, we have individually tested dY/dE and $dY/d\vartheta$, replacing the full spectra with $\Delta_{j,k}$ with $k = 1, 2, 3$, for particles with $Z \leq 5$, provided that the statistics in the model is sufficient (typically less than five counts). We have also evaluated discrete observables such as $Y(^A Z)$ for $Z = 1-5$,

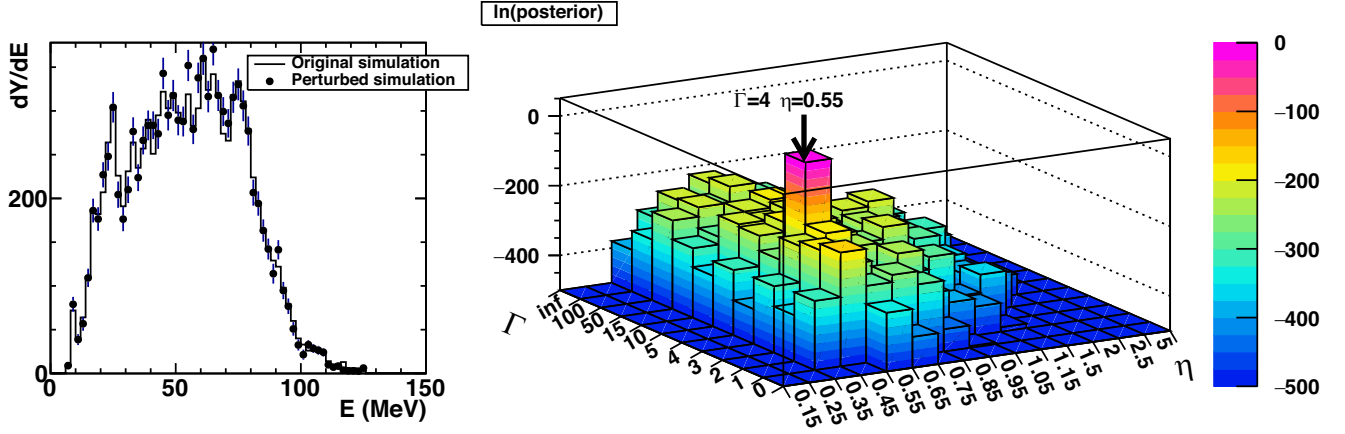


FIG. 4. AMD + HFI simulations for the system $^{20}\text{Ne} + ^{12}\text{C}$ at 50 MeV/nucleons. Left panel: kinetic energy distribution of protons in the laboratory reference frame obtained with model parameters $\Gamma = 4$ and $\eta = 0.55$. The histograms with line and full dots correspond to the original results of the simulation and to the modified ones after a Gaussian perturbation of the proton kinetic energy spectrum, respectively (see text). The modified simulation was used as reference data for the Bayesian analysis. Right panel: Result of the Bayesian analysis obtained using the AMD + HFI simulation with parameters $\eta = 0.55$ and $\Gamma = 4$ as reference data. The posterior distribution on the (η, Γ) plane shown in the picture was obtained from the likelihood distribution $L^T(\{p_i\})$ calculated with the kinetic energy distribution of protons and ^4He in the laboratory reference frame and the polar angle distribution in the c.m. of ^7Li . The prior distribution of Fig. 3 was used. The black arrow corresponds to the position of the reference simulation on the (η, Γ) plane.

$Y(Z)$ for all the ejectiles and the average multiplicity $\langle M \rangle$ for light products.

In the table, for all the individually tested observables, the maximum value of the ln-likelihood distribution [calculated with the Gaussian likelihood of Eq. (2)], the position of the maximum of the likelihood and of the posterior distribution (calculated with the prior distribution of Fig. 3) on the (η, Γ) plane are summarized. For some experimentally detected light products such as ^6He , ^8Li , ^9Li , ^7Be , ^{11}Be , ^{12}Be , ^8B , ^{12}B and ^{13}B there is not enough statistics in the simulation to individually study their kinetic energy spectra and polar angle distributions. It is important to note that a significant increase of the number of simulated primary events (6000) on the adopted grid is not conceivable for the present analysis due to the extremely long computational time required.

An example of the obtained ln-likelihood distribution for $dY(^{10}\text{Be})/d\vartheta$, replaced by $\Delta_{j,k}$ with $k = 1, 2, 3$, is shown in Fig. 5 top panel. In this plot the normalization to the modal value of the likelihood distribution is removed; in this way it is possible to observe that the maximum of the ln-likelihood, located at $\eta = 1.05$ and $\Gamma = 2$, has an absolute value of -0.51 . When the gap between experimental data and simulation is of the same order of magnitude as the statistical errors, by definition the ln-likelihood tends towards the expected value $-\frac{1}{2}N_{\text{dof}}$. N_{dof} is the number of degrees of freedom, equal to the number of bins for discrete observables [e.g., the number of considered isotopes for $Y(^AZ)$ or the number of considered elements in case of $Y(Z)$ for $Z = (Z_{\text{min}}, Z_{\text{max}})$] and to the number of considered moments for the continuous differential ones. The standard deviation of the ln-likelihood distribution STD is equal to $\sqrt{\frac{N_{\text{dof}}}{2}}$, as given by the χ^2 distribution. A maximum of the ln-likelihood falling within $3 \times \text{STD}$ with respect to the expected value $-\frac{1}{2}N_{\text{dof}}$, as in case of Fig. 5 top panel, where $N_{\text{dof}}=3$, indicates that the model is

able to reproduce the experimental data within the statistical errors for the corresponding combination of the η and Γ parameters. This happens for the observables listed in part A of Table I.

When the disagreement between the experimental data and the simulation is significantly larger than the statistical errors, the maximum of the ln-likelihood differs for more than three times STD from the expected value. This is the case of $dY(^4\text{He})/dE$, where $N_{\text{dof}} = 3$, as shown in Fig. 5 bottom panel. In this case the absolute maximum, located at $\eta = 5$ and $\Gamma = 15$, is very negative (ln-likelihood around -1130). The second maximum (located at $\eta = 5$ and $\Gamma = \infty$) is about -1600 , producing a very peaked ln-likelihood distribution. A similar behavior is observed also for the observables listed in part B and part C of Table I. In Part C redundant observables, whose information is already included in the observables of Part A and Part B, thus not included in the calculation of the total likelihood equation (11), are listed.

The $(\eta_{\text{like}}, \Gamma_{\text{like}})$ position of the maximum of the likelihood on the (η, Γ) plane is often equal to the $(\eta_{\text{Post}}, \Gamma_{\text{Post}})$ position of the maximum of the posterior obtained with the prior shown in Fig. 3; the presence of secondary maxima may be responsible for the difference between $(\eta_{\text{like}}, \Gamma_{\text{like}})$ and $(\eta_{\text{Post}}, \Gamma_{\text{Post}})$. The investigated set of observables includes many of the physical quantities, which in Ref. [7] were found to be particularly sensitive to the parametrization of the clustering in the model (i.e., $\Gamma = 0$ vs $\Gamma = \infty$).

A first glance at Table I shows that, although a common position for the maximum of the likelihood among the different observables does not exist, not even for those in Part A, some trends clearly emerge: a higher value of the η parameter with respect to the standard value of 0.85, used in all the previous comparisons of AMD with FAZIA and INDRA-FAZIA data [7,10–14], is preferred and $\Gamma > 10$ is dominant. As already stated, it is important to note that values of η close to 5 do

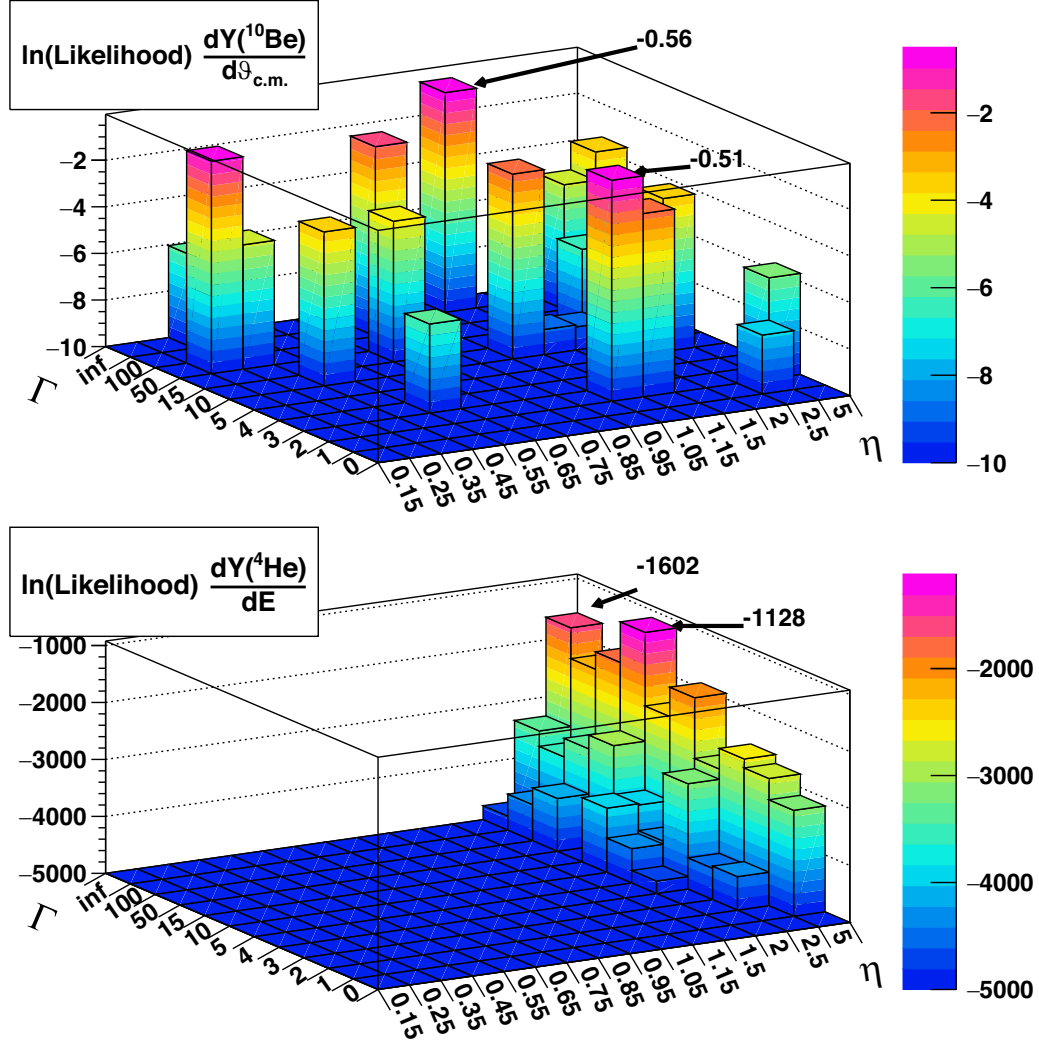


FIG. 5. Results of the Bayesian analysis performed comparing the experimental data of the system $^{20}\text{Ne} + ^{12}\text{C}$ at 50 MeV/nucleon with AMD + HFI simulations, aimed at tuning η and Γ parameters (see text). In-likelihood for the c.m. polar angle distribution of ^{10}Be $\frac{dY(^{10}\text{Be})}{d\Omega_{\text{c.m.}}}$ (which is an observable belonging to Table I Part A) (top) and for the kinetic energy spectrum of ^4He $\frac{dY(^4\text{He})}{dE}$ (which is an observable belonging to Table I Part B) (bottom) on the (η, Γ) plane. For both observables in the calculation of the likelihood the original spectra have been replaced by their mean, standard deviation and skewness ($\Lambda_{j,k}$ for $k = 1, 2, 3$).

not mean that the nucleon-nucleon collisions are ruled by the free cross section, because the collision cross section is given by Eq. (12), where η is an ingredient included only in the calculation of the matrix element M . However, a higher η corresponds to a larger number of collisions, as shown in Fig. 2. The fact that the clustering is necessary to reproduce the experimental data (i.e., that Γ should be considerably > 1) is an evidence already supported in the literature [7,25].

2. Combination of observables

In the light of these observations and taking into account the limits of the procedure due to the fact that in many cases the gap between data and simulation is greater than the statistical errors, we apply a Bayesian inference combining the available observables, first those of Part A only, second those of Part A + B. This was done in order to see if there

is a region of parameter values able to further improve the global agreement between experimental data and simulation. The consequence may be a worsening of the concordance for the single observables, considering the fact that the individual likelihoods of the chosen observables, even those of Part A of Table I, exhibit some tension on the most probable parameter values.

The In-posterior distribution obtained combining all the observables of Part A of Table I, achieved with the prior of Fig. 3 and using the Gaussian likelihood of Eq. (2), is shown in Fig. 6 top panel. The absolute maximum of both the In-posterior and the In-likelihood (not shown) distributions is located at $\eta = 5$ and $\Gamma = 4$. Other lower maxima are all located in the region of high η ; for example the second highest peak is located at $\eta = 5$ and $\Gamma = 2$. The absolute maximum value of the likelihood distribution is indeed very negative (ln-likelihood around -1100 ; the second highest maximum is around -1200).

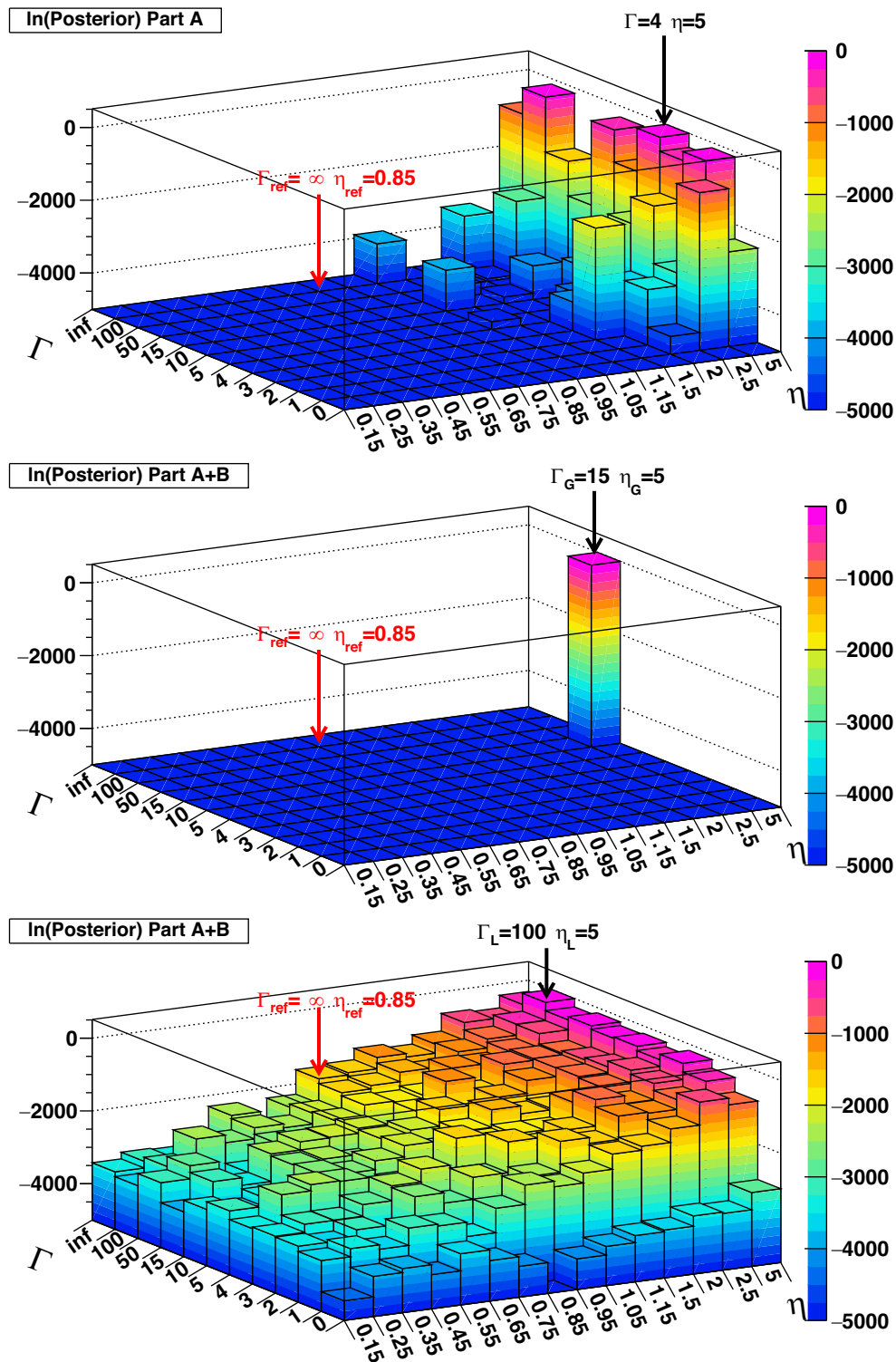


FIG. 6. Results of the Bayesian analysis performed on the base of AMD + HFI simulations compared with experimental data for the colliding system $^{20}\text{Ne} + ^{12}\text{C}$ at 50 MeV/nucleon, aimed at the tuning of η and Γ parameters (see text) Top: In-posterior distributions in the plane of the AMD parameters η and Γ obtained including all the observables listed in Part A of Table I, using the Gaussian likelihood distribution of Eq. (2). Middle: the same, but including all the available observables of Part A and B of Table I. Bottom: In-posterior for all the observables of Part A and B using the Laplacian likelihood distribution of Eq. (9). Black arrows indicate the position of the maximum of the posterior distributions, while red arrows indicate the parameter values used in previous studies, e.g., Refs. [9–14].

Such low values of the maximum of the likelihood suggest a weak agreement between the model and the experimental data even when combining the observables of the only Part A of Table I. However, the indication emerging from this plot confirms the trend already highlighted by the single observables in Table I: a higher number of collisions with respect to those produced by the standard value of $\eta = 0.85$ is preferred and the clustering is necessary. Concerning this last point, a reduction of the probability of cluster formation with respect to the pure superposition of the wave functions (i.e., $\Gamma = \infty$) seems to be favored.

In Fig. 6 middle panel the ln-posterior distribution obtained including all the observables of Part A and B of Table I is shown. In this case the maximum of both the posterior and the likelihood (not shown) is located at $\eta_G = 5$ and $\Gamma_G = 15$ (where the subscript G stands for Gaussian), with a very much lower secondary maximum (not shown due to the limited range of the z axis) at $\eta = 5$ and $\Gamma = 50$. Due to the shape of the likelihood, the use of a milder prior distribution with respect to the one of Fig. 3 does not shift the position of the maximum of the posterior distribution. As expected, the absolute maximum of the likelihood obtained including the observables of both part A and B is even lower than in the previous case of part A only (ln-likelihood around -28000). Anyhow, the message emerging from this plot is exactly the same as that of Fig. 6 top row: more collisions with respect to the standard values used in the literature improve the global agreement between data and simulation.

In the Appendix some tests demonstrating the robustness of these results will be shown.

A strongly peaked posterior distribution as the one presented in Fig. 6 middle row is the fruit of a poor agreement (with respect to the statistical errors) between data and simulation. It is worth noting that a similar posterior distribution prevents any further application of the Bayesian inference; in fact, if such a distribution is used as new prior when new data are available, the new posterior will hardly move from it, whatever be the shape of the likelihood distribution. The use of a Laplacian likelihood distribution, as the one of Eq. (9), reduces this problem, as shown in Fig. 6 bottom row, where all the observables of part A and B of Table I are included (as in Fig. 6 middle row).

The maximum of both the ln-likelihood (not shown) and the ln-posterior is located at $\eta_L = 5$, $\Gamma_L = 100$ (where the subscript L stands for Laplacian), quite similar to what we obtained with the Gaussian likelihood (i.e., $\eta_G = 5$ and $\Gamma_G = 15$). However, the distribution is much wider and smoother and the obtained posterior distribution can be easily used in an iterative way in the Bayesian inference as new prior in presence of new experimental information. Therefore we believe that this is the correct approach once the error of the model gives the major contribution to the likelihood value.¹

For the Bayesian approach the maximum of the ln-posterior located at the edge of the explored range of η is a

weak result. This result should suggest to widen the interval for this parameter. However, in this particular case, as shown in Fig. 1, the σ function of Eq. (13) saturates at σ_{free} for $\eta = 5$, therefore going beyond the upper limit of the investigated range would not make any improvement. We can thus conclude that we are facing a limit of the model: either the model intrinsically fails in simulating the details of the experimental data, or these same details might be better reproduced acting on other parameters, not tested in this analysis, which are not generally modified by the users.

In order to check the effect of the obtained results in terms of the observable spectra, in Fig. 7 some examples are presented. These pictures correspond to $\frac{dY(p)}{d\vartheta_{\text{cm}}}$ (top left), $\frac{dY(^3\text{He})}{dE}$ (top right), $Y(^A\text{B})$ (bottom left) and to the model-to-experimental ratio of $\langle M \rangle_{\text{restricted}}$ (bottom right). In all panels of Fig. 7 (except for bottom right) black symbols are the experimental data, while red continuous histograms are the standard simulation with $\eta_R = 0.85$ and $\Gamma_R = \infty$ (where the subscript R stands for reference); the continuous blue line corresponds to the maximum of the likelihood obtained including all the observables of part A and B of Table I using the Gaussian likelihood of Eq. (2) (as in Fig. 6 middle row, $\eta = 5$, $\Gamma = 15$), the dotted blue line is the same but obtained with the Laplacian likelihood of Eq. (9) (as in Fig. 6 bottom row), while the black dashed line corresponds to the maximum of the likelihood obtained considering only the presented observable. In the bottom right panel of Fig. 7 red circles correspond to the simulation-to-experimental ratio of $\langle M \rangle_{\text{restricted}}$ obtained with the standard simulation, black squares correspond to the ratio obtained for the simulation maximizing the likelihood for $\langle M \rangle_{\text{restricted}}$, while blue triangles and open diamond are obtained for the simulation maximizing the total likelihood using the Gaussian and the Laplacian likelihood, respectively.

These plots show that, although there is not a strong difference among the results produced with different simulations, the best agreement is usually (quite obviously) obtained when only the considered observable is used for the likelihood calculation (black continuous lines and black squares). The simulation maximizing the total likelihood behaves better than the standard one mainly for $\langle M \rangle_{\text{restricted}}$ and for $\frac{dY(p)}{d\vartheta_{\text{cm}}}$, while for the other spectra the gain seems to be absent. Anyway, all the presented simulations (including the standard one) give reasonable results for all the shown observables.

As matter of fact, as published in the literature [7–14], the standard simulation is already able to reproduce in a reasonable way the experimental data; the simulation selected with the Bayesian inference offers only a small improvement of the agreement.

A particular mention must be made to boron isotopes $Y(^A\text{B})$ (bottom left panel of Fig. 7): it is clearly evident that $Y(^8\text{B})$ is strongly underestimated by the model, as shown in Ref. [25]. The use of the Bayesian inference for observables beyond the model applicability requires caution in the choice of the likelihood functional form; the Laplace distribution of Eq. (9) might be a better choice with respect to the gaussian one to treat outliers such as this light cluster as single observable.

¹When experimental and model errors are comparable, the correct likelihood would be the convolution of the Laplace likelihood for the model and the Gaussian one for the experimental case.

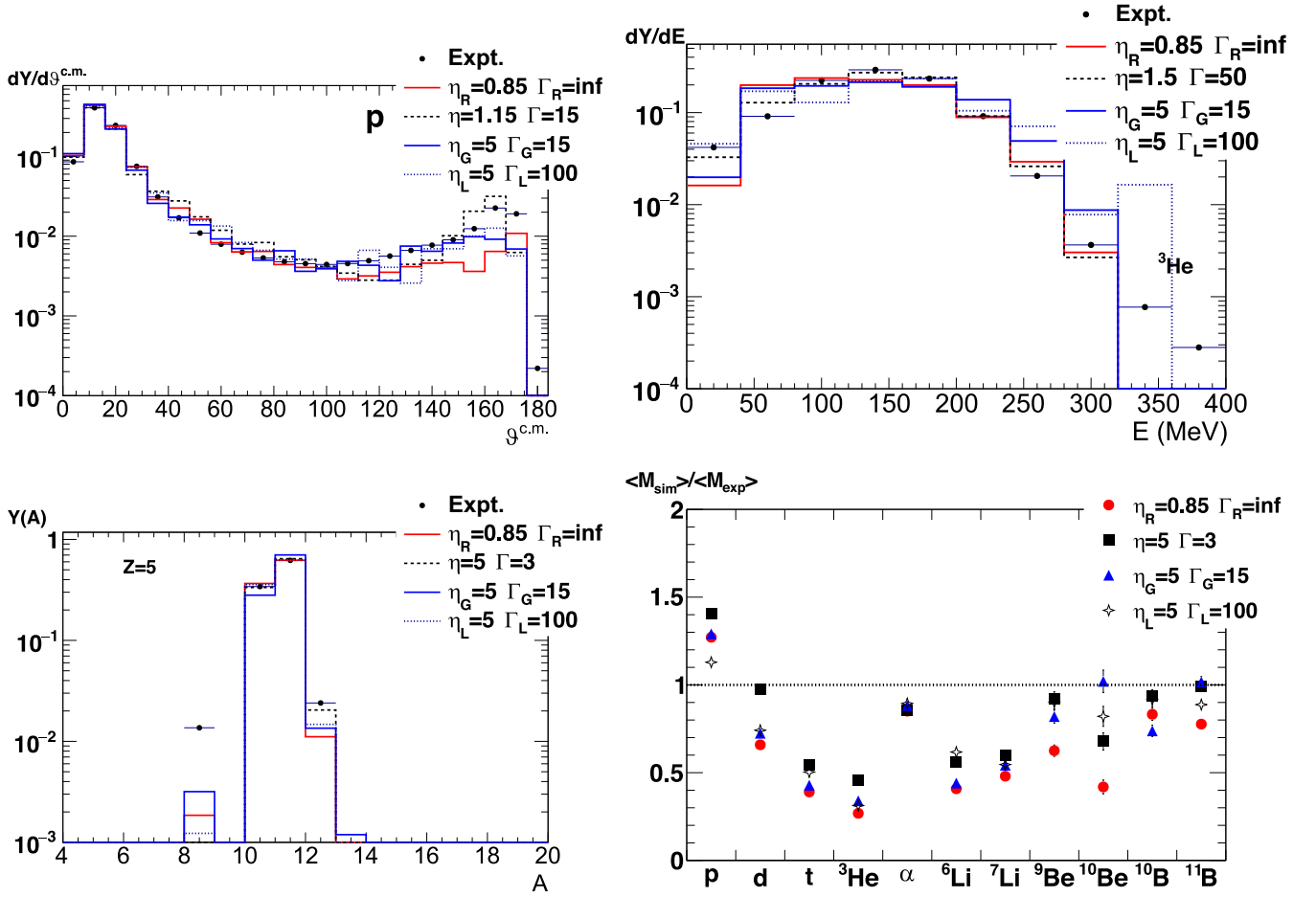


FIG. 7. Comparison of experimental data for the system $^{20}\text{Ne} + ^{12}\text{C}$ at 50 MeV/nucleon with the results of AMD + HFI simulations run with different values of η and Γ parameters (see text for their description). Top left panel $\frac{dY(p)}{dg_{\text{c.m.}}}$, top right panel $\frac{dY(^3\text{He})}{dE}$, bottom left panel $Y(A)$, bottom right panel simulated-to-experimental ratio of $\langle M \rangle_{\text{restricted}}$. In the top and bottom left panels black symbols are the experimental data, the red histogram corresponds to the simulation with $\Gamma_R = \infty$ and $\eta_R = 0.85$ (the standard one); the dashed black histogram corresponds to the simulation which maximizes the likelihood separately for the observable reported in each panel (see Table I), while the continuous blue one ($\eta_G = 5$, $\Gamma_G = 15$), corresponds to the simulation maximizing the likelihood for all the observables of Table I part A and B using the Gaussian likelihood of Eq. (2); the dotted one is the same obtained with the Laplacian likelihood of Eq. (9), i.e., $\eta_L = 5$, $\Gamma_L = 100$. Bottom right panel: red circles correspond to the standard simulation ($\eta = 0.85$, $\Gamma = \infty$), black squares to the simulation maximizing the likelihood including only particle multiplicities $\langle M \rangle_{\text{restricted}}$, blue triangles correspond to the simulation maximizing the total likelihood using the Gaussian likelihood, open diamonds are the same but with the Laplacian one.

IV. SUMMARY, CONCLUSIONS, AND PERSPECTIVES

In this work a Bayesian analysis applied to experimental data collected with a reduced version of the FAZIA setup for the system $^{20}\text{Ne} + ^{12}\text{C}$ at 50 MeV/nucleon has been discussed. Data were compared with the AMD simulation, coupled to HFI as an afterburner, run with many different values of the Γ and η parameters, related to clustering and to in-medium nucleon-nucleon cross section. Every simulation has been filtered via a software replica of the setup to take into account the geometrical coverage and the identification thresholds of the setup.

Since we aimed at globally reproducing the experimental events, the Bayesian analysis has been applied to as many observables as possible, compatibly with the model statistics. The result of the Bayesian inference shows a preference for

η higher than the standard value of 0.85 used in previous analysis (e.g., Ref. [7] for this same dataset), suggesting that a higher number of collisions improves the agreement between experimental and simulated data. Concerning the Γ parameter related to the clustering, this analysis confirms that the introduction of a certain amount of clustering, i.e., $\Gamma > 0$, is mandatory to reproduce the experimental data, while suggesting a finite value for Γ , i.e., not the standard value of ∞ used in Ref. [7] and in all the previous analysis. These results demonstrated to be sufficiently robust, being basically stable with different selections of the observable dataset used for the analysis.

The analysis discussed in this paper can be extended to other parameters of the simulation; for example, different cross section formulas can be investigated. Moreover, at least

for central multifragmentation events, the Mott critical density can be studied, although the dataset here discussed is not the best one for such studies: among those available at present in the INDRA-FAZIA Collaboration, the best choice might be $^{36}\text{Ar} + ^{58}\text{Ni}$ at 74 MeV/nucleon, as it is, to date, the smallest system investigated with a setup with high geometrical coverage. However, in this case the size of the system is considerably bigger, making it more time consuming to simulate.

The presented analysis has been done on a grid of simulations run with different values of the two investigated parameters, which have been varied with more or less wide finite steps. It is at present under study the possibility to reduce the step size for the simulated data by means of machine-learning-based emulator, training the algorithm to reproduce the output of the model for the investigated observables as a function of η and Γ . The test of the algorithm can be done excluding from the training the simulated data corresponding to some values of the grid of parameters. In this way it would be possible to predict the observables for parameter combinations not included in the original grid, thus refining the Bayesian analysis. This would open the possibility to apply the present analysis also to heavier systems producing a sparse mesh, thus partially overcoming the problem of time consumption associated to the simulation of a reaction between heavy partners.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: DISCUSSION ON THE ROBUSTNESS OF THE OBTAINED RESULTS

The comparison of the plots shown in Fig. 2 with the results on the posterior distributions shown in Sec. III C indicates that an average number of collisions higher than that associated to the standard value of $\eta = 0.85$ is preferred for all the combinations of observables, even if this is not always true when the physical quantities are considered separately (e.g., for the kinetic energies of d and t). On the contrary, the obtained results are not indicating a strong preference for a particular Γ value, although it is clear that a significant amount of clustering is mandatory; in fact, under no circumstances there is an observable, alone or in combination, for which the maximum of the likelihood is at $\Gamma = 0$, with the possible exception of $dY(p)/dE$ and of $Y(Z)$ for $Z \geq 6$.

In order to verify the robustness of these conclusions we have performed some tests, all of them using the Gaussian likelihood of Eq. (2).

First of all, we have investigated the effect of using GEMINI++ instead of HFI as an afterburner; the obtained In-likelihood distribution for all the observables of part A and B of Table I, analogous to the In-posterior of Fig. 6 middle panel (essentially identical to the corresponding In-likelihood)

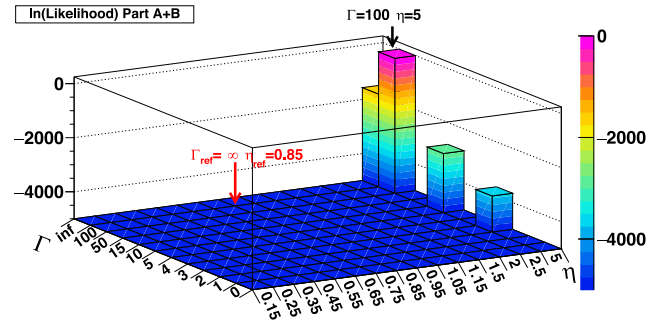


FIG. 8. Result of the Bayesian analysis performed on experimental data of $^{20}\text{Ne} + ^{12}\text{C}$ at 50 MeV/nucleon, using the AMD model coupled to GEMINI++ as an afterburner. In-likelihood distributions on the (η, Γ) plane obtained including all the observables of part A and B of Table I. The distribution is normalized to its modal value. The black arrow indicates the position of the maximum of the likelihood distribution, while the read arrow indicates the parameter values used in previous studies, e.g., Refs. [9–14].

produced with HFI, is shown in Fig. 8. Although the details of the In-likelihood depend on the adopted afterburner, the emerging indication towards an increased average number of collisions per events is not contradicted; in fact in this case the maximum of the distribution is located at $\eta = 5$ and $\Gamma = 100$, with a secondary maximum at $\eta = 5$, $\Gamma = \infty$ and the same is true for the posterior distribution, to be compared with the values $\eta = 5$ and $\Gamma = 15$ for the absolute maximum of the In-likelihood and In-posterior associated to the HFI afterburner. A second test concerns the error treatment. As already indicated, in this analysis only the statistical errors both on the model and the experimental data have been taken into account. Further sources of errors may be systematic errors on the experimental data, but also errors on the model associated to the parameters not investigated in this analysis, e.g., the effective interaction

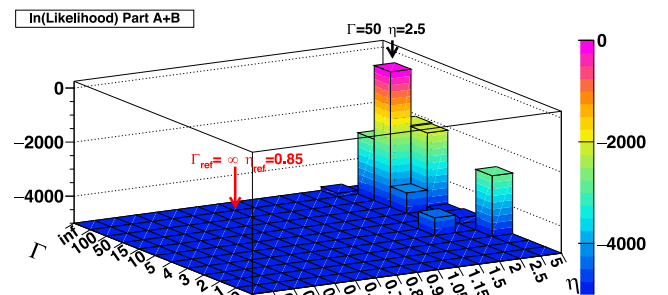


FIG. 9. Result of the Bayesian analysis performed on experimental data of $^{20}\text{Ne} + ^{12}\text{C}$ at 50 MeV/nucleon, using the AMD + HFI model. In-likelihood distribution on the (η, Γ) plane obtained including all the observables of part A and B in Table I (as in Fig. 6 middle panel) using the full spectra (reduced to few bins), normalized to unity, for continuous differential observables. The distribution is normalized to its modal value. The black arrow indicates the position of the maximum of the likelihood distribution, while the read arrow indicates the parameter values used in previous studies, e.g., Refs. [9–14].

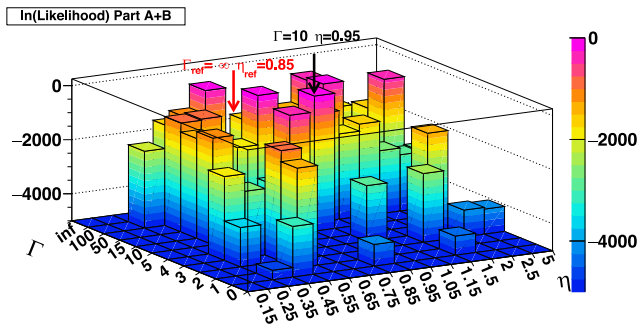


FIG. 10. Result of the Bayesian analysis performed on experimental data of $^{20}\text{Ne} + ^{12}\text{C}$ at 50 MeV/nucleon, using the AMD + HFI model. In-likelihood distribution on the (η, Γ) plane obtained including all the observables of part A and B in Table I (as in Fig. 6 bottom left panel) using the full spectra (reduced to a few bins), normalized to unity, for the continuous differential observables and adding an error equal to 10% of the simulated counts for the model. The distribution is normalized to its modal value. The black arrow indicates the position of the maximum of the likelihood distribution, while the red arrow indicates the parameter values used in previous studies, e.g., Refs. [9–14].

and the symmetry energy functional. Moreover, the correlation among the $\Lambda_{j,k}$ for $k = 1, 2, 3$ for continuous differential observables has not been included in the estimate of the error.

In order to see the effect on the achieved results associated to some of these contributions and starting from the last one, Fig. 9 shows the ln-likelihood distribution obtained including all the possible observables of Part A and B of Table I (as in Fig. 6 bottom row left side), using a proper binning also for the continuous differential ones instead of their moments. In this way the contribution to the error of the correlations among the $\Lambda_{j,k}$ for each given observable j is removed. In this case for continuous differential

observables $\chi_j^2(\{p_i\}) = \sum_{k=1}^{N_{\text{bins}}} \left[\frac{y_{j,k}^{\text{exp}} - y_{j,k}^{\text{mod}}(\{p_i\})}{\sigma_{j,k}} \right]^2$, where N_{bins} is the number of considered bins, $y_{j,k}^{\text{exp(mod)}}$ is the content of the k th bin for the j observable in the experimental (model) case and $\sigma_{j,k}$ its uncertainty, see Eq. (4), with $\sigma_{j,k}^{\text{exp(mod)}} = \sqrt{y_{j,k}^{\text{exp(mod)}}$. The number of bins was reduced to around 5 in order to avoid both statistics problems and a significant change in the relative weight of continuous and discrete observables with respect to the use of the first three moments for the former ones. Moreover, the spectra corresponding to continuous differential observables are normalized to their integral. The maximum of the obtained ln-likelihood is located at $\eta = 2.5$ and $\Gamma = 50$, i.e., slightly shifted with respect to the original one of Fig. 6 middle row, but still pointing towards an increase of the number of collisions.

We then quadratically added to the statistical errors an error equal to a fixed percentage of the counts for the model, to keep into account the contribution due to the model parameters not included in the analysis. The obtained ln-likelihood distribution is shown in Fig. 10 for an error in the model equal to 10%; the introduction of such an error shifts the maximum position to $\eta = 0.95$, $\Gamma = 10$ and this is true also when the percent error is decreased down to 6%. Below 6% the maximum position shifts back to $\eta = 2.5$ and $\Gamma = 50$.

Therefore the final message emerging from these plots again confirms a preference towards an higher number of collisions per event with respect to the previously used simulation and the needing for a strong clustering effect.

The posterior distribution associated to the likelihood of Fig. 10 can be used in an iterative way as new prior, as established by the Bayesian inference, if new measurements become available.

In short, the indication emerging from all the performed tests towards an increase in the number of collisions with respect to the standard simulation (i.e., the simulation with $[\eta = 0.85, \Gamma = \infty]$) proves to be a quite robust message.

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