

Exclusive heavy-vector-meson photoproduction in pp and pPb collisions within the generalized parton distribution approach: A phenomenological analysis

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(Received 31 July 2025; accepted 5 November 2025; published 15 December 2025)

The exclusive J/ψ photoproduction and the exclusive Υ photoproduction in proton-proton (pp) and proton-lead (pPb) collisions at the energies available at the CERN Large Hadron Collider are investigated using the generalized parton distribution approach. Assuming the Goloskokov-Kroll model, we estimate the corresponding total cross sections and rapidity distributions considering different parametrizations for the unpolarized gluon distribution of the proton. In particular, we compare the linear predictions, associated with Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution, with those derived taking into account the leading nonlinear corrections resulting from gluon recombination. The dependence of our predictions on the choice for the factorization scale is discussed.

DOI: [10.1103/8zkq-zll1](https://doi.org/10.1103/8zkq-zll1)

I. INTRODUCTION

The possibility of improving our understanding of the quantum three-dimensional (3D) imaging of partons inside protons and nuclei, and constraining the behavior of the gluon distribution at high energies, has strongly motivated the study of the exclusive vector-meson photoproduction in ultraperipheral collisions (UPCs) at the CERN Large Hadron Collider (LHC) [1–3]. Over the past years, this process has been analyzed considering distinct theoretical approaches, based on different assumptions and treatments for meson wave functions and descriptions of the QCD dynamics (see, e.g., Refs. [4–24]). In particular, in Refs. [18–22], the authors have considered the collinear factorization including the scale evolution of the generalized parton distributions (GPDs), which we denote as the GPD approach hereafter. Such studies have demonstrated the importance of the exclusive heavy-meson photoproduction to advance our understanding of the 3D content of the proton.

In this paper, we assume the GPD approach and investigate the exclusive J/ψ and Υ photoproduction in proton-proton (pp) and proton-lead (pPb) collisions at the energies available

at the LHC. The process is represented in Fig. 1(a). We have that the cross section for the ultraperipheral hadronic collision can be described in terms of the photon flux associated with the incident proton or the lead nucleus and the photon-proton cross section. The amplitude for the γp interaction can be factorized in terms of the hard part that describes the transition of the photon into a vector meson and the GPDs that depend on the longitudinal momentum of the proton (x), the skewness (ξ), and the momentum transfer t [see Fig. 1(b)]. In our study, we assume the Goloskokov-Kroll model [25–27], which allows us to express the gluon GPD in terms of the standard gluon parton distribution function (PDF), and we derive the corresponding total cross sections and rapidity distributions assuming different parametrizations for the unpolarized gluon distribution of the proton. Our analysis is motivated by the results derived in Refs. [21,22], which have demonstrated that the data for the J/ψ production at HERA and the pp collisions at the LHC can be described using this approach. In particular, we extend these previous studies for pPb collisions and for the Υ production. Moreover, we present, for the first time, the predictions associated with a gluon distribution derived taking into account the leading nonlinear corrections to the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution resulting from the gluon recombination obtained in Ref. [28]. Finally, as our calculation will be performed at leading order, we also investigate the dependence of our predictions on the choice for the factorization scale assumed in the calculation of the cross sections.

This paper is organized as follows. In Sec. II, we present a brief review of the formalism needed to describe the exclusive vector-meson photoproduction in ultraperipheral hadronic collisions within the GPD approach. In addition, we discuss

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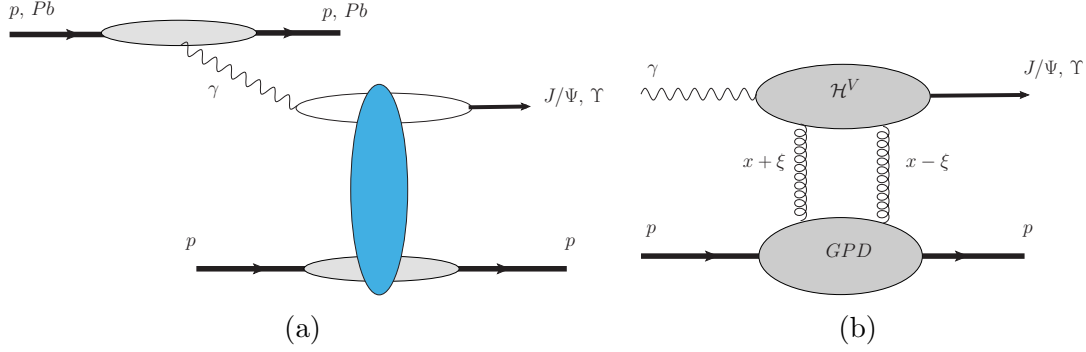


FIG. 1. (a) Exclusive heavy-vector-meson photoproduction in pp and $p\text{Pb}$ collisions. (b) Scattering amplitude for the γp interaction within the GPD approach.

the main ingredients used in our calculations. In Sec. III we present our predictions for the γp cross sections and for the rapidity distributions associated with the exclusive J/ψ and Υ photoproduction in pp and $p\text{Pb}$ collisions at energies available at the LHC, derived assuming different parametrizations for the unpolarized gluon distribution of the proton. A comparison with the current experimental data is performed. Moreover, we also estimate the impact of different choices for the factorization scale on the results derived using the gluon distribution obtained in Ref. [28], which takes into account the leading nonlinear corrections to the QCD dynamics. Finally, in Sec. IV, we summarize our main results and conclusions.

II. FORMALISM

In the equivalent photon approximation [29], the rapidity distribution for the exclusive heavy-vector-meson photoproduction in proton-hadron (ph) interactions, with $h = p$ or Pb , can be factorized in terms of the photon flux of one of the incident hadrons and the photoproduction cross section as follows:

$$\begin{aligned} \frac{d\sigma^{p+h \rightarrow p+h+V}}{dy} &= S_{ph}^2(W_+) \left(k_+ \frac{dn_p}{dk_+} \right) \sigma_+(\gamma h \rightarrow Vh) \\ &+ S_{ph}^2(W_-) \left(k_- \frac{dn_h}{dk_-} \right) \sigma_-(\gamma p \rightarrow Vp), \end{aligned} \quad (1)$$

where we have considered the possibility that both incident hadrons can be the source of photons. The factor S_{ph}^2 represents the survival factor, which is energy dependent, and takes into account the probability that the rapidity gap between the vector meson and the hadron target is not populated by soft interactions that would destroy the exclusivity of the event. In our analysis of pp collisions, we use the survival factors derived in Refs. [14,30]. In contrast, for $p\text{Pb}$ collisions, we assume $S_{p\text{Pb}}^2 = 1$, which is expected to be a reasonable first approximation. Moreover, the photon energies k_i can be expressed in terms of the rapidity y and the mass M_V of the vector meson by the relation $k_{\pm} = (M_V/2)e^{\pm y}$. Moreover, the photon-hadron center-of-mass energies are given by $W_{\pm}^2 = M_V \sqrt{s_{NN}} e^{\pm y}$. The flux of photons emitted from a proton is

given by [31]

$$\begin{aligned} \frac{dn_p(k)}{dk} &= \frac{\alpha_{\text{em}}}{2\pi k} \left[1 + \left(1 - \frac{2k}{\sqrt{s}} \right)^2 \right] \\ &\times \left(\ln \Omega - \frac{11}{6} + \frac{3}{\Omega} - \frac{3}{2\Omega^2} + \frac{1}{3\Omega^3} \right), \end{aligned} \quad (2)$$

with $\Omega = 1 + 0.71 \text{ GeV}^2/Q_{\text{min}}^2$, where $Q_{\text{min}}^2 \approx (k/\gamma_L)^2$ and γ_L is the Lorentz factor. On the other hand, for a nucleus, one has [2]

$$\frac{dn_A(k)}{dk} = \frac{2Z^2\alpha_{\text{em}}}{\pi k} \left[\xi K_1(\xi) K_0(\xi) - \frac{\xi^2}{2} [K_1^2(\xi) - K_0^2(\xi)] \right], \quad (3)$$

where $\xi = kb_{\text{min}}/\gamma_L$, with $b_{\text{min}} = R_p + R_A$, and $K_0(x)$ and $K_1(x)$ are modified Bessel functions. As the photoproduction cross section increases with the energy, the first (second) term in Eq. (1) is the dominant contribution for positive (negative) rapidities, when we have the collision between identical particles. For asymmetric collisions, as, e.g., in $p\text{Pb}$ collisions, the dependence of the photon flux on the nuclear charge implies that the contribution associated with γp interactions becomes dominant.

The total cross section for the exclusive vector-meson production in photon-hadron interactions is given by

$$\sigma^{\gamma+p \rightarrow V+p}(W) = \int dt \frac{d\sigma_T}{dt} = \int dt \frac{1}{16\pi W^4} |\mathcal{A}(W, t)|^2, \quad (4)$$

where the scattering amplitude $\mathcal{A}$ is represented in Fig. 1(b). We have that the two exchanged gluons carry different fractions of the incoming hadron momentum, which implies that the description of the process involves a generalized (skewed) gluon distribution. As the formation time $\tau_f \approx 2k/M_V^2$ is much greater than the $q\bar{q}$ -hadron interaction time, the amplitude can be factorized in terms of the function $\mathcal{H}^V$, which describes the photon-vector-meson transition, and the gluon GPDs. In our analysis, we use the function $\mathcal{H}^V$ derived in Ref. [21] at leading order. In particular, we have that the squared scattering amplitude can be expressed as follows:

$$|\mathcal{A}(W, t)|^2 = [|\mathcal{M}_{++,++}|^2 + |\mathcal{M}_{+,-,++}|^2], \quad (5)$$

where the helicity amplitudes $\mathcal{M}$ are defined by [21]

$$\mathcal{M}_{\mu'+,\mu+} = \frac{e}{2} C_V \int_0^1 \frac{dx}{(x+\xi)(x-\xi+i\epsilon)} \times \{ \mathcal{H}_{\mu',\mu}^{V+} H_g(x, \xi, t, \mu_F) + \mathcal{H}_{\mu',\mu}^{V-} \tilde{H}_g(x, \xi, t, \mu_F) \} \quad (6)$$

and

$$\mathcal{M}_{\mu'-,\mu+} = -\frac{e}{2} C_V \frac{\sqrt{-t}}{2m} \int_0^1 \frac{dx}{(x+\xi)(x-\xi+i\epsilon)} \times \{ \mathcal{H}_{\mu',\mu}^{V+} E_g(x, \xi, t, \mu_F) + \mathcal{H}_{\mu',\mu}^{V-} \tilde{E}_g(x, \xi, t, \mu_F) \}, \quad (7)$$

with $H_g(x, \xi, t, \mu_F)$ and $E_g(x, \xi, t, \mu_F)$ [$\tilde{H}_g(x, \xi, t, \mu_F)$ and $\tilde{E}_g(x, \xi, t, \mu_F)$] being the unpolarized [polarized] gluon GPDs. In the vector-meson production, the dominant contri-

bution is $H_g(x, \xi, t, \mu_F)$, thus the other GPD functions can be neglected in the following calculations. The flavor factor C_V is equal to 2/3 (1/3) for the J/ψ (Υ) meson. The functions $\mathcal{H}_{\mu',\mu}^{V\pm}$ are determined in terms of a sum or difference between the amplitudes for different gluon helicities, which implies

$$\begin{aligned} \mathcal{H}_{\mu',\mu}^{V\pm} &= [\mathcal{H}_{\mu'+,\mu+}^V \pm \mathcal{H}_{\mu'-,\mu-}^V] \\ &= 64\pi^2 \alpha_s(\mu_R) \int_0^1 d\tau \int \frac{d^2 \mathbf{k}_\perp}{16\pi^3} \psi^V(\tau, \mathbf{k}_\perp) \mathcal{F}_{\mu',\mu}^\pm \\ &\quad \times (\tau, x, \xi, \mathbf{k}_\perp^2), \end{aligned} \quad (8)$$

where $\alpha_s(\mu^2)$ is QCD running coupling, given by $\alpha_s(\mu^2) = 4\pi/\beta_0 \log(\mu^2/\Lambda^2)$, with $\Lambda = 0.22$ GeV. In our calculations, we assume $\mu = m_V$ and $m_q = m_V/2$. Moreover, $\psi^V(\tau, \mathbf{k}_\perp)$ is the wave function of the vector meson and $\mathcal{F}_{\mu',\mu}^\pm$ can be calculated perturbatively taking into account the transverse momentum $\mathbf{k}_\perp$ of the quark (and antiquark) and is given by [21]

$$\mathcal{F}_{\mu',\mu}^\pm = \frac{f_{\mu',\mu}^\pm}{(2\mathbf{k}_\perp^2 + m_V^2)[4\xi\mathbf{k}_\perp^2 + m_V^2(\xi - x) + i\epsilon][4\xi\mathbf{k}_\perp^2 + m_V^2(\xi + x)]}, \quad (9)$$

with $f_{+,+}^+ = 64m_V^5(x^2 - \xi^2)$ and $f_{+,+}^- = -256m_V^3\mathbf{k}_\perp^2 x\xi$. Following Refs. [21,22], the wave function will be written as

$$\psi^V(\tau, \mathbf{k}_\perp) = N_V \exp\left(-a_V^2 \frac{\mathbf{k}_\perp^2}{\tau(1-\tau)}\right), \quad (10)$$

with $\tau = 1/2$. In particular, for the J/ψ case, $N_{J/\psi} = 0.025 \text{ GeV}^{-1}$. As shown in Ref. [22], it implies $a_V^2 = 0.091 \text{ GeV}^{-2}$. In our analysis of the Υ case, we assume $a_V^2 = a_{\psi}^2$, and the value of N_Υ is fixed using the HERA data. In this work, $N_\Upsilon = 0.192 \text{ GeV}^{-1}$.

In order to estimate the gluon GPDs, present in Eq. (7), we consider the model proposed and detailed in Refs. [25–27], usually called the Goloskokov-Kroll (GK) model, which is based on fits of meson electroproduction data. In this model,

the gluon GPDs are expressed as follows:

$$F_g(x, \xi, t) = \int_1^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(\beta + \xi\alpha - x) f_g(\beta, \alpha, t), \quad (11)$$

where the double distribution $f_g(\beta, \alpha, t)$ for gluons, introduced originally in Refs. [32,33], is given by

$$f_g(\beta, \alpha, t) = e^{B_V t} \beta g(\beta, \mu^2) \frac{15 [(1-|\beta|)^2 - \alpha^2]^2}{16 (1-|\beta|)^5}. \quad (12)$$

Here, $g(\beta, \mu^2)$ is the usual unpolarized gluon PDF at the hard scale μ^2 . As a consequence, using this approach, the energy dependence of the photon-hadron cross section will be sensitive to the x dependence of $g(x, \mu^2)$, which is determined by the QCD dynamics.

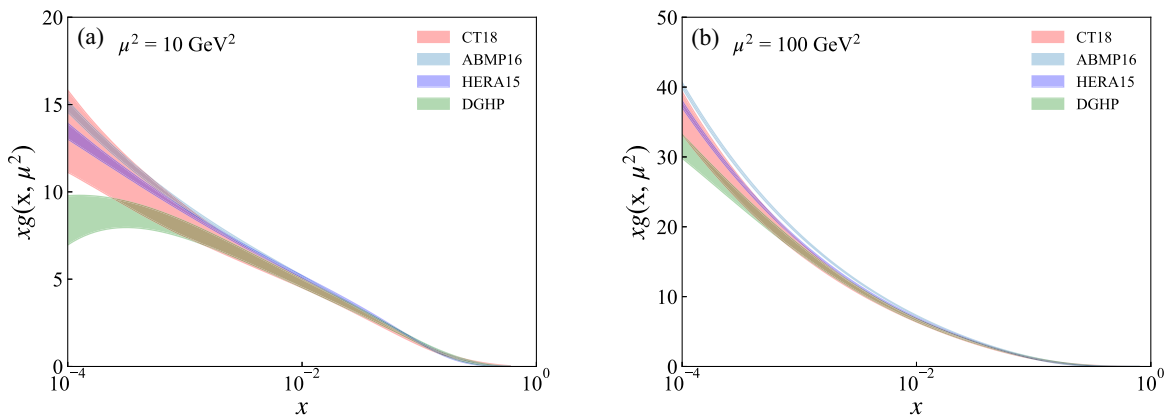


FIG. 2. Predictions of the different PDF sets considered in our study for the x dependence of the gluon distribution, $xg(x, \mu^2)$, for two different values of the hard scale: $\mu^2 = 10 \text{ GeV}^2$ (a) and $\mu^2 = 100 \text{ GeV}^2$ (b).

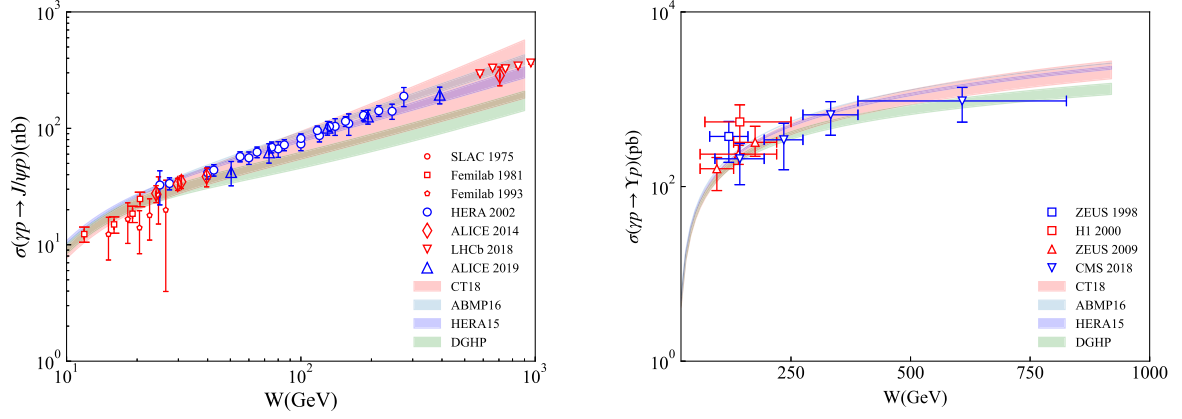


FIG. 3. Energy dependence of the exclusive J/ψ (a) and Υ (b) production cross sections in photon-proton interactions. The measured cross sections for the J/ψ and Υ photoproduction are from Refs. [37–44] and [41,45–47], respectively.

In our calculations, we assume the approximation $d\sigma/dt \simeq (1/2B_V)d\sigma/dt(t=0)$, with the slope B_V determined by fitting the experimental data. Following Ref. [21], we take for the J/ψ case that $B_V = b_0 + \alpha' \ln[W^2/m_V^2]$, with $\alpha' = 0.185 \text{ GeV}^{-2}$ and $b_0 = 1.06 \text{ GeV}^{-2}$. For the Υ production, we assume $2B_V = 4.63 \text{ GeV}^{-2} +$

$0.24 \text{ GeV}^{-2} \log(W/90.0 \text{ GeV})$, as in Ref. [11]. In addition, as discussed above, the normalization of the Υ wave function was adjusted in order to describe the current HERA data.

The last ingredient in our calculations is the description of the proton gluon distribution, presented in Eq. (12). As in Refs. [21,22], we consider three different PDF sets,

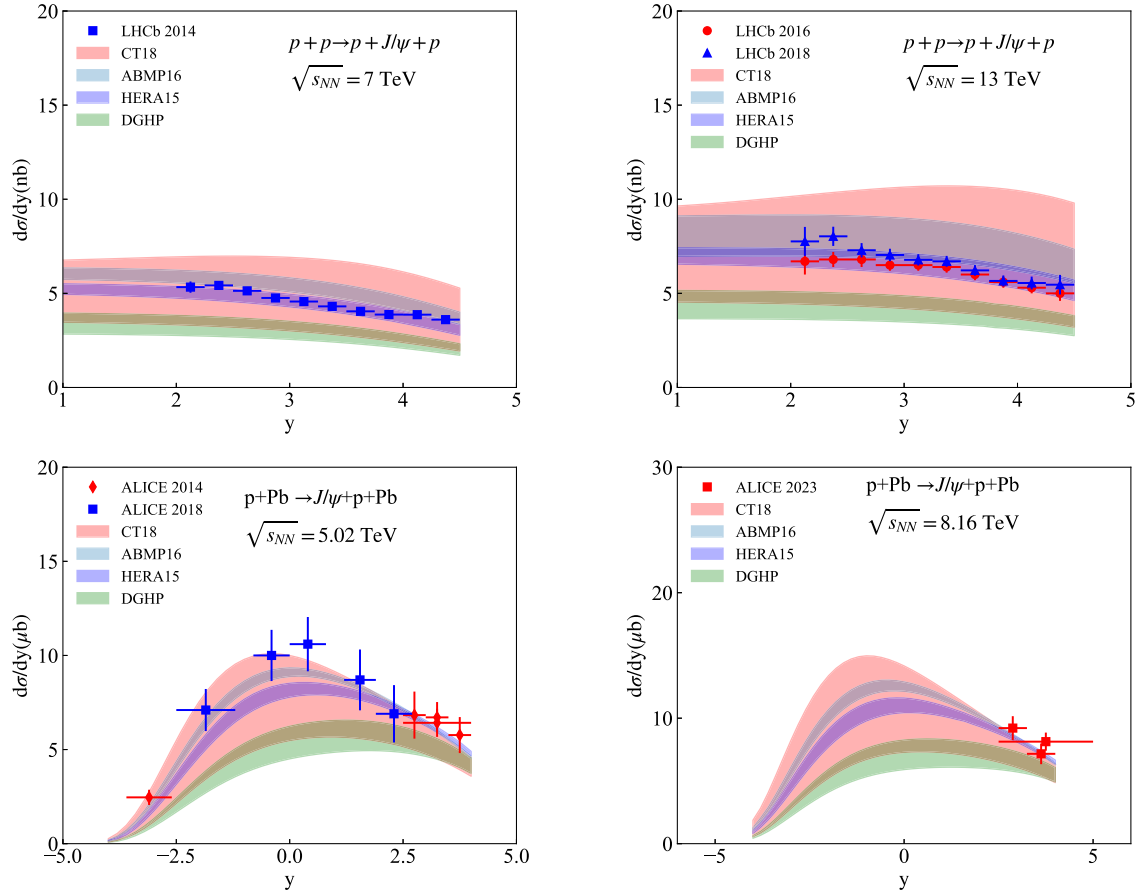


FIG. 4. Rapidity distributions for the exclusive J/ψ photoproduction in pp (upper panels) and $p\text{Pb}$ (lower panels) collisions. Experimental data of pp are from Refs. [42,48,49], while experimental data of $p\text{Pb}$ are from Refs. [43,44,50].

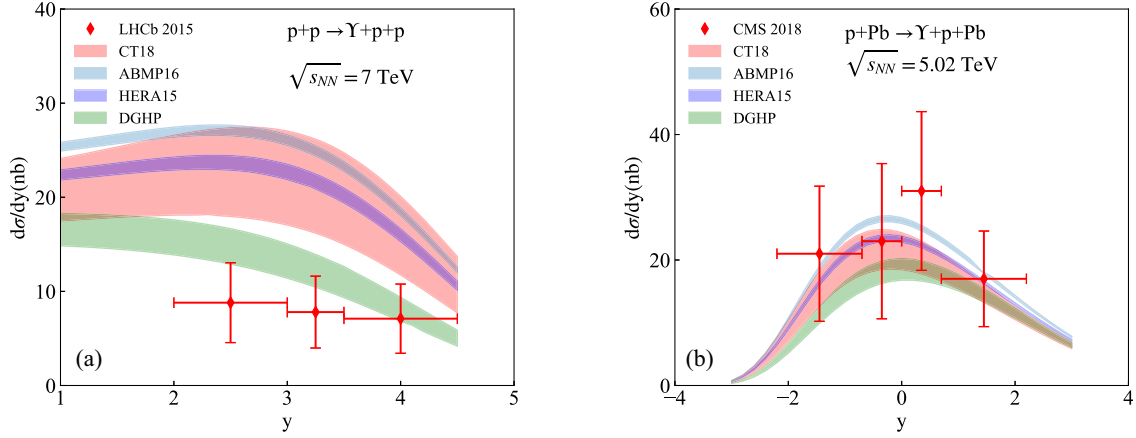


FIG. 5. Rapidity distributions for the exclusive Υ photoproduction in pp (a) and pPb (b) collisions. Experimental data are from Refs. [47,51].

ABMP16 [34], CT18 [35], and HERA15 [36], which perform a global fit of the current experimental data using the DGLAP evolution equations. In addition, we consider the parametrization obtained in Ref. [28], denoted DGHP hereafter, by the performance of a fitting of the current lepton-proton deep-inelastic-scattering data using nonlinear evolution equations, which take into account the leading nonlinear corrections to the DGLAP equations associated with the gluon recombination effects. In particular, we use the R1 set for the DGHP parametrization. In Fig. 2 we present a comparison between the predictions of these different PDF sets for the gluon distribution, considering two different values for the hard scale. We have that the predictions are similar for large x , but the inclusion of the nonlinear effects in the QCD dynamics implies a smaller DGHP gluon distribution at smaller values of x . As expected, the impact of the nonlinear effects decreases with the increasing of the hard scale.

III. NUMERICAL RESULTS AND DISCUSSION

In what follows, we present our predictions for the exclusive J/ψ and Υ photoproduction considering the gluon PDFs discussed in the previous section, considering the as-

sociated uncertainties in the parametrizations provided by the different groups. Initially, we estimate the cross sections assuming that the hard scale is given the mass of the vector meson ($\mu^2 = m_V^2$). In Fig. 3 we compare our predictions for the energy dependence of the exclusive J/ψ [panel (a)] and Υ [panel (b)] production cross sections in photon-proton interactions with the current experimental data [37–41]. As already observed in Refs. [21,22], the predictions associated with the ABMP16 [34], CT18 [35], and HERA15 [36] parametrizations, based on the linear DGLAP equations, are able to describe the J/ψ data. Our results indicate that such a conclusion is also valid in the Υ case. The predictions associated with the linear evolution gluon densities are in good agreement with Υ photoproduction data [41,45–47]. On the other hand, the DGHP predictions describe the Υ data and underestimate the J/ψ data at high energies.

In Fig. 4 we present a comparison between our predictions for the rapidity distribution of exclusive J/ψ photoproduction in pp and pPb collisions at the energies available at the LHC and the LHCb [48,49] and ALICE [43,44,50] data. In this case, the contribution associated with γPb interactions was disregarded, since it is very small in the rapidity range of the data. As expected from the results presented in Fig. 3,

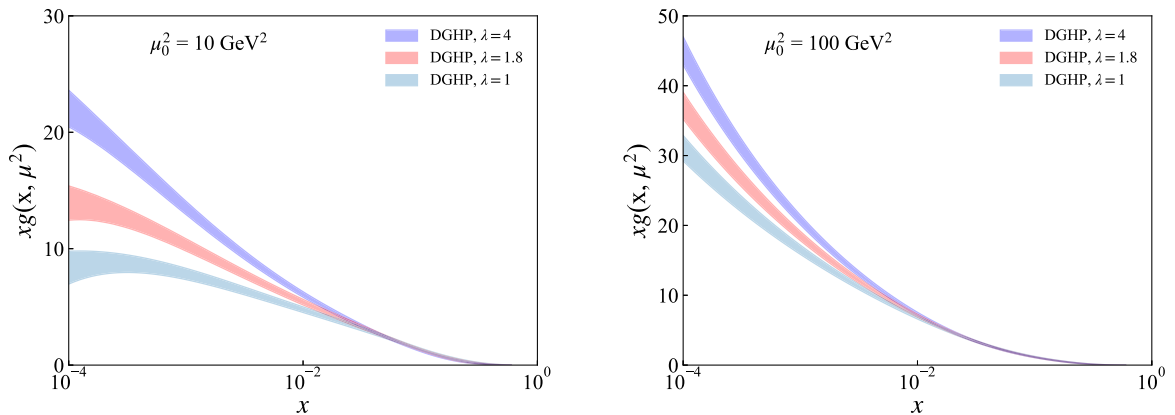


FIG. 6. Predictions of the DGHP parametrization for the gluon distribution evaluated for different values of the hard scale $\mu^2 = \lambda\mu_0^2$.

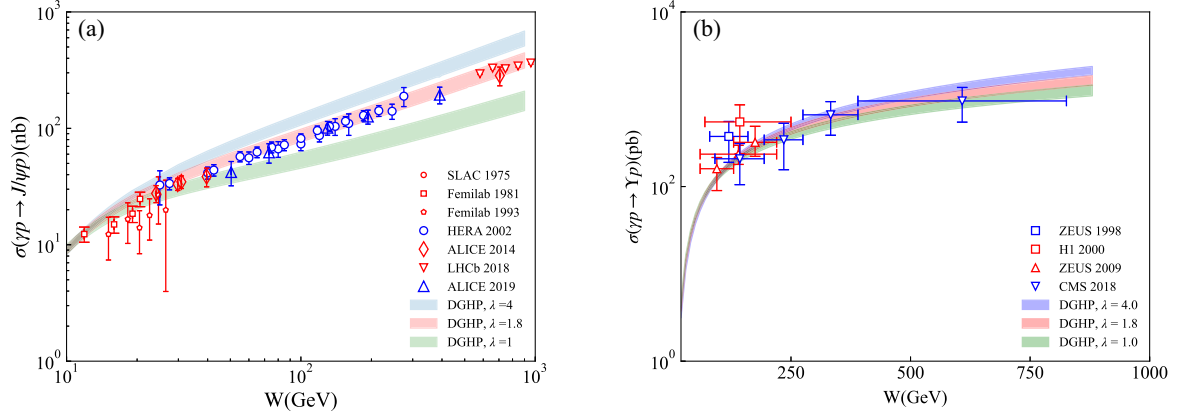


FIG. 7. Energy dependence of the exclusive J/ψ (a) and γ (b) production cross sections in photon-proton interactions, derived using the DGHP parametrization and different values of the hard scale $\mu^2 = \lambda m_V^2$. The measured cross sections of J/ψ are Refs. [37–44], and the measured cross sections of γ are from Refs. [41,45–47].

the DGLAP predictions provide a satisfactory description of the current data, while they are underestimated when the nonlinear effects, as described by the DGHP parametrization, are taken into account. In contrast, the results for the γ production presented in Fig. 5, indicate that the linear predictions

are not able to describe the pp data [51]. For pPb collisions, the current data have a large uncertainty [47], which does not allow discriminating between the distinct predictions. As a consequence, a stronger conclusion about the preferred description of the gluon distribution is still not possible.

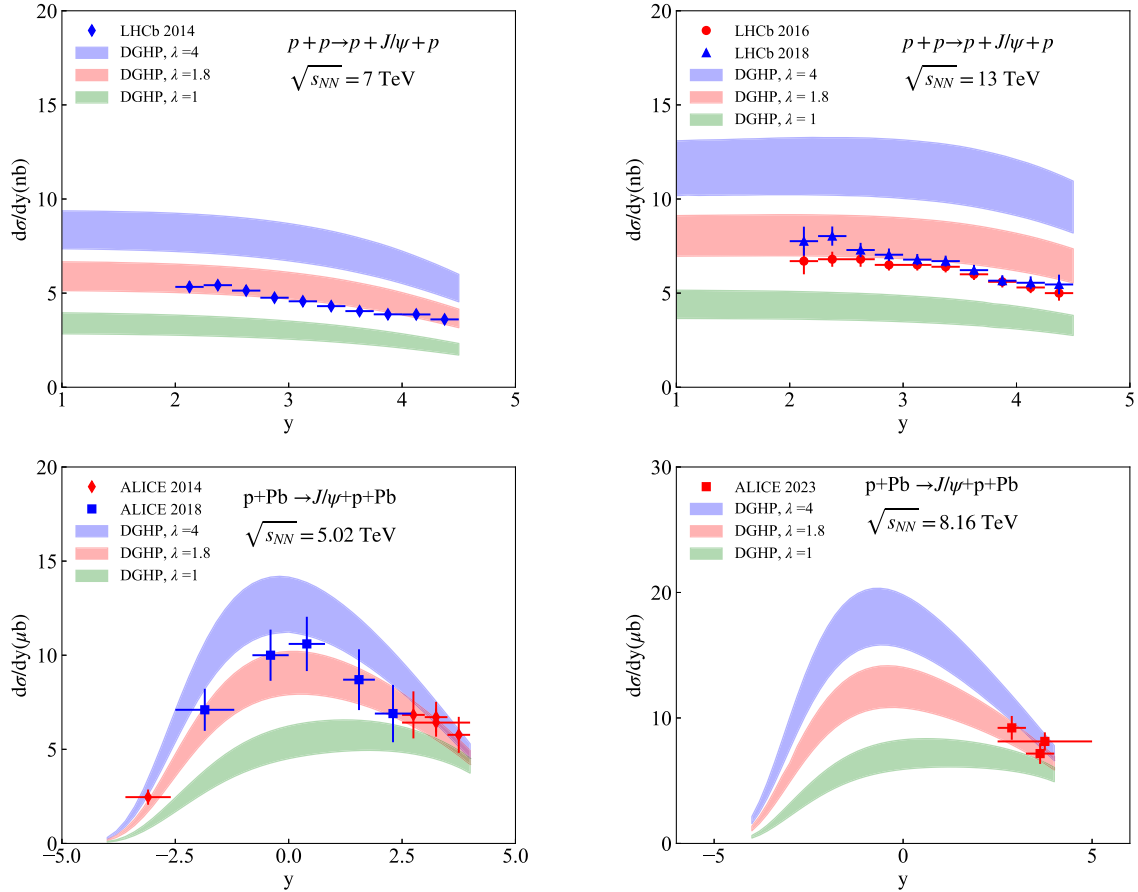


FIG. 8. Rapidity distributions for the exclusive J/ψ photoproduction in pp (upper panels) and pPb (lower panels) collisions, derived using the DGHP parametrization and different values of the hard scale $\mu^2 = \lambda m_V^2$. Experimental data of pp are from Refs. [42,48,49], and experimental data of pPb are from Refs. [43,44,50].

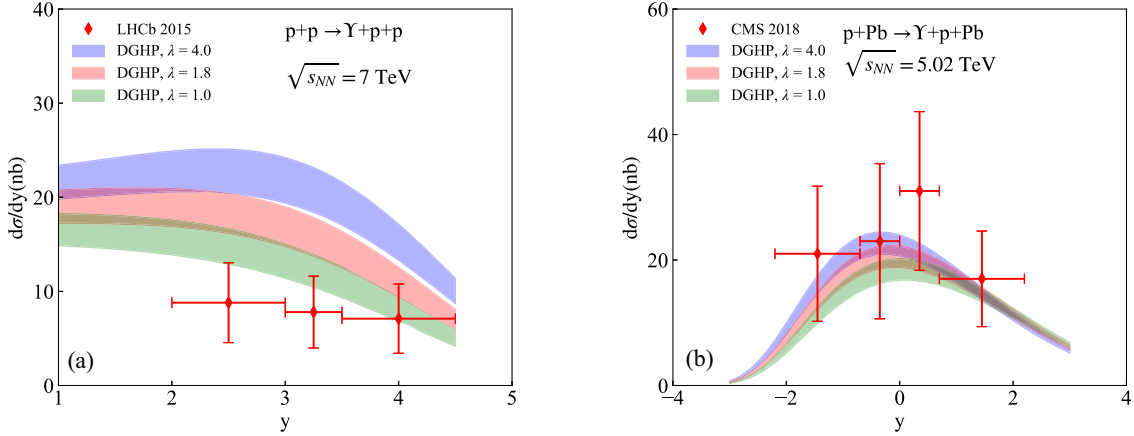


FIG. 9. Rapidity distributions for the exclusive Υ photoproduction in pp (a) and pPb (b) collisions, derived using the DGHP parametrization and different values of the hard scale $\mu^2 = \lambda m_V^2$. The experimental data are from Refs. [47,51].

Previous studies of the exclusive vector-meson photoproduction using the GPD model have demonstrated that the associated predictions are strongly sensitive to the choice of the hard scale assumed to describe the process [21]. In order to estimate the dependence of our predictions on this choice, in what follows we perform a comparison of the DGHP results, derived assuming $\mu^2 = m_V^2$, with those obtained for $\mu^2 = \lambda m_V^2$, with $\lambda = 1.8$ and 4 . The impact of these difference values of μ^2 on the gluon distribution are presented in Fig. 6. We have found that the behavior of the gluon distribution at small x is strongly dependent on the value assumed for λ , which becomes steeper with the increasing of the hard scale. Such a behavior occurs since the contribution of the nonlinear corrections are larger for smaller values of the hard scale. The increasing of λ offsets the nonlinear effect of the gluon PDF.

In Fig. 7 we present the energy dependence of the exclusive J/ψ [panel (a)] and Υ [panel (b)] production cross section in photon-proton interactions, derived using the DGHP parametrization and different values of the hard scale $\mu^2 = \lambda m_V^2$. As expected from Fig. 6, the increasing of λ implies a greater energy dependence of the cross section. In particular, for $\lambda = 1.8$, we have a very good description of the J/ψ data. In addition, such a choice implies that the current data for the rapidity distributions associated with exclusive J/ψ photoproduction in pp and pPb collisions are quite well described, as verified in Fig. 8.

Finally, in Fig. 9 we present our results for the rapidity distributions associated with the exclusive Υ photoproduction in pp and pPb collisions, derived using the DGHP parametrization and different values of the hard scale $\mu^2 = \lambda m_V^2$. In this case, we have that the increasing of λ implies a worse description of the pp data [51]. In particular, if the value $\lambda = 1.8$ is assumed, only the data for very forward rapidities are described. On the other hand, the current pPb data [47] are not able to discriminate the predictions derived assuming different values for the hard scale.

IV. SUMMARY

In this paper, the exclusive J/ψ photoproduction and the exclusive Υ photoproduction were investigated within the GPD approach. We have assumed the GK model and estimated the corresponding total cross sections and rapidity distributions, considering different parametrizations for the unpolarized gluon distribution of the proton. In particular, three different parametrizations based on the linear DGLAP equations were adopted in our calculations, as well as a recent parametrization that takes into account the leading nonlinear corrections resulting from gluon recombination. A detailed comparison with the data was performed and the dependence of the predictions on the choice for the hard scale was estimated. Our results indicate that the GPD model is able to describe the current data, but a stronger conclusion about the more adequate description of the gluon distribution is still not possible. However, future data, in particular for the Υ production, could shed light on this problem and allow us to improve the description of the QCD dynamics at high energies.

ACKNOWLEDGMENTS

Y.-P.X. gives many thanks to Sergey Goloskokov for the communication. V.P.G. was partially supported by CNPq, FAPERGS, and INCT-FNA (Process No. 464898/2014-5). This work is partially supported by the National Natural Science Foundation of China (Grant No. 12293061) and the National Key R&D Program of China (Grant No. 2024YFA1611000).

DATA AVAILABILITY

No data were created or analyzed in this study.

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