

γZ -exchange contribution in elastic ep scattering by perturbative QCD

Qian-Qian Guo^{1,2,*}, Hui-Yun Cao,³ and Hai-Qing Zhou^{2,†}

¹*Department of Mathematics and Physics, Luoyang Institute of Science and Technology, Luoyang 471023, China*

²*School of Physics, Southeast University, Nanjing 211189, China*

³*School of Physics and Electronic Science, Hubei Normal University, Huangshi 435002, China*



(Received 24 April 2026; revised 21 June 2026; accepted 27 July 2026; published 14 August 2026)

In this study, we calculate the γZ -exchange contribution to elastic ep scattering at large momentum transfer within perturbative quantum chromodynamics (pQCD). We present analytical expressions for the γZ -exchange contributions to the amplitudes. We also estimate the asymptotic behaviors of the amplitude contributions and of the physical quantity A_{PV} at high momentum transfer. These asymptotic behaviors determine the subtraction order in the dispersion relations (DRs) satisfied by the amplitudes. At high Q^2 beyond the forward limit, our pQCD analysis shows that this subtraction order is fixed by the high- ν behavior of the amplitude coefficients $c_i^{\gamma Z, PV}$, and that $c_{3,A}^{\gamma Z, PV}$ requires a once-subtracted DR. Within the present pQCD framework and the adopted proton distribution amplitudes, these high-energy results also provide nontrivial constraints on low-energy DR assumptions.

DOI: [10.1103/8j1r-3yxl](https://doi.org/10.1103/8j1r-3yxl)

I. INTRODUCTION

The two-photon-exchange (TPE) contribution has been shown to have a sizable effect on the extraction of electromagnetic form factors (FFs) in unpolarized ep scattering. Similarly, the γZ -exchange contribution also plays an important role in extracting the proton weak charge and strange FFs. Many methods have been applied to estimate these two-boson-exchange (TBE) contributions, including hadronic models [1–4], general parton distributions [5–7], perturbative quantum chromodynamics (pQCD) [8–10], dispersion relation (DR) methods [11–14], and lattice quantum chromodynamics (QCD) [15–19]. Among these methods, pQCD provides a reliable prediction of the high-energy behavior of these contributions. These behaviors are directly useful at high energy and also indicate the subtraction properties in the DR method. The TPE contribution in elastic ep scattering has been studied in Ref. [9], and the corresponding γZ -exchange contribution in pQCD was investigated in Ref. [20]. We extend those studies by presenting the results directly in terms of the invariant amplitudes $c_i^{\gamma Z, PV}$, including the full set of parity-violating coefficients and their high- ν behavior at large Q^2 . For the DR analysis, we focus on these amplitude coefficients rather than on the combinations $\square_{\gamma Z}^{V,A}$ that enter A_{PV} . As shown

in Ref. [21], DRs written directly for $\square_{\gamma Z}^{V,A}$ are reliable only in the forward approximation, whereas DRs for $c_i^{\gamma Z, PV}$ or for the helicity amplitudes provide the appropriate starting point at finite Q^2 .

In this work, we extend the pQCD calculation of the TPE contribution in ep scattering to the γZ -exchange contribution at large Q^2 , and we analyze the high- ν behavior of the amplitude coefficients $c_i^{\gamma Z, PV}$ and the implications for their DRs beyond the forward limit. In Sec. II, we present the basic formalism, including the amplitude decomposition, the relevant couplings, and the pQCD setup for TPE and γZ exchange. In Sec. III, we give the analytical results for the invariant amplitudes. In Sec. IV, we present the numerical analysis, extract the high-energy asymptotic behaviors, and discuss the implications for DRs of $c_i^{\gamma Z, PV}$. Section V summarizes the main conclusions.

II. BASIC FORMALISM

A. Expressions for two-photon-exchange contribution in elastic ep within pQCD

The TPE contribution in elastic ep scattering is discussed in Ref. [9] using the pQCD method. Here, we review the basic formalism used in this method. Within pQCD for ep scattering, at leading order in the coupling, there are 24 Feynman diagrams, and some of them are shown in Fig. 1. The momenta of the incoming electron, outgoing electron, incoming proton, and outgoing proton are labeled as $p_{1,3,2,4}$, respectively. m_e is the electron mass and m_N is the proton mass. For convenience, we define

$$P \equiv \frac{1}{2}(p_2 + p_4), \quad K \equiv \frac{1}{2}(p_1 + p_3), \quad q \equiv p_1 - p_3, \\ Q^2 \equiv -q^2, \quad \nu \equiv K \cdot P, \quad s \equiv (p_1 + p_2)^2. \quad (1)$$

*Contact author: guoqq@lit.edu.cn

†Contact author: zhouhq@seu.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

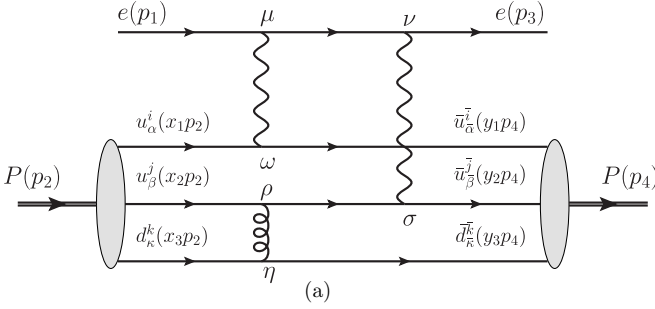


FIG. 1. One of the Feynman diagrams for the TPE contribution in $ep \rightarrow ep$, where the two photons are connected to two u quarks, and the gluon is connected to the u and d quarks.

The corresponding amplitude for the Feynman diagram in Fig. 1 is

$$\begin{aligned} \mathcal{M}_{2\gamma}^{(a)} = & -i \int_0^1 [dx dy] \bar{u}_3 \Gamma_{\gamma ee}^\nu S_e(p_1 - y_1 p_4 + x_1 p_2) \\ & \times \Gamma_{\gamma ee}^\mu u_1 \Phi_P^{\text{fin}}(\bar{i}, \bar{j}, \bar{k}, \bar{\alpha}, \bar{\beta}, \bar{\kappa}, p_4, \lambda_4) [\Gamma_{\gamma uu}^\omega]_{\bar{\alpha}\alpha} \delta_{ii} \\ & \times [\Gamma_{\gamma uu}^\sigma S_q((x_2 + x_3)p_2 - y_3 p_4) \Gamma_{\gamma uu}^\rho]_{\bar{\beta}\beta} T_{jj}^a \\ & \times [\Gamma_{\gamma dd}^\eta]_{\bar{\kappa}\kappa} T_{kk}^a \Phi_P^{\text{ini}}(i, j, k, \alpha, \beta, \kappa, p_2, \lambda_2) \\ & \times D_{g, \rho\eta} [y_3 p_4 - x_3 p_2] D_{\gamma, \mu\omega}(y_1 p_4 - x_1 p_2) \\ & \times D_{\gamma, \nu\sigma}((y_2 + y_3)p_4 - (x_2 + x_3)p_2), \end{aligned} \quad (2)$$

where $x_{1,2,3}, y_{1,2,3}$ are the corresponding momentum factors of the quarks in the proton and

$$[dx dy] \equiv dx_1 dx_2 dx_3 dy_1 dy_2 dy_3 \delta(x_1 + x_2 + x_3 - 1) \delta(y_1 + y_2 + y_3 - 1) \quad (3)$$

with

$$\begin{aligned} x_1 + x_2 + x_3 &= 1, \quad 0 \leq x_l \leq 1, \\ y_1 + y_2 + y_3 &= 1, \quad 0 \leq y_l \leq 1. \end{aligned} \quad (4)$$

The indices i, j, k and $\bar{i}, \bar{j}, \bar{k}$ denote quark color, α, β, κ and $\bar{\alpha}, \bar{\beta}, \bar{\kappa}$ are Dirac indices, a, b are gluon color indices, $\lambda_{2,4}$ are the helicities of the incoming and outgoing proton, and $\Phi_P^{\text{fin/ini}}$ are the distribution amplitudes of the outgoing and incoming protons in pQCD. The propagators $S_e(k)$, $D_{\gamma, \mu\nu}(k)$, $S_q(k)$, and $D_{g, \mu\nu}(k)$ for the electron, photon, quark, and gluon are

$$\begin{aligned} D_{\gamma, \mu\nu}(k) &= \frac{-i}{k^2 + i\epsilon} g_{\mu\nu}, \quad S_e(k) = \frac{i(\not{k} + m_e)}{k^2 - m_e^2 + i\epsilon}, \\ D_{g, \mu\nu}(k) &= \frac{-i}{k^2 + i\epsilon} g_{\mu\nu}, \quad S_q(k) = \frac{i\not{k}}{k^2 - m_q^2 + i\epsilon}, \end{aligned} \quad (5)$$

The vertices are

$$\begin{aligned} \Gamma_{\gamma ee}^\mu &= -ie\gamma^\mu, \quad \Gamma_{\gamma uu}^\mu = iQ_u e\gamma^\mu, \\ \Gamma_{\gamma dd}^\mu &= iQ_d e\gamma^\mu, \quad \Gamma_{gdd/uu}^\mu = -ig_s \gamma^\mu, \end{aligned} \quad (6)$$

where e is the electromagnetic interaction coupling constant, g_s is the strong interaction coupling constant, and $Q_u = 2/3$ and $Q_d = -1/3$ are the quark charges. The distribution amplitudes are expressed as

$$\begin{aligned} \Phi_P^{\text{ini}}[i, j, k, \alpha, \beta, \kappa, p, \lambda] &= \frac{1}{24} \epsilon_{ijk} \{ V[pC]_{\alpha\beta} [\gamma_5 u(p, \lambda)]_\kappa \\ &+ A[\not{p}\gamma_5 C]_{\alpha\beta} [u(p, \lambda)]_\kappa \\ &+ T[\sigma_{\mu\nu} p^\nu C]_{\alpha\beta} [\gamma^\mu \gamma_5 u(p, \lambda)]_\kappa \}, \\ \Phi_P^{\text{fin}}[i, j, k, \alpha, \beta, \kappa, p, \lambda] &= (\Phi_P^{\text{ini}}[i, j, k, \alpha, \beta, \kappa, p, \lambda])^*, \end{aligned} \quad (7)$$

where C is the charge-conjugation matrix, $u^+(p, \lambda)$ is the plus component of the Dirac spinor, and the functions A, V, T are scalar functions. The above distribution amplitudes correspond to the matrix element in Ref. [9]:

$$\begin{aligned} \epsilon_{ijk} \Phi_P^{\text{ini}}[i, j, k, \alpha, \beta, \gamma, p, \lambda] \\ \sim 4 \langle 0 | \epsilon_{ijk} u_\alpha^i(x_1 n) u_\beta^j(x_2 n) d_\kappa^k(x_3 n) | p, \lambda \rangle \\ = V p^+ [(\frac{1}{2} \bar{n} \cdot \gamma) C]_{\alpha\beta} [\gamma_5 N^+]_\sigma + A p^+ [(\frac{1}{2} \bar{n} \cdot \gamma) \gamma_5 C]_{\alpha\beta} \\ \times [N^+]_\sigma + T p^+ [\frac{1}{2} i \sigma_{\perp \bar{n}} C]_{\alpha\beta} [\gamma^\perp \gamma_5 N^+]_\sigma, \end{aligned} \quad (8)$$

where the incoming-proton momentum satisfies $\frac{1}{2} \bar{n} p^+ \sim p$ and the spinor satisfies $N^+ \sim u^+(p, \lambda)$. In the practical calculation, we take the functions A, V, T to be the same as those used in Ref. [9]:

$$\begin{aligned} V(x_i) &= 120 x_1 x_2 x_3 f_N [1 + r_+ (1 - 3x_3)], \\ A(x_i) &= 120 x_1 x_2 x_3 f_N r_- (x_2 - x_1), \\ T(x_i) &= 120 x_1 x_2 x_3 f_N [1 + 1/2 (r_- r_+) (1 - 3x_3)]. \end{aligned} \quad (9)$$

The parameters $f_N, r_\pm$ are listed in Table I, where three models are used: COZ [22], BLW [23], and QCDSF [24].

For the running coupling constant $\alpha_s(Q^2) \equiv g_s^2/(4\pi)$, we use

$$\alpha_s(Q^2) = \frac{1}{b_0 t} \left(1 - \frac{b_1 l}{b_0^2 t} \right), \quad (10)$$

where

$$\begin{aligned} l &= \ln[t], \quad b_0 = \frac{33 - 2n_f}{12\pi}, \\ t &= \ln \left[\frac{Q^2}{\Lambda_{\text{QCD}}^2} \right], \quad b_1 = \frac{153 - 19n_f}{24\pi^2} \end{aligned} \quad (11)$$

with $\Lambda_{\text{QCD}} = 0.2 \text{ GeV}$ and $n_f = 4$.

TABLE I. Parameters f_N and $r_\pm$ in the COZ [22], BLW [23], and QCDSF [24] models.

	$f_N (10^{-3}) \text{ GeV}^2$	r_-	r_+
COZ	5.0 ± 0.5	4.0 ± 1.5	1.1 ± 0.3
BLW	5.0 ± 0.5	1.37	0.35
QCDSF	$3.23 \pm 0.06 \pm 0.09$	$1.06 \pm 0.09 \pm 0.31$	$0.33 \pm 0.03 \pm 0.11$

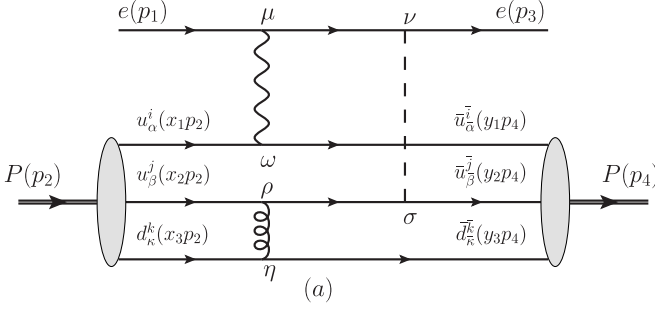


FIG. 2. One of the Feynman diagrams for γZ exchange in $ep \rightarrow ep$, where the photon and the Z boson are attached to two u quarks, and the gluon is attached to the u and d quarks.

We note that, at the leading order considered here, $(\alpha_e^2 \alpha_s)$, the box diagrams in Figs. 1 and 2 contain no explicit infrared divergence. Infrared divergences from additional gluon exchanges appear only at the next order $(\alpha_e^2 \alpha_s^2)$, which is beyond the scope of the present work.

B. γZ -exchange contribution in elastic ep scattering

Similar to the TPE case, the γZ -exchange contribution can also be estimated by pQCD at high energy, and there are 48 Feynman diagrams for this process at leading order. We show one diagram in Fig. 2, where the notations are the same as those used in the TPE case. The corresponding amplitude is

$$\begin{aligned} \mathcal{M}_{\gamma Z}^{(a)} = & -i \int_0^1 [dx dy] \bar{u}_3 \Gamma_{Zee}^\nu S_e(p_1 - y_1 p_4 + x_1 p_2) \Gamma_{\gamma ee}^\mu u_1 \Phi_P^{\text{fin}}(\bar{i}, \bar{j}, \bar{k}, \bar{\alpha}, \bar{\beta}, \bar{\kappa}, p_4, \lambda_4) [\Gamma_{\gamma uu}^\omega]_{\bar{\alpha}\alpha} \delta_{\bar{i}\bar{i}} \\ & \times [\Gamma_{Zuu}^\sigma S_q((x_2 + x_3)p_2 - y_3 p_4) \Gamma_{guu}^\rho]_{\bar{\beta}\beta} T_{\bar{j}j}^a [\Gamma_{gdd}^\eta]_{\bar{\kappa}\kappa} T_{\bar{k}k}^a \Phi_P^{\text{ini}}[i, j, k, \alpha, \beta, \kappa, p_2, \lambda_2] \\ & \times D_{g,\rho\eta}(y_3 p_4 - x_3 p_2) D_{\gamma,\mu\omega}(y_1 p_4 - x_1 p_2) D_{Z,\mu\sigma}((y_2 + y_3)p_4 - (x_2 + x_3)p_2), \end{aligned} \quad (12)$$

where the vertices are

$$\begin{aligned} \Gamma_{Zee}^\mu &= -i(\bar{g}_V^e \gamma^\mu + \bar{g}_A^e \gamma^\mu \gamma_5), \\ \Gamma_{Zuu}^\mu &= -i(\bar{g}_V^u \gamma^\mu + \bar{g}_A^u \gamma^\mu \gamma_5), \\ \Gamma_{Zdd}^\mu &= -i(\bar{g}_V^d \gamma^\mu + \bar{g}_A^d \gamma^\mu \gamma_5) \end{aligned} \quad (13)$$

with

$$\bar{g}_{V,A}^{e,u,d} = -\frac{e}{4 \sin \theta_W \cos \theta_W} g_{V,A}^{e,u,d}, \quad (14)$$

and θ_W the Weinberg angle, and

$$\begin{aligned} g_V^e &= -1 + 4 \sin^2 \theta_W, & g_A^e &= -1, \\ g_V^u &= 1 - 8/3 \sin^2 \theta_W, & g_A^u &= 1, \\ g_V^d &= -1 + 4/3 \sin^2 \theta_W, & g_A^d &= -1. \end{aligned} \quad (15)$$

The propagator of Z boson is taken as

$$\begin{aligned} D_{Z,\mu\nu}(k) &\equiv \left(g_{\mu\nu} - (1 - \xi_Z) \frac{k_\mu k_\nu}{k^2 - \xi_Z M_Z^2 + i\epsilon} \right) \\ &\times \frac{-i}{k^2 - M_Z^2 + i\epsilon} \\ &= \frac{-i}{k^2 - M_Z^2 + i\epsilon} g_{\mu\nu} \end{aligned} \quad (16)$$

with $\xi_Z = 1$.

C. Invariant amplitudes

To discuss the TBE contribution at the amplitude level, we express the amplitude in a general form as

$$\mathcal{M} = \mathcal{M}^{\text{PC}} + \mathcal{M}^{\text{PV}}, \quad (17)$$

where the indices PC and PV refer to the parity-conserving and parity-violating parts, respectively. In the practical

calculation, we choose

$$\mathcal{M}^{\text{PC,PV}} = \sum_{i=1}^3 c_i^{\text{PC,PV}} \mathcal{M}_i^{\text{PC,PV}} \quad (18)$$

with

$$\begin{aligned} \mathcal{M}_1^{\text{PC}} &\equiv [\bar{u}_3 \gamma_\mu u_1][\bar{u}_4 \gamma^\mu u_2], \\ \mathcal{M}_2^{\text{PC}} &\equiv \frac{1}{Q} [\bar{u}_3 \not{p} u_1][\bar{u}_4 u_2], \\ \mathcal{M}_3^{\text{PC}} &\equiv \frac{1}{Q^2} [\bar{u}_3 \not{p} u_1][\bar{u}_4 \not{K} u_2], \end{aligned} \quad (19)$$

and

$$\begin{aligned} \mathcal{M}_1^{\text{PV}} &\equiv [\bar{u}_3 \gamma_\mu \gamma_5 u_1][\bar{u}_4 \gamma^\mu u_2], \\ \mathcal{M}_2^{\text{PV}} &\equiv \frac{1}{Q} [\bar{u}_3 \gamma_\mu \gamma_5 u_1][\bar{u}_4 i\sigma^{\mu\nu} q_\nu u_2], \\ \mathcal{M}_3^{\text{PV}} &\equiv \frac{1}{Q^2} [\bar{u}_3 \not{p} \gamma_5 u_1][\bar{u}_4 \not{K} u_2]. \end{aligned} \quad (20)$$

Here, we note that since we focus on the behavior of the amplitudes in the limit $M_N/Q \rightarrow 0$, we use Q (rather than M_N as in Refs. [9,20]) to scale the invariant amplitudes.

For the one-photon-exchange diagram, we have

$$\begin{aligned} \mathcal{M}_{1\gamma} &= -i\bar{u}_3(-ie\gamma^\mu)u_1 \frac{-ig_{\mu\nu}}{(p_1 - p_3)^2} \bar{u}_4 \\ &\times \left[ie \left(F_1 \gamma^\nu + F_2 \frac{i\sigma^{\alpha\nu}}{2M_N} (p_4 - p_2)_\alpha \right) \right] u_2, \end{aligned} \quad (21)$$

where F_1 and F_2 are the Dirac and Pauli form factors. The amplitude is

$$\mathcal{M}_{1\gamma} \equiv \sum_{i=1}^3 c_i^{1\gamma} \mathcal{M}_i^{\text{PC}}. \quad (22)$$

Then, the corresponding coefficients are

$$c_1^{1\gamma} = \frac{e^2(F_1 + F_2)}{Q^2}, \quad c_2^{1\gamma} = \frac{-e^2 F_2}{2M_N Q}, \quad c_3^{1\gamma} = 0. \quad (23)$$

In Ref. [9], the full amplitude for OPE + TPE diagrams is written as

$$\mathcal{M} = \frac{e^2}{Q^2} \bar{u}_3 \gamma_\mu u_1 \times \bar{u}_4 \left(\tilde{G}_M \gamma^\mu - \tilde{F}_2 \frac{P^\mu}{M_N} + \tilde{F}_3 \frac{\gamma \cdot K P^\mu}{M_N^2} \right) u_2, \quad (24)$$

where

$$\begin{aligned} \tilde{G}_M &\equiv G_M + \delta \tilde{G}_M, \\ \tilde{F}_2 &\equiv F_2 + \delta \tilde{F}_2, \end{aligned} \quad (25)$$

and

$$G_M \equiv F_1 + F_2, \quad G_E \equiv F_1 - \tau F_2. \quad (26)$$

Comparing with our notations, we have

$$\begin{aligned} \frac{e^2}{Q^2} \delta \tilde{G}_M &= c_1^{(2\gamma)}, \\ \frac{e^2}{M_N Q} \delta \tilde{F}_2 &= -c_2^{(2\gamma)}, \\ \frac{e^2}{M_N^2} \tilde{F}_3 &= c_3^{(2\gamma)}. \end{aligned} \quad (27)$$

D. Physical quantities expressed by c_i

In principle, when $c_i^{1\gamma}$, c_i^{1Z} , $c_i^{2\gamma}$, and $c_i^{\gamma Z}$ are known, all OBE and TBE contributions to unpolarized and polarized observables are determined. In this subsection, we use these coefficients to express the unpolarized cross sections and A_{PV} .

For the unpolarized cross section, in the leading order of α_e with $\alpha_e \equiv e^2/(4\pi)$, one has

$$\frac{d\sigma_{un}^{1\gamma \otimes 1\gamma}}{d\Omega} = \frac{1}{64\pi^2 s} \frac{E'}{E} \frac{1}{4} \sum_{\text{helicity}} \mathcal{M}_{1\gamma} \mathcal{M}_{1\gamma}^*, \quad (28)$$

where $\mathcal{M}_{1\gamma}$ is given in Eq. (21), and E' and E are the energies of the initial and final electrons in the laboratory frame, respectively. This gives the usual form

$$\sigma_R^{1\gamma \otimes 1\gamma} \equiv \frac{1}{R} \frac{d\sigma_{un}^{1\gamma \otimes 1\gamma}}{d\Omega} = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2), \quad (29)$$

where

$$R \equiv \frac{1}{64\pi^2 s} \frac{E'}{E} \frac{2e^4}{(1-\varepsilon)}, \quad \tau \equiv \frac{Q^2}{4M_N^2}. \quad (30)$$

For the TPE contribution, where the TPE amplitude is written as

$$\mathcal{M}_{2\gamma} \equiv \sum_{i=1}^3 c_i^{2\gamma} \mathcal{M}_i^{\text{PC}}, \quad (31)$$

we have

$$\frac{d\sigma_{un}^{1\gamma \otimes 2\gamma}}{d\Omega} = \frac{1}{64\pi^2 s} \frac{E'}{E} \frac{1}{4} \sum_{\text{helicity}} 2\text{Re}[\mathcal{M}^{1\gamma*} \mathcal{M}_{2\gamma}], \quad (32)$$

which gives

$$\sigma_R^{1\gamma \otimes 2\gamma} \simeq 2G_M(\delta \tilde{G}_M + \tilde{F}_3 \varepsilon N) + \frac{2G_E \varepsilon}{\tau} (\delta \tilde{G}_M + \tilde{F}_3 N) \quad (33)$$

with

$$N \equiv \frac{\sqrt{(1-\varepsilon^2)\tau(1+\tau)}}{1-\varepsilon}. \quad (34)$$

Here, the contribution from $\delta \tilde{F}_2$ has been neglected since it is zero at the leading order of pQCD.

For the asymmetry quantity A_{PV} , which is defined as

$$A_{PV} \equiv \frac{\sum_{\text{helicity}} (\mathcal{M}_+ \mathcal{M}_+^* - \mathcal{M}_- \mathcal{M}_-^*)}{\sum_{\text{helicity}} (\mathcal{M}_+ \mathcal{M}_+^* + \mathcal{M}_- \mathcal{M}_-^*)}, \quad (35)$$

where $\mathcal{M}_{+,-}$ are the helicity amplitudes for incoming-electron helicities $+$, $-$, respectively, the γZ -exchange contribution can be written as

$$\begin{aligned} A_{PV}^{\gamma \otimes \gamma Z} &= \frac{1}{e^2 \sigma} \sum_{i=1}^3 \mathcal{N}_i \text{Re} [c_i^{\gamma Z, \text{PV}}] \\ &= \frac{1}{e^2 \sigma} \sum_{i=1}^3 \mathcal{N}_i \{ \text{Re} [c_{i,V}^{\gamma Z, \text{PV}}] g_e^A + \text{Re} [c_{i,A}^{\gamma Z, \text{PV}}] g_e^V \} \\ &\equiv \text{Re} [\square_{\gamma Z}^V] + \text{Re} [\square_{\gamma Z}^A], \end{aligned} \quad (36)$$

where

$$\begin{aligned} \sigma &= 4F_1^2 M_N^2 (16v^2 - 4M_N^2 Q^2 + Q^4) + 16F_1 F_2 M_N^2 Q^4 \\ &\quad + F_2^2 Q^2 (16v^2 + 4M_N^2 Q^2 - Q^4), \end{aligned} \quad (37)$$

and

$$\begin{aligned} \mathcal{N}_1 &= 8M_N^2 Q^2 [(16v^2 - 4M_N^2 Q^2 + Q^4)F_1 + 2Q^4 F_2], \\ \mathcal{N}_2 &= 2M_N Q^3 [8M_N^2 Q^2 F_1 + (16v^2 - 4M_N^2 Q^2 + Q^4)F_2], \\ \mathcal{N}_3 &= 8M_N^2 v (16v^2 - 4M_N^2 Q^2 + Q^4)F_1, \end{aligned} \quad (38)$$

$c_{i,V}^{\gamma Z, \text{PV}}$, $c_{i,A}^{\gamma Z, \text{PV}}$, and $c_i^{\gamma Z, \text{PV}}$ are defined as

$$\begin{aligned} \mathcal{M}_{\gamma Z} &\equiv \sum_{i=1}^3 (c_i^{\gamma Z, \text{PC}} \mathcal{M}_i^{\text{PC}} + c_i^{\gamma Z, \text{PV}} \mathcal{M}_i^{\text{PV}}), \\ c_i^{\gamma Z, \text{PV}} &\equiv c_{i,V}^{\gamma Z, \text{PV}} g_e^A + c_{i,A}^{\gamma Z, \text{PV}} g_e^V. \end{aligned} \quad (39)$$

Here, we note that the definitions of P , K , v , and c_i are slightly different from those in Ref. [25].

III. ANALYTICAL RESULTS

In the large- Q^2 limit, where we approximately take $m_e \rightarrow 0$ and $M_N/Q \rightarrow 0$, the TPE contribution from Fig. 1 to the

coefficients is

$$\begin{aligned}\delta\tilde{G}_M^{(a)} &= \int_0^1 [dxdy] \frac{-e^2 Q_u^2 g_s^2 s [4TT' + (A - V)(A' - V')]}{36Q^4 x_3(x_2 + x_3)^2 y_1 y_3 (y_2 + y_3) (s(x_1 - y_1) + Q^2(x_2 + x_3)y_1 + i\varepsilon)}, \\ \delta\tilde{F}_2^{(a)} &= 0, \\ \frac{\nu}{M_N^2} \tilde{F}_3^{(a)} &= \int_0^1 dx_i dy_i \frac{-e^2 Q_u^2 g_s^2 s [4TT' + (A - V)(A' - V')]}{18Q^4 x_1 x_3 (x_2 + x_3) y_1 y_3 (y_2 + y_3) (s(x_1 - y_1) + Q^2(x_2 + x_3)y_1 + i\varepsilon)},\end{aligned}\quad (40)$$

where the variables in V, T, A and V', T', A' are (x_1, x_2, x_3) and (y_1, y_2, y_3) , respectively.

Similarly, the coefficients for all Feynman diagrams are

$$\begin{aligned}\delta\tilde{G}_M &= -\frac{\alpha_{em}\alpha_s}{Q^4} \frac{16\pi^2}{9} (2\zeta - 1) \int_0^1 [dxdy] \frac{x_2 y_2}{D} \{Q_u^2 [4TT' + (V + A)(V' + A')](3, 2, 1) \\ &\quad + Q_u Q_d [4TT' + (V + A)(V' + A')](1, 2, 3) + 2Q_u Q_d [VV' + AA'] (1, 3, 2)\}, \\ \delta\tilde{F}_2 &= 0, \\ \frac{\nu}{M_N^2} \tilde{F}_3 &= -\frac{\alpha_{em}\alpha_s}{Q^4} \frac{8\pi^2}{9} (2\zeta - 1) \int_0^1 [dxdy] \frac{(x_2 \bar{y}_2 + \bar{x}_2 y_2)}{D} \{Q_u^2 [4TT' + (V + A)(V' + A')](3, 2, 1) \\ &\quad + Q_u Q_d [4TT' + (V + A)(V' + A')](1, 2, 3) + 2Q_u Q_d [VV' + AA'] (1, 3, 2)\},\end{aligned}\quad (41)$$

where some variable transformations are used. The symbol (1,2,3) means that the variables in V, A, T and V', A', T' are $x_{1,2,3}$ and $y_{1,2,3}$ in order, $\zeta \equiv s/Q^2$ with $\zeta \geq 1$, and $\bar{\zeta} \equiv 1 - \zeta$. We define

$$D = (y_1 y_2 \bar{y}_2)(x_1 x_2 \bar{x}_2) [x_2 \bar{\zeta} + y_2 \zeta - x_2 y_2 + i\varepsilon] [x_2 \zeta + y_2 \bar{\zeta} - x_2 y_2 + i\varepsilon]. \quad (42)$$

The above results are the same as those given in Ref. [9]. In this work, we also calculate the γZ -exchange contribution to the corresponding PV coefficients, which are

$$\begin{aligned}c_{1,A}^{\gamma Z, PV} &= \frac{16\pi^3 \alpha_e^2 \alpha_s}{9Q^4 \sin^2 2\theta_W} \int_0^1 [dxdy] (x_2 y_2 - 2(x_2 + y_2)\zeta \bar{\zeta}) [M_{(3,2,1)}^A - M_{(1,2,3)}^A + M_{(1,3,2)}^A], \\ c_{3,A}^{\gamma Z, PV} &= \frac{32\pi^3 \alpha_e^2 \alpha_s}{9Q^4 \sin^2 2\theta_W} \int_0^1 [dxdy] (x_2 + y_2)(2\zeta - 1) [-M_{(3,2,1)}^A + M_{(1,2,3)}^A - M_{(1,3,2)}^A], \\ c_{1,V}^{\gamma Z, PV} &= \frac{16\pi^3 \alpha_e^2 \alpha_s}{9Q^4 \sin^2 2\theta_W} \int_0^1 [dxdy] x_2 y_2 (2\zeta - 1) [M_{(3,2,1)}^V + M_{(1,2,3)}^V + M_{(1,3,2)}^V], \\ c_{3,V}^{\gamma Z, PV} &= \frac{32\pi^3 \alpha_e^2 \alpha_s}{9Q^4 \sin^2 2\theta_W} \int_0^1 [dxdy] (\bar{x}_2 y_2 + x_2 \bar{y}_2) [M_{(3,2,1)}^V + M_{(1,2,3)}^V + M_{(1,3,2)}^V]\end{aligned}\quad (43)$$

with

$$\begin{aligned}M_{(3,2,1)}^A &= Q_u g_A^\mu \left[\frac{4TT' - (A + V)(A' + V')}{D_1} + \frac{4TT' + (A + V)(A' + V')}{D_2} \right], \\ M_{(1,2,3)}^A &= \frac{Q_u g_A^d [4TT' - (A + V)(A' + V')]}{D_1} - \frac{Q_d g_A^\mu [4TT' + (A + V)(A' + V')]}{D_2}, \\ M_{(1,3,2)}^A &= \frac{2Q_d g_A^\mu [A'V + AV']}{D_1} + \frac{2Q_u g_A^d [AA' + VV']}{D_2}, \\ M_{(3,2,1)}^V &= Q_u g_V^\mu \left[\frac{1}{D_1} + \frac{1}{D_2} \right] [4TT' + (A + V)(A' + V')], \\ M_{(1,2,3)}^V &= \left[\frac{Q_u g_V^d}{D_1} + \frac{Q_d g_V^\mu}{D_2} \right] [4TT' + (A + V)(A' + V')], \\ M_{(1,3,2)}^V &= \left[\frac{2Q_d g_V^\mu}{D_1} + \frac{2Q_u g_V^d}{D_2} \right] [AA' + VV'],\end{aligned}\quad (44)$$

and

$$\begin{aligned}D_1 &= x_1 x_2 y_1 y_2 (M_Z^2 + Q^2 \bar{x}_2 \bar{y}_2) (x_2 \zeta + y_2 \bar{\zeta} - x_2 y_2 + i\varepsilon) (x_2 \bar{\zeta} + y_2 \zeta - x_2 y_2 + i\varepsilon), \\ D_2 &= x_1 \bar{x}_2 y_1 \bar{y}_2 (M_Z^2 + Q^2 x_2 y_2) (x_2 \zeta + y_2 \bar{\zeta} - x_2 y_2 + i\varepsilon) (x_2 \bar{\zeta} + y_2 \zeta - x_2 y_2 + i\varepsilon),\end{aligned}\quad (45)$$

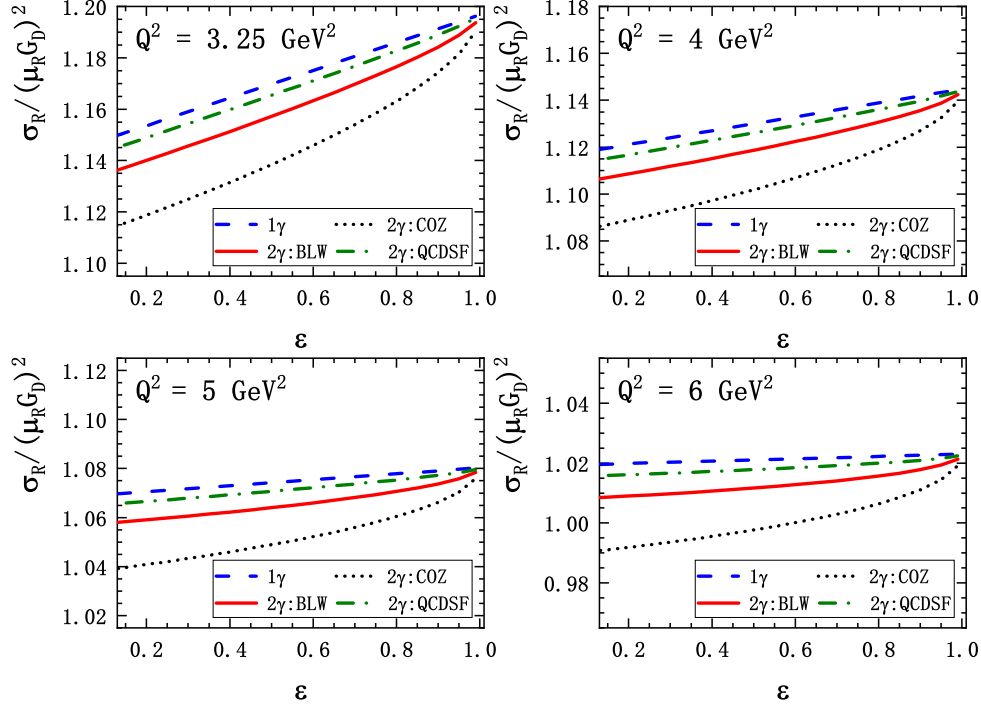


FIG. 3. Numerical results for $\sigma_R/(\mu_p G_D)^2$. The dashed blue curves refer to the 1γ -exchange contribution. The dotted black, solid red, and dash-dotted olive curves refer to results with the COZ, BLW, and QCDSF models, respectively.

where labels such as (1,2,3) denote that the variables in V , T , A and V' , T' , A' are (x_1, x_2, x_3) and (y_1, y_2, y_3) , respectively.

We note that a similar γZ -exchange contribution in pQCD is also calculated in the PhD thesis [20]. Reference [20] expresses the γZ box in terms of effective form-factor-like quantities such as $\delta \tilde{G}_M^Z$, whereas we work directly with the invariant amplitudes $c_{i,V/A}^{\gamma Z, PV}$ defined in Eq. (39). As one explicit example, the $Q_u g_V^d$ term in $\delta \tilde{G}_M^Z$ of Ref. [20] contains the operator structure $(V' - A')(V - A) + 4T'T$ with the quark-label ordering (1,3,2), whereas the corresponding term in our $c_{1,V}^{\gamma Z, PV}$ contains $(V' + A')(V + A) + 4T'T$ with ordering (3,2,1) or $(V' - A')(V - A) + 4T'T$ with ordering (3,1,2). Similar differences appear in the relative signs and in the quark permutations of the other terms. In addition, our analytical expressions retain the full Z -boson mass dependence through the propagator denominators D_1 and D_2 , whereas the comparison formula in Ref. [20] is organized in the high- Q^2 limit. These differences reflect different amplitude decompositions and conventions, not merely a change of notation. Our results reduce to the same high- Q^2 scaling.

IV. NUMERICAL RESULTS

To numerically calculate the integrations in Eqs. (41) and (43), one can reduce the denominators by applying the change $x_1 \leftrightarrow x_2$, $y_1 \leftrightarrow y_2$, and then integrate x_3, y_3, x_1, y_1 analytically after substituting the functions V, T, A . For the remaining integrals over x_2, y_2 , there is a pole in y_2 in the physical region

$$y_2^{\text{pole}} = \frac{s x_2}{s + Q^2(-1 + x_2)} + i\epsilon. \quad (46)$$

After the pole position is known, the integrals can be done directly.

A. TPE contributions to unpolarized cross section

For comparison, we present the TPE contributions to $\sigma_R^{1\gamma \otimes 1\gamma}/(\mu_p G_D)^2$ and $\sigma_R^{1\gamma \otimes 2\gamma}/(\mu_p G_D)^2$ as functions of ϵ in Fig. 3, where

$$G_D \equiv \frac{1}{(1 + Q^2/0.71)^2}. \quad (47)$$

Here, $\mu_p = 2.79$ is the proton magnetic moment, and

$$\epsilon = [1 + 2(1 + \tau \tan^2(\theta_e/2))]^{-1}, \quad (48)$$

$$\tau = \frac{Q^2}{4M_N^2}$$

with θ_e the scattering angle in the laboratory frame.

Although the analytical expressions are equivalent to those in Ref. [9], our numerical results are slightly different. This difference may come from the values of G_E, G_M used in $\sigma_R^{1\gamma \otimes 1\gamma}$. In our calculation, we take G_E, G_M from Refs. [26–28]. As a validation benchmark, our TPE numerical implementation still reproduces the expected magnitude and kinematic trend of the known pQCD contribution to the reduced unpolarized cross section.

B. γZ -exchange contributions $c_{i,V}^{\gamma Z, PV}$ and $c_{i,A}^{\gamma Z, PV}$

The numerical results for the coefficients $c_{i,V}^{\gamma Z, PV}$ and $c_{i,A}^{\gamma Z, PV}$ as functions of Q^2 are presented in Fig. 4, where we take $\epsilon = 0.3$ as an example and show only the real parts. The

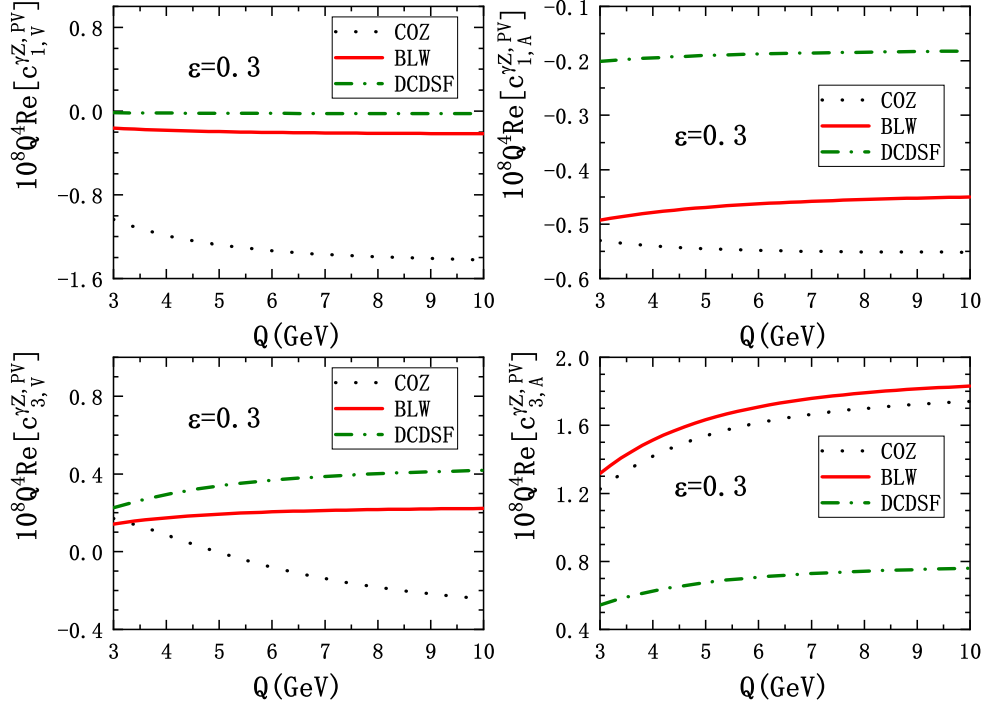


FIG. 4. Numerical results for $\text{Re}[c_{1,V}^{\gamma Z, PV}]$, $\text{Re}[c_{3,V}^{\gamma Z, PV}]$, $\text{Re}[c_{1,A}^{\gamma Z, PV}]$, and $\text{Re}[c_{3,A}^{\gamma Z, PV}]$ as functions of Q at $\varepsilon = 0.3$. The dotted black, solid red, and dash-dotted olive curves refer to results with the COZ, BLW, and QCDSF models, respectively.

dotted black, solid red, and dash-dotted olive curves refer to results using the COZ [22], BLW [23], and QCDSF [24] models, respectively. Figure 4 provides numerical evidence for this asymptotic behavior:

$$c_{i,A}^{\gamma Z, PV} \sim c_{1,V}^{\gamma Z, PV} \sim Q^{-4}. \quad (49)$$

Having examined the Q^2 dependence in Fig. 4, we now discuss the fixed- Q^2 DRs for the amplitude coefficients at high energy. At low energy, DRs for the combinations $\square_{\gamma Z}^{V,A}$ that enter A_{PV} are widely used [11–14]. For example, DRs of the following form are used:

$$\begin{aligned} \text{Re}[X_{\gamma Z}(\nu, Q^2)] &= \frac{2\nu}{\pi} \text{P} \left[\int_{\nu_{th}}^{\infty} \frac{\text{Im}[X_{\gamma Z}(\bar{\nu}^+, Q^2)]}{\bar{\nu}^2 - \nu^2} d\bar{\nu} \right], \\ \text{Re}[Y_{\gamma Z}(\nu, Q^2)] &= \frac{2}{\pi} \text{P} \left[\int_{\nu_{th}}^{\infty} \frac{\bar{\nu} \text{Im}[Y_{\gamma Z}(\bar{\nu}^+, Q^2)]}{\bar{\nu}^2 - \nu^2} d\bar{\nu} \right], \end{aligned} \quad (50)$$

where $\bar{\nu}^+ \equiv \bar{\nu} + i\epsilon$ denotes the integration variable taken on the physical sheet, P denotes the Cauchy principal value of the integral, X refers to $\square_{\gamma Z}^V$, and Y refers to $\square_{\gamma Z}^A$. Ref. [21] shows that, at finite Q^2 , the appropriate DRs are those for $c_i^{\gamma Z, PV}$ rather than for $\square_{\gamma Z}^{V,A}$, which are reliable only in the forward approximation. At the P2 design kinematics ($Q^2 = 0.0045 \text{ GeV}^2$, $E = 0.155 \text{ GeV}$), for example, the DR written directly for $\square_{\gamma Z}^V$ is significantly affected, whereas that for $c_i^{\gamma Z, PV}$ remains reliable. Accordingly, the same DR form in Eq. (50) applies with $X = c_1^{\gamma Z, PV}$ and $Y = c_3^{\gamma Z, PV}$, and these DRs remain reliable at finite Q^2 .

The high- ν asymptotic behavior of the amplitudes determines the convergence of the DR integrals, and the

convergence properties determine the required subtraction order. For the X -type DR, the large- $\bar{\nu}$ kernel behaves as $\sim 1/\bar{\nu}^2$ (at fixed external ν), so convergence is controlled by the integral of $\text{Im}X_{\gamma Z}(\bar{\nu})/\bar{\nu}^2$. For the Y -type DR, the kernel behaves as $\sim 1/\bar{\nu}$, so convergence is controlled by the integral of $\text{Im}Y_{\gamma Z}(\bar{\nu})/\bar{\nu}$. Therefore, if $\text{Im}Y_{\gamma Z}$ approaches a constant at high energy, the unsubtracted Y -type DR develops a logarithmic ultraviolet divergence and a subtraction is required. In contrast, quantities with sufficiently suppressed high- ν behavior remain compatible with the unsubtracted form.

Since the physical DRs should be the same at low and high energy, we test the DRs in Eq. (50) for $c_i^{\gamma Z, PV}$ in the high- Q^2 regime. The present paper complements the low-energy analysis of Ref. [21] by using pQCD to determine the high- ν behavior of $c_i^{\gamma Z, PV}$ and thereby the subtraction order required beyond the forward limit.

The high- ν behavior indicates that not all quantities satisfy the commonly used unsubtracted DR form. In Figs. 5 and 6, we present the numerical results for the coefficients $c_{i,V}^{\gamma Z, PV}$ and $c_{i,A}^{\gamma Z, PV}$ as functions of ν at $Q = 5 \text{ GeV}$. The dotted black, solid red, and dash-dotted olive curves refer to results obtained with the COZ, BLW, and QCDSF models, respectively.

The results clearly show the high-energy behavior when $\nu \rightarrow \infty$:

$$\begin{aligned} \text{Re}[c_{1,A}^{\gamma Z, PV}] &\sim \nu^0, \\ \text{Re}[c_{1,V}^{\gamma Z, PV}] &\sim \text{Re}[c_{3,A}^{\gamma Z, PV}] \sim \nu^{-1}, \\ \text{Re}[c_{3,V}^{\gamma Z, PV}] &\sim \nu^{-2} \end{aligned} \quad (51)$$

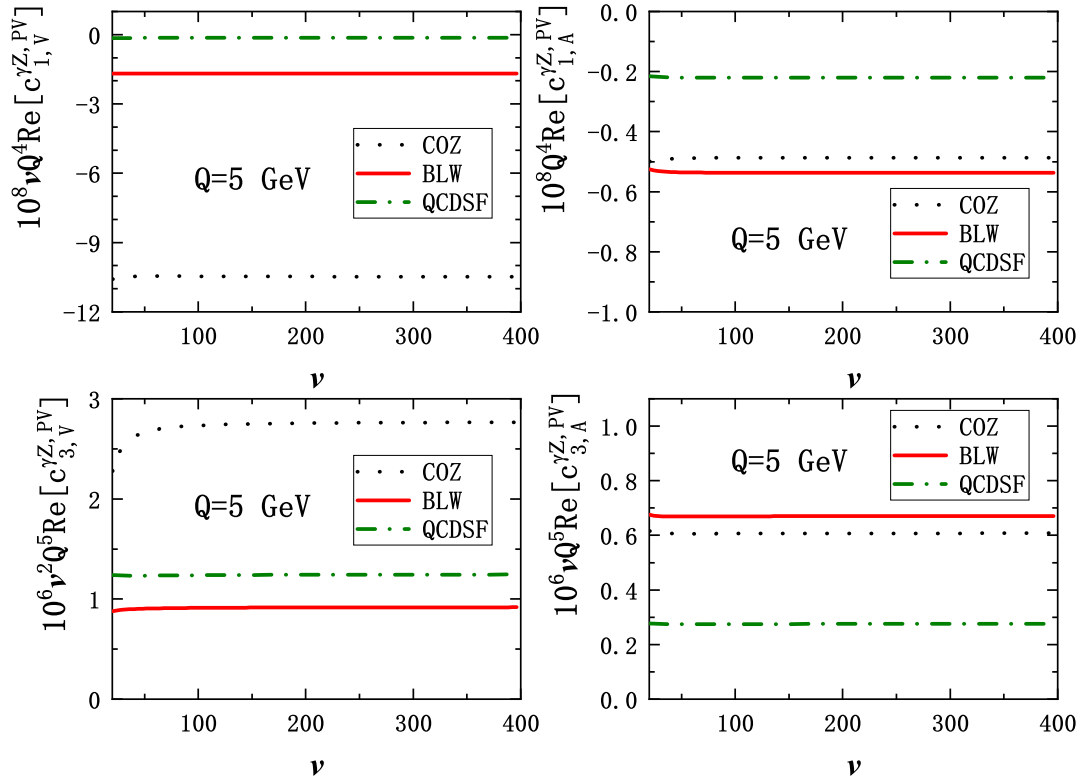


FIG. 5. Numerical results for $\text{Re}[c_{i,V}^{\gamma Z, PV}]$ and $\text{Re}[c_{i,A}^{\gamma Z, PV}]$ as functions of ν at $Q = 5$ GeV. The dotted black, solid red, and dash-dotted olive curves refer to results obtained with the COZ, BLW, and QCDSF models, respectively.

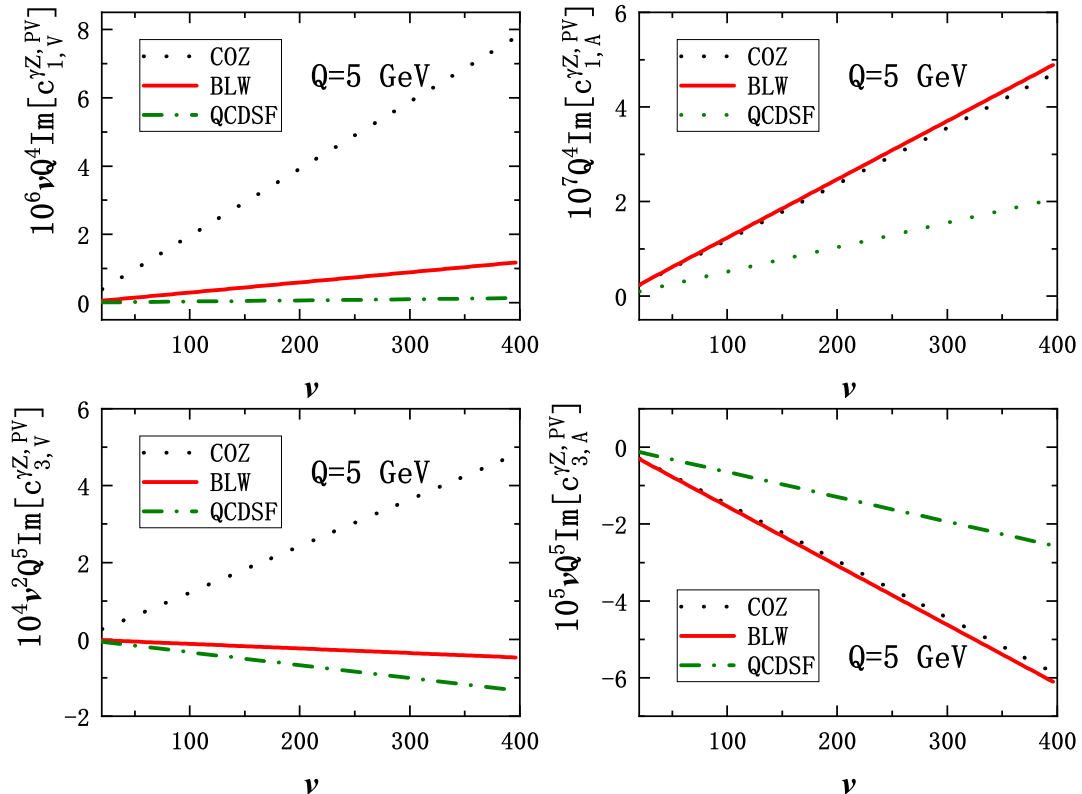


FIG. 6. Numerical results for $\text{Im}[c_{i,V}^{\gamma Z, PV}]$ and $\text{Im}[c_{i,A}^{\gamma Z, PV}]$ as functions of ν at $Q = 5$ GeV. The dotted black, solid red, and dash-dotted olive curves refer to results obtained with the COZ, BLW, and QCDSF models, respectively.

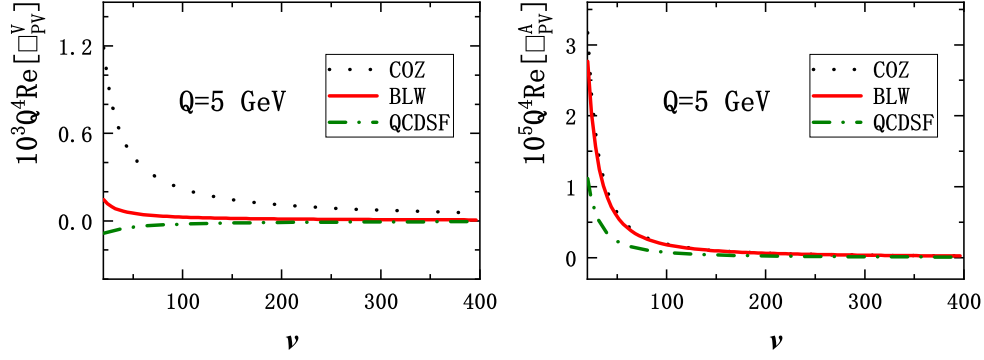


FIG. 7. Numerical results for $\text{Re}[\square_{\gamma Z}^{V,A}]$ as functions of ν at $Q = 5$ GeV. The dotted black, solid red, and dash-dotted olive curves refer to results with the COZ, BLW, and QCDSF models, respectively.

and

$$\begin{aligned} \text{Im}[c_{1,A}^{\gamma Z, PV}] &\sim \nu, \\ \text{Im}[c_{1,V}^{\gamma Z, PV}] &\sim \text{Im}[c_{3,A}^{\gamma Z, PV}] \sim \nu^0, \\ \text{Im}[c_{3,V}^{\gamma Z, PV}] &\sim \nu^{-1}. \end{aligned} \quad (52)$$

These behaviors mean that, at high Q^2 , $c_{1,A}^{\gamma Z, PV}$, and $c_{3,A}^{\gamma Z, PV}$ do not satisfy the corresponding DRs in Eq. (50), and subtraction terms should be added.

C. γZ -exchange contributions to $\square_{\gamma Z}^{V,A}$

For completeness, Figs. 7 and 8 also show the high- ν behavior of $\square_{\gamma Z}^{V,A}$, but the DR analysis in this section remains centered on $c_i^{\gamma Z, PV}$.

At low Q^2 , unsubtracted DRs similar to Eq. (50) are usually written for $\square_{\gamma Z}^{V,A}$. As discussed above, Ref. [21] shows that such DRs are reliable only in the forward approximation.

These DRs can also be tested in the high-energy case using the above results. We present the high- ν behaviors for $\square_{\gamma Z}^{V,A}$ in Figs. 7 and 8, where we take $Q = 5$ GeV as an example. Here, F_1 and F_2 are approximated as

$$\begin{aligned} F_1 &\approx \frac{1 + \mu_p \tau}{(1 + \tau)} G_D, \\ F_2 &\approx \frac{\mu_p - 1}{(1 + \tau)} G_D. \end{aligned} \quad (53)$$

The results in Figs. 7 and 8 clearly show that when $\nu \rightarrow \infty$, we have

$$\begin{aligned} \text{Re}[\square_{\gamma Z}^V] &\sim \text{Re}[\square_{\gamma Z}^A] \sim 0, \\ \text{Im}[\square_{\gamma Z}^V] &\sim \text{Im}[\square_{\gamma Z}^A] \sim \nu^0. \end{aligned} \quad (54)$$

These behaviors mean that the DR in Eq. (50) applied directly to $\square_{\gamma Z}^A$ is shown in Figs. 7 and 8 for illustration only. The subtracted form required at high Q^2 applies to $c_{3,A}^{\gamma Z, PV}$, as discussed above.

V. SUMMARY

In this work, we calculate the γZ -exchange contribution at the amplitude level in elastic ep scattering within pQCD. At fixed Q^2 , the high- ν asymptotic behavior of the coefficients $c_i^{\gamma Z, PV}$ provides a direct link between pQCD scaling and the convergence properties of the DR integrals, thereby determining the required subtraction order in the regime beyond the forward limit ($s/Q^2 \rightarrow \infty$). Within the present pQCD framework and adopted distribution amplitudes, we find that $c_{3,A}^{\gamma Z, PV}$ requires a once-subtracted DR at high Q^2 . Following Ref. [21], the DRs are applied to the amplitude coefficients rather than directly to $\square_{\gamma Z}^{V,A}$, which are reliable only in the forward approximation.

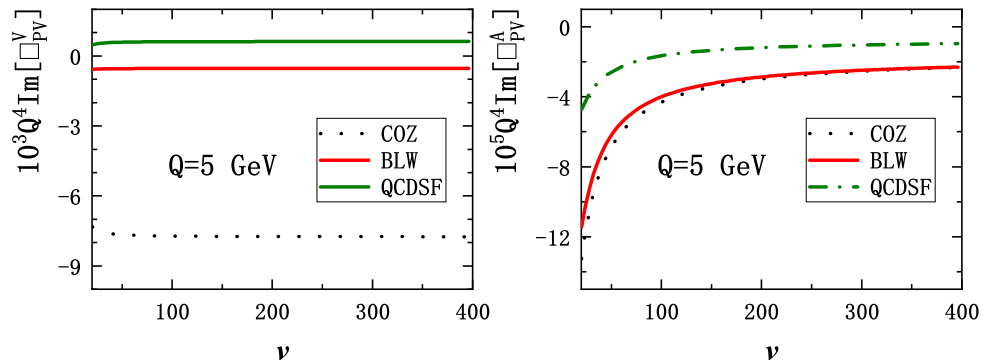


FIG. 8. Numerical results for $\text{Im}[\square_{\gamma Z}^{V,A}]$ as functions of ν at $Q = 5$ GeV. The dotted black, solid red, and dash-dotted olive curves refer to results with the COZ, BLW, and QCDSF models, respectively.

ACKNOWLEDGMENTS

H.Q.Z. would like to thank Zhi-Hui Guo for helpful discussions. H.Q.Z. is supported by the National Natural Science Foundation of China under Grants No. 12150013 and No. 12075058. H.Y.C. is supported by the Science and Technology Research Project of Hubei Provincial Education Department under Grant No. 20222502, the Hubei Provincial Natural Science Foundation under Grant No. 2023AFB443, and the

National Natural Science Foundation of China under Grant No. 12405153.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

-
- [1] H. Q. Zhou, C. W. Kao, and S. N. Yang, Two-photon-exchange and γZ -exchange corrections to parity-violating elastic electron-proton scattering, *Phys. Rev. Lett.* **99**, 262001 (2007).
 - [2] K. Nagata, H. Q. Zhou, C. W. Kao, and S. N. Yang, Contribution of two-boson exchange with $\Delta(1232)$ excitation to parity-violating elastic electron-proton scattering, *Phys. Rev. C* **79**, 062501 (2009).
 - [3] J. A. Tjon and W. Melnitchouk, Effect of two-boson exchange on parity-violating $e-p$ scattering, *Phys. Rev. Lett.* **100**, 082003 (2008).
 - [4] J. A. Tjon, P. G. Blunden, and W. Melnitchouk, Detailed analysis of two-boson exchange in parity-violating $e-p$ scattering, *Phys. Rev. C* **79**, 055201 (2009).
 - [5] Y. C. Chen, C. W. Kao, and M. Vanderhaeghen, Partonic calculation of γZ exchange corrections to parity-violating elastic electron-proton scattering, [arXiv:0903.1098](https://arxiv.org/abs/0903.1098).
 - [6] Y. C. Chen, A. Afanasev, S. J. Brodsky, C. E. Carlson, and M. Vanderhaeghen, Partonic calculation of the two-photon exchange contribution to elastic electron-proton scattering at large momentum transfer, *Phys. Rev. Lett.* **93**, 122301 (2004).
 - [7] A. Afanasev and C. E. Carlson, Two-photon-exchange correction to parity-violating elastic electron-proton scattering, *Phys. Rev. Lett.* **94**, 212301 (2005).
 - [8] D. Borisyuk and A. Kobushkin, Perturbative QCD predictions for two-photon exchange, *Phys. Rev. D* **79**, 034001 (2009).
 - [9] N. Kivel and M. Vanderhaeghen, Two-photon exchange in elastic electron-proton scattering: A QCD factorization approach, *Phys. Rev. Lett.* **103**, 092004 (2009).
 - [10] J. Guttman, N. Kivel, and M. Vanderhaeghen, Study of two-photon corrections in the $p\bar{p} \rightarrow e^+e^-$ process: Hard rescattering mechanism, *Phys. Rev. D* **83**, 094021 (2011).
 - [11] M. Gorchtein and C. J. Horowitz, Dispersion γZ -box correction to the weak charge of the proton, *Phys. Rev. Lett.* **102**, 091806 (2009).
 - [12] A. Sibirtsev, P. G. Blunden, W. Melnitchouk, and A. W. Thomas, γZ corrections to forward-angle parity-violating ep scattering, *Phys. Rev. D* **82**, 013011 (2010).
 - [13] M. Gorchtein, C. J. Horowitz, and M. J. Ramsey-Musolf, Model dependence of the γZ dispersion correction to the parity-violating asymmetry in elastic ep scattering, *Phys. Rev. C* **84**, 015502 (2011).
 - [14] J. Erler, M. Gorchtein, O. Koshchii, C. Y. Seng, and H. Spiesberger, Reduced uncertainty of the axial γZ -box diagram correction to the proton's weak charge, *Phys. Rev. D* **100**, 053007 (2019).
 - [15] F. Yang, X. Feng, L. C. Jin, and C. F. Lu, Lattice QCD calculation of the two-photon exchange contribution to the muonic-hydrogen Lamb shift, *Phys. Rev. Lett.* **128**, 172002 (2022).
 - [16] W. Z. Xiong and C. Peng, Proton electric charge radius from lepton scattering, *Universe*, **9**, 182 (2023).
 - [17] K. Pachucki, V. Lensky, F. Hagelstein, S. S. Li Muli, S. Bacca, and R. Pohl, Comprehensive theory of the Lamb shift in light muonic atoms, *Rev. Mod. Phys.* **96**, 015001 (2024).
 - [18] M. Salg, D. Djukanovic, G. M. von Hippel, H. B. Meyer, K. Otnad, and H. Wittig, Proton radii for muonic hydrogen spectroscopy from lattice QCD, *PoS LATTICE* **2024**, 315 (2025).
 - [19] Z. L. Zhang, X. Feng, M. Gorchtein, L. C. Jin, C. Liu, and C. Y. Seng, Lattice QCD determination of the γZ box contribution to the proton weak charge, [arXiv:2601.06582](https://arxiv.org/abs/2601.06582).
 - [20] J. Guttman, Nucleon electromagnetic structure studies in the spacelike and timelike regions, Ph.D. thesis, Universität Mainz, 2013.
 - [21] Q. Q. Guo and H. Q. Zhou, Corrections to the forward-limit dispersion relations for γZ -exchange contributions, *Phys. Rev. C* **109**, 014308 (2024).
 - [22] V. L. Chernyak, A. A. Ogloblin, and I. R. Zhitnitsky, Wave functions of octet baryons, *Z. Phys. C* **42**, 569 (1989).
 - [23] V. M. Braun, A. Lenz, and M. Wittmann, Nucleon form factors in QCD, *Phys. Rev. D* **73**, 094019 (2006).
 - [24] M. Gockeler *et al.*, Nucleon distribution amplitudes from lattice QCD, *Phys. Rev. Lett.* **101**, 112002 (2008).
 - [25] Q. Q. Guo and H. Q. Zhou, γZ -exchange contributions in low-energy parity-violating ep scattering, *Phys. Rev. C* **108**, 035501 (2023).
 - [26] M. K. Jones *et al.* (Jefferson Lab Hall A Collaboration), G_{Ep}/G_{Mp} ratio by polarization transfer in $\bar{e}p \rightarrow e\bar{p}$, *Phys. Rev. Lett.* **84**, 1398 (2000).
 - [27] O. Gayou *et al.* (Jefferson Lab Hall A Collaboration), Measurement of G_{Ep}/G_{Mp} in $\bar{e}p \rightarrow e\bar{p}$ to $Q^2 = 5.6 \text{ GeV}^2$, *Phys. Rev. Lett.* **88**, 092301 (2002).
 - [28] V. Punjabi *et al.*, Proton elastic form factor ratios to $Q^2 = 3.5 \text{ GeV}^2$ by polarization transfer, *Phys. Rev. C* **71**, 055202 (2005).