

Entanglement entropy for π^+p elastic scattering using a spin-density matrix

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(Received 18 June 2025; revised 1 July 2025; accepted 8 December 2025; published 21 January 2026)

The π^+p elastic scattering process is studied, using an effective Lagrangian approach that incorporates the s -, u -, and t -channel amplitudes, including $\Delta^{++}(1232)$, $\Delta^0(1232)$, neutron, and ρ^0 contributions. By constructing the spin-density matrices from the scattering amplitudes, the von Neumann (entanglement) entropy is derived and associated with the spin degrees of freedom of the initial and final state particles. The entropy is computed by performing a partial trace over the spin subsystems, and its behavior is analyzed as a function of the center-of-mass energy W and scattering angle θ . It is found that entropy exhibits nontrivial angular and energy dependences, reflecting the interplay among resonance contributions and background amplitudes. In particular, the Δ^{++} region shows relatively low entanglement, suggesting spin coherence dominated by the specific resonant contribution, while the background contributions increase entropy due to mixed spin configurations. These results indicate that entanglement entropy can serve as a novel probe into the quantum structure of hadronic scattering amplitudes, providing complementary insights beyond conventional cross-section analyses.

DOI: [10.1103/7vps-vt18](https://doi.org/10.1103/7vps-vt18)

I. INTRODUCTION

Elastic scattering processes involving pions and nucleons provide essential insights into the nonperturbative regime of quantum chromodynamics (QCD) and the structure of baryonic resonances. Among them, the π^+p elastic scattering is particularly notable because the $\Delta(1232)$ resonance dominates at low energies, playing a key role in understanding hadronic excitation mechanisms. Numerous experimental and theoretical studies have sought to describe this process using an effective Lagrangian approach and partial-wave analyses [1,2]. While conventional analyses focus on cross-sections and resonance parameters, recent developments in quantum information theory have introduced new observables that probe deeper aspects of scattering dynamics. In particular, the von Neumann entropy, a measure of quantum entanglement, can be applied to hadronic reactions to study spin correlations and coherence [3–5]. This approach allows for an extended characterization of the scattering process beyond classical observables. Recent studies have directly linked entanglement entropy with particle creation in high-energy processes. Lin *et al.* [6] investigated how expanding background fields generate entangled particle-antiparticle pairs, using von Neumann entropy to distinguish between entropy production and entanglement dynamics in quantum field theory.

In this study, $\pi^+p \rightarrow \pi^+p$ elastic scattering is investigated using a tree-level effective Lagrangian approach that includes

s -, u -, and t -channel contributions with the Δ^{++} , Δ^0 , neutron, and ρ^0 intermediates. The primary motivation for choosing this process is that all involved particles can be experimentally traced due to their electric charge. Both the total and differential cross sections are computed, and the spin-density matrix (SDM) is constructed based on canonical spin quantization. From the SDM, the von Neumann entropy of the initial and final proton spin states is extracted to analyze quantum entanglement induced by hadronic interactions. The behavior of the entropy across different kinematic regimes is examined, with particular emphasis on how it reflects the interplay between resonant and nonresonant contributions. By comparing entropy distributions obtained from various diagram combinations, the quantum coherence structure encoded in the spin degrees of freedom is identified. The results provide a novel quantum information-theoretic perspective on elastic scattering and open avenues for future studies of inelastic processes, polarization observables, and entangled baryonic states. The SDM formulation employed here is not merely an imported formalism from quantum information theory, but rather a natural extension that emerges from the intrinsic spin structure of the πN scattering amplitude itself.

Specifically, the numerical results show that the total cross section for π^+p elastic scattering is strongly dominated by the Δ^{++} resonance near $W \approx 1230$ MeV, with background contributions from Δ^0 , neutron, and ρ^0 channels being significantly suppressed. The differential cross section exhibits clear forward and backward peaking behavior at $W \approx 1285$ MeV, consistent with p -wave dominance. The SDM, constructed from the scattering amplitudes based on canonical spin quantization in the center-of-mass (c.m.) frame, reveals coherent spin structures in the resonance-dominated region and mixed configurations elsewhere. The von Neumann entropies S_{SDM} computed from reduced SDMs show strong angular and energy dependences, remaining relatively low in the region

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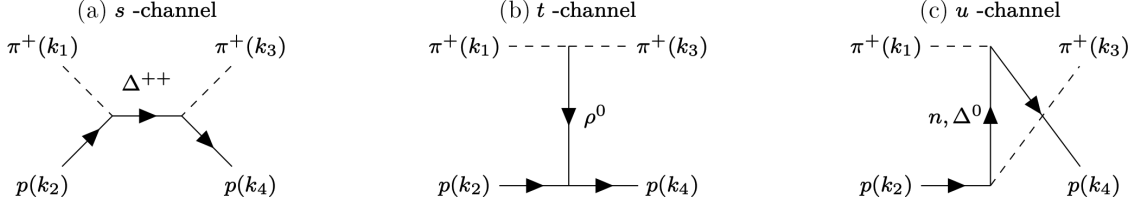


FIG. 1. Relevant Feynman diagrams for $\pi^+ p$ elastic scattering: (a) Δ^{++} -pole diagram in the s channel, (b) ρ^0 exchange in the t channel, and (c) n and Δ^0 exchanges in the u channel. The four-momenta for the involved particles are also defined by $k_{1,2,3,4}$.

of Δ^{++} dominance and increasing when nonresonant amplitudes interfere. The entropy distributions for selected channel combinations demonstrate that background processes, such as neutron and ρ^0 exchange, substantially modify the spin-entanglement structure beyond the Δ^{++} region. In particular, they generate regions of both enhanced mixed-spin configurations and localized suppression of entropy, depending on the interference pattern induced by the dominant Δ^{++} amplitude. These results provide quantum-informational insights into how resonant and nonresonant components redistribute spin coherence, extending beyond what is captured by conventional cross-section observables. Beyond being a formal construction, the SDM and corresponding entanglement entropy are, in principle, experimentally reconstructible from polarization and spin-transfer observables in πN scattering.

The remainder of this paper is organized as follows. In Sec. II, the theoretical framework of the effective Lagrangian approach used to compute the $\pi^+ p$ elastic scattering process is briefly described, together with the construction of the von Neumann entropy from the corresponding SDMs. Section III presents the numerical results for the total and angular-dependent differential cross sections, SDMs, and von Neumann entropies, along with detailed discussions. Finally, Sec. IV summarizes the conclusions and provides perspectives for future studies.

II. THEORETICAL FRAMEWORK

A. Effective Lagrangians, scattering amplitudes, and form factors

In this section, a brief introduction to the effective Lagrangian approach for the $\pi^+ p$ elastic scattering process is provided. Within the tree-level Born approximation, the relevant Feynman diagrams are shown in Fig. 1: (a) the Δ^{++} -pole diagram in the s channel, (b) ρ^0 exchange in the t channel, and (c) n and Δ^0 exchanges in the u channel. The solid and dashed lines denote baryons (nucleon and Δ) and pions, respectively, and the four-momenta of the particles are defined accordingly. For the interaction vertices, the effective Lagrangians corresponding to each channel are introduced. In particular, the effective Lagrangian for the $\pi N \Delta$ vertex associated with Figs. 1(a) and 1(c) is expressed within the Rarita-Schwinger field formalism [7]:

$$\mathcal{L}_{\pi N \Delta} = \frac{f_{\pi N \Delta}}{M_\pi} \bar{\Delta}^\mu \partial_\mu (S \cdot \boldsymbol{\pi}) N + \text{H.c.}, \quad (1)$$

where Δ_μ and N are the Δ baryon and nucleon fields, respectively, and S the isospin transition operator between the

isospin 1/2 and 3/2 fields. Thus, the scattering amplitudes are calculated straightforwardly as follows:

$$i\mathcal{M}_s^{\Delta^{++}} = -\frac{if_{\pi N \Delta}^2}{M_\pi^2} \bar{u}(k_4) k_{3\mu} G^{\mu\nu}(q_{1+2}) k_{1\nu} u(k_2),$$

$$i\mathcal{M}_u^{\Delta^0} = -\frac{if_{\pi N \Delta}^2}{3M_\pi^2} \bar{u}(k_4) k_{1\mu} G^{\mu\nu}(q_{2-3}) k_{3\nu} u(k_2), \quad (2)$$

where $q_{i\pm j} \equiv k_i \pm k_j$, and $f_{\pi N \Delta}$ denotes the $\pi N \Delta$ coupling constant. The Mandelstam variables are defined as $s = q_{1+2}^2$ and $u = q_{2-3}^2$. The spinor $u(k_i)$ represents the proton spinor in the Bjorken-Drell (canonical) representation [8]. The Δ -baryon propagator $G^{\mu\nu}(q)$ appearing in Eq. (2) is defined as

$$G^{\mu\nu}(q) = \frac{q + M_\Delta}{q^2 - M_\Delta^2 + iM_\Delta \Gamma_\Delta} \left[-g^{\mu\nu} + \frac{1}{3} \gamma^\mu \gamma^\nu + \frac{2q^\mu q^\nu}{3M_\Delta^2} - \frac{q^\mu \gamma^\nu - q^\nu \gamma^\mu}{3M_\Delta} \right]. \quad (3)$$

For Fig. 1(b), the relevant interaction Lagrangians are given by

$$\mathcal{L}_{\rho\pi\pi} = g_{\rho\pi\pi} \vec{\rho}^\mu \cdot (\vec{\pi} \times \partial_\mu \vec{\pi})$$

$$= ig_{\rho\pi\pi} \rho^{0\mu} (\pi^+ \partial_\mu \pi^- - \pi^- \partial_\mu \pi^+), \quad (4)$$

$$\mathcal{L}_{\rho NN} = -g_{\rho NN} \bar{N} \left[\gamma^\mu \vec{\tau} \cdot \vec{\rho}_\mu - \frac{\kappa_\rho}{2M_N} \sigma^{\mu\nu} \vec{\tau} \cdot \partial_\nu \vec{\rho}_\mu \right] N. \quad (5)$$

Here, ρ and π denote the corresponding meson fields, and κ_ρ denotes the anomalous magnetic moment of the ρ meson. Then, the following scattering amplitude is obtained using the Mandelstam variable $t = q_{1-3}^2$:

$$\mathcal{M}_t^{\rho^0} = -g_{\rho\pi\pi} g_{\rho NN} \bar{u}(k_4) \left[\frac{q_{1+2}^\mu}{t - M_\rho^2} \right] \left[g_{\mu\nu} - \frac{q_{1-3\mu} q_{1-3\nu}}{M_\rho^2} \right]$$

$$\times \left[\gamma^\nu + \frac{i\kappa_\rho}{2M_N} \sigma^{\nu\lambda} q_{1-3\lambda} \right] u(k_2). \quad (6)$$

Finally, for Fig. 1(c), the effective Lagrangian for the πNN interaction in terms of the pseudovector scheme reads

$$\mathcal{L}_{\pi NN} = -\frac{f_{\pi NN}}{M_\pi} \bar{N} \gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi} N + \text{H.c.}, \quad (7)$$

where $f_{\pi NN}$ indicates the pseudovector πNN coupling constant. Using the effective Lagrangian in Eq. (7), the scattering amplitude reads

$$i\mathcal{M}_u^n = -\frac{2if_{\pi NN}^2}{3M_\pi^2} \bar{u}(k_4) \gamma_5 k_1 \left[\frac{q_{2-3} + M_N}{u - M_N^2} \right] \gamma_5 k_3 u(k_2). \quad (8)$$

The spin-averaged differential cross section, summing all the (s, t, u) -channel scattering amplitudes, reads

$$\frac{d\sigma_{\pi^+p}}{d\Omega} = \frac{1}{128\pi^2 s} \frac{|\mathbf{k}_3|}{|\mathbf{k}_1|} \sum_{\text{spin}} |\mathcal{M}_{\pi^+p}^{\text{total}}|^2, \quad (9)$$

where $\mathbf{k}_1$ and $\mathbf{k}_3$ represent the c.m. three-momenta of the particles in the initial and final states, respectively, and the total amplitude with the phenomenological form factors can be written as

$$i\mathcal{M}_{\pi^+p}^{\text{total}} = i\mathcal{M}_s^{\Delta^{++}} F_s^{\Delta^{++}} + i\mathcal{M}_t^{\rho^0} F_t + [i\mathcal{M}_u^{\Delta^0} F_u^{\Delta^0} + i\mathcal{M}_u^n F_u^n]. \quad (10)$$

The form factors, which are responsible for incorporating the spatial extension of the hadrons, are defined for each channel by

$$F_x^h(x) = \frac{\Lambda_{\text{cutoff}}^4}{\Lambda_{\text{cutoff}}^4 + (x - M_h^2)^2}, \quad (11)$$

where x and h denote the Mandelstam variable (s, t, u) and corresponding off-shell hadrons, respectively. All the relevant parameters, including couplings and cutoff Λ , will be discussed in Sec. III.

III. SPIN-DENSITY MATRIX AND ENTANGLEMENT ENTROPY FORMALISM

In this work, quantum spin entanglement in π^+p elastic scattering is analyzed through the SDM formalism. The dynamical information contained in the scattering amplitude is mapped to a density operator acting on the spin degrees of freedom, enabling a quantum-information interpretation of hadronic spin correlations.

A. Spin basis and SDM construction

The scattering amplitudes are written as

$$M_{s_i s_f} \equiv \langle s_f | \mathcal{M} | s_i \rangle, \quad s_{i,f} = \uparrow, \downarrow, \quad (12)$$

where s_i (s_f) labels the spin of the initial (final) proton. The canonical spin basis, in which spin is quantized in the rest frame along a fixed direction, is adopted, and the spinor is subsequently boosted to the c.m. frame. This is in contrast to the helicity basis, where spin is quantized along the momentum direction; the two bases differ by a momentum-dependent Wigner rotation. Consequently, parity constraints take different explicit forms in the two bases.

For the amplitude matrix

$$\mathcal{M}_{s_i s_f} = (\mathcal{M}_{\uparrow\uparrow}, \mathcal{M}_{\uparrow\downarrow}, \mathcal{M}_{\downarrow\uparrow}, \mathcal{M}_{\downarrow\downarrow}) \equiv (a, b, c, d), \quad (13)$$

the 4×4 SDM for the proton spin system is constructed as

$$\rho_{\text{SDM}} = \frac{1}{\mathcal{N}} \begin{pmatrix} |a|^2 & ab^* & ac^* & ad^* \\ a^*b & |b|^2 & bc^* & bd^* \\ a^*c & b^*c & |c|^2 & cd^* \\ a^*d & b^*d & c^*d & |d|^2 \end{pmatrix}, \quad \mathcal{N} = |a|^2 + |b|^2 + |c|^2 + |d|^2. \quad (14)$$

By construction, $\text{Tr}[\rho_{\text{SDM}}] = 1$. The diagonal elements encode spin-state probabilities, while the off-diagonal elements capture quantum coherence between spin configurations.

In the helicity basis and in the c.m. frame, parity relates the amplitudes via $a = d$ and $b = -c$. In the canonical basis, however, these equalities do not hold elementwise due to Wigner rotations. Nevertheless, parity still guarantees that only two independent physical amplitudes remain, ensuring consistency with the symmetry structure of πN scattering. The canonical basis preserves rest-frame spin alignment under boosts, producing Wigner rotations. Therefore, parity relations differ from the helicity basis at the matrix-element level, while the number of independent amplitudes remains two by symmetry. For more detailed explanations, see the Appendix.

B. Reduced density matrices and von Neumann entropy

To quantify spin entanglement, one traces over either the initial or final spin index to obtain a 2×2 reduced SDM:

$$\begin{aligned} \rho_{\text{SDM}}^i &= \text{Tr}_f(\rho_{\text{SDM}}) = \frac{1}{\mathcal{N}} \begin{pmatrix} |a|^2 + |b|^2 & ac^* + bd^* \\ a^*c + b^*d & |c|^2 + |d|^2 \end{pmatrix}, \\ \rho_{\text{SDM}}^f &= \text{Tr}_i(\rho_{\text{SDM}}) = \frac{1}{\mathcal{N}} \begin{pmatrix} |a|^2 + |c|^2 & ab^* + cd^* \\ a^*b + c^*d & |b|^2 + |d|^2 \end{pmatrix}. \end{aligned} \quad (15)$$

The eigenvalues of $\rho_{\text{SDM}}^{i,f}$ are

$$\lambda_{\pm}^{i,f} = \frac{1}{2} (1 \pm \sqrt{1 - 4 \text{Det}[\rho_{\text{SDM}}^{i,f}]}), \quad (16)$$

leading to the von Neumann entropy

$$S_{\text{SDM}}^{i,f} = - \sum_{\alpha=\pm} \lambda_{\alpha}^{i,f} \ln \lambda_{\alpha}^{i,f}. \quad (17)$$

Here, S_{SDM} quantifies the degree of entanglement between proton spin and all other degrees of freedom (orbital motion, isospin, etc.). A vanishing entropy indicates a pure spin state with no entanglement, whereas a nonzero entropy reflects mixing induced by the scattering dynamics. A comparison of S_{SDM}^i and S_{SDM}^f reveals how quantum information is redistributed between initial and final spin subsystems during the elastic process. It is noted that the SDM elements ρ_{ij} are not abstract quantities: in elastic πN scattering, they correspond to measurable polarization and spin-transfer observables, such as the single-spin asymmetry $P(A_y)$, and double-polarization coefficients D_{ij} and C_{ij} . Thus, $\rho_{ij} \leftrightarrow (P, A_y, D_{ij}, C_{ij})$ provides an operational bridge to experimental analysis, ensuring that the entropy constructed from ρ remains, in principle, experimentally reconstructible [9–13].

IV. NUMERICAL RESULTS AND DISCUSSIONS

A. Cross sections and angular distributions

In this section, the numerical results and their corresponding discussions are given. In Table I, all the relevant parameters for the numerical calculations from experimental [14] and theoretical information are listed [15]. Note that some masses and widths of the hadrons are modified to fit the data, e.g., those for Δ . More detailed computations

TABLE I. Relevant parameters for the numerical calculations.

$f_{\pi NN}$	$f_{\pi N\Delta}$	$g_{\rho\pi\pi}$	$g_{\rho NN}$	κ_ρ	M_p	$(M, \Gamma)_{\Delta^{++},0}$	M_{π^+}	$(M, \Gamma)_{\rho^0}$	Λ_{cutoff}
0.989	2.2	6.0	3.25	6.1	939.272 MeV	(1210,80) MeV	139.570 MeV	(763,72.5) MeV	750 MeV

and discussions on the model can be found in our previous work [16].

Figure 2(a) shows the total cross section σ as a function of the c.m. energy W (in MeV). The Δ^{++} resonance dominates in the region around $W \approx 1230$ MeV, producing a sharp peak that aligns well with experimental data from the Particle Data Group (PDG) and Swiss Institute of Nuclear Research (SINR) [17,18]. The background contributions from Δ^0 , neutron, and ρ^0 exchanges are significantly suppressed but are included

to provide completeness and to study their effects on entanglement properties. The theoretical predictions qualitatively reproduce the energy dependence of the cross section. This result confirms that the Δ resonance captures the essential dynamics of low-energy π^+p scattering.

To investigate the angular dependence of the reaction, Fig. 2(b) presents the differential cross section $d\sigma/d\Omega$ at $W \approx 1285$ MeV as a function of the scattering angle θ in the c.m. frame. The result exhibits a pronounced forward-

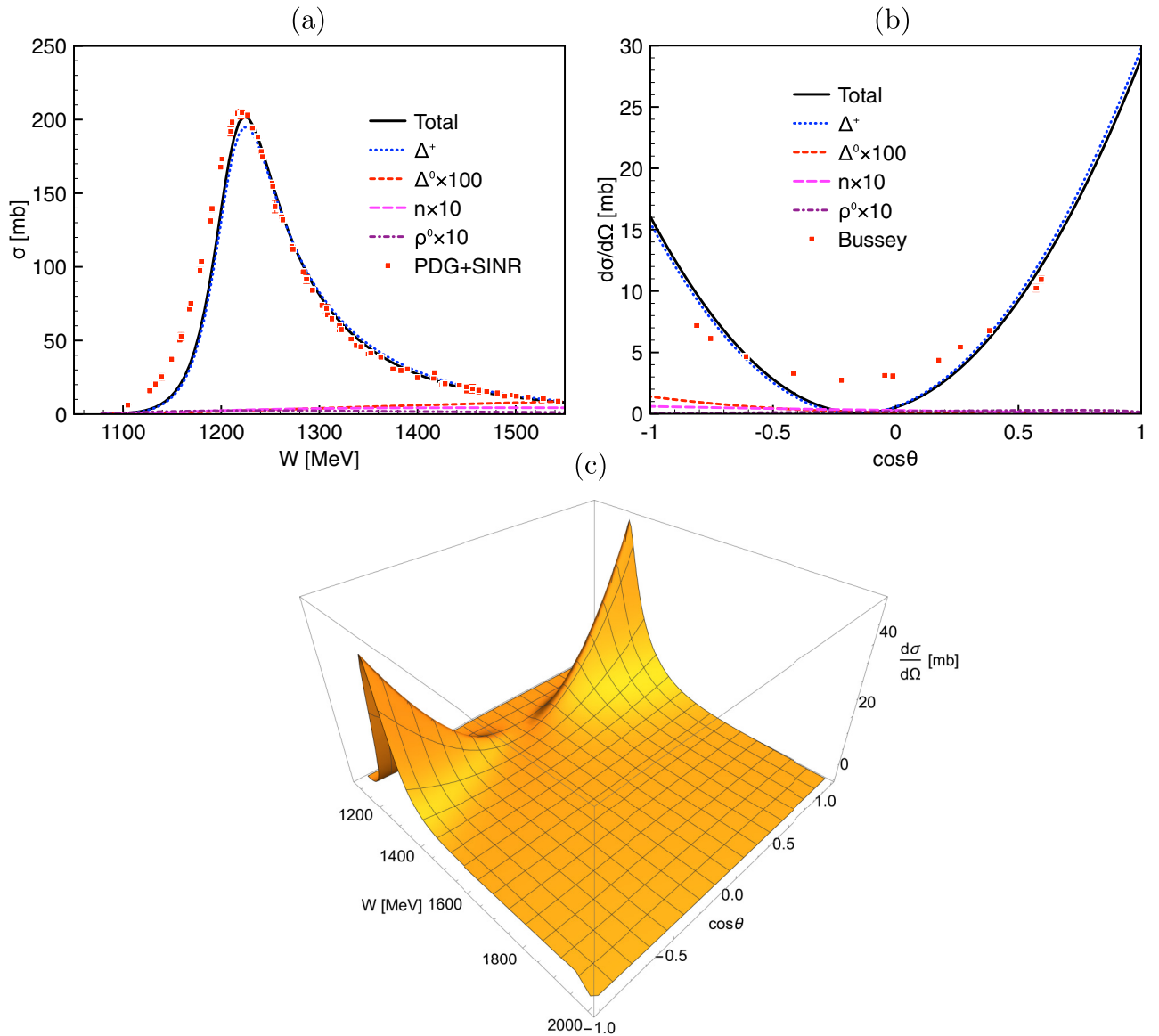


FIG. 2. (a) Total cross section for the π^+p elastic scattering process as a function of c.m. energy W (in MeV). Also shown are separate contributions and experimental data from PDG and SINR [17,18]. (b) Angular-dependent differential cross section as a function of the scattering angle θ of the outgoing π^+p in the c.m. frame at $W \approx 1.285$ GeV with the experimental data from Ref. [19]. (c) Differential cross section as a function of W and $\cos\theta$.

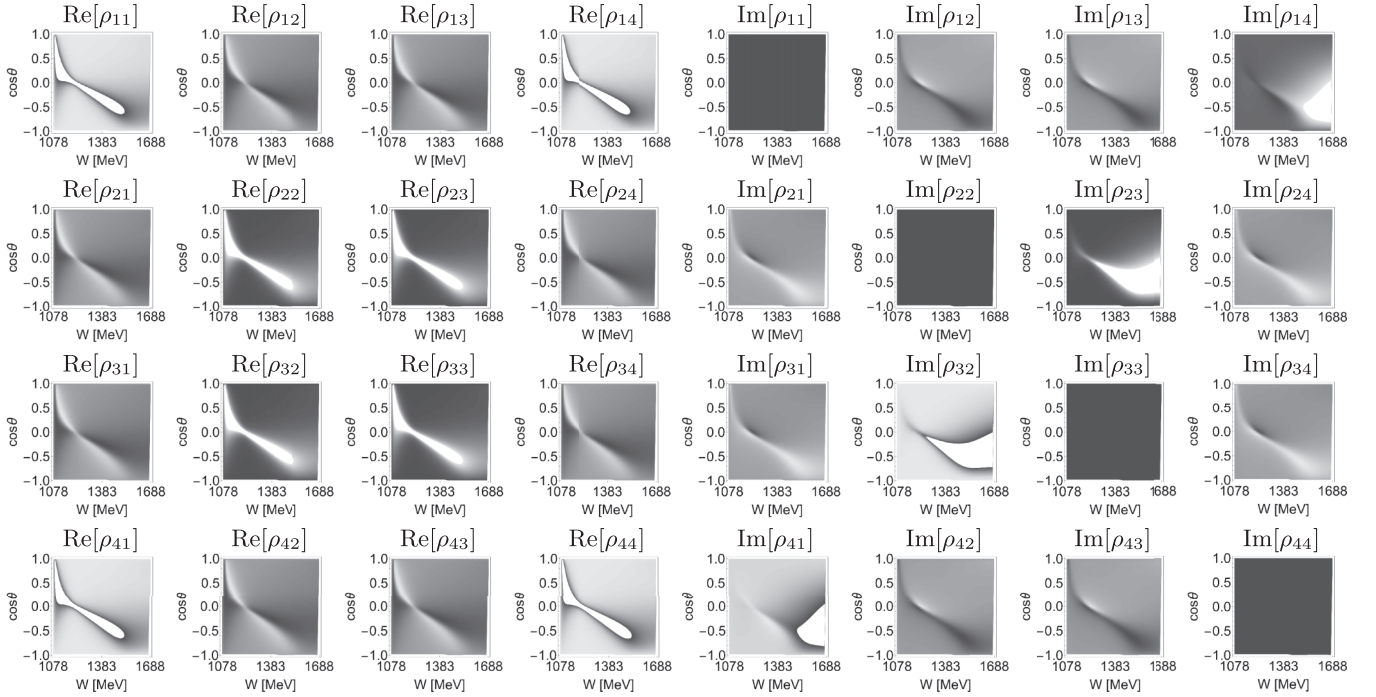


FIG. 3. Real and imaginary matrix elements of ρ_{SDM} in separate panels as functions of $W = (W_{\text{th}} - 2.0)$ GeV and $|\cos \theta| \leq 1$ for the horizontal and vertical axes, respectively. $W_{\text{th}} = M_{\pi^+} + M_p$ denotes the c.m. threshold energy.

backward asymmetry, consistent with dominant p -wave contributions from the Δ resonance in the s channel. The agreement with experimental data, particularly in the forward region, is encouraging and validates the underlying effective interaction vertices. The angular structure arises from interference among multiple spin-parity amplitudes and plays a key role in shaping the spin-entanglement structure discussed below.

In Fig. 2(c), a two-dimensional visualization of the differential cross section as a function of W and $\cos \theta$ is shown, offering a comprehensive picture of the interplay between energy and angle across the scattering phase space. The plot reveals regions of enhanced cross section corresponding to the Δ resonance and highlights the forward- and backward-peaking behavior. Outside the resonance region, the cross section becomes relatively smooth and flat. This information is crucial for identifying kinematic domains in which spin coherence or decoherence may arise due to underlying dynamical mechanisms.

B. Spin-density matrices and entanglement entropy

To explore the quantum spin structure, one constructs the 4×4 SDM ρ_{SDM} for the initial and final state protons. Figure 3 presents the real and imaginary parts of all matrix elements as functions of W (horizontal axis) and $\cos \theta$ (vertical axis). The matrix is normalized to have trace unity and encodes the probability distributions and coherence of spin transitions during scattering. The physical meaning of these elements is crucial: the diagonal elements $\text{Re}[\rho_{ii}]$ quantify the probability of finding the proton in a specific spin state, while the off-diagonal elements ($\text{Re}[\rho_{ij}]$ and $\text{Im}[\rho_{ij}]$ for

$i \neq j$) signify the presence and strength of quantum interference between different spin states, indicating the degree of coherence or entanglement. Note that the diagonal elements of $\text{Im}[\rho_{\text{SDM}}]$ are zero by construction, as probabilities are real. The interpretation of the results shows that variations in these elements across kinematic regions signal transitions between coherent and mixed spin configurations; for instance, a region with dominant diagonal elements and small off-diagonal elements suggests high spin coherence, often associated with a single dominant scattering channel, whereas a significant presence of nonzero off-diagonal elements implies a mix of spin states and active interference, reflecting the presence of a background, as in the present work, or the interplay of multiple scattering amplitudes. The von Neumann entropies $S_{\text{SDM}}^{i,f}$ for the initial and final spin subsystems are computed from the eigenvalues of the corresponding reduced density matrices, obtained via partial trace operations. It is found that the von Neumann entropies of the initial and final spin subsystems are numerically identical across the entire kinematic domain: $S_{\text{SDM}}^i = S_{\text{SDM}}^f = S_{\text{SDM}}$. This indicates that the reduced spin entropies of the initial and final proton subsystems are identical, reflecting the fact that the spin degrees of freedom share the same entanglement spectrum with the remaining degrees of freedom at each kinematic point. Physically, this indicates that the scattering acts as a unitary transformation on the spin space, redistributing quantum information without generating or destroying it. For a pure bipartite state, the reduced density matrices of the two subsystems have identical nonzero eigenvalues [20]. Thus, for elastic scattering with a pure initial state, i.e., $S_{\text{SDM}}^i = S_{\text{SDM}}^f$, consistent with unitary evolution in the spin Hilbert space.

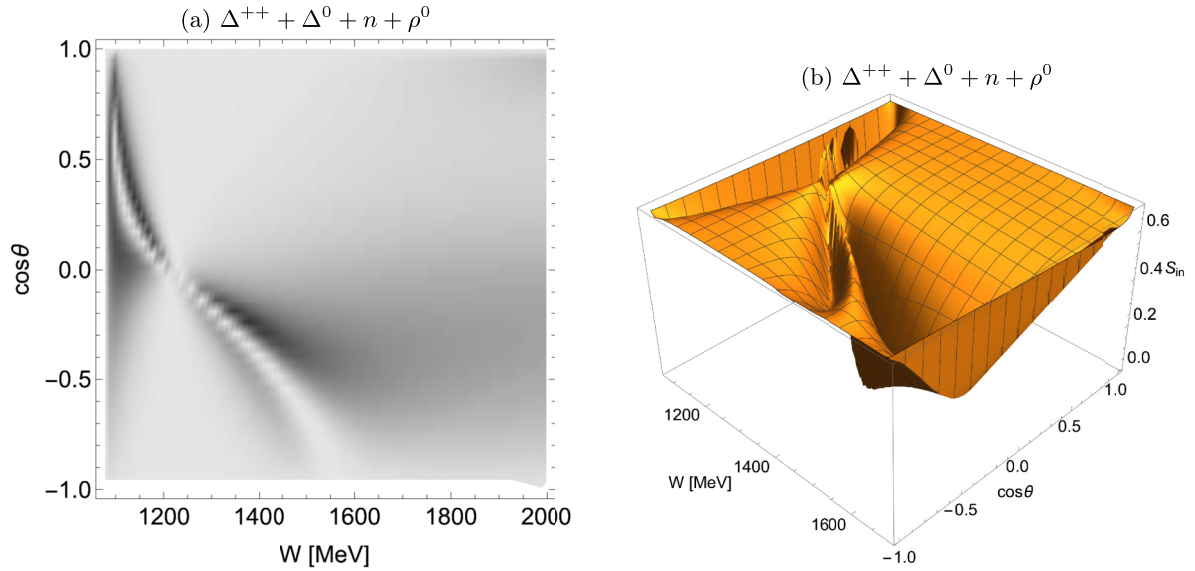


FIG. 4. von Neumann entropy for the SDM, i.e., S_{SDM} as a function of W (in MeV) and $\cos \theta$, including all the contributions, Δ^{++} , Δ^0 , n , and ρ^0 .

Figure 4 displays the entropy value as a function of W and $\cos \theta$, including contributions from all relevant diagrams. The entropy presents a nontrivial structure in the Δ^{++} -dominant region, indicating that complicated (dis)entanglement occurs from interference with different spin contributions (n and ρ^0). For instance, a small but finite forward scattering enhancement is generated by the ρ^0 contribution. In contrast, beyond the Δ^{++} region, the entropy increases and becomes relatively flat, where multiple amplitudes overlap and interfere strongly, i.e., stronger entanglements, especially those involving background channels such as n and ρ^0 . This clearly shows how spin entanglement responds to the dynamical structure of the scattering amplitude. As discussed in detail below, the nontrivial valleylike dip indicates a reduced entanglement between the proton spin and background spin configurations, leading to a partial restoration of spin coherence. To understand the specific roles of different channels in generating spin entanglement, one analyzes entropy distributions for selected amplitude combinations in Fig. 5. For instance, when only Δ^{++} and Δ^0 are included, the entropy remains almost flat across most kinematic regions, as depicted in Fig. 5(a), indicating strong entanglement between the same spin-3/2 states. It is verified that the smaller-entropy region (spot-like) at $W \lesssim 1.6$ GeV and $\cos \theta \lesssim 0$ is mainly produced by the Δ^{++} contribution. However, the addition of neutron or ρ^0 exchange channels significantly alters the entropy structures, as shown in Figs. 5(b) and 5(c), making the entropy much smaller around that spot. This observation indicates that the dominant Δ^{++} spin configuration becomes less entangled with background contributions such as n and ρ^0 . Other combinations also exhibit nontrivial and distinctive entropy structures that differ from those with the Δ^{++} contribution, as shown in Figs. 5(d), 5(e), and 5(f). Thus, the von Neumann entropy effectively captures the transition from relatively pure to mixed spin states as background contributions become significant. Therefore, combinations involving particles with

different spins tend to reduce the entropy, leading to disentanglement and energy- and angular-dependent dynamics. In contrast, combinations of particles with the same spin increase the entropy due to stronger entanglement. Depending on the interference pattern, background amplitudes can either enhance mixed-spin entanglement or locally restore spin coherence, leading to characteristic dips in entropy.

Hence, the numerical analysis demonstrates that the von Neumann entropy derived from SDMs provides a novel and insightful observable for hadronic scattering processes, unlike traditional observables, such as cross sections. The entropy probes internal spin correlations and coherence effects that would otherwise remain hidden.

V. SUMMARY AND FUTURE PERSPECTIVES

In this work, the π^+p elastic scattering process is investigated, using an effective Lagrangian approach that incorporates s -, t -, and u -channel contributions involving Δ^{++} , Δ^0 , neutron, and ρ^0 . Tree-level scattering amplitudes were constructed and used to compute both total and differential cross-sections. The numerical results show qualitatively good agreement with existing experimental data, particularly around the Δ^{++} resonance peak near $W \approx 1230$ MeV. Beyond conventional observables, the von Neumann entropy was introduced as a quantum information measure to probe the spin dynamics of the scattering process. A 4×4 SDM was built from the scattering amplitudes, based on canonical spin quantization in the c.m. frame, and partial traces were performed to extract 2×2 reduced density matrices for the initial and final proton spin states. The corresponding von Neumann entropies quantify the degree of spin entanglement with other kinematic and internal variables.

The entropies exhibit a clear dependence on the c.m. energy W and scattering angle θ . In regions dominated by the Δ^{++} resonance, entropy is relatively low, leading to

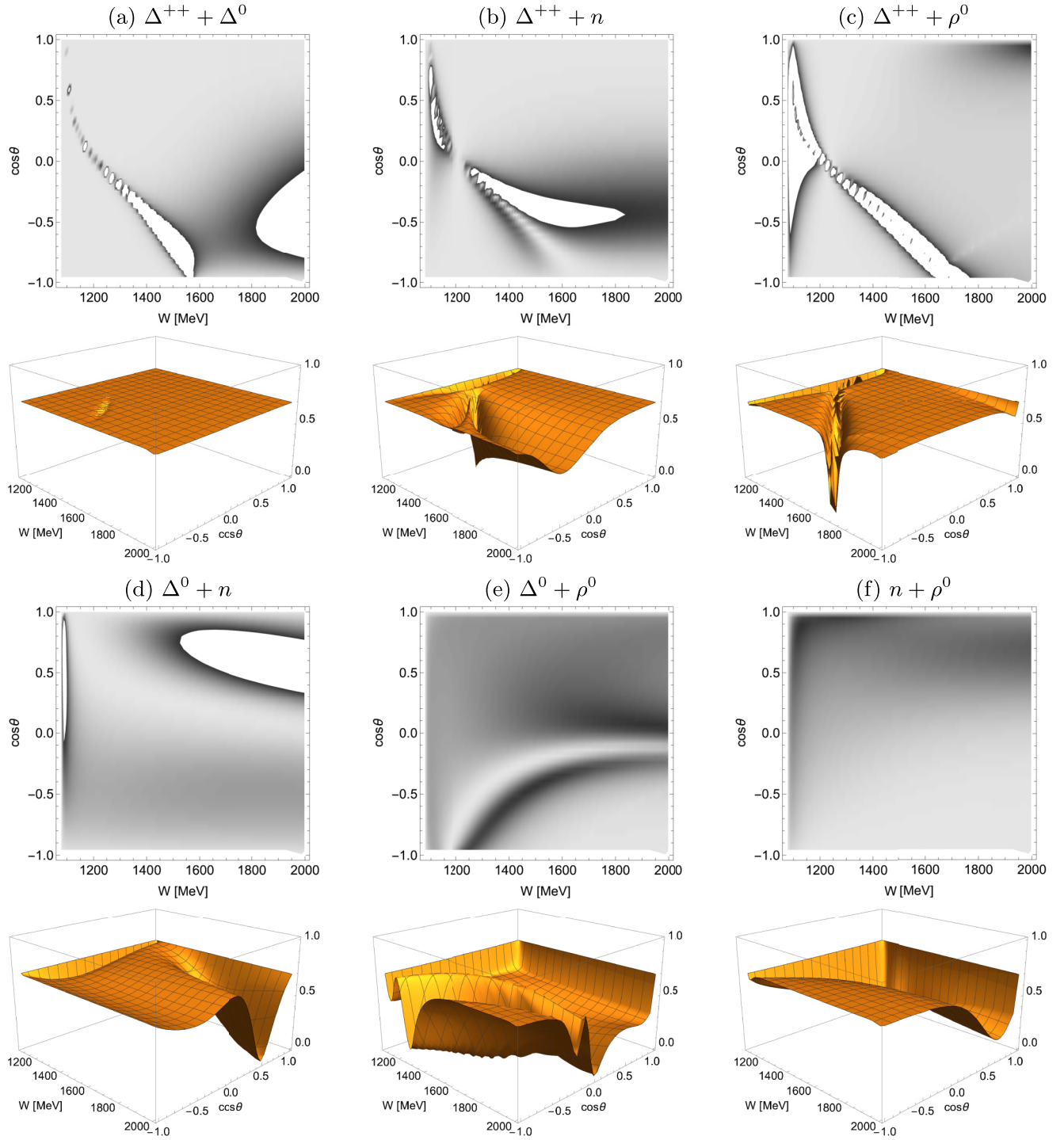


FIG. 5. Similar to Fig. 4; S_{SDM} as a function of W (in MeV) and $\cos \theta$ for the (a) $\Delta^{++} + \Delta^0$, (b) $\Delta^{++} + n$, (c) $\Delta^{++} + \rho^0$, (d) $\Delta^0 + n$, (e) $\Delta^0 + \rho^0$, and (f) $n + \rho^0$ contributions.

complex structures. In contrast, higher entropy values are observed when multiple channels interfere, particularly involving background contributions, such as neutron and ρ^0 exchange, indicating mixed spin states and enhanced quantum entanglement among different spin states. To further explore channel-specific effects, the total amplitude is decomposed into diagram subsets, and the entropy is computed for each case. It was observed that combinations involving

only Δ resonances maintain almost flat entropy, whereas the inclusion of non-resonant backgrounds, such as n and ρ^0 , significantly alters the entropy structure, decreasing it. This again confirms the role of background amplitudes in generating spin decoherence, as evidenced by the reduced entanglement between the proton spin and background degrees of freedom. Overall, the results suggest that the von Neumann entropy extracted from SDMs serves as a novel and

insightful observable in hadronic scattering. It complements traditional cross-section analysis by providing direct access to the quantum coherence and entanglement properties of spin subsystems.

As observed in the valleylike dip structures, the von Neumann entropy of the spin system can serve as a quantum diagnostic tool for searching for resonances and exotic states in hadronic scattering. For instance, resonance formations dynamically favor specific spin states, thereby decreasing entropy and signaling a reduction in entanglement between the spin and other degrees of freedom, such as background contributions. Such entropy dips, as functions of W and θ , may reveal hidden resonant and exotic structures even when conventional observables are ambiguous. While challenging, experimental validation of the entropy structure is conceivable through future polarization-sensitive measurements. Observable quantities derived from the SDM, such as spin correlation coefficients, polarization transfer, or entanglement witnesses, could connect theory with data and make the framework operationally testable. Altogether, these enhancements would not only improve the quantitative precision of the model but also deepen its relevance to nonperturbative QCD and the emergent quantum features of hadronic systems.

Although SAID-based partial-wave analyses provide data-driven amplitudes [21,22], the present work focuses on a model-based construction of SDMs, thereby enabling direct control over specific diagrammatic contributions and coherence structures. Nonetheless, it is intended to complement the present formal development study with a numerical analysis using SAID partial-wave amplitudes. This will provide an explicit mapping between helicity amplitudes extracted from data and the canonical-basis amplitudes used here, serving as an external consistency check of the Wigner-rotation treatment and the two-independent-amplitude structure implied by parity. Such a comparison will be reported in a dedicated followup work. Related works will appear elsewhere.

ACKNOWLEDGMENTS

The author is grateful for the interesting discussions on von Neumann entropy and quantum entanglement with Tet-suo Hyodo (RCNP/Osaka University). This work is supported by grants from the National Research Foundation of Korea (NRF), funded by the Korean government (MSIT) (Grant No. RS-2025-16065906). The partial support from the Institute for Radiation Science and Technology of PKNU (IRST/PKNU) is also acknowledged.

DATA AVAILABILITY

The data that support the findings of this article are openly available [17].

APPENDIX

In this appendix, I provide technical details regarding the spin conventions, the relation between canonical and helicity spinors via the Wigner rotation, the implementation of parity constraints, and the construction of reduced SDMs used in the text.

1. Canonical and helicity spinors

Canonical (or instant-form) proton spinors are defined by boosting rest-frame spin eigenstates to the desired momentum:

$$u_s(k) = S(\Lambda_k) u_s(0), \quad s = \uparrow, \downarrow, \quad (\text{A1})$$

where $S(\Lambda_k)$ is the Dirac representation of the standard Lorentz boost Λ_k . Helicity spinors are instead defined as eigenstates of $\Sigma \cdot \hat{\mathbf{k}}$:

$$u_\lambda(k) = D^{1/2}[R(\hat{\mathbf{k}})] u_\lambda(0), \quad \lambda = \pm \frac{1}{2}, \quad (\text{A2})$$

where $R(\hat{\mathbf{k}})$ is the rotation that aligns the z axis with $\hat{\mathbf{k}} = \mathbf{k}/|\mathbf{k}|$.

The two spin bases are related by a momentum-dependent Wigner rotation R_W :

$$u_s(k) = \sum_{\lambda} D_{s\lambda}^{1/2}[R_W(k)] u_\lambda(k), \quad (\text{A3})$$

where

$$R_W(k) = L^{-1}(k) R(\hat{\mathbf{k}}) L(k), \quad (\text{A4})$$

where $L(k)$ is the rotation-free boost to momentum k . In the c.m. frame, for scattering in the xz plane, the Wigner matrix takes the explicit SU(2) form

$$D^{1/2}(R_W) = \begin{pmatrix} \cos \frac{\omega}{2} & -\sin \frac{\omega}{2} e^{-i\phi} \\ \sin \frac{\omega}{2} e^{i\phi} & \cos \frac{\omega}{2} \end{pmatrix}, \quad (\text{A5})$$

where ω is the Wigner angle, and ϕ is the azimuthal scattering angle. For collinear configurations ($\omega = 0$), canonical and helicity bases coincide.

2. Parity constraints

In the helicity basis, parity invariance imposes the well-known relation

$$M_{\lambda_i \lambda_f}(\theta) = \eta_P (-1)^{\lambda_i - \lambda_f} M_{-\lambda_i, -\lambda_f}(\theta), \quad (\text{A6})$$

reducing the number of independent πN amplitudes to 2. In the canonical basis, the scattering matrix elements read

$$M_{s_i s_f} = \sum_{\lambda_i, \lambda_f} D_{s_i \lambda_i}^{1/2}[R_W] M_{\lambda_i \lambda_f} D_{s_f \lambda_f}^{1/2*}[R_W], \quad (\text{A7})$$

which generally do *not* satisfy $M_{\uparrow\uparrow} = M_{\downarrow\downarrow}$ or $M_{\uparrow\downarrow} = -M_{\downarrow\uparrow}$ elementwise due to $R_W(k) \neq 1$ for noncollinear boosts.

Nevertheless, inserting Eq. (A6) into Eq. (A7) shows that the rank of the canonical-basis amplitude matrix remains

$$\text{rank}(\mathcal{M}) = 2, \quad (\text{A8})$$

ensuring that only two physical invariant amplitudes exist, as required by parity symmetry. Thus, the canonical and helicity descriptions are fully equivalent at the level of physical degrees of freedom.

3. Reduced spin-density matrices and entropies

The 4×4 SDM for the proton spin system is

$$\rho_{s_i s_f; s'_i s'_f} = \frac{1}{N} M_{s_i s_f} M_{s'_i s'_f}^*, \quad N = \sum_{s_i, s_f} |M_{s_i s_f}|^2, \quad (\text{A9})$$

ensuring $\text{Tr}[\rho_{\text{SDM}}] = 1$. Reduced density matrices for initial and final spins follow from partial traces:

$$(\rho_i)_{s_i s'_i} = \sum_{s_f} \rho_{s_i s_f; s'_i s_f}, \quad (\rho_f)_{s_f s'_f} = \sum_{s_i} \rho_{s_i s_f; s'_i s_f}. \quad (\text{A10})$$

Their eigenvalues and the von Neumann entropy are

$$\lambda_{\pm} = \frac{1}{2}(1 \pm \sqrt{1 - 4 \det \rho_{i,f}}), \quad S = - \sum_{\alpha=\pm} \lambda_{\alpha} \ln \lambda_{\alpha}. \quad (\text{A11})$$

For completeness, let $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$. Then, one has

$$\begin{aligned} |\Psi\rangle &= \sum_{i,j} c_{ij} |i\rangle_A |j\rangle_B, \quad \rho = |\Psi\rangle\langle\Psi| \\ &= \sum_{i,j,k,l} c_{ij} c_{kl}^* (|i\rangle_A \langle k|_A) \otimes (|j\rangle_B \langle l|_B). \end{aligned} \quad (\text{A12})$$

Tracing out subsystem B yields

$$\rho^A = \text{Tr}_B(\rho) = \sum_m \langle m|_B \rho |m\rangle_B = \sum_{i,k} \left(\sum_m c_{im} c_{km}^* \right) |i\rangle_A \langle k|_A,$$

which is Eq. (A10) in the present context.

In the helicity basis, parity invariance implies that all spin information is encoded in two independent helicity amplitudes, say H_1 and H_2 . Using the canonical transform,

$$M_{s_i s_f} = \sum_{\lambda_i, \lambda_f} D_{s_i \lambda_i}^{1/2}(R_W) M_{\lambda_i \lambda_f} D_{s_f \lambda_f}^{1/2*}(R_W), \quad (\text{A13})$$

and the helicity-parity constraint

$$M_{\lambda_i \lambda_f} = \eta_P (-1)^{\lambda_i - \lambda_f} M_{-\lambda_i, -\lambda_f}, \quad (\text{A14})$$

one can always express the canonical matrix as a linear combination

$$M = H_1 \mathcal{U}_1 + H_2 \mathcal{U}_2, \quad (\text{A15})$$

where $\mathcal{U}_{1,2}$ are 2×2 matrices built from Wigner rotation elements. Hence $\text{rank}(M) \leq 2$. Since nontrivial πN scattering contains at least two independent amplitudes, one obtains

$$\text{rank}(M) = 2,$$

demonstrating that parity restricts the canonical amplitude to two independent components, consistent with the helicity formalism.

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