

Fermi-liquid view of viscosity in cold and dense nucleon matter

Jianing Li ^{1,2,3,*}, Weiyao Ke ^{3,†}, and Jin Hu ^{4,‡}

¹*Physikalisches Institut, Universität Heidelberg, 69120 Heidelberg, Germany*

²*GSI Helmholtzzentrum für Schwerionenforschung, 64291 Darmstadt, Germany*

³*Key Laboratory of Quark and Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China*

⁴*Department of Physics, Fuzhou University, Fuzhou 350116, China*



(Received 16 December 2025; accepted 18 March 2026; published 4 June 2026)

We develop a framework to calculate transport properties in cold, dense relativistic quasiparticle system within the Fermi-liquid theory at the mean-field level. Building on our previous study J. Li *et al.* [*Phys. Rev. C* **111**, 044904 (2025)], we start from the linearized relativistic Boltzmann equation tailored to quasiparticles with medium-dependent dispersion relation and implement Landau matching conditions, proving that the bulk viscosity is manifestly nonnegative. A low-temperature expansion then yields leading-order (T/μ^*) expressions for the shear (η) and bulk (ζ) viscosities, where the behavior $\zeta/\eta \propto (T/\mu^*)^4$ in the degenerate regime is found to be robust against quasiparticle mass correction. We couple the kinetic framework to a Walecka-type mean-field equation of state and compute η and ζ for cold, dense nucleon matter. The transport properties of nucleonic matter in the degenerate regime can be relevant for intermediate beam-energy nuclear experiments.

DOI: [10.1103/77kj-8v36](https://doi.org/10.1103/77kj-8v36)

I. INTRODUCTION

The transport phenomena is of vital importance in the study of physical systems out of equilibrium and often shows universal behaviors across orders of magnitude—from the microscopic relaxation in nuclear collisions to the dissipation processes in macroscopic astrophysical systems [1–3]. Since the discovery of the color-deconfined matter of quantum chromodynamics (QCD) in ultrarelativistic heavy-ion collisions [4,5], the precise characterization of its transport properties has always been a central focus [6]. The understanding of the nonequilibrium process plays indispensable roles in identifying the partonic degrees of freedom, extracting shear and bulk viscosity, and constraining the QCD equation of state of the hot QCD matter from data [7]. Many developments of the nonequilibrium theory to study the hot QCD rely on the predominant temperature as the solely relevant energy scale that determines various microscopic timescales [8–11]. In contrast, transport inside cold and dense nucleonic matter is governed by quasiparticle excitations near a sharp Fermi surface, set by the Fermi energy (or Fermi momentum), and dissipation involving hadronic interactions among the these

particles [12]. A controlled description of nucleonic transport is now especially timely.

The compression and excitation of nuclear matter at low temperature relative to the baryon chemical potential $T \ll \mu_B$ can be achieved in intermediate-energy heavy-ion facilities [13–16], creating a unique environment to study its transport properties and to pursue hydrodynamic and other collective behavior in this regime. Furthermore, the search for collective effects has expanded to include ultraperipheral collisions, which also requires a precise understanding of transport at high baryon density [17–19]. Upcoming facilities like the Electron-Ion Collider (EIC) in the United States [20] and the proposed Electron-Ion Collider in China (EicC) [21] promise to probe this cold, dense matter in another way, potentially allowing us to observe the propagation of shocks through it. However, interpreting the resulting signals, which are sensitive to density gradients, compressional flows, and entropy production, presents a significant challenge. Similar mechanisms are probed in other low-energy nuclear experiments where the system remains far from deconfinement yet exhibits nonequilibrium dynamics on hadronic scales [22–24]. These considerations underscore the need for a dedicated framework for nucleonic transport at low temperature. It requires transport coefficients, specifically the shear and bulk viscosity (η and ζ), that are not merely phenomenological parameters but are fundamentally consistent with the symmetries, thermodynamics, and microscopic interactions of nuclear matter.

Such cold/warm and dense nuclear matter is also realized in neutron stars and neutron star merger events, where its transport properties play an important role in stellar cooling [25–28]. In that context, however, dissipation is dominated by slow electroweak processes [29,30]. This work aims to

*Contact author: ljianing@physi.uni-heidelberg.de

†Contact author: weiyaoke@ccnu.edu.cn

‡Contact author: hu-j23@fzu.edu.cn

establish the essential link between transport related to nuclear experimental observables and the intrinsic properties of cold, dense nuclear matter. In particular, we focus on strong-interaction processes acting on short timescales [31], where many-body scattering near the Fermi surface governs the attenuation of collective modes [32,33].

In our previous work [33], we analyzed cold, dense nucleon matter within a mean-field Fermi-liquid framework and computed transport coefficients in the Walecka model [34]. That study revealed a gap in the understanding: In a generic quasiparticle picture, the sign of ζ is not automatically guaranteed positive without further constraints. This is because, in our previous analysis, an appropriate matching condition for such a quasiparticle fermionic system was not yet fully established. In the present work, we remedy this limitation by establishing consistency between transport theory and hydrodynamics. Specifically, we develop a general, self-consistent scheme for dense and cold quasiparticle systems in which ζ is rigorously non-negative under transparent assumptions tied to thermodynamic consistency and microscopic reversibility. Within this framework, the symmetry constraints of the linearized collision kernel, and their restoration within the relaxation-time approximation (RTA) treatment, are encoded in a set of Landau matching conditions. A direct implication is that in cold Fermi systems, ζ is strongly suppressed at low temperature, rendering bulk dissipation parametrically smaller than shear dissipation. We can conclude that the effect of mean-field dressed mass will not alter the general behavior of Fermi liquid that $\zeta/\eta \propto (T/\mu^*)^4$ at low temperature.

This paper is organized as follows. In Sec. II A, we formulate the linearized relativistic Boltzmann equation for a Fermi liquid of quasiparticles and derive the expressions for η and ζ within the RTA. In Sec. II B, we implement Landau matching conditions adapted to quasiparticles and prove a general positivity result for ζ , while Sec. II C develops the low-temperature expansion scheme. In Sec. III, we apply this framework to a Walecka-type mean-field equation of state and present numerical results for cold, dense nucleon matter. Section IV summarizes the main conclusions and discusses implications for phenomenology and future directions.

II. TRANSPORT IN FERMIL-LIQUID

A. Linearized relativistic Boltzmann equation framework

To investigate the transport properties of a quasifermion system, we begin with the linearized relativistic Boltzmann equation,

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}}{E_p^*} \cdot \nabla_{\mathbf{x}} - \nabla_{\mathbf{x}} E_p^* \cdot \nabla_{\mathbf{p}} \right) f_{\text{eq}}(x, p^*) = \mathcal{C}[\delta f]. \quad (1)$$

Here $f_{\text{eq}}(x, p^*)$ denotes the local equilibrium distribution function, and $\mathcal{C}[\delta f]$ is the corresponding linearized collision kernel, obtained by expanding the full nonlinear collision kernel $\mathcal{C}_{\text{full}}[f]$ to first order in the deviation δf around f_{eq} . The nonequilibrium correction $\delta f(x, p^*)$ is defined by

$$f(x, p^*) = f_{\text{eq}}(x, p^*) + \delta f(x, p^*). \quad (2)$$

The equilibrium distribution takes the Fermi-Dirac form,¹

$$f_{\text{eq}}(x, p^*) = \frac{1}{e^{(u \cdot p^* - \mu^*)/T} + 1}, \quad (3)$$

where T denotes the temperature, u^μ the fluid four-velocity, μ^* the effective chemical potential, i.e., the energy required to add one more particle to the local system. Because of repulsive part of the mean-field interactions, μ^* is different from μ , with the latter being the thermodynamic conjugate to the particle number density. The mean field interaction also modifies the quasiparticle dispersion relation to be $E_p^* = \sqrt{m^{*2} + \mathbf{p}^2}$ with effective mass m^* and momentum $\mathbf{p}$, and the corresponding four-momentum is $p^{*\mu} = (E_p^*, \mathbf{p})$. Both m^* and μ^* are functions of μ .

The nonequilibrium correction associated with a deviation of μ can be assumed as

$$\delta f = \tilde{f}_{\text{eq}} \phi, \quad (4)$$

with

$$\tilde{f}_{\text{eq}}(x, p^*) \equiv \frac{\partial f_{\text{eq}}(x, p^*)}{\partial \mu^*}. \quad (5)$$

The function ϕ can be decomposed to pieces containing different tensor structure of the flow gradients

$$\phi = \phi_1 p^{*\mu} p^{*\nu} \sigma_{\mu\nu} - \phi_2 \vartheta, \quad (6)$$

with

$$\begin{aligned} \sigma_{\mu\nu} &= D_\mu u_\nu + D_\nu u_\mu - \frac{2}{3} \Delta_{\mu\nu} \vartheta, \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu, \\ D_\mu &= \Delta_{\mu\nu} \partial^\nu, \quad \vartheta = \partial \cdot u, \end{aligned} \quad (7)$$

where we have used the mostly plus metric $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$.

The energy-momentum tensor of the system can be expanded near thermal equilibrium

$$T^{\mu\nu} = -p g^{\mu\nu} + (p + \varepsilon) u^\mu u^\nu + \delta T_{\text{diss}}^{\mu\nu}, \quad (8)$$

where p and ε denote pressure and energy density, respectively. To the first order in gradients, the dissipative correction is

$$\delta T_{\text{diss}}^{\mu\nu} = \eta \sigma^{\mu\nu} - \zeta \Delta^{\mu\nu} \vartheta, \quad (9)$$

with η and ζ representing shear and bulk viscosities. In kinetic theory, the same tensor can be expressed with the distribution function of the quasiparticles

$$\begin{aligned} T^{\mu\nu} &= \int_p \frac{p^{*\mu} p^{*\nu}}{E_p^*} f(x, p^*) \\ &= \int_p \frac{p^{*\mu} p^{*\nu}}{E_p^*} f_{\text{eq}}(x, p^*) + \int_p \frac{p^{*\mu} p^{*\nu}}{E_p^*} \delta f(x, p^*), \end{aligned} \quad (10)$$

with $\int_p = \int d^3\mathbf{p}/(2\pi)^3$. The first term gives the equilibrium part of the energy-momentum tensor, then the dissipative part

¹Throughout this work, we consider a dense fermionic system in which particle contributions strongly dominate over antiparticles; antiparticle effects are therefore neglected.

follows:

$$\delta T_{\text{diss}}^{\mu\nu} = \int_p \frac{p^{*\mu} p^{*\nu}}{E_p^*} \delta f(x, p^*). \quad (11)$$

Combining Eq. (11) with Eq. (6) in local rest frame, one obtains the expression for bulk viscosity in kinetic theory:

$$\eta = \frac{2}{15} \int_p \frac{p^4}{E_p^*} \phi_1 \tilde{f}_{\text{eq}}, \quad \zeta = \frac{1}{3} \int_p \frac{p^2}{E_p^*} \phi_2 \tilde{f}_{\text{eq}}. \quad (12)$$

B. Quasiparticle matching condition for the relaxation-time approximation

In principle, one can obtain the viscosities η and ζ by inverting Eq. (1) to solve for δf , thus determining the perturbations ϕ_1 and ϕ_2 . However, the linearized version of any valid collision operator should possess the following properties:

$$\mathcal{C}[a + b^\mu p_\mu^*] = 0, \quad (13)$$

for arbitrary constant number a and vector b^μ . They are consequences of particle-number and energy-momentum conservation of the microscopic two-body collision processes [35]. Consequently, 1 and $p^{*\mu}$ constitute zero modes of the collision operator. Therefore, we can only invert Eq. (1) up to an arbitrary linear combination of the zero modes. This means that the same out-of-equilibrium $T^{\mu\nu}$ can be decomposed in multiple ways into equilibrium and dissipative contributions, i.e., solutions of $\{\phi_1, \phi_2\}$ to Eq. (1) are not necessarily unique unless a convention is chosen to fix the state. The Landau matching conditions provide precisely such a prescription. They require that deviations of the local thermodynamic quantities do not alter the local energy-momentum density or conserved charge density n^μ , i.e.,

$$u_\nu u_\mu \delta T^{\mu\nu} = 0, \quad u_\mu \delta n^\mu = 0. \quad (14)$$

Correspondingly, they also define the flow velocity u^μ via the energy current $u_\nu T^{\mu\nu} = \varepsilon u^\mu$ [36].

For cold and dense fermionic matter, we assume that δf arises solely from spacetime variations of μ , while fluctuations of T are neglected. Within the RTA, the collision term is simplified as $\mathcal{C}[\delta f] = -\delta f/\tau_{\text{rel}}$ with relaxation time τ_{rel} .² The corresponding particular solutions for the coefficients are [33]³

$$\phi_1(p^*) = \frac{\tau_{\text{rel}}}{2E_p^*}, \quad \phi_2^{\text{par}}(p^*) = \frac{\alpha}{E_p^*} [(u \cdot p^*)^2 \gamma - m^{*2}]. \quad (15)$$

²The RTA does not by itself preserve the zero modes of the linearized collision operator, i.e., breaking Eq. (13). In practice, this violation is removed by imposing the matching conditions. Equivalently, one may view this as adding counterterms that ensure the collision operator annihilates the zero modes. The final results are therefore fully consistent with the symmetries of the microscopic theory. The implementation will be shown later.

³At low temperature, the entropy contribution is negligible compared with the density effect and is omitted during the formulation. Further details can be found in Ref. [33].

Here v_s denotes the adiabatic speed of sound, and for later use we define

$$\alpha = \frac{\tau_{\text{rel}}}{3}, \quad \gamma = 1 - 3v_s^2 \left(1 - \frac{m^*}{\mu^*} \kappa^*\right), \quad \kappa^* = \frac{dm^*}{d\mu^*}. \quad (16)$$

Note that ϕ_1 is the unique solution, whereas ϕ_2^{par} denotes a particular solution of ϕ_2 . In other words, the nonuniqueness induced by the zero modes is associated solely with ζ . This is because $\sigma_{\mu\nu}$ is traceless and thus orthogonal to the conserved scalar quantities. In the local rest frame, owing to $\sigma_{0\nu} = 0$ and $\delta^{ij}\sigma_{ij} = 0$, the contribution proportional to $p^i p^j \sigma_{ij}$ in the isotropic system vanishes in the LL conditions. Thus, the shear term associated with ϕ_1 always gives zero in the LL conditions, and the arbitrariness arises only in bulk one.

To obtain the full solution, we supplement the particular solution by deducting the zero-mode components $\{1, p^{*\mu}\}$ in the local rest frame [35],

$$\phi_2 = \phi_2^{\text{par}} - a - bE_p^*. \quad (17)$$

The coefficients a and b are determined uniquely by the Landau matching conditions in Eq. (14). The deviation of the total energy density becomes

$$\delta T^{00} = \int_p E_p^* \delta f. \quad (18)$$

Similarly, the deviation of the total single-component conserved charge number density in the local rest frame is

$$\delta n^0 = \int_p \delta f. \quad (19)$$

For convenience, we define an inner product weighted by $\tilde{f}_{\text{eq}}$ as

$$\langle X, Y \rangle \equiv \int_p X(p) Y(p) \tilde{f}_{\text{eq}}. \quad (20)$$

Then, the first Landau matching condition, enforcing energy density conservation in the local rest frame, reads

$$\langle E_p^*, \phi_2 \rangle = 0. \quad (21)$$

The second matching condition, enforcing particle number conservation, gives

$$\langle 1, \phi_2 \rangle = 0, \quad (22)$$

Using Eq. (12), the bulk viscosity can be expressed as

$$\zeta = \frac{1}{3} \langle \mathcal{R}_p, \phi_2 \rangle, \quad \mathcal{R}_p \equiv \frac{p^2}{E_p^*} = \frac{E_p^{*2} - m^{*2}}{E_p^*}. \quad (23)$$

In general, ϕ_2 admits the expansion $\phi_2 = \alpha \mathcal{R}_p + \lambda_0 + \lambda_1 E_p^*$. The Landau matching conditions imply that the coefficient vector $\lambda = (\lambda_0, \lambda_1)^T$ must satisfy

$$H\lambda = -\alpha \mathcal{R}, \quad (24)$$

with $H_{ij} \equiv \langle \psi_i, \psi_j \rangle$ and $\mathcal{R}_i = \langle \psi_i, \mathcal{R}_p \rangle$ for $\psi_0 = 1$ and $\psi_1 = E_p^*$. Since H is a Gram matrix of linearly independent variables, it is invertible [37], and then Eq. (24) always admits a solution for λ . Combining this with the Landau matching

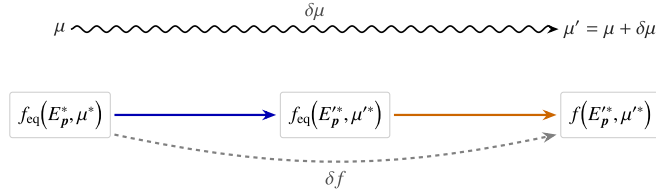


FIG. 1. Separation of equilibrium relabeling induced by $\delta\mu$ from the nonequilibrium correction δf obtained from the kinetic equation, where $E_p'^* = \sqrt{m^*(\mu + \delta\mu) + \mathbf{p}^2}$ and $\mu'^* = \mu^*(\mu + \delta\mu)$.

conditions in Eqs. (21) and (22), the bulk viscosity becomes

$$\zeta = \frac{1}{3\alpha} \langle (\phi_2 - \lambda_0 - \lambda_1 E_p^*), \phi_2 \rangle = \frac{1}{3\alpha} \int_p \tilde{f}_{\text{eq}} \phi_2^2 \geq 0, \quad (25)$$

which is manifestly non-negative.

Finally, we note that the quasiparticle picture differs subtly from the particle picture. Since both m^* and μ^* depend on μ , an intermediate equilibrium state is established, as illustrated in Fig. 1. To maintain consistency with hydrodynamics, the Landau matching conditions still retain the same form as in the particle picture [38]. Some related discussions can also be found in Refs. [39,40].

C. Low temperature approximation

For later convenience, we define

$$\begin{aligned} \mathcal{S}_{-1} &\equiv \langle 1, 1/E^* \rangle, & \mathcal{S}_0 &\equiv \langle 1, 1 \rangle, \\ \mathcal{S}_1 &\equiv \langle 1, E^* \rangle, & \mathcal{S}_2 &\equiv \langle E^*, E^* \rangle. \end{aligned} \quad (26)$$

The Landau matching conditions then yield

$$\begin{aligned} a &= \frac{\alpha m^{*2}}{\Delta} (\mathcal{S}_2 \mathcal{S}_{-1} - \mathcal{S}_1 \mathcal{S}_0), \\ b &= \alpha \gamma + \frac{\alpha m^{*2}}{\Delta} (\mathcal{S}_0^2 - \mathcal{S}_1 \mathcal{S}_{-1}), \end{aligned} \quad (27)$$

with $\Delta = \mathcal{S}_1^2 - \mathcal{S}_2 \mathcal{S}_0$.

We apply the Sommerfeld expansion to the equilibrium distribution in the local rest frame at low temperature [41],⁴

$$\lim_{T \rightarrow 0^+} f_{\text{eq}} \approx \Theta(\mu^* - E^*) - \sum_{i=1}^{\infty} \kappa_i \frac{d^{2i-1}}{dE^{*2i-1}} \delta(E^* - \mu^*), \quad (28)$$

with prefactor $\kappa_i = 2(1 - 2^{1-2i})T^{2i}\zeta_{2i}$. Here, $\zeta_x \equiv \zeta(x)$ is the Riemann zeta function, $\Theta(x)$ the Heaviside function, and $\delta(x)$ the Dirac delta. Differentiating gives

$$\lim_{T \rightarrow 0^+} \tilde{f}_{\text{eq}} \approx \delta(\mu^* - E^*) + \sum_{i=1}^{\infty} \kappa_i \frac{d^{2i}}{dE^{*2i}} \delta(E^* - \mu^*), \quad (29)$$

in agreement with the canonical symmetry of the transport kernel in the Boltzmann equation at leading order (LO) [32].

For LO, the viscosities are⁵

$$\eta_{\text{LO}} = \frac{p_F^{*5}}{30\pi^2 \mu^*} \tau_{\text{rel}}, \quad \zeta_{\text{LO}} = \frac{8\pi^2 m^{*4} p_F^{*4}}{405 \mu^{*5}} \tau_{\text{rel}}, \quad (30)$$

so

$$\frac{\zeta_{\text{LO}}}{\eta_{\text{LO}}} \propto \left(\frac{T}{\mu^*} \right)^4. \quad (31)$$

Thus, apart from the singular point $m^* = \mu^*$, the ζ is parametrically negligible compared to η in relativistic cold, degenerate Fermi liquids, as in the nonrelativistic case [42].

Typically, τ_{rel} is proportional to T^{-2} [33,42–44]. Consequently, as $T \rightarrow 0^+$ the two viscosities behave in opposite ways: η diverges while ζ vanishes. Physically, η and ζ measure resistance to shape and volume deformations, respectively. In the regime near zero temperature, transport is dominated by states in an infinitesimal shell around the Fermi surface [32,33]. Owing to the conservations of energy and conserved charge number, an isotropic compression or expansion simply rescales the Fermi sphere, producing no energy dissipation. Since the state still remains on an equilibrium manifold, the process is reversible and generates no entropy, hence the bulk channel is inactive and $\zeta = 0$. On the other hand, for a scale-invariant fluid, ζ is strictly zero, as enforced by the form of the stress-energy tensor and the associated Ward identity. A finite ζ , therefore, signals the breaking of scale invariance in the underlying microscopic theory. In some cases, such a breaking is dynamically generated by fluctuations beyond a mean-field description in the nonrelativistic system [45]. These fluctuations are Pauli-blocked by conservation laws at $T = 0$. Shear flow, by contrast, distorts the Fermi sphere into a quadrupolar anisotropy. Relaxing this anisotropy requires momentum-randomizing collisions. However, at $T = 0$, elastic isotropic scattering is absent, so the relaxation time diverges. In hydrodynamics, the entropy production density produced in shear channel is $T \partial_\mu s^\mu \sim \eta \sigma^{\mu\nu} \sigma_{\mu\nu}$. With the mean free path scaling as $\ell \sim p_F^* \tau_{\text{rel}} / m^*$, set by the effective Fermi momentum p_F^* , we obtain $\ell \propto T^{-2}$. Hydrodynamic consistency then requires the allowed wave numbers to satisfy $k \ll \ell^{-1} \propto T^2$. Hence $\sigma^{\mu\nu} \sigma_{\mu\nu} \sim (k u^\mu)^2$ scales as T^4 , and the entropy production vanishes as $T \rightarrow 0^+$. This establishes that bulk dissipation disappears while shear dissipation becomes parametrically suppressed at the edge of hydrodynamic applicability.

III. DENSE NUCLEON MATTER AT LOW TEMPERATURE

In general, one of the key applications of Fermi-liquid theory is the description of static nucleon matter. In this work, we adopt the well-known Walecka model [34], in which the system consists of neutrons (n) and protons (p) interacting

⁴In the Sommerfeld expansion, the lower integration limit is extended from $-\mu^*/T$ to $-\infty$, so the result is asymptotic rather than exact.

⁵In the Sommerfeld expansion, because of the μ^*/T dependence in both the numerators and Δ through a and b , the truncation order in Eq. (29) does not directly match the order of the viscosity results. Higher terms must be retained.

through the exchange of scalar and vector mesons. The Lagrangian density is

$$\mathcal{L}_W = \mathcal{L}_N + \mathcal{L}_m + U_m. \quad (32)$$

The nucleonic part reads

$$\mathcal{L}_N = \sum_{N=n,p} \bar{\psi}_N (i\not{\partial} - m_N + g_\sigma \sigma - g_\omega \omega + \mu_B \gamma_0) \psi_N, \quad (33)$$

where ψ_N denotes the nucleon fields. For simplicity, we assume the isospin symmetry, so neutrons and protons share the same mass $m_N = 939$ MeV and baryon chemical potential μ_B .⁶ The scalar meson field σ , with mass $m_\sigma = 550$ MeV and coupling $g_\sigma = 8.69$, generates the long-range attractive interaction, while the vector meson field ω^μ , with mass $m_\omega = 783$ MeV and coupling $g_\omega = 8.65$, provides the short-range repulsion. The mesonic dynamics are governed by

$$\mathcal{L}_m = \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_\omega^2 \omega^\mu \omega_\mu. \quad (34)$$

In addition, an effective self-interaction potential

$$U_m = \frac{1}{3}\kappa_3 m_N (g_\sigma \sigma)^3 + \frac{1}{4}\kappa_4 (g_\sigma \sigma)^4, \quad (35)$$

is introduced to reproduce the empirical saturation properties of nuclear matter with $\kappa_3 = 7.95 \times 10^{-3}$ and $\kappa_4 = 6.95 \times 10^{-4}$ [46].

We decompose the meson fields into condensate and fluctuations, $\sigma = \bar{\sigma} + \delta\sigma$ and $\omega^\mu = \bar{\omega}_0 + \delta\omega^\mu$, and neglect the fluctuations $\delta\sigma$ and $\delta\omega^\mu$ in the mean-field approximation (MFA). Using the standard imaginary-time path-integral formalism [46], the effective thermodynamic potential at finite temperature is obtained as [46,47]

$$\bar{\mathcal{V}}_W(T, \mu_B) = -2p_0(T, \mu_B^*, m_N^*) + \frac{1}{2}m_\sigma^2 \bar{\sigma}^2 - \frac{1}{2}m_\omega^2 \bar{\omega}_0^2 + \bar{U}_m, \quad (36)$$

where the quasiparticle nucleon mass m_N^* and effective baryon chemical potential μ_B^* are shifted by the condensates according to

$$\begin{aligned} m_N^*[\bar{\sigma}] &= m_N - g_\sigma \bar{\sigma}(T, \mu_B), \\ \mu_B^*[\bar{\omega}_0] &= \mu_B - g_\omega \bar{\omega}_0(T, \mu_B), \end{aligned} \quad (37)$$

and $\bar{U}_m$ denotes the potential term in MFA.

The pressure of a free fermion species with spin s is

$$p_0(T, \mu, m) = \frac{2s+1}{3\pi^2} \int_p \frac{p^2}{E_p} f_{\text{eq}}(\mathbf{p}, \{T, \mu, m\}). \quad (38)$$

Minimizing $\bar{\mathcal{V}}_W$ leads to the MFA gap equations

$$\begin{aligned} m_\omega^2 \bar{\omega}_0 - 2g_\omega n_0(T, \mu_B^*, m_N^*) &= 0, \\ m_\sigma^2 \bar{\sigma} + \frac{\partial \bar{U}_m}{\partial \bar{\sigma}} - 2g_\sigma n_s(T, \mu_B^*, m_N^*) &= 0, \end{aligned} \quad (39)$$

⁶In what follows, the effective quasiparticle mass and chemical potential will be denoted explicitly for nucleons and baryon.

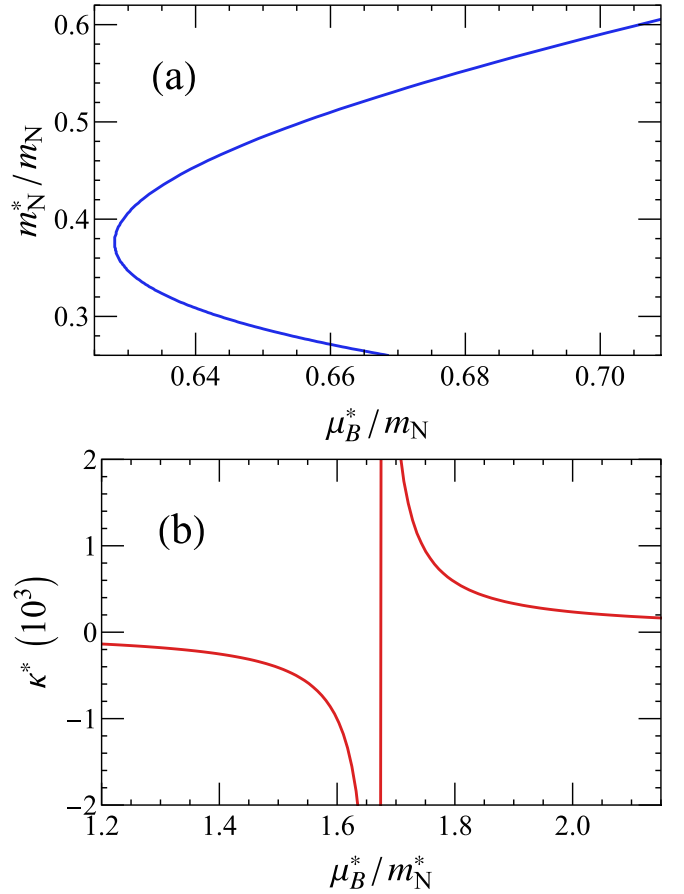


FIG. 2. Input from the mean-field solution at $T = 8$ MeV. (a) Quasinucleon mass m_N^* and effective baryon chemical potential μ_B^* from the gap equations Eq. (39). (b) The response coefficient κ^* obtained from Eq. (41).

where n_0 and n_s denote the free particle and scalar densities:

$$\begin{aligned} n_0(T, \mu, m) &= (2s+1) \int_p f_{\text{eq}}(\mathbf{p}, \{T, \mu, m\}), \\ n_s(T, \mu, m) &= (2s+1) \int_p \frac{m}{E_p} f_{\text{eq}}(\mathbf{p}, \{T, \mu, m\}). \end{aligned} \quad (40)$$

In MFA, the response coefficient κ^* can be derived as [33]

$$\kappa^* = -\frac{g_\sigma}{g_\omega} \frac{\partial^2 \bar{\mathcal{V}}_W}{\partial \bar{\omega}_0 \partial \bar{\sigma}} \left(\frac{\partial^2 \bar{\mathcal{V}}_W}{\partial \bar{\sigma}^2} \right)^{-1}. \quad (41)$$

The sign of κ^* cannot be determined from Eq. (41) alone, although this ambiguity does not affect the positivity of ζ . As noted in Ref. [33], the solution of Eq. (39) corresponds to a saddle point of $\bar{\mathcal{V}}_W$, which can yield a nonmonotonic κ^* with sign changes and apparent divergences (see Fig. 2). In fact, at finite density, taking $\bar{\mathcal{V}}_W$ to be imaginary provides a refinement that resolves this artifact [48,49]. A detailed treatment of this point lies beyond the scope of the present work.

Although the fluctuations $\delta\sigma$ and $\delta\omega^\mu$ do not modify the equation of state at MFA, they obey the same dynamics as Eq. (34) at $\mathcal{O}(\delta\sigma^2)$ and mediate nucleon scattering [33]. Thus,

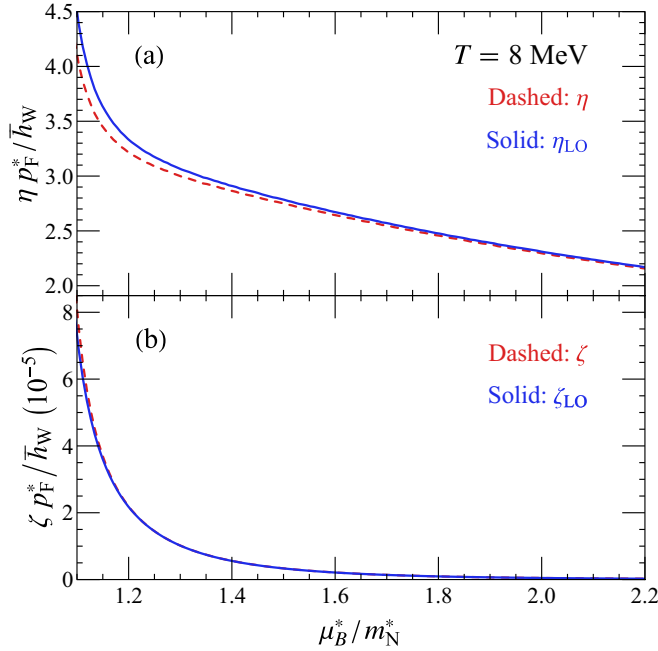


FIG. 3. Dimensionless viscosities at $T = 8$ MeV. (a) Shear viscosity η and (b) bulk viscosity ζ , each scaled by the mean-field enthalpy $\bar{h}_W$ and effective Fermi momentum p_F^* , shown versus μ_B^*/m_N^* . Solid (blue) lines are for LO result calculated from Eq. (30). Dashed (red) lines are from full calculations using Eqs. (12).

dissipation arises only beyond the strict MFA. At $T = 8$ MeV, this temperature is insufficient to induce thermal excitations [12]. Obtained as the inverse of the LO $2 \leftrightarrow 2$ scattering rate, τ_{rel} can be parameterized as [33]⁷

$$\tau_{rel}(x) = 388 \frac{x^4 - 2.37x^3 + 5.44x^2 - 5.63x + 1.90}{x^5 + 2.25x^4 - 3.60x^2} \text{ fm}, \quad (42)$$

with $x = \mu_B^*/m_N^*$.

For dense matter, the appropriate measure of fluidity is the enthalpy h rather than the entropy density [50,51]. Within the Walecka model at MFA, the enthalpy is

$$\bar{h}_W(T, \mu_B^*, m_N^*) = \bar{p}_W(T, \mu_B^*, m_N^*) + \bar{\varepsilon}_W(T, \mu_B^*, m_N^*), \quad (43)$$

where the pressure and energy density are

$$\begin{aligned} \bar{p}_W &= -\bar{\mathcal{V}}_W, \\ \bar{\varepsilon}_W &= 2\varepsilon_0(T, \mu_B^*, m_N^*) + \frac{1}{2}m_\sigma^2\bar{\sigma}^2 + \frac{1}{2}m_\omega^2\bar{\omega}_0^2 + \bar{U}_m, \end{aligned} \quad (44)$$

with the free-fermion energy density

$$\varepsilon_0(T, \mu, m) = (2s + 1) \int_p E_p f_{eq}(\mathbf{p}, \{T, \mu, m\}). \quad (45)$$

⁷A beyond tree-level scattering calculation based on the Brueckner-Hartree-Fock approach, in which the effective mean free paths relevant for shear viscosity are calculated, is presented in Ref. [27].

The resulting dimensionless viscosities, rescaled by p_F^* and $\bar{\varepsilon}_W$, are shown in Fig. 3. As anticipated, dissipation in the bulk channel is negligible compared to the shear channel.

IV. SUMMARY AND OUTLOOK

In this work, we developed a kinetic framework for cold, dense quasifermion matter. Solving the linearized relativistic Boltzmann equation within RTA tailored to quasiparticles and enforcing Landau matching, we fixed zero-mode ambiguities and proved that the bulk viscosity ζ is non-negative. A low-temperature analysis yielded compact expressions, $\eta \sim p_F^{*5}\tau_{rel}/\mu^*$ and $\zeta/\eta \propto (T/\mu^*)^4$, establishing that bulk dissipation is parametrically subleading to shear in the degenerate regime. Coupling the framework to the Walecka model at mean field, we presented numerical results over densities relevant to cold, dense nucleon matter, where shear dominates the attenuation of collective motion. We also clarified that apparent pathologies in the bulk channel are artifacts of mean-field stationarity (a saddle point of the effective potential) rather than genuine instabilities of the properly matched hydrodynamics.

These MFA expressions can also be applied to phases with quark and gluon degrees of freedom in the cold and degenerate limit, such as color-superconducting matter [52]. Looking ahead, it will be important to replace the RTA by microscopic collision integrals that include in-medium scattering and anisotropic angular structure, to quantify mesonic fluctuations beyond mean field, and to incorporate isospin asymmetry [53–56]. To sharpen transport inputs for electron-nucleus programs, one should also employ more realistic equations of state and account for the natural deformation of finite nuclei [57,58]. Finally, cold atom systems offer a complementary arena in which quasiparticle transport can be probed under controlled conditions, providing valuable benchmarks for both hydrodynamic and nonhydrodynamic aspects of the nuclear case [59–62].

ACKNOWLEDGMENTS

We gratefully acknowledge insightful discussions with M. Buballa, X.-G. Huang, F. Rennecke, and Y. Yin, fruitful comments from L. von Smekal, as well as editorial assistance from ChatGPT-5. J.L. appreciates the warm hospitality of Fabian Rennecke during his visit to the Institute for Theoretical Physics at Justus-Liebig-Universität Gießen. J.L. is supported by the DFG Collaborative Research Centre SFB 1225 (ISOQUANT). J.H. is financially supported by the National Natural Science Foundation of China under Grant No.12505149.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

- [1] S. R. de Groot and P. Mazur, *Non-Equilibrium Thermodynamics* (Courier Dover Publications, Mineola, New York, 2013).
- [2] W. Botermans and R. Malfliet, Quantum transport theory of nuclear matter, *Phys. Rept.* **198**, 115 (1990).
- [3] S. L. Shapiro and S. A. Teukolsky, *Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects* (Wiley, New York, 1983).
- [4] K. Adcox, S. S. Adler *et al.* (PHENIX Collaboration), Formation of dense partonic matter in relativistic nucleus-nucleus collisions at RHIC: Experimental evaluation by the PHENIX collaboration, *Nucl. Phys. A* **757**, 184 (2005).
- [5] K. Aamodt *et al.* (ALICE Collaboration), Elliptic flow of charged particles in Pb-Pb collisions at 2.76 TeV, *Phys. Rev. Lett.* **105**, 252302 (2010).
- [6] U. Heinz and R. Snellings, Collective flow and viscosity in relativistic heavy-ion collisions, *Annu. Rev. Nucl. Part. Sci.* **63**, 123 (2013).
- [7] R. Kumar *et al.* (MUSES Collaboration), Theoretical and experimental constraints for the equation of state of dense and hot matter, *Living Rev. Rel.* **27**, 3 (2024).
- [8] R. Baier, A. H. Mueller, D. Schiff, and D. T. Son, “Bottom up” thermalization in heavy ion collisions, *Phys. Lett. B* **502**, 51 (2001).
- [9] P. B. Arnold, G. D. Moore, and L. G. Yaffe, Effective kinetic theory for high temperature gauge theories, *J. High Energy Phys.* **01** (2003) 030.
- [10] P. Romatschke and U. Romatschke, *Relativistic Fluid Dynamics in and Out of Equilibrium*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, UK, 2019).
- [11] J. Berges, M. P. Heller, A. Mazeliauskas, and R. Venugopalan, QCD thermalization: *Ab initio* approaches and interdisciplinary connections, *Rev. Mod. Phys.* **93**, 035003 (2021).
- [12] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer Science & Business Media, Berlin, 2004).
- [13] J. C. Yang, J. W. Xia, G. Q. Xiao, H. S. Xu, H. W. Zhao, X. H. Zhou, X. W. Ma, Y. He, L. Ma, D. Gao, *et al.*, High intensity heavy ion accelerator facility (HIAF) in China, *Nucl. Instrum. Methods Phys. Res. Sect. B* **317**, 263 (2013).
- [14] P. Senger, Nuclear matter physics at NICA, *Eur. Phys. J. A* **52**, 217 (2016).
- [15] A. Bzdak, S. Esumi, V. Koch, J. Liao, M. Stephanov, and N. Xu, Mapping the phases of quantum chromodynamics with beam energy scan, *Phys. Rep.* **853**, 1 (2020).
- [16] K. Agarwal and for the CBM Collaboration (CBM), The compressed baryonic matter (CBM) experiment at FAIR—physics, status and prospects, *Phys. Scr.* **98**, 034006 (2023).
- [17] W. Zhao, C. Shen, and B. Schenke, Collectivity in ultraperipheral Pb+Pb collisions at the large hadron collider, *Phys. Rev. Lett.* **129**, 252302 (2022).
- [18] C. Shen and B. Schenke, Longitudinal dynamics and particle production in relativistic nuclear collisions, *Phys. Rev. C* **105**, 064905 (2022).
- [19] B. Schenke, C. Shen, and W. Zhao, Collectivity in ultraperipheral heavy-ion and $e + A$ collisions, *Acta Phys. Pol. B Proc. Suppl.* **18**, 1 (2025).
- [20] A. Accardi, J. L. Albacete, M. Anselmino, N. Armesto, E. C. Aschenauer, A. Bacchetta, D. Boer, W. K. Brooks, T. Burton, N.-B. Chang, *et al.*, Electron ion collider: The next QCD frontier: Understanding the glue that binds us all, *Eur. Phys. J. A* **52**, 268 (2016).
- [21] D. P. Anderle, V. Bertone, X. Cao, L. Chang, N. Chang, G. Chen, X. Chen, Z. Chen, Z. Cui, L. Dai, *et al.*, Electron-ion collider in China, *Front. Phys.* **16**, 64701 (2021).
- [22] J. Aichelin, “Quantum” molecular dynamics a dynamical microscopic n -body approach to investigate fragment formation and the nuclear equation of state in heavy ion collisions, *Phys. Rept.* **202**, 233 (1991).
- [23] M. N. Harakeh and A. van der Woude, *Giant Resonances: Fundamental High-Frequency Modes of Nuclear Excitation* (Oxford University Press, Oxford, 2001).
- [24] T. Galatyuk (HADES Collaboration), HADES overview, *Nucl. Phys. A* **931**, 41 (2014).
- [25] F. Douchin and P. Haensel, A unified equation of state of dense matter and neutron star structure, *Astron. Astrophys.* **380**, 151 (2001).
- [26] A. Schmitt and P. Shternin, Reaction rates and transport in neutron stars, *Astrophys. Space Sci. Libr.* **457**, 455 (2018).
- [27] P. Shternin and M. Baldo, Transport coefficients of nucleon neutron star cores for various nuclear interactions within the Brueckner-Hartree-Fock approach, *Phys. Rev. D* **102**, 063010 (2020).
- [28] C. Y. Tsang, M. B. Tsang, W. G. Lynch, R. Kumar, and C. J. Horowitz, Determination of the equation of state from nuclear experiments and neutron star observations, *Nat. Astron.* **8**, 328 (2024).
- [29] C. Hoyos, N. Jokela, M. Jarvinen, J. G. Subils, J. Tarrio, and A. Vuorinen, Transport in strongly coupled quark matter, *Phys. Rev. Lett.* **125**, 241601 (2020).
- [30] J. Cruz Rojas, T. Gorda, C. Hoyos, N. Jokela, M. Järvinen, A. Kurkela, R. Paatelainen, S. Säppi, and A. Vuorinen, Estimate for the bulk viscosity of strongly coupled quark matter using perturbative QCD and holography, *Phys. Rev. Lett.* **133**, 071901 (2024).
- [31] J. J. Sakurai, Theory of strong interactions, *Ann. Phys.* **11**, 1 (1960).
- [32] L. V. Delacretaz, Y.-H. Du, U. Mehta, and D. T. Son, Nonlinear bosonization of Fermi surfaces: The method of coadjoint orbits, *Phys. Rev. Res.* **4**, 033131 (2022).
- [33] J. Li and W. Ke, Transport coefficients of dense nucleon matter at low temperature, *Phys. Rev. C* **111**, 044904 (2025).
- [34] J. D. Walecka, A theory of highly condensed matter, *Ann. Phys.* **83**, 491 (1974).
- [35] S. R. de Groot, W. A. van Leeuwen, and C. G. van Weert, *Relativistic Kinetic Theory: Principles and Applications* (North-Holland, Amsterdam, 1980).
- [36] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics*, 2nd ed., Course of Theoretical Physics, Vol. 6 (Butterworth-Heinemann, Oxford, UK, 1987).
- [37] R. A. Horn and C. R. Johnson, *Matrix Analysis* (Cambridge University Press, Cambridge, UK, 1985).
- [38] K. Dusling and T. Schäfer, Bulk viscosity and conformal symmetry breaking in the dilute Fermi gas near unitarity, *Phys. Rev. Lett.* **111**, 120603 (2013).
- [39] P. Chakraborty and J. I. Kapusta, Quasi-particle theory of shear and bulk viscosities of hadronic matter, *Phys. Rev. C* **83**, 014906 (2011).
- [40] M. Albright and J. I. Kapusta, Quasiparticle theory of transport coefficients for hadronic matter at finite temperature and baryon density, *Phys. Rev. C* **93**, 014903 (2016).

- [41] A. Sommerfeld, On the electron theory of metals based on Fermi statistics, *Z. Phys.* **47**, 1 (1928).
- [42] J. Sykes and G. A. Brooker, The transport coefficients of a Fermi liquid, *Ann. Phys.* **56**, 1 (1970).
- [43] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics*, Course of Theoretical Physics, Vol. 10 (Pergamon Press, Oxford, 1981).
- [44] G. A. Brooker and J. Sykes, Transport properties of a Fermi liquid, *Phys. Rev. Lett.* **21**, 279 (1968).
- [45] T. Enss, Bulk viscosity and contact correlations in attractive Fermi gases, *Phys. Rev. Lett.* **123**, 205301 (2019).
- [46] J. I. Kapusta and C. Gale, *Finite-Temperature Field Theory: Principles and Applications*, 2nd ed., Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, UK, 2006).
- [47] J. Li, T. Guo, J. Zhao, and L. He, Do we need dense matter equation of state in curved spacetime for neutron stars? *Phys. Rev. D* **106**, 083021 (2022).
- [48] H. Nishimura, M. C. Ogilvie, and K. Pangeni, Liquid-gas phase transitions and $\mathcal{C}\mathcal{N}$ symmetry in quantum field theories, *Phys. Rev. D* **95**, 076003 (2017).
- [49] M. Haensch, F. Rennecke, and L. von Smekal, Medium induced mixing, spatial modulations, and critical modes in QCD, *Phys. Rev. D* **110**, 036018 (2024).
- [50] J. Liao and V. Koch, On the fluidity and super-criticality of the QCD matter at RHIC, *Phys. Rev. C* **81**, 014902 (2010).
- [51] L. Du, A. Sorensen, and M. Stephanov, The QCD phase diagram and beam energy scan physics: A theory overview, *Int. J. Mod. Phys. E* **33**, 2430008 (2024).
- [52] B. C. Barrois, Superconducting quark matter, *Nucl. Phys. B* **129**, 390 (1977).
- [53] D. B. Kaplan, M. J. Savage, and M. B. Wise, Nucleon-nucleon scattering from effective field theory, *Nucl. Phys. B* **478**, 629 (1996).
- [54] A. B. Larionov and L. von Smekal, Effects of chiral symmetry restoration on meson and dilepton production in relativistic heavy-ion collisions, *Phys. Rev. C* **105**, 034914 (2022).
- [55] J. A. Fotakis, E. Molnár, H. Niemi, C. Greiner, and D. H. Rischke, Multicomponent relativistic dissipative fluid dynamics from the Boltzmann equation, *Phys. Rev. D* **106**, 036009 (2022).
- [56] J. Hu and S. Shi, Multicomponent second-order dissipative relativistic hydrodynamics with binary reactive collisions, *Phys. Rev. D* **106**, 014007 (2022).
- [57] P. Wen, J. W. Holt, and M. Li, Generative modeling of nucleon-nucleon interactions, *Phys. Rev. Lett.* **133**, 252501 (2024).
- [58] J. Jia, G. Giacalone, B. Bally, J. D. Brandenburg, U. Heinz, S. Huang, D. Lee, Y.-J. Lee, W. Li, C. Loizides, *et al.*, Imaging the initial condition of heavy-ion collisions and nuclear structure across the nuclide chart, *Nucl. Sci. Tech.* **35**, 220 (2024).
- [59] S. Brandstetter, P. Lunt, C. Heintze, G. Giacalone, L. H. Heyen, M. Gałka, K. Subramanian, M. Holten, P. M. Preiss, S. Floerchinger, and S. Jochim, Emergent interaction-driven elliptic flow of few fermionic atoms, *Nat. Phys.* **21**, 52 (2025).
- [60] W. Ke and Y. Yin, Does a Quark-Gluon plasma feature an extended hydrodynamic regime? *Phys. Rev. Lett.* **130**, 212303 (2023).
- [61] W. Ke and Y. Yin, Non-hydrodynamic response in QCD-like plasma, *J. High Energy Phys.* **05** (2024) 171.
- [62] J. Berges, S. Brandstetter, J. Brewer, G. Bruun, T. Enss, S. Floerchinger, K. Fujii, M. Gałka, G. Giacalone, Q. Guan, *et al.*, Few is different: Deciphering many-body dynamics in mesoscopic quantum gases, [arXiv:2509.05049](https://arxiv.org/abs/2509.05049).