

Phase-space excluded-volume approach for light clusters in a nuclear medium

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We present a phase-space excluded-volume approach applicable to both nuclear matter and nonequilibrium processes, such as heavy-ion collisions, to account for in-medium effects on light clusters. In this approach, light clusters can exist only if the nucleon one-body phase-space occupation of the surrounding nuclear medium—including explicit contributions from light clusters—is sufficiently low. The nucleon occupation is determined self-consistently by accounting for the interplay between in-medium effects and thermodynamic properties, thereby improving upon the conventional treatment based on an uncorrelated medium (i.e., an overall nucleonic Fermi-Dirac distribution) typically employed in studies of light clusters in nuclear matter. We apply the approach to evaluate the Mott momenta and the fractions of light clusters in nuclear matter. Their differences from results obtained under the uncorrelated-medium assumption are found to be moderate. The main advantage of the present approach lies in its ability to be integrated into dynamical models, enabling more accurate studies of in-medium effects on light clusters, based on their measured yields in heavy-ion collisions.

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I. INTRODUCTION

Few-body correlations among nucleons (including neutrons n and protons p) may lead to the formation of light clusters, such as deuterons (d) and α particles, in nuclear matter. Understanding the composition and thermodynamic properties of clustered nuclear matter is of crucial importance for a wide range of topics in both nuclear physics and astrophysics. These include the structure of finite nuclei [1,2], the dynamics of heavy-ion collisions [3], the behavior of supernovae explosions [4–6], the properties of neutron stars and their mergers [7], and, more generally, our knowledge of the nuclear equation of state [8–15].

Reliable descriptions of these phenomena require careful treatment of the in-medium modifications of light-cluster properties. In general, the clustering correlations of nucleons embedded in the surrounding nuclear medium can be studied within the thermodynamic Green's function formalism [16–18], where the light-cluster energy is provided by

the in-medium many-body Schrödinger equation. Solving this equation yields reduced binding energies of light clusters in the nuclear medium compared with their vacuum values, mainly due to Pauli blocking effects from the surrounding nucleons, including those bound in clusters. Therefore, the presence of light clusters is suppressed as the density of the medium increases, giving rise to the Mott effect [19]. This suppression can be effectively incorporated into various mean-field approaches, e.g., by parametrizing the in-medium binding energies and/or by introducing effective meson-cluster couplings. These methods have been employed to study the composition of nuclear matter [11,20–24], leading to the formulation of several nuclear equations of state with light-cluster ingredients. We note that studies about nuclear clustering based on *ab initio* methods, such as the lattice effective field theory, have also emerged in recent years [25–28].

In addition to theoretical descriptions, significant efforts have also been devoted to extract empirical information on in-medium properties of light clusters from heavy-ion data, such as their Mott densities [29] and abundances [30–32] in nuclear matter, as well as the effective meson-cluster couplings [32,33]. Such information, incorporated within mean-field models, is indispensable for the description of heavy-ion collisions and large-scale astrophysical simulations of, e.g., supernova explosions and neutron star mergers [4–6].

The above empirical studies, based on heavy-ion collision data around the Fermi energy [34], assume an intermediate-velocity source in global thermal equilibrium, where nucleons and light clusters freeze out. Since the estimated freeze-out density is very low—less than one-third of the nuclear saturation density ρ_0 —these analyses allowed access to the in-medium properties of light clusters in dilute nuclear

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matter. In contrast, in-medium properties at higher densities may be explored through heavy-ion collisions at higher beam energies. However, since only local—rather than global [35,36]—thermal equilibrium may be reached in such collisions, dynamical approaches, such as transport theories that properly account for in-medium effects on light clusters [37–40], are required.

In principle, the in-medium Schrödinger equation can be coupled to transport approaches [41–44], such as the Boltzmann-Uehling-Uhlenbeck (BUU) equation and the quantum-molecular-dynamic approach, to achieve a consistent description of the dynamical evolution of light clusters in a nuclear medium. However, since these dynamical models deal with the time-dependent, spatially resolved evolution of the nuclear system, the in-medium effects should be treated locally in both space and time. This involves solving the in-medium Schrödinger equation in each spatial cell and at every time step, which is, however, rather computationally demanding. In order to circumvent this difficulty, a phase-space excluded-volume approach for light clusters has been proposed as an effective way to incorporate the in-medium effects on their dynamical evolution in heavy-ion collisions [37,39,45]. The method captures the main feature of the more sophisticated in-medium Schrödinger equation, but with significantly reduced computational cost, by introducing a criterion to determine the survival of light clusters directly from the phase-space density of the surrounding nuclear medium.

In the present work, we generalize the phase-space excluded-volume approach and employ it to study the properties of clustered nuclear matter. In particular, we include the contribution of nucleons bound in light clusters in the evaluation of in-medium effects, and consistently account for the interplay between these effects and thermodynamic prop-

erties of both nucleons and light clusters—an aspect that is missing in existing models. The present approach provides a viable and effective way for studying in-medium effects on light clusters within mean-field models. It also serves as a connection between the in-medium properties of light clusters in nuclear matter, such as their abundances, and in-medium effects on their evolution in dynamical processes, thus paving the way for studying the former through light-nuclei observables in intermediate-energy heavy-ion collisions.

We introduce the phase-space excluded-volume approach for light clusters and the formalism adopted to describe clustered nuclear matter in Sec. II. The in-medium properties of light clusters, i.e., Mott momenta, Mott densities, and light-cluster fractions in nuclear matter, are presented in Sec. III, where their sensitivities to the neutron-proton effective mass splitting are also explored. In Sec. IV, we compare, for n pd matter, the results obtained from the phase-space excluded-volume approach with those obtained by solving the in-medium Schrödinger equation. Conclusions and a brief outlook are given in Sec. V. In the Appendix, we outline the equations related to nuclear effective masses within Skyrme energy-density functionals and discuss various possible approximations to the current approach.

II. THEORETICAL FRAMEWORK

A. In-medium Schrödinger equation

For a (bound) light cluster of species ν that consists of A nucleons, with center-of-mass momentum $\vec{P}$, its wave function $\Psi_{\nu,\vec{P}}(1, 2, \dots, A)$ is the eigenstate of the in-medium Schrödinger equation [16–18,46] with the corresponding eigenvalue $\epsilon_\nu(\vec{P})$, i.e.,

$$\begin{aligned} & [\epsilon_{\tau_1}(\vec{p}_1) + \epsilon_{\tau_2}(\vec{p}_2) + \dots + \epsilon_{\tau_A}(\vec{p}_A) - \epsilon_\nu(\vec{P})] \Psi_{\nu,\vec{P}}(1, 2, \dots, A) \\ & + \sum_{i < j} [1 - f_{\tau_i}^{\text{tot}}(\vec{p}_i) - f_{\tau_j}^{\text{tot}}(\vec{p}_j)] \int \prod_{l=1', 2', \dots, A'} \frac{d\vec{p}_l}{(2\pi\hbar)^3} V(ij, i'j') \prod_{k \neq i, j} \delta_{kk'} \Psi_{\nu,\vec{P}}(1', 2', \dots, A') = 0. \end{aligned} \quad (1)$$

In the above equation, $i, j, k = 1, 2, \dots, A$ label the momentum and isospin state of the constituent nucleons (the spin degrees of freedom can be included analogously), and ϵ_τ denotes the single-nucleon quasiparticle energies for isospin $\tau = n$ or p . We have simplified the many-body interaction within the A -particle cluster to a sum of two-body interactions, i.e., $V(1, 2, \dots, A; 1', 2', \dots, A') \approx \sum_{i < j} V(ij, i'j') \prod_{k \neq i, j} \delta_{kk'}$. Note that $V(ij, i'j')$ vanishes unless $\vec{p}_i + \vec{p}_j = \vec{p}_i' + \vec{p}_j'$, and one has nonzero $\Psi_{\nu,\vec{P}}(1, 2, \dots, A)$ only when $\vec{p}_1 + \vec{p}_2 + \dots + \vec{p}_A = \vec{P}$. The factor $1 - f_{\tau_i}^{\text{tot}}(\vec{p}_i) - f_{\tau_j}^{\text{tot}}(\vec{p}_j)$ accounts for the Pauli blocking effect of the nuclear medium on the constituent nucleons inside the cluster, with $f_{\tau}^{\text{tot}}(\vec{p})$ representing the total nucleon phase-space occupation of the nuclear medium,

$$f_{\tau}^{\text{tot}}(\vec{p}) = f_{\tau}(\vec{p}) + f_{\tau}^{\text{clu}}(\vec{p}). \quad (2)$$

In the above, $f_{\tau}(\vec{p})$ denotes the phase-space occupation of quasifree nucleons (namely, nucleons not bound in clusters), and $f_{\tau}^{\text{clu}}(\vec{p})$ denotes that solely from constituent nucleons inside light clusters. The latter is given by

$$f_n^{\text{clu}}(\vec{p}) = \sum_{\nu}^{\text{clu}} N \int \frac{d\vec{P}}{(2\pi\hbar)^3} f_{\nu}(\vec{P}) |\phi_{\nu,\vec{P}}(\vec{p})|^2, \quad (3)$$

$$f_p^{\text{clu}}(\vec{p}) = \sum_{\nu}^{\text{clu}} Z \int \frac{d\vec{P}}{(2\pi\hbar)^3} f_{\nu}(\vec{P}) |\phi_{\nu,\vec{P}}(\vec{p})|^2, \quad (4)$$

where N and Z denote the neutron and proton numbers, respectively, of the light-cluster species ν , and the summations run over the light-cluster species considered (see also Refs. [21,46]). In the above equations, $f_{\nu}(\vec{P})$ represents the occupation of light clusters in momentum space. The quantity $|\phi_{\nu,\vec{P}}(\vec{p})|^2$ is the normalized one-body probability distribution

of $\Psi_{v,\vec{P}}(1, 2, \dots, A)$ in momentum space,

$$|\phi_{v,\vec{P}}(\vec{p})|^2 \propto \int \prod_{i=1}^{A-1} \frac{d\vec{p}_i}{(2\pi\hbar)^3} |\Psi_{v,\vec{P}}(1, 2, \dots, A)|^2, \quad (5)$$

where, for brevity, we have omitted the distinction between the neutron and proton components.

The binding energy of the light cluster is defined as the difference between $\epsilon_v(\vec{P})$ and the continuum edge of the cluster, i.e., $[N\epsilon_n(\vec{P}/A) + Z\epsilon_p(\vec{P}/A)]$. A bound-state solution of Eq. (1) can only be found when $\epsilon_v(\vec{P})$ lies below the continuum edge. In nuclear matter, this condition defines the Mott momentum P_v^{Mott} : light clusters of species v can exist only when the magnitude of their center-of-mass momentum $|\vec{P}|$ exceeds P_v^{Mott} .

In perturbative calculations, i.e., for given $f_n^{\text{tot}}(\vec{p})$ and $f_p^{\text{tot}}(\vec{p})$, Eq. (1) can be solved independently. However, within a fully consistent treatment, Eq. (1) is coupled with Eqs. (2)–(4), and they need to be solved iteratively, which is a significant computational challenge. Owing to this difficulty, for nuclear matter, an effective treatment to consider the light-cluster contribution is usually employed, i.e., to approximate $f_v^{\text{tot}}(\vec{p})$ in Eq. (1) by a Fermi-Dirac distribution normalized to the *total* nucleon densities ρ_n^{tot} . This uncorrelated-medium assumption decouples the in-medium Schrödinger equation from Eqs. (2)–(4), and reduces the computational complexity substantially. We evaluate the viability of this assumption in Appendix B, based on the phase-space excluded-volume approach presented below.

B. Phase-space excluded-volume approach

Motivated by the copious production of light nuclei in intermediate-energy heavy-ion collisions [47], dynamical descriptions of light clusters within kinetic approaches or transport models have been developed [37–40,48–50]. In particular, to incorporate in-medium effects on light clusters within kinetic approaches, which is essential for a proper dynamical description, especially considering the transient dense nuclear medium produced in those collisions, a phase-space excluded-volume approach has been proposed [37,39,45].

In this approach, a criterion is introduced to determine the survival of light clusters in the nuclear medium directly from the average nucleon occupation within the phase-space volume corresponding to the free-space wave function of the cluster. Specifically, a light cluster of species v with mass number A and center-of-mass momentum $\vec{P}$ can exist only if both $\langle f_n \rangle_v(\vec{P})$ and $\langle f_p \rangle_v(\vec{P})$ are less than a cutoff value¹ F_A^{cut} , i.e.,

$$\langle f_\tau \rangle_v(\vec{P}) \equiv \int f_\tau^{\text{tot}}(\vec{p}) |\tilde{\phi}_{v,\vec{P}}(\vec{p})|^2 \frac{d\vec{p}}{(2\pi\hbar)^3} < F_A^{\text{cut}}, \quad (6)$$

where $|\tilde{\phi}_{v,\vec{P}}(\vec{p})|^2$ denotes the probability distribution $|\phi_{v,\vec{P}}(\vec{p})|^2$ in free space, which is usually approximated

by a Gaussian form (see Sec. III). More commonly, Eq. (6) is written in terms of the momentum of the constituent nucleons $\vec{q}$ inside the cluster, with $\vec{p} = \frac{\vec{P}}{A} + \vec{q}$. The phase-space excluded-volume approach essentially describes the Pauli blocking effect exerted by the nuclear medium on the constituent nucleons inside light clusters, which is the main origin of the in-medium effect on light clusters. It thus captures the main features of the in-medium Schrödinger equation while requiring much less computational effort. Most notably, as in the case of the in-medium Schrödinger equation, the above criterion defines a Mott momentum. In nuclear matter, $f_\tau^{\text{tot}}(\vec{p})$ decreases monotonously as $|\vec{p}|$ increases,² and consequently $\langle f_\tau \rangle_v(\vec{P})$ also decreases with increasing $|\vec{P}|$. Hence, the Mott momentum satisfies the following relations:

$$P_v^{\text{Mott}} = 0 \quad \text{if } \langle f_\tau \rangle_v(\vec{P} = 0) < F_A^{\text{cut}}, \quad \tau = n, p, \\ \max_{\tau=n,p} [\langle f_\tau \rangle_v(P_v^{\text{Mott}})] = F_A^{\text{cut}} \quad \text{if } P_v^{\text{Mott}} \neq 0. \quad (7)$$

The parameter F_A^{cut} characterizes the strength of the in-medium effect. A smaller F_A^{cut} indicates that the presence of light clusters in the nuclear medium is less favorable, and thus corresponds to a stronger Mott effect. In principle, different values of F_A^{cut} may be required to better reproduce the realistic Mott momenta and fractions of light clusters for nuclear matter at various densities and temperatures. Nevertheless, since the density and temperature dependence is mild (as can be seen in Sec. IV, where, for a simplified case, we compare the Mott momentum obtained from the phase-space excluded-volume approach with that from the in-medium Schrödinger equation), and one can always focus on a specific density and temperature region in a certain process, we keep F_A^{cut} constant and explore the sensitivity to its choice.

In previous transport-model studies [37,39,45], the contribution of light clusters to the nuclear medium is neglected, i.e., the total nucleon occupation $f_\tau^{\text{tot}}(\vec{p})$ in Eq. (6) is replaced by that of quasifree nucleons, $f_\tau(\vec{p})$, in the spirit of treating light clusters as completely independent degrees of freedom. This leads to a smaller $\langle f_\tau \rangle_v(\vec{P})$, and thus the presence of light clusters is less suppressed for a given value of F_A^{cut} . To reproduce similar in-medium properties, one may reduce the cutoff parameter to compensate for the omission of the light-cluster contribution to $f_\tau^{\text{tot}}(\vec{p})$. However, from a more consistent point of view, the Pauli blocking originating from nucleons bound in light clusters should be treated on the same footing as that from quasifree nucleons, as we aim to do in this work.

Finally, it is important to note that, in the context of heavy-ion collisions, where the phase-space excluded-volume approach was first applied, the total nucleon distribution function used in Eq. (6) does not necessarily represent (local) equilibrium conditions. In contrast, our aim here is to apply this formalism to equilibrated nuclear matter. Studying

¹In principle, one can introduce separate cutoff values for $\langle f_n \rangle_v(\vec{P})$ and $\langle f_p \rangle_v(\vec{P})$.

²Note that if P_v^{Mott} is sufficiently large, $f_\tau^{\text{clu}}(\vec{p})$ exhibits a peak around P_v^{Mott}/A (see Appendix B), and it is possible that $f_\tau^{\text{tot}}(\vec{p})$ becomes nonmonotonic. We omit this scenario, since it arises only when F_A^{cut} is unreasonably large.

the properties of light clusters in this context, especially in comparison with results from more sophisticated methods, can also help build confidence in extending this approach to out-of-equilibrium scenarios.

C. Nuclear matter with light clusters

The phase-space distribution functions of quasifree nucleons and light clusters in thermal equilibrium are derived resorting to the BUU equation [51].

We first recall how the Fermi-Dirac distribution in unclustered nuclear matter is obtained. In nuclear matter composed purely of nucleons, at thermal equilibrium, the gain and the loss terms in the collision integral of the BUU equation cancel each other, and the nucleon occupation f_τ satisfies

$$f_{\tau_1} f_{\tau_2} \bar{f}_{\tau_3} \bar{f}_{\tau_4} = f_{\tau_3} f_{\tau_4} \bar{f}_{\tau_1} \bar{f}_{\tau_2}, \quad (8)$$

where $\bar{f}_\tau = 1 - f_\tau$ accounts for the Pauli principle. Dividing both sides by the product of all $\bar{f}_\tau$ and applying the energy conservation condition $\epsilon_{\tau_1} + \epsilon_{\tau_2} = \epsilon_{\tau_3} + \epsilon_{\tau_4}$, we obtain

$$\frac{f_\tau}{1 - f_\tau} \sim e^{-\beta \epsilon_\tau}, \quad (9)$$

which leads to the Fermi-Dirac distribution for nucleons,

$$f_\tau(\vec{p}) = \frac{1}{\exp\{\beta[\epsilon_\tau(\vec{p}) - \mu_\tau]\} + 1}, \quad (10)$$

with μ_τ being the nucleon chemical potential and β the inverse of the temperature T .

For nuclear matter consisting of nucleons and light clusters, since the quasifree nucleons can “feel” the Pauli blocking from the constituent nucleons inside light clusters, their occupation deviates from the Fermi-Dirac distribution. In this sense, one could modify $\bar{f}_\tau$ to include the Pauli blocking from the constituent nucleons inside light clusters, i.e., $1 - f_\tau \rightarrow 1 - f_\tau - f_\tau^{\text{clu}}$, and correspondingly Eq. (9) becomes

$$\frac{f_\tau}{1 - f_\tau - f_\tau^{\text{clu}}} \sim e^{-\beta \epsilon_\tau}. \quad (11)$$

We recall that f_τ^{clu} represents the contribution to the total nucleon occupation f_τ^{tot} from the nucleons bound in light clusters [see Eqs. (3) and (4)]. We then obtain the distribution function of quasifree nucleons,

$$f_\tau(\vec{p}) = \frac{1 - f_\tau^{\text{clu}}(\vec{p})}{\exp\{\beta[\epsilon_\tau(\vec{p}) - \mu_\tau]\} + 1}. \quad (12)$$

Adding $f_\tau(\vec{p})$ with $f_\tau^{\text{clu}}(\vec{p})$ leads to the total nucleon occupation,

$$f_\tau^{\text{tot}}(\vec{p}) = \frac{1 + f_\tau^{\text{clu}}(\vec{p}) \exp\{\beta[\epsilon_\tau(\vec{p}) - \mu_\tau]\}}{\exp\{\beta[\epsilon_\tau(\vec{p}) - \mu_\tau]\} + 1}. \quad (13)$$

Under equilibrium conditions, the phase-space distribution function of light clusters, $f_v(\vec{P})$, which is needed in Eqs. (3) and (4) to calculate f_τ^{clu} , follows either the Fermi-Dirac or Bose-Einstein distribution, as it can still be derived from the collision integral of the BUU equation, when one includes many-body scatterings involving light clusters [37,39,45], e.g., $nnp \leftrightarrow nd$. The in-medium effects enter the light-cluster distribution function by means of the Mott momentum;

namely, a light cluster can only exist if the magnitude of its momentum is greater than the Mott momentum. Finally, one has

$$f_v(\vec{P}) = \frac{H(|\vec{P}| - P_v^{\text{Mott}})}{\exp\{\beta[\epsilon_v(\vec{P}) - \mu_v]\} \pm 1}, \quad (14)$$

where the plus (minus) sign is for fermions (bosons), and H represents the Heaviside step function (equal to 1 for $|\vec{P}| > P_v^{\text{Mott}}$).

In principle, light-cluster wave functions are needed to evaluate $f_\tau^{\text{clu}}(\vec{p})$. Since they are not known within the phase-space excluded-volume approach, the probability distributions $|\phi_{v,\vec{P}}(\vec{p})|^2$ in Eqs. (3) and (4) are approximated by their free-space forms, $|\tilde{\phi}_{v,\vec{P}}(\vec{p})|^2$, for simplicity. Accordingly, the binding energy (or the mass) of the light clusters that enters the energy $\epsilon_v(\vec{P})$ is taken as its value in free space. Although the internal wave functions of light clusters are modified in the medium according to the in-medium Schrödinger equation, the corresponding changes in the phase-space distribution $f_\tau^{\text{clu}}(\vec{p})$ are less pronounced. Taking deuterons as an example, the solution of the in-medium Schrödinger equation shows that the momentum-space width of their internal wave function decreases and eventually vanishes with increasing medium density. Since this width is already small in free space compared to the smearing induced by the deuteron momentum distribution $f_d(\vec{P})$, the overall effect on $f_\tau^{\text{clu}}(\vec{p})$ remains modest. In principle, the present phase-space excluded-volume approach can be extended to include medium-dependent modifications of binding energies and internal wave functions, which is a further effect arising from the in-medium Schrödinger equation. Such an extension would involve assigning different binding-energy values and distribution widths according to the average one-body phase-space density $\langle f_\tau \rangle_v$ surrounding a light cluster. This will be pursued in the future.

Finally, for given total nucleon density ρ_τ^{tot} and temperature T , the nucleon chemical potentials μ_n and μ_p can be determined by imposing the relation between ρ_τ^{tot} and the total nucleon phase-space occupation $f_\tau^{\text{tot}}(\vec{p})$,

$$g \int \frac{d\vec{p}}{(2\pi\hbar)^3} f_\tau^{\text{tot}}(\vec{p}) = \rho_\tau^{\text{tot}}, \quad (15)$$

with g being the nucleon spin degeneracy. This involves solving coupled equations (2)–(4), (7), and (13)–(15) simultaneously, with the Mott momentum P_v^{Mott} determined self-consistently. Under chemical equilibrium conditions, the light-cluster chemical potentials in Eq. (14) are given by $\mu_v = N\mu_n + Z\mu_p$. Examples of the behavior of $f_\tau(\vec{p})$, $f_\tau^{\text{tot}}(\vec{p})$, and $f_\tau^{\text{clu}}(\vec{p})$, obtained in certain density and temperature conditions, can be found in Appendix B.

In nonrelativistic frameworks, the quasiparticle energy ϵ can be divided into the kinetic part ϵ^{kin} and the potential part ϵ^{pot} . In the present work, the nucleon single-particle potential $\epsilon_\tau^{\text{pot}}(\vec{p})$ is obtained with the standard Skyrme energy-density functional [52,53] (see Appendix A). The light-cluster quasiparticle potential energies $\epsilon_v^{\text{pot}}(\vec{P})$ are assumed to obey the scaling

$$\epsilon_v^{\text{pot}}(\vec{P}) = N\epsilon_n^{\text{pot}}(\vec{P}/A) + Z\epsilon_p^{\text{pot}}(\vec{P}/A). \quad (16)$$

With the standard Skyrme interaction, in nuclear matter, the momentum dependence of $\epsilon_\tau^{\text{pot}}$ can be recast into the kinetic part, leading to an effective kinetic energy $\epsilon_\tau^{\text{kin},*}$. The latter is obtained by replacing the free-space nucleon mass m in $\epsilon_\tau^{\text{kin}}$ with the effective mass m_τ^* , defined as

$$\frac{m}{m_\tau^*} \equiv 1 + m \frac{\vec{p} \cdot \nabla_{\vec{p}} \epsilon_\tau^{\text{pot}}(\vec{p})}{p^2}. \quad (17)$$

We note that, within standard Skyrme energy-density functionals, the effective mass only depends on the (total) nucleon density and not on the single-particle momentum (see Appendix A). The remaining (momentum-independent) part of $\epsilon_\tau^{\text{pot}}$ is a constant for given ρ_n^{tot} and ρ_p^{tot} , and can be absorbed into the chemical potential μ_τ in Eqs. (12) and (13).

The effective mass of light clusters can be obtained by applying Eq. (16),

$$\begin{aligned} \frac{m_v}{m_v^*} &\equiv 1 + m_v \frac{\vec{p} \cdot \nabla_{\vec{p}} \epsilon_v^{\text{pot}}(\vec{p})}{p^2} \\ &= 1 + \frac{m_v}{A^2} \left[N \frac{\vec{p} \cdot \nabla_{\vec{p}} \epsilon_n^{\text{pot}}(\vec{p})}{p^2} + Z \frac{\vec{p} \cdot \nabla_{\vec{p}} \epsilon_p^{\text{pot}}(\vec{p})}{p^2} \right], \end{aligned} \quad (18)$$

where m_v is the light-cluster mass in free space and $\vec{p} = \frac{\vec{P}}{A}$. Note that Eq. (16) also guarantees that the relation $\mu_v = N\mu_n + Z\mu_z$ still holds after absorbing the momentum-independent part of ϵ^{pot} into the chemical potentials.

In the present work, we employ the Skyrme interaction MSL1 [54] to calculate the quasiparticle potential energies $\epsilon_\tau^{\text{pot}}(\vec{p})$ and $\epsilon_v^{\text{pot}}(\vec{P})$. To explore the influence of the neutron-proton effective mass splitting $m_{n-p}^* \equiv m_n^* - m_p^*$ on light-cluster properties in nuclear matter, we introduce another Skyrme interaction, MSL1-m, which differs from MSL1 in the value of m_{n-p}^* . The Skyrme parameters of these two interactions are provided in Appendix A.

III. RESULTS

In the present work, we consider nuclear matter consisting of n , p , d , tritons (t), helium-3 (h), and α particles, within the phase-space excluded-volume approach. We define three cutoff parameters, namely, F_2^{cut} for d , F_3^{cut} for t and h , and F_4^{cut} for α . To study the sensitivities of light-cluster in-medium properties to the choice of their values, we choose three sets of $\mathbf{F}^{\text{cut}} \equiv (F_2^{\text{cut}}, F_3^{\text{cut}}, F_4^{\text{cut}})$, as given in Table I. We approximate the free-space one-body probability distribution $|\phi_{v,\vec{p}}(\vec{p})|^2$ by a Gaussian form,

$$|\phi_{v,\vec{p}}(\vec{p})|^2 \propto \exp\left[-\frac{2(\vec{p} - \frac{\vec{P}}{A})^2}{\hbar^2} \sigma_v^2\right], \quad (19)$$

TABLE I. The $\mathbf{F}^{\text{cut}} \equiv (F_2^{\text{cut}}, F_3^{\text{cut}}, F_4^{\text{cut}})$ used in the present work.

$\mathbf{F}^{\text{cut}}$	F_2^{cut}	F_3^{cut}	F_4^{cut}
set (i)	0.10	0.15	0.25
set (ii)	0.20	0.25	0.35
set (iii)	0.10	0.15	0.15

with σ_v determined by the rms radius of light nuclei in free space, namely, σ_d , σ_t , σ_h , and σ_α are equal to 2.26, 1.59, 1.76, and 1.54 fm, respectively.

A. Mott momenta and Mott densities

To illustrate the characteristics of the Mott momenta obtained with the present approach, we consider nuclear matter with different temperatures and isospin asymmetries, which are typical in heavy-ion collisions. Their sensitivity to the strength of the in-medium effect, i.e., the magnitude of F_A^{cut} , is also examined. We present in Fig. 1 the P_v^{Mott} for d , t , h and α as a function of the baryon density $\rho_B = \rho_n^{\text{tot}} + \rho_p^{\text{tot}}$, for symmetric nuclear matter obtained with $\mathbf{F}^{\text{cut}}$ sets (i) and (ii). We also include in Fig. 1 the results for asymmetric matter with isospin asymmetry $\delta = (\rho_n^{\text{tot}} - \rho_p^{\text{tot}}) = 0.13$, employing $\mathbf{F}^{\text{cut}}$ set (i), for comparison. For all cases, two different temperatures, $T = 5$ and 20 MeV, are considered.

From Fig. 1, we notice that P_v^{Mott} becomes nonzero starting from a certain ρ_B . This defines the Mott densities ρ_v^{Mott} of light clusters, above which light clusters can reside only in part of the whole momentum space. The overall larger $\mathbf{F}^{\text{cut}}$ (ii), which corresponds to a weaker in-medium effect, allows light clusters with a higher $\langle f_\tau \rangle_v(\vec{P})$ to bind, thereby decreasing the Mott momenta. A positive δ leads to a larger neutron density and subsequently a larger $\langle f_n \rangle_v(\vec{P})$. It thus increases the Mott momenta, though the effect is not that pronounced. As the temperature increases, the total nucleon occupation $f_\tau^{\text{tot}}(\vec{p})$ exhibits a broader momentum distribution. Therefore, at

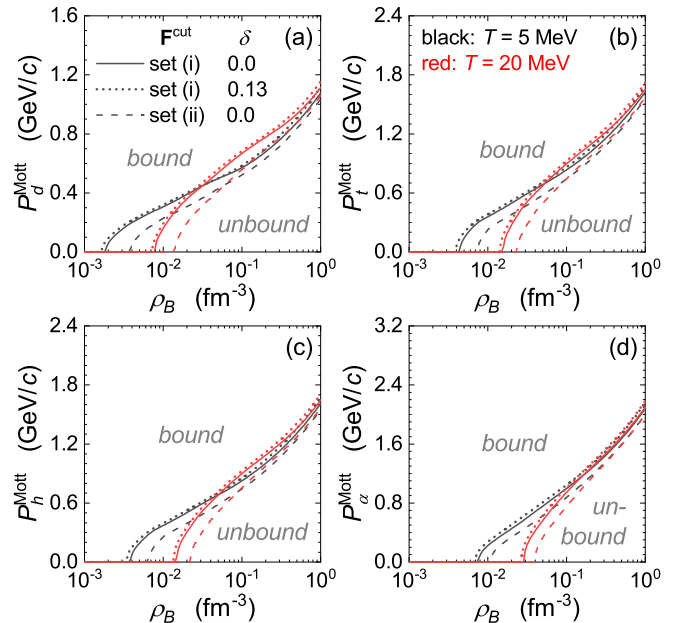


FIG. 1. Mott momenta P_v^{Mott} for (a) d , (b) t , (c) h , and (d) α as functions of baryon density ρ_B obtained with the phase-space excluded-volume approach for symmetric nuclear matter with $\mathbf{F}^{\text{cut}}$ (i) = (0.10, 0.15, 0.25) and $\mathbf{F}^{\text{cut}}$ (ii) = (0.20, 0.25, 0.35), and for nuclear matter with isospin asymmetry $\delta = 0.13$ with $\mathbf{F}^{\text{cut}}$ (i), at temperatures $T = 5$ and 20 MeV, respectively. All calculations are obtained by employing the Skyrme interaction MSL1.

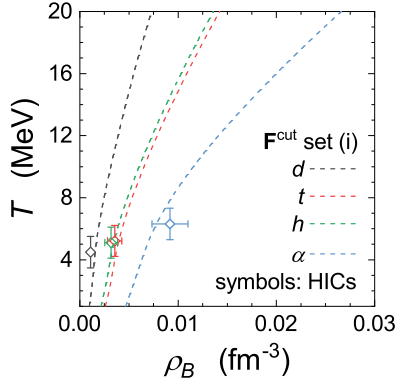


FIG. 2. Mott lines of d , t , h , and α for nuclear matter with isospin asymmetry $\delta = 0.13$ in ρ_B - T phase diagram obtained by the phase-space excluded-volume approach with $\mathbf{F}^{\text{cut}}$ (i) = (0.10, 0.15, 0.25) and the Skyrme interaction MSL1. Symbols with error bars represent the Mott densities for light clusters deduced from light-nuclei yields in heavy-ion collisions (HICs) around the Fermi energy [29].

$T = 20$ MeV, we obtain smaller P_v^{Mott} for lower densities and larger P_v^{Mott} for higher densities, with respect to the results at $T = 5$ MeV.

In Fig. 2, we display, in the ρ_B - T phase diagram, the Mott lines, i.e., the Mott density as a function of temperature, for d , t , h , and α , for asymmetric nuclear matter with $\delta = 0.13$, obtained employing the phase-space excluded-volume approach with $\mathbf{F}^{\text{cut}}$ (i). As a reference, we include in the figure the Mott densities extracted from an experimental analysis of heavy-ion collisions around the Fermi energy [29]. The isospin asymmetry $\delta = 0.13$ roughly corresponds to the average isospin asymmetry of the reaction systems, i.e., ^{40}Ar and ^{64}Zn bombarding $^{112,124}\text{Sn}$, analyzed in Ref. [29]. In particular, there the Mott densities were deduced by fitting light-cluster emission from the (thermally equilibrated) midrapidity region with a statistical model.

We notice in Fig. 2 that the phase-space excluded-volume approach with $\mathbf{F}^{\text{cut}}$ (i) gives rise to Mott densities of light clusters consistent with those deduced from Fermi-energy heavy-ion collisions [29]. The latter can be regarded as a probe of the strength of in-medium effects on light clusters, and can be used to determine $\mathbf{F}^{\text{cut}}$ suitable for calculations at low baryon densities. However, they offer limited insight into the strength of the Mott effect and the magnitude of $\mathbf{F}^{\text{cut}}$ at higher densities. To explore this density range, one may resort to heavy-ion collisions at higher beam energies, where light clusters are primarily produced in warm and dense nuclear matter formed transiently during the compression stage of the reaction [39].

B. Light-cluster fractions

The light-cluster fractions in nuclear matter, X_v , are given by

$$X_v = \frac{A}{\rho_B} \int g_v f_v(\vec{P}) \frac{d\vec{P}}{(2\pi\hbar)^3}, \quad (20)$$

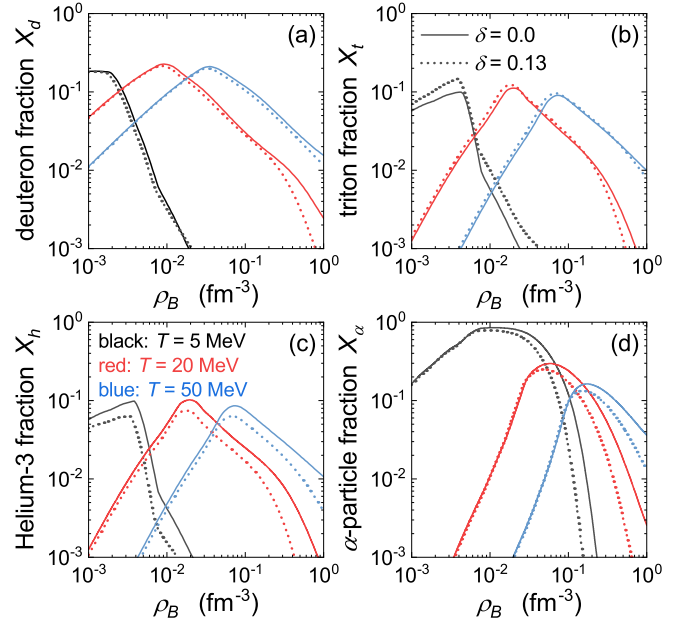


FIG. 3. Fractions of (a) d , (b) t , (c) h , and (d) α of nuclear matter as functions of baryon density ρ_B , calculated using the phase-space excluded-volume approach for two isospin asymmetries at various temperatures. Results are obtained with $\mathbf{F}^{\text{cut}}$ (i) = (0.10, 0.15, 0.25) and the Skyrme interaction MSL1.

with g_v being the spin degeneracy of light-cluster species v . We plot in Fig. 3 the light-cluster fractions X_v of d , t , h , and α for nuclear matter of various densities, at temperatures $T = 5, 20$, and 50 MeV, obtained using the phase-space excluded-volume approach with $\mathbf{F}^{\text{cut}}$ (i) and the Skyrme interaction MSL1. Two isospin asymmetries, $\delta = 0$ and 0.13 , are considered.

The light-cluster fractions X_v presented in Fig. 3 exhibit a rise-and-fall pattern. Below the Mott density, X_v increases, in line with the predictions of conventional statistical approaches [11]. The increase of X_v starts to slow down when ρ_B reaches the Mott density, above which the phase space allowed for light clusters to exist is reduced. Eventually, X_v is significantly suppressed in the high-density region. Another feature emerging from Fig. 3 is that the light-cluster fractions in dense nuclear matter increase significantly with increasing temperature. At high temperature, more nucleons populate high-momentum regions in phase space, where light clusters are allowed to exist. Therefore, a weaker in-medium suppression of light clusters is observed. In the case of $\delta = 0.13$, the helium-3 fraction is suppressed, as expected, due to a lower μ_p compared with the symmetric case. For d and α particles, since their chemical potentials are less affected by the isospin asymmetry, the larger Mott momenta observed in Fig. 1 for asymmetric matter lead to the suppression of their fractions, especially for α particles as observed in Fig. 3(d).

Sharing the same underlying concept of the Mott momentum, the present framework can be regarded as a phenomenological version of the quantum statistical approach [11,46], in which the parameters governing the strength of in-medium effects, i.e., $\mathbf{F}^{\text{cut}}$, are determined through

benchmarking against experimental data. The present approach is therefore expected to yield light-cluster fractions similar to those obtained from the quantum statistical approach. However, in the results shown in Ref. [11], the particle fractions include the (negative) contribution from scattering states, leading to their stronger suppression at densities near saturation. In the present approach, which can be regarded as an approximation to a more exact treatment of the correlations, these scattering-state contributions are not taken into account. As a result, the suppression of light-cluster fractions at high densities is weaker.

One of the implications of the present work is to propose a connection between the in-medium modifications of light-cluster properties in nuclear matter and the light-nuclei productions in heavy-ion collisions at intermediate energies. Based on dynamical models, it has been shown that light clusters are mainly produced in the compression stage of the collision [39]. As a consequence, the final-state light-nuclei yields are largely related to the light-cluster fractions in warm and dense nuclear matter, produced transiently in the compression stage. Therefore, we focus on warm and dense nuclear matter and examine the influence of the strength of the in-medium effect, i.e., the magnitude of F_A^{cut} , on light-cluster fractions.

The fractions of d , t , h , and α in dense symmetric nuclear matter at temperatures $T = 20$ and 50 MeV, obtained with the $\mathbf{F}^{\text{cut}}$ sets (i), (ii), and (iii), are compared in Fig. 4. A considerable influence of the magnitude of F_A^{cut} on the light-cluster fractions is observed. $\mathbf{F}^{\text{cut}}$ (iii), which

adopts a smaller F_4^{cut} for α with respect to $\mathbf{F}^{\text{cut}}$ (i), leads to significantly lower α -particle fractions, and subsequently increases slightly the fractions of the other light-cluster species, due to baryon number conservation. The overall larger $\mathbf{F}^{\text{cut}}$ (ii) results in larger fractions for all light-cluster species at high densities (say, beyond ρ_0). However, in the case of $T = 50$ MeV, a suppression of the α -particle fraction is observed at low densities, due to the more enhanced deuteron fraction, indicating the interplay between the F_A^{cut} values of different light-cluster species. This sensitivity to F_A^{cut} has already been observed for the light-nuclei yields in heavy-ion collisions, based on transport models adopting previous formulations of the phase-space excluded-volume approach [39,45]. By implementing the present, more refined, approach into dynamical models and comparing with experimental light-nuclei yields in intermediate-energy heavy-ion collisions, the magnitude of F_A^{cut} , which represents the strength of the in-medium effects on light clusters, can be deduced, and thus information about the light-cluster fractions in warm and dense nuclear matter can be accessed. These studies are in progress and will be reported elsewhere [55].

C. Sensitivity to neutron-proton effective mass splitting

The concept of nucleon effective mass is widely used to characterize the momentum and/or energy dependence of the single-nucleon potential or the real part of the nucleon self-energy in the nuclear medium. In asymmetric nuclear matter, the difference between neutron and proton potentials may lead to the splitting of their effective masses. Its dependence on the matter asymmetry is usually characterized by the linear isospin splitting coefficient $\Delta m_1^* = \frac{\partial m_{n-p}^*(\rho_0)}{\partial \delta}|_{\delta=0}$. Empirically, the isoscalar effective mass m_s^* —the nucleon effective mass in symmetric nuclear matter—and m_{n-p}^* can be determined from analyses of finite nuclei structure [56] and collective motions [57–61], and nuclear reactions [62–65] using mean-field and/or dynamical models, as well as optical model analyses of nucleon-nucleus scattering data [66]. One may refer to Ref. [67] for a recent review on nucleon effective masses.

Since the effective mass enters the phase-space distribution function, the isoscalar effective mass, as well as the linear isospin splitting coefficient Δm_1^* , influences the occupations of different particle species. We concentrate here on the effect of the isospin splitting coefficient, which is currently more debated. We employ Skyrme interactions with distinct Δm_1^* values, namely, MSL1 and MSL1-m, to explore this Δm_1^* sensitivity of various properties of nuclear matter with light clusters. At the nuclear saturation density, $\Delta m_1^* = 0.23m$ and $\Delta m_1^* = -0.17m$ for MSL1 and MSL1-m, respectively. Details of the nucleon effective masses with the standard Skyrme energy-density functional can be found in Appendix A. In the left-hand panel of Fig. 5, we show the light-cluster fractions X_ν as functions of baryon density ρ_B for asymmetric nuclear matter with $\delta = 0.13$ at temperature $T = 20$ MeV, obtained with the phase-space excluded-volume approach employing the Skyrme interactions MSL1 and MSL1-m. In these calculations, $\mathbf{F}^{\text{cut}}$ set (i) is adopted. It is seen from the figure that the influence of Δm_1^* on particle fractions is negligible, due to

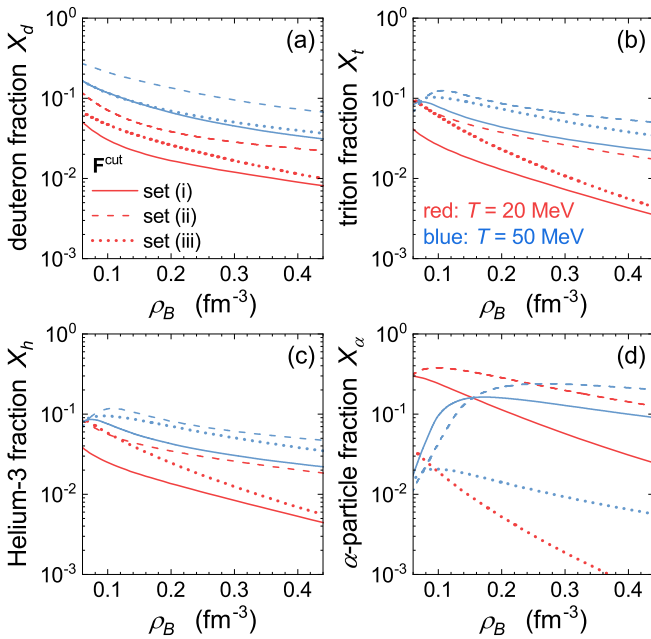


FIG. 4. Fractions of (a) d , (b) t , (c) h , and (d) α of dense symmetric nuclear matter as functions of baryon density ρ_B at temperatures $T = 20$ and 50 MeV obtained with the phase-space excluded-volume approach. We employ the Skyrme interaction MSL1, and three $\mathbf{F}^{\text{cut}}$ sets in the calculations, namely, $\mathbf{F}^{\text{cut}}$ (i) = (0.10, 0.15, 0.25), $\mathbf{F}^{\text{cut}}$ (ii) = (0.20, 0.25, 0.35), and $\mathbf{F}^{\text{cut}}$ (iii) = (0.10, 0.15, 0.15).

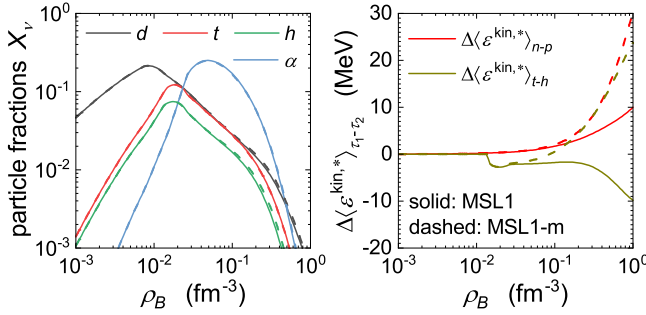


FIG. 5. The light-cluster fractions X_v (left) and the difference between the average single-particle effective kinetic energies of two particle species $\Delta\langle\epsilon^{\text{kin},*}\rangle_{\tau_1-\tau_2}$ (right) as functions of ρ_B for nuclear matter with isospin asymmetry $\delta = 0.13$ at temperature $T = 20$ MeV, obtained with the phase-space excluded-volume approach. $\mathbf{F}^{\text{cut}}$ (i) = (0.10, 0.15, 0.25), and two Skyrme interactions, i.e., MSL1 and MSL1-m, are employed. The nonsmooth behavior of $\Delta\langle\epsilon^{\text{kin},*}\rangle_{t-h}$ in the right-hand panel is caused by the emergence of nonzero Mott momentum.

the fixed isospin asymmetry. The same situation also applies to the obtained Mott momenta.

We further examine the influence of Δm_1^* on the average single-particle effective kinetic energies $\langle\epsilon^{\text{kin},*}\rangle$ of certain particle species. The difference in $\langle\epsilon^{\text{kin},*}\rangle$ between n and p , i.e., $\Delta\langle\epsilon^{\text{kin},*}\rangle_{n-p}$, and that between t and h , i.e., $\Delta\langle\epsilon^{\text{kin},*}\rangle_{t-h}$ are presented, as functions of ρ_B , in the right-hand panel of Fig. 5. The figure exhibits two interesting features. In the first, $\Delta\langle\epsilon^{\text{kin},*}\rangle_{t-h}$ shows greater sensitivity to Δm_1^* than $\Delta\langle\epsilon^{\text{kin},*}\rangle_{n-p}$. Moreover, a large negative value of $\Delta\langle\epsilon^{\text{kin},*}\rangle_{t-h}$ is observed in the high-density region in the case of MSL1. These two phenomena can be understood as follows.

The average single-particle effective kinetic energies are roughly related to the average single-particle kinetic energies by $\langle\epsilon^{\text{kin},*}\rangle \approx \frac{m}{m^*}\langle\epsilon^{\text{kin}}\rangle$ (exact in the nonrelativistic limit). Noting that the dependence of $\langle\epsilon^{\text{kin}}\rangle$ on Δm_1^* is relatively small, for a fixed m_s^* , changing Δm_1^* from negative to positive values leads to a decrease of $\langle\epsilon^{\text{kin},*}\rangle$ for n and t , and an increase of $\langle\epsilon^{\text{kin},*}\rangle$ for p and h . Therefore, the positive Δm_1^* of MSL1 results in the reduction of both $\Delta\langle\epsilon^{\text{kin},*}\rangle_{n-p}$ and $\Delta\langle\epsilon^{\text{kin},*}\rangle_{t-h}$, with respect to the case of MSL1-m (where Δm_1^* is negative), as can be observed in the right-hand panel of Fig. 5. For tritons and helium-3, their $\langle\epsilon^{\text{kin}}\rangle$ are enhanced due to their nonzero Mott momenta (see Fig. 1). Accordingly, the deviation of $\Delta\langle\epsilon^{\text{kin},*}\rangle_{t-h}$ between the cases MSL1 and MSL1-m is amplified, compared with that of $\Delta\langle\epsilon^{\text{kin},*}\rangle_{n-p}$. This amplification is already pronounced at nuclear saturation density, where the triton and helium-3 fractions remain non-negligible. It therefore indicates that observables of tritons and helium-3 (for instance, final-state average kinetic energies in heavy-ion collisions) might be more efficient probes for constraining Δm_1^* or neutron-proton effective mass splitting than those of neutrons and protons.

The second feature, i.e., the large negative value of $\Delta\langle\epsilon^{\text{kin},*}\rangle_{t-h}$ for MSL1, can be understood similarly. At low densities (say, below 0.1 fm^{-3}) where the influence of the effective mass is still less pronounced, for both interactions,

the magnitude of $\langle\epsilon^{\text{kin},*}\rangle$ for h is larger than that for t . This is in contrast with the behavior observed for n and p , which arises from the larger μ_n with respect to μ_p in neutron-rich matter (resulting in a larger $\langle\epsilon^{\text{kin}}\rangle_n$ compared with $\langle\epsilon^{\text{kin}}\rangle_p$). This observation can be attributed to a larger Mott momentum of h compared with t at low densities, which drives h to populate higher-momentum regions in phase space. This larger Mott momentum of h (though barely noticeable in Fig. 1) stems from the more compact momentum distribution of the constituent nucleons within h ($\sigma_h > \sigma_t$). For MSL1-m, $\Delta\langle\epsilon^{\text{kin},*}\rangle_{t-h}$ changes its sign at higher densities due to the negative Δm_1^* . In contrast, for MSL1, because of the positive Δm_1^* , the $\Delta\langle\epsilon^{\text{kin},*}\rangle_{t-h}$ remains negative. The boost of the magnitude of $\Delta\langle\epsilon^{\text{kin},*}\rangle_{t-h}$ is again due to the enhanced $\langle\epsilon^{\text{kin}}\rangle_v$ of t and h caused by the Mott effect (recall the rough relation $\langle\epsilon^{\text{kin},*}\rangle \approx \frac{m}{m^*}\langle\epsilon^{\text{kin}}\rangle$). This large negative value of $\Delta\langle\epsilon^{\text{kin},*}\rangle_{t-h}$ obtained with interactions featuring a positive m_{n-p}^* is in line with observations in heavy-ion collisions often referred to as the “ ^3He puzzle” [68–70]; i.e., the average kinetic energy of the emitted helium-3 is larger than that of tritons. That is to say, the in-medium effects on tritons and helium-3, along with a positive m_{n-p}^* , may offer a potential resolution to this puzzle. Detailed studies along this line will be pursued in the future.

IV. COMPARISON WITH IN-MEDIUM SCHRÖDINGER EQUATION

To compare the phase-space excluded-volume approach with the in-medium Schrödinger equation in describing light-cluster properties—such as the Mott momentum—we focus on the two-body case, thus restricting our analysis to nuclear matter composed only of nucleons and deuterons. We then solve Eq. (1) and compare the resulting Mott momentum and deuteron fraction with those obtained from the phase-space excluded-volume approach. When solving the in-medium Schrödinger equation, as usual, the uncorrelated-medium assumption is adopted; i.e., the total nucleon occupations $f_\tau^{\text{tot}}(\vec{p})$ in Eq. (1) are approximated by a Fermi-Dirac distribution,

$$f_\tau^{\text{tot}}(\vec{p}) \approx \tilde{f}_\tau^{\text{tot}}(\vec{p}) \equiv \frac{1}{\exp\{\beta[\epsilon_\tau(\vec{p}) - \tilde{\mu}_\tau]\} + 1}, \quad (21)$$

with $\tilde{\mu}_\tau$ determined by requiring that $\int g \tilde{f}_\tau^{\text{tot}}(\vec{p}) \frac{d\vec{p}}{(2\pi\hbar)^3}$ yields the total nucleon density ρ_τ^{tot} . For the single-nucleon quasi-particle energies in Eq. (1), in this section, we approximate the effective mass by the bare nucleon mass, for simplicity. As for the two-body potential in Eq. (1), we choose a simple Gaussian-form separable potential,

$$\begin{aligned} V_{np}(\vec{p}_i, \vec{p}_j; \vec{p}'_i, \vec{p}'_j) \\ = \lambda_d \delta(\vec{p}'_i + \vec{p}'_j - \vec{p}_i - \vec{p}_j) \exp\left[-\frac{(\vec{p}_i - \vec{p}_j)^2}{4\gamma_d^2 \hbar^2}\right] \\ \times \exp\left[-\frac{(\vec{p}'_i - \vec{p}'_j)^2}{4\gamma_d^2 \hbar^2}\right], \end{aligned} \quad (22)$$

with $\lambda_d = -1.2874 \text{ GeV fm}^3$ and $\gamma_d = 1.474 \text{ fm}^{-1}$ determined by the binding energy and the rms radius of deuterons in free space [71].

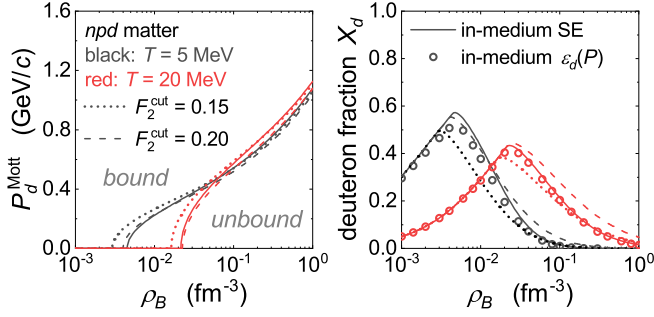


FIG. 6. The deuteron Mott momentum P_d^{Mott} (left) and fraction X_d (right) as functions of ρ_B for symmetric npd matter at two different temperatures, obtained by solving the in-medium Schrödinger equation [in-medium SE and in-medium $\epsilon_d(P)$], and from the phase-space excluded-volume approach with two different F_2^{cut} .

By means of Eq. (21), the in-medium Schrödinger equation [Eq. (1)] is decoupled from Eqs. (2)–(4) and becomes manageable to solve to obtain the Mott momentum P_d^{Mott} . Similarly, within the phase-space excluded-volume approach, P_d^{Mott} can be obtained solely from Eq. (6). For given P_d^{Mott} , one then determines the chemical potentials, imposing that, from $f_\tau(\vec{p})$ in Eq. (12) and $f_d(\vec{p})$ in Eq. (14), the total nucleon density ρ_τ^{tot} is obtained, under the chemical equilibrium condition. Subsequently, the particle fractions are evaluated. In this process, we further neglect the $f_\tau^{\text{clu}}(\vec{p})$ in Eq. (12) for simplicity. We discuss the validity of this approximation, as well as the uncorrelated-medium assumption, in Appendix B. Note that the above two approximations are only adopted in the calculations presented in this Section.

We present in Fig. 6 the deuteron Mott momentum P_d^{Mott} and fraction X_d , obtained within the two approaches, as functions of ρ_B , for nuclear matter at temperatures $T = 5$ and 20 MeV. In the case of the phase-space excluded-volume approach, two F_2^{cut} values are employed. For the in-medium Schrödinger equation, we consider two cases, the one (circles) that includes the in-medium correction on the single-particle energy $\epsilon_d(\vec{p})$ entering Eq. (14), and the other one (solid lines) where we simply employ the free-space binding energy, as done in the phase-space excluded-volume approach. The overall effect of the in-medium correction of $\epsilon_d(\vec{p})$ is not that pronounced, especially in the high-temperature case. Comparing the results from the two approaches, it is found that, for both temperatures, the obtained P_d^{Mott} and X_d from the in-medium Schrödinger equation tend to be better reproduced by a larger F_2^{cut} for small ρ_B , and a smaller F_2^{cut} for large ρ_B . Since this density dependence of F_2^{cut} is not that remarkable, a rough overall agreement in terms of P_d^{Mott} and X_d can still be achieved for both temperatures with a constant value of F_2^{cut} lying in between 0.15 and 0.20. In addition, in certain nuclear processes, e.g., heavy-ion collisions, one typically does not need to consider nuclear matter over such a wide density range spanning several orders of magnitude, which also legitimizes the use of a constant F_A^{cut} . To reproduce, with a fixed F_A^{cut} , the Mott momentum obtained by solving the in-medium Schrödinger equation, additional momentum-dependent factors might need to be introduced in the integral of Eq. (6),

thereby affecting the density dependence of the resulting Mott momenta. For deuterons, for instance, solving the in-medium Schrödinger equation suggests introducing an additional p^2 dependence in the momentum integral. This would lead to a prescription physically corresponding to a constraint on the Pauli-blocking energy shift, which still ensures that the light clusters remain bound (rather than a constraint on the phase-space occupation). Work is underway in this direction.

In the end, we want to mention that the results obtained from solving the in-medium Schrödinger equation, displayed in Fig. 6, should not be regarded as a strict reference, since they actually depend on the choice of the many-body interaction $V(1, 2, \dots, A; 1', 2', \dots, A')$. For a realistic, nonseparable potential, the equation becomes increasingly complicated in nuclear matter, let alone in dynamical processes. In this sense, the empirical information deduced from experiments, based on effective approaches such as the one presented in this work, is still needed to get access to the in-medium properties of light clusters.

V. SUMMARY AND OUTLOOK

We have employed a phase-space excluded-volume approach for light clusters to study their in-medium properties in nuclear matter. In this approach, the total nucleon phase-space occupation around a light cluster, with contributions from both quasifree nucleons and light clusters, is compared with a cut-off parameter F_A^{cut} to determine whether the light cluster can survive in the nuclear medium. This criterion is inspired by the more sophisticated in-medium Schrödinger equation, but its implementation requires significantly less computational effort. The parameter F_A^{cut} can be regarded as a surrogate of the strength of in-medium effects on light clusters. The present approach improves upon the conventional treatment of the one-body nucleon phase-space occupation based on an uncorrelated medium, typically adopted in studies of light clusters in nuclear matter. The Mott momenta and light-cluster fractions in nuclear matter have been obtained, with the nuclear mean-field potential modeled using standard Skyrme energy-density functionals. Our study allows to assess the sensitivity of these in-medium properties to various conditions of density, temperature, isospin asymmetry, and strength of in-medium effects represented by different F_A^{cut} values. We have further shown that the effects of the present excluded-volume approach are quite comparable with those obtained from the in-medium Schrödinger equation in a simplified case. When exploring the kinetic properties of light clusters, it is also found that, in nuclear matter, the difference in the average effective kinetic energies between tritons and helium-3, i.e., $\Delta\langle\epsilon^{\text{kin},*}\rangle_{t-h}$, is quite sensitive to the neutron-proton effective mass splitting m_{n-p}^* , mainly due to the Mott effect. A weaker sensitivity is seen for the neutron-proton average effective kinetic energies difference $\Delta\langle\epsilon^{\text{kin},*}\rangle_{n-p}$, implying that light-nuclei observables (mainly available in heavy-ion collisions) could be more efficient probes of isospin-dependent quantities than nucleon observables.

As a further development, the present phase-space excluded-volume approach can be employed to calculate the equation of state of nuclear matter with light clusters at both

zero and finite temperatures. Moreover, since the approach can be applied to the nuclear medium with arbitrary phase-space distributions, it bridges the in-medium properties of light clusters in nuclear matter, e.g., light-cluster fractions, with those in nonequilibrium processes like heavy-ion collisions. The present work thus paves the way for probing the in-medium properties of light clusters in warm and dense nuclear matter through intermediate-energy heavy-ion collisions. These studies are being pursued and will be reported elsewhere [55].

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: SKYRME ENERGY-DENSITY FUNCTIONALS AND NUCLEON EFFECTIVE MASSES

Within the standard Skyrme energy-density functional [52,53], the nucleon single-particle potential $\epsilon_\tau^{\text{pot}}(\vec{p})$ can be expressed as

$$\begin{aligned} \epsilon_\tau^{\text{pot}}(\vec{p}) = & \epsilon_\tau^{\text{pot},0}(\rho_n, \rho_p) \\ & + \frac{C^{[2]}}{8\hbar^2} \int \frac{d\vec{p}'}{(2\pi\hbar)^3} (\vec{p} - \vec{p}')^2 g[f_n^{\text{tot}}(\vec{p}') + f_p^{\text{tot}}(\vec{p}')] \\ & + \frac{D^{[2]}}{8\hbar^2} \int \frac{d\vec{p}'}{(2\pi\hbar)^3} (\vec{p} - \vec{p}')^2 g f_\tau^{\text{tot}}(\vec{p}'), \end{aligned} \quad (\text{A1})$$

with $\epsilon_\tau^{\text{pot},0}(\rho_n, \rho_p)$ being the density-dependent part, and $C^{[2]}$ and $D^{[2]}$ being combinations of the Skyrme parameters t_1 , x_1 , t_2 , and x_2 , defined as

$$C^{[2]} = t_1(x_1 + 2) + t_2(x_2 + 2), \quad (\text{A2})$$

$$D^{[2]} = -t_1(2x_1 + 1) + t_2(2x_2 + 1). \quad (\text{A3})$$

Note that ρ_τ in Eq. (A1) should be understood as ρ_τ^{tot} . The advantage of employing the standard Skyrme interaction is that, in nuclear matter, the momentum dependence of $\epsilon_\tau^{\text{pot}}$ can be recast into the kinetic part $\epsilon_\tau^{\text{kin}}$ by replacing the mass with the effective mass m^* . This can be seen by expanding the integrand in $C^{[2]}$ and $D^{[2]}$ terms:

$$\int \frac{d\vec{p}'}{(2\pi\hbar)^3} (\vec{p} - \vec{p}')^2 g f_\tau^{\text{tot}}(\vec{p}') = p^2 \rho_\tau^{\text{tot}} + \langle p^2 \rangle_\tau. \quad (\text{A4})$$

TABLE II. Skyrme parameters of MSL1 and MSL1-m. Note that only two free parameters among t_1 , x_1 , t_2 , and x_2 are needed for describing nuclear matter, namely, their combinations $C^{[2]}$ and $D^{[2]}$ given in Eqs. (A2) and (A3), respectively.

Skyrme parameter	MSL1	MSL1-m
t_0 (MeV fm ³)	−1963.23	−1962.93
t_1 (MeV fm ⁵)	379.845	379.795
t_2 (MeV fm ⁵)	−394.554	−714.113
t_3 (MeV fm ^{3+3σ})	12174.9	12174.3
x_0	0.320770	0.529806
x_1	0.344849	0.134762
x_2	−0.847304	−1.02737
x_3	0.321930	0.865856
σ	0.269359	0.269417

Note that the $\vec{p} \cdot \vec{p}'$ term vanishes because of the spherical symmetry of the momentum distribution in nuclear matter. Since the p -dependent part, i.e., $p^2 \rho_\tau^{\text{tot}}$, does not explicitly depend on $f_\tau^{\text{tot}}(\vec{p})$, it can be combined with the kinetic part $\epsilon_\tau^{\text{kin}}$ to define the effective kinetic energy $\epsilon_\tau^{\text{kin},*}$. With Eq. (A1), the nucleon effective mass in Eq. (17) can be expressed as

$$\begin{aligned} \frac{m}{m_\tau^*} & \equiv 1 + m \frac{\vec{p} \cdot \nabla_{\vec{p}} \epsilon_\tau^{\text{pot}}(\vec{p})}{p^2} \\ & = 1 + \frac{m}{4\hbar^2} [C^{[2]}(\rho_n^{\text{tot}} + \rho_p^{\text{tot}}) + D^{[2]} \rho_\tau^{\text{tot}}]. \end{aligned} \quad (\text{A5})$$

Effective masses of light clusters expressed in Eq. (18) can be obtained similarly. It can be seen that, for standard Skyrme interactions, the effective mass only depends on the (total) nucleon density and not on the single-particle momentum. More generally, for Skyrme interactions extended to include higher-order derivative terms (relative-momentum terms) [72–78], this condition does not hold any more.

The Skyrme parameters of MSL1 and MSL1-m used in the present work are provided in Table II. Note that only parameters $C^{[2]}$ and $D^{[2]}$ given in Eqs. (A2) and (A3), respectively, are relevant for the calculations presented in this work. For the corresponding macroscopic quantities [79], such as the incompressibility coefficient K_0 and the symmetry energy $E_{\text{sym}}(\rho_0)$ at nuclear saturation density, one may refer to Ref. [54]. These quantities given by MSL1 and MSL1-m differ only in the isoscalar effective mass $m_{v,0}^*$ (related to Δm_1^*).

APPENDIX B: APPROXIMATIONS ON TOTAL NUCLEON OCCUPATION

As we have mentioned in the main text, an uncorrelated-medium assumption, i.e., replacing the total nucleon occupation $f_\tau^{\text{tot}}(\vec{p})$ in the in-medium Schrödinger equation [Eq. (1)] with a Fermi-Dirac distribution $\tilde{f}_\tau^{\text{tot}}(\vec{p})$ given in Eq. (21), is usually adopted to reduce computational complexity in nuclear matter calculations. With the uncorrelated-medium assumption, one can first obtain the Mott momenta of light clusters solely from Eq. (1) or Eq. (7), and then treat the nuclear matter as a standard thermal equilibrium system composed of multiple particle species.

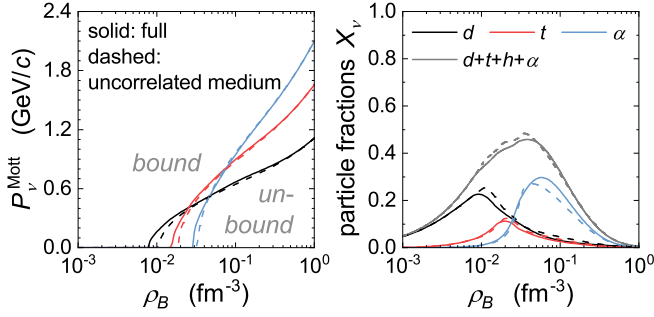


FIG. 7. Mott momenta P_v^{Mott} (left) and fractions X_v (right) of light clusters of symmetric nuclear matter as functions of baryon density ρ_B at temperature $T = 20$ MeV obtained using the phase-space excluded-volume approach with $\mathbf{F}^{\text{cut}}$ (i) = (0.10, 0.15, 0.25) and Skyrme interaction MSL1. Dashed lines represent the results where the uncorrelated-medium assumption is employed.

To examine the validity of this assumption, we employ it in the phase-space excluded-volume approach, i.e., replacing the $f_\tau^{\text{tot}}(\vec{p})$ in Eq. (6) with $\tilde{f}_\tau^{\text{tot}}(\vec{p})$, and compare the obtained Mott momenta P_v^{Mott} and fractions X_v of light clusters with those from the full calculations presented in Secs. III A and III B. The comparison is shown in Fig. 7 for d , t , and α . Calculations are performed for symmetric nuclear matter at temperature $T = 20$ MeV, with $\mathbf{F}^{\text{cut}}$ set (i) and the Skyrme interaction MSL1. The results for the full phase-space excluded-volume calculation are the same as those in Figs. 1 and 3. It is seen from the figure that the application of the uncorrelated-medium assumption only causes slight deviations with respect to the full calculations for both P_v^{Mott} and X_v .

We then go into the detailed nucleon occupations in momentum space obtained within the phase-space excluded-volume approach. We present in Fig. 8 the total neutron occupation $f_n^{\text{tot}}(\vec{p})$ [Eq. (13)], that from quasifree neutrons, $f_n(\vec{p})$ [Eq. (12)], and that from neutrons bound in light clusters, $f_n^{\text{clu}}(\vec{p})$ [Eq. (3)] (top panels), and the momentum distribution of deuterons, tritons, and α particles [Eq. (14)] (center panels), corresponding to the calculations shown in Fig. 7 at two baryon densities. It is seen from the figure that the light-cluster distributions shift toward higher momenta as the density increases, according to the values of the Mott momenta represented in the left-hand panel of Fig. 7, thereby displacing the $f_n^{\text{clu}}(\vec{p})$ to larger momenta. Also shown in the figure is the Fermi-Dirac distribution $\tilde{f}_\tau^{\text{tot}}(\vec{p})$ in Eq. (21) employed in the uncorrelated-medium assumption (or the nucleon distribution in pure nucleon matter), whose difference from the exact $f_n^{\text{tot}}(\vec{p})$ is clearly seen. To better illustrate the importance of including nucleon clustering, the occupations shown in the top panels are replotted after being multiplied by the square of the wave vector $k = \frac{p}{\hbar}$ (bottom panels). In nuclear matter at higher densities, where light clusters populate higher-momentum regions, the total nucleon occupation $f_n^{\text{tot}}(\vec{p})$ shifts toward larger momenta, leading to an increase in the kinetic energy of the system and affecting its equation of state. However, employing $\tilde{f}_\tau^{\text{tot}}(\vec{p})$ is found to be effective for evaluating Mott momenta. Since $\tilde{f}_\tau^{\text{tot}}(\vec{p})$ roughly coincides with the exact $f_\tau^{\text{tot}}(\vec{p})$ when they become small, to

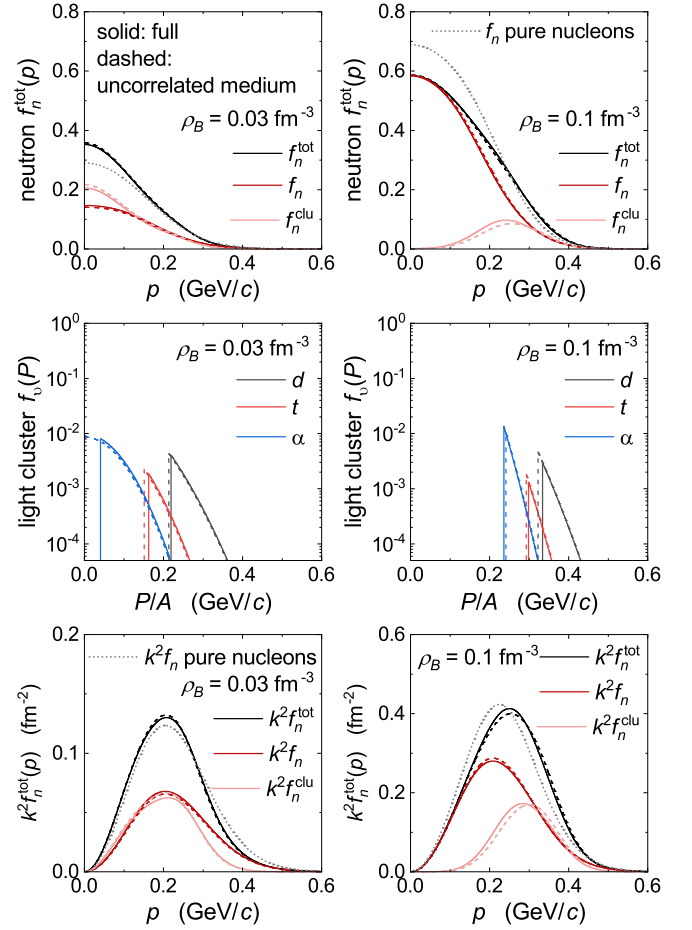


FIG. 8. Neutron occupations $f_n^{\text{tot}}(\vec{p})$, $f_n(\vec{p})$, and $f_n^{\text{clu}}(\vec{p})$ (top), momentum distributions for deuterons, tritons, and α particles (center), and neutron occupations multiplied by the square of the wave vector k (bottom), corresponding to the calculations shown in Fig. 7 for two different ρ_B . The dotted gray lines represent $\tilde{f}_\tau^{\text{tot}}(\vec{p})$, or the neutron distribution in pure nucleon matter at the indicated temperature and density.

which the Mott momenta are more relevant, they finally lead to similar Mott momenta and fractions of light clusters in nuclear matter, as seen in Fig. 7. This finding justifies to some extent approximating $f_\tau^{\text{tot}}(\vec{p})$ by $\tilde{f}_\tau^{\text{tot}}(\vec{p})$ in the in-medium Schrödinger equation. Nonetheless, the uncorrelated-medium assumption is applicable only in calculations of nuclear matter. In nonequilibrium processes, $f_\tau^{\text{tot}}(\vec{p})$ should be evaluated self-consistently, most likely numerically, depending on specific conditions.

The other point we want to discuss concerns the correction on the occupation of quasifree nucleons from light clusters. In Eq. (12), we propose that the occupation of quasifree nucleons in nuclear matter should be modified due to the presence of light clusters and thus deviate from the Fermi-Dirac distribution, since in principle the Pauli-blocking effect on quasifree nucleons from constituent nucleons in light clusters should also be taken into account. To determine the nucleon chemical potential μ_τ by normalizing Eq. (13) usually involves iteration procedures. The presence of the additional term $f_\tau^{\text{clu}}(\vec{p})$ in the numerator of Eq. (12) obliges

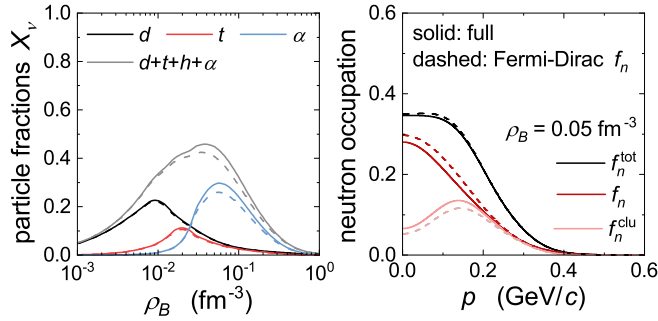


FIG. 9. Left: light-cluster fractions X_v of symmetric nuclear matter as functions of baryon density ρ_B at temperature $T = 20$ MeV using the phase-space excluded-volume approach with $\mathbf{F}^{\text{cut}}$ (i) = (0.10, 0.15, 0.25) and Skyrme interaction MSL1. Right: the corresponding neutron occupations $f_n^{\text{tot}}(\vec{p})$, $f_n(\vec{p})$, and $f_n^{\text{clu}}(\vec{p})$ at $\rho_B = 0.05 \text{ fm}^{-3}$. Dashed lines represent the results where the $f_\tau^{\text{clu}}(\vec{p})$ in Eq. (12) is neglected.

one to calculate the integrals in Eqs. (3) and (4) for each trial μ_τ , causing substantial extra computational effort. We therefore examine whether this additional term plays an important role in calculating light-cluster fractions.

We calculate the light-cluster fractions in nuclear matter at $T = 20$ MeV using $\mathbf{F}^{\text{cut}}$ (i) and the Skyrme interaction MSL1 (the same as those in Fig. 7), employing the phase-space excluded-volume approach but neglecting the $f_\tau^{\text{clu}}(\vec{p})$ in

Eq. (12) [imposing a Fermi-Dirac $f_\tau(\vec{p})$]. Note the distinction between this Fermi-Dirac $f_\tau(\vec{p})$ and $f_\tau^{\text{tot}}(\vec{p})$, with the former representing the occupation of quasifree nucleons and the latter being an approximation of the total nucleon occupation. Apart from the light-cluster fractions X_v given in the left-hand panel of Fig. 9, we show in the right-hand panel the corresponding neutron occupations $f_n^{\text{tot}}(\vec{p})$, $f_n(\vec{p})$, and $f_n^{\text{clu}}(\vec{p})$ at $\rho_B = 0.05 \text{ fm}^{-3}$. The correction $f_\tau^{\text{clu}}(\vec{p})$ in Eq. (12) decreases the magnitude of $f_\tau(\vec{p})$. It therefore results in a systematic increase in light-cluster fractions, as can be seen in the left-hand panel of Fig. 9. However, the influence of the correction on the total nucleon occupation f_τ^{tot} is negligible, mainly due to the particle-number conservation. It therefore means that the Mott momenta obtained with and without the correction are almost identical to each other, which is the reason why we only show the light-cluster fractions in Fig. 9.

In general, the effect of omitting $f_\tau^{\text{clu}}(\vec{p})$ in Eq. (12), as well as adopting the uncorrelated-medium assumption, is not that pronounced in the cases shown here, mainly due to the constraint of a fixed density. However, these two approximations may break down when the light-cluster fractions are large enough, for example, when larger values of F_A^{cut} are used. However, the treatment presented in this article is more practically relevant for realistic transport calculations of heavy-ion collisions, since determining local temperatures, and hence applying the uncorrelated-medium assumption, is generally difficult in such calculations.

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