

Radiative corrections to the parity-violating spin asymmetry

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The parity-violating spin asymmetry A_{pv} for elastic electron scattering from spin-zero nuclei, together with its QED corrections, is evaluated nonperturbatively within the phase-shift analysis. Dispersion corrections, taking into account low-lying transient nuclear excitations, are estimated with the help of the lowest-order γZ box diagrams. Collision energies in the range 5–500 MeV are considered, and results are provided for the ^{12}C and ^{208}Pb target nuclei. In addition, we have evaluated A_{pv} at GeV energies and small scattering angles—relevant for the Pb Radius Experiment (PREx)—revealing that the low-lying nuclear excited states give no measurable dispersive contribution. However, they are important at lower energies and backward angles.

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I. INTRODUCTION

High-precision measurements of the parity-violating spin asymmetry A_{pv} at GeV collision energies have become feasible during recent years [1–3], improving earlier experiments [4]. Since A_{pv} depends strongly on the neutron ground-state distribution and on the Weinberg mixing angle, it serves to extract the target neutron skin [5–7], but also provides information on the weak neutral current couplings and the weak charge radius [8–10]. For this application of A_{pv} it is mandatory for theory to take into account any possible corrections as precisely as possible. Here we consider the radiative corrections from vacuum polarization and the vertex plus self-energy correction, as well as from dispersion, based on the excitation of low-lying nuclear states. Dispersion estimates are usually obtained from the γZ box diagrams [11,12] and are used to correct the weak charge Q_w which enters into the normalization of the weak potential A_w [5,13]. Also the universal radiative corrections, estimated in the Born approximation, are included in the modification of Q_w [11,14]. An exact calculation of dispersion involves the sum over all transient excited nuclear states. Several approximations have been in use. The earliest one is the closure approximation where all nuclear excitations are taken at the same energy, which lies preferably in the giant dipole resonance region

[15]. For lower collision energies it is more adequate to consider explicitly the most collective excited states with small angular momentum and extending up to about 30 MeV [16]. A different approach, applicable for higher collision energies above the pion production threshold (135 MeV), relies on the relation between dispersion, the forward nuclear Compton amplitude, and the experimental photoproduction cross section [17–19]. An application to the γZ box diagram can be found in [12,20]. Alternatively, the relation between dispersion and the structure functions from parity-violating deep inelastic scattering can be employed [11].

The present work is aimed at an interpretation of future precision experiments at the MESA facility on the parity-violating asymmetry for low-energy electron scattering [21]. Hence the QED effects are treated nonperturbatively by expressing them in terms of potentials [22–24] which are included in the Dirac equation for the electronic scattering states. Moreover, in the γZ box diagrams, the low-lying nuclear excitations are considered explicitly in terms of their transition densities as obtained from nuclear Energy Density Functionals [25–27]. By inserting the dispersion correction directly into the differential scattering cross section, one is independent of the widely used Born representation of A_{pv} .

The paper is organized as follows: Sec. II describes the theory for A_{pv} and its radiative corrections. Results for the ^{12}C and ^{208}Pb target nuclei are presented in Sec. III. Concluding remarks follow in Sec. IV. Atomic units ($\hbar = m_e = e = 1$) are used unless indicated otherwise.

II. THEORY

In this section the theory for A_{pv} is briefly recapitulated. Subsequently, the treatment of the QED corrections is presented. Finally dispersion is considered to lowest order.

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A. Parity-violating spin asymmetry

For collision energies well above 10 MeV, the electron mass m_e can usually be neglected. Hence the electrons polarized longitudinally, i.e., parallel (+) or antiparallel (−) to their direction of motion, are described by the helicity eigenstates ψ_+ or ψ_- , respectively, which are solutions to the Dirac equation

$$[-ic\boldsymbol{\alpha}\nabla + V(r) \pm A(r)] \psi_{\pm}(\mathbf{r}) = E \psi_{\pm}(\mathbf{r}), \quad (2.1)$$

where, to start with, $V(r) = V_T(r)$ is taken as the Coulombic nuclear target potential and $A(r) = A_w(r)$ as the weak potential which is related to the weak density $\varrho_w(r)$ by means of [5]

$$A_w(r) = \frac{G_F c}{2\sqrt{2}} \varrho_w(r), \quad (2.2)$$

where $G_F = 1.16638 \times 10^{-5} \text{ GeV}^{-2}$, equivalent to $G_F c = 6.2283 \times 10^{-5} \text{ fm}^2$, is the Fermi coupling constant.

The weak density is calculated from the convolution of the pointlike proton (ϱ_p , normalized to the number of protons Z) and neutron (ϱ_n , normalized to the number N of neutrons) nuclear density distributions with the proton (G_p^w) and neutron (G_n^w) weak form factors as in [6,7,28]

$$\varrho_w(\mathbf{r}) = \int d\mathbf{r}' [G_p^w(\mathbf{r}') \varrho_p(\mathbf{r} - \mathbf{r}') + G_n^w(\mathbf{r}') \varrho_n(\mathbf{r} - \mathbf{r}')]. \quad (2.3)$$

The latter ones can be written in terms of the electromagnetic form factors ($G_{n,p}^{\text{em}}$) and the weak charges of the neutron (Q_n^w) and the proton (Q_p^w) in the usual way,

$$G_p^w = Q_p^w G_p^{\text{em}} + Q_n^w G_n^{\text{em}}, \quad (2.4)$$

$$G_n^w = Q_n^w G_p^{\text{em}} + Q_p^w G_n^{\text{em}}, \quad (2.5)$$

where at tree level in the Standard Model, with θ_w the Weinberg mixing angle,

$$Q_p^w = 1 - 4 \sin^2 \theta_w, \quad (2.6)$$

$$Q_n^w = -1. \quad (2.7)$$

The parity-violating (or “weak”) spin asymmetry is obtained from the relative cross section difference,

$$A_{\text{pv}} = \frac{d\sigma/d\Omega(+)-d\sigma/d\Omega(-)}{d\sigma/d\Omega(+)+d\sigma/d\Omega(-)}, \quad (2.8)$$

where the differential scattering cross section for the helicity (+) and (−) states is given by

$$\frac{d\sigma_{\text{coul}}}{d\Omega}(\pm) = \frac{|\mathbf{k}_f|}{|\mathbf{k}_i|} \frac{1}{f_{\text{rec}}} \sum_{\sigma_f} |A_{fi}^{\text{coul}}(\psi_{\pm})|^2. \quad (2.9)$$

We have attached the superscript “coul” to the scattering amplitude A_{fi} to indicate that the potential $V(r)$ used in the Dirac equation is solely the Coulombic nuclear potential. The momenta of the incoming and scattered electrons are denoted by k_i and k_f , respectively, σ_f is the final spin polarization, and f_{rec} is a recoil factor close to unity [29].

B. QED corrections

The scattering amplitude A_{fi}^{QED} , which includes the QED effects, is obtained from the wave functions $\psi_{\pm}$, which are

solutions to (2.1) in the modified potentials,

$$V(r) = V_T(r) + V_{\text{vac}}(r) + V_{\text{vs}}(r),$$

$$A(r) = A_w(r) + A_{\text{vs}}^{\text{ax}}(r). \quad (2.10)$$

The potential V_{vac} for vacuum polarization is taken to be the Uehling potential as parametrized in [30]. The potential V_{vs} for the vector vertex plus self-energy (vs) correction is defined in terms of the integral representation [23],

$$V_{\text{vs}}(r) = -\frac{2Z}{\pi} \int_0^\infty d|q| \frac{\sin(|q|r)}{|q|r} F_L(|q|) F_1^{\text{vs}}(-q^2), \quad (2.11)$$

where $q^2 = (E_i - E_f)^2/c^2 - \mathbf{q}^2$ with $\mathbf{q} = \mathbf{k}_i - \mathbf{k}_f$ is the squared four-momentum transfer to the nucleus. E_i and E_f are, respectively, the total energies of the incoming and scattered electrons. F_L is the nuclear ground-state charge form factor and F_1^{vs} is the electric form factor describing the vs correction in the first-order Born approximation [31,32].

The potential $A_{\text{vs}}^{\text{ax}}$ for the axial-vector vs correction is, in analogy to (2.11), defined by means of the integral representation [24],

$$A_{\text{vs}}^{\text{ax}}(r) = \frac{G_F c}{4\sqrt{2}\pi^2} \int_0^\infty q^2 d|q| \frac{\sin(|q|r)}{|q|r} \times F_w(|q|) F_1^{\text{w,vs}}(-q^2), \quad (2.12)$$

where F_w is the form factor relating to the weak density and $F_1^{\text{w,vs}}$ is the weak vs form factor describing the axial-vector vs correction in the first-order Born approximation. Its large- q limit is given by [33]

$$F_1^{\text{w,vs}}(-q^2) = \frac{1}{2\pi c} \left[-\frac{1}{2} \ln^2 \left(\frac{-q^2}{c^2} \right) + \frac{3}{2} \ln \left(\frac{-q^2}{c^2} \right) + \frac{\pi^2}{6} - 1 \right],$$

$$-q^2/c^2 \gg 1. \quad (2.13)$$

In order to check the relevance of low momentum transfer in (2.12), it is assumed that the exact q dependence is the same as for the electric vs form factor F_1^{vs} . Hence with

$$F_0(-q^2) = \frac{\nu^2 + 1}{4\nu} \left(\ln \frac{\nu + 1}{\nu - 1} \right) \left(\ln \frac{\nu^2 - 1}{4\nu^2} \right) + \frac{2\nu^2 + 1}{2\nu} \ln \frac{\nu + 1}{\nu - 1} + \frac{\nu^2 + 1}{2\nu} \times \left\{ \text{Li} \left(\frac{\nu + 1}{2\nu} \right) - \text{Li} \left(\frac{\nu - 1}{2\nu} \right) \right\}, \quad (2.14)$$

where $\nu = \sqrt{1 - 4c^2/q^2}$ and $\text{Li}(x) = -\int_0^\infty dt \frac{\ln|1-t|}{t}$ is the Spence function, one has

$$F_1^{\text{vs}}(-q^2) = \frac{1}{2\pi c} [F_0(-q^2) - 2],$$

$$F_1^{\text{w,vs}}(-q^2) = \frac{1}{2\pi c} [F_0(-q^2) - 1]. \quad (2.15)$$

It turns out that, due to its decrease as $|q| \rightarrow 0$, the small- q part of the integrand in (2.12) can be neglected except for very

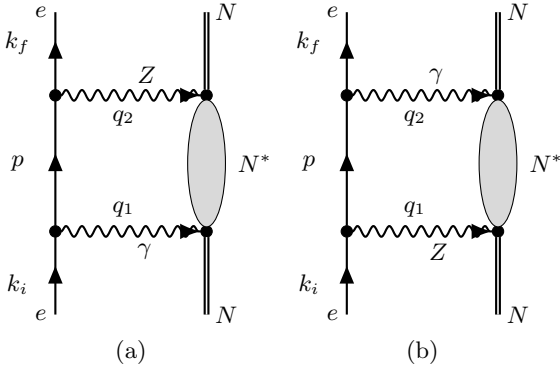


FIG. 1. Feynman box diagrams for dispersion. Diagram (a) describes the electron (e) nucleus (N) coupling by a photon (γ) with momentum q_1 and subsequently by a Z boson with momentum q_2 . N^* indicates the excitation of the nucleus N while k_i and k_f are the momenta of the incoming and scattered electrons. In diagram (b), the order is reversed.

low collision energies or near-forward scattering angles (note that $-q^2/c^2 < 100$ corresponds to $|q| < 0.026 \text{ fm}^{-1}$).

In concord with (2.9), the differential cross section is given by

$$\frac{d\sigma^{\text{QED}}}{d\Omega}(\pm) = \frac{|k_f|}{|k_i|} \frac{1}{f_{\text{rec}}} \sum_{\sigma_f} |A_{fi}^{\text{QED}}(\psi_{\pm})|^2. \quad (2.16)$$

The corresponding parity-violating spin asymmetry, termed $A_{\text{pv}}^{\text{QED}}$, is defined by (2.8), with $d\sigma/d\Omega$ replaced by $d\sigma^{\text{QED}}/d\Omega$.

C. Dispersion effects

In lowest-order Born approximation, dispersion is calculated from the two Feynman box diagrams shown in Fig. 1, involving one virtual photon and one Z boson. The contributions from the crossed-box diagrams are found to be small, and also the seagull diagrams can be disregarded according to [15].

The initial and final electronic helicity eigenstates are described by plane waves,

$$\begin{aligned} \psi_{i,\pm}^{(0)} &= u_{k_i}^{(\pm)}, \\ \psi_{f,+}^{(0)} &= \cos \frac{\theta}{2} u_{k_f}^{(+)} + \sin \frac{\theta}{2} u_{k_f}^{(-)}, \\ \psi_{f,-}^{(0)} &= -\sin \frac{\theta}{2} u_{k_f}^{(+)} + \cos \frac{\theta}{2} u_{k_f}^{(-)}, \end{aligned} \quad (2.17)$$

where $u_k^{(\pm)}$ are plane-wave four-spinors (with the prefactor $\sqrt{(E+c^2)/(2E)}$, retaining the electron mass) and θ is the scattering angle. A coordinate system is chosen where the z axis is along k_i , while k_f lies in the (x, z) plane. The cross section with inclusion of the dispersion amplitude A_{fi}^{box} , as calculated from the box diagrams of Fig. 1, is to lowest order

given by

$$\frac{d\sigma^{\text{box}}}{d\Omega}(\pm) = \frac{|k_f|}{|k_i|} \frac{1}{f_{\text{rec}}} \sum_{\sigma_f} [|A_{fi}^{\text{coul}}(\psi_{\pm})|^2 + 2 \text{Re}\{A_{fi}^{\text{coul}*}(\psi_{\pm}) A_{fi}^{\text{box}}(\psi_{\pm}^{(0)})\}], \quad (2.18)$$

$$\begin{aligned} A_{fi}^{\text{box}}(\psi_{\pm}^{(0)}) &= 2\sqrt{\frac{E_i E_f}{c^2}} \sum_{L, \omega_L} [M_{fi}^{Z\gamma}(L, \omega_L, \pm) \\ &\quad + M_{fi}^{\gamma Z}(L, \omega_L, \pm)], \end{aligned}$$

where L is the angular momentum and ω_L the corresponding energy of the transient nuclear excited state. The transition matrix element $M_{fi}^{Z\gamma}$ corresponds to the first and $M_{fi}^{\gamma Z}$ to the second Feynman diagram of Fig. 1. The evaluation of $M_{fi}^{Z\gamma}$ and $M_{fi}^{\gamma Z}$ proceeds as in the case of the beam-normal spin asymmetry [16]. Details are given in Appendixes A and B.

The contribution of the axial part of the Z -nucleus vertex is suppressed by the small electron's weak charge $Q_e^w = -Q_p^w \approx -0.05$ [12]. Moreover, it represents magnetic transitions to excited nuclear states which are in general only relevant at the backmost angles, i.e., in a small portion of the phase space of the electron in its intermediate state. Therefore this axial contribution is disregarded.

III. RESULTS

In this section we present estimates for the parity-violating spin asymmetry and its radiative corrections. To this end, the Dirac equation is solved with the help of the Fortran code RADIAL [34]. The spin asymmetry modification ΔA_{pv} is defined by

$$\Delta A_{\text{pv}}^{\text{QED/box}} = A_{\text{pv}}^{\text{QED/box}} - A_{\text{pv}}, \quad (3.1)$$

where the term A_{pv} denotes the spin asymmetry relating to the Coulombic field, i.e., $V(r) = V_T(r)$ in (2.1). For low collision energies or small scattering angles (when A_{pv} is well away from zero), it is reasonable to define the relative spin asymmetry change dA_{pv} ,

$$dA_{\text{pv}} = \frac{\Delta A_{\text{pv}}}{A_{\text{pv}}}. \quad (3.2)$$

For dispersion, the relative spin asymmetry changes $dA_{\text{pv}}^{\text{box}}(L, \omega_L)$, pertaining to a given excited nuclear state with angular momentum L and excitation energy ω_L , are additive,

$$dA_{\text{pv}}^{\text{box}} \approx \sum_{L, \omega_L} dA_{\text{pv}}^{\text{box}}(L, \omega_L). \quad (3.3)$$

This additivity is based on the fact that the relative cross section changes by dispersion are negligibly small (well below 10^{-6}).

When the two kinds of radiative corrections are of comparable size, they have to be combined. However, a simple addition is not possible since the relative cross section change $\Delta\sigma^{\text{QED}} = \frac{d\sigma^{\text{QED}}}{d\Omega} / \frac{d\sigma^{\text{coul}}}{d\Omega} - 1$ by the vacuum polarization and the vs effect is in general large (up to -20%). Therefore the weak

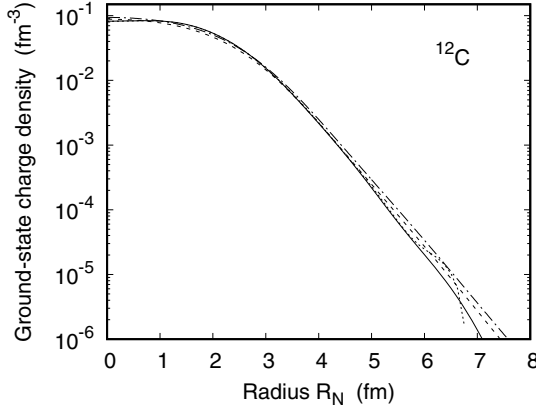


FIG. 2. Ground-state charge density ρ_0 of ^{12}C as function of the distance R_N from the nuclear center. Shown are the Gauss parametrization (—), the Fourier-Bessel fit (.....), and the result from the nuclear model (- · - · -). Included is the negative value of the weak density ρ_w (---).

spin asymmetry which includes the radiative corrections has to be calculated from

$$A_{pv}^{\text{box+QED}} \approx A_{pv}^{\text{QED}} + A_{pv} dA_{pv}^{\text{box}} \frac{1}{1 + \Delta\sigma^{\text{QED}}}, \quad (3.4)$$

and correspondingly its relative change

$$dA_{pv}^{\text{box+QED}} \approx dA_{pv}^{\text{QED}} + dA_{pv}^{\text{box}} \frac{1}{1 + \Delta\sigma^{\text{QED}}}. \quad (3.5)$$

Two spin-zero nuclei, ^{12}C and ^{208}Pb , are considered in what follows.

A. The ^{12}C nucleus

For the ground-state charge distribution ρ_0 , from which the nuclear potential V_T is generated, three different prescriptions were considered: a parametrization (Gauss) with a sum of Gaussians [35], a Fourier-Bessel (FB) parametrization [35], both obtained from a fit to elastic scattering data, and a numerical ρ_0 calculated from the SkM* nuclear model [36]. They are displayed in Fig. 2. The shapes of the FB and the Gauss parametrizations are nearly identical (up to 5.5 fm), with a minimum near $R_N = 0$. However, there are considerable deviations between these two densities and the theoretical one.

Figure 3 shows the angular and energy distribution of the differential cross section for the elastic scattering of unpolarized electrons, as obtained by means of the phase-shift analysis [37]. At low collision energies $E_{i,\text{kin}} = E_i - c^2$ [Fig. 3(a)] the difference arising from the choice of the Gauss ρ_0 and the theoretical ρ_0 is small, mostly below 10% at 155 MeV and less ($\lesssim 3\%$) at 55 MeV. However, in the vicinity of the first diffraction minimum [at a momentum transfer $q \approx 1.84 \text{ fm}^{-1}$, Fig. 3(b)] these deviations are formidable, the more so the higher the collision energy. The above momentum transfer corresponds to a mean electron-nucleus distance $R_N \approx q^{-1} = 0.54 \text{ fm}$, where the respective choices of ρ_0 are very different.

The weak density ρ_w is calculated from (2.3) as predicted by the SkM* [36]. The weak charge of a nucleus with N

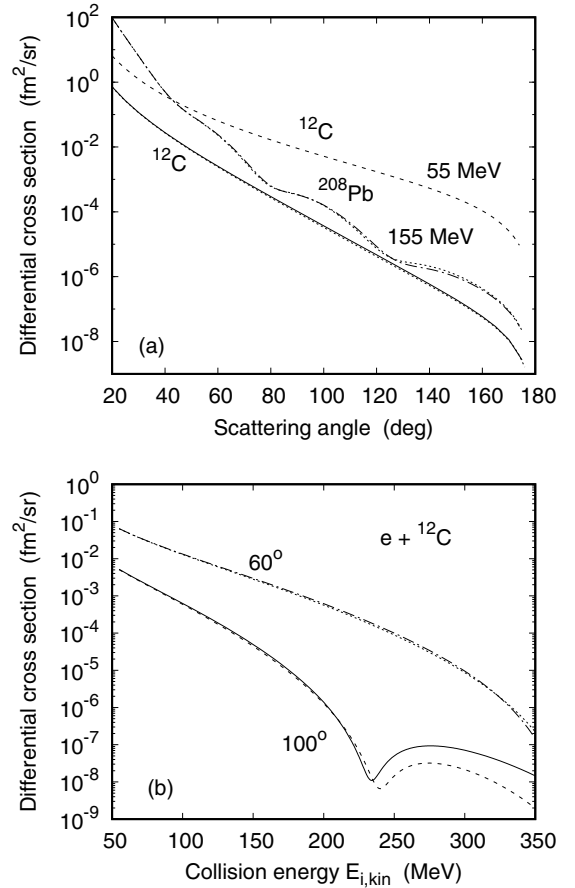


FIG. 3. Differential cross section $\frac{d\sigma_{\text{coul}}}{d\Omega}$ for unpolarized electrons (a) scattering from ^{12}C at 55 MeV (---) and at 155 MeV (—), as well as from ^{208}Pb at 155 MeV (- · - · -) as function of the scattering angle θ , using the Gauss charge distribution. Included are the results from the numerical ρ_0 for 155 MeV (.....). (b) Scattering from ^{12}C at $\theta = 60^\circ$ (- · - · -) and 100° (—) as function of collision energy $E_{i,\text{kin}}$, using the Gauss ρ_0 . Included are the results from the numerical ρ_0 for 60° (.....) and 100° (---).

neutrons and Z protons as defined by [5]

$$Q_w^{N,Z} = \int d\mathbf{r} \rho_w(r) = NQ_n^w + ZQ_p^w \quad (3.6)$$

is -5.55 for ^{12}C at tree level and -5.51 taking into account radiative corrections to $Q_p^w = 0.0719 \pm 0.0045$ and $Q_n^w = -0.9890 \pm 0.0007$ [14]. The absolute value of ρ_w for ^{12}C is depicted in Fig. 2. Due to the fact that it is an $N = Z$ nucleus, the neutron and proton densities are very similar and thus, as can be seen, the electric and weak charge density distributions are also very similar in magnitude but have opposite signs. While our calculations are performed with $Q_w^{N,Z} = -5.50$, a renormalization by a factor $5.55/5.50$ reduces A_{pv} by up to 5% at a collision energy of 155 MeV and angles up to 175° .

The influence of ρ_0 on the spin asymmetry A_{pv} is displayed in Fig. 4. The deviations are larger than for the differential cross section, particularly in the region of the diffraction minimum where A_{pv} shows a large oscillatory structure [Fig. 4(b)].

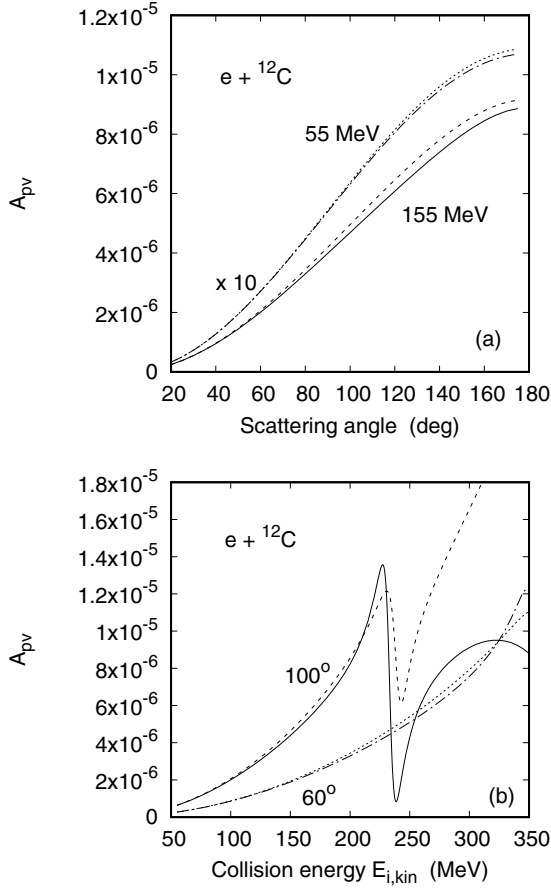


FIG. 4. Parity-violating asymmetry A_{pv} for electron scattering from ^{12}C (a) at 55 MeV (---, multiplied by a factor of 10) and 155 MeV (—) as function of scattering angle θ and (b) at 60° (---) and 100° (—) as function of collision energy, using the Gauss charge distribution. The results from the numerical charge density are also shown, at 55 MeV (·····, multiplied by 10) and 155 MeV (----) in (a) and at 60° (·····) and 100° (----) in (b).

Beyond the diffraction minimum even the shape of the energy distribution differs strongly. This sensitivity to the model densities at higher momentum transfers has to be kept in mind as a much larger source of uncertainty than the radiative corrections. In the subsequent calculations the representation Gauss of the ground-state charge density is used.

Concerning the dispersive corrections, we have considered the most relevant excited states with angular momentum $L \leq 3$ and excitation energy $\omega_L < 30$ MeV as predicted by the SkM* effective interaction [36] solved via the Hartree-Fock plus Random Phase Approximation [27]. The six selected excited states are three 1^- states at 19.6 (isovector), 20.9 (isoscalar), and 27.8 MeV (isoscalar); two 2^+ states at 4.439 (isoscalar) and 9.84 MeV (isovector); and one 3^- state at 9.64 MeV (isoscalar). The proton and neutron transition densities are calculated consistently within the same nuclear model. The corresponding transition form factors together with their weak counterparts are displayed in Fig. 5 for the 19.6 MeV (1^-) and 4.439 MeV (2^+) states. The isoscalar

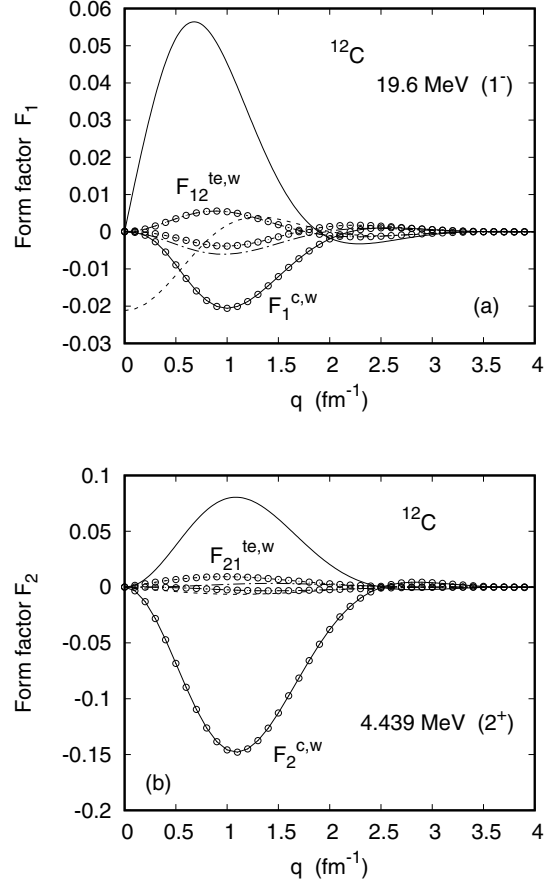


FIG. 5. ^{12}C transition form factors as function of momentum transfer (a) for the isovector 1^- state at 19.6 MeV and (b) for the isoscalar 2^+ state at 4.439 MeV. Shown are the charge form factor F_L^c (—), the weak form factor $F_L^{c,w}$ (---), the transverse electric form factors $F_{L,L+1}^{te}$ (---) and $F_{L,L-1}^{te}$ (---), as well as their weak counterparts $F_{L,L+1}^{te,w}$ (---) and $F_{L,L-1}^{te,w}$ (---) for (a) $L = 1$ and (b) $L = 2$.

states are characterized by a large weak charge form factor [Fig. 5(b)], while for the isovector states the magnetic weak form factors gain importance [Fig. 5(a)].

The influence of the radiative effects on the weak spin asymmetry is shown in Fig. 6 at one forward and one backward angle. At small angles, dispersion and also the QED perturbation are mostly negligible [Fig. 6(a)], while near and above the diffraction minimum, accessible at the larger angles, dispersion is strongly dominating [Fig. 6(b)].

Let us now consider the relative changes by the QED effects (dA_{pv}^{QED}) and by dispersion (dA_{pv}^{box}). Their energy distribution at 30° is displayed in Fig. 7. It is seen that both radiative effects vanish when the collision energy tends to zero. In Fig. 7(a) separate dispersion results are shown for the 19.6 and 27.8 MeV dipole, the 4.439 MeV quadrupole, and the 9.64 MeV octupole excitations (whereas the 20.9 and the 9.84 MeV contributions can be neglected). It follows that the main contribution to dispersion is due to the lowest quadrupole excitation, while the dipole states are of minor importance. This is opposite to the results found for the Sherman

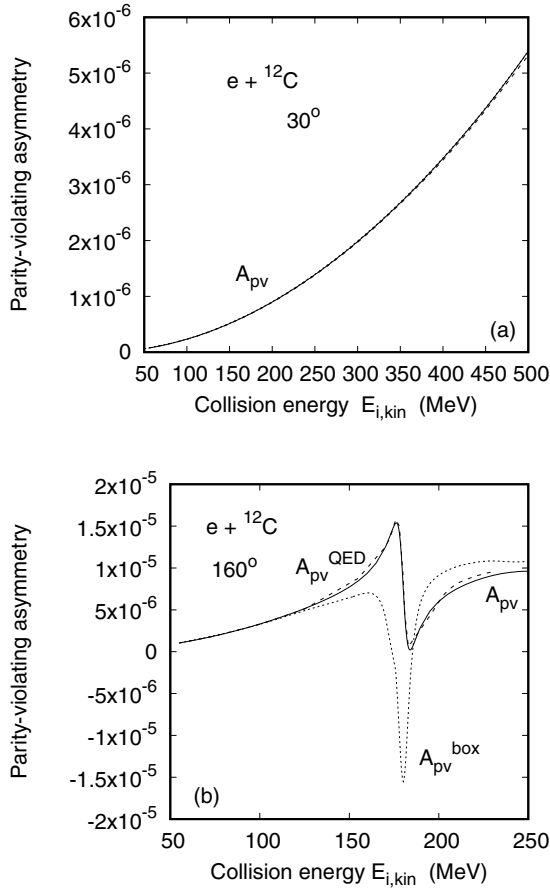


FIG. 6. Parity-violating asymmetry A_{pv} (—) in $e + {}^{12}\text{C}$ collisions, corrected for QED effects (A_{pv}^{QED} , ---) and for dispersion (A_{pv}^{box} , ...) (a) at 30° and (b) at 160° as function of collision energy. In (a), A_{pv}^{box} coincides with A_{pv} .

function where dispersion originates predominantly from the dipole states, but that could also be due to a different choice of model [16] for the dipole transition densities. The present result can be understood in terms of the form factors (Fig. 5). While for the 2^+ state the weak charge form factor is enhanced by a factor of 2 as compared to F_L^c , it is reduced by a factor of 3 in the case of the 1^- state. Also the magnetic weak form factors for this state are smaller than their protonic counterparts. We note that the above-mentioned renormalization of q_w (by the factor 5.55/5.50) changes $\Delta A_{pv}^{\text{box}}$ by at most 0.1%.

The correction of A_{pv} by the QED effects [Fig. 7(b)] is similar in magnitude to dispersion at this forward angle, except at high energies. The vector vs correction leads to a large effect on A_{pv} , but is mostly compensated by the axial-vector vs contribution, with a net effect well below 1%. At backward angles, however, dispersion is largely dominating. This is shown for 160° in Fig. 8. Due to the diffraction oscillations of A_{pv} at this angle, where the weak spin asymmetry nearly vanishes, the definition of the relative spin asymmetry change is no longer meaningful. Therefore, the spin asymmetry modifications $\Delta A_{pv}^{\text{box}}$ and $\Delta A_{pv}^{\text{QED}}$ from (3.1) are displayed instead. The figure indicates that dispersion is nearly exclusively due to the lowest quadrupole state, showing a large excursion near

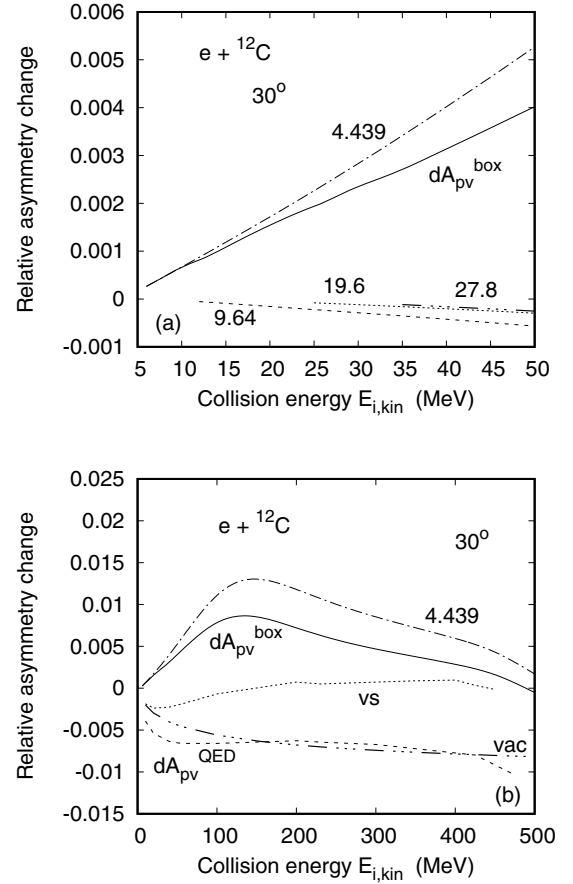


FIG. 7. Relative asymmetry change dA_{pv} in $e + {}^{12}\text{C}$ collisions at $\theta = 30^\circ$ as function of collision energy. Shown are dA_{pv}^{box} (summed over all excited states, —) and $dA_{pv}^{\text{box}}(L, \omega_L)$ for the 2^+ state at 4.439 MeV (---). Included in (a) is $dA_{pv}^{\text{box}}(L, \omega_L)$ for the 3^- state at 9.64 MeV (---) and for the 1^- states at 19.6 MeV (.....) and 27.8 MeV (---). In (b), the changes by the vertex plus self-energy correction (.....), by the vacuum polarization (---), and by both QED effects (dA_{pv}^{QED} , ---) are shown in addition.

the location of the diffraction minimum. The contributions from dipole and octupole states mostly cancel each other, even for still higher energies.

The angular distribution of the parity-violating asymmetry and its radiative changes for the two energies of 55 and 155 MeV is depicted in Fig. 9. While the QED effects are dominating at 55 MeV for all angles [Fig. 9(a)], there is a crossover between dA_{pv}^{QED} and $|dA_{pv}^{\text{box}}|$ near 100° at the higher energy.

As seen from Fig. 9(b) the $L = 2$ contribution is for 155 MeV dominating at all angles, while the changes by dipole and octupole excitations are at most in the percent region.

We have also studied the radiative corrections in the high-energy regime. As a test case we have considered an energy of 953 MeV and a scattering angle of 4.7° , a geometry which has been selected for A_{pv} precision experiments on the lead nucleus [1]. For A_{pv} , being equal to 5.033×10^{-7} , the

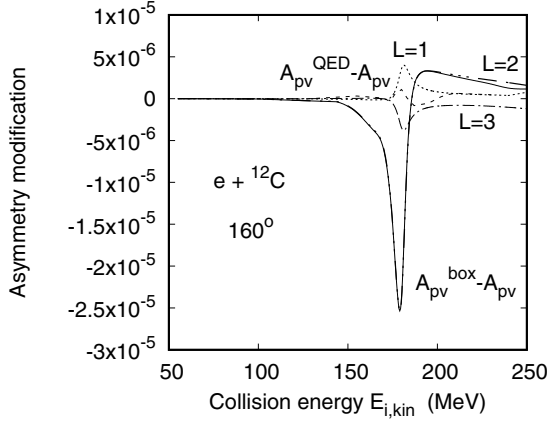


FIG. 8. Spin asymmetry modification ΔA_{pv} for electrons colliding with ^{12}C at 160° as function of collision energy. Shown are the dispersive modifications $\Delta A_{pv}^{\text{box}} = A_{pv}^{\text{box}} - A_{pv}$ when summed over the dipole states ($\cdots$), over the quadrupole states ($-\cdots-$), over the octupole states ($-\cdots-$), as well as over $L \leq 3$ (—). Included is the asymmetry modification by the QED effects, $\Delta A_{pv}^{\text{QED}} = A_{pv}^{\text{QED}} - A_{pv}$ (---).

relative change by the QED effects is $dA_{pv}^{\text{QED}} \approx -0.51\%$ and by dispersion $dA_{pv}^{\text{box}} = -0.19\%$. Thus the dispersive effects from low-lying excited states can safely be neglected at such geometries and incident electron energies, and also the QED corrections play only a minor role.

B. The ^{208}Pb nucleus

As for the carbon nucleus, the Gauss parametrization [35] of the experimentally determined nuclear ground-state charge distribution ϱ_0 is employed in the subsequent calculations. A comparison with the Fourier-Bessel charge distribution [35] shows a basic agreement except for very small nuclear distances, as seen in Fig. 10. In contrast, the numerically obtained ϱ_0 from the nuclear model SkP [38] deviates from the Gauss ϱ_0 in large regions of space. The effect of these deviations on the differential cross section for elastic scattering is negligibly small, as displayed in Fig. 3(a) for a collision energy of 155 MeV, except at the larger angles. The weak density for ^{208}Pb , normalized to $Q_w = -118.76$ (which includes the universal radiative corrections [14]), is also shown in Fig. 10, again with a global minus sign for the sake of shape comparison. In contrast to the results for ^{12}C , the absolute value of ϱ_w is about 30% higher than ϱ_0 for $R_N \lesssim 5$ fm, and even more in the outer region due to the large neutron skin of the lead nucleus.

In order to account for dispersion, ten dominant excited states with $L \leq 3$ and $\omega_L < 30$ MeV are considered. These are the five 1^- states at energies 5.512 MeV (isoscalar) and 12.6, 14.3, 14.6, and 15.3 MeV (isovector). In addition, the three quadrupole states at 4.085 and 10.9 MeV (isoscalar) and at 21.6 MeV (isovector), as well as two octupole states at 2.615 MeV (isoscalar) and 27.9 MeV (isovector), are included. They are consistently calculated from the same (SkP) model [38]. It turns out that the most important ones are the 10.9 MeV 2^+ and the 2.615 MeV 3^- states. Their transition

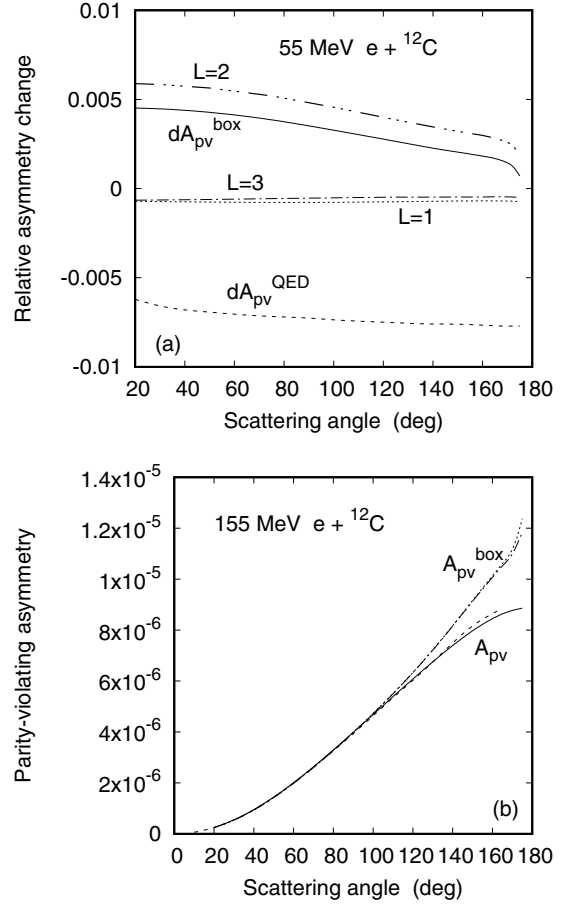


FIG. 9. Parity-violating spin asymmetry and its radiative corrections in $e + ^{12}\text{C}$ collisions (a) at 55 MeV and (b) at 155 MeV as function of scattering angle θ . Shown in (a) are the dispersive changes dA_{pv}^{box} when summed over the dipole states ($\cdots$), over the quadrupole states ($-\cdots-$), over the octupole states ($-\cdots-$), as well as over $L \leq 3$ (—). Included is the change dA_{pv}^{QED} by the QED effects (---). Shown in (b) are A_{pv} (—), A_{pv}^{QED} (---), and A_{pv}^{box} ($\cdots$). Included is the spin asymmetry corrected only for the $L = 2$ dispersive effects ($-\cdots-$).

form factors are given in Fig. 11, together with the corresponding weak form factors. It is seen that for both states the weak charge form factors ($F_L^{c,w}$) are about a factor of 2 larger than the corresponding charge form factors (F_L^c). As for the isoscalar states of ^{12}C , the magnetic form factors are much smaller, particularly for the 2.615 MeV state. For the sake of comparison, the form factors of the most relevant of the dipole states, the one at 14.3 MeV, are also shown [Fig. 11(a)]. For all isovector 1^- states, proton and weak form factors have basically the same shape (but opposite sign).

The parity-violating spin asymmetry together with its radiative corrections is shown in Fig. 12. The angular distribution of A_{pv} at 155 MeV [Fig. 12(a)] exhibits an oscillatory behavior in concord with the diffraction structures of the differential cross section [see Fig. 3(a)], which, however, are quite shallow. The QED effects slightly reduce the spin

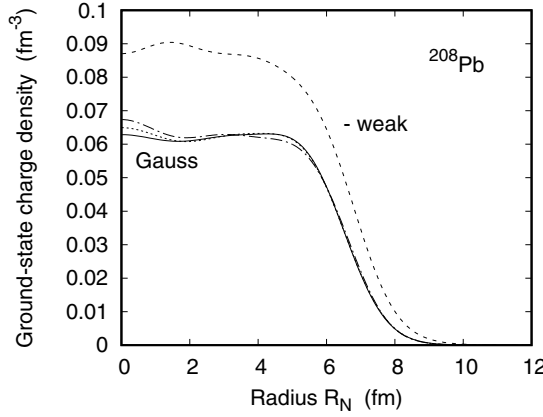


FIG. 10. Ground-state charge density ρ_0 of ^{208}Pb as function of the distance from the nuclear center. Shown are the Gauss parametrization (—), the FB fit (·····), and the result from the nuclear model (— · — ·). Included is the negative of the weak density ρ_w (— — —).

asymmetry, except around 140° . Dispersion increases the spin asymmetry, well below 1% for $\theta < 60^\circ$, but up to 10% for the backward angles.

The behavior of A_{pv} and its radiative corrections with energy is displayed in Fig. 12(b) for the forward scattering angle of 30° . The energy dependence of the diffraction structures is much alike their angular dependence, confirming that A_{pv} is basically dependent on momentum transfer, which increases in a similar way with angle and with energy.

Again the dispersive corrections enhance the weak spin asymmetry, but they are unimportant at this angle except for energies above 300 MeV. The QED effects are small throughout.

Figure 13 shows the relative change of A_{pv} by the radiative corrections at 155 MeV. For dispersion [Fig. 13(a)], the dominant contribution at the larger angles originates from the $L = 2$ and $L = 3$ excited states, with the 2.615 MeV state prevailing beyond 140° . The dipole states determine the angular distribution only at angles below 60° , and this basically because the quadrupole and octupole contributions cancel each other in this angular region.

Dispersion is particularly important at the higher angles. The structures seen in the angular distribution conform with the oscillations of A_{pv} [Fig. 12(a)]. Each minimum in $\frac{d\sigma_{\text{conl}}}{d\Omega}$ or in A_{pv} produces a large excursion of both dA_{pv}^{QED} and dA_{pv}^{box} . Included in Fig. 13(b) are the two constituents of dA_{pv}^{QED} : the change by vacuum polarization (amounting to about 1%) and by the vs correction which is of opposite sign. The QED corrections are very small because vacuum polarization mostly compensates the vs correction. This latter correction is intrinsically small because also here the vector vs and the axial-vector vs contributions tend to cancel each other except at the larger angles.

The energy distribution of the relative spin asymmetry change is provided in Fig. 14 for a forward scattering angle. The dispersive corrections [(Fig. 14(a)] are well below 1% at energies below 300 MeV. However, in contrast to ^{12}C at this angle (Fig. 7), they increase with energy, showing a maximum

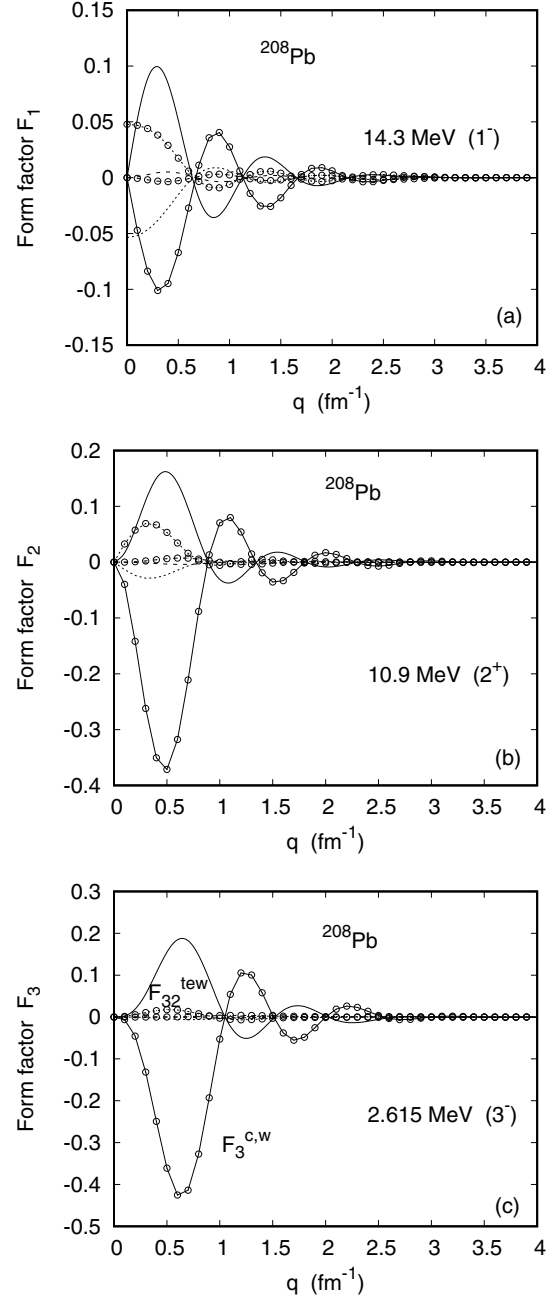


FIG. 11. Transition form factors as function of momentum transfer, (a) for the ^{208}Pb isovector 1^- state at 14.3 MeV, (b) for the isoscalar 2^+ state at 10.9 MeV, and (c) for the isoscalar 3^- state at 2.615 MeV. Shown are the charge form factor F_L^c (—), the weak form factor $F_L^{c,w}$ (—○—), the transverse electric form factors $F_{L,L+1}^{\text{te}}$ (— — —) and $F_{L,L-1}^{\text{te}}$ (·····), as well as their weak counterparts $F_{L,L+1}^{\text{te,w}}$ (—○—○—) and $F_{L,L-1}^{\text{te,w}}$ (·····○·····) for (a) $L = 1$, (b) $L = 2$, and (c) $L = 3$.

of nearly 5% in the second diffraction minimum, basically produced by the two isoscalar 2^+ excitations. The QED corrections [Fig. 14(b)] are at most 1% at this angle, irrespective of energy.

For the high-energy geometry, 953 MeV at 4.7° , our results are $A_{pv} = 5.898 \times 10^{-7}$, $dA_{pv}^{\text{QED}} \approx 8.79 \times 10^{-4}$, and

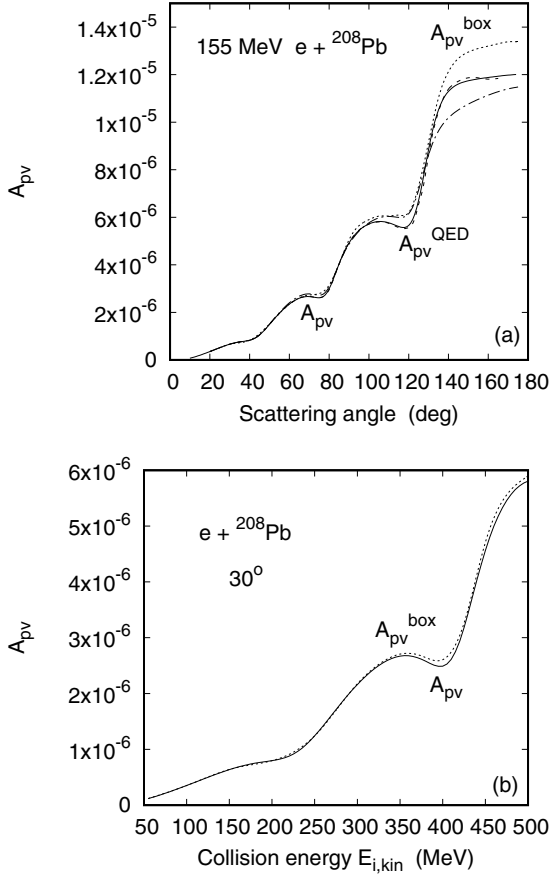


FIG. 12. Parity-violating asymmetry A_{pv} (—) in $e + ^{208}\text{Pb}$ collisions, corrected for QED effects (A_{pv}^{QED} , ---) and for dispersion (A_{pv}^{box} , ...) (a) at 155 MeV as function of scattering angle θ and (b) at 30° as function of collision energy $E_{i,\text{kin}}$. In (a), also A_{pv} resulting from the numerical ϱ_0 is shown (- · - · -). In (b), A_{pv}^{QED} coincides with A_{pv} .

$dA_{pv}^{\text{box}} = -1.07 \times 10^{-3}$, confirming that dispersion as well as the QED effects are negligible at GeV energies for small angles.

IV. CONCLUSION

We have estimated the radiative corrections to the parity-violating spin asymmetry for electron scattering from ^{12}C and ^{208}Pb nuclei by treating the QED effects nonperturbatively and by estimating the dispersive corrections from low-lying excited states by evaluating the γZ box diagrams in the second-order Born approximation. We have found that, at small scattering angles and low collision energies, the dispersive changes of A_{pv} are of little importance.

At backward angles, on the other hand, dispersion is largely dominating at high energies and may affect A_{pv} by more than 10% at a collision energy of 155 MeV. In general it will be large in the diffraction minima of A_{pv} . For carbon, when the diffraction effects do not obscure the energy dependence of the spin asymmetry, we have found that dispersion decreases with energy beyond 150 MeV at forward angles. This is in concord with the expectation that the contribution

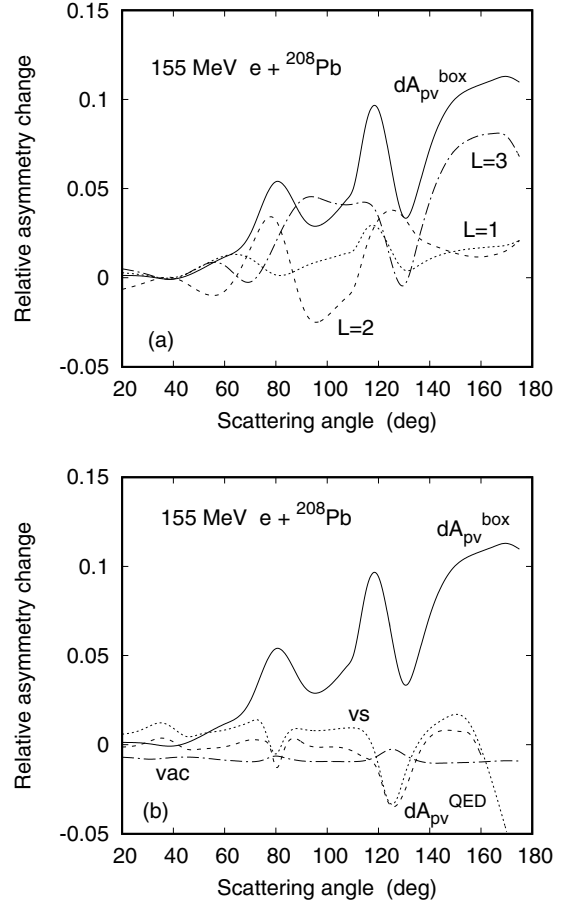


FIG. 13. Relative spin asymmetry change dA_{pv} in 155 MeV $e + ^{208}\text{Pb}$ collisions as function of scattering angle θ . Shown in (a) are the dispersive changes dA_{pv}^{box} when summed over the dipole states (.....), over the quadrupole states (---), over the octupole states (- · - · -), and over all states with $L \leq 3$ (dA_{pv}^{box} , —). Shown in (b) are the QED changes by vacuum polarization (- · - · -), by the vertex plus self-energy correction (.....), and by both QED effects (dA_{pv}^{QED} , ---). Included is dA_{pv}^{box} (—) from (a).

from the low-lying excited states is suppressed according to $1/E_i^2$ for energies well above the pion production threshold [39]. On the other hand, the QED corrections are small and nearly independent of collision energy, being usually below 1%. For the test study at 953 MeV and 4.7° , the geometry selected in [1], it is found that for ^{12}C the two radiative effects add to each other, but still affect A_{pv} by less than 1%. For ^{208}Pb they tend to compensate each other such that the result is one order of magnitude smaller than for ^{12}C .

For the design of precision experiments it is important to choose small scattering angles, well below the onset of the diffraction structures. One reason is that the radiative corrections, calculated within a given nuclear model, are tiny at small angles but increase with angle, the more so the higher the collision energy. Moreover, also the dependence of the ground-state charge density and of the parity-violating asymmetry on the underlying nuclear models is quite strong for the larger angles. Also, whenever dispersion is important,

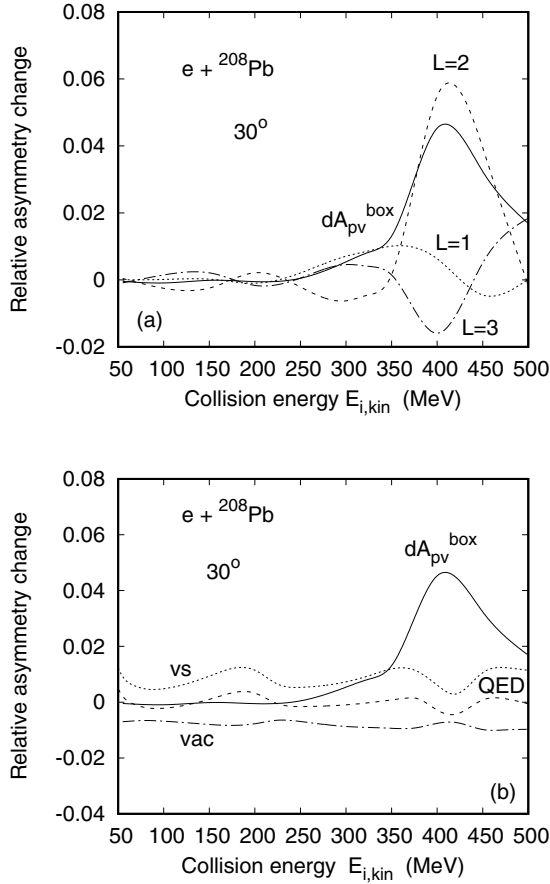


FIG. 14. Relative spin asymmetry change dA_{pv} in $e + {}^{208}\text{Pb}$ collisions at $\theta = 30^\circ$ as function of collision energy. Shown in (a) are the dispersive changes dA_{pv}^{box} when summed over the dipole states ($\cdots$), over the quadrupole states ($-\cdots-$), over the octupole states ($-\cdots-\cdots$), and over all states with $L \leq 3$ (dA_{pv}^{box} , $---$). Shown in (b) are the QED changes by vacuum polarization ($\cdots$), by the vertex plus self-energy correction ($-\cdots-$), and by both QED effects (dA_{pv}^{QED} , $---$). Included is dA_{pv}^{box} ($---$) from (a).

the model dependence of the transition densities cannot be neglected.

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DATA AVAILABILITY

The data that support the findings of this article are available within this article.

APPENDIX A: CORRESPONDENCE BETWEEN ELECTROMAGNETIC AND WEAK SCATTERING

In this appendix we give some details on the correspondence between electromagnetic (two-photon) scattering and weak (photon-Z boson) scattering.

For the weak interaction, the photon propagator is replaced by the Z-boson propagator,

$$\frac{g^{\mu\nu}}{q^2 + i\epsilon} \mapsto \frac{g^{\mu\nu} - q^\mu q^\nu / (M_z^2 c^2)}{q^2 - M_z^2 c^2 + i\epsilon} \approx \frac{g^{\mu\nu}}{-M_z^2 c^2}, \quad \mu, \nu = 0-3, \quad (\text{A1})$$

where $g^{\mu\nu}$ is the metric tensor, q the momentum of the photon (left hand side) or the Z boson (right hand side) where the q dependence is eliminated due to the large mass $M_z \gg q/c$ of the Z boson.

Moreover, the electron- γ vertex is substituted by the electron-Z vertex, relying on the Standard Model [40]:

$$\frac{ie}{\sqrt{c}} [\bar{u}_f \gamma_\mu u_i] \mapsto -\frac{i}{4} g_z [\bar{u}_f \gamma_\mu (4 \sin^2 \theta_w - 1 + \gamma_5) u_i], \quad (\text{A2})$$

where u_f, u_i are plane-wave four-spinors, γ_μ are Dirac matrices and $\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is the 4×4 chirality matrix. We have introduced the weak coupling constant g_z of the Z boson,

$$g_z = \frac{g_w}{\cos \theta_w}, \quad g_w = \sqrt{\frac{8M_w^2 c^4 G_F}{2}}, \quad (\text{A3})$$

where g_w is the coupling constant of the W boson, θ_w is the Weinberg mixing angle ($\sin^2 \theta_w = 0.23857$ [41]), and $M_w c^2$ is the rest energy of the W boson.

Likewise, the γ -nucleus vertex is replaced by the Z-nucleus vertex,

$$-\frac{ie}{\sqrt{c}} [\bar{\phi}_n \gamma_0 J_{\text{em}}^\mu \phi_n] \mapsto -\frac{i}{4} g_z [\bar{\phi}_n \gamma_0 J_w^\mu \phi_n], \quad (\text{A4})$$

where J_{em} is the electromagnetic transition current, J_w the weak transition current, and ϕ_n a nuclear state.

Proton (ϕ_P) as well as neutron (ϕ_N) parts of the nuclear wave function ϕ_n contribute to the matrix element of J_w^μ . For a single proton $|P\rangle$ or neutron $|N\rangle$, the isospin rotation model provides the correspondence between J_w^μ and the electromagnetic current, if the axial (magnetic) contribution to J_w^μ is disregarded [12,42]:

$$\begin{aligned} \langle P | J_w^\mu | P \rangle &= (1 - 4 \sin^2 \theta_w) \langle P | J_{\text{em}}^\mu | P \rangle - \langle N | J_{\text{em}}^\mu | N \rangle, \\ \langle N | J_w^\mu | N \rangle &= (1 - 4 \sin^2 \theta_w) \langle N | J_{\text{em}}^\mu | N \rangle - \langle P | J_{\text{em}}^\mu | P \rangle. \end{aligned} \quad (\text{A5})$$

Generalizing (A5) by means of $\langle \phi_P | J^\mu | \phi_P \rangle = Z \langle P | J^\mu | P \rangle$ and $\langle \phi_N | J^\mu | \phi_N \rangle = N \langle N | J^\mu | N \rangle$, where the prefactors Z and N are the numbers of protons and neutrons in the nucleus, one obtains

$$\begin{aligned} \langle \phi_n | J_w^\mu | \phi_n \rangle &= \langle \phi_P | J_w^\mu | \phi_P \rangle + \langle \phi_N | J_w^\mu | \phi_N \rangle \\ &= \left(1 - 4 \sin^2 \theta_w - \frac{N}{Z} \right) \langle \phi_P | J_{\text{em}}^\mu | \phi_P \rangle \\ &\quad + \left(1 - 4 \sin^2 \theta_w - \frac{Z}{N} \right) \langle \phi_N | J_{\text{em}}^\mu | \phi_N \rangle. \end{aligned} \quad (\text{A6})$$

In the special case of an $N = Z$ nucleus (such as ^{12}C), the Z -nucleus vertex simplifies to

$$-\frac{i}{4} g_z [\bar{\phi}_n \gamma_0 J_w^\mu \phi_n] = i g_z \sin^2 \theta_w \times [\langle \phi_p | J_{\text{em}}^\mu | \phi_p \rangle + \langle \phi_n | J_{\text{em}}^\mu | \phi_n \rangle]. \quad (\text{A7})$$

APPENDIX B: EVALUATION OF THE TRANSITION MATRIX ELEMENT $M_{fi}^{Z\gamma}$ FOR DIAGRAM (a) OF FIG. 1

Substituting in the electromagnetic formalism [16] the Z -boson propagator and vertices, one obtains

$$M_{fi}^{Z\gamma}(L, \omega_L, \pm) = -\frac{4\pi}{(2\pi)^3} \frac{e^2}{c} \left(\frac{g_z}{4}\right)^2 \times \int d\mathbf{p} \frac{1}{(-M_z^2 c^2)} \frac{1}{q_1^2 + i\epsilon} \times \sum_{\mu\nu} t_{\mu\nu}^{Z\gamma}(\pm) \sum_M (T_n^{Z\gamma})^{\mu\nu}, \quad (\text{B1})$$

where $q_1 = k_i - p$ and the sum runs over the magnetic quantum number M of the nuclear state with angular momentum L .

When helicity is conserved (corresponding to electron mass $m_e = 0$), the electronic transition matrix element $t_{\mu\nu}^{Z\gamma}$ reads

$$t_{\mu\nu}^{Z\gamma}(\pm) = c \left[\psi_{f,\pm}^{(0)+} (4 \sin^2 \theta_w - 1 + \gamma_5) \gamma_0 \gamma_\mu \times \frac{E_p + c\boldsymbol{\alpha}\mathbf{p} + \gamma_0 c^2}{E_p^2 - \mathbf{p}^2 c^2 - c^4 + i\epsilon} \gamma_0 \gamma_\nu \psi_{i,\pm}^{(0)} \right], \quad (\text{B2})$$

where $E_p = E_i - \omega_L - \sqrt{\mathbf{q}_1^2 c^2 + M_T^2 c^4} + M_T c^2$ is the energy of the electron in its intermediate state, with M_T the target mass. In the present calculations the electron mass is retained, allowing for spin-flip (where $\psi_{f,\pm}^{(0)}$ has to be replaced by $\psi_{f,\mp}^{(0)}$). However, it only plays any role at the lowest energies or backmost angles considered (for example, in 55 MeV $e + ^{12}\text{C}$ collisions for $\theta = 175^\circ$, the spin-flip contribution to the relative spin asymmetry change is actually 3%).

The nuclear matrix element $(T_n^{Z\gamma})^{\mu\nu}$ for the transient transition from the ground state $|0\rangle$ to an excited state $|n\rangle$ which is characterized by the quantum numbers L and M , is given by

$$(T_n^{Z\gamma})^{\mu\nu} = \langle 0 | J_w^\mu(\mathbf{q}_2) | n \rangle \langle n | J_p^\nu(\mathbf{q}_1) | 0 \rangle, \\ J_w^\mu(\mathbf{q}) = \left(1 - 4 \sin^2 \theta_w - \frac{N}{Z} \right) J_p^\mu(\mathbf{q}) \\ + \left(1 - 4 \sin^2 \theta_w - \frac{Z}{N} \right) J_N^\mu(\mathbf{q}), \quad (\text{B3})$$

where J_w^μ is the weak transition density, constructed in analogy to (A6) from the superposition of the proton (J_p^μ) and neutron (J_N^μ) parts of the charge ($\mu = 0$) and magnetic ($\mu = 1, 2, 3$) transition densities of the target nucleus. Its matrix elements can be expressed in terms of weak transition form

factors [16],

$$\langle n | J_w^\mu(\mathbf{q}) | 0 \rangle = \begin{cases} 4\pi i^L F_L^{c,w}(|\mathbf{q}|) Y_{LM}^*(\hat{\mathbf{q}}), & \mu = 0, \\ -4\pi i \sum_{\lambda=L\pm 1} i^\lambda F_{L\lambda}^{\text{te},w}(|\mathbf{q}|) [Y_{L\lambda}^{M*}(\hat{\mathbf{q}})]^m, & \mu = m \end{cases} \quad (\text{B4})$$

with $m = 1, 2, 3$, and Y_{LM} and $Y_{L\lambda}^M$ are, respectively, a spherical harmonic function and the vector spherical harmonics. The weak form factors $F_L^{c,w}$ and $F_{L\lambda}^{\text{te},w}$ are the Fourier transforms of the weak transition densities. The photon-induced matrix elements $\langle n | J_p^\nu | 0 \rangle$ are also obtained from (B4) by replacing, respectively, the weak form factors with the charge (F_L^c) and the transverse electric ($F_{L\lambda}^{\text{te}}$) form factors.

For the further evaluation of $M_{fi}^{Z\gamma}$ the integrand in (B1) is written in terms of the photon propagator in Feynman gauge,

$$I(q) \equiv \frac{1}{q_1^2 + i\epsilon} (T_n^{Z\gamma})^{\mu\nu} = \sum_{\varrho=0}^3 g^{\mu\varrho} \sum_{\tau=0}^3 \frac{g^{\nu\tau}}{q_1^2 + i\epsilon} (T_n^{Z\gamma})_{\varrho\tau}. \quad (\text{B5})$$

Upon decomposing the photon propagator into Coulombic and transverse terms [43],

$$\frac{g^{\nu\tau}}{q^2 + i\epsilon} = -\frac{1}{\mathbf{q}^2} \delta_{\nu 0} \delta_{\tau 0} - \frac{\delta_{mk} - \hat{q}^m \hat{q}^k}{q^2 + i\epsilon} \delta_{\nu m} \delta_{\tau k}, \quad m, k = 1-3, \quad (\text{B6})$$

where $\hat{q}^m$ is the m th component of the unit vector in direction of $\mathbf{q}$, one obtains

$$\sum_{\mu\nu=0}^3 t_{\mu\nu}^{Z\gamma} I(q) = -\frac{1}{q_1^2} \sum_{\mu=0}^3 (t^{Z\gamma})_\mu^0 (T_n^{Z\gamma})_{\mu 0} - \frac{1}{q_1^2 + i\epsilon} \times \left[\sum_{m=1}^3 \sum_{\mu=0}^3 (t^{Z\gamma})_\mu^m \left((T_n^{Z\gamma})_{\mu m} - \hat{q}_1^m \sum_{k=1}^3 \hat{q}_1^k (T_n^{Z\gamma})_{\mu k} \right) \right]. \quad (\text{B7})$$

Corresponding to the two-term expression (B7) the transition matrix element is split into two parts, $M_{fi}^{Z\gamma} = M_1 + M_2$. Let us first concentrate on the further evaluation of M_1 . The singularity from the factor $1/q_1^2$ in the first term of (B7) is omitted by changing the integration variable over the electronic intermediate states from $\mathbf{p}$ to $\mathbf{p}' = \mathbf{p} - \mathbf{k}_i = -\mathbf{q}_1$. Upon insertion of (B2) and (B7) into (B1) one obtains (with $|\mathbf{p}'| \equiv p'$)

$$M_1 = -\frac{g_z^2}{32\pi^2 M_z^2 c^2} \int_0^\infty dp' \int d\Omega_{p'} \times \frac{1}{E_p^2 - (\mathbf{p}' + \mathbf{k}_i)^2 c^2 - c^4 + i\epsilon} \sum_{\mu=0}^3 S_1(\mathbf{p}'), \quad (\text{B8})$$

where $S_1(\mathbf{p}')$ is a well-behaved function,

$$S_1(\mathbf{p}') = [\psi_{f,\pm}^{(0)+} (4 \sin^2 \theta_w - 1 + \gamma_5) \gamma_0 \gamma^\mu \times (E_p + c\alpha(\mathbf{p}' + \mathbf{k}_i) + \gamma_0 c^2) \psi_{i,\pm}^{(0)}] \sum_M (T_n^{Z\gamma})_{\mu 0}. \quad (\text{B9})$$

The structure of M_2 is analogous to (B8), except for the additional factor $p'^2/[(E_i - E_p)^2/c^2 - p'^2 + i\epsilon]$, which leads to a second singularity near $p' = \omega_L/c$. This means that the evaluation of $M_{fi}^{Z\gamma}$ is done with the same techniques that are applied to the transition amplitudes M_{fi}^c and M_{fi}^{te1} of the two-photon box diagram. The basic difference is that the innermost

integral (over the azimuthal angle of $\mathbf{p}'$) does not contain any singularity. This speeds up convergence and avoids the logarithmic singularity that occurs in the cross section for the two-photon box diagram. Nevertheless, the absence of $1/(q_2^2 + i\epsilon)$ with $q_2 = p - k_f$ in the integrand leads to a poorer convergence with respect to the upper integration limit $p_{\max}$ of the radial integral, which therefore relies solely on the vanishing of the transition form factors at large momentum transfers.

Diagram (b) of Fig. 1 is treated in the same way, but changing variables from $\mathbf{p}$ to $\mathbf{p}' = \mathbf{p} - \mathbf{k}_f = \mathbf{q}_2$. Therefore, also here, the same techniques can be used as for the evaluation of the transition amplitudes M_{fi}^{te2} and M_{fi}^{te3} of the two-photon box diagram [16].

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