

Boost-invariant and cylindrically symmetric perfect spin hydrodynamics

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Equations of a boost-invariant and cylindrically symmetric perfect hydrodynamics are solved numerically for initial conditions inspired by the wounded nucleon model. The energy-momentum and spin tensors are used in the form that describes a relativistic massive gas governed by Boltzmann statistics. In contrast to one-dimensional boost-invariant expansion, we find a coupling between the azimuthal and longitudinal components of the electric and magnetic components of the spin polarization tensor. This feature is similar to that found earlier for the Gubser symmetry; however, our treatment allows for a more general form of initial conditions and expansion geometry. Defining the freeze-out hypersurface by the constant temperature condition, we evaluate the Pauli-Lubanski four-vector and find that for the assumed geometry the only nonzero total polarization may be induced by the longitudinal component of the magnetic part of the spin polarization tensor, coupled with the azimuthal electric component. The obtained results may serve as a reference point for more realistic models of hydrodynamic expansion.

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I. INTRODUCTION

The experimental measurements of hyperon spin polarization [1–9] and vector-meson spin alignment [10–12] inspired a great deal of interest in the spin phenomena that may occur in the collisions of heavy ions at relativistic energies. For recent reviews see [13–15], and for various related theoretical studies see [16–69].

One of the important theoretical developments related to this topic is the construction of spin hydrodynamics. In this paper, we analyze some formal aspects of this theory, namely, we construct its boost-invariant and cylindrically symmetric solutions. We restrict ourselves to the case of perfect spin hydrodynamics in the form defined in our recent papers [70], i.e., we assume that the spin part of the total angular momentum is separately conserved and adopt the so-called GLW (de Groot–van Leeuwen–van Weert) versions of the energy–momentum and spin tensors [71,72]. Moreover, we do not include the feedback of the spin variables on the hydrodynamic evolution of the spinless background.

Our current analysis can be viewed as an extension of earlier works that included a one-dimensional boost invariant expansion [73–79] and a Gubser hydrodynamic flow [80,81];

see also [82–85]. In contrast to [80,81,86], our transverse dynamics is not constrained by any symmetry except the cylindrical one. The solutions in the transverse plane are obtained numerically for physically motivated initial conditions; since cylindrical symmetry is appropriate for central collisions, the initial transverse profiles of the entropy density are expressed by the Woods-Saxon nucleon distributions.

With our two symmetries constraining the system dynamics, we cannot address the two phenomena that are commonly discussed in the context of spin physics, namely, the global and local longitudinal polarizations. The global polarization is connected with the rotation about the y axis (out-of-plane axis), which is absent in boost-invariant systems, whereas the longitudinal polarization is connected with the formation of the elliptic flow and does not appear in a cylindrically symmetric case. However, in our opinion, our analysis is important for the very development of spin hydrodynamics, since very little is known about the properties of its solutions. On the one hand, one-dimensional boost invariant and two-dimensional Gubser solutions have been constructed, and on the other hand the first numerical solution has been found for a full 3D dynamics [63,65]. Our present study is the first attempt to fill this gap and analyze 2D dynamics; however, still with the constraint of cylindrical symmetry. Future, less restrictive analyzes may benefit from our findings, since they should reduce to them in some special cases. Moreover, the Gubser-flow approach is known to combine inconsistently the hydrodynamic background (conformal, massless limit) and spin dynamics (approach valid for massive particles only). Thus, our approach has the advantage of using for the first time a consistent boost-invariant and cylindrically symmetric framework, derived uniformly from the kinetic theory for these two aspects of system dynamics.

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The paper is organized as follows. In Sec. II we define the structure of perfect spin hydrodynamics used in our analysis. In particular, we discuss splitting of the system dynamics into a spinless hydrodynamic background and a spin evolution in this background. Boost invariance and cylindrical symmetry are implemented in Sec. III. Section IV describes the hydrodynamic equations of a spinless background. Numerical results are shown for the equation of state that describes a gas of massive particles. Equations describing the time evolution of the components of the spin polarization tensor are introduced in Sec. V. In Sec. VI we present our calculation of the Pauli-Lubański four-vector of particles with a given momentum, emitted from a system on the freeze-out hypersurface defined by the condition of constant temperature. We summarize in Sec. VII. Details of the calculations are given in the Appendixes.

Notation and conventions. For the Levi-Civita tensor $\epsilon^{\mu\nu\alpha\beta}$ we follow the convention $\epsilon^{0123} = -\epsilon_{0123} = +1$. The metric tensor is of the form $g_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$. We use boldface symbols to denote three vectors and their scalar products are written as $\mathbf{a} \cdot \mathbf{b}$. The particles are always considered on the mass shell with the energy $E_p = \sqrt{m^2 + \mathbf{p}^2}$. Throughout the text, we make use of natural units, $\hbar = c = k_B = 1$.

II. PERFECT SPIN HYDRODYNAMICS

The perfect spin hydrodynamics is based on the conservation laws for the baryon number, energy, linear momentum, and the spin part of the total angular momentum (for a recent summary of this approach, see [70]). These conservation laws take the differential form

$$\partial_\mu N^\mu = 0, \quad \partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu S^{\mu,\alpha\beta} = 0, \quad (1)$$

where N^μ is the baryon current, $T^{\mu\nu}$ is the energy-momentum tensor and $S^{\mu,\alpha\beta}$ is the spin tensor. Having in mind application to a gas of spin-1/2 particles in local equilibrium, we use the so-called GLW versions of $T^{\mu\nu}$ and $S^{\mu,\alpha\beta}$ [72]. In this case (1) become eleven equations for temperature (T), baryon chemical potential (μ), three independent components of the fluid four-velocity ($\mathbf{u}$), and six components of the spin polarization tensor ($\omega_{\mu\nu}$).

The GLW form of the energy-momentum tensor is symmetric, which implies that the corresponding GLW spin tensor is conserved. As a consequence, there is no transfer between the spin and orbital parts of the total angular momentum. Such a transfer is possible if we go beyond the description of a perfect fluid. In this case, the energy-momentum tensor receives asymmetric gradient corrections that imply that the divergence of the spin tensor does not vanish any longer.

It is important to distinguish between gradient corrections that are dissipative and corrections that result from the expansion of various tensors in the magnitude of the spin polarization tensor ω . The latter do not introduce dissipation in a way similar to that of expansion in the dimensionless ratio μ/T (low baryon-number density expansion). Since the expansion in ω of the baryon current and energy-momentum tensor starts with quadratic terms, while that of the spin tensor starts with linear terms, it is natural to consider the case where one solves spinless hydrodynamic evolution first and then

solves the spin dynamics in the given hydrodynamic background. This corresponds formally to an expansion including only first-order terms in ω . This scheme is adopted in this work. An extension to higher orders in ω is planned for future works.

We note that our treatment rigorously follows from the kinetic theory; hence, it leads to thermodynamically consistent description of the expanding system.

III. SYMMETRY IMPLEMENTATION

In this section, we implement cylindrical symmetry and boost invariance in the formalism of perfect spin hydrodynamics. We follow the treatment of [73,87]: we first introduce cylindrically symmetric and boost-invariant basis four-vectors and subsequently use them to construct the energy-momentum and spin tensors.

A. Choice of basis

The hydrodynamic flow is described by the four-vector

$$U^\mu = \gamma(1, \mathbf{v}) = \gamma(1, v_x, v_y, v_z), \quad \gamma = (1 - v^2)^{-1/2}. \quad (2)$$

For boost-invariant and cylindrically symmetric systems, we can use the following parametrization [73,87]:

$$\begin{aligned} U^0 &= \cosh \theta \cosh \eta, & U^1 &= \sinh \theta \cos \phi, \\ U^2 &= \sinh \theta \sin \phi, & U^3 &= \cosh \theta \sinh \eta, \end{aligned} \quad (3)$$

where θ is the transverse fluid rapidity defined by the formula

$$v_\perp = \sqrt{v_x^2 + v_y^2} = \tanh \theta, \quad (4)$$

η is the space-time rapidity,¹

$$\eta = \frac{1}{2} \ln \frac{t + z}{t - z}, \quad (5)$$

and ϕ is the azimuthal angle,

$$\phi = \arctan \frac{y}{x}. \quad (6)$$

The scalar function θ depends on the proper time

$$\tau = \sqrt{t^2 - z^2} \quad (7)$$

and the radial distance

$$r = \sqrt{x^2 + y^2}. \quad (8)$$

It is straightforward to verify in this case that U has a cylindrically symmetric and boost-invariant form, i.e., it satisfies the condition $U'^\mu(x') = L^\mu_\nu U^\nu(x) = U^\mu(x')$, where L^μ_ν represents a Lorentz boost along the z axis or a rotation about the z axis, and $x' = Lx$.²

In addition to U^μ we define three other symmetric four-vectors. The first one, Z^μ , defines the longitudinal direction

¹For simplicity of notation we use the variables θ and η rather than previously used $\theta_\perp$ and $\eta_\parallel$.

²The boost invariance of vector fields is discussed in more detail in Ref. [88].

that plays a special role owing to the initial geometry of the collision,

$$Z^0 = \sinh \eta, \quad Z^1 = 0, \quad Z^2 = 0, \quad Z^3 = \cosh \eta. \quad (9)$$

The second four-vector, R^μ , defines a radial direction with respect to the beam,

$$\begin{aligned} R^0 &= \sinh \theta \cosh \eta, & R^1 &= \cosh \theta \cos \phi, \\ R^2 &= \cosh \theta \sin \phi, & R^3 &= \sinh \theta \sinh \eta, \end{aligned} \quad (10)$$

while the third four-vector, Φ^μ , defines the azimuthal direction,

$$\Phi^0 = 0, \quad \Phi^1 = -\sin \phi, \quad \Phi^2 = \cos \phi, \quad \Phi^3 = 0. \quad (11)$$

The four-vector U^μ is timelike, while the four-vectors Z^μ , R^μ , Φ^μ are spacelike. In addition, they are all orthogonal to each other,

$$\begin{aligned} U^2 &= 1, & Z^2 &= R^2 = \Phi^2 = -1, \\ U \cdot Z &= 0, & U \cdot R &= 0, & U \cdot \Phi &= 0, \\ Z \cdot R &= 0, & R \cdot \Phi &= 0, & \Phi \cdot Z &= 0. \end{aligned} \quad (12)$$

B. Energy-momentum tensor

In the formalism of hydrodynamics one uses the operator $\Delta^{\mu\nu} = g^{\mu\nu} - U^\mu U^\nu$ that projects on the three-dimensional space orthogonal to U^μ . It can be shown that

$$\Delta^{\mu\nu} = -R^\mu R^\nu - \Phi^\mu \Phi^\nu - Z^\mu Z^\nu. \quad (13)$$

Using Eqs. (12) we find that Z^μ , R^μ , and Φ^μ are eigenvectors of $\Delta^{\mu\nu}$,

$$\Delta^{\mu\nu} R_\nu = R^\mu, \quad \Delta^{\mu\nu} \Phi_\nu = \Phi^\mu, \quad \Delta^{\mu\nu} Z_\nu = Z^\mu. \quad (14)$$

The energy-momentum tensor of the perfect spin fluid has the following structure:

$$T^{\mu\nu} = \varepsilon U^\mu U^\nu + P(R^\mu R^\nu + \Phi^\mu \Phi^\nu + Z^\mu Z^\nu). \quad (15)$$

Since we consider boost-invariant and cylindrically symmetric systems, ε and P (as scalars) may depend only on the (longitudinal) proper time and radial distance; see Eqs. (7) and (8).

In this work, we consider a classical relativistic gas where

$$\varepsilon = \frac{g}{2\pi^2} T m^2 (e^{\mu/T} + e^{-\mu/T}) [3T K_2(z) + m K_1(z)] \quad (16)$$

and

$$P = \frac{g}{2\pi^2} T^2 m^2 (e^{\mu/T} + e^{-\mu/T}) K_2(z). \quad (17)$$

Here $K_n(z)$ denotes the modified Bessel function of the second kind of the argument $z = m/T$. The two exponential functions correspond to the contributions of the particles and antiparticles (below we use also the notation $\xi = \mu/T$). The factor g denotes the number of internal degrees of freedom. Since our results are independent of g , we ignore it in further expressions. In the numerical calculations, we consider two values of the particle mass: $m = 1$ GeV and $m = 0.3$ GeV. We note that spin polarization considered in this work is defined in the particle rest frame; hence, it is well defined for massive

particles. Consequently, the limit $m \rightarrow 0$ cannot be taken as it is not well defined.

In order to characterize the propagation of perturbations within the fluid, it is essential to determine the speed of sound c_s [89]. For a perfect fluid, the square of the speed of sound is defined as the derivative of pressure with respect to energy density at a constant entropy per net baryon $s = \sigma/n$,

$$c_s^2 = \left(\frac{\partial P}{\partial \varepsilon} \right)_s. \quad (18)$$

Using standard thermodynamic relations and the properties of Jacobians presented in [90], this derivative can be expanded in terms of our independent variables (T, μ) as

$$c_s^2 = \frac{\left(\frac{\partial P}{\partial T} \right)_\mu + \left(\frac{\partial P}{\partial \mu} \right)_T \left(\frac{d\mu}{dT} \right)_s}{\left(\frac{\partial \varepsilon}{\partial T} \right)_\mu + \left(\frac{\partial \varepsilon}{\partial \mu} \right)_T \left(\frac{d\mu}{dT} \right)_s}, \quad (19)$$

where the condition $ds = 0$ determines the derivative $(d\mu/dT)_s$.

Using (16) and (17), along with the baryon density

$$n = \frac{g}{\pi^2} T^3 z^2 \sinh(\xi) K_2(z), \quad (20)$$

and utilizing the recurrence relations for the modified Bessel functions, one can derive an analytical expression for the speed of sound. Introducing a dimensionless auxiliary function $\alpha(z)$,

$$\alpha(z) \equiv 4 + z \frac{K_1(z)}{K_2(z)}, \quad (21)$$

the exact square of the speed of sound for a massive gas at finite chemical potential takes the form

$$c_s^2(z, \xi) = \frac{\alpha(z)^2 \coth^2 \xi - 2\alpha(z)^2 + 5\alpha(z) + z^2}{\alpha(z)[(3\alpha(z) + z^2) \coth^2 \xi - (\alpha(z) - 1)^2]}. \quad (22)$$

Furthermore, in the regime of vanishing baryon density ($\mu \rightarrow 0$, which implies $\xi \rightarrow 0$ and $\coth^2 \xi \rightarrow \infty$), the terms proportional to $\coth^2 \xi$ dominate the expression. In this limit (22) simplifies to the formula for a strictly thermal relativistic massive gas (equivalent to Eq. (16) of [91]),

$$\lim_{\mu \rightarrow 0} c_s^2(z) = \frac{\alpha(z)}{3\alpha(z) + z^2} = \frac{4K_2(z) + zK_1(z)}{(12 + z^2)K_2(z) + 3zK_1(z)}. \quad (23)$$

The contour plots of the function $c_s^2(T, \mu)$ are shown in Fig. 1 for two values of the particle mass: $m = 1$ GeV (a) and $m = 0.3$ GeV (b). As expected, for smaller values of the mass, the values of the function $c_s^2(T, \mu)$ are larger. In the limit $m \rightarrow 0$, one has $c_s^2 \rightarrow 1/3$. In the hydrodynamic model, larger values of the speed of sound lead to faster initial cooling and expansion of matter. We will observe this effect in Sec. IV C.

C. Spin polarization tensor

Following the method used in relativistic magnetohydrodynamics, we decompose the spin polarization tensor into an

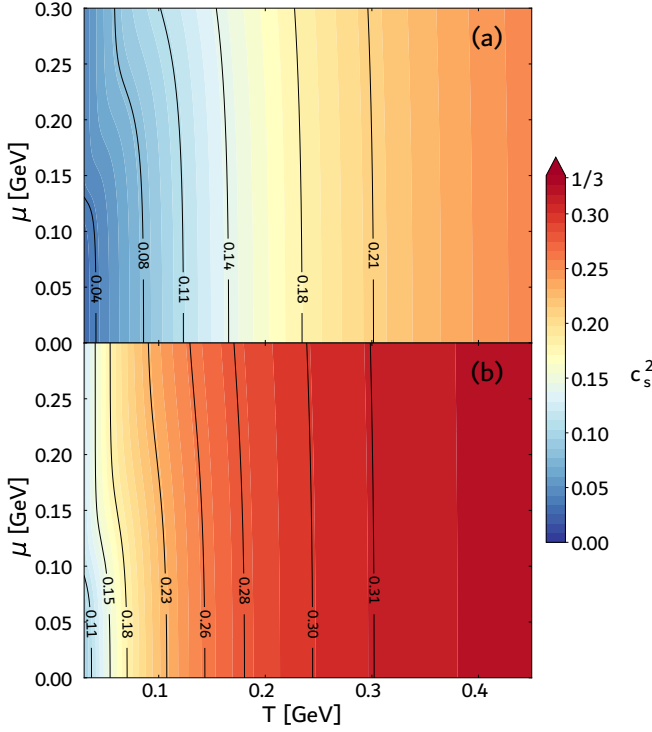


FIG. 1. Contour plots of the square of the speed of sound, c_s^2 , in the temperature and baryon chemical potential (T, μ) plane. The upper panel (a) shows the results for a particle mass of $m = 1$ GeV, while the lower panel (b) corresponds to a lower mass of $m = 0.3$ GeV. The solid black lines represent contours of constant c_s^2 .

“electric” and a “magnetic” part,³

$$\omega_{\mu\nu} = k_\mu U_\nu - k_\nu U_\mu + t_{\mu\nu} = k_\mu U_\nu - k_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta. \quad (24)$$

The four-vectors k and ω (the electriclike and magneticlike components, respectively) satisfy the orthogonality conditions

$$k_\mu U^\mu = 0, \quad \omega_\mu U^\mu = 0. \quad (25)$$

Equation (24) implicitly defines the tensor $t_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta$.

Since the four-vectors k and ω are orthogonal to the flow vector U , we can decompose them as follows:

$$\begin{aligned} k^\mu &= C_{kr} R^\mu + C_{k\phi} \Phi^\mu + C_{kz} Z^\mu, \\ \omega^\mu &= C_{\omega r} R^\mu + C_{\omega\phi} \Phi^\mu + C_{\omega z} Z^\mu, \end{aligned} \quad (26)$$

where the coefficient functions C depend only on τ and r . For convenience, we introduce a three-vector notation

$$\mathbf{C}_k = (C_{kr}, C_{k\phi}, C_{kz}), \quad \mathbf{C}_\omega = (C_{\omega r}, C_{\omega\phi}, C_{\omega z}). \quad (27)$$

A straightforward calculation gives

$$\begin{aligned} \omega_{\mu\nu} &= C_{kz}(Z_\mu U_\nu - Z_\nu U_\mu) + C_{kr}(R_\mu U_\nu - R_\nu U_\mu) \\ &\quad + C_{k\phi}(\Phi_\mu U_\nu - \Phi_\nu U_\mu) \\ &\quad + \epsilon_{\mu\nu\alpha\beta} U^\alpha (C_{\omega z} Z^\beta + C_{\omega r} R^\beta + C_{\omega\phi} \Phi^\beta), \end{aligned} \quad (28)$$

and for the dual tensor we find

$$\begin{aligned} \tilde{\omega}_{\mu\nu} &= C_{\omega z}(Z_\mu U_\nu - Z_\nu U_\mu) + C_{\omega r}(R_\mu U_\nu - R_\nu U_\mu) \\ &\quad + C_{\omega\phi}(\Phi_\mu U_\nu - \Phi_\nu U_\mu) \\ &\quad - \epsilon_{\mu\nu\alpha\beta} U^\alpha (C_{kz} Z^\beta + C_{kr} R^\beta + C_{k\phi} \Phi^\beta). \end{aligned} \quad (29)$$

We note that the dual tensor is obtained by the replacements $\mathbf{C}_k \rightarrow \mathbf{C}_\omega$ and $\mathbf{C}_\omega \rightarrow -\mathbf{C}_k$.

D. Spin tensor

Following our earlier works, we use the GLW definition of the spin tensor [23],

$$\begin{aligned} S^{\lambda,\mu\nu} &= U^\lambda [A(k^\mu U^\nu - k^\nu U^\mu) + A_1 t^{\mu\nu}] \\ &\quad + \frac{A}{2} (t^{\lambda\mu} U^\nu - t^{\lambda\nu} U^\mu + \Delta^{\lambda\mu} k^\nu - \Delta^{\lambda\nu} k^\mu), \end{aligned} \quad (30)$$

where the functions A and A_1 are defined by the expressions

$$A = -\frac{4s^2 \cosh \xi}{3\pi^2} z T^3 K_3(z), \quad (31)$$

$$A_1 = \frac{2s^2 \cosh \xi}{3\pi^2} z T^3 [z K_2(z) + 2K_3(z)]. \quad (32)$$

Here $s^2 = 3/4$ is the eigenvalue of the Casimir operator for $SU(2)$.

All tensors that have been introduced so far are defined in the GLW pseudogauge. A transition to the canonical spin tensor is made using the formula (see Eq. (117) in [72])

$$S_{\text{can}}^{\lambda,\mu\nu} = S^{\lambda,\mu\nu} + S^{\mu,\nu\lambda} + S^{\nu,\lambda\mu}. \quad (33)$$

We note that $S_{\text{can}}^{\lambda,\mu\nu}$ defined in this way is totally antisymmetric in all three indices. Substituting (30) into (33) one finds

$$S_{\text{can}}^{\lambda,\mu\nu} = (A + A_1)(t^{\lambda\mu} U^\nu - t^{\lambda\nu} U^\mu + t^{\mu\nu} U^\lambda) = \frac{\tilde{n}}{4} \epsilon^{\lambda\mu\nu\rho} \omega_\rho. \quad (34)$$

Interestingly, the spin tensor in the canonical pseudogauge depends only on the magnetic-like components of the spin polarization tensor. It has been argued in [92] that the canonical spin tensor is preferable to represent spin observables; it is directly connected to the generators of rotations. The quantity $\tilde{n}$ in (34) is the sum of particle and antiparticle densities ($\tilde{n} = n/\tanh \xi$, with n given by (20) with $g = 2$).

The canonical spin tensor is not conserved, and its divergence equals

$$\partial_\lambda S_{\text{can}}^{\lambda,\mu\nu} = -\partial_\lambda S^{\mu,\lambda\nu} + \partial_\lambda S^{\nu,\lambda\mu} = 2 T_{\text{can}}^{[\nu\mu]}, \quad (35)$$

where the second equality in (35) was derived in [72]. It is important to note that (35) is equivalent to the spin conservation in the form $\partial_\lambda S^{\lambda,\mu\nu} = 0$. We thus see that the transition between the spin and orbital parts of the total angular momentum may depend on the pseudogauge used (GLW vs canonical); however, this dependence does not change the real dynamics of the system (the equations of motion for T , μ , $\mathbf{u}$, and $\omega_{\mu\nu}$).

IV. SPINLESS HYDRODYNAMIC BACKGROUND

With dissipation effects neglected and components of the spin polarization tensor included up to the first order, the

³Here we use the original notation from Ref. [23].

system of equations of spin hydrodynamics splits into separate equations for a spinless hydrodynamic background and equations that determine the evolution of the spin polarization tensor coupled to this background. Hence, a natural method to deal with such a system is to first solve the standard hydrodynamic equations and then switch to the equation for the spin tensor. In this section, following several earlier works, most importantly [93], we describe our treatment of hydrodynamic equations for cylindrically symmetric and boost-invariant systems.

A. Baryon current

For cylindrically symmetric and boost-invariant systems the baryon number conservation law can be cast into the form

$$\left(\frac{\partial}{\partial \tau} + \tanh \theta \frac{\partial}{\partial r}\right)n + n \left[\left(\frac{1}{\tau} + \frac{\partial \theta}{\partial r}\right) + \tanh \theta \left(\frac{1}{r} + \frac{\partial \theta}{\partial \tau}\right)\right] = 0. \quad (36)$$

We recall that the theta parameter denotes transverse rapidity. If the transverse expansion is neglected, $\theta = 0$, then from (36) we find that $dn/d\tau + n/\tau = 0$, with a scaling solution $n = n_0 \tau_0 / \tau$, where n_0 and τ_0 are integration constants.

B. Energy-momentum tensor

Because of the cylindrical symmetry and boost invariance, the four equations expressing the conservation of energy and linear momentum are reduced to only two equations,

$$\left(\frac{\partial}{\partial \tau} + \tanh \theta \frac{\partial}{\partial r}\right)\varepsilon + (\varepsilon + P) \times \left[\left(\frac{1}{\tau} + \frac{\partial \theta}{\partial r}\right) + \tanh \theta \left(\frac{1}{r} + \frac{\partial \theta}{\partial \tau}\right)\right] = 0 \quad (37)$$

and

$$\left(\tanh \theta \frac{\partial}{\partial \tau} + \frac{\partial}{\partial r}\right)P + (\varepsilon + P) \left(\tanh \theta \frac{\partial \theta}{\partial r} + \frac{\partial \theta}{\partial \tau}\right) = 0. \quad (38)$$

Using the thermodynamic relations $\varepsilon + P = T\sigma + \mu n$ and $d\varepsilon = Td\sigma + \mu dn$ (where σ is the entropy density), it can be checked that Eq. (37) represents the conservation of entropy. On the other hand, Eq. (38) is the relativistic Euler equation that determines the dynamics of the transverse expansion. In the limit $\theta \rightarrow 0$ and for transversely uniform systems, Eq. (38) is automatically fulfilled, while Eq. (37) is reduced to $d\sigma/d\tau + \sigma/\tau = 0$, again leading to a scaling solution $\sigma = \sigma_0 \tau_0 / \tau$.

Equations (36)–(38) are three equations for three unknown functions: T , μ , and θ . In the next section we turn to a discussion of their numerical solutions.

C. Numerical results

To solve the equations of the spinless hydrodynamic background, we specify the initial conditions at the proper time

$\tau_0 = 1.0$ fm. The initial transverse thermodynamic profiles are initialized proportionally to the Woods-Saxon nuclear density distribution,

$$\rho(r) = \frac{1}{1 + \exp\left(\frac{r-R}{a}\right)}, \quad (39)$$

where the nuclear radius $R = 7.0$ fm and the skin thickness $a = 0.54$ fm are chosen to reflect the standard geometric properties of heavy nuclei commonly used in experiments, such as Au or Pb.

Following this geometric profile, the initial temperature and baryon chemical potential are parametrized as $T(\tau_0, r) = T_0 \rho(r)^{1/3} + T_{\text{vac}}$ and $\mu(\tau_0, r) = \mu_0 \rho(r)$. Here $T_0 = 0.3$ GeV is a central temperature while the parameter $T_{\text{vac}} = 0.03$ GeV acts as a small ambient vacuum temperature necessary to ensure numerical stability at large radii where the fluid density becomes very small. The central value of the baryon chemical potential is set to $\mu_0 = 0.03$ GeV. Notably, μ_0 is chosen to be exactly an order of magnitude smaller than the central temperature, placing our system in a regime of high temperature and low net-baryon density. The initial transverse fluid rapidity is set to a small constant value $\theta(\tau_0, r) = 0.01$ instead of strictly zero to avoid numerical singularities and ensure a stable initialization of the solver.

Coupled partial differential equations are solved on a one-dimensional radial grid extending up to $r_{\text{max}} = 20.0$ fm with a uniform spatial resolution of $dr = 0.05$ fm. The temporal evolution is computed up to a final proper time of $\tau_{\text{max}} = 10.0$ fm. Spatial gradients are evaluated utilizing second-order finite differences. For the proper-time integration, we employ the Backward Differentiation Formula (BDF).

To maintain high computational efficiency during the time-stepping process, the thermodynamic quantities and their exact derivatives—which depend on the modified Bessel functions $K_n(z)$ as defined in Sec. III B—are precomputed over a fine temperature grid and evaluated continuously using cubic spline interpolation. Additionally, a small numerical viscosity coefficient is introduced via a Laplacian operator to suppress spurious numerical oscillations near steep thermodynamic gradients, without distorting the underlying physical expansion flow.

The results of our numerical calculations are presented in Fig. 2. Panels (a) and (b) show the spatial profiles of temperature for four values of proper time: $\tau = 1.0, 4.0, 7.0$ and 10.0 fm. Similarly, panels (c) and (d) show the profiles of the baryon chemical potential. The left (right) panels correspond to the case $m = 1$ GeV ($m = 0.3$ GeV). The transverse expansion is faster in the case where the particle mass is smaller. In this case, the speed of sound is larger, and the rarefaction wave entering the system is faster.

D. Comparison with earlier results

To validate our numerical framework, we benchmark our generalized hydrodynamics code against the results obtained in [93] for a strictly massless, cylindrically symmetric expanding fluid. Here the initial temperature profile

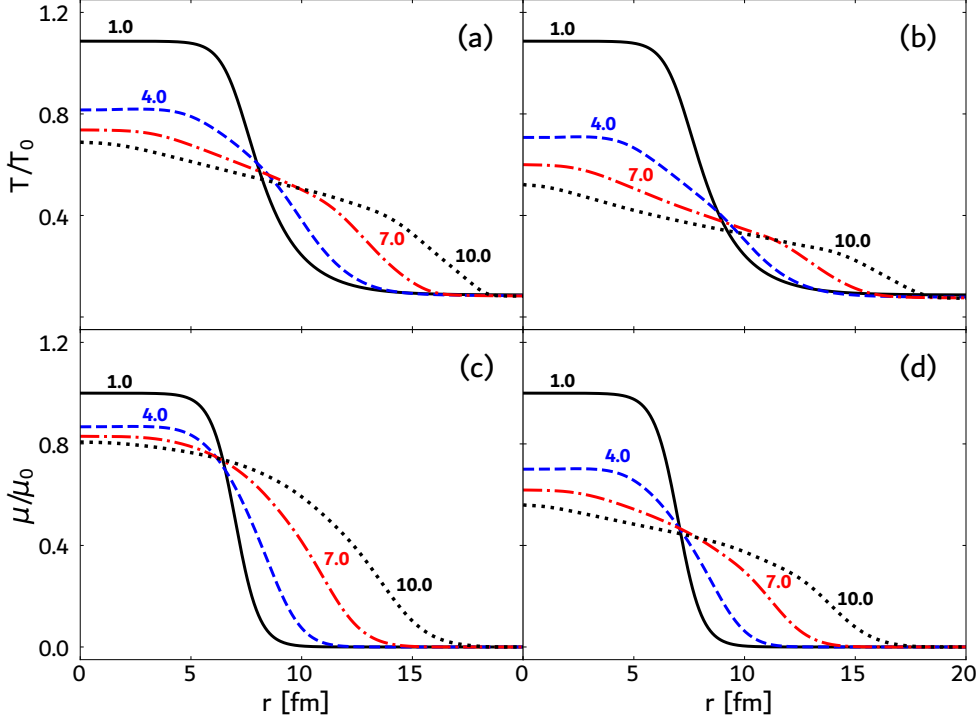


FIG. 2. Transverse spatial profiles of the scaled temperature T/T_0 (top row) and scaled baryon chemical potential μ/μ_0 (bottom row) at different proper times τ . The left panels, (a) and (c), display the hydrodynamic evolution for a heavy constituent particle mass of $m = 1$ GeV. The right panels, (b) and (d), correspond to a lighter particle mass of $m = 0.3$ GeV. The initial conditions at $\tau = 1.0$ fm are depicted by the solid black lines. The subsequent expansion stages are shown for $\tau = 4.0$ fm (dashed blue lines), $\tau = 7.0$ fm (dash-dotted red lines), and $\tau = 10.0$ fm (dotted black lines). A central initial temperature $T_0 = 0.3$ GeV and chemical potential $\mu_0 = 0.03$ GeV are used.

was assumed to directly follow the nuclear density profile, $T(r) \propto \rho(r)$. For a strictly massless gas obeying the Stefan-Boltzmann law ($\varepsilon \propto T^4$), this assumption leads to an initial energy density profile heavily peaked at the center: $\varepsilon(r) \propto \rho(r)^4$.

To provide a more physically motivated description, we adopt the wounded nucleon model (WNM) [94]. This approach is grounded in the experimental observation that the multiplicity of particles in relativistic heavy-ion collisions scales linearly with the number of wounded nucleons. Because the final particle multiplicity is directly related to the total initial entropy of the system, it is physically appealing to scale the initial entropy density $\sigma(r)$, rather than the temperature, with the nuclear density profile, yielding $\sigma(r) \propto \rho(r)$.

Since the entropy density of a massless gas scales as $\sigma \propto T^3$, the WNM implies that the initial temperature profile must follow $T(r) \propto \rho(r)^{1/3}$. Consequently, the initial energy density profile in our validation framework is defined as

$$\varepsilon(r, \tau_0) = \varepsilon_0 \left[\frac{\rho(r)}{\rho_0} \right]^{4/3}. \quad (40)$$

While exact formulation presented in [93] strictly relies on the equation of state for a massless gas ($\varepsilon = 3P$) with zero net-baryon density, our numerical scheme is fundamentally constructed for a gas of massive particles with

an explicitly conserved baryon current. Therefore, testing the massless limit requires a careful numerical setup. To reproduce such a limit within our generalized code, we instantiate our model with an exceptionally small particle mass of $m = 0.002$ GeV.

To satisfy the requirements of the baryon current conservation equations when testing the massless limit, the initial baryon chemical potential is set to a minimal, nonzero spatial profile $\mu(\tau_0, r) = \mu_0 \rho(r)$, where $\mu_0 = 2 \times 10^{-6}$ GeV. Furthermore, a small ambient vacuum temperature T_{vac} is included in the energy density profile to prevent numerical singularities at large radii.

Comparisons between our results and those obtained using the approach of Ref. [93] are shown in Fig. 3. We observe a very good agreement between the results obtained with two methods.

V. SPIN DENSITIES

The conservation of the spin part of the total angular momentum yields six equations that determine the spacetime dependence of the six independent coefficients of the spin polarization tensor, C_k and C_ω . For boost-invariant systems that are uniform in the transverse plane, these equations decouple and become six independent equations for each coefficient C appearing in the decomposition of the spin polarization tensor (provided that the energy-momentum tensor is included

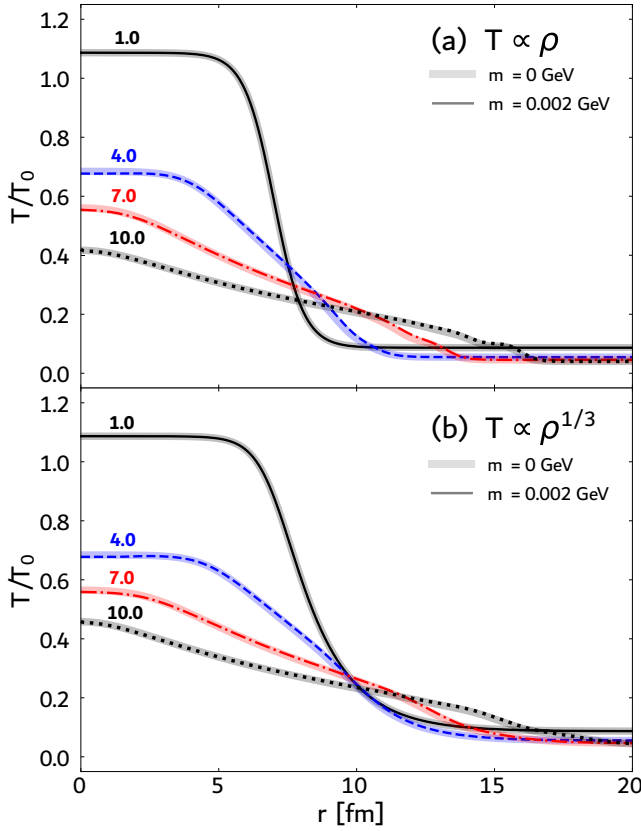


FIG. 3. Comparison of the transverse spatial profiles of the scaled temperature T/T_0 to validate the massless limit under two distinct initial density scalings: (a) $T \propto \rho$ and (b) $T \propto \rho^{1/3}$, which is in agreement with the wounded nucleon model (WNM). The thick semitransparent lines represent the solution for a strictly massless gas ($m = 0$ GeV) using the method outlined in [93]. The superimposed patterned lines show the numerical results from our massive hydrodynamic solver, evaluated for an exceptionally small particle mass ($m = 0.002$ GeV) and a minimal baryon chemical potential. The initial conditions at $\tau = 1.0$ fm are shown as solid black lines, while the subsequent expansion stages are depicted at proper times $\tau = 4.0$ (dashed blue), 7.0 (dash-dotted red), and 10.0 fm (dotted black).

in the leading order in ω). For cylindrically symmetric and boost-invariant systems, there appears a coupling between the longitudinal and azimuthal components, which allows us to consider four independent cases denoted below by the symbols (a), (b), (c), and (d).

(a) The radial electriclike sector defines the dynamics of the coefficient C_{kr} (obtained from the projection $U_\mu R_\nu \partial_\lambda S^{\lambda,\mu\nu} = 0$),

$$\begin{aligned} A(\dot{C}_{kr} + C_{kr}' \tanh \theta) \\ = -C_{kr}(\dot{A} + A' \tanh \theta) \\ - C_{kr}A\left(\dot{\theta} \tanh \theta + \theta' + \frac{3}{2\tau} + \frac{3 \tanh \theta}{2r}\right). \end{aligned} \quad (41)$$

Here and in the following, the dot denotes a partial derivative with respect to τ , whereas the prime denotes the partial derivative with respect to the coordinate r . The combination $\dot{f} + f' \tanh \theta$ can be interpreted as the convective derivative along the stream lines $df/d\tau$.

(b) The radial magnetic-like sector defines the dynamics of $C_{\omega r}$ (from $\Phi_\mu \partial_\nu \partial_\lambda S^{\lambda,\mu\nu} = 0$),

$$\begin{aligned} A_1(\dot{C}_{\omega r} + C_{\omega r}' \tanh \theta) \\ = -C_{\omega r}(\dot{A}_1 + A_1' \tanh \theta) \\ - C_{\omega r}\left[A_1\left(\dot{\theta} \tanh \theta + \theta' + \frac{1}{\tau} + \frac{\tanh \theta}{r}\right) \right. \\ \left. - \frac{A}{2}\left(\frac{1}{\tau} + \frac{\tanh \theta}{r}\right)\right]. \end{aligned} \quad (42)$$

(c) The longitudinal magnetic-like sector defines the coupled dynamics of $C_{k\phi}$ and $C_{\omega z}$ (from the projections $U_\mu \Phi_\nu \partial_\lambda S^{\lambda,\mu\nu} = 0$ and $R_\mu \Phi_\nu \partial_\lambda S^{\lambda,\mu\nu} = 0$),

$$\begin{aligned} 0 = A(\dot{C}_{k\phi} + C_{k\phi}' \tanh \theta) - \frac{A}{2}(\dot{C}_{\omega z} \tanh \theta + C_{\omega z}') \\ + C_{k\phi}(\dot{A} + A' \tanh \theta) - \frac{C_{\omega z}}{2}(\dot{A} \tanh \theta + A') \\ + A\left[\frac{3}{2}C_{k\phi}\left(\theta' + \dot{\theta} \tanh \theta + \frac{1}{\tau} + \frac{2 \tanh \theta}{3r}\right) \right. \\ \left. - \frac{C_{\omega z}}{2}\left(\theta' \tanh \theta + \dot{\theta} + \frac{\tanh \theta}{\tau}\right)\right] \\ + A_1 C_{\omega z}(\theta' \tanh \theta + \dot{\theta}), \end{aligned} \quad (43)$$

$$\begin{aligned} 0 = A_1(\dot{C}_{\omega z} + C_{\omega z}' \tanh \theta) + \frac{A}{2}(C_{k\phi}' + \dot{C}_{k\phi} \tanh \theta) \\ + \frac{C_{k\phi}}{2}(\dot{A} \tanh \theta + A') + C_{\omega z}(\dot{A}_1 + A_1' \tanh \theta) \\ + A\left[\frac{3}{2}C_{k\phi}\left(\theta' \tanh \theta + \dot{\theta} + \frac{1}{3r}\right) \right. \\ \left. - \frac{C_{\omega z}}{2}\left(\theta' + \dot{\theta} \tanh \theta + \frac{\tanh \theta}{r}\right)\right] \\ + A_1 C_{\omega z}\left(\theta' + \dot{\theta} \tanh \theta + \frac{1}{\tau} + \frac{\tanh \theta}{r}\right). \end{aligned} \quad (44)$$

(d) The longitudinal electric-like sector defines the coupled dynamics of C_{kz} and $C_{\omega\phi}$ (obtained from the projections $U_\mu Z_\nu \partial_\lambda S^{\lambda,\mu\nu} = 0$ and $Z_\mu R_\nu \partial_\lambda S^{\lambda,\mu\nu} = 0$),

$$\begin{aligned} 0 = A(\dot{C}_{kz} + C_{kz}' \tanh \theta) + \frac{A}{2}(\dot{C}_{\omega\phi} \tanh \theta + C_{\omega\phi}') \\ + C_{kz}(\dot{A} + A' \tanh \theta) + \frac{C_{\omega\phi}}{2}(\dot{A} \tanh \theta + A') \\ + A\left[\frac{3}{2}C_{kz}\left(\theta' + \dot{\theta} \tanh \theta + \frac{2}{3\tau} + \frac{\tanh \theta}{r}\right) \right. \\ \left. + \frac{C_{\omega\phi}}{2}\left(\theta' \tanh \theta + \dot{\theta} + \frac{1}{r}\right)\right] \\ - A_1 C_{\omega\phi}(\theta' \tanh \theta + \dot{\theta}), \end{aligned} \quad (45)$$

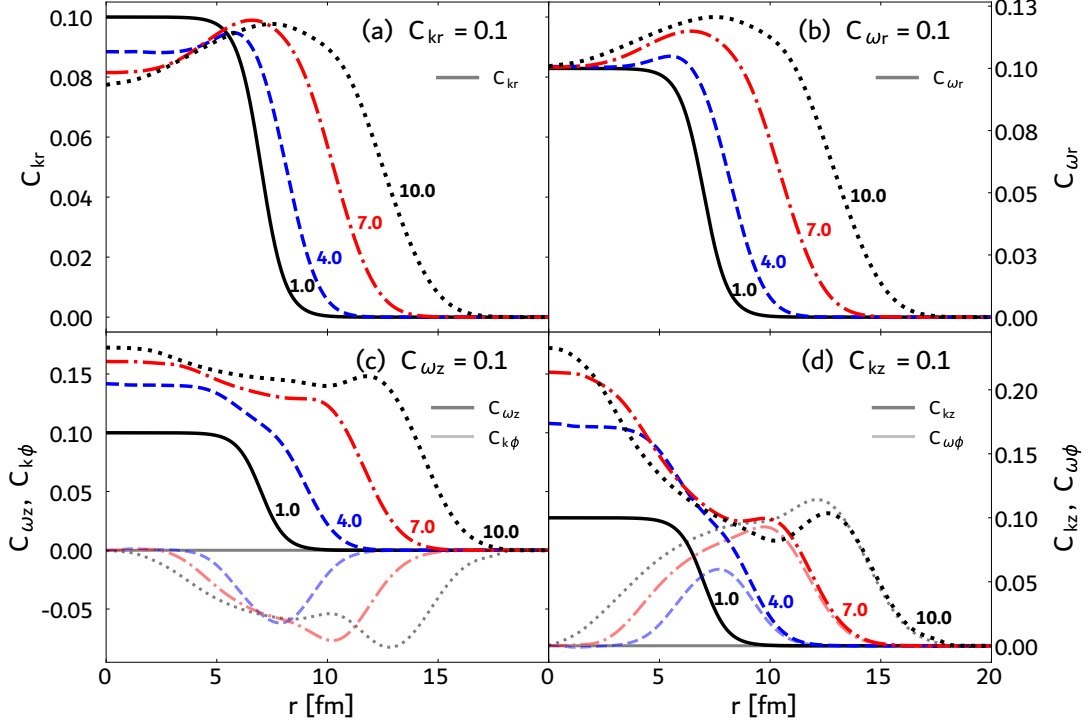


FIG. 4. Time evolution of the spin polarization tensor components as a function of the radial distance r for a heavy particle mass of $m = 1$ GeV. The initial conditions at $\tau = 1.0$ fm are defined by a modified Woods-Saxon profile (solid black lines). Subsequent expansion stages are shown for $\tau = 4.0$ fm (dashed blue lines), 7.0 fm (dash-dotted red lines), and 10.0 fm (dotted black lines). Panels (a) and (b) demonstrate the independent evolution of the radial components C_{kr} and $C_{\omega r}$. Panels (c) and (d) present the coupled dynamics: initializing $C_{\omega z}$ induces $C_{k\phi}$, and initializing C_{kz} induces $C_{\omega\phi}$. The initialized components are plotted with thick lines, while the dynamically induced components are shown with semitransparent, thinner lines.

$$\begin{aligned}
 0 = & A_1(\dot{C}_{\omega\phi} + C_{\omega\phi}' \tanh \theta) - \frac{A}{2}(\dot{C}_{kz} \tanh \theta + C_{kz}') \\
 & + C_{\omega\phi}(\dot{A}_1 + A_1' \tanh \theta) - \frac{C_{kz}}{2}(\dot{A} \tanh \theta + A') \\
 & - A \left[\frac{3}{2} C_{kz} \left(\theta' \tanh \theta + \dot{\theta} + \frac{\tanh \theta}{3\tau} \right) \right. \\
 & \left. + \frac{C_{\omega\phi}}{2} \left(\theta' + \dot{\theta} \tanh \theta + \frac{1}{\tau} \right) \right] \\
 & + A_1 C_{\omega\phi} \left(\theta' + \dot{\theta} \tanh \theta + \frac{1}{\tau} + \frac{\tanh \theta}{r} \right). \quad (46)
 \end{aligned}$$

The coupling between the C_{kz} and $C_{\omega\phi}$ components, as in (d), and between the $C_{\omega z}$ and $C_{k\phi}$ components, as in (c), was previously found in the analysis of the Gubser flow [80,81]. However, our finding is more general because we do not assume conformal symmetry used in the Gubser flow. Our initial profiles of C_k and C_ω in the transverse plane are arbitrary.

It is important to note that in cases (c) and (d) the terms $C_{\omega\phi}/r$ and $C_{k\phi}/r$ appear.⁴ This indicates that the coefficients

$C_{\omega\phi}(\tau, r)$ and $C_{k\phi}(\tau, r)$ should vanish for $r = 0$. This is a natural assumption, since the azimuthal components of vectors cannot be finite at the origin of the polar coordinates. As a consequence, in the numerical calculations we always assume the initial conditions $C_{\omega\phi}(\tau_0, r = 0) = 0$ and $C_{k\phi}(\tau_0, r = 0) = 0$. It turns out that the hydrodynamic evolution preserves these conditions at later times.

As explained above, to solve the spin evolution equations numerically, we decouple the problem by first evaluating the spinless hydrodynamic background and then solving the spin dynamics using the precalculated background variables (T , μ , and θ) as interpolating functions. The solution method was adapted from the hydrodynamic background. The spatial derivatives are computed using finite differences on a one-dimensional radial grid, while the time integration is performed using the BDF. To suppress spurious numerical oscillations near steep thermodynamic gradients without altering the physical dynamics, a small numerical viscosity term is explicitly added to the evolution equations.

To illustrate the dynamics of the spin polarization tensor, we initialize the system by setting only one specific component to a nonzero value at the initial proper time $\tau_0 = 1.0$ fm, while keeping all other components strictly zero. The initial nonzero spin density profiles are assumed to be proportional to the nuclear density distribution, $C_i(\tau_0, r) = 0.1\rho(r)$. Based

⁴They are discussed in more detail in Appendix A.

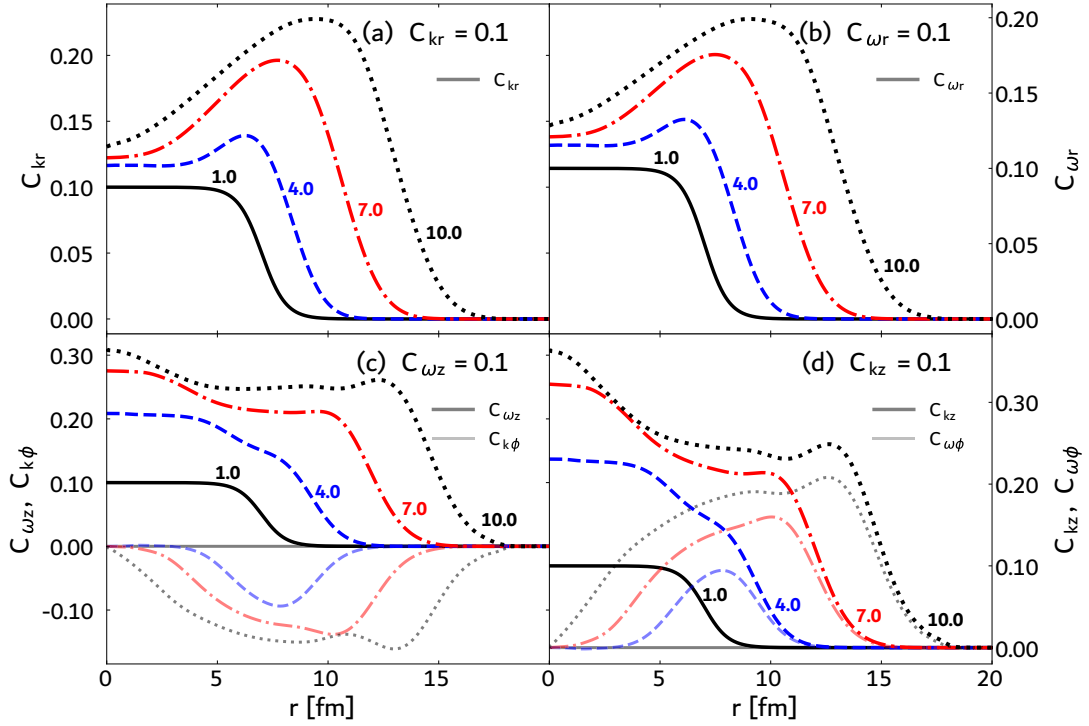


FIG. 5. Time evolution of the spin polarization tensor components as a function of the radial distance r , evaluated for a lighter particle mass of $m = 0.3$ GeV. The layout, initial conditions, and line styles correspond exactly to those in Fig. 4.

on the structure of the evolution equations introduced above, we distinguish four independent cases, which are presented in panels (a)–(d) of Figs. 4 and 5. In panels (a) and (b), we initialize the radial electriclike (C_{kr}) and magneticlike ($C_{\omega r}$) components, respectively. Their evolution is completely decoupled from the rest of the spin tensor, expanding radially without inducing any other spin components. In contrast, panels (c) and (d) demonstrate coupled dynamics. Initializing the longitudinal magnetic-like component $C_{\omega z}$ dynamically induces the azimuthal electriclike component $C_{k\phi}$ (c), whereas initializing the longitudinal electriclike component C_{kz} leads to the dynamic generation of the azimuthal magneticlike component $C_{\omega\phi}$ (d).

Figures 4 and 5 correspond to the cases $m = 1$ GeV and $m = 0.3$ GeV, respectively. The mass difference affects mainly the evolution of the coefficients C_{kr} , whose central value decreases for $m = 1$ GeV and increases for $m = 0.3$ GeV. In other cases, we observe a moderate increase of the coefficients with time, and the outwards shift of the maximum, an effect caused by the transverse expansion. An increase of the coefficients C_i in time has been observed previously in perfect spin hydrodynamics in the boost-invariant transversely homogeneous geometry [78,79]. It is likely that an inclusion of dissipation would change this picture and cause those coefficients to decrease. This may be verified by future studies of dissipative spin hydrodynamics.

In the coupled channels, we find some interesting features: the increase of the longitudinal magnetic component $C_{\omega z}$ produces a negative azimuthal electric component $C_{k\phi}$, while the increase of the longitudinal electric component C_{kz} produces

a positive azimuthal magnetic component $C_{\omega\phi}$. This behavior shares some similarities with the Maxwell equations.

VI. PAULI-LUBAŃSKI VECTOR

In this section, we calculate the spin polarization of particles described by our hydrodynamic model. We assume that, at some fixed freeze-out temperature, the system undergoes a transition from fluidlike behavior to free streaming. The mean spin polarization of the particles with four-momentum p is obtained from the Cooper-Frye formula [19,72]

$$\langle \pi_\mu \rangle = \frac{E_p \frac{d\Pi_\mu(p)}{d^3p}}{E_p \frac{d\mathcal{N}(p)}{d^3p}}, \quad (47)$$

where the numerator defines the momentum density of the Pauli-Lubański vector [19,72],

$$E_p \frac{d\Pi_\mu(p)}{d^3p} = -\frac{\cosh(\xi)}{(2\pi)^3 m} \int d\Sigma_\lambda p^\lambda e^{-\beta \cdot p} \tilde{\omega}_{\mu\beta} p^\beta, \quad (48)$$

while the denominator gives the momentum density of particles,

$$E_p \frac{d\mathcal{N}(p)}{d^3p} = \frac{4 \cosh(\xi)}{(2\pi)^3} \int d\Sigma_\lambda p^\lambda e^{-\beta \cdot p}. \quad (49)$$

Here we include both particles and antiparticles, which is reflected in the common prefactor $\cosh \xi$. However, this factor is canceled in the expression for the mean spin polarization (as

a consequence of using the classical statistics),

$$\langle \pi_\mu \rangle = -\frac{1}{4m} \frac{\int d\Sigma_\lambda p^\lambda e^{-\beta \cdot p} \tilde{\omega}_{\mu\beta} p^\beta}{\int d\Sigma_\lambda p^\lambda e^{-\beta \cdot p}}. \quad (50)$$

We note that the formula (48) is the same for the GLW and canonical versions of the spin tensor, as demonstrated in [24]. The expression $\tilde{\omega}_{\mu\nu} p^\nu$ that appears in (48) can be written as

$$\begin{aligned} \tilde{\omega}_{\mu\nu} p^\nu &= (C_{\omega z} Z_\mu + C_{\omega r} R_\mu + C_{\omega\phi} \Phi_\mu) U \\ &\cdot p - U_\mu (C_{\omega z} Z \cdot p + C_{\omega r} R \cdot p + C_{\omega\phi} \Phi \cdot p) \\ &- \epsilon_{\mu\nu\alpha\beta} p^\nu U^\alpha (C_{kz} Z^\beta + C_{kr} R^\beta + C_{k\phi} \Phi^\beta). \end{aligned} \quad (51)$$

The explicit components of this vector are given in Appendix C.

A. Freeze-out hypersurface

The hypersurface Σ , invariant under boosts and cylindrically symmetric, is parameterized with the help of three coordinates: the azimuthal angle ϕ , the spacetime rapidity η , and an additional variable ζ [88],

$$\Sigma^\mu = (\tau(\zeta) \cosh \eta, r(\zeta) \cos \phi, r(\zeta) \sin \phi, \tau(\zeta) \sinh \eta). \quad (52)$$

See also Appendix B. Here ζ is used to parametrize the freeze-out hypersurface (strictly speaking, its projection) in the τ - r plane. A standard choice is to determine the curve $(\tau(\zeta), r(\zeta))$ by the condition of constant temperature, $T(\tau, \zeta) = \text{const}$.

The volume element of the hypersurface (52) is then [88]

$$\begin{aligned} d\Sigma^\mu &= \left(\frac{dr(\zeta)}{d\zeta} \cosh \eta, \frac{d\tau(\zeta)}{d\zeta} \cos \phi, \frac{d\tau(\zeta)}{d\zeta} \sin \phi, \frac{dr(\zeta)}{d\zeta} \sinh \eta \right) \\ &\times r(\zeta) \tau(\zeta) d\zeta d\eta d\phi. \end{aligned} \quad (53)$$

With the standard momentum parametrization in terms of rapidity y and transverse momentum p_T (the transverse mass being $m_T = \sqrt{m^2 + p_T^2}$)

$$p^\mu = (m_T \cosh(y), p_T \cos(\phi_p), p_T \sin(\phi_p), m_T \sinh(y)), \quad (54)$$

we find

$$\begin{aligned} p \cdot d\Sigma &= \left[m_T \cosh(y - \eta) \frac{dr}{d\zeta} - p_T \cos(\phi - \phi_p) \frac{d\tau}{d\zeta} \right] \\ &\times r(\zeta) \tau(\zeta) d\zeta d\eta d\phi. \end{aligned} \quad (55)$$

We also have

$$p \cdot U = m_T \cosh(\theta) \cosh(y - \eta) - p_T \sinh(\theta) \cos(\phi - \phi_p). \quad (56)$$

Equations (55) and (56) can be used in (50) to obtain the Pauli-Lubański four-vector.

B. Hypersurface integrals

In the case of Boltzmann statistics, the distribution function $e^{-\beta \cdot p}$ factorizes into a product of functions depending on $\cosh(y - \eta)$ and $\cos(\phi - \phi_p)$. As a consequence, one can analytically perform the integrals over spacetime rapidity η and azimuthal angle ϕ . The first integrals can be expressed by the modified Bessel function of the second kind $K_n(m_T \cosh \theta / T)$, while the second integrals yield the modified Bessel function of the first kind $I_n(p_T \sinh \theta / T)$.

For example, the integral in the numerator of Eq. (50) has a structure

$$\begin{aligned} \int d\Sigma_\lambda p^\lambda e^{-\beta \cdot p} \tilde{\omega}_{\mu\beta} p^\beta &= \int \left(m_T F_\mu \frac{dr}{d\zeta} - p_T G_\mu \frac{d\tau}{d\zeta} \right) \\ &\times r(\zeta) \tau(\zeta) d\zeta, \end{aligned} \quad (57)$$

where (for $y = 0$) the four-vectors F_μ and G_μ are defined by the integrals

$$\begin{aligned} F_\mu &= \int \cosh(\eta) \exp \left[-\frac{m_T}{T} \cosh(\theta) \cosh(\eta) \right] \\ &\times \left[\int \exp \left[\frac{p_T}{T} \sinh(\theta) \cos(\phi - \phi_p) \right] \tilde{\omega}_{\mu\beta} p^\beta d\phi \right] d\eta \end{aligned} \quad (58)$$

and

$$\begin{aligned} G_\mu &= \int \exp \left[-\frac{m_T}{T} \cosh(\theta) \cosh(\eta) \right] \\ &\times \left[\int \cos(\phi - \phi_p) \exp \left[\frac{p_T}{T} \sinh(\theta) \cos(\phi - \phi_p) \right] \right. \\ &\times \tilde{\omega}_{\mu\beta} p^\beta d\phi \left. \right] d\eta. \end{aligned} \quad (59)$$

Let us introduce a shorthand notation

$$\mathbf{m} = \frac{m_T \cosh \theta}{T}, \quad \mathbf{p} = \frac{p_T \sinh \theta}{T}. \quad (60)$$

The integrals over η and ϕ give

$$\begin{aligned} F_0 &= 2\pi C_{\omega r} p_T \kappa I_1(\mathbf{p}), \quad F_1 = 4\pi p_T \sin(\phi_p) (C_{kz} \cosh \theta - C_{\omega\phi} \sinh \theta) K_1(\mathbf{m}) I_0(\mathbf{p}) - 2\pi m_T \kappa I_1(\mathbf{p}) [C_{kz} \sinh \theta \sin(\phi_p) \\ &\quad - C_{\omega\phi} \cosh(\theta) \sin(\phi_p) + C_{\omega r} \cos(\phi_p)], \\ F_2 &= 4\pi p_T \cos(\phi_p) (C_{\omega\phi} \sinh \theta - C_{kz} \cosh \theta) K_1(\mathbf{m}) I_0(\mathbf{p}) - 2\pi m_T \kappa I_1(\mathbf{p}) [-C_{kz} \sinh \theta \cos(\phi_p) + C_{\omega\phi} \cosh \theta \cos(\phi_p) \\ &\quad + C_{\omega r} \sin(\phi_p)], \\ F_3 &= 2\pi p_T (C_{k\phi} \cosh \theta + C_{\omega z} \sinh \theta) \kappa I_1(\mathbf{p}) - 4\pi m_T [C_{k\phi} \sinh \theta + C_{\omega z} \cosh(\theta)] K_1(\mathbf{m}) I_0(\mathbf{p}), \end{aligned} \quad (61)$$

where we have put

$$\kappa = K_0(\mathbf{m}) + K_2(\mathbf{m}). \quad (62)$$

One can explicitly check that $p^\mu F_\mu = 0$. Similarly, the components of the four-vector G_μ are

$$\begin{aligned} G_0 &= 4\pi C_{\omega r} K_1(\mathbf{m}) [p_T I_2(\mathbf{p}) + T \operatorname{csch}(\theta) I_1(\mathbf{p})], \\ G_1 &= \frac{4\pi}{p_T} I_1(\mathbf{p}) \{ p_T^2 \sin(\phi_p) (C_{kz} \cosh \theta - C_{\omega\phi} \sinh \theta) K_0(\mathbf{m}) \\ &\quad - m_T T K_1(\mathbf{m}) [\sin(\phi_p) (C_{kz} - C_{\omega\phi} \coth \theta) + C_{\omega r} \operatorname{csch}(\theta) \cos(\phi_p)] \} \\ &\quad - 4\pi m_T K_1(\mathbf{m}) I_2(\mathbf{p}) [C_{kz} \sinh \theta \sin(\phi_p) \\ &\quad - C_{\omega\phi} \cosh \theta \sin(\phi_p) + C_{\omega r} \cos(\phi_p)], \\ G_2 &= \frac{4\pi}{p_T} I_1(\mathbf{p}) \left\{ p_T^2 \cos(\phi_p) (C_{\omega\phi} \sinh \theta - C_{kz} \cosh \theta) K_0(\mathbf{m}) \right. \\ &\quad \left. + m_T T K_1(\mathbf{m}) [\cos(\phi_p) (C_{kz} - C_{\omega\phi} \coth \theta) - C_{\omega r} \operatorname{csch}(\theta) \sin(\phi_p)] \right\} \\ &\quad - 4\pi m_T K_1(\mathbf{m}) I_2(\mathbf{p}) [-C_{kz} \sinh \theta \cos(\phi_p) + C_{\omega\phi} \cosh \theta \cos(\phi_p) + C_{\omega r} \sin(\phi_p)], \\ G_3 &= 4\pi \sinh \theta [(C_{k\phi} \coth \theta + C_{\omega z}) K_1(\mathbf{m}) [p_T I_2(\mathbf{p}) + T \operatorname{csch}(\theta) I_1(\mathbf{p})] - m_T (C_{k\phi} + C_{\omega z} \coth \theta) K_0(\mathbf{m}) I_1(\mathbf{p})]. \end{aligned} \quad (63)$$

In this case we can also verify that $p^\mu G_\mu = 0$.

To evaluate the mean spin polarization vector from the Cooper-Frye formalism, we dynamically extract the relevant isothermal freeze-out contour from the background hydrodynamic evolution (see Fig. 6). The spacetime profiles of the background thermodynamic variables, the fluid velocity, and all the spin polarization tensor components are mapped onto this one-dimensional curve utilizing a high-resolution regular grid interpolation. To ensure numerical stability during the integration, appropriate safeguards are implemented near the origin ($r \rightarrow 0$), where L'Hôpital's rule is applied

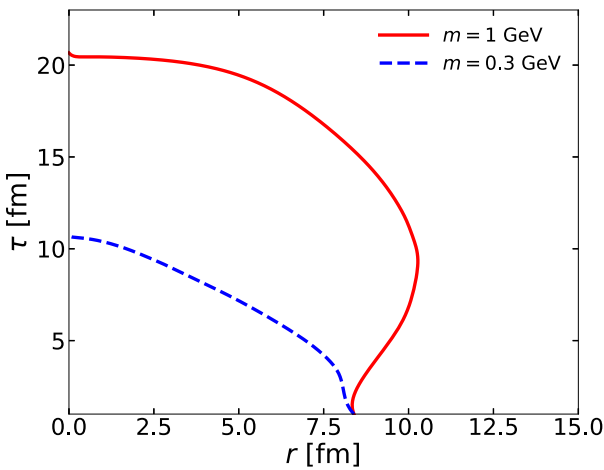


FIG. 6. Projection ($\eta = 0$ and $\phi = 0$) of the isothermal freeze-out hypersurfaces corresponding to a constant temperature of $T = 0.15$ GeV. The solid black line indicates the expansion profile for a fluid composed of massive particles with $m = 1$ GeV, while the dashed line represents the lower mass case of $m = 0.3$ GeV. The curves display the mapping $\zeta \rightarrow (\tau(\zeta), r(\zeta))$.

to properly evaluate divergent-looking geometric source terms.

The analytical hypersurface integrands of the momentum density of the Pauli-Lubański vector and the momentum density of particles given by Eqs. (48) and (49)—which explicitly depend on the modified Bessel functions K_n and I_n —are evaluated at each point along the extracted curve. This evaluation is performed over a uniformly discretized grid of transverse momenta (p_x, p_y) spanning the range from -4.0 to 4.0 GeV. The continuous integration over the freeze-out hypersurface is carried out numerically via a discrete summation, strictly accounting for the appropriate cylindrical volume measure. The mean Pauli-Lubański four-vector is then obtained by Eq. (47).

To ensure the accuracy of this semianalytical approach, we independently cross-checked our results using a fully numerical integration. Instead of utilizing the analytically preintegrated formulas with Bessel functions, the integrals in Eqs. (48) and (49) over the spacetime rapidity η and the azimuthal angle ϕ were evaluated explicitly over a highly resolved, two-dimensional discrete grid: $\eta \in [-4.0, 4.0]$ and $\phi \in [0, 2\pi)$. In this alternative scheme, the covariant components of the Pauli-Lubański vector were computed directly from their definitions. This involved explicitly constructing the local basis four-vectors at each integration node, calculating the exact four-vector scalar products, and dynamically evaluating the local Boltzmann weights. This fully numerical integration scheme yielded exact agreement with the semianalytical method, providing a validation of our computational framework.

C. Boost to the particle rest frame

Finally, the mean spin four-vector computed in the laboratory frame, $\langle \pi_{\text{LAB}}^\mu \rangle$, must be transformed to the particle rest frame (PRF) to yield the physical, observable

polarization components. In our numerical implementation, this transformation is executed point by point over the discrete two-dimensional transverse momentum grid (p_x, p_y) . For each grid node, we evaluate the transverse momentum magnitude $p_T = \sqrt{p_x^2 + p_y^2}$ and the corresponding transverse mass $m_T = \sqrt{m^2 + p_T^2}$, which acts as the effective energy of the particle in the transverse plane. Considering particles produced at midrapidity ($y = 0$), the longitudinal momentum vanishes ($p_z = 0$). Consequently, the boost velocity $\mathbf{v}$ from the laboratory frame to the PRF is purely transverse. Its Cartesian components are numerically defined as $\mathbf{v} = (p_x/m_T, p_y/m_T, 0)$, and the corresponding Lorentz factor simplifies directly to the ratio $\gamma = m_T/m$. Utilizing these specific kinematic variables, the spatial components of the mean polarization vector in the PRF are derived through the standard Lorentz boost,

$$\langle \pi^* \rangle = \langle \pi_{\text{LAB}} \rangle + \frac{\gamma^2}{\gamma + 1} \mathbf{v}(\mathbf{v} \cdot \langle \pi_{\text{LAB}} \rangle) - \gamma \mathbf{v} \langle \pi_{\text{LAB}}^0 \rangle. \quad (64)$$

Numerically, this vector equation is explicitly decomposed into its Cartesian components, where the scalar product $\mathbf{v} \cdot \langle \pi_{\text{LAB}} \rangle = v_x \langle \pi_x^{\text{LAB}} \rangle + v_y \langle \pi_y^{\text{LAB}} \rangle$ is evaluated using the contravariant spatial components of the laboratory-frame vector. These resulting PRF spatial components, $\langle \pi_x^* \rangle$, $\langle \pi_y^* \rangle$, and $\langle \pi_z^* \rangle$, directly constitute the two-dimensional transverse momentum polarization maps presented in Figs. 7 and 8.

Since the boost to PRF is performed in the transverse plane, it does not affect the components F_3 and G_3 . Hence, one may conclude that the longitudinal (z) component of the PL four-vector will be expressed by F_3 and G_3 . Because these components depend on $C_{\omega z}$ and $C_{k\phi}$ coefficients only, we conclude that the longitudinal component of the PL four-vector can be different from zero only in the case (c). This is supported by our numerical results presented in Figs. 7 and 8. Moreover, since F_3 and G_3 are independent of the angle ϕ_p , we expect that the longitudinal component of the PL four-vector will be cylindrically symmetric in the transverse plane, which is again confirmed by our numerical results shown in Figs. 7 and 8.

VII. SUMMARY

We have numerically solved boost-invariant and cylindrically symmetric equations of perfect spin hydrodynamics. The energy-momentum and spin tensors have been taken in the form that describes a relativistic massive gas governed by Boltzmann statistics. We have found that radial expansion leads to a coupling between the azimuthal and longitudinal components of the electric and magnetic components of the spin polarization tensor. Although this feature was found earlier for the Gubser symmetry, our treatment allows for a more general form of initial conditions and expansion geometry. Moreover, it yields regular solutions that are finite at the origin of the coordinate system. Defining the freeze-out hypersurface by the constant temperature condition, we have evaluated the Pauli-Lubański four-vector and found that for the assumed geometry the only nonzero total polarization may be induced

by the longitudinal component of the magnetic part of the spin polarization tensor.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this article. Requests for further information or data should be sent to the authors.

APPENDIX A: DISCUSSION OF THE $1/r$ TERMS

Two of the six equations that follow from the conservation of spin contain terms proportional to $1/r$ that do not vanish in the limit $\theta \rightarrow 0$, namely, Eq. (44) contains $\frac{AC_{k\phi}}{2r}$, and Eq. (45) contains $\frac{AC_{\omega\phi}}{2r}$.

Let us introduce the following notation for the two sectors of the spin tensor (30) that are responsible for those terms,

$$S_{\Delta k}^{\lambda, \mu\nu} = \frac{A}{2} (\Delta^{\lambda\mu} k^\nu - \Delta^{\lambda\nu} k^\mu), \quad (A1)$$

$$S_{tu}^{\lambda, \mu\nu} = \frac{A}{2} (t^{\lambda\mu} U^\nu - t^{\lambda\nu} U^\mu). \quad (A2)$$

Then it can be verified that

$$R_\mu \Phi_\nu \partial_\lambda S_{\Delta k}^{\lambda, \mu\nu} = \frac{A}{2r} (\partial_\phi C_{kr} - C_{k\phi}), \quad (A3)$$

$$U_\mu Z_\nu \partial_\lambda S_{tu}^{\lambda, \mu\nu} = \frac{A}{2r} (C_{\omega\phi} - \partial_\phi C_{\omega r}). \quad (A4)$$

If the coefficients C_{kr} and $C_{\omega r}$ are independent of ϕ , as we have assumed, then we are left with precisely the terms that we mentioned above.⁵

The fact that the boost-invariant transversely homogeneous equations considered in previous studies [78,79] are not recovered by setting $\theta \rightarrow 0$ is natural and comes from the fact that a vector field that is constant and nonzero in the transverse plane is not a special case of a cylindrically symmetric vector field. The geometries are different. To recover the previous case, one would have to change the setting by introducing an explicit dependence of the coefficients C_i on ϕ and relate the coefficients considered in the present work to coefficients in a basis that is Cartesian in the transverse directions,

$$\begin{aligned} k^\mu &= C_{kx} X^\mu + C_{ky} Y^\mu + C_{kz} Z^\mu, \\ \omega^\mu &= C_{\omega x} X^\mu + C_{\omega y} Y^\mu + C_{\omega z} Z^\mu, \end{aligned} \quad (A5)$$

via

$$\begin{aligned} C_{kr}(\phi) &= C_{kx} \cos \phi + C_{ky} \sin \phi, \\ C_{k\phi}(\phi) &= -C_{kx} \sin \phi + C_{ky} \cos \phi, \end{aligned} \quad (A6)$$

⁵The flipped sign in (44) comes from the fact that Eq. (44), as written, was multiplied by -1 .

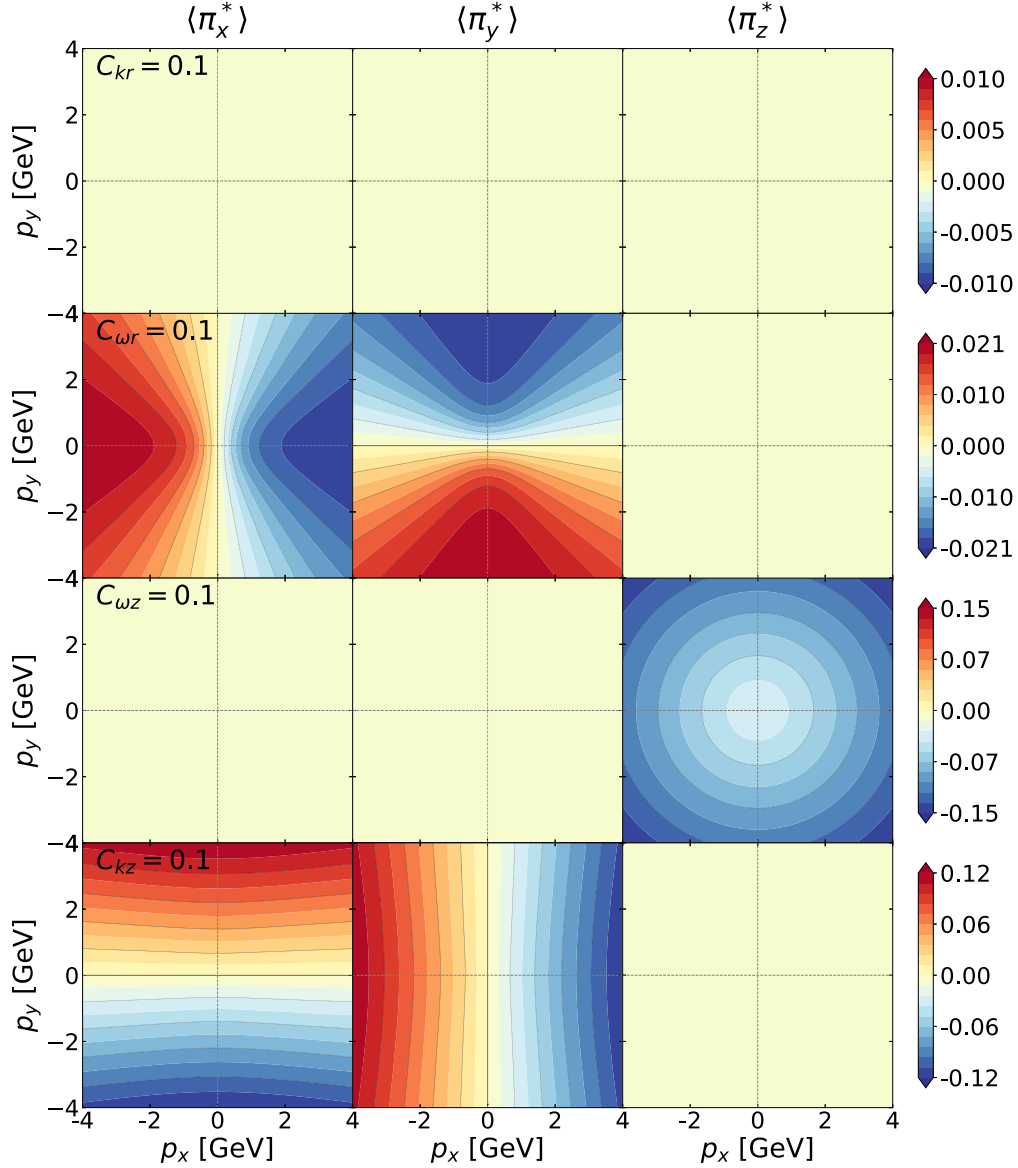


FIG. 7. Momentum dependence of the mean spin polarization vector components $\langle \pi_x^* \rangle$, $\langle \pi_y^* \rangle$, and $\langle \pi_z^* \rangle$ in the transverse momentum plane (p_x, p_y) , evaluated for a heavy particle mass of $m = 1$ GeV at the freeze-out temperature $T = 0.15$ GeV. The columns correspond to the Cartesian components of the polarization vector in the particle rest frame. The four rows display the results for the four independent initial configurations: $C_{kr} = 0.1\rho(r)$ (first row), $C_{\omega r} = 0.1\rho(r)$ (second row), $C_{\omega z} = 0.1\rho(r)$ (third row), and $C_{kz} = 0.1\rho(r)$ (fourth row). Empty panels indicate polarization components vanishing exactly owing to the underlying symmetry constraints.

$$\begin{aligned} C_{\omega r}(\phi) &= C_{\omega x} \cos \phi + C_{\omega y} \sin \phi, \\ C_{\omega \phi}(\phi) &= -C_{\omega x} \sin \phi + C_{\omega y} \cos \phi. \end{aligned} \quad (\text{A7})$$

Then one would have

$$\partial_\phi C_{kr} = C_{k\phi}, \quad \partial_\phi C_{\omega r} = C_{\omega \phi}, \quad (\text{A8})$$

and both $1/r$ coefficients in Eqs. (A3) and (A4) would vanish,

$$\partial_\phi C_{kr} - C_{k\phi} = 0, \quad C_{\omega \phi} - \partial_\phi C_{\omega r} = 0. \quad (\text{A9})$$

APPENDIX B: FREEZE-OUT HYPERSURFACE

The four-vector Σ^μ that defines the freeze-out hypersurface via Eq. (52) possesses a simple decomposition in the considered basis,

$$\begin{aligned} \Sigma^\mu &= [\tau(\zeta) \cosh \theta - r(\zeta) \sinh \theta] U^\mu + [r(\zeta) \cosh \theta \\ &\quad - \tau(\zeta) \sinh \theta] R^\mu. \end{aligned} \quad (\text{B1})$$

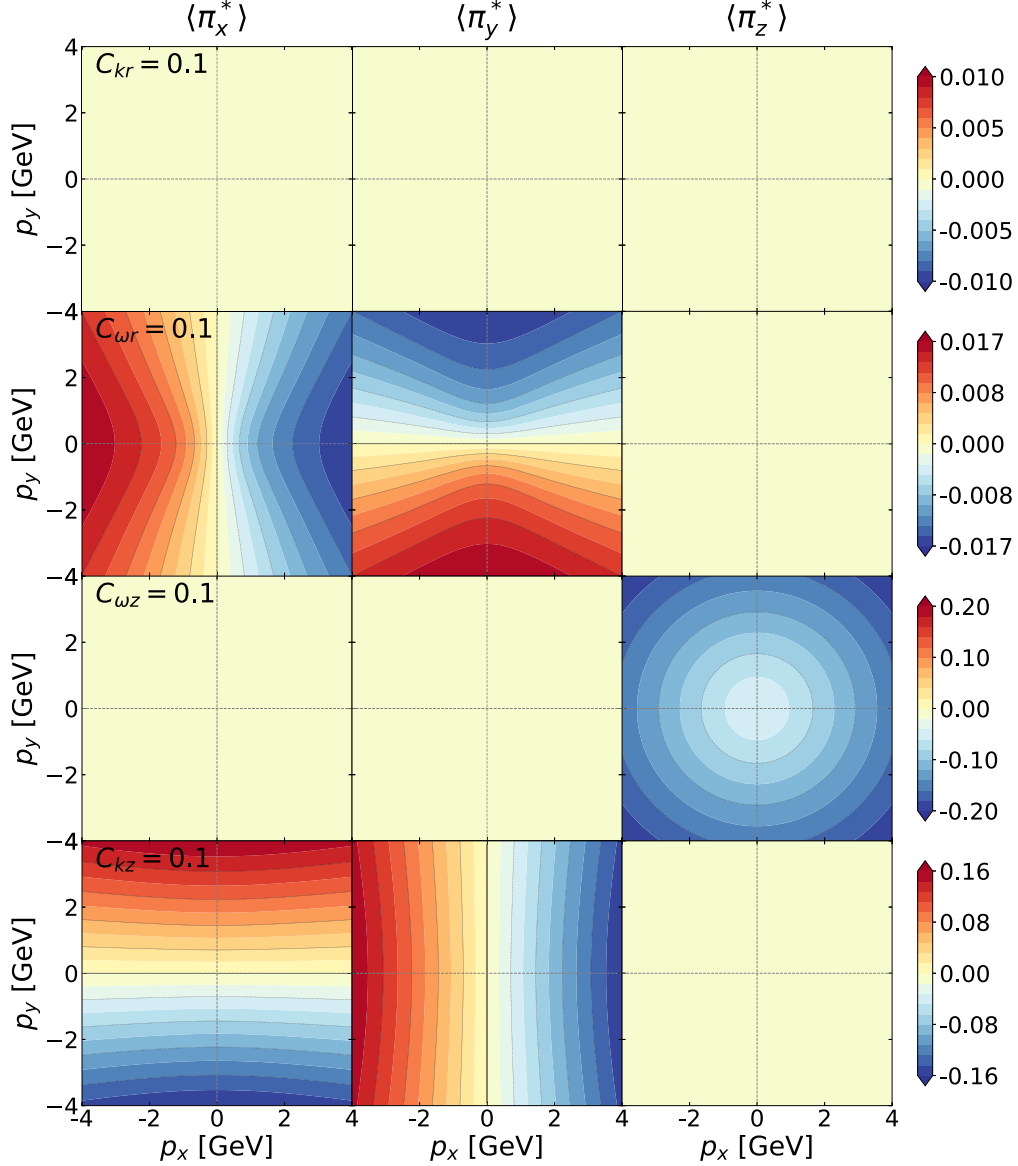


FIG. 8. Momentum dependence of the mean spin polarization vector components $\langle \pi_x^* \rangle$, $\langle \pi_y^* \rangle$, and $\langle \pi_z^* \rangle$ in the transverse momentum plane (p_x, p_y) , evaluated for a lighter particle mass of $m = 0.3$ GeV at the freeze-out temperature $T = 0.15$ GeV. The layout of the panels and the initial conditions are identical to those in Fig. 7.

Analogously, the volume element from Eq. (53) can be written as

$$d\Sigma^\mu = \left[\left(\frac{dr}{d\zeta} \cosh \theta - \frac{d\tau}{d\zeta} \sinh \theta \right) U^\mu + \left(\frac{d\tau}{d\zeta} \cosh \theta - \frac{dr}{d\zeta} \sinh \theta \right) R^\mu \right] r(\zeta) \tau(\zeta) d\zeta d\eta d\phi. \quad (\text{B2})$$

APPENDIX C: CONTRACTIONS OF FOUR-MOMENTUM WITH THE DUAL SPIN POLARIZATION TENSOR

Through straightforward algebraic manipulations, we obtain the explicit expressions for the vector $\tilde{\omega}_{\mu\nu} p^\nu$ that appears in (48),

$$\begin{aligned} \tilde{\omega}_{0\nu} p^\nu &= p_T \cosh(\eta) [C_{\omega r} \cos(\phi - \phi_p) - C_{\omega\phi} \cosh \theta \sin(\phi - \phi_p)] + C_{\omega z} [m_T \cosh \theta \sinh(y) - p_T \sinh(\eta) \sinh \theta \cos(\phi - \phi_p)] \\ &\quad + \sinh \theta [C_{k\phi} m_T \sinh(y) + C_{kz} p_T \cosh(\eta) \sin(\phi - \phi_p)] - p_T \sinh(\eta) [C_{k\phi} \cosh \theta \cos(\phi - \phi_p) + C_{kr} \sin(\phi - \phi_p)], \end{aligned} \quad (\text{C1})$$

$$\begin{aligned}\tilde{\omega}_{1v}p^v &= m_T \cosh(y - \eta)[C_{\omega\phi} \cosh \theta \sin(\phi) - C_{\omega r} \cos(\phi)] - \sinh \theta [C_{\omega\phi} p_T \sin(\phi_p) + C_{\omega z} m_T \cos(\phi) \sinh(y - \eta)] \\ &\quad + \cosh \theta [C_{kz} p_T \sin(\phi_p) - C_{k\phi} m_T \cos(\phi) \sinh(y - \eta)] - m_T \sin(\phi) [C_{kr} \sinh(y - \eta) + C_{kz} \sinh \theta \cosh(y - \eta)],\end{aligned}\quad (C2)$$

$$\begin{aligned}\tilde{\omega}_{2v}p^v &= \sinh \theta [C_{\omega\phi} p_T \cos(\phi_p) - C_{\omega z} m_T \sin(\phi) \sinh(y - \eta)] - m_T \cosh(y - \eta) [C_{\omega\phi} \cosh \theta \cos(\phi) + C_{\omega r} \sin(\phi)] \\ &\quad + m_T \cos(\phi) [C_{kr} \sinh(y - \eta) + C_{kz} \sinh \theta \cosh(y - \eta)] - \cosh \theta [C_{k\phi} m_T \sin(\phi) \sinh(y - \eta) + C_{kz} p_T \cos(\phi_p)],\end{aligned}\quad (C3)$$

$$\begin{aligned}\tilde{\omega}_{3v}p^v &= p_T \sinh(\eta) [C_{\omega\phi} \cosh \theta \sin(\phi - \phi_p) - C_{\omega r} \cos(\phi - \phi_p)] - C_{\omega z} [m_T \cosh \theta \cosh(y) - p_T \cosh(\eta) \sinh \theta \cos(\phi - \phi_p)] \\ &\quad + p_T \cosh(\eta) [C_{k\phi} \cosh \theta \cos(\phi - \phi_p) + C_{kr} \sin(\phi - \phi_p)] - \sinh \theta [C_{k\phi} m_T \cosh(y) + C_{kz} p_T \sinh(\eta) \sin(\phi - \phi_p)].\end{aligned}\quad (C4)$$

In the case $\theta = \phi = 0$, Eqs. (C1)–(C4) reduce to Eq. (61) in [73], if we make identifications $R^\mu = X^\mu$ and $\Phi^\mu = Y^\mu$.

APPENDIX D: BOOST TO PRF

1. Case (a)

If only the C_{kr} component is different from zero, one can check that $F_\mu^* = 0$ and $G_\mu^* = 0$, which is confirmed by the numerical results presented in Figs. 7 and 8 (the top rows show that all the components of the Pauli-Lubański four-vector vanish).

2. Case (b)

In this case, the only nonzero component of the spin polarization tensor is $C_{\omega r}$. The boost to the particle rest frame gives

$$F_\mu^* = 4\pi C_{\omega r} I_1(\mathbf{p}) [K_0(\mathbf{m}) m_T + K_1(\mathbf{m}) T \operatorname{sech} \theta] \frac{m}{m_T} (0, -\cos \phi_p, -\sin \phi_p, 0) \quad (D1)$$

and

$$G_\mu^* = 4\pi C_{\omega r} K_1(\mathbf{m}) [I_2(\mathbf{p}) p_T + I_1(\mathbf{p}) T \operatorname{csch} \theta] \frac{m}{p_T} (0, -\cos \phi_p, -\sin \phi_p, 0). \quad (D2)$$

For conciseness, the arguments of the Bessel functions use the notation introduced in Eq. (60).

3. Case (c)

In this case the only nonzero components of the spin polarization tensor are $C_{\omega z}$ and $C_{k\phi}$. The boost to PRF gives

$$F_0^* = F_1^* = F_2^* = 0, \quad F_3^* = 2\pi \{ \sinh(\theta) [C_{\omega z} I_1(\mathbf{p}) \kappa p_T - 2C_{k\phi} I_0(\mathbf{p}) K_1(\mathbf{m}) m_T] + \cosh(\theta) [C_{k\phi} I_1(\mathbf{p}) \kappa p_T - 2C_{\omega z} I_0(\mathbf{p}) K_1(\mathbf{m}) m_T] \} \quad (D3)$$

and

$$\begin{aligned}G_0^* &= G_1^* = G_2^* = 0, \quad G_3^* = 4\pi \sinh(\theta) \{ K_1(\mathbf{m}) [C_{k\phi} \coth(\theta) + C_{\omega z}] [I_1(\mathbf{p}) T \operatorname{csch}(\theta) + I_2(\mathbf{p}) p_T] - I_1(\mathbf{p}) K_0(\mathbf{m}) m_T \\ &\quad \times [C_{k\phi} + C_{\omega z} \coth(\theta)] \}.\end{aligned}\quad (D4)$$

4. Case (d)

In this case, the only nonzero components are C_{kz} and $C_{\omega\phi}$. A straightforward calculation gives

$$\begin{aligned}F_0^* &= 0, \\ F_1^* &= 4\pi \sin(\phi_p) \{ I_0(\mathbf{p}) K_1(\mathbf{m}) p_T [C_{kz} \cosh(\theta) - C_{\omega\phi} \sinh(\theta)] \\ &\quad + I_1(\mathbf{p}) \operatorname{sech}(\theta) [C_{\omega\phi} \cosh(\theta) - C_{kz} \sinh(\theta)] [K_0(\mathbf{m}) m_T \cosh(\theta) + K_1(\mathbf{m}) T] \}, \\ F_2^* &= 4\pi \cos(\phi_p) \{ I_0(\mathbf{p}) K_1(\mathbf{m}) p_T [C_{\omega\phi} \sinh(\theta) - C_{kz} \cosh(\theta)] \\ &\quad - I_1(\mathbf{p}) [C_{\omega\phi} - C_{kz} \tanh(\theta)] [K_0(\mathbf{m}) m_T \cosh(\theta) + K_1(\mathbf{m}) T] \}, \\ F_3^* &= 0,\end{aligned}\quad (D5)$$

and

$$\begin{aligned}
G_0^* &= 0, \\
G_1^* &= \frac{4\pi}{p_T} \sin(\phi_p) \{-p_T \sinh(\theta) [C_{kz} I_2(p) K_1(m) m_T + C_{\omega\phi} I_1(p) K_0(m) p_T] \\
&\quad + p_T \cosh(\theta) [C_{kz} I_1(p) K_0(m) p_T + C_{\omega\phi} I_2(p) K_1(m) m_T] \\
&\quad + I_1(p) K_1(m) m_T T [C_{\omega\phi} \coth(\theta) - C_{kz}]\}, \\
G_2^* &= \frac{4\pi}{p_T} \cos(\phi_p) \{p_T \sinh(\theta) [C_{kz} I_2(p) K_1(m) m_T + C_{\omega\phi} I_1(p) K_0(m) p_T] \\
&\quad - p_T \cosh(\theta) [C_{kz} I_1(p) K_0(m) p_T + C_{\omega\phi} I_2(p) K_1(m) m_T] \\
&\quad + I_1(p) K_1(m) m_T T [C_{kz} - C_{\omega\phi} \coth(\theta)]\}, \\
G_3^* &= 0.
\end{aligned} \tag{D6}$$

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- [1] L. Adamczyk *et al.* (STAR Collaboration), Global Λ hyperon polarization in nuclear collisions, *Nature (London)* **548**, 62 (2017).
- [2] J. Adam *et al.* (STAR Collaboration), Global polarization of Λ hyperons in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV, *Phys. Rev. C* **98**, 014910 (2018).
- [3] J. Adam *et al.* (STAR Collaboration), Polarization of $\Lambda(\bar{\Lambda})$ hyperons along the beam direction in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV, *Phys. Rev. Lett.* **123**, 132301 (2019).
- [4] S. Acharya *et al.* (ALICE Collaboration), Global polarization of Λ and $\bar{\Lambda}$ hyperons in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV, *Phys. Rev. C* **101**, 044611 (2020); **105**, 029902(E) (2022).
- [5] J. Adam *et al.* (STAR Collaboration), Global polarization of Ξ and Ω hyperons in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV, *Phys. Rev. Lett.* **126**, 162301 (2021); **131**, 089901(E) (2023).
- [6] S. Acharya *et al.* (ALICE Collaboration), Polarization of Λ and $\bar{\Lambda}$ hyperons along the beam direction in Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, *Phys. Rev. Lett.* **128**, 172005 (2022).
- [7] M. S. Abdallah *et al.* (STAR Collaboration), Global Λ -hyperon polarization in Au + Au collisions at $\sqrt{s_{NN}} = 3$ GeV, *Phys. Rev. C* **104**, L061901 (2021).
- [8] R. Abou Yassine *et al.* (HADES Collaboration), Measurement of global polarization of Λ hyperons in few-GeV heavy-ion collisions, *Phys. Lett. B* **835**, 137506 (2022).
- [9] M. I. Abdulhamid *et al.* (STAR Collaboration), Global polarization of Λ and $\bar{\Lambda}$ hyperons in Au + Au collisions at $\sqrt{s_{NN}} = 19.6$ and 27 GeV, *Phys. Rev. C* **108**, 014910 (2023).
- [10] S. Acharya *et al.* (ALICE Collaboration), Evidence of spin-orbital angular momentum interactions in relativistic heavy-ion collisions, *Phys. Rev. Lett.* **125**, 012301 (2020).
- [11] M. S. Abdallah *et al.* (STAR Collaboration), Pattern of global spin alignment of ϕ and K^{*0} mesons in heavy-ion collisions, *Nature (London)* **614**, 244 (2023).
- [12] S. Acharya *et al.* (ALICE Collaboration), Measurement of the J/ψ polarization with respect to the event plane in Pb-Pb collisions at the LHC, *Phys. Rev. Lett.* **131**, 042303 (2023).
- [13] F. Becattini, M. Buzzegoli, T. Niida, S. Pu, A.-H. Tang, and Q. Wang, Spin polarization in relativistic heavy-ion collisions, *Int. J. Mod. Phys. E* **33**, 2430006 (2024).
- [14] T. Niida and S. A. Voloshin, Polarization phenomenon in heavy-ion collisions, *Int. J. Mod. Phys. E* **33**, 2430010 (2024).
- [15] J.-H. Chen, Z.-T. Liang, Y.-G. Ma, X.-L. Sheng, and Q. Wang, Vector meson's spin alignments in high energy reactions, *Sci. China Phys. Mech. Astron.* **68**, 211001 (2025).
- [16] Z.-T. Liang and X.-N. Wang, Globally polarized quark-gluon plasma in noncentral A + A collisions, *Phys. Rev. Lett.* **94**, 102301 (2005); **96**, 039901(E) (2006).
- [17] Z.-T. Liang and X.-N. Wang, Spin alignment of vector mesons in non-central A + A collisions, *Phys. Lett. B* **629**, 20 (2005).
- [18] F. Becattini and L. Tinti, The ideal relativistic rotating gas as a perfect fluid with spin, *Ann. Phys. (NY)* **325**, 1566 (2010).
- [19] F. Becattini, V. Chandra, L. Del Zanna, and E. Grossi, Relativistic distribution function for particles with spin at local thermodynamical equilibrium, *Ann. Phys. (NY)* **338**, 32 (2013).
- [20] Y.-G. Yang, R.-H. Fang, Q. Wang, and X.-N. Wang, Quark coalescence model for polarized vector mesons and baryons, *Phys. Rev. C* **97**, 034917 (2018).
- [21] D. Montenegro, L. Tinti, and G. Torrieri, Ideal relativistic fluid limit for a medium with polarization, *Phys. Rev. D* **96**, 056012 (2017); **96**, 079901(E) (2017).
- [22] D. Montenegro, L. Tinti, and G. Torrieri, Sound waves and vortices in a polarized relativistic fluid, *Phys. Rev. D* **96**, 076016 (2017).
- [23] W. Florkowski, B. Friman, A. Jaiswal, and E. Speranza, Relativistic fluid dynamics with spin, *Phys. Rev. C* **97**, 041901 (2018).
- [24] W. Florkowski, B. Friman, A. Jaiswal, R. Ryblewski, and E. Speranza, Spin-dependent distribution functions for relativistic hydrodynamics of spin- $\frac{1}{2}$ particles, *Phys. Rev. D* **97**, 116017 (2018).
- [25] F. Becattini and I. Karpenko, Collective longitudinal polarization in relativistic heavy-ion collisions at very high energy, *Phys. Rev. Lett.* **120**, 012302 (2018).
- [26] W. Florkowski, A. Kumar, and R. Ryblewski, Thermodynamic versus kinetic approach to polarization-vorticity coupling, *Phys. Rev. C* **98**, 044906 (2018).
- [27] Y. Sun and C. M. Ko, Azimuthal angle dependence of the longitudinal spin polarization in relativistic heavy ion collisions, *Phys. Rev. C* **99**, 011903 (2019).
- [28] X.-L. Sheng, L. Oliva, and Q. Wang, What can we learn from the global spin alignment of ϕ mesons in heavy-ion collisions? *Phys. Rev. D* **101**, 096005 (2020); **105**, 099903(E) (2022).

- [29] K. Hattori, M. Hongo, X.-G. Huang, M. Matsuo, and H. Taya, Fate of spin polarization in a relativistic fluid: An entropy-current analysis, *Phys. Lett. B* **795**, 100 (2019).
- [30] Y. Xie, D. Wang, and L. P. Csernai, Fluid dynamics study of the Λ polarization for Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV, *Eur. Phys. J. C* **80**, 39 (2020).
- [31] N. Weickgenannt, X.-L. Sheng, E. Speranza, Q. Wang, and D. H. Rischke, Kinetic theory for massive $spin - 1/2$ particles from the Wigner-function formalism, *Phys. Rev. D* **100**, 056018 (2019).
- [32] W. Florkowski, A. Kumar, R. Ryblewski, and A. Mazeliauskas, Longitudinal spin polarization in a thermal model, *Phys. Rev. C* **100**, 054907 (2019).
- [33] N. Weickgenannt, E. Speranza, X.-L. Sheng, Q. Wang, and D. H. Rischke, Generating spin polarization from vorticity through nonlocal collisions, *Phys. Rev. Lett.* **127**, 052301 (2021).
- [34] S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar, and R. Ryblewski, Dissipative spin dynamics in relativistic matter, *Phys. Rev. D* **103**, 014030 (2021).
- [35] S. Shi, C. Gale, and S. Jeon, From chiral kinetic theory to relativistic viscous spin hydrodynamics, *Phys. Rev. C* **103**, 044906 (2021).
- [36] C. Yi, S. Pu, and D.-L. Yang, Reexamination of local spin polarization beyond global equilibrium in relativistic heavy ion collisions, *Phys. Rev. C* **104**, 064901 (2021).
- [37] M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov, and H.-U. Yee, Relativistic spin hydrodynamics with torsion and linear response theory for spin relaxation, *J. High Energy Phys.* **11** (2021) 150.
- [38] D.-L. Wang, X.-Q. Xie, S. Fang, and S. Pu, Analytic solutions of relativistic dissipative spin hydrodynamics with radial expansion in Gubser flow, *Phys. Rev. D* **105**, 114050 (2022).
- [39] S. Y. F. Liu and Y. Yin, Spin polarization induced by the hydrodynamic gradients, *J. High Energy Phys.* **07** (2021) 188.
- [40] B. Fu, S. Y. F. Liu, L. Pang, H. Song, and Y. Yin, Shear-induced spin polarization in heavy-ion collisions, *Phys. Rev. Lett.* **127**, 142301 (2021).
- [41] F. Becattini, M. Buzzegoli, and A. Palermo, Spin-thermal shear coupling in a relativistic fluid, *Phys. Lett. B* **820**, 136519 (2021).
- [42] F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, and A. Palermo, Local polarization and isothermal local equilibrium in relativistic heavy ion collisions, *Phys. Rev. Lett.* **127**, 272302 (2021).
- [43] X.-L. Sheng, L. Oliva, Z.-T. Liang, Q. Wang, and X.-N. Wang, Spin alignment of vector mesons in heavy-ion collisions, *Phys. Rev. Lett.* **131**, 042304 (2023).
- [44] X.-L. Sheng, L. Oliva, Z.-T. Liang, Q. Wang, and X.-N. Wang, Relativistic spin dynamics for vector mesons, *Phys. Rev. D* **109**, 036004 (2024).
- [45] M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov, and H.-U. Yee, Spin relaxation rate for heavy quarks in weakly coupled QCD plasma, *J. High Energy Phys.* **08** (2022) 263.
- [46] N. Weickgenannt, D. Wagner, E. Speranza, and D. H. Rischke, Relativistic second-order dissipative spin hydrodynamics from the method of moments, *Phys. Rev. D* **106**, 096014 (2022).
- [47] G. Sarwar, M. Hasanujjaman, J. R. Bhatt, H. Mishra, and J.-e. Alam, Causality and stability of relativistic spin hydrodynamics, *Phys. Rev. D* **107**, 054031 (2023).
- [48] D. Wagner, N. Weickgenannt, and D. H. Rischke, Lorentz-covariant nonlocal collision term for spin-1/2 particles, *Phys. Rev. D* **106**, 116021 (2022).
- [49] D. Wagner, N. Weickgenannt, and E. Speranza, Generating tensor polarization from shear stress, *Phys. Rev. Res.* **5**, 013187 (2023).
- [50] A. Kumar, B. Müller, and D.-L. Yang, Spin polarization and correlation of quarks from the glasma, *Phys. Rev. D* **107**, 076025 (2023).
- [51] R. Biswas, A. Daher, A. Das, W. Florkowski, and R. Ryblewski, Relativistic second-order spin hydrodynamics: An entropy-current analysis, *Phys. Rev. D* **108**, 014024 (2023).
- [52] N. Weickgenannt, Linearly stable and causal relativistic first-order spin hydrodynamics, *Phys. Rev. D* **108**, 076011 (2023).
- [53] M. Kiamari, N. Sadooghi, and M. S. Jafari, Relativistic magnetohydrodynamics of a spinful and vortical fluid: Entropy current analysis, *Phys. Rev. D* **109**, 036024 (2024).
- [54] S. Fang, S. Pu, and D.-L. Yang, Spin polarization and spin alignment from quantum kinetic theory with self-energy corrections, *Phys. Rev. D* **109**, 034034 (2024).
- [55] A. Palermo and F. Becattini, Exact spin polarization of massive and massless particles in relativistic fluids at global equilibrium, *Eur. Phys. J. Plus* **138**, 547 (2023).
- [56] A. Palermo, Exact polarization of particles of any spin at global equilibrium, *EPJ Web Conf.* **296**, 04008 (2024).
- [57] A. Kumar, D.-L. Yang, and P. Gubler, Spin alignment of vector mesons by second-order hydrodynamic gradients, *Phys. Rev. D* **109**, 054038 (2024).
- [58] A. Kumar, B. Müller, and D.-L. Yang, Spin alignment of vector mesons by glasma fields, *Phys. Rev. D* **108**, 016020 (2023).
- [59] J. Chen, Z.-T. Liang, Y.-G. Ma, and Q. Wang, Global spin alignment of vector mesons and strong force fields in heavy-ion collisions, *Sci. Bull.* **68**, 874 (2023).
- [60] K. J. Gonçalves, G. Torrieri, and R. Ryblewski, Meson spin alignment and baryon polarization from coalescence with spin-vorticity nonequilibrium, *Phys. Rev. C* **112**, 014901 (2025).
- [61] E. Grossi, A. Palermo, and I. Zahed, Polarized ϕ meson rates with viscous corrections at energies available at the BNL Relativistic Heavy Ion Collider, *Phys. Rev. C* **111**, 014914 (2025).
- [62] F. Sun, J. Shao, R. Wen, K. Xu, and M. Huang, Chiral phase transition and spin alignment of vector mesons in the polarized-Polyakov-loop Nambu–Jona-Lasinio model under rotation, *Phys. Rev. D* **109**, 116017 (2024).
- [63] S. K. Singh, R. Ryblewski, and W. Florkowski, Spin dynamics with realistic hydrodynamic background for relativistic heavy-ion collisions, *Phys. Rev. C* **111**, 024907 (2025).
- [64] S. Banerjee, S. Bhadury, W. Florkowski, A. Jaiswal, and R. Ryblewski, Longitudinal spin polarization in a thermal model with dissipative corrections, *Phys. Rev. C* **111**, 064912 (2025).
- [65] Sapna, S. K. Singh, and D. Wagner, Spin polarization of Λ hyperons from dissipative spin hydrodynamics, *Phys. Rev. C* **112**, 054902 (2025).
- [66] E. Grossi, A. Palermo, and I. Zahed, Polarization of the ϕ meson in the hadronic phase with nucleon scatterings and a viscous hydrodynamic background, *Phys. Rev. C* **112**, 064910 (2025).
- [67] A. Arslan, W.-B. Dong, C. Gale, S. Jeon, Q. Wang, and X.-Y. Wu, In-plane transverse polarization in heavy-ion collisions, *Phys. Rev. C* **113**, 034920 (2026).

- [68] Z. Liu, Z. Li, W. Zha, and Z. Tang, Energy dependence of rescattering effect on vector mesons spin alignment at RHIC, *Phys. Lett. B* **873**, 140161 (2026).
- [69] W. Florkowski, S. K. Kar, and V. Mykhaylova, Spin alignment, tensor polarizabilities, and local equilibrium for spin-1 particles, *Acta Phys. Polon. B* **57**, 6-A4 (2026).
- [70] S. Bhadury, Z. Drogosz, W. Florkowski, and V. Mykhaylova, Spin hydrodynamics – recent developments, *J. Subatomic Part. Cosmol.* **4**, 100218 (2025).
- [71] S. R. De Groot, in *Relativistic Kinetic Theory: Principles and Applications*, edited by W. A. Van Leeuwen and C. G. Van Weert (North-Holland, Amsterdam, 1980).
- [72] W. Florkowski, A. Kumar, and R. Ryblewski, Relativistic hydrodynamics for spin-polarized fluids, *Prog. Part. Nucl. Phys.* **108**, 103709 (2019).
- [73] W. Florkowski, A. Kumar, R. Ryblewski, and R. Singh, Spin polarization evolution in a boost-invariant hydrodynamical background, *Phys. Rev. C* **99**, 044910 (2019).
- [74] W. Florkowski, R. Ryblewski, R. Singh, and G. Sophys, Spin polarization dynamics in the non-boost-invariant background, *Phys. Rev. D* **105**, 054007 (2022).
- [75] R. Singh, M. Shokri, and R. Ryblewski, Spin polarization dynamics in the Bjorken-expanding resistive MHD background, *Phys. Rev. D* **103**, 094034 (2021).
- [76] D.-L. Wang, S. Fang, and S. Pu, Analytic solutions of relativistic dissipative spin hydrodynamics with Bjorken expansion, *Phys. Rev. D* **104**, 114043 (2021).
- [77] R. Biswas, A. Daher, A. Das, W. Florkowski, and R. Ryblewski, Boost invariant spin hydrodynamics within the first order in derivative expansion, *Phys. Rev. D* **107**, 094022 (2023).
- [78] Z. Drogosz, W. Florkowski, N. Łygan, and R. Ryblewski, Boost-invariant spin hydrodynamics with spin feedback effects, *Phys. Rev. C* **111**, 024909 (2025).
- [79] Z. Drogosz and N. Łygan, Boost-invariant perfect Fermi-Dirac spin hydrodynamics, *Acta Phys. Polon. B* **57**, 9-A4 (2026).
- [80] R. Singh, G. Sophys, and R. Ryblewski, Spin polarization dynamics in the Gubser-expanding background, *Phys. Rev. D* **103**, 074024 (2021).
- [81] R. Singh and A. Soloviev, Spin hydrodynamics on a hyperbolic expanding background, [arXiv:2603.02296](#).
- [82] D. Rindori, L. Tinti, F. Becattini, and D. H. Rischke, Relativistic quantum fluid with boost invariance, *Phys. Rev. D* **105**, 056003 (2022).
- [83] N. Weickgenannt and J.-P. Blaizot, Spin polarization of an expanding and rotating system, *Phys. Rev. D* **111**, 076007 (2025).
- [84] S. Singh, S. Dey, A. Das, H. Mishra, and A. Jaiswal, Dissipative spin hydrodynamics in Bjorken flow and thermal dilepton production, [arXiv:2604.04533](#).
- [85] A. Palermo and D. Roselli, Exact expectation values in a boost-invariant fluid of Dirac fermions with finite spin density, *JHEP* **07** (2026) 180.
- [86] G.-H. Li, X. Ren, D.-L. Wang, and S. Pu, Late-time attractors in relativistic spin hydrodynamics in Gubser flow, [arXiv:2603.27182](#).
- [87] W. Florkowski and R. Ryblewski, Projection method for boost-invariant and cylindrically symmetric dissipative hydrodynamics, *Phys. Rev. C* **85**, 044902 (2012).
- [88] W. Florkowski, *Phenomenology of Ultra-Relativistic Heavy-Ion Collisions* (World Scientific, Singapore, 2010).
- [89] M. Chojnacki and W. Florkowski, Temperature dependence of sound velocity and hydrodynamics of ultra-relativistic heavy-ion collisions, *Acta Phys. Pol. B* **38**, 3249 (2007).
- [90] L. D. Landau and E. M. Lifshitz, *Statistical Physics, Part 1*, Course of Theoretical Physics Vol. 5 (Butterworth-Heinemann, Oxford, 1980).
- [91] P. Castorina, J. Cleymans, D. E. Miller, and H. Satz, The speed of sound in hadronic matter, *Eur. Phys. J. C* **66**, 207 (2010).
- [92] S. Dey, W. Florkowski, A. Jaiswal, and R. Ryblewski, Pseudo-gauge freedom and the SO(3) algebra of spin operators, *Phys. Lett. B* **843**, 137994 (2023).
- [93] G. Baym, B. L. Friman, J. P. Blaizot, M. Soyeur, and W. Czyz, Hydrodynamics of ultra-relativistic heavy ion collisions, *Nucl. Phys. A* **407**, 541 (1983).
- [94] A. Bialas, M. Bleszynski, and W. Czyz, Multiplicity distributions in nucleus-nucleus collisions at high energies, *Nucl. Phys. B* **111**, 461 (1976).