

Constraints on superconducting pairing in altermagnets

Debmalya Chakraborty^{1,2,3,4,*} and Annica M. Black-Schaffer^{1,†}

¹*Department of Physics and Astronomy, Uppsala University, Box 516, S-751 20 Uppsala, Sweden*

²*Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Straße 38, 01187 Dresden, Germany*

³*Department of Physics, Birla Institute of Technology and Science - Pilani,*

K. K. Birla Goa Campus, NH-17B, Zuarinagar, Sancoale, Goa 403726, India

⁴*Department of Physical Sciences, Indian Institute of Science Education and Research (IISER) Mohali, Sector 81, S.A.S. Nagar, Manauli PO 140306, India*



(Received 8 August 2024; revised 1 July 2025; accepted 7 July 2025; published 22 July 2025)

Superconductivity in the recently discovered altermagnetic materials hosts large prospects for both fundamental physics and technological applications. In this work, we show that a characteristic spin-sublattice locking in altermagnets puts severe constraints on possible superconducting pairing. In particular, by considering the archetypical two-band model for a $d_{x^2-y^2}$ -wave altermagnet on a square lattice, we uncover that the most common form of superconductivity, uniform s -wave spin-singlet pairing is not possible to achieve in altermagnets. Instead, we find that the most likely forms of spin-singlet pairing have $d_{x^2-y^2}$ - or extended s -wave symmetry. We also find that the simplest form of equal-spin-triplet p -wave pairing is not allowed, but only a mixed-spin-triplet p -wave state can exist. We verify these constraints on the superconducting pairing also within an interaction-induced model of altermagnetism, where we further establish their validity for finite-momentum pairing. Additionally we discuss the possible pairing symmetries for odd-frequency superconducting pairing. Due to the generality of our results, they are applicable to both intrinsic superconductivity and proximity-induced superconductivity in altermagnet-superconductor hybrid junctions.

DOI: [10.1103/physrevb.112.014516](https://doi.org/10.1103/physrevb.112.014516)

I. INTRODUCTION

Magnetism and superconductivity are the two most celebrated quantum phases of matter and have a “friend-foe” dichotomous relation. While ferromagnetism intuitively becomes a foe to spin-singlet superconductivity due to the latter’s opposite spin alignments of the two electrons in the superconducting Cooper pairs, proximity to an antiferromagnetism phase can give rise to unconventional spin-singlet superconductivity [1–3]. Still, spin-singlet superconductivity can sometimes coexist with ferromagnetism by forming Cooper pairs with a finite center-of-mass momentum, resulting in finite-momentum superconductivity [4,5]. Hybrid superconducting structures offer an additional way to combine magnetism and superconductivity, by explicitly combining materials with either magnetic and superconducting properties. Such a setup gives the possibility of also realizing Majorana fermions through achieving topological superconductivity [6–10], as well as produces interesting prospects for energy efficient spintronic technologies [11,12].

Recently, a new form of magnetism, called altermagnetism, has been proposed theoretically [13–23] and observed experimentally [24–32] in a large number of materials [33], beyond the previously well-established ferro- and antiferromagnetic possibilities. In contrast to both ferro- and antiferromagnets, altermagnets break the Kramer’s spin degeneracy but with a momentum dependent spin splitting, still keeping zero net magnetization. Another essential feature that makes altermagnets different is that the system breaks into two spin-sublattices, which cannot be mapped onto each other by translation or inversion [34]. Since the symmetry transformation that connects the two spin-sublattices does not leave the electronic spectra invariant, the electron bands end up with the characteristic momentum dependent spin splitting [34].

The most common mechanism of generating altermagnetism is through the presence of an electric crystal field splitting and spin-exchange splitting [16,17]. Such a mechanism does not require a Fermi surface instability to form a magnetic state and can be straightforwardly captured within first-principles density functional theory (DFT) calculations [16,17]. Alternate mechanisms for generating altermagnetism through electron-electron interactions generating a Fermi surface instability has also recently been proposed theoretically [35–40]. Such interaction-induced altermagnetism can be distinguished into two broad classes, one where electron-electron repulsion leads to an onsite magnetization, which along with a rotational symmetry breaking band structure gives altermagnetism [35–39], while another mechanism dates longer back originally introduced under the name of a spin-Pomeranchuk instability, where magnetization lives on bonds [41]. However

*Contact author: debmalya@iisermohali.ac.in

†Contact author: annica.black-schaffer@physics.uu.se

experimental indications of such a spin-Pomeranchuk mechanism is still lacking [17].

Due to the unique spin structure of altermagnets, its interplay with superconductivity was already early on anticipated to be intriguing [42] and has since then generated significant attention. For example, when altermagnetism and superconductivity coexist intrinsically in the same material, it has theoretically been predicted that finite-momentum superconductivity can appear even in the absence of any net magnetization [43–48]. Experimentally, hints of such coexistence also exists in the altermagnetic candidate material RuO₂ under strain [49,50] and monolayer FeSe [51]. Moreover, the parent cuprate material La₂CuO₄, which upon doping displays high-temperature superconductivity, is also proposed to host altermagnetism [16]. In a different direction, theoretical investigations considering superconductor-altermagnet hybrid junctions have also uncovered great prospects for exotic phenomena, such as intriguing Josephson effects [52–56], phase-shifted Andreev levels [57], prospects for stray-field-free memory devices [58], controllable [59] and perfect [60] superconducting diode effect, dissipationless spin-splitting and filtering effects [61], magnetoelectric effect [62], realizing topological superconductivity [63–65], stray-field-free thermoelectric response [66], and spin-polarized specular Andreev reflections [67]. However, despite this large number of theoretical works on the interplay of altermagnetism and superconductivity, most works [47,48,52,54–57,59,62–65,67] seemingly miss to consider the crucial aspect that spin and sublattice symmetries are intrinsically coupled in altermagnets.

In this work, we show that the strong intrinsic coupling of spin and sublattice in altermagnets has a decisive effect on the nature of superconducting pairing symmetries in altermagnets, irrespective of the origin of superconductivity. By investigating the archetypical four-band model of altermagnetism, proposed in Ref. [16], we first identify that the widely used [47,48,52,54–57,59,62–65,67] two low-energy bands relevant for superconductivity have an essentially complete spin-sublattice locking. We then uncover that this spin-sublattice locking imposes severe constraints on the superconducting symmetries in this two-band model of altermagnets. In particular, we find that the two most commonly studied superconducting pairing symmetries, spin-singlet onsite, or uniform, *s*-wave, and spin-triplet nearest-neighbor *p*-wave pairing are not allowed by symmetries. In fact, for a $d_{x^2-y^2}$ -wave altermagnet, the most commonly studied altermagnet, we find that the only possible spin-singlet pairing must have $d_{x^2-y^2}$ -wave or extended *s*-wave (i.e., beyond onsite) symmetry, unless considering higher harmonics or angular momentum, which are both much less likely to appear. However, also allowing for unequal-time Cooper pairs, we show that further possibilities emerge with odd-frequency pairing, including onsite *s*-wave equal-spin-triplet odd-frequency pairing. We additionally explore a model for interaction-induced altermagnetism through a Fermi surface instability and find that its spin Fermi surfaces are also fully sublattice polarized and thus generates the same constraints for the superconducting pairing as in the archetypical two-band model. Using this interaction-induced altermagnet model, we explicitly show that our analysis of the superconducting pairing

symmetries holds even when allowing for finite momentum pairing.

We organize the rest of this paper in the following way. In Sec. II, we discuss the archetypical model for altermagnetism. We first show the construction of the minimal two-band model in Sec. II A. We then find the possible superconducting pairing symmetries in Sec. II B. Next, in Sec. III, we explore a different, interaction-induced altermagnet model where we first demonstrate altermagnetism within a mean-field analysis of the electronic interaction in Sec. III A and then calculate different superconducting order parameters self-consistently allowing also for finite momentum pairing in Sec. III B. Finally, we summarize our results in Sec. IV.

II. ARCHETYPICAL MODEL OF ALTERMAGNETISM

Altermagnetism can appear in materials in two ways. First, the most studied model for altermagnetism was originally proposed in Ref. [16], where a combination of electric crystal field splitting and large spin-exchange splitting directly generates a momentum dependent spin-splitting of the electronic band structure. Here, altermagnetism occur mainly due to certain broken symmetries of the spin arrangement in the crystal and is thus not due to a Fermi surface instability. In this work, we simply call this the archetypical model of magnetism. It can be straightforwardly captured within first-principles DFT calculations [16,17]. A second alternative is altermagnetism instead originating from electronic interactions through a Fermi surface instability, providing a very different but equally valid mechanism [35–41]. In this section, we consider first, the archetypical model for altermagnetism, while, later in Sec. III we consider an interaction-induced model with altermagnetism generated by a Fermi surface instability.

A. Construction of minimal two-band model

The original archetypical model of altermagnets can be well-captured in a four-band model on the square lattice with $d_{x^2-y^2}$ -wave altermagnetism [16],

$$H_{4b} = \sum_k \Psi^\dagger \hat{H}_{4b} \Psi, \quad (1)$$

with

$$\begin{aligned} \hat{H}_{4b} = & (-2t(\cos k_x + \cos k_y) - \mu)\sigma_0\tau_0 \\ & - \frac{t_{\text{sub}}}{2}(\cos k_x - \cos k_y)\sigma_0\tau_z - J\sigma_z\tau_z, \end{aligned} \quad (2)$$

using the basis $\Psi^\dagger = (c_{kA\uparrow}^\dagger, c_{kB\uparrow}^\dagger, c_{kA\downarrow}^\dagger, c_{kB\downarrow}^\dagger)$, where $c_{kA/B\sigma}^\dagger$ ($c_{kA/B\sigma}$) are the creation (annihilation) operators of an electron with spin σ and momentum k on the sublattice *A/B*, while σ and τ are Pauli matrices in spin and sublattice space, respectively. Further, *t* is the nearest-neighbor hopping amplitude, t_{sub} denotes the sublattice symmetry breaking originating from the electric crystal field splitting [16], *J* is the momentum independent spin-exchange interaction, while μ is the chemical potential tuned to fix the average density of electrons. It is important to highlight a few features of the Hamiltonian in Eq. (2). First, the exchange splitting is opposite for different sublattices due to the τ_z term, which is an essential feature in order to generate altermagnetism in the model Eq. (2).

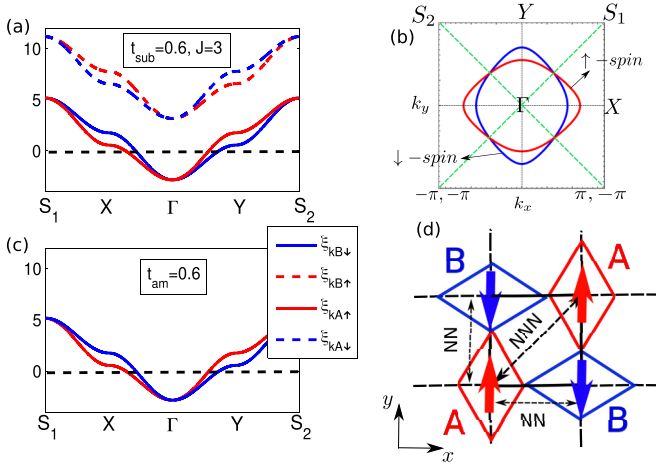


FIG. 1. Energy bands in the four-band model in Eq. (1) (a) and extracted low-energy two-band model in Eq. (3) (c). (b) Fermi surface of both four-band and two-band models with $\downarrow$ -spin band (blue) and $\uparrow$ -spin band (red) and with indicated high-symmetry points, Γ , X , Y , S_1 , and S_2 , of the first BZ. Green dashed lines indicate the nodes of the altermagnet. (d) Real-space projection of the low-energy bands on the underlying square lattice. Square box with black solid lines indicate the unit cell, dashed black lines indicate that the unit cell is repeated to form the lattice. Red and blue colors denote $\uparrow$ -spin and $\downarrow$ -spin band projections in real space, respectively. Distortions of the diamond lattice sites A and B signify that they have different local environments and hence different crystal field splittings, i.e., $t_{\text{sub}} \neq 0$ leading to $t_{\text{am}} \neq 0$. Nearest-neighbor (NN) and next-nearest-neighbor (NNN) sites are indicated by dashed double-arrows.

Secondly, in this four-band model, t_{sub} term is not spin-dependent, as expected for electric crystal field splitting.

To understand the origin of altermagnetism, we plot in Fig. 1(a) the four energy bands of Eq. (1), which we label $\xi_{kA\uparrow}$, $\xi_{kA\downarrow}$, $\xi_{kB\uparrow}$, and $\xi_{kB\downarrow}$, for $t = 1$ (i.e., all energies are in units of t) and a fixed average electron density $\rho = 0.6$. Since the Hamiltonian in Eq. (1) is diagonal, the four bands can be labeled by the same sublattice and spin indices as in the chosen Ψ basis. This basis and labeling of the four bands using sublattice and spin indices is motivated by the findings of Ref. [16]. For $t_{\text{sub}} = 0$ and $J = 0$, all the four bands are degenerate. With finite $t_{\text{sub}} = 0.6$, but still $J = 0$, the A and B bands split, indicating sublattice symmetry breaking, but there is still spin degeneracy. Then, also a finite $J = 3$ results in spin symmetry breaking. We here consider J to be large, just as in Ref. [16]. Hence a momentum dependent spin-splitting appears for both finite t_{sub} and J . As seen in Fig. 1(a), the spin-splitting occurs in opposite direction for the two different sublattices. Consequently, the two lowest energy bands become $\xi_{kA\uparrow}$ and $\xi_{kB\downarrow}$ with spin and sublattice degree of freedoms intrinsically coupled. The corresponding Fermi surfaces in the first Brillouin zone (BZ) are shown in Fig. 1(b), which also illustrates the nodal lines of the altermagnet (green dashed lines), where the band spin-splitting is zero. The other two bands, $\xi_{kA\downarrow}$ and $\xi_{kB\uparrow}$, are split off from these two low-energy states by an energy scale of $J = 3$, which is known to be large [16].

For the purpose of considering superconducting pairing, it is particularly important to here note that different spin bands

are associated to different sublattices. The main reason behind this is that, as shown in Ref. [16], for altermagnetic materials the red solid line representing the low-energy $\uparrow$ -spin (and blue dashed line representing high-energy $\downarrow$ -spin) band in Fig. 1, when projected into real space, have maximum weight on sublattice A, while the blue solid line representing the low-energy $\downarrow$ -spin (and red dashed line representing high-energy $\uparrow$ -spin) band, when projected in real space, have maximum weight on sublattice B. We schematically illustrate this in the real-space schematic in Fig. 1(d). The differently distorted diamonds on the square lattice (black lines) indicate the different local crystal field environments for sublattices A and B, set by the t_{sub} term in Eq. (2).

With only two bands close to the Fermi level, we can, for any low-energy phenomena and particularly superconductivity, to a very good approximation consider an effective two-band model consisting of only the energy bands $\xi_{kA\uparrow}$ and $\xi_{kB\downarrow}$. The corresponding two-band Hamiltonian is then given by

$$H_{2b} = \sum_k \Psi_{2b}^\dagger \hat{H}_{2b} \Psi_{2b}, \quad (3)$$

with

$$\hat{H}_{2b} = (-2t(\cos k_x + \cos k_y) - \mu)s_0 - \frac{t_{\text{am}}}{2}(\cos k_x - \cos k_y)s_z, \quad (4)$$

now using the basis $\Psi_{2b}^\dagger = (c_{kA\uparrow}^\dagger, c_{kB\downarrow}^\dagger)$. Note that here s is now a 2×2 Pauli matrix in the joint sublattice and spin basis given by Ψ_{2b} . In this two-band model t_{am} by itself encodes for the full momentum dependent spin-splitting with nodes along $|k_x| = |k_y|$. By using the joint spin-sublattice Pauli matrices s , we are in Eq. (4) able to limit ourselves to a single parameter t_{am} to capture altermagnetism, instead of the previous two, t_{sub} and J . Value-wise, we need to set $t_{\text{am}} = t_{\text{sub}}$ in order to perfectly match the band structures in Figs. 1(a) and 1(c). The energy bands of Eq. (3) are shown in Fig. 1(c). These bands are exactly same as the two low-energy bands of Fig. 1(a) by construction and hence give the same Fermi surface as in Fig. 1(b).

B. Superconducting pairing

Since superconductivity is a low-energy phenomenon, it is sufficient to consider only the two bands closest to the Fermi level, as given by the Hamiltonian Eq. (3). In most of the current literature [47,48,52,54–57,59,62–65,67] on superconductivity in altermagnets, the sublattice index in Eq. (3) has been dropped in the two-band model of altermagnetism. However, we should be careful with this sublattice index when discussing superconductivity, since both spatial and spin symmetries are crucial for determining the possible superconducting pairing symmetries [68]. Superconductivity occurs due to the formation of Cooper pairs made of two electronlike quasiparticles. Because of the fermionic nature of the constituents, there are restrictions on the nature of the Cooper pairs. In particular, under a joint operation of spin permutation (S), momentum exchange or parity (P), and relative time permutation (T) of the individual electrons, superconducting pairing needs to satisfy the condition $SPT = -1$, for a single orbital system [69–72].

TABLE I. Possible occurrence of superconducting pairing symmetries in a $d_{x^2-y^2}$ -wave altermagnet on a square lattice, Eq. (1), indicated by their spin symmetry, momentum structure or parity, corresponding form factor, and simplest real-space structure, considering up to NNN pairing in real space. Only equal-time (or equivalently even-frequency) BCS-like pairings are shown.

Spin symmetry (S)	Momentum symmetry (P)	Form factor	Real-space structure	Occurrence
Spin-singlet ($S = -1$)	s -wave ($P = 1$)	1	Onsite	×
	Extended s -wave ($P = 1$)	$\cos k_x + \cos k_y$	NN	✓
	$d_{x^2-y^2}$ -wave ($P = 1$)	$\cos k_x - \cos k_y$	NN	✓
	d_{xy} -wave ($P = 1$)	$\sin k_x \sin k_y$	NNN	×
Equal-spin-triplet ($S = 1$)	p -wave ($P = -1$)	$\sin k_x, \sin k_y$	NN	×
Mixed-spin-triplet ($S = 1$)	p -wave ($P = -1$)	$\sin k_x, \sin k_y$	NN	✓

Let us begin our discussion on the pairing symmetries possible in the archetypical model of altermagnets, Eq. (3) by first considering only equal-time pairing, as is the standard (BCS) form of superconductivity. Equal-time pairing automatically means $T = 1$, since there is no consequences of exchanging the time coordinate of the individual electrons in the Cooper pair. In this scenario, only spin and spatial symmetries decide the nature of the pairing. Then, in order to satisfy $SPT = -1$, spin-singlet pairing ($S = -1$) has to be even-parity in k space ($P = 1$), while spin-triplet pairing ($S = 1$) has to be odd-parity ($P = -1$), as is standard for superconductivity.

1. Spin-singlet pairing

Focusing first on spin-singlet pairing, the simplest, and most common, is a uniform or isotropic s -wave symmetry (also known as conventional) pairing. This leads to pairing of opposite spins and on the same sublattice, since the uniformness of the s -wave pairing in k -space leads to onsite pairing in real space. However, in Eq. (3) and explicitly shown in Fig. 1, opposite spins of the same sublattice are not present in the two-band minimal model. Consequently, onsite s -wave pairing is not allowed in altermagnets. If we next expand and consider spin-singlet pairing on nearest-neighbor (NN) sites in real space, the possible spin-singlet candidates are $d_{x^2-y^2}$ -wave and extended (i.e., beyond onsite) s -wave, given by the form factors in Table I. As NN sites belong to different sublattices and thus have opposite spins, both of these pairing symmetries are allowed in an altermagnet. Further considering next-nearest-neighbor (NNN) spin-singlet pairing, we recover also the d_{xy} -wave symmetry. However, since NNN pairing occurs between diagonal sites on the square lattice, which, as shown in Fig. 1 carry equal spins in an altermagnet, spin-singlet d_{xy} -wave pairing is not allowed. Consequently, the only possible spin-singlet pairing symmetries up to NNN in real space are the $d_{x^2-y^2}$ -wave and extended s -wave symmetries. We summarize these results in Table I. Considering longer-range pairing in real space results in either (specific) higher harmonics of the same symmetries or higher order symmetries (e.g., g , i -wave symmetries), all of which contain more nodes in the BZ and are thus often energetically less favorable as the superconducting condensation energy decreases when more low-energy quasiparticles are present. We thus conclude that for spin-singlet superconducting pairing, we should generally only consider $d_{x^2-y^2}$ -wave and extended s -wave symmetries in the archetypical model, Eq. (1), of altermagnets. This explicitly excludes the most common form of superconductivity, the spin-singlet uniform s -wave state,

commonly assumed to exist for all phonon-driven superconductors. At the same time, it allows for the $d_{x^2-y^2}$ -wave state present in the high-temperature cuprate superconductors [1]. Interestingly, at least the parent cuprate material La_2CuO_4 has also been proposed to host altermagnetism [16]. We also note that the $d_{x^2-y^2}$ -wave has the same nodal structure as the $d_{x^2-y^2}$ -wave nodes of altermagnetism in Eq. (1).

Pure spin-singlet pairing further requires that the pairing is of the form $(kA \uparrow, -kB \downarrow) - (kA \downarrow, -kB \uparrow)$. While $(kA \uparrow, -kB \downarrow)$ pairs are present in the two low energy bands in Fig. 1(c), $(kA \downarrow, -kB \uparrow)$ pairs belong to the two higher energy bands. Hence, pairing forced to consist of only $(kA \downarrow, -kB \uparrow)$ in the two low energy bands is a mixture of spin-singlet form $(kA \uparrow, -kB \downarrow) - (kA \downarrow, -kB \uparrow)$ and a mixed-spin-triplet form $(kA \uparrow, -kB \downarrow) + (kA \downarrow, -kB \uparrow)$, in the same spirit as in Ising superconductors [73]. However, pure spin-singlet pairing is possible when both the low-energy and high-energy bands are allowed in the four-band model in Fig. 1(a). We note that this condition for pure spin-singlet pairing involving both low- and high-energy bands is different from the arguments presented in the previous paragraph. Here, individual electrons in a Cooper pair are still at the same energy. In contrast, in the previous paragraph we discuss prohibition of pairing symmetries where constituent electrons of a single pair are at different energies. Forming Cooper pairs where the constituent electrons lie at different energy levels is highly unlikely compared to both of them lying at the same energy level [74]. In Sec. III B, we show numerically in a model, where all the four bands are kept, that pure singlet d -wave pairing is indeed possible to stabilize.

2. Spin-triplet pairing

Next, considering spin-triplet pairing, it can be of two forms, equal-spin-triplet and mixed-spin-triplet pairing depending on the spin projection along the spin axis. Equal-spin-triplet pairing would necessarily mean pairing A-sublattice with A sublattice in the low-energy two band model, Eq. (3). This would be possible if the pairing occurs onsite, but this would mean $P = 1$ and $S = 1$ pairing which does not satisfy $SPT = -1$ and is thus not allowed. The only possibility to achieve equal-spin-triplet pairing is to construct pairing between different A sublattice sites, because the standard NN p -wave pairing is blocked due to the spin structure. The shortest range possibility is pairing on second nearest neighbors, or $2x$ or $2y$ bonds in Fig. 1(d). However, this is longer-range pairing compared to both NN and NNN pairing and will therefore contain more nodes in k space, leading often to less

TABLE II. Possible occurrence of odd-frequency superconducting pairing symmetries in a $d_{x^2-y^2}$ -wave altermagnet on a square lattice, Eq. (1), indicated by their spin symmetry and momentum structure or parity, considering up to NNN pairing in real space.

Possible odd-frequency pairings ($T = -1$)	
Spin symmetry (S)	Momentum symmetry (P)
Spin-singlet ($S = -1$)	p -wave ($P = -1$)
Equal-spin-triplet ($S = 1$)	s -wave ($P = 1$)
	d_{xy} -wave ($P = 1$)
Mixed-spin-triplet ($S = 1$)	$d_{x^2-y^2}$ -wave ($P = 1$)
	Extended s -wave ($P = 1$)

condensation energy. Or alternatively, viewed in real space, longer-range pairing requires longer range interactions, which tend to be decaying with distance, such that they become subdominant. As a consequence, we deem equal-spin pairing to be unlikely in an altermagnet. The only possible spin-triplet pairing is therefore a mixed-spin-triplet state. This state can occur on NN sites, and then have a p -wave symmetry. We summarize all possibilities for spin-triplet pairing up to NNN in Table I. As discussed in the last section, a combination of spin-singlet and mixed-spin-triplet pairings may appear when strictly only two low energy bands are considered.

3. Odd-frequency pairing

We finally discuss the scenario where also unequal-time pairing is considered, i.e., the two electrons can also pair at unequal times, τ_1 and τ_2 . If the pairing is even in the relative time coordinate, $\tau_1 - \tau_2$, then $T = 1$ and the possible pairing symmetries are exactly same as discussed in the previous two subsections and summarized in Table I. However, if we also allow for the pairing to be odd in the relative time coordinate, or equivalently odd in frequency pairing, additional distinct possibilities emerge due to $T = -1$. Such odd-frequency pairing [70,71,75–87] has been studied extensively in both superconductor-ferromagnet hybrid junctions [11,88–94] and superconductor-antiferromagnet hybrid junctions [95–97], also considering spintronics applications [11,12]. For superconductor-altermagnet hybrid junctions, the spintronics applications can be even more technologically beneficial as the zero net magnetization in altermagnets result in no stray fields [17]. Hence, we also explore the possible superconducting pairing symmetries for odd-frequency pairing.

Due to the oddness in relative time symmetry $T = -1$, $SPT = -1$ can now be satisfied for spin-singlet pairing ($S = -1$) with odd-parity ($P = -1$) or spin-triplet pairing ($S = 1$) with even-parity ($P = 1$). Consequently, the even-parity pairing symmetries not allowed with equal-time pairing, on-site s -wave and d_{xy} -wave pairing, are now allowed with an equal-spin-triplet configuration. The other even-parity pairings previously considered, $d_{x^2-y^2}$ -wave and extended s -wave, will now instead appear in a mixed-spin-triplet configuration, due to them connecting different sublattices. Finally, spin-singlet odd-parity is also allowed as a NN p -wave pairing. We summarize these possible odd-frequency pairing symmetries in Table II.

To conclude, in this section, we show that the coupled nature of spin and sublattice, essential in generating the momentum dependent spin splitting in noninteracting altermagnets, puts severe constraints on the possible superconducting pairing in the widely used [47,48,52,54–57,59,62–65,67] two-band model of altermagnets, as summarized in Table I and extended in Table II for odd-frequency superconducting pairing. Notably, these constraints are independent on the strength of the altermagnet term t_{am} . However, if the value of the spin-exchange interaction J in Eq. (2) is much smaller than we consider, all four bands in Fig. 1 may instead be present at low energies. It would then be necessary to consider all four bands when introducing superconductivity, and hence new constraints on the superconductivity may emerge. Such calculations are however beyond the scope of the present work. The results in this section stem from general symmetry arguments of altermagnets and superconducting pairing and thus do not assume any specific form for the superconducting pairing mechanism. Hence, the analysis is applicable to any type of intrinsic superconductivity as well as for proximity-induced superconducting pairing in hybrid structures.

III. INTERACTION-INDUCED ALTERMAGNETISM

In the last section, we considered a model for altermagnetism where the spin arrangement on the crystal and the resulting band structure gives rise to the characteristic momentum dependent spin-splitting. There also exist other models where altermagnetism is alternatively generated by electronic interactions providing a Fermi surface instability [35–41]. In this section, we study one such model where electronic interactions combined with band structure effects can generate altermagnetism [35]. However, the discussion in this section is also applicable to other interaction-induced models of altermagnetism. In terms of considering how altermagnetism and superconductivity interact with each other, this distinction between different origins of altermagnetism may be important, because when altermagnetism is generated by a Fermi instability, as in this section, superconductivity has to compete with the altermagnetism for the accessible quasiparticles at the Fermi surface. Instead, when altermagnetism is generated due to a specific spin arrangement, as in the archetypical model of altermagnetism studied in the previous section, there is no such direct competition for quasiparticles to generate the superconducting instability. Hence, it is not a priori clear that the two different origins of altermagnetism will result in similar behavior for superconductivity.

A. Normal state

Following Ref. [35], we again consider the square lattice with two sublattices, but now with the noninteracting, or kinetic energy given by

$$H_0 = \sum_{k,\sigma} \xi_{AA}(k) c_{kA\sigma}^\dagger c_{kA\sigma} + \xi_{BB}(k) c_{kB\sigma}^\dagger c_{kB\sigma} + \epsilon_{AB}(k) (c_{kA\sigma}^\dagger c_{kB\sigma} + \text{H.c.}), \quad (5)$$

where again $c_{kA/B\sigma}^\dagger$ ($c_{kA/B\sigma}$) are the creation (annihilation) operators of an electron with spin σ and momentum k on the

sublattice A/B , and with

$$\begin{aligned}\xi_{AA}(k) &= -2t_1 \cos(2k_x) - 2t_2 \cos(2k_y) - \mu, \\ \xi_{BB}(k) &= -2t_2 \cos(2k_x) - 2t_1 \cos(2k_y) - \mu, \\ \epsilon_{AB}(k) &= -2t(\cos(k_x) + \cos(k_y)).\end{aligned}\quad (6)$$

Here t is the NN hopping amplitude, $t_{1/2}$ are second neighbor hopping amplitudes, and μ is the chemical potential tuned to fix the average density of electrons. We then add interactions in the form of an onsite repulsive Hubbard interaction,

$$H_U = \sum_i U n_{i\uparrow} n_{i\downarrow}, \quad (7)$$

where $n_{i\sigma}$ is the number operator and U is the onsite Coulomb repulsion. We achieve magnetism by mean-field decoupling of this Hubbard interaction in the Hartree channel resulting in

$$H_{U,\text{mf}} = \sum_{i\sigma} \frac{1}{2} U (\rho n_{i\sigma} - m_i \sigma n_{i\sigma}) + \text{constant}, \quad (8)$$

where ρ is the average electron density per sublattice and $m_i = \langle n_{i\uparrow} - n_{i\downarrow} \rangle$ is the magnetization. We here assume a homogeneous bulk solution of the form $m_i = m_A = -m_B \equiv m$, which generates a sublattice dependence of the magnetization, and, if nonzero, gives either antiferromagnetism or altermagnetism.

The total Hamiltonian becomes $H_{\text{int}} = H_0 + H_{U,\text{mf}}$ and can be written in a matrix form using the same basis as in Sec. II, $\Psi^\dagger = (c_{kA\uparrow}^\dagger, c_{kB\uparrow}^\dagger, c_{kA\downarrow}^\dagger, c_{kB\downarrow}^\dagger)$, as

$$H_{\text{int}} = \sum_k \Psi^\dagger \hat{H}_{\text{int}} \Psi, \quad (9)$$

with

$$\begin{aligned}\hat{H}_{\text{int}} &= (\xi_{AA}(k) + \frac{1}{2}\xi_{BB}(k))\sigma_0\tau_0 \\ &+ (\xi_{AA}(k) - \frac{1}{2}\xi_{BB}(k))\sigma_0\tau_z \\ &+ \epsilon_{AB}(k)\sigma_0\tau_x - M\sigma_z\tau_z,\end{aligned}\quad (10)$$

where we define $M = \frac{1}{2}Um$, while the spin-independent Hartree shift term $\frac{1}{2}U\rho$ of Eq. (8) is absorbed into an effective μ . As shown in Ref. [35], altermagnetism appears in Eq. (9) for finite M and when there is a sublattice asymmetry with $\xi_{AA}(k) \neq \xi_{BB}(k)$. From Eq. (6), we see that the condition $\xi_{AA}(k) \neq \xi_{BB}(k)$ is equivalent to $t_1 \neq t_2$.

Beyond the same basis, Eq. (10) also has a very similar matrix form to Eq. (2) for archetypical model of altermagnetism. The magnetization M in Eq. (10) plays the role of the spin-exchange splitting J in Eq. (2), although M is now an order parameter obtained self-consistently from a Hubbard interaction U . Here, $t_1 - t_2$ plays the role of the sublattice symmetry breaking t_{sub} in Eq. (2). However, in contrast to Eq. (2), Eq. (10) has an additional hybridization between the two sublattices through the term $\epsilon_{AB}(k)\sigma_0\tau_x$. Because of this hybridization, Eq. (10) is no longer diagonal and hence the four diagonal bands cannot automatically be labeled by the same indices as in the basis Ψ . However, as we will see later, the spin bands will still be strongly coupled to sublattice in the altermagnetic state, just as in Eq. (2).

To proceed with finding interaction-driven altermagnetism, we solve the Hamiltonian Eq. (9) self-consistently for the

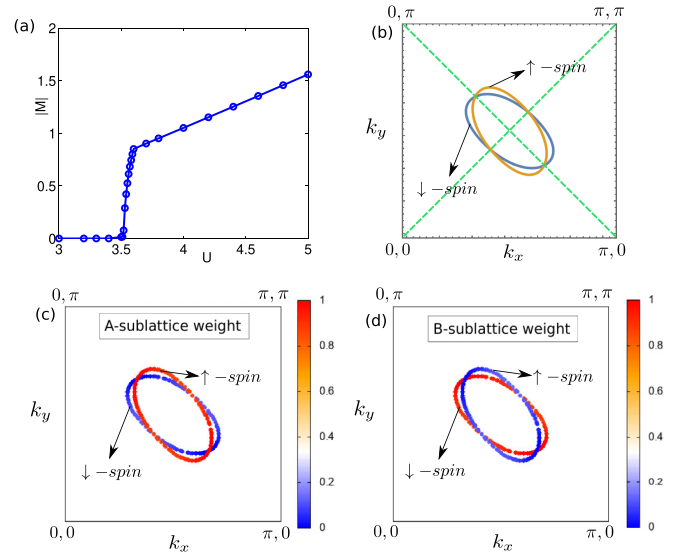


FIG. 2. (a) Magnetization per sublattice $|M|$ as a function of U for the interaction-driven altermagnet model in Eq. (9) with extracted Fermi surface for $U = 4.6$ (b). Green dashed lines in (b) indicate the nodes of the altermagnet. [(c) and (d)] Normalized sublattice weight calculated for the low-energy bands crossing the Fermi surface. In (b)–(d) only the positive quadrant of the first BZ is shown for a better visibility. Other quadrants are obtained by four-fold rotational symmetry.

order parameter M . In the following we report results for $t_1 = 0.2$, $t_2 = 0.4$ giving $t_1 - t_2 = 0.2$, using again $t = 1$ as the energy unit, and for a fixed average density per sublattice $\rho = 0.9$ and we use a system size of 1000×1000 k points in momentum space. Larger system sizes do not change the results. In Fig. 2(a), we show the obtained value of $|M|$ with varying U . For $U \leq 3.52$, there is no magnetization, $|M| = 0$. At a critical $U = 3.53$, $|M|$ acquires a finite large value and then $|M|$ increases linearly with increasing U . The corresponding Fermi surfaces in the ordered phase are shown in 2(b) in the positive quadrant of the first BZ. Due to four-fold rotational symmetry each quadrant hold a similar Fermi surface. Due to the finite $|M|$ and asymmetry $t_1 - t_2$, the Fermi surfaces of $\uparrow$ spin (yellow) and $\downarrow$ spin (blue) are split, except along the nodal lines $|k_x| = |k_y|$ and $|k_x \pm k_y| = \pi$ (green dashed). These results establish that a finite interaction U generates altermagnetism with nodes along both $|k_x| = |k_y|$ and $|k_x \pm k_y| = \pi$, the latter generating additional nodal lines compared to the band-structure altermagnetism in Sec. II. For $t_1 - t_2 = 0$, a finite $|M|$ instead generates antiferromagnetism, with a nonsplit Fermi surface (not shown). We further comment on the value of the model parameters. For a fixed ρ , a larger (smaller) $t_1 - t_2$ simply results in a larger (smaller) critical U . Moreover, our choice of $\rho = 0.9$ is motivated by two reasons. First, at half filling per sublattice, $\rho = 1.0$, the altermagnetic state is metallic only for a very narrow window of U [35]. To get superconductivity a metallic altermagnetic state is required and we therefore choose $\rho = 0.9 \neq 1.0$ in order to achieve a doped metallic altermagnet for a larger window of U . Secondly, filling levels $\rho \ll 1.0$ and $\rho \gg 1.0$

are also detrimental to interaction-induced altermagnetism since such densities need very large U to get finite M .

As mentioned above, in contrast to in Sec. II, Eq. (10) is no longer a diagonal matrix. Hence, in order to identify the superconducting symmetries it is important to find out the sublattice weights of the low-energy bands in this model as they are not necessarily inherited from the original basis Ψ . In Figs. 2(c) and 2(d), we plot the normalized projection of the eigenvectors of the two low-energy diagonal bands on the A sublattice (c) and the B sublattice (d). We only show the sublattice weights for k points on the Fermi surface since only the Fermi surface is essential to superconductivity. As seen, the $\uparrow$ -spin Fermi surface is purely A-sublattice polarized, while the $\downarrow$ -spin Fermi surface is purely B-sublattice polarized. This establishes that even an interaction-induced model for altermagnetism has the same basic properties of spin-sublattice locking as discussed in the archetypical four-band model of altermagnetism used in Sec. II.

B. Superconducting pairing

Due to the same nature of spin-sublattice locking in the interaction-induced altermagnet model Eq. (9) as in the archetypical altermagnet model Eq. (1), all the discussion of superconducting pairing symmetries in Sec. II B, with results summarized in Tables I and II, is also valid for the interaction-induced model, with exactly the same constraints on the pairing symmetries. In this subsection, we demonstrate this explicitly by calculating three different examples of spin-singlet pairing, uniform s -wave, extended s -wave, and $d_{x^2-y^2}$ -wave (called d -wave pairing from here on) symmetries, and now also by allowing for the possibility of finite-momentum superconductivity.

To be able to study superconductivity, we need to provide an effective attraction in the system. For this purpose, we add to Eq. (9) a NN attractive interaction V_{sc} for generating d -wave and extended s -wave pairing, and an onsite attractive interaction U_{sc} for generating onsite s -wave pairing,

$$H_{sc} = \sum_{\langle ij \rangle \sigma \sigma'} -V_{sc} n_{i\sigma} n_{j\sigma'} - \sum_i U_{sc} n_{i\uparrow} n_{i\downarrow}. \quad (11)$$

We here note that both V_{sc} and U_{sc} should be viewed as encoding effective attractions, for example generated by electron-phonon coupling, and should not be confused or mixed up with the repulsive U term in Eq. (7) that is the source of altermagnetism. We reach a superconducting state by mean-field decoupling H_{sc} in the spin-singlet Cooper channel. We further avoid adding these attractive interactions to the normal mean-field channel, in order to keep the magnetism being purely generated by U . The primary motivation for this treatment is two-fold. First, it provides the simplest possibility to explicitly capture the competition between magnetism and superconductivity. Secondly, and most importantly, the origin of H_{sc} in Eq. (11) and H_U in Eq. (7) are likely very different. For example, an onsite attraction in Eq. (11) often effectively arises due to electron-phonon coupling, whereas H_U is purely stemming from Coulomb repulsion. Hence, it is also relevant to treat them separately at the mean-field level in order to capture the competition between magnetism and superconductivity. It should also be noted that our motiva-

tion in this work is not to address the microscopic origin of altermagnetism or superconductivity, but to understand their competition, once they are present. Moreover, when the effect of screening due to electron-electron interaction on electron-phonon coupling is considered, the electron-phonon coupling is known to be renormalized only leading to a rescaling of the resulting superconducting order [98–100].

Upon mean-field decoupling H_{sc} in the spin-singlet Cooper channel, allowing for a finite center-of-mass momentum of the Cooper pairs Q , we arrive at [44]

$$\begin{aligned} H_{sc, \text{mf}} = & \sum_k (\Delta_k^Q c_{(-k+Q/2)A\downarrow} c_{(k+Q/2)B\uparrow} \\ & - \Delta_k^Q c_{(k+Q/2)A\uparrow} c_{(-k+Q/2)B\downarrow} + \text{H.c.}) \\ & + (\Delta_s^Q c_{(-k+Q/2)A\downarrow} c_{(k+Q/2)A\uparrow} \\ & + \Delta_s^Q c_{(-k+Q/2)B\downarrow} c_{(k+Q/2)B\uparrow} + \text{H.c.}) \\ & + \text{constant}, \end{aligned} \quad (12)$$

where Δ_k^Q and Δ_s^Q are NN and onsite spin-singlet superconducting order parameters, respectively, obtained from the self-consistency relations

$$\begin{aligned} \Delta_k^Q = & \sum_{k'} \frac{1}{2} V_{k,k'} \langle c_{(k'+Q/2)A\uparrow}^\dagger c_{(-k'+Q/2)B\downarrow}^\dagger \\ & - c_{(-k'+Q/2)A\downarrow}^\dagger c_{(k'+Q/2)B\uparrow}^\dagger \rangle, \\ \Delta_s^Q = & - \sum_{k'} U_{sc} \langle c_{(k'+Q/2)A/B\uparrow}^\dagger c_{(-k'+Q/2)A/B\downarrow}^\dagger \rangle. \end{aligned} \quad (13)$$

Here the interaction $V_{k,k'}$ is given by

$$V_{k,k'} = -V_{sc}(\gamma(k)\gamma(k') + \eta(k)\eta(k')), \quad (14)$$

with $\gamma(k) = \cos(k_x) + \cos(k_y)$ and $\eta(k) = \cos(k_x) - \cos(k_y)$ being the two form factors for NN interaction on a square lattice. Incorporating the momentum dependence of $V_{k,k'}$, we can finally write $\Delta_k^Q = \Delta_d^Q \eta(k) + \Delta_{\text{ext}-s}^Q \gamma(k)$, with Δ_d^Q being the d -wave superconducting order parameter and $\Delta_{\text{ext}-s}^Q$ being the extended s -wave superconducting order parameter [101]. Here, the nodes of the d -wave state lie along the $k_x = \pm k_y$ lines which partially matches with the altermagnet nodes obtained in Fig. 2(b). By allowing for a finite Q , finite-momentum pairing becomes a possibility, while $Q = 0$ corresponds to (zero-momentum) BCS pairing. Note that all the three superconducting order parameters, onsite Δ_s^Q , Δ_d^Q , and $\Delta_{\text{ext}-s}^Q$, parametrically depend on Q .

We formulate the total Hamiltonian $H = H_0 + H_{U, \text{mf}} + H_{sc, \text{mf}}$ using Eqs. (5), (8), and (12) in a Nambu basis and solve self-consistently for both the superconducting order parameters Δ_s^Q , Δ_d^Q , and $\Delta_{\text{ext}-s}^Q$, as well as the normal state order parameter M for a particular Q and fixed ρ . Then, we obtain the true ground state by minimizing the ground state energy with respect to Q . This allows us to capture any putative finite-momentum pairing state. For simplicity, we use the same parameters as in Sec. III A, i.e., $\rho = 0.9$, $t_1 - t_2 = 0.2$, and a system size of 1000×1000 .

We start by investigating at the possibility of uniform s -wave pairing by setting $U_{sc} = 2$ and $V_{sc} = 0$. In Figs. 3(a) and 3(c), we plot $|M|$ and Δ_s^Q as a function of varying U . Note

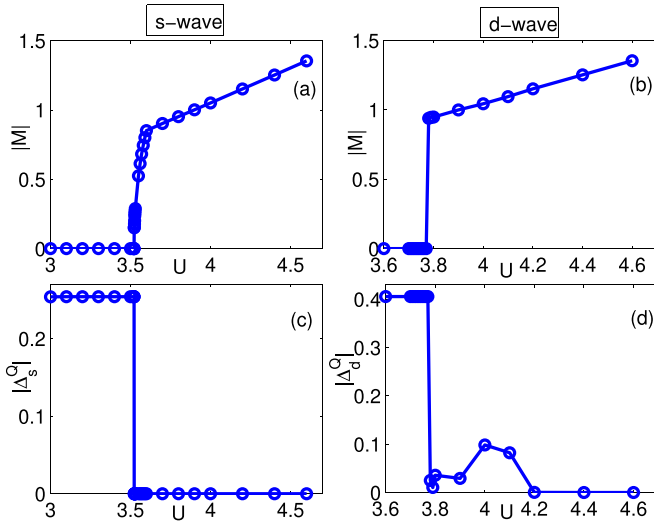


FIG. 3. Magnetization per sublattice $|M|$ [(a) and (b)], superconducting order parameters Δ_s^Q (c), and Δ_d^Q (d) as a function of U . In (a) and (c), only onsite s -wave pairing is considered by setting $U_{sc} = 2$ and $V_{sc} = 0$, whereas in (b) and (d), NN pairing is considered by setting $U_{sc} = 0$ and $V_{sc} = 2$.

that it is not necessary that the behavior of $|M|$ in Fig. 3(a) will be same as in Fig. 2(a) due to the possible competition with Δ_s^Q . However, despite this, we find the same behavior of $|M|$ with U as in Fig. 2(a). In terms of Δ_s^Q , we find it to only be finite for $U \leq 3.523$, when also $|M| = 0$. In fact, Δ_s^Q sharply goes to zero as soon as $|M|$ acquires a finite value. Hence, onsite s -wave pairing does not survive in the presence of altermagnetism. This explicitly demonstrates that the spin-sublattice locking in the altermagnet state, seen in Fig. 2, entirely prevents developing an onsite s -wave pairing, a result also fully consistent with the results in Table I. Note that this result also holds for all finite-momentum s -wave superconducting states. We further illustrate the importance of this incompatibility of onsite s -wave pairing with altermagnetism by comparing it with a conventional antiferromagnet in Appendix, which, in contrast, allow for coexisting s -wave pairing.

Next, to show that NN spin-singlet pairing can occur in altermagnets, we focus on d -wave and extended s -wave pairings by setting $V_{sc} = 2$ and $U_{sc} = 0$. In Fig. 3(b), we first show $|M|$ as a function of U . We find that with the competition of NN pairing, the critical U for altermagnetism to appear is shifted to $U = 3.78$, i.e., slightly greater than the critical U in Fig. 2(a). For the superconducting order we find in Fig. 3(d) a large Δ_d^Q for $U \leq 3.77$ with $Q = 0$ as the ground state, i.e., where $|M| = 0$. Within the altermagnetic phase marked by a finite $|M|$, Δ_d^Q still survives over a range $3.78 < U < 4.2$, but now the ground state is at finite Q , meaning a finite-momentum Fulde-Ferrell (FF) state [4] is formed. This result establishes that NN d -wave pairing is feasible in altermagnets, a result again fully in line with Table I. Further increasing U eventually results in $\Delta_d^Q = 0$ due to a large value of $|M|$. We further find that the obtained values of $\Delta_{\text{ext-s}}^Q$ are always very small compared to Δ_d^Q (not shown). We note that this smallness of $\Delta_{\text{ext-s}}^Q$ is not related to altermagnetism but persists

also when $|M| = 0$. Instead, the relative strength of $\Delta_{\text{ext-s}}^Q$ and Δ_d^Q depends on the density-dependent Fermi surface, and for the chosen value of $\rho = 0.9$ per sublattice in this work, $\Delta_{\text{ext-s}}^Q$ is small. Hence, with a different ρ , we have checked that a finite $\Delta_{\text{ext-s}}^Q$ can also be achieved. We note that we here choose to not explicitly calculate results for NNN d_{xy} -wave or spin-triplet pairing symmetries, in order to avoid unnecessary computational complexity resulting from larger matrices and multiple order parameters. However, the similarities of our results in this section and Table I suggest that these pairings will also not appear if calculated self-consistently.

To summarize, in this section we show that interaction-induced altermagnetism also exhibits a spin-sublattice locking, similar to the one present in the archetypical model of Sec. II. Hence, we find exactly the same fundamental constraints on superconducting pairing as in Sec. II. We further explicitly demonstrate these constraints by self-consistently calculating the spin-singlet superconducting order parameters up to NNs along with the magnetization responsible for altermagnetism, also allowing for finite-momentum pairing. Our results demonstrate that the constraints on superconducting pairing symmetries established in Table I are fully generic and apply to widely different models of altermagnetism, including considering finite-momentum pairing.

IV. CONCLUSIONS

In conclusion, in this work we highlight the importance of the coupled spin and sublattice character of the low-energy bands in altermagnets by exploring two different models of a prototypical $d_{x^2-y^2}$ -wave altermagnet, one where the spin arrangement on the crystal alone gives altermagnetism, often treated as an archetypical model of altermagnetism [16], and another where electronic interactions, driving a Fermi surface instability, along with the band structure generates altermagnetism [35]. We uncover that the characteristic spin-sublattice locking in both of these models put the same severe constraints on the nature of the superconducting pairing in altermagnets. In particular, we establish that the most common form of superconductivity, conventional or spin-singlet uniform, or onsite, s -wave pairing is not feasible in altermagnets. We further show that nearest-neighbor equal-spin-triplet p -wave pairing is also not possible to achieve. In fact, considering pairing up to next-nearest-neighbors in real space, our analysis reveals that the only allowed superconducting pairing symmetries in a $d_{x^2-y^2}$ -wave altermagnet are spin-singlet $d_{x^2-y^2}$ -wave and extended s -wave pairing, and mixed spin-triplet p -wave pairing. Longer-range pairing may also be allowed, but results most often in more nodes, resulting in a reduced superconducting condensation energy. The only exception to these constraints is if we also allow for odd-frequency pairing [70,71,79], then uniform s -wave pairing becomes allowed in altermagnets as a spin-triplet odd-frequency superconducting state. Similarly, for odd-frequency superconductivity also spin-singlet p -wave, equal-spin-triplet onsite s - and d_{xy} -wave, and mixed spin-triplet $d_{x^2-y^2}$ - and extended s -wave pairing symmetries become allowed. These results are compactly summarized in Tables I and II. Having used two very different setups to model the origin of al-

termagnetism, our results should be generally applicable to altermagnets described within a two-band low-energy model.

In this work, we only consider the case of a $d_{x^2-y^2}$ -wave altermagnet on a square lattice with a single orbital. However, our analysis can be easily extended to other forms of altermagnets on different lattices and also for multiorbital systems. The key is to carefully consider the characteristic spin-sublattice coupling of altermagnets, which will generate constraints on the superconducting pairing. For example, if the spin-exchange interaction J is smaller, our extracted two-band models might not be fully accurate, but instead a four or more band model has to be analyzed. Moreover, the analysis of this work is very general and does not depend on whether superconductivity is intrinsic to the material or proximity-induced. Hence, our findings are applicable both to systems exhibiting coexistence of superconductivity and altermagnetism, and to hybrid structures, such as altermagnet-superconductor junctions. Thus the results in this work are important to a large class of systems. Possible future directions of our work are many, also including investigations of bound states in altermagnet-superconductor junctions in the same spirit as for antiferromagnet-superconductor junctions [102–104], as well as investigations of pairing symmetries in ferrimagnets where sublattice symmetries are important but in the presence of a net magnetization.

ACKNOWLEDGMENTS

We gratefully acknowledge financial support from the Knut and Alice Wallenberg Foundation through the Wallenberg Academy Fellows program, KAW 2019.0309, and the Swedish Research Council (Vetenskapsrådet) Grant Agreement No. 2022-03963. The computations were enabled by resources provided by the National Academic Infrastructure for Supercomputing in Sweden (NAISS), partially funded by the Swedish Research Council through Grant Agreement No. 2022-06725.

DATA AVAILABILITY

The data are available from the corresponding author (D.C.) upon reasonable request.

APPENDIX: COMPARISON OF ALTERMAGNETISM AND ANTIFERROMAGNETISM

In this Appendix, we compare the possibility of onsite s -wave superconducting pairing in systems with altermagnetism and antiferromagnetism, in order to complement the results in Sec. III and illustrate the stark difference between these two magnetic orders when it comes to their influence

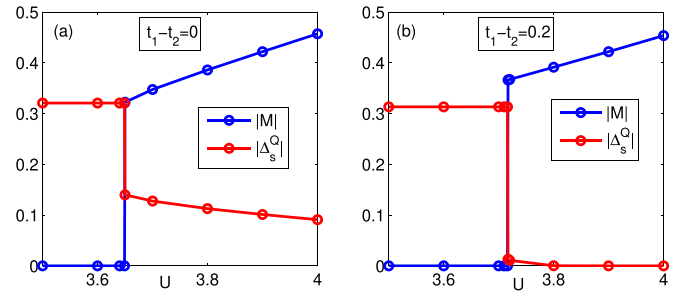


FIG. 4. Magnetization per sublattice $|M|$ (blue) and superconducting order parameter Δ_s^Q (red) as a function of U for $t_1 - t_2 = 0$ (a) and $t_1 - t_2 = 0.2$ (b), giving antiferromagnetism and altermagnetism, respectively.

on superconductivity. Just as in Sec. III, we again study the total Hamiltonian $H = H_0 + H_{U, \text{mf}} + H_{sc, \text{mf}}$ using Eqs. (5), (8), and (12) in a Nambu basis and solve self-consistently for the superconducting order parameter Δ_s^Q , also allowing for finite-momentum pairing, as well as the normal state magnetic order parameter M . To focus only on the s -wave pairing, we set $U_{sc} = 2$ and $V_{sc} = 0$, just as in Figs. 3(a) and 3(c). We here, however, choose the electron density per sublattice $\rho = 0.7$, which is different from Fig. 3, because we here want to have a larger region of U giving relatively small values of $|M|$, such that we can focus on the possibility of onsite s -wave pairing when the magnetization is weak and hence at least putatively allowing for superconductivity. In order to compare altermagnetism and antiferromagnetism we study two cases, one where we set $t_1 = t_2 = 0.3$ giving $t_1 - t_2 = 0$ which gives antiferromagnetic order for finite $|M|$, and the other where we set $t_1 = 0.2$, $t_2 = 0.4$ giving $t_1 - t_2 = 0.2$, which is the same as in Fig. 3 and giving altermagnetic order for finite $|M|$.

In Fig. 4, we plot the magnitude of the magnetism $|M|$ and superconducting order parameter Δ_s^Q as a function of varying U , for antiferromagnetism with $t_1 - t_2 = 0$ (a) and altermagnetism $t_1 - t_2 = 0.2$ (b). As seen in Fig. 4(b), Δ_s^Q becomes insignificantly low as soon as $|M|$ becomes finite for $U \geq 3.717$. This behavior is the same as in Figs. 3(a) and 3(c), but now for a different $\rho = 0.7$. Such an effectively full suppression of Δ_s^Q when $|M|$ becomes finite further elucidates the importance of the spin-sublattice locking in the altermagnet state. In contrast, for the antiferromagnetic state, generated by $t_1 - t_2 = 0$ in Fig. 4(a), there is no sublattice symmetry breaking and no spin-split Fermi surface, and hence spin-sublattice coupling is not a relevant constraint for superconductivity. As a consequence, as clearly seen in Fig. 4(a), Δ_s^Q remains finite even when $|M|$ becomes finite for $U \geq 3.65$. This illustrates that onsite s -wave pairing can easily survive in an antiferromagnet, while easily become forbidden in an altermagnet with strong spin-sublattice coupling.

- [1] D. Scalapino, The case for $d_{x^2-y^2}$ pairing in the cuprate superconductors, *Phys. Rep.* **250**, 329 (1995).
- [2] P. J. Hirschfeld, M. M. Korshunov, and I. I. Mazin, Gap symmetry and structure of Fe-based superconductors, *Rep. Prog. Phys.* **74**, 124508 (2011).

- [3] A. V. Chubukov, D. Pines, and J. Schmalian, A Spin Fluctuation Model for d-Wave Superconductivity, in *Superconductivity Conventional and Unconventional Superconductors*, edited by K. H. Bennemann and J. B. Ketterson (Springer, Berlin, Heidelberg, 2008), pp. 1349–1413.

- [4] P. Fulde and R. A. Ferrell, Superconductivity in a strong spin-exchange field, *Phys. Rev.* **135**, A550 (1964).
- [5] A. I. Larkin and Y. N. Ovchinnikov, Nonuniform state of superconductors, *Zh. Eksp. Teor. Fiz.* **47**, 1136 (1964).
- [6] J. Alicea, New directions in the pursuit of Majorana fermions in solid state systems, *Rep. Prog. Phys.* **75**, 076501 (2012).
- [7] C. Beenakker, Search for majorana fermions in superconductors, *Annu. Rev. Condens. Phys.* **4**, 113 (2013).
- [8] R. Aguado, Majorana quasiparticles in condensed matter, *Riv. Nuovo Cim.* **40**, 523 (2017).
- [9] R. M. Lutchyn, E. P. A. M. Bakkers, L. P. Kouwenhoven, P. Krogstrup, C. M. Marcus, and Y. Oreg, Majorana zero modes in superconductor–semiconductor heterostructures, *Nat. Rev. Mater.* **3**, 52 (2018).
- [10] K. Flensberg, F. von Oppen, and A. Stern, Engineered platforms for topological superconductivity and Majorana zero modes, *Nat. Rev. Mater.* **6**, 944 (2021).
- [11] M. Eschrig and T. Löfwander, Triplet supercurrents in clean and disordered half-metallic ferromagnets, *Nat. Phys.* **4**, 138 (2008).
- [12] J. Linder and J. W. A. Robinson, Superconducting spintronics, *Nat. Phys.* **11**, 307 (2015).
- [13] L. Šmejkal, R. González-Hernández, T. Jungwirth, and J. Sinova, Crystal time-reversal symmetry breaking and spontaneous Hall effect in collinear antiferromagnets, *Sci. Adv.* **6**, eaaz8809 (2020).
- [14] L.-D. Yuan, Z. Wang, J.-W. Luo, E. I. Rashba, and A. Zunger, Giant momentum-dependent spin splitting in centrosymmetric low-Z antiferromagnets, *Phys. Rev. B* **102**, 014422 (2020).
- [15] I. I. Mazin, K. Koepnick, M. D. Johannes, R. González-Hernández, and L. Šmejkal, Prediction of unconventional magnetism in doped FeSb₂, *Proc. Natl. Acad. Sci. USA* **118**, e2108924118 (2021).
- [16] L. Šmejkal, J. Sinova, and T. Jungwirth, Beyond conventional ferromagnetism and antiferromagnetism: A phase with nonrelativistic spin and crystal rotation symmetry, *Phys. Rev. X* **12**, 031042 (2022).
- [17] L. Šmejkal, J. Sinova, and T. Jungwirth, Emerging research landscape of altermagnetism, *Phys. Rev. X* **12**, 040501 (2022).
- [18] S. Hayami, Y. Yanagi, and H. Kusunose, Momentum-dependent spin splitting by collinear antiferromagnetic ordering, *J. Phys. Soc. Jpn.* **88**, 123702 (2019).
- [19] S. Hayami, Y. Yanagi, and H. Kusunose, Bottom-up design of spin-split and reshaped electronic band structures in antiferromagnets without spin-orbit coupling: Procedure on the basis of augmented multipoles, *Phys. Rev. B* **102**, 144441 (2020).
- [20] I. V. Solov'yev, Magneto-optical effect in the weak ferromagnets LaMO₃ (M = Cr, Mn, and Fe), *Phys. Rev. B* **55**, 8060 (1997).
- [21] Y. Noda, K. Ohno, and S. Nakamura, Momentum-dependent band spin splitting in semiconducting MnO₂: A density functional calculation, *Phys. Chem. Chem. Phys.* **18**, 13294 (2016).
- [22] T. Okugawa, K. Ohno, Y. Noda, and S. Nakamura, Weakly spin-dependent band structures of antiferromagnetic perovskite LaMO₃ (M = Cr, Mn, Fe), *J. Phys.: Condens. Matter* **30**, 075502 (2018).
- [23] M. Naka, S. Hayami, H. Kusunose, Y. Yanagi, Y. Motome, and H. Seo, Spin current generation in organic antiferromagnets, *Nat. Commun.* **10**, 4305 (2019).
- [24] Z. Feng, X. Zhou, L. Šmejkal, L. Wu, Z. Zhu, H. Guo, R. González-Hernández, X. Wang, H. Yan, P. Qin, X. Zhang, H. Wu, H. Chen, Z. Meng, L. Liu, Z. Xia, J. Sinova, T. Jungwirth, and Z. Liu, An anomalous Hall effect in altermagnetic ruthenium dioxide, *Nat. Electron.* **5**, 735 (2022).
- [25] R. D. G. Betancourt, J. Zubáć, R. Gonzalez-Hernandez, K. Geishendorf, Z. Šobán, G. Springholz, K. Olejník, L. Šmejkal, J. Sinova, T. Jungwirth, S. T. B. Goennenwein, A. Thomas, H. Reichlová, J. Železný, and D. Kriegner, Spontaneous anomalous Hall effect arising from an unconventional compensated magnetic phase in a semiconductor, *Phys. Rev. Lett.* **130**, 036702 (2023).
- [26] H. Bai, Y. C. Zhang, Y. J. Zhou, P. Chen, C. H. Wan, L. Han, W. X. Zhu, S. X. Liang, Y. C. Su, X. F. Han, F. Pan, and C. Song, Efficient spin-to-charge conversion via altermagnetic spin splitting effect in antiferromagnet RuO₂, *Phys. Rev. Lett.* **130**, 216701 (2023).
- [27] O. Fedchenko, J. Minár, A. Akashdeep, S. W. D'Souza, D. Vasilyev, O. Tkach, L. Odenbreit, Q. Nguyen, D. Kutnyakhov, N. Wind, L. Wenthaus, M. Scholz, K. Rossnagel, M. Hoesch, M. Aeschlimann, B. Stadtmüller, M. Kläui, G. Schönhense, T. Jungwirth, A. B. Hellènes *et al.*, Observation of time-reversal symmetry breaking in the band structure of altermagnetic RuO₂, *Sci. Adv.* **10**, eadj4883 (2024).
- [28] Y.-P. Zhu, X. Chen, X.-R. Liu, Y. Liu, P. Liu, H. Zha, G. Qu, C. Hong, J. Li, Z. Jiang, X.-M. Ma, Y.-J. Hao, M.-Y. Zhu, W. Liu, M. Zeng, S. Jayaram, M. Lenger, J. Ding, S. Mo, K. Tanaka *et al.*, Observation of plaid-like spin splitting in a noncoplanar antiferromagnet, *Nature (London)* **626**, 523 (2024).
- [29] J. Krempaský, L. Šmejkal, S. W. D'Souza, M. Hajlaoui, G. Springholz, K. Uhlířová, F. Alarab, P. C. Constantinou, V. Strocov, D. Usanov, W. R. Pudelko, R. González-Hernández, A. B. Hellènes, Z. Jansa, H. Reichlová, Z. Šobán, R. D. G. Betancourt, P. Wadley, J. Sinova, D. Kriegner *et al.*, Altermagnetic lifting of Kramers spin degeneracy, *Nature (London)* **626**, 517 (2024).
- [30] B. Jiang, M. Hu, J. Bai, Z. Song, C. Mu, G. Qu, W. Li, W. Zhu, H. Pi, Z. Wei, Y. Sun, Y. Huang, X. Zheng, Y. Peng, L. He, S. Li, J. Luo, Z. Li, G. Chen, H. Li *et al.*, Discovery of a metallic room-temperature *d*-wave altermagnet KV₂Se₂O, *Nat. Phys.* **21**, 754 (2025).
- [31] F. Zhang, X. Cheng, Z. Yin, C. Liu, L. Deng, Y. Qiao, Z. Shi, S. Zhang, J. Lin, Z. Liu, M. Ye, Y. Huang, X. Meng, C. Zhang, T. Okuda, K. Shimada, S. Cui, Y. Zhao, G.-H. Cao, S. Qiao *et al.*, Crystal-symmetry-paired spin-valley locking in a layered room-temperature antiferromagnet, *Nat. Phys.* **21**, 760 (2025).
- [32] L. Han, X. Fu, R. Peng, X. Cheng, J. Dai, L. Liu, Y. Li, Y. Zhang, W. Zhu, H. Bai, Y. Zhou, S. Liang, C. Chen, Q. Wang, X. Chen, L. Yang, Y. Zhang, C. Song, J. Liu, and F. Pan, Electrical 180° switching of Néel vector in spin-splitting antiferromagnet, *Sci. Adv.* **10**, eadn0479 (2024).
- [33] L. Bai, W. Feng, S. Liu, L. Šmejkal, Y. Mokrousov, and Y. Yao, Altermagnetism: Exploring new frontiers in magnetism and spintronics, *Adv. Funct. Mater.* **34**, 2409327 (2024).
- [34] I. Mazin, Altermagnetism then and now, *Physics* **17**, 4 (2024).
- [35] T. A. Maier and S. Okamoto, Weak-coupling theory of neutron scattering as a probe of altermagnetism, *Phys. Rev. B* **108**, L100402 (2023).

- [36] V. Leeb, A. Mook, L. Šmejkal, and J. Knolle, Spontaneous formation of altermagnetism from orbital ordering, *Phys. Rev. Lett.* **132**, 236701 (2024).
- [37] B. Brekke, A. Brataas, and A. Sudbø, Two-dimensional altermagnets: Superconductivity in a minimal microscopic model, *Phys. Rev. B* **108**, 224421 (2023).
- [38] M. Roig, A. Kreisel, Y. Yu, B. M. Andersen, and D. F. Agterberg, Minimal models for altermagnetism, *Phys. Rev. B* **110**, 144412 (2024).
- [39] Y. Yu, H.-G. Suh, M. Roig, and D. F. Agterberg, Altermagnetism from coincident Van Hove singularities: Application to κ -Cl, *Nat. Commun.* **16**, 2950 (2025).
- [40] S. Giuli, C. Mejuto-Zaera, and M. Capone, Altermagnetism from interaction-driven itinerant magnetism, *Phys. Rev. B* **111**, L020401 (2025).
- [41] C. Wu, K. Sun, E. Fradkin, and S.-C. Zhang, Fermi liquid instabilities in the spin channel, *Phys. Rev. B* **75**, 115103 (2007).
- [42] I. I. Mazin, Notes on altermagnetism and superconductivity, *AAPPS Bull.* **35**, 18 (2025).
- [43] R. Soto-Garrido and E. Fradkin, Pair-density-wave superconducting states and electronic liquid-crystal phases, *Phys. Rev. B* **89**, 165126 (2014).
- [44] D. Chakraborty and A. M. Black-Schaffer, Zero-field finite-momentum and field-induced superconductivity in altermagnets, *Phys. Rev. B* **110**, L060508 (2024).
- [45] S. Sumita, M. Naka, and H. Seo, Fulde-ferrell-larkin-ovchinnikov state induced by antiferromagnetic order in κ -type organic conductors, *Phys. Rev. Res.* **5**, 043171 (2023).
- [46] A. Bose, S. Vadnais, and A. Paramakanti, Altermagnetism and superconductivity in a multiorbital t - J model, *Phys. Rev. B* **110**, 205120 (2024).
- [47] G. Sim and J. Knolle, Pair density waves and supercurrent diode effect in altermagnets, *Phys. Rev. B* **112**, L020502 (2025).
- [48] S. Hong, M. J. Park, and K.-M. Kim, Unconventional p-wave and finite-momentum superconductivity induced by altermagnetism through the formation of Bogoliubov Fermi surface, *Phys. Rev. B* **111**, 054501 (2025).
- [49] M. Uchida, T. Nomoto, M. Musashi, R. Arita, and M. Kawasaki, Superconductivity in uniquely strained RuO_2 films, *Phys. Rev. Lett.* **125**, 147001 (2020).
- [50] L. Šmejkal, A. Marmodoro, K.-H. Ahn, R. González-Hernández, I. Turek, S. Mankovsky, H. Ebert, S. W. D'Souza, O. Šípr, J. Sinova, and T. Jungwirth, Chiral magnons in altermagnetic RuO_2 , *Phys. Rev. Lett.* **131**, 256703 (2023).
- [51] I. Mazin, R. González-Hernández, and L. Šmejkal, Induced monolayer altermagnetism in $\text{MnP}(\text{S}, \text{Se})_3$ and FeSe , *arXiv:2309.02355*.
- [52] S.-B. Zhang, L.-H. Hu, and T. Neupert, Finite-momentum cooper pairing in proximitized altermagnets, *Nat. Commun.* **15**, 1801 (2024).
- [53] J. A. Ouassou, A. Brataas, and J. Linder, dc Josephson effect in altermagnets, *Phys. Rev. Lett.* **131**, 076003 (2023).
- [54] M. Papaj, Andreev reflection at the altermagnet-superconductor interface, *Phys. Rev. B* **108**, L060508 (2023).
- [55] H.-P. Sun, S.-B. Zhang, C.-A. Li, and B. Trauzettel, Field-enhanced critical current and tunable skewness in altermagnetic Josephson junctions, *Phys. Rev. B* **111**, 165406 (2025).
- [56] B. Lu, K. Maeda, H. Ito, K. Yada, and Y. Tanaka, φ Josephson junction induced by altermagnetism, *Phys. Rev. Lett.* **133**, 226002 (2024).
- [57] C. W. J. Beenakker and T. Vakhitel, Phase-shifted Andreev levels in an altermagnet Josephson junction, *Phys. Rev. B* **108**, 075425 (2023).
- [58] H. G. Giil and J. Linder, Superconductor-altermagnet memory functionality without stray fields, *Phys. Rev. B* **109**, 134511 (2024).
- [59] S. Banerjee and M. S. Scheurer, Altermagnetic superconducting diode effect, *Phys. Rev. B* **110**, 024503 (2024).
- [60] D. Chakraborty and A. M. Black-Schaffer, Perfect superconducting diode effect in altermagnets, *Phys. Rev. Lett.* **135**, 026001 (2025).
- [61] H. G. Giil, B. Brekke, J. Linder, and A. Brataas, Quasi-classical theory of superconducting spin-splitter effects and spin-filtering via altermagnets, *Phys. Rev. B* **110**, L140506 (2024).
- [62] A. A. Zyuzin, Magnetoelectric effect in superconductors with d -wave magnetization, *Phys. Rev. B* **109**, L220505 (2024).
- [63] Y.-X. Li, Realizing tunable higher-order topological superconductors with altermagnets, *Phys. Rev. B* **109**, 224502 (2024).
- [64] S. A. A. Ghorashi, T. L. Hughes, and J. Cano, Altermagnetic Routes to Majorana modes in zero net magnetization, *Phys. Rev. Lett.* **133**, 106601 (2024).
- [65] D. Zhu, Z.-Y. Zhuang, Z. Wu, and Z. Yan, Topological superconductivity in two-dimensional altermagnetic metals, *Phys. Rev. B* **108**, 184505 (2023).
- [66] P. O. Sukhachov, E. W. Hodt, and J. Linder, Thermoelectric effect in altermagnet-superconductor junctions, *Phys. Rev. B* **110**, 094508 (2024).
- [67] Y. Nagae, A. P. Schnyder, and S. Ikegaya, Spin-polarized specular Andreev reflections in altermagnets, *Phys. Rev. B* **111**, L100507 (2025).
- [68] M. Sigrist and K. Ueda, Phenomenological theory of unconventional superconductivity, *Rev. Mod. Phys.* **63**, 239 (1991).
- [69] C. Triola and A. V. Balatsky, Odd-frequency superconductivity in driven systems, *Phys. Rev. B* **94**, 094518 (2016).
- [70] J. Linder and A. V. Balatsky, Odd-frequency superconductivity, *Rev. Mod. Phys.* **91**, 045005 (2019).
- [71] C. Triola, J. Cayao, and A. M. Black-Schaffer, The role of odd-frequency pairing in multiband superconductors, *Ann. Phys. (NY)* **532**, 1900298 (2020).
- [72] Alternatively, we may treat the sublattice index as an orbital degree of freedom [81] within an extended unit cell consisting of both an A and B site. This results in instead using the condition $SPOT = -1$, where O stands for orbital parity, while the spatial parity becomes $P = 1$ within the unit cell. This results in the same conclusions as in Tables I and II but the resulting spatial symmetries (form factors) are less straightforward to extract.
- [73] D. Wickramaratne, S. Khmelevskiy, D. F. Agterberg, and I. I. Mazin, Ising superconductivity and magnetism in NbSe_2 , *Phys. Rev. X* **10**, 041003 (2020).
- [74] T. Löthman and A. M. Black-Schaffer, Universal phase diagrams with superconducting domes for electronic flat bands, *Phys. Rev. B* **96**, 064505 (2017).
- [75] V. L. Berezinskii, New model of the anisotropic phase of superfluid He^3 , *Pisma Zh. Eksp. Teor. Fiz.* **20**, 628 (1974).

- [76] T. R. Kirkpatrick and D. Belitz, Disorder-induced triplet superconductivity, *Phys. Rev. Lett.* **66**, 1533 (1991).
- [77] A. Balatsky and E. Abrahams, New class of singlet superconductors which break the time reversal and parity, *Phys. Rev. B* **45**, 13125 (1992).
- [78] J. R. Schrieffer, A. V. Balatsky, E. Abrahams, and D. J. Scalapino, Odd frequency pairing in superconductors, *J. Supercond.* **7**, 501 (1994).
- [79] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Odd triplet superconductivity and related phenomena in superconductor-ferromagnet structures, *Rev. Mod. Phys.* **77**, 1321 (2005).
- [80] T. Yokoyama, Y. Tanaka, and N. Nagaosa, Anomalous Meissner effect in a normal-metal-superconductor junction with a spin-active interface, *Phys. Rev. Lett.* **106**, 246601 (2011).
- [81] A. M. Black-Schaffer and A. V. Balatsky, Odd-frequency superconducting pairing in multiband superconductors, *Phys. Rev. B* **88**, 104514 (2013).
- [82] M. Alidoust, K. Halterman, and J. Linder, Meissner effect probing of odd-frequency triplet pairing in superconducting spin valves, *Phys. Rev. B* **89**, 054508 (2014).
- [83] Y. Tanaka, M. Sato, and N. Nagaosa, Symmetry and topology in superconductors –odd-frequency pairing and edge states–, *J. Phys. Soc. Jpn.* **81**, 011013 (2012).
- [84] J. Cayao, C. Triola, and A. M. Black-Schaffer, Odd-frequency superconducting pairing in one-dimensional systems, *Eur. Phys. J. Spec. Top.* **229**, 545 (2020).
- [85] D. Chakraborty and A. M. Black-Schaffer, Odd-frequency pair density wave correlations in underdoped cuprates, *New J. Phys.* **23**, 033001 (2021).
- [86] D. Chakraborty and A. M. Black-Schaffer, Quasiparticle interference as a direct experimental probe of bulk odd-frequency superconducting pairing, *Phys. Rev. Lett.* **129**, 247001 (2022).
- [87] D. Chakraborty and A. M. Black-Schaffer, Interplay of finite-energy and finite-momentum superconducting pairing, *Phys. Rev. B* **106**, 024511 (2022).
- [88] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Josephson current in superconductor-ferromagnet structures with a nonhomogeneous magnetization, *Phys. Rev. B* **64**, 134506 (2001).
- [89] A. I. Buzdin, Proximity effects in superconductor-ferromagnet heterostructures, *Rev. Mod. Phys.* **77**, 935 (2005).
- [90] A. Di Bernardo, S. Diesch, Y. Gu, J. Linder, G. Divitini, C. Ducati, E. Scheer, M. G. Blamire, and J. W. A. Robinson, Signature of magnetic-dependent gapless odd frequency states at superconductor/ferromagnet interfaces, *Nat. Commun.* **6**, 8053 (2015).
- [91] A. Di Bernardo, Z. Salman, X. L. Wang, M. Amado, M. Egilmez, M. G. Flokstra, A. Suter, S. L. Lee, J. H. Zhao, T. Prokscha, E. Morenzoni, M. G. Blamire, J. Linder, and J. W. A. Robinson, Intrinsic paramagnetic meissner effect due to *s*-wave odd-frequency superconductivity, *Phys. Rev. X* **5**, 041021 (2015).
- [92] S. H. Jacobsen, I. Kulagina, and J. Linder, Controlling superconducting spin flow with spin-flip immunity using a single homogeneous ferromagnet, *Sci. Rep.* **6**, 23926 (2016).
- [93] J. A. Ouassou, S. H. Jacobsen, and J. Linder, Conservation of spin supercurrents in superconductors, *Phys. Rev. B* **96**, 094505 (2017).
- [94] V. Perrin, F. L. N. Santos, G. C. Ménard, C. Brun, T. Cren, M. Civelli, and P. Simon, Unveiling odd-frequency pairing around a magnetic impurity in a superconductor, *Phys. Rev. Lett.* **125**, 117003 (2020).
- [95] L. G. Johnsen, S. H. Jacobsen, and J. Linder, Magnetic control of superconducting heterostructures using compensated antiferromagnets, *Phys. Rev. B* **103**, L060505 (2021).
- [96] E. H. Fyhn, A. Brataas, A. Qaiumzadeh, and J. Linder, Superconducting proximity effect and long-ranged triplets in dirty metallic antiferromagnets, *Phys. Rev. Lett.* **131**, 076001 (2023).
- [97] T. Salamone, M. Skjærpe, H. G. Hugdal, M. Amundsen, and S. H. Jacobsen, Interface probe for antiferromagnets using geometric curvature, *Phys. Rev. B* **109**, 094508 (2024).
- [98] J. Schrieffer, *Theory of Superconductivity* (WA Benjamin, Inc., New York, 1964).
- [99] G. D. Mahan, *Many-Particle Physics* (Springer, New York, 2000).
- [100] X. Wu, D. Chakraborty, A. P. Schnyder, and A. Greco, Crossover between electron-electron and electron-phonon mediated pairing on the kagome lattice, *Phys. Rev. B* **109**, 014517 (2024).
- [101] K. Fossheim and A. Sudbø, *Superconductivity: Physics and Applications* (Wiley, Hoboken, NJ, 2005).
- [102] I. V. Bobkova, P. J. Hirschfeld, and Y. S. Barash, Spin-dependent quasiparticle reflection and bound states at interfaces with itinerant antiferromagnets, *Phys. Rev. Lett.* **94**, 037005 (2005).
- [103] B. M. Andersen, I. V. Bobkova, P. J. Hirschfeld, and Y. S. Barash, Bound states at the interface between antiferromagnets and superconductors, *Phys. Rev. B* **72**, 184510 (2005).
- [104] A. Kamra, A. Rezaei, and W. Belzig, Spin splitting induced in a superconductor by an antiferromagnetic insulator, *Phys. Rev. Lett.* **121**, 247702 (2018).