

Chiral magnetic excitations and domain textures of *g*-wave altermagnets

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Altermagnets (AMs) constitute a novel class of spin-compensated materials in which opposite-spin sublattices are connected by a crystal rotation, causing their electronic isoenergy surfaces to be spin split. While cubic and tetragonal crystal symmetries tend to produce AMs in which the splitting of *electronic* isoenergy surfaces has *d*-wave symmetry, hexagonal AMs, such as CrSb and MnTe, are *g*-wave AMs. Here we investigate the purely *magnetic* modes and spin textures of *g*-wave AMs and show that they are drastically different for easy-axial (CrSb) and easy-planar (MnTe) materials. We show that in CrSb the splitting of the chiral magnon branches possesses *g*-wave symmetry, with each branch carrying a fixed momentum-independent magnetic moment. The altermagnetic splitting is not affected by the easy-axial anisotropy and is the same as that in the nonrelativistic limit. The magnon splitting of MnTe, however, does not strictly possess *g*-wave symmetry due to its easy-planar anisotropy. Instead, the magnetic moment of each branch becomes momentum dependent, with a distribution that is of *g*-wave symmetry. To generalize the concept of the altermagnetic splitting beyond the nonrelativistic limit, we introduce an alternative, directly observable splitting parameter that encompasses both the magnon eigenenergy and its magnetic moment (a component along the ground-state Néel vector), and exhibits *g*-wave symmetry in both easy-axial and easy-planar cases. The associated altermagnetic domain walls in easy-axial CrSb possess a net magnetization with an amplitude that depends on their orientation.

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I. INTRODUCTION

The observation of unconventional time-reversal symmetry breaking in several collinear compensated magnets [1–5] has led to the recent discovery of altermagnets (AMs) [6,7], a new third class of collinear compensated magnetism rigorously delimited and classified by spin symmetries. AMs exhibit unconventional spin order with *d*, *g*, or *i* partial wave parity in the band structure. This anisotropic order gives rise to a series of unusual electric and optical properties, including the possibility of an anomalous Hall effect governed by the direction of the antiferromagnetic Néel vector [1,8,9]. Recently, it has also been established that altermagnetism has very interesting repercussions for other types of charge neutral magnetic elementary excitations. In *d*-wave AMs, with for instance tetragonal rutile structures, a *d*-wave splitting of the *magnon*

branches arises, which induces several novel physical effects: a net magnetization induced by nonuniformities of the Néel vector [10], deformation and Walker breakdown for moving domain walls [10], fluctuation-induced piezomagnetism [11,12], spin currents induced by the temperature gradients [12], and a curvature-induced net magnetization in AM films [13].

The experimentally very fertile hexagonal altermagnetic materials do not exhibit *d*- but *g*-wave symmetry, in particular metallic CrSb [7,14–18], semiconducting MnTe [7,19–22], and semimetallic VNb₃S₆ [7]. These hexagonal AMs with NiAs-type structure belong to the crystallographic group 6/*mmm*. Their *g*-wave altermagnetic splitting of the electron bands has been convincingly demonstrated experimentally by angle-resolved photoemission spectroscopy, in both MnTe [20–22] and CrSb [14,15,23], which in addition harbors electronic Weyl nodes and topological surface states [15].

Here we show that the purely magnetic excitations and spin textures of these *g*-wave AMs have properties very distinct from *d*-wave ones. Depending on the magnetocrystalline anisotropy being easy-axial (as for CrSb) or easy-planar (as for MnTe) magnon spectra are drastically different (see Fig. 1). While the splitting of the magnon bands of CrSb is not affected by the anisotropy and possesses a *g*-wave symmetry,

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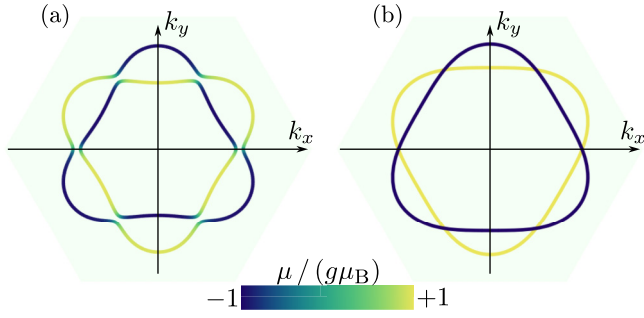


FIG. 1. Schematic representation of the isolines of constant energy $\omega_v = \text{const}$ for MnTe (a) and CrSb (b) made for a constant $k_z > 0$. Distribution of the amplitude of the magnon magnetic moment μ along the isolines is shown by the color scheme. In both cases $\mu \parallel \mathbf{n}_0$ where $\mathbf{n}_0$ is the ground-state Néel vector. For details of the magnon magnetic moment computation, see Sec. III B. The green background hexagon indicates the size of the first Brillouin zone.

as for the nonrelativistic case [see Fig. 1(b)], the splitting of the magnon bands of MnTe is strongly affected by the anisotropy and, strictly speaking, does not possess g -wave symmetry [see Fig. 1(a)] because of the presence of a finite gap between the branches at any momentum.

We will show that each chiral magnon branch in CrSb carries a quantized magnetic moment that is parallel or antiparallel to the ground-state Néel vector $\mathbf{n}_0$, which points along its easy axis. Also, magnons in MnTe carry a magnetic moment μ collinear to the $\mathbf{n}_0$ oriented in plane, but the magnitude and orientation of μ depend on momentum [see Fig. 1(a)]. The magnetic moment in MnTe is an altermagnetic property and is not expected for simple conventional easy-planar antiferromagnets. For each magnon branch, the distribution of the magnon magnetic moment within the first Brillouin zone possesses the g -wave symmetry. We show that in spite of this difference in the properties of the chiral magnons for CrSb and MnTe, the presence of the magnon moment allows the definition of an universal g -wave symmetry characteristic

$$\lambda = \frac{1}{\omega_0 g \mu_B} \sum_{\sigma=\pm} (\mu_{\sigma})_{\text{gs}} \omega_{\sigma} \quad (1)$$

that comprises both cases and generalizes the concept of spin splitting of altermagnetic magnon bands beyond the non-relativistic limit. Here σ numerates the magnon branches, $(\mu_{\sigma})_{\text{gs}} = \mu_{\sigma} \cdot \mathbf{n}_0$ is the magnon magnetic moment along the ground state, ω_0 is some characteristic frequency, $g \approx 2$ is the electron g factor, and μ_B is the Bohr magneton.

From analyzing the corresponding AM continuum Hamiltonian we find that magnetic domain wall (DW) textures in CrSb possess a net magnetization, in analogy to d -wave altermagnets [10], with an amplitude that depends on the DW orientation in the crystal, and we determined 12 orientations that correspond to the maximal magnetization.

II. MAGNETIC HAMILTONIAN AND ITS GROUND STATES

The magnetic structure of CrSb and MnTe consists of ferromagnetic (001) planes which are coupled

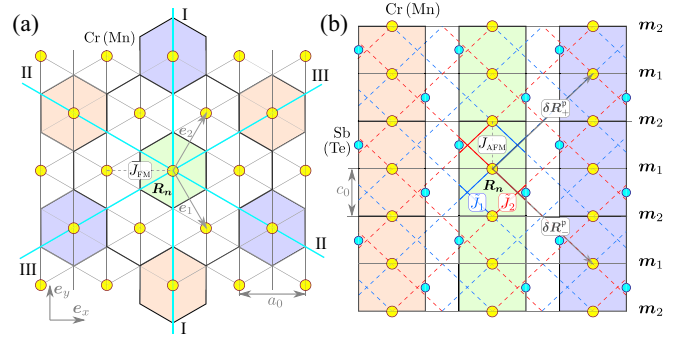


FIG. 2. Schematic crystal structure and exchange interactions in hexagonal CrSb and MnTe-type altermagnets. (a) The (001) plane of magnetic atoms (yellow disks: Cr or Mn) which form a triangular lattice with basis $\mathbf{e}_1$, $\mathbf{e}_2$. Thin lines connect next-nearest neighbors while thick lines border the hexagonal 2D Wigner-Seitz cells. Ferromagnetic Heisenberg exchange of strength J_{FM} couples nearest neighbors in each layer. (b) Cross section of one of the vertical planes I, II, or III. The antiferromagnetic exchange J_{AFM} couples nearest magnetic atoms in two neighboring layers at distance c_0 . Altermagnetic Heisenberg bonds $\tilde{J}_{1(2)}$ are shown by blue (red) lines. The structure of the altermagnetic bonds is unique for all planes I, II, and III if the orientation of the cross section (b) is consistent with the coloring of the unit cells (orange, green, magenta).

antiferromagnetically (see Fig. 2). Their magnetic Hamiltonian comprises five contributions: $\mathcal{H} = \mathcal{H}_{\text{AFM}} + \mathcal{H}_{\text{FM}} + \mathcal{H}_{\text{ALT}} + \mathcal{H}_{\text{AN}} + \mathcal{H}_z$. Here $\mathcal{H}_{\text{AFM}}$ is the antiferromagnetic Heisenberg exchange interaction between the nearest (001) planes illustrated by the bond J_{AFM} in Fig. 2(b). $\mathcal{H}_{\text{FM}}$ takes into account the nearest-neighbor ferromagnetic exchange within each (001) plane [see the bond J_{FM} in Fig. 2(a)]. The altermagnetic properties are encoded in $\mathcal{H}_{\text{ALT}}$ employing the additional Heisenberg exchanges of strengths $\tilde{J}_{1,2}$ shown in Fig. 2(b) by red and blue lines. Orientations of the altermagnetic bonds are determined by the displacement vectors $\delta \mathbf{R}_{\pm}^{\mathbf{p}}$ with $\mathbf{p} \in \{\text{i, ii, iii}\}$ denoting the cross-section plane comprising the displacement vector [see cyan lines in Fig. 2(a)]. Here $\delta \mathbf{R}_{\pm}^{\text{i}} = -\mathbf{e}_1 + \mathbf{e}_2 \pm \mathbf{e}_3$, $\delta \mathbf{R}_{\pm}^{\text{ii}} = 2\mathbf{e}_1 + \mathbf{e}_2 \pm \mathbf{e}_3$, $\delta \mathbf{R}_{\pm}^{\text{iii}} = -\mathbf{e}_1 - 2\mathbf{e}_2 \pm \mathbf{e}_3$ with $\mathbf{e}_1 = \frac{a_0}{2}(\mathbf{e}_x - \sqrt{3}\mathbf{e}_y)$, $\mathbf{e}_2 = \frac{a_0}{2}(\mathbf{e}_x + \sqrt{3}\mathbf{e}_y)$, and $\mathbf{e}_3 = 2c_0\mathbf{e}_z$ being the basis vectors. Note the very different relative locations of the red and blue bonds concerning the nonmagnetic atoms Sb or Te indicated by blue dots: red bonds pass in close proximity to the nonmagnetic atoms. Due to the superexchange interaction realized through the nonmagnetic atoms we have $\tilde{J}_1 \neq \tilde{J}_2$ [24]. In the following we introduce the symmetrization $\tilde{J}_1 = \tilde{J} - \delta\tilde{J}$, $\tilde{J}_2 = \tilde{J} + \delta\tilde{J}$ and show that the altermagnetic effects are proportional to $\delta\tilde{J}$. The explicit expressions for the exchange contributions $\mathcal{H}_{\text{AFM}}$, $\mathcal{H}_{\text{FM}}$, and $\mathcal{H}_{\text{ALT}}$ are presented in Eqs. (A1), (A2), and (A3), respectively. For details, see Appendix A. Additionally, we take into account the uniaxial anisotropy $\mathcal{H}_{\text{AN}} = -K \sum_{\mathbf{R}_n} [m_z^2(\mathbf{R}_n) + m_z^2(\mathbf{R}_n + c_0\mathbf{e}_z)]$ with $K > 0$ and $K < 0$ for CrSb and MnTe, respectively. Here $\mathbf{m}_v(\mathbf{r}_n)$ with $v = 1, 2$ is a unit vector showing the direction of the magnetic moment of v th sublattice located in position $\mathbf{r}_n$. The lattice vectors $\mathbf{R}_n = n_1\mathbf{e}_1 + n_2\mathbf{e}_2 + n_3\mathbf{e}_3$ with $n_i \in \mathbb{Z}$ numerate the hexagonal unit cells the primitive magnetic unit cells in the form of hexagonal

TABLE I. Parameters of the discrete model estimated for CrSb and MnTe. The values for the exchange and anisotropy constants of CrSb and MnTe are based on our own DFT calculations (see Appendix B) and taken from Ref. [24], respectively.

	J_{AFM} (meV)	J_{FM} (meV)	$\tilde{J}$ (meV)	$\delta\tilde{J}$ (meV)	K (meV)	μ_s	a_0 (Å)	c_0 (Å)
CrSb	35.86	13.53	0.5	0.5	0.49	$3 \mu_B$ [25]	4.12 [26]	2.73 [26]
MnTe	3.99	0.12	0.023	0.045	-0.048 [24]	$5 \mu_B$ [27]	4.17 [28,29]	3.375 [28,29]

prisms bordered by thick lines in Fig. 2. The interaction with external magnetic field $\mathbf{B} = B\mathbf{e}_z$ is captured by the Hamiltonian $\mathcal{H}_z = -B\mu_s \sum_{\mathbf{R}_n} [m_{1z}(\mathbf{R}_n) + m_{2z}(\mathbf{R}_n + c_0\mathbf{e}_z)]$, where μ_s is magnetic moment of a magnetic atom.

Parameters of the introduced model are listed in Table I. This model is minimal in the sense that further (longer-range) magnetic coupling might be present. However, these will not change the AM symmetries nor the mathematical structure of low-energy continuum description that we will derive next, in order to investigate magnetic domain wall properties. Since the antiferromagnetic exchange strongly dominates the rest of the interactions for both CrSb and MnTe, the ground state is a uniform AFM collinear magnetization oriented parallel and perpendicular to the anisotropy axis [001] for CrSb and MnTe, respectively. An applied magnetic field $\mathbf{B} = B\mathbf{e}_z$ does not change the ground state of CrSb $\mathbf{m}_v^0 = (-1)^{v-1}\mathbf{e}_z$, if $|B| < B_{\text{sf}}$ where $B_{\text{sf}} \approx \sqrt{8KJ_{\text{AFM}}/\mu_s} \approx 68$ T is the spin-flop field.¹ Since in our case $K \ll J_{\text{AFM}}$, the field region in the vicinity of B_{sf} which corresponds to the metastable collinear state [30] is negligibly narrow.

In the case of MnTe, the applied field leads to the canting of the magnetic moments resulting in the net magnetization along the field. Thus, the ground state for MnTe is $\mathbf{m}_v^0 = (-1)^{v-1} \sin \theta_0 (\mathbf{e}_x \cos \phi_0 + \mathbf{e}_y \sin \phi_0) + \cos \theta_0 \mathbf{e}_z$, where ϕ_0 is an arbitrary constant and $\cos \theta_0 = B/B_0$ with $B_0 \approx 4J_{\text{AFM}}/\mu_s$ being the field of spin flip. Here we neglect possible weak in-plane anisotropies which may fix the orientation of $\mathbf{m}_v^0$ in a real crystal.

III. DISPERSION AND POLARIZATION OF SPIN-WAVE EIGENMODES

Dynamics of each magnetic moment $\mathbf{m}_v(\mathbf{R}_n)$ is described by the Landau-Lifshitz equation

$$\dot{\mathbf{m}}_v(\mathbf{R}_n) = \frac{\gamma}{\mu_s} \left[\mathbf{m}_v(\mathbf{R}_n) \times \frac{\partial \mathcal{H}}{\partial \mathbf{m}_v(\mathbf{R}_n)} \right], \quad (2)$$

where $\gamma = g\mu_B/\hbar > 0$ is the electron gyromagnetic ratio. The set of equations (2) coupled via Hamiltonian $\mathcal{H}$ describes the collective dynamics of all magnetic moments. To describe the dynamics of deviations from a ground state $\mathbf{m}_v^0 = \sin \Theta_v (\mathbf{e}_x \cos \Phi_v + \mathbf{e}_y \sin \Phi_v) + \cos \Theta_v \mathbf{e}_z$, it is convenient to introduce the complex-valued function ψ_v [31]:

$$\mathbf{m}_v = \mathbf{m}_v^0 (1 - |\psi_v|^2) + \sqrt{2 - |\psi_v|^2} (\mathbf{T}_v \psi_v + \mathbf{T}_v^* \psi_v^*), \quad (3)$$

where the form of null vector $\mathbf{T}_v = \frac{1}{2}(\cos \Theta_v \cos \Phi_v - i \sin \Theta_v) \mathbf{e}_x + \frac{1}{2}(\cos \Theta_v \sin \Phi_v + i \cos \Theta_v) \mathbf{e}_y - \frac{1}{2} \sin \Theta_v \mathbf{e}_z$

guarantees fulfillment of the constraint $|\mathbf{m}_v| = 1$.² Note that for the particular case of the CrSb ground state, Eq. (3) becomes a classical analog of the Holstein-Primakoff representation [32]. In terms of ψ_v functions, the Landau-Lifshitz equations (2) take the Schrödinger-type form

$$i\dot{\psi}_v(\mathbf{R}_n) = \frac{\gamma}{\mu_s} \frac{\partial \mathcal{H}}{\partial \psi_v^*(\mathbf{R}_n)}. \quad (4)$$

A. Magnon dispersion relation

The linearization of Eqs. (4) with respect to small ψ_v and the subsequent solution of the eigenvalue problem for a periodic lattice enables us to obtain the dispersion relation for CrSb (see Appendix C):

$$\omega_{\pm}^{\text{CrSb}} = \omega_0 \left[\sqrt{(F_k + \kappa)^2 - \cos^2(k_z c_0)} \pm \delta \varepsilon \Omega_k^- \right] \pm \gamma B, \quad (5)$$

where $F_k = 1 + \eta \Omega_k^{\text{FM}} - \varepsilon \Omega_k^+$. The dispersions $\Omega_k^{\text{FM}} = \sin^2(\mathbf{k} \cdot \mathbf{e}_1/2) + \sin^2(\mathbf{k} \cdot \mathbf{e}_2/2) + \sin^2(\mathbf{k} \cdot (\mathbf{e}_1 + \mathbf{e}_2)/2)$, and $\Omega_k^{\pm} = \sum_{\mathbf{p} \in \{\text{i, ii, iii}\}} [\sin^2(\mathbf{k} \cdot \delta \mathbf{R}_{\mathbf{p}}^{\pm}/2) \pm \sin^2(\mathbf{k} \cdot \delta \mathbf{R}_{\mathbf{p}}^{\mp}/2)]$ originate from the ferromagnetic and altermagnetic contributions, respectively. The parameters are defined as $\omega_0 = 2\gamma J_{\text{AFM}}/\mu_s$, $\kappa = K/J_{\text{AFM}}$, $\eta = 4J_{\text{FM}}/J_{\text{AFM}}$, $\varepsilon = 2\tilde{J}/J_{\text{AFM}}$, and $\delta \varepsilon = 2\delta\tilde{J}/J_{\text{AFM}}$. In the vicinity of Γ point, dispersion relation (5) simplifies to

$$\omega_{\pm}^{\text{CrSb}} \approx \sqrt{\omega_{\text{AFMR}}^2 + \mathbf{c}_{\perp}^2 k_{\perp}^2 + \mathbf{c}_z^2 k_z^2} + O(k^4) \pm [\Lambda k_z k_y (k_y^2 - 3k_x^2) + \gamma B]. \quad (6)$$

Here $k_{\perp}^2 = k_x^2 + k_y^2$ and $\omega_{\text{AFMR}} = \gamma B_{\text{sf}} \approx 1.2 \times 10^{13}$ rad/s (1.9 THz) is the frequency of the uniform antiferromagnetic resonance. $\mathbf{c}_{\perp} = \omega_{\text{AFMR}} \ell_{\perp}$ and $\mathbf{c}_z = \omega_{\text{AFMR}} \ell_z$ are the magnon speeds in xy plane and along z axis, respectively, where the typical length scales are $\ell_{\perp} = a_0[(3J_{\text{FM}} - 9\tilde{J})/(2K)]^{1/2} \approx 2.5$ nm and $\ell_z = c_0[(J_{\text{AFM}} - 24\tilde{J})/(2K)]^{1/2} \approx 1.3$ nm. Parameter $\Lambda = 3\sqrt{3} \frac{\gamma}{\mu_s} \delta\tilde{J} a_0^3 c_0$ represents the altermagnetic strength and determines the splitting between branches. The latter is of fourth order in k , in contrast to the second-order splitting in d -wave altermagnets [10].

The magnon spectra in CrSb have the same properties as the electron spectra in this material in terms of spin splitting of the dispersive branches [14–18]. For $B = 0$, the altermagnetism, which manifests through the parameter $\delta\tilde{J}$, lifts the Kramers' degeneracy of the magnon branches (see the upper row in Fig. 3). Note that the surfaces of constant energies for magnons shown in Fig. 3(c) are similar to

¹Because of a large estimated value of B_{sf} , we do not consider the spin-flop phase of CrSb in this paper.

²One should take into account the following properties: $\mathbf{m}_v^0 \cdot \mathbf{T}_v = 0$, $\mathbf{T}_v \cdot \mathbf{T}_v = 0$, and $\mathbf{T}_v \cdot \mathbf{T}_v^* = \frac{1}{2}$.

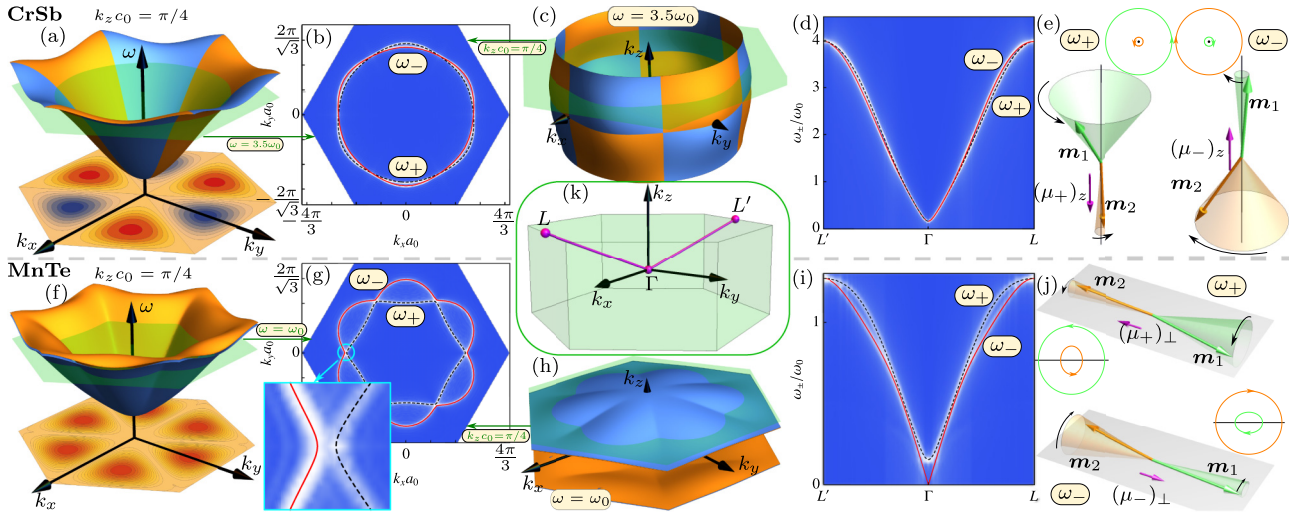


FIG. 3. Comparison of the magnon spectra (5) for CrSb (top row) and (7) for MnTe (bottom row). The figures are built for the values of the parameters listed in Table I and for $B = 0$. (a), (f) Show the evolution of two branches ω_+ (orange) and ω_- (blue) within the first Brillouin zone (1BZ) for $k_z = \text{const}$. The value of the splitting $\Delta\omega = \omega_+ - \omega_-$ is shown by the color coding at the bottom. Surfaces of constant energy $\omega = \text{const}$ are shown on (c) and (h) for CrSb and MnTe, respectively. (b), (g) Aggregate panels (a) and (c) and (f) and (h), respectively, showing the surfaces of the constant energy for the constant k_z . On (b), (d), (g), and (i), the analytical dispersion (lines) is compared to the spectra extracted from the spin-lattice simulations (color density). (k) Shows the 1BZ and the path $L'\Gamma L$ along which we computed the dispersions presented on (d) and (i). (e), (j) Show precession of the neighboring magnetic moments of different sublattices for a spin wave with wave vector $(k_x a_0, k_y a_0, k_z c_0) = (0, \pi/\sqrt{3}, \pi/4)$ for the case of CrSb and MnTe, respectively. The insets show the trajectories swept by the ends of the vectors $\mathbf{m}_\nu$.

the Fermi surfaces recently computed for electrons in CrSb [15,18]. The magnon branches of CrSb possess splitting of a bulk g -wave symmetry: $\Delta\omega_{\mathbf{k}}^{\text{CrSb}} = 2\omega_0 \delta\epsilon \Omega_{\mathbf{k}}^- = -\frac{16\gamma \delta\tilde{J}}{\mu_s} \times \sin(2k_z c_0) \sin(\frac{\sqrt{3}}{2} k_y a_0) [\cos(\frac{3}{2} k_x a_0) - \cos(\frac{\sqrt{3}}{2} k_y a_0)]$ (see Fig. 4 which demonstrates the similarity of the surfaces of constant splitting $\Delta\omega_{\mathbf{k}}^{\text{CrSb}} = \pm \text{const}$ with a g orbital of a hydrogen atom). The splitting is absent if the $\mathbf{k}$ vector lies in either the horizontal plane (001) or in one of three vertical nodal planes containing vector $\mathbf{e}_1$, or $\mathbf{e}_2$ or $\mathbf{e}_1 + \mathbf{e}_2$ (see Fig. 2). The maximal splitting (6.9 meV) is achieved when the $\mathbf{k}$ vector lies in one of the vertical planes I, II or III (see Fig. 2) making angle $\vartheta_{\text{max}} = \arctan \frac{16c_0}{3\sqrt{3}a_0} \approx 63.9^\circ$ with the vertical

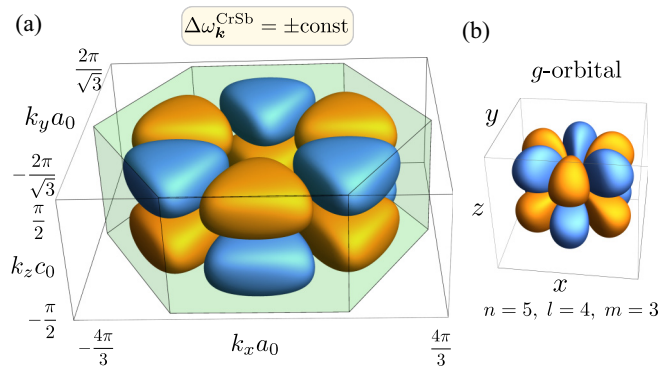


FIG. 4. (a) Surfaces of constant magnon splitting $\Delta\omega_{\mathbf{k}}^{\text{CrSb}} = \pm 8\gamma \delta\tilde{J}/\mu_s$ of the magnon branches of CrSb in the first BZ compared to (b) the hydrogen g orbital with the corresponding quantum numbers (n, l, m) . Orange and blue colors correspond to $\Delta\omega_{\mathbf{k}}^{\text{CrSb}} > 0$ and $\Delta\omega_{\mathbf{k}}^{\text{CrSb}} < 0$, respectively.

direction [001] and having the absolute value $k_{\text{max}} = \pi [\frac{16}{27a_0^2} + \frac{1}{16c_0^2}]^{1/2} \approx 0.65 \text{ rad/\AA}$. Note that the value of $\Delta\omega_{\mathbf{k}}^{\text{CrSb}}$ is determined by the altermagnetic strength $\delta\tilde{J}$ and is not influenced by the anisotropy.

Similarly, considering the corresponding ground state, we obtain the dispersion relation for the magnons in MnTe (see Appendix C)

$$\begin{aligned} \frac{\omega_{\pm}^{\text{MnTe}}}{\omega_0} &= \{F_{\mathbf{k}}(F_{\mathbf{k}} + \kappa \sin^2 \theta_0) + \cos 2\theta_0 \cos^2(k_z c_0) + (\delta\epsilon \Omega_{\mathbf{k}}^-)^2 \\ &\pm [\cos^2(k_z c_0) \{[\kappa + \cos^2 \theta_0 (2F_{\mathbf{k}} - \kappa)]^2 - 4(\delta\epsilon \Omega_{\mathbf{k}}^-)^2 \sin^2 \theta_0\} \\ &+ (\delta\epsilon \Omega_{\mathbf{k}}^-)^2 (2F_{\mathbf{k}} + \kappa \sin^2 \theta_0)^2]^{\frac{1}{2}}\}^{\frac{1}{2}}, \end{aligned} \quad (7)$$

where $\kappa = |K|/J_{\text{AFM}}$ and definitions of the constants ω_0 , η , ϵ , $\delta\epsilon$ are the same as in (5). Due to the different ground states, the spectra of CrSb and MnTe are drastically different. The ground state of MnTe is continuously degenerate with respect to its orientation within the easy plane. The latter circumstance leads to the emergence of the Goldstone mode [see the spectra behavior in Γ point in Fig. 3(i)]. In contrast to CrSb, the splitting $\Delta\omega_{\mathbf{k}}^{\text{MnTe}}$ of the magnon branches of MnTe is strongly influenced by the anisotropy and, strictly speaking, does not possess g -wave symmetry. The magnon branches $\omega_{\pm}^{\text{MnTe}}$ are not degenerate even if $B = 0$ and $\delta\tilde{J} = 0$. The introduction of the altermagnetism ($\delta\tilde{J} \neq 0$) leads to the sixfold modulations of the magnon branches (see the bottom row in Fig. 3). Nevertheless, the magnon branches do not intersect and the splitting $\Delta\omega_{\mathbf{k}}^{\text{MnTe}}$ does not alternate sign. However, for the particular values of the parameters listed in Table I the distance between two magnon branches can be

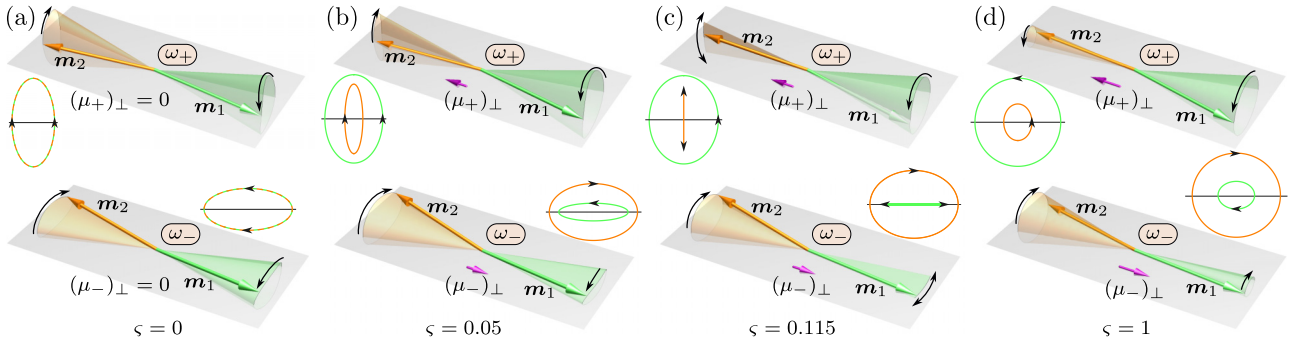


FIG. 5. The altermagnetically induced emergence of the chirality and magnetic moment in spin waves for MnTe. The precession dynamics is reconstructed by means of Eq. (C8) for the wave vector $(k_x a_0, k_y a_0, k_z c_0) = (0, \pi/\sqrt{3}, \pi/4)$. To illustrate the role of the altermagnetism, we introduce the scaling factor ζ for the altermagnetic parameters ε and $\delta\varepsilon$ of MnTe and show the precession dynamics for $(\varepsilon', \delta\varepsilon') = \zeta(\varepsilon, \delta\varepsilon)$ with ζ varying from 0 to 1. The easy plane is shown by gray.

rather small [see Fig. 3(g)] and, therefore, may be difficult to resolve experimentally [24].

In the limit case $K = 0$, $B = 0$, the dispersion (7) is simplified to $\omega_{\pm}^{\text{MnTe}} = \omega_0[\sqrt{F_k^2 - \cos^2(k_z c_0)} \pm |\delta\varepsilon\Omega_k^-|]$. In the considered limit, $\omega_{\pm}^{\text{MnTe}}$ reproduces $\omega_{\pm}^{\text{CrSb}}$ in (5), but $\Delta\omega_k^{\text{MnTe}} = |\Delta\omega_k^{\text{CrSb}}|$. However, both spectra become identical if the magnetic moment of the magnon is used as a criterion for distinguishing modes [see Figs. 9(d) and 9(h)]. The evolution of both spectra with K can be traced in Fig. 9.

Solving the eigenvalue problem, we reconstruct the dynamics of the magnetic moments $\mathbf{m}_v(\mathbf{R}_n)$ using eigenvectors and Eq. (C8). The obtained dynamics of two neighboring moments $\mathbf{m}_1$ and $\mathbf{m}_2$ is shown in Fig. 3(e). The dynamics is typical for easy-axial antiferromagnets [33], i.e., moments $\mathbf{m}_v$ demonstrate circular precession about the anisotropy easy axis shown by the vertical gray line. For a given spin wave, the direction of precession is the same for both sublattices $\mathbf{m}_1$ and $\mathbf{m}_2$, however, it is opposite for the spin waves belonging to different branches ω_+ and ω_- . The latter allows us to characterize magnons by *chirality*, that is, by the direction of precession clockwise (branch ω_+) or counterclockwise (branch ω_-) relative to the direction $\mathbf{m}_1^0$. In the branch ω_+ (ω_-), the magnetic moment $\mathbf{m}_2$ ($\mathbf{m}_1$) precesses in the direction opposite to the natural precession direction, which would appear in the absence of the AFM coupling between the sublattices. Thus, one can say that the moment $\mathbf{m}_1$ enforces the unfavorable direction of precession for the moment $\mathbf{m}_2$ for the branch ω_+ and vice versa for the branch ω_- . The amplitudes of the unfavorable precessions are smaller and the difference in the precession amplitudes of moments $\mathbf{m}_1$ and $\mathbf{m}_2$ gives rise to the uncompensated magnetic moment $(\mu_{\pm})_z$. The latter has opposite directions for different branches and, therefore, it correlates with magnon chirality, i.e., the magnons of opposite chiralities have opposite magnetic moments. In the next section we provide a rigorous calculation of the magnon magnetic moment.

The altermagnetism of CrSb does not change qualitatively the precession dynamics described above. This is in strong contrast to MnTe, where altermagnetism drastically influences the precession of the magnetic moments. Similarly to all easy-planar AFMs, magnetic moments of MnTe demonstrate

elliptical precession with the semimajor axes parallel and perpendicular to the easy plane for the lower (ω_-) and higher (ω_+) branches, respectively [see Fig. 3(j)]. For vanishing altermagnetic interactions [see Fig. 5(a)], the precession directions of $\mathbf{m}_1$ and $\mathbf{m}_2$ are opposite, meaning that this type of dynamics can not be characterized by chirality. Each magnetic moment precesses in its favorite direction, and the precession amplitudes of $\mathbf{m}_1$ and $\mathbf{m}_2$ are equal [33]. The latter results in zero averaged magnetic moment. Thus, magnons in easy-planar AFMs are not chiral and do not possess a magnetic moment. With the increase of the altermagnetic parameter, one of the moments changes the direction of the precession by passing through the linear polarization regime (see Fig. 5). Finally, both moments precess in the same direction, which can be associated with a certain chirality. This is the mechanism of the altermagnetically induced emergence of the magnon chirality. Similarly to the case of CrSb, the amplitude of the precession in the unfavorable direction is smaller, this results in the uncompensated averaged magnetic moment $(\mu_{\pm})_z$. In the next section we rigorously introduce the magnetic moment of magnons and their computation.

B. Magnetic moment of spin-wave eigenmodes

Let us now compute the magnetic moment carried by one magnon in a vanishing magnetic field [34]

$$\boldsymbol{\mu}_\sigma = -\hbar(\partial\omega_\sigma/\partial\mathbf{B})|_{B=0}. \quad (8)$$

In this regard, we note the principally different dependence of the spectra of CrSb and MnTe on the magnetic field applied in the z direction: while $\omega_{\pm}^{\text{CrSb}}$ is linear in B , for MnTe, one has $\omega_{\pm}^{\text{MnTe}} = \omega_{\pm}^{\text{MnTe}}(B^2)$. This difference originates from the different orientation of the magnetic field with respect to the ground state. Consequently, according to (8), $(\mu_{\pm}^{\text{CrSb}})_z = \mp g\mu_B$ while $(\mu_{\pm}^{\text{MnTe}})_z = 0$. The behavior of the perpendicular components is opposite: while $(\mu_{\pm}^{\text{CrSb}})_\perp = 0$, magnons in MnTe possess finite magnetic moment oriented along

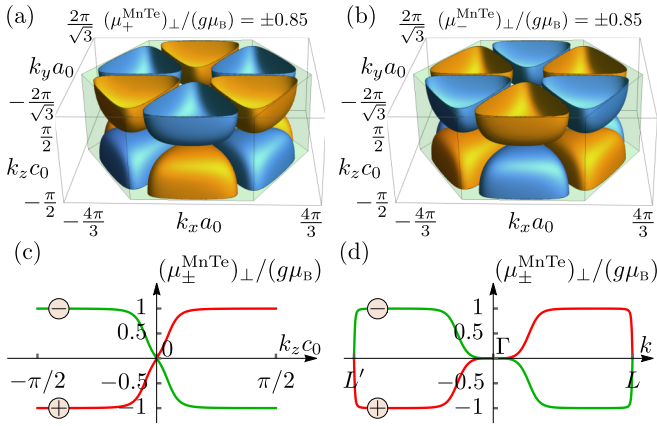


FIG. 6. Magnetic moment carried by one magnon in MnTe determined by Eq. (C11). The moment is oriented along the MnTe ground state and has the amplitude $(\mu_{\pm}^{\text{MnTe}})_{\perp}$, where the subscripts “+” or “−” correspond to the higher and lower magnon branches, respectively. (a), (b) Show the distribution of magnon magnetic moment within the BZ by means of the orange [corresponds to $(\mu_{\pm}^{\text{MnTe}})_{\perp} > 0$] and blue [corresponds to $(\mu_{\pm}^{\text{MnTe}})_{\perp} < 0$] isosurfaces. (a), (b) Correspond to the higher and lower branches, respectively. (c), (d) Show the evolution of the magnetic moment along paths $-\pi/2 \leq k_z c_0 \leq \pi/2$ with $k_x a_0 = 0$, $k_y a_0 = \pi/\sqrt{3}$, and $L\Gamma L$, respectively. Positive and negative value of $(\mu_{\pm}^{\text{MnTe}})_{\perp}$ corresponds to the magnetic moment orientation along $\mathbf{m}_1^0$ and $\mathbf{m}_2^0$, respectively.

the ground state.³ In the limit $\delta\epsilon \ll \kappa \ll 1$ (equivalent to $\delta\tilde{J} \ll |K| \ll J_{\text{AFM}}$),⁴ we estimate

$$(\mu_{\pm}^{\text{MnTe}})_{\perp} \approx \mp g\mu_B \frac{2\delta\epsilon}{\kappa} \frac{\Omega_k^-}{\cos(c_0 k_z)} \sqrt{F_k^2 - \cos^2(c_0 k_z)}. \quad (9)$$

For the exact expression, see Eq. (C11). The subscripts “+” and “−” correspond to the higher and lower magnon branches, respectively. Note that within the accepted assumptions [zero Dzyaloshinskii-Moriya interaction (DMI) and zero higher-order anisotropies] the magnetic moments of the eigenmodes (9), as well as (C11), disappear in the limit $\delta\epsilon \rightarrow 0$. The behavior of the magnon magnetic moments within the BZ is analyzed in Fig. 6. One can see that for each magnon branch the distribution of the magnon magnetic moment over the BZ has the form of a domain structure with 12 regions: 6 regions exhibit magnetization along $\mathbf{m}_1^0$ (orange isosurfaces), while the other 6 regions show the opposite orientation along $\mathbf{m}_2^0$ (blue isosurfaces), as shown in Figs. 6(a) and 6(b). This distribution has a symmetry of a g orbital [compare to Fig. 4(b)]. The transition region between two domains can be treated as a domain wall in k space: it has a kinklike structure [see Fig. 6(c)]. The magnon magnetic moment vanishes on the side

³In order to compute $(\mu_v)_{\perp}$ we obtain the dispersion relation for the magnetic field applied within xy plane and use Eq. (8). For the case of MnTe, we fix the orientation of the ground state by introducing the additional easy-axial anisotropy $\tilde{K}$ with the easy axis lying within the easy plane. Then we compute magnetic moment (8) and take the limit $\tilde{K} \rightarrow 0$. For details see Appendix C.

⁴Although this limit is not fulfilled for MnTe, the simplified expression for magnetic moment (9) is clear and easy to analyze.

surface of the BZ but not on the top and bottom surfaces [see Figs. 6(c) and 6(d)]. The distribution of $(\mu_{\pm}^{\text{MnTe}})_{\perp}$ qualitatively reproduces the distribution of the neutron chiral factor for the magnons in MnTe [24]. Note that the sign of the neutron chiral factor distinguishes modes with opposite polarizations [35], which in our terminology corresponds to the opposite orientation of μ .

It was recently shown that the similar k -dependent spin polarization of electronic bands in MnTe occurs due to the spin-orbit coupling [20]. In contrast to the magnetic moment of a magnon, the electron spin has a fixed length, so it necessarily rotates when moving from one domain to another. The latter leads to the appearance of the z component of the electron spin polarization [20]. The magnon magnetic moment $\mu_{\pm}^{\text{MnTe}}$, however, is always collinear to the ground state and vanishes on the domain wall.

It is instructive to compare the distributions of the magnon magnetic moment along the isolines of the constant energy $\omega_{\pm}(k_x, k_y, k_z = \text{const}) = \text{const}$ for the easy-planar (MnTe) and easy-axial (CrSb) cases (see Figs. 1 and 7). One can see that this distribution is very similar for both cases, except for the fact that the degeneracy points for the easy-axial CrSb are gapped out for the easy-planar MnTe. The gaps, however, vanish for the vanishing easy-planar anisotropy [see Fig. 7(c)]. Based on the symmetry of the distribution of the magnon magnetic moment, we introduce a quantity λ defined in Eq. (1) which possesses the g -wave symmetry in both cases, regardless of the sign and magnitude of the anisotropy (see the bottom row of Fig. 7). Note that parameter λ has the same analytical form $\lambda = -2\delta\epsilon \Omega_k^-$ for CrSb, as well as for MnTe.

Ignoring the altermagnetic part of the model results in the vanishing of the magnon magnetic moment for MnTe and degeneracy of the spectra for CrSb. In both cases the spectra possess C_6 rotational symmetry. This reflects the C_6 symmetry of the part of the lattice composed solely of magnetic atoms. Taking into account the position of the nonmagnetic atoms replaces C_6 axis by a screw axis, such that the symmetry operation is a combination of three actions: rotation by 60° , translation along the vertical axis by half of the vertical size of the unit cell, and permutation of the magnetic sublattices. The latter results in the inversion of the magnon magnetic moment. One can see that Figs. 1 and 7 are invariant with respect to the combined action of the C_6 rotation and inversion of the color denoting the magnetic moment. The application of the screw-symmetry operation twice is equivalent to the simple C_3 rotation, thus, all magnon spectra possess also C_3 symmetry.

IV. MAGNETIZATION OF MAGNETIC TEXTURES

Applying the Taylor expansion to terms $\mathbf{m}_v(\mathbf{R}_n + \delta\mathbf{R})$ and keeping terms up to the fourth order in δR_i we proceed to the continuum approximation of the Hamiltonian $\mathcal{H}$ by performing the replacement $\sum_{\mathbf{R}_n}(\dots) \rightarrow \frac{1}{V_{\text{puc}}} \int(\dots) d\mathbf{r}$ with $V_{\text{puc}} = \sqrt{3}a_0^2 c_0$ being the volume of the primitive magnetic unit cell. The continuum approximation of the altermagnetic part is

$$\mathcal{H}_{\text{ALT}} = \frac{\mathcal{B}_{\text{ALT}}}{2} \int (\mathbf{m}_1 \cdot \hat{\mathbf{D}} \mathbf{m}_1 - \mathbf{m}_2 \cdot \hat{\mathbf{D}} \mathbf{m}_2) d\mathbf{r}, \quad (10)$$

where $\mathcal{B}_{\text{ALT}} = 3a_0 \delta\tilde{J}$ and $\hat{\mathbf{D}} = \partial_{zy}(\partial_{yy} - 3\partial_{xx})$. Note that the form of the differential operator $\hat{\mathbf{D}}$ of the g -wave

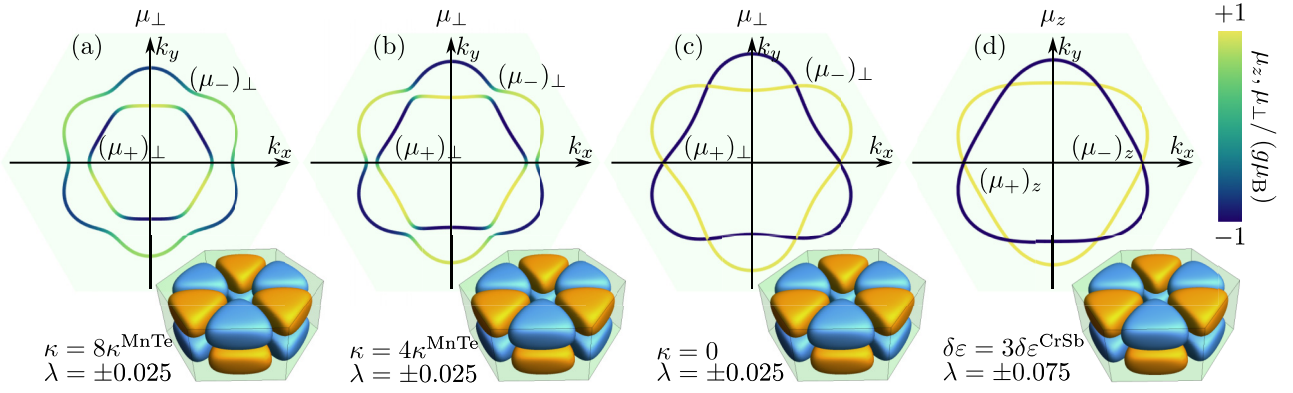


FIG. 7. Distribution of amplitude of the magnon magnetic moment along the isolines of constant energy $\omega_{\pm} = \text{const}$ for easy-planar (a)–(c) and easy-axial (d) anisotropies, respectively. All isolines are built for constant $k_z = \pi/(4c_0)$. (a)–(c) Show evolution of the isolines as anisotropy decreases to zero. Parameters of (a)–(c) are the same as for MnTe [see Fig. 3(g)] but anisotropy. Parameters for (d) are the same as for CrSb [see Fig. 3(b)] but the enhanced altermagnetic strength $\delta\epsilon \rightarrow 3\delta\epsilon$ (to visually increase the altermagnetic splitting). The bottom row demonstrates distribution of the altermagnetic parameter (1) over the Brillouin zone by means of the isosurfaces $\lambda = \pm \text{const}$.

altermagnetic Hamiltonian (10) is very different to the one for d -wave altermagnets [10]. Note that the fourth order of the Taylor expansion is the lowest order which leads to the nonvanishing $\mathcal{H}_{\text{ALT}}$. The complete expression for $\mathcal{H}$ is provided in Eq. (E1). Introducing the Néel order parameter $\mathbf{n} = (\mathbf{m}_1 - \mathbf{m}_2)/2$ and the vector of the net magnetization $\mathbf{m} = (\mathbf{m}_1 + \mathbf{m}_2)/2$, the magnetization can be shown to be

$$\mathbf{m} \approx \frac{1}{\gamma B_{\text{ex}}} \dot{\mathbf{n}} \times \mathbf{n} + \frac{1}{B_{\text{ex}}} \mathbf{n} \times \mathbf{B} \times \mathbf{n} - \mu_{\text{ALT}} \mathbf{n} \times \hat{\mathbf{D}} \mathbf{n} \times \mathbf{n}, \quad (11)$$

in the exchange approximation ($|\mathbf{m}| \ll 1$), where $B_{\text{ex}} = 4J_{\text{AFM}}/\mu_s$ is the exchange field and $\mu_{\text{ALT}} = \frac{3\sqrt{3}}{4} \frac{\delta J}{J_{\text{AFM}}} a_0^3 c_0$.

A. Domain wall in CrSb

The effective continuum Hamiltonian allows a domain wall solution for the easy-axial CrSb. In terms of the continuous Néel vector represented in the angular parametrization $\mathbf{n} = \sin\theta(\mathbf{e}_x \cos\phi + \mathbf{e}_y \sin\phi) + \mathbf{e}_z \cos\theta$ the domain wall solution is $\theta = 2 \arctan e^{p\xi/x_{\text{DW}}}$ with $p = \pm 1$ being the domain wall topological charge and $\phi = \phi_0 = \text{const}$ determines the domain wall chirality, e.g., $\phi_0 = \pm\pi/2$ for Bloch domain walls of opposite chiralities. Here ξ is coordinate along some arbitrary direction determined by a unit vector $\mathbf{e}_{\xi} = \sin\vartheta(\mathbf{e}_x \cos\phi + \mathbf{e}_y \sin\phi) + \mathbf{e}_z \cos\vartheta$, i.e., $\xi = \mathbf{r} \cdot \mathbf{e}_{\xi}$. The domain wall width $x_{\text{DW}} = (\ell_{\perp}^2 \sin^2\vartheta + \ell_z^2 \cos^2\vartheta)^{1/2}$ depends on the domain wall orientation. Applying (11) we obtain the following magnetization of a static domain wall in the case $B = 0$:

$$\mathbf{m} = \mu_{\text{ALT}} \Upsilon(\vartheta, \varphi) \mathcal{R}(\xi/x_{\text{DW}}) [\sinh(\xi/x_{\text{DW}}) \mathbf{e}_0 + p \mathbf{e}_z], \quad (12)$$

where $\mathbf{e}_0 = \mathbf{e}_x \cos\phi_0 + \mathbf{e}_y \sin\phi_0$ is direction of the staggered field $\mathbf{n}$ at the domain wall position. Function $\Upsilon(\vartheta, \varphi) = \tan^3\vartheta \sin\varphi (1 - 4\cos^2\varphi) / (\ell_{\perp}^2 \tan^2\vartheta + \ell_z^2)^2$ determines the angular distribution of the magnetization magnitude concerning the domain wall orientation, and function $\mathcal{R}(x) = \tanh x (12 - \cosh^2 x) / \cosh^4 x$ determines the magnetization distribution in the direction perpendicular to domain wall. The dependencies $n_z(\xi)$ and $m_z(\xi)$ for $p = 1$ are shown in Fig. 8(d). Function $\Upsilon(\vartheta, \varphi)$ has extrema for $\vartheta = \vartheta^* = \pm \arctan(\sqrt{3}\ell_z/\ell_{\perp}) \approx 43^\circ$ and $\varphi = \varphi^* =$

$\pm\pi/2, \pm\pi/6, \pi \pm \pi/6$. The extreme values φ^* correspond to planes I, II, III shown in Figs. 2(a) and 8(a). The extreme value is $\Upsilon(\vartheta^*, \varphi^*) = 3\sqrt{3}/(16\ell_{\perp}^3 \ell_z)$. Using this and maximizing $\mathcal{R}(x)$ we estimate $m_z^{\text{max}} \approx \mu_{\text{ALT}}/(\ell_{\perp}^3 \ell_z) \approx 1.64 \times 10^{-5}$. By introducing the magnetic moment density $\mathbf{M} = 2\mathbf{m}_s/V_{\text{puc}}$ we estimate $\mu_0 M_z^{\text{max}} \approx 14 \mu\text{T}$.

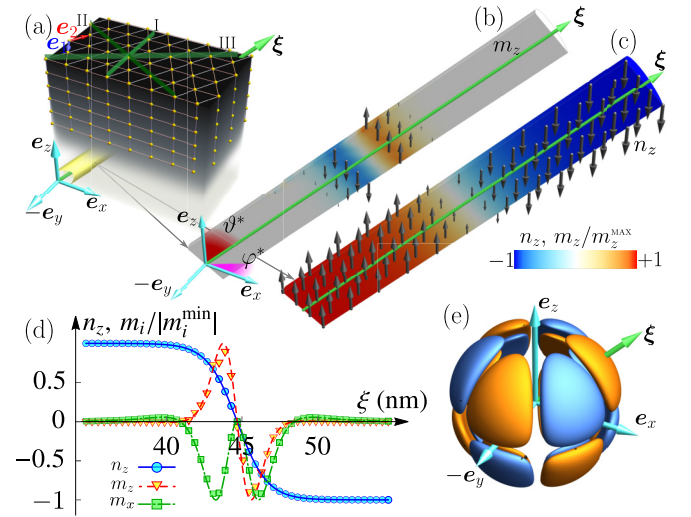


FIG. 8. Domain wall magnetization: simulations vs theory. (a) Shows the orientation of the domain wall direction ξ relative to the crystal structure (yellow points denote Cr atoms). Vector $\mathbf{e}_{\xi}$ lies within plane III, its orientation is determined by the spherical angles ϑ^* and $\varphi^* = \pi/6$ which correspond to the maximal domain wall magnetization. Spin-lattice simulations were performed for Cr atoms located within a cylindrical tube of radius $3x_{\text{DW}}$ and oriented along $\mathbf{e}_{\xi}$. Cross sections of the simulated tubes with radius $0.5x_{\text{DW}}$ are shown on (b) and (c) which demonstrate distributions of the magnetization and Néel vector, respectively. Arrows show the projection of $\mathbf{m}$ or $\mathbf{n}$ on the z axis. (d) Compares the profiles n_z , m_x , and m_z obtained analytically (lines) and using the simulations (symbols). (e) Shows isosurfaces $m_z = \pm 10^{-3}$ for a hypotheticalal spherical domain wall $n_z = -\tanh[(\sqrt{x^2 + y^2 + z^2} - R)/x_{\text{DW}}]$, $n_y/n_x = \tan\phi_0$ with $R = 7x_{\text{DW}}$ and $x_{\text{DW}} = 3$.

Total magnetic moment per unit area of the domain wall is

$$\mathfrak{M}_{\text{DW}} = \int_{-\infty}^{\infty} \mathbf{M} d\xi = \frac{2\pi\mu_s\mu_{\text{ALT}}x_{\text{DW}}}{V_{\text{puc}}} \Upsilon(\vartheta, \varphi) \mathbf{e}_0. \quad (13)$$

For the optimal domain wall orientation, e.g., for $\vartheta = \vartheta^*$ and $\varphi = \varphi^*$ we estimate $|\mathfrak{M}_{\text{DW}}^*| \approx 2.5 \times 10^{-3} \mu_{\text{B}}/\text{nm}^2$. Although the averaged magnetic moment density $\mu_0|\mathfrak{M}_{\text{DW}}^*|/x_{\text{DW}}^* \approx 15 \mu\text{T}$ with $x_{\text{DW}}^* = 2\ell_{\perp}\ell_z/\sqrt{\ell_{\perp}^2 + 3\ell_z^2} \approx 1.9 \text{ nm}$ being the width of the optimally oriented domain wall, is rather small, a large-area domain wall can accumulate a measurable total magnetic moment.

The stabilization of a domain wall in easy-planar MnTe requires the higher-order anisotropies to be taken into account. Although such an extension of the model will allow us to obtain solutions in the form of domain walls, which were experimentally observed in MnTe [36], it will clutter the study of the basic magnonic properties of this material. For this reason, the altermagnetic properties of domain walls in MnTe will be presented in a separate paper.

V. SUMMARY AND CONCLUSIONS

Based on simple Heisenberg models which capture the g -wave altermagnetic properties, we carried out the comparative analysis of the magnon spectra of easy-axial CrSb and easy-planar MnTe. We find that the altermagnetic splitting of the magnon branches of CrSb possesses g -wave symmetry and is the same as in the nonrelativistic limit (i.e., for vanishing anisotropy). In the case of MnTe, the easy-planar anisotropy qualitatively changes the magnon properties, and a generalization of the concept of altermagnetic splitting is required. We propose a universal altermagnetic characteristic (1) that involves both the magnon energy and its magnetic moment and has g -wave symmetry in both cases, i.e., it is independent of the sign and magnitude of the anisotropy.

We expect that the described difference between the magnon properties in the easy-axial and easy-planar g -wave altermagnets also takes place for d -wave altermagnets. Namely, we expect the altermagnetic-induced magnon magnetic moment in easy-planar d -wave altermagnets, e.g., in NiF₂, whose distribution over the first Brillouin zone possesses d -wave symmetry for each magnon branch.

In this study, we neglected DMI, assuming that its influence on the spectrum is small. Considering DMI in the form proposed in Ref. [27], we found dispersion relations for CrSb [see Eq. (D3)] as well as for MnTe [see Eq. (D5)] with DMI taken into account. As it follows from Eq. (D3), DMI does not influence the altermagnetic splitting in CrSb. Both magnon branches of CrSb gain the same DMI-induced correction $\delta\omega_{\text{DMI}}^{\text{CrSb}} \sim -\gamma D_z^2/(\mu_s J_{\text{AFM}})$, whose k dependence is of the order $O(k^8)$ for small wave vectors. Here D_z is the length of the projections of the DMI vectors on direction [001] (see Appendix D 1 and Ref. [27] for details). Thus, $\delta\omega_{\text{DMI}}^{\text{CrSb}}$ can be neglected as compared to the altermagnetic splitting which is of the order $O(k^4)$ [see Eq. (6)]. In MnTe, the magnon branches gain similar corrections $\delta\omega_{\text{DMI}}^{\text{MnTe}} \sim -\gamma D_{\perp}^2/(\mu_s J_{\text{AFM}})$, however, its k dependence depends on the orientation of the ground state $\mathbf{n}_0$. Here $D_{\perp}$ is the length of the projections of the DMI vectors on plane (001) (see

Appendix D 2). It is important to note that although DMI leads to some deformation of the magnon branches, the splitting pattern does not change qualitatively, i.e., the branches do not intersect for the nonvanishing κ . The latter circumstance justifies the neglect of DMI in the context of this paper.

DMI also contributes to the domain magnetization. However, the DMI-induced magnetization has the opposite parity compared to the altermagnetically induced magnetization (see Appendix E for details). The latter property can be used to distinguish these two contributions.

Similarly to d -wave altermagnets [10], static domain walls in CrSb possess altermagnetically induced magnetization which strongly depends on the domain wall orientation concerning the crystallographic axes.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: EXCHANGE HAMILTONIAN

Here we present the explicit form of the exchange part of the Hamiltonian. The antiferromagnetic interaction ($J_{\text{AFM}} > 0$) between layers (001) has the following explicit form:

$$\mathcal{H}_{\text{AFM}} = J_{\text{AFM}} \sum_{\mathbf{R}_n} \mathbf{m}_1(\mathbf{R}_n) \cdot [\mathbf{m}_2(\mathbf{R}_n + c_0 \mathbf{e}_z) + \mathbf{m}_2(\mathbf{R}_n - c_0 \mathbf{e}_z)]. \quad (A1)$$

The ferromagnetic exchange ($J_{\text{FM}} > 0$) between the nearest neighbors within (001) layer is

$$\mathcal{H}_{\text{FM}} = -J_{\text{FM}} \sum_{\mathbf{R}_n} \sum_{\delta \mathbf{R}_{\perp}} \sum_{\nu} \mathbf{m}_{\nu}(\mathbf{R}_n + \boldsymbol{\zeta}_{\nu}) \cdot [\mathbf{m}_{\nu}(\mathbf{R}_n + \boldsymbol{\zeta}_{\nu} + \delta \mathbf{R}_{\perp}) + \mathbf{m}_{\nu}(\mathbf{R}_n + \boldsymbol{\zeta}_{\nu} - \delta \mathbf{R}_{\perp})], \quad (A2)$$

where $\delta \mathbf{R}_{\perp} \in \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_1 + \mathbf{e}_2\}$, $\boldsymbol{\zeta}_{\nu} = (\nu - 1)c_0 \mathbf{e}_z$, and $\nu \in \{1, 2\}$ enumerates sublattices. We present the altermagnetic

Hamiltonian as a sum of symmetrical and asymmetrical parts $\mathcal{H}_{\text{ALT}} = \mathcal{H}_{\text{ALT,S}} + \mathcal{H}_{\text{ALT,A}}$, where

$$\begin{aligned} \mathcal{H}_{\text{ALT,S}} = & \frac{\tilde{J}}{2} \sum_{\mathbf{R}_n} \sum_{\mathbf{p} \in \{\text{i,ii,iii}\}} \sum_{\nu} \mathbf{m}_{\nu}(\mathbf{R}_n + \boldsymbol{\zeta}_{\nu}) \\ & \cdot [\mathbf{m}_{\nu}(\mathbf{R}_n + \boldsymbol{\zeta}_{\nu} + \delta \mathbf{R}_{+}^{\text{p}}) + \mathbf{m}_{\nu}(\mathbf{R}_n + \boldsymbol{\zeta}_{\nu} - \delta \mathbf{R}_{+}^{\text{p}}) \\ & + \mathbf{m}_{\nu}(\mathbf{R}_n + \boldsymbol{\zeta}_{\nu} + \delta \mathbf{R}_{-}^{\text{p}}) + \mathbf{m}_{\nu}(\mathbf{R}_n + \boldsymbol{\zeta}_{\nu} - \delta \mathbf{R}_{-}^{\text{p}})], \end{aligned} \quad (\text{A3a})$$

$$\begin{aligned} \mathcal{H}_{\text{ALT,A}} = & \frac{\delta \tilde{J}}{2} \sum_{\mathbf{R}_n} \sum_{\mathbf{p} \in \{\text{i,ii,iii}\}} \sum_{\nu} (-1)^{\nu} \mathbf{m}_{\nu}(\mathbf{R}_n + \boldsymbol{\zeta}_{\nu}) \\ & \cdot [\mathbf{m}_{\nu}(\mathbf{R}_n + \boldsymbol{\zeta}_{\nu} + \delta \mathbf{R}_{+}^{\text{p}}) + \mathbf{m}_{\nu}(\mathbf{R}_n + \boldsymbol{\zeta}_{\nu} - \delta \mathbf{R}_{+}^{\text{p}}) \\ & - \mathbf{m}_{\nu}(\mathbf{R}_n + \boldsymbol{\zeta}_{\nu} + \delta \mathbf{R}_{-}^{\text{p}}) - \mathbf{m}_{\nu}(\mathbf{R}_n + \boldsymbol{\zeta}_{\nu} - \delta \mathbf{R}_{-}^{\text{p}})]. \end{aligned} \quad (\text{A3b})$$

Table I lists the numerical values of the considered model's parameters.

APPENDIX B: DFT CALCULATIONS

Magnetic scalar-relativistic DFT calculations were performed using the generalized gradient approximation (GGA) [37] implemented in the full-potential code FPLO [38], version 22. Since this code does not support magnetic symmetries, the antiferromagnetic structure of CrSb is represented within the space group $P\bar{3}m1$ (a subgroup of $P6_3/mmc$), where Cr occupies two independent Wyckoff positions $1a$ and $1b$. While different lattice constants have been reported in the literature, we followed Ref. [39] and adopted $a = 4.12$ Å and $c = 5.47$ Å that are nearly identical to the experimental values in Ref. [26].

To estimate the isotropic (Heisenberg) magnetic exchanges between Cr atoms, we constructed a rectangular ($2a, \sqrt{3}a, 2c$) supercell comprising 16 formula units and calculated GGA total energies of 36 different magnetic configurations. Since the average magnitude of magnetic moments localized on Cr atoms is close to $3 \mu_{\text{B}}$, we mapped these energies onto a classical $S = \frac{3}{2}$ Heisenberg model. In this model, we consider seven short-range exchange interactions with $d_{\text{Cr...Cr}} \leq 8$ Å, as well as the average $\tilde{J}$ of two altermagnetic exchanges $\tilde{J}_1$ and $\tilde{J}_2$. In the absence of information on individual contributions (which is only possible by further increasing the already large supercell), we further assume that $\tilde{J}_2 = 2\tilde{J}$ and $\tilde{J}_1 = 0$, justified by the disparity of the respective exchange paths: while $\tilde{J}_2$ exchange runs through two Sb atoms, no ligand atoms are entailed by the $\tilde{J}_1$ path.

Values of exchange integrals obtained by a least-squares solution of the resulting redundant linear problem are provided in Table II. As expected, we find the leading antiferromagnetic interplane exchange J_1 and the subleading ferromagnetic intraplane exchange J_2 . In addition, we find appreciable antiferromagnetic exchange between second-neighbor planes (~ 3.5 meV) and second-neighbor ferromagnetic intraplane exchange (~ -2.3 meV). Note that the latter underpins the A-type antiferromagnetic structure of CrSb, while the former gives rise to small frustration. For the average of altermagnetic exchanges $\tilde{J}$, we find a rather small value of 0.6 meV.

TABLE II. Magnetic exchange integrals in CrSb estimated by mapping the GGA total energies onto a $S = \frac{3}{2}$ Heisenberg model. The solution of a redundant system of 36 linear equations was obtained by a least-squares fit. Error bars were estimated by bootstrapping with replacement. Interatomic vectors $\mathbf{R}_{\text{Cr...Cr}}$ pertain to the crystallographic unit cell, z is the number of J bonds per Cr atom (only one out of z symmetry-equivalent $\mathbf{R}$ is given); distances $d_{\text{Cr...Cr}}$ correspond for $a = 4.12$ Å and $c = 5.47$ Å.

$\mathbf{R}_{\text{Cr...Cr}}, (a, a, c)$	z	$d_{\text{Cr...Cr}}, \text{Å}$	Notation	J (meV)
$(0, 0, \frac{1}{2})$	1	2.7350	J_{AFM}	33.3(9)
$(1, 0, 0)$	3	4.1200	J_{FM}	-11.9(5)
$(1, 0, \frac{1}{2})$	6	4.9452		0.3(5)
$(0, 0, 1)$	1	5.4700		3.5(9)
$(1, 0, 1)$	6	6.8480		-1.0(3)
$(\sqrt{3}, 0, 0)$	3	7.1360		-2.3(4)
$(\sqrt{3}, 0, \frac{1}{2})$	6	7.6422		0.5(4)
$(\sqrt{3}, 0, 1)$	3	8.9913	$2\tilde{J} \equiv \tilde{J}_2$	0.6(5)

To assess the accuracy of the computed exchanges, we performed bootstrapping with replacement. The error bars (shown in brackets after J values in Table II) are sizable, and for small exchanges they are comparable with respective J values. Since the number of magnetic configurations is not small (36 for 9 fitted parameters), these error bars reflect an intrinsic shortcoming of the mapping procedure: CrSb is a metal, and the Heisenberg model (as any localized-spin model) neglects the itinerant component of its magnetism. The relevance of the itinerant component is further corroborated by a rather large standard deviation ($0.16 \mu_{\text{B}}$) on the mean value of Cr local moments ($3.08 \mu_{\text{B}}$) across all calculated configurations.

Finally, we compare our estimates of exchange interactions with literature data. Reference [40] reports $J_{\text{AFM}} \simeq 23$ meV, which is about 30% smaller than our estimate, and $J_{\text{FM}} \simeq -12$ meV, in an excellent match with our calculations. The agreement with Korringa-Kohn-Rostoker band structure calculations in Ref. [41] is even better: $J_{\text{AFM}} \simeq 36$ meV and $J_{\text{FM}} \simeq -14$ meV are very close to our estimates. The recent DFT study [42] reports significantly smaller values: $J_{\text{AFM}} \simeq 12$ meV and $J_{\text{FM}} \simeq -2$ meV. Although the cause of discrepancy is not fully clear, these estimates seem too low to account for the magnetic ordering temperature of CrSb.

APPENDIX C: MAGNON SPECTRUM CALCULATION

1. Dispersion relations

Here we utilize the Fourier transforms on the periodic lattice

$$\begin{aligned} \psi_{\nu}(\mathbf{R}_n) &= \frac{1}{\sqrt{N}} \sum_{\mathbf{k} \in \text{1BZ}} \hat{\psi}_{\nu}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{R}_n}, \\ \hat{\psi}_{\nu}(\mathbf{k}) &= \frac{1}{\sqrt{N}} \sum_{\mathbf{R}_n} \psi_{\nu}(\mathbf{R}_n) e^{-i\mathbf{k} \cdot \mathbf{R}_n} \end{aligned} \quad (\text{C1})$$

supplemented with the completeness relation $\sum_{\mathbf{R}_n} e^{i(\mathbf{k}-\mathbf{k}') \cdot \mathbf{R}_n} = N \delta_{\mathbf{k},\mathbf{k}'}$. Here N is the number of magnetic moments in one sublattice. Applying the Fourier transform

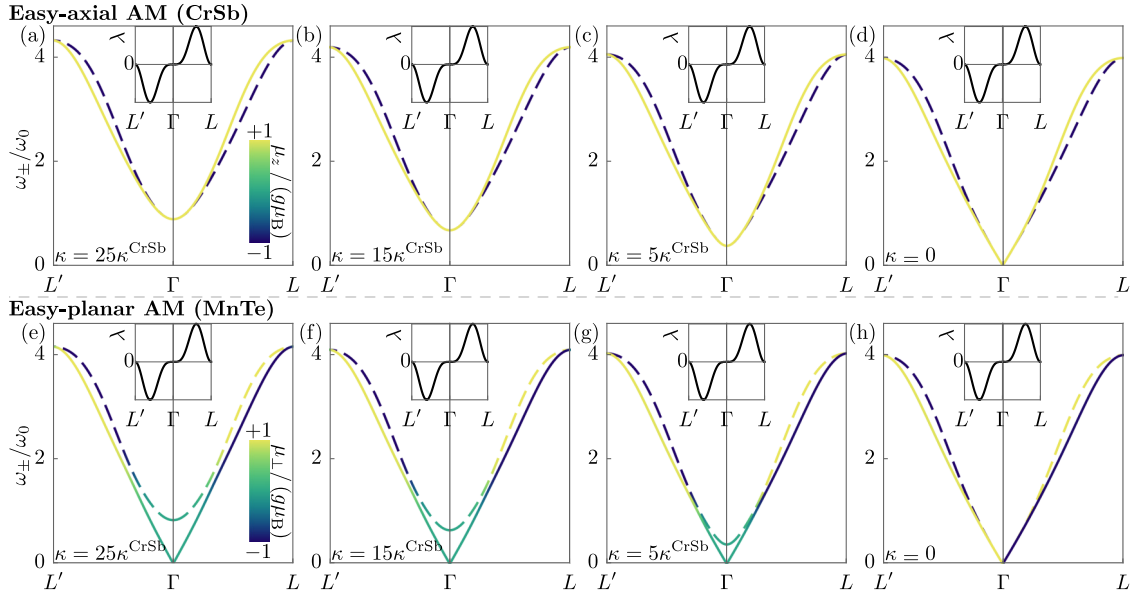


FIG. 9. Comparison of the magnon spectra (5) for easy-axis AM (top row, CrSb) and (7) for easy-planar AM (bottom row, MnTe) for different anisotropy strength. Color scheme corresponds to the amplitude of the magnon magnetic moment, which for the case MnTe was computed using (C11). The figures are built for the following values $\varepsilon = \varepsilon^{\text{CrSb}}$, $\delta\varepsilon = 3\delta\varepsilon^{\text{CrSb}}$, $\eta = \eta^{\text{CrSb}}$, and $B = 0$. Insets in the figures show the g -wave symmetry characteristic λ for given anisotropy strength.

(C1) to linearized Eq. (4), we obtain

$$i\dot{\hat{\psi}}_v(\mathbf{k}) = \frac{\gamma}{\mu_s} \frac{\partial \mathcal{H}^{(2)}}{\partial \hat{\psi}_v^*(\mathbf{k})}, \quad (\text{C2})$$

where $\mathcal{H}^{(2)} = \mathcal{H}_{\text{AFM}}^{(2)} + \mathcal{H}_{\text{FM}}^{(2)} + \mathcal{H}_{\text{ALT}}^{(2)} + \mathcal{H}_{\text{AN}}^{(2)} + \mathcal{H}_z^{(2)}$ is harmonic (with respect to $\hat{\psi}_v$) part of Hamiltonian $\mathcal{H}$. The form of $\mathcal{H}^{(2)}$ depends on the ground state.

For CrSb the ground state is $\mathbf{m}_1^0 = (-1)^{v-1} \mathbf{e}_z$, with $v = 1, 2$. Without loss of generality one can take $\mathbf{T}_v = [(-1)^{v-1} \mathbf{e}_x + i\mathbf{e}_y]/2$.⁵ Now using (3), the form of Hamiltonian $\mathcal{H}$, and Fourier transform (C1), we derive the Hamiltonian components in the harmonic approximation

$$\begin{aligned} \mathcal{H}_{\text{AFM}}^{(2)} &= 2J_{\text{AFM}} \sum_{\mathbf{k} \in 1\text{BZ}} \{ \hat{\mathcal{E}}_{\mathbf{k}}^+ - \cos(k_z c_0) [\hat{\psi}_1(\mathbf{k}) \hat{\psi}_2(-\mathbf{k}) \\ &\quad + \hat{\psi}_1^*(\mathbf{k}) \hat{\psi}_2^*(-\mathbf{k})] \}, \\ \mathcal{H}_{\text{AN}}^{(2)} &= 2K \sum_{\mathbf{k} \in 1\text{BZ}} \hat{\mathcal{E}}_{\mathbf{k}}^+, \quad \mathcal{H}_z^{(2)} = \mu_s B \sum_{\mathbf{k} \in 1\text{BZ}} \hat{\mathcal{E}}_{\mathbf{k}}^-, \\ \mathcal{H}_{\text{FM}}^{(2)} + \mathcal{H}_{\text{ALT}}^{(2)} &= 4 \sum_{\mathbf{k} \in 1\text{BZ}} [(2J_{\text{FM}} \Omega_{\mathbf{k}}^{\text{FM}} - \tilde{J} \Omega_{\mathbf{k}}^+) \hat{\mathcal{E}}_{\mathbf{k}}^+ + \delta \tilde{J} \Omega_{\mathbf{k}}^- \hat{\mathcal{E}}_{\mathbf{k}}^-], \end{aligned} \quad (\text{C3})$$

⁵In general case, $\mathbf{T}_v = e^{i\Phi_v} [(-1)^{v-1} \mathbf{e}_x + i\mathbf{e}_y]/2$, where Φ_v are constant phases. One can always get rid of the constant phase factors by the change of variables $\tilde{\psi}_v = e^{i\Phi_v} \psi_v$ which keeps the equation (4) unchanged.

where $\hat{\mathcal{E}}_{\mathbf{k}}^{\pm} = |\hat{\psi}_1(\mathbf{k})|^2 \pm |\hat{\psi}_2(\mathbf{k})|^2$. Thus, the total harmonic Hamiltonian can be written in the form

$$\begin{aligned} \mathcal{H}^{(2)} &= \omega_0 \frac{\mu_s}{\gamma} \sum_{\mathbf{k} \in 1\text{BZ}} \{ \mathcal{A}_{\mathbf{k}} \hat{\mathcal{E}}_{\mathbf{k}}^+ + \mathcal{B}_{\mathbf{k}} \hat{\mathcal{E}}_{\mathbf{k}}^- - \cos(k_z c_0) \\ &\quad \times [\hat{\psi}_1(\mathbf{k}) \hat{\psi}_2(-\mathbf{k}) + \hat{\psi}_1^*(\mathbf{k}) \hat{\psi}_2^*(-\mathbf{k})] \}, \end{aligned} \quad (\text{C4})$$

where $\mathcal{A}_{\mathbf{k}} = 1 + \eta \Omega_{\mathbf{k}}^{\text{FM}} - \varepsilon \Omega_{\mathbf{k}}^+ + \kappa = F_{\mathbf{k}} + \kappa$, and $\mathcal{B}_{\mathbf{k}} = \frac{\gamma B}{\omega_0} + \delta \varepsilon \Omega_{\mathbf{k}}^-$. With Hamiltonian (C4), the couple of equations (C2) together with their complex-conjugated counterparts form a closed set of four equations

$$\begin{aligned} i\dot{\hat{\Psi}} &= \mathbb{M} \hat{\Psi}, \\ \frac{\mathbb{M}}{\omega_0} &= \begin{bmatrix} \mathcal{A}_{\mathbf{k}} + \mathcal{B}_{\mathbf{k}} & 0 & 0 & -\cos(k_z c_0) \\ 0 & \mathcal{A}_{\mathbf{k}} - \mathcal{B}_{\mathbf{k}} & -\cos(k_z c_0) & 0 \\ 0 & \cos(k_z c_0) & -(\mathcal{A}_{\mathbf{k}} + \mathcal{B}_{\mathbf{k}}) & 0 \\ \cos(k_z c_0) & 0 & 0 & -(\mathcal{A}_{\mathbf{k}} - \mathcal{B}_{\mathbf{k}}) \end{bmatrix} \end{aligned} \quad (\text{C5})$$

for each given $\mathbf{k}$. Here $\hat{\Psi} = [\hat{\psi}_1(\mathbf{k}), \hat{\psi}_2(\mathbf{k}), \hat{\psi}_1^*(-\mathbf{k}), \hat{\psi}_2^*(-\mathbf{k})]^t$ and we used that $\mathcal{A}_{\mathbf{k}} = \mathcal{A}_{-\mathbf{k}}$, $\mathcal{B}_{\mathbf{k}} = \mathcal{B}_{-\mathbf{k}}$. The set (C5) has solution $\hat{\Psi} = \hat{\Psi}_0 e^{-i\omega t}$, which is nontrivial ($\hat{\Psi}_0 \neq 0$) when ω is an eigenvalue of $\mathbb{M}$. This gives the dispersion relation (5).

For MnTe, the ground state is $\mathbf{m}_v^0 = (-1)^{v-1} \sin \theta_0 (\mathbf{e}_x \cos \phi_0 + \mathbf{e}_y \sin \phi_0) + \mathbf{e}_z \cos \theta_0$ and, consequently, $\mathbf{T}_v = \frac{(-1)^{v-1}}{2} (\cos \theta_0 \cos \phi_0 - i \sin \theta_0 \sin \phi_0) \mathbf{e}_x + \frac{(-1)^{v-1}}{2} (\cos \theta_0 \sin \phi_0 + i \sin \theta_0 \cos \phi_0) \mathbf{e}_y - \frac{1}{2} \sin \theta_0 \mathbf{e}_z$. The harmonic

part of the Hamiltonian has the following contributions:

$$\begin{aligned}\mathcal{H}_{\text{AFM}}^{(2)} &= J_{\text{AFM}} \sum_{\mathbf{k} \in \text{IBZ}} \{-\cos 2\theta_0 \hat{\mathbf{e}}_{\mathbf{k}}^+ + 2 \cos(k_z c_0) [\sin^2 \theta_0 \hat{\psi}_1(\mathbf{k}) \hat{\psi}_2(-\mathbf{k}) - \cos^2 \theta_0 \hat{\psi}_1(\mathbf{k}) \hat{\psi}_2^*(\mathbf{k})] + \text{c.c.}\}, \\ \mathcal{H}_{\text{AN}}^{(2)} &= |K| \sum_{\mathbf{k} \in \text{IBZ}} \sum_{\nu} \left\{ \frac{\sin^2 \theta_0}{2} \hat{\psi}_{\nu}(\mathbf{k}) \hat{\psi}_{\nu}(-\mathbf{k}) + \frac{1 - 3 \cos^2 \theta_0}{2} |\hat{\psi}_{\nu}|^2 + \text{c.c.} \right\}, \quad \mathcal{H}_z^{(2)} = B \mu_s \cos \theta_0 \sum_{\mathbf{k} \in \text{IBZ}} \hat{\mathbf{e}}_{\mathbf{k}}^+.\end{aligned}\quad (\text{C6})$$

The expressions for $\mathcal{H}_{\text{ALT}}^{(2)}$ and $\mathcal{H}_{\text{FM}}^{(2)}$ are the same as in (C3). For each $\mathbf{k}$, Eq. (C2) together with their complex-conjugated counterparts form the set (C5) with matrix

$$\mathbb{M} = \omega_0 \begin{bmatrix} F_{\mathbf{k}} + \frac{\kappa}{2} \sin^2 \theta_0 - \delta \varepsilon \Omega_{\mathbf{k}}^- & -\cos^2 \theta_0 \cos(k_z c_0) & \frac{\kappa}{2} \sin^2 \theta_0 & \sin^2 \theta_0 \cos(k_z c_0) \\ -\cos^2 \theta_0 \cos(k_z c_0) & F_{\mathbf{k}} + \frac{\kappa}{2} \sin^2 \theta_0 + \delta \varepsilon \Omega_{\mathbf{k}}^- & \sin^2 \theta_0 \cos(k_z c_0) & \frac{\kappa}{2} \sin^2 \theta_0 \\ -\frac{\kappa}{2} \sin^2 \theta_0 & -\sin^2 \theta_0 \cos(k_z c_0) & -(F_{\mathbf{k}} + \frac{\kappa}{2} \sin^2 \theta_0 - \delta \varepsilon \Omega_{\mathbf{k}}^-) & \cos^2 \theta_0 \cos(k_z c_0) \\ -\sin^2 \theta_0 \cos(k_z c_0) & -\frac{\kappa}{2} \sin^2 \theta_0 & \cos^2 \theta_0 \cos(k_z c_0) & -(F_{\mathbf{k}} + \frac{\kappa}{2} \sin^2 \theta_0 + \delta \varepsilon \Omega_{\mathbf{k}}^-) \end{bmatrix}, \quad (\text{C7})$$

whose eigenvalues determine the dispersion relation (7). Here $\kappa = |K|/J_{\text{AFM}}$ and definitions of the constants ω_0 , η , ε , $\delta \varepsilon$ are the same as previously.

Solving the eigenvalue problem we compute also the eigenvector $\hat{\Psi}_0(\mathbf{k}) = [\hat{\psi}_{01}(\mathbf{k}), \hat{\psi}_{02}(\mathbf{k}), \hat{\psi}_{01}^*(-\mathbf{k}), \hat{\psi}_{02}^*(-\mathbf{k})]^T$ which enables us to reconstruct dynamics of a magnetic moment sitting in the site $\mathbf{R}_n$ when a spin wave with the wave vector $\mathbf{k}$ is excited:

$$\mathbf{m}_{\nu} \approx \mathbf{m}_{\nu}^0 + \sqrt{\frac{2}{N}} \text{Re}[e^{i(\mathbf{k} \cdot \mathbf{R}_n - \omega_{\mathbf{k}} t + \varphi_0)} (\mathbf{T}_{\nu} \hat{\psi}_{0\nu}(\mathbf{k}) + \mathbf{T}_{\nu}^* \hat{\psi}_{0\nu}^*(-\mathbf{k}))]. \quad (\text{C8})$$

Here φ_0 is an arbitrary phase. Examples of the reconstructed dynamics are shown in Figs. 3(e) and 3(j) and 5.

2. Magnetic moment of a magnon in MnTe

As was shown in the previous subsection, the field dependence of the eigenfrequencies has a general form $\omega_{\nu} = \omega_{\nu}(B^2)$ if the magnetic field is applied perpendicularly to the ground state. According to (8), this leads to the vanishing of the component of the magnetic moment perpendicular to the ground state. To compute magnetic moment in the direction of the ground state of MnTe, we introduce an auxiliary easy-axial anisotropy with the easy axis lying within the easy plane

$$\tilde{\mathcal{H}}_{\text{AN}} = -\tilde{K} \sum_{\mathbf{R}_n} [m_{1x}^2(\mathbf{R}_n) + m_{2x}^2(\mathbf{R}_n + c_0 \mathbf{e}_z)]. \quad (\text{C9})$$

Due to (C9), the orientation of the ground state within the easy plane is fixed in the x direction and application of a small enough magnetic field in this direction does not lead to the spin-flop reorientation. Now, we apply the technique described in Appendix C 1 to the MnTe Hamiltonian with the additional term (C9) and with the Zeeman term in the form $\mathcal{H}_z = -B \mu_s \sum_{\mathbf{R}_n} [m_{1x}(\mathbf{R}_n) + m_{2x}(\mathbf{R}_n + c_0 \mathbf{e}_z)]$ and obtain the following dispersion relation for magnons excited on the top of the ground state $\mathbf{m}_{\nu}^0 = (-1)^{\nu+1} \mathbf{e}_x$:

$$\begin{aligned}\omega_{\pm} &= \omega_0 \sqrt{\tilde{A}_{\mathbf{k}}^2 + \tilde{B}_{\mathbf{k}}^2 - \cos^2(k_z c_0) - \frac{\kappa^2}{4}} \pm 2\varpi_{\mathbf{k}}, \\ \varpi_{\mathbf{k}} &= \sqrt{\tilde{A}_{\mathbf{k}}^2 \tilde{B}_{\mathbf{k}}^2 - \cos^2(k_z c_0) (\tilde{B}_{\mathbf{k}}^2 - \kappa^2/4)}. \end{aligned} \quad (\text{C10})$$

Here $\tilde{A}_{\mathbf{k}} = F_{\mathbf{k}} + \frac{\kappa}{2} + \tilde{\kappa}$ with $\tilde{\kappa} = \tilde{K}/J_{\text{AFM}}$, and $\tilde{B}_{\mathbf{k}} = \delta \varepsilon \Omega_{\mathbf{k}}^- + \frac{\gamma B}{\omega_0}$. Note that dispersions (C10) and (7) coincide for the case $B = 0$ and $\tilde{\kappa} = 0$. Using Eq. (8) and applying the limit $\tilde{\kappa} \rightarrow 0$ we obtain

$$\begin{aligned}(\mu_{\pm}^{\text{MnTe}})_{\perp} &= \frac{-g \mu_B \delta \varepsilon \Omega_{\mathbf{k}}^- [1 \pm \frac{(F_{\mathbf{k}} + \frac{\kappa}{2})^2 - \cos^2(k_z c_0)}{\varpi_{\mathbf{k}}^0}]}{\sqrt{F_{\mathbf{k}}(F_{\mathbf{k}} + \frac{\kappa}{2}) + (\delta \varepsilon \Omega_{\mathbf{k}}^-)^2 - \cos^2(k_z c_0) \pm \varpi_{\mathbf{k}}^0}}, \\ \varpi_{\mathbf{k}}^0 &= \sqrt{\left(F_{\mathbf{k}} + \frac{\kappa}{2}\right)^2 (\delta \varepsilon \Omega_{\mathbf{k}}^-)^2 - \cos^2(k_z c_0) \left[(\delta \varepsilon \Omega_{\mathbf{k}}^-)^2 - \frac{\kappa^2}{4}\right]}.\end{aligned} \quad (\text{C11})$$

meaning that the total moment is $\mu_{\pm}^{\text{MnTe}} = (\mu_{\pm}^{\text{MnTe}})_{\perp} \mathbf{e}_x$. In the general case, magnons with the same $\mathbf{k}$ belonging to different branches have magnetic moments of different amplitudes, i.e., $|(\mu_{+}^{\text{MnTe}})_{\perp}| \neq |(\mu_{-}^{\text{MnTe}})_{\perp}|$. However, for the parameters of MnTe (see Table I) one has $(\mu_{+}^{\text{MnTe}})_{\perp} \approx -(\mu_{-}^{\text{MnTe}})_{\perp}$ (see Fig. 6). The magnetic moment (C11) is shown in Figs. 9(e)–9(h) by means of the color scheme. The straightforward calculation of parameter λ with the use of (C10) and (C11) results in $\lambda = -2\delta \varepsilon \Omega_{\mathbf{k}}^-$.

APPENDIX D: DMI-INDUCED CORRECTIONS IN MAGNON SPECTRUM

Here, we consider the influence of the DMI, in the form proposed in Ref. [27], on the magnon spectrum for CrSb and MnTe. The DMI Hamiltonian has the form

$$\begin{aligned}\mathcal{H}_{\text{DMI}} &= \frac{1}{2} \sum_{\mathbf{R}_n} \sum_{i=1}^6 \mathbf{D}_i \cdot \mathbf{m}_1(\mathbf{R}_n) \times \left[\mathbf{m}_2\left(\mathbf{R}_n + \delta \mathbf{R}_i + \frac{\mathbf{e}_3}{2}\right) \right. \\ &\quad \left. + \mathbf{m}_2\left(\mathbf{R}_n - \delta \mathbf{R}_i - \frac{\mathbf{e}_3}{2}\right) \right],\end{aligned} \quad (\text{D1})$$

where $\mathbf{D}_i = \mathbf{D}_i^{\perp} + (-1)^i D_z \mathbf{e}_z$ is a DMI vector with $\mathbf{D}_1^{\perp} = \mathbf{D}_4^{\perp} = D_{\perp}(\mathbf{e}_1 + \mathbf{e}_2)$, $\mathbf{D}_2^{\perp} = \mathbf{D}_5^{\perp} = -D_{\perp} \mathbf{e}_2$, and $\mathbf{D}_3^{\perp} = \mathbf{D}_6^{\perp} = -D_{\perp} \mathbf{e}_1$. The displacement vector $\delta \mathbf{R}_i$ defines the position of the nearest-neighbor magnetic moment, with $\delta \mathbf{R}_1 = -\delta \mathbf{R}_4 = \mathbf{e}_1 + \mathbf{e}_2$, $\delta \mathbf{R}_2 = -\delta \mathbf{R}_5 = \mathbf{e}_2$, and $\delta \mathbf{R}_6 = -\delta \mathbf{R}_3 = \mathbf{e}_1$.

In the following, we perform the same procedure as described in Appendix C 1 to write the DMI Hamiltonian in the harmonic approximation.

1. Easy-axial CrSb

For CrSb the DMI Hamiltonian has the form

$$\mathcal{H}_{\text{DMI}}^{(2)} = 8iD_z \sum_{\mathbf{k} \in \text{1BZ}} \Omega_{\mathbf{k}}^{\text{DMI}} [\hat{\psi}_1^*(\mathbf{k}) \hat{\psi}_2^*(-\mathbf{k}) - \hat{\psi}_1(\mathbf{k}) \hat{\psi}_2(-\mathbf{k})], \quad (\text{D2})$$

where $\Omega_{\mathbf{k}}^{\text{DMI}} = \sin[\mathbf{k} \cdot (\mathbf{e}_1 + \mathbf{e}_2)/2] \prod_{i=1}^3 \sin[\mathbf{k} \cdot \mathbf{e}_i/2]$. With the DMI Hamiltonian (D2) we can rewrite the equations (C5) with new matrix $\mathbb{M} \rightarrow \tilde{\mathbb{M}} = \mathbb{M} + \mathbb{M}^{\text{DMI}}$, where $\mathbb{M}^{\text{DMI}} = i\omega_0 d_z \Omega_{\mathbf{k}}^{\text{DMI}} \text{antidiag}(1, 1, 1, 1)$ is a DMI-induced correction with $d_z = 4D_z/J_{\text{AFM}}$. The resulting dispersion relation for the magnons with a DMI and a zero magnetic field has a form

$$\omega_{\pm}^{\text{CrSb+DMI}} = \omega_0 [\sqrt{A_k^2 - \cos^2(k_z c_0) - d_z^2 (\Omega_{\mathbf{k}}^{\text{DMI}})^2} \pm \delta \varepsilon \Omega_{\mathbf{k}}^-]. \quad (\text{D3})$$

2. Easy-planar MnTe

For MnTe the DMI Hamiltonian has the form

$$\mathcal{H}_{\text{DMI}}^{(2)} = iD_{\perp} \sum_{\mathbf{k} \in \text{1BZ}} \Omega_{\mathbf{k}}^{\text{DMI}} [\hat{\psi}_1(\mathbf{k}) \hat{\psi}_2(-\mathbf{k}) - \hat{\psi}_1^*(\mathbf{k}) \hat{\psi}_2^*(-\mathbf{k})], \quad (\text{D4})$$

where $\Omega_{\mathbf{k}}^{\text{DMI}} = (\varpi_c \cos \phi_0 + \varpi_s \sin \phi_0) \cos(k_z c_0)$ with $\varpi_c = \cos \mathbf{k} \cdot \mathbf{e}_1 + \cos \mathbf{k} \cdot \mathbf{e}_2 - 2 \cos \mathbf{k} \cdot (\mathbf{e}_1 + \mathbf{e}_2)$ and $\varpi_s = -\sqrt{3}(\cos \mathbf{k} \cdot \mathbf{e}_1 - \cos \mathbf{k} \cdot \mathbf{e}_2)$. Note that $\Omega_{\mathbf{k}}^{\text{DMI}}$

depends on the orientation of the ground-state Néel vector, and for small k it can be presented as $\Omega_{\mathbf{k}}^{\text{DMI}} = \frac{3}{32}(k_x^2 + k_y^2)a_0^2 \cos(2\chi + \phi_0) + O(k^4)$ where $\tan \chi = k_y/k_x$. Similarly with the previous case, here we can rewrite Eq. (C5) with new matrix $\mathbb{M} \rightarrow \tilde{\mathbb{M}} = \mathbb{M} + \mathbb{M}^{\text{DMI}}$, where $\mathbb{M}^{\text{DMI}} = -i\omega_0 d_{\perp} \Omega_{\mathbf{k}}^{\text{DMI}} \text{antidiag}(1, 1, 1, 1)$ is a DMI-induced correction with $d_{\perp} = D_{\perp}/(2J_{\text{AFM}})$. The resulting dispersion relation for the magnons with a DMI and a zero magnetic field has a form

$$\frac{\omega_{\pm}^{\text{MnTe+DMI}}}{\omega_0} = \{F_{\mathbf{k}}(F_{\mathbf{k}} + \kappa) - \cos^2(k_z c_0) + (\delta \varepsilon \Omega_{\mathbf{k}}^-)^2 - (d_{\perp} \Omega_{\mathbf{k}}^{\text{DMI}})^2 \pm \{[\cos^2(k_z c_0)[\kappa^2 - 4(\delta \varepsilon \Omega_{\mathbf{k}}^-)^2] + (\delta \varepsilon \Omega_{\mathbf{k}}^-)^2[(2F_{\mathbf{k}} + \kappa)^2 - 4(d_{\perp} \Omega_{\mathbf{k}}^{\text{DMI}})^2]\}^{\frac{1}{2}}\}^{\frac{1}{2}}. \quad (\text{D5})$$

APPENDIX E: CONTINUOUS APPROXIMATION

To obtain the continuous approximation of the Hamiltonian $\mathcal{H}$ we utilize the Taylor expansion up to the fourth order $\mathbf{m}_v(\mathbf{R}_n + \delta \mathbf{R}) \approx \mathbf{m}_v(\mathbf{R}_n) + \delta R_i \partial_i \mathbf{m}_v(\mathbf{R}_n) + \frac{1}{2} \delta R_i \delta R_j \partial_{ij} \mathbf{m}_v(\mathbf{R}_n) + \frac{1}{3!} \delta R_i \delta R_j \delta R_k \partial_{ijk} \mathbf{m}_v(\mathbf{R}_n) + \frac{1}{4!} \delta R_i \delta R_j \delta R_k \delta R_l \partial_{ijkl} \mathbf{m}_v(\mathbf{R}_n)$ and perform the replacement $\sum_{\mathbf{R}_n}(\dots) \rightarrow \frac{1}{V_{\text{puc}}} \int(\dots) d\mathbf{r}$ with $V_{\text{puc}} = \sqrt{3}a_0^2 c_0$ being the volume of the primitive magnetic unit cell. Finally,

$$\mathcal{H} = \int \left[\frac{1}{2} B_{\text{ex}} M_s \mathbf{m}_1 \cdot \mathbf{m}_2 - A_{\text{AFM}} \partial_z \mathbf{m}_1 \cdot \partial_z \mathbf{m}_2 + \frac{1}{2} A_{\text{FM}}^{\perp} \partial_{\alpha} \mathbf{m}_v \cdot \partial_{\alpha} \mathbf{m}_v + \frac{1}{2} A_{\text{FM}}^z \partial_z \mathbf{m}_v \cdot \partial_z \mathbf{m}_v - \frac{1}{2} B_{\text{an}} M_s (m_{1z}^2 + m_{2z}^2) - M_s \mathbf{B} \cdot (\mathbf{m}_1 + \mathbf{m}_2) + \frac{1}{2} \mathcal{B}_{\text{ALT}} (\mathbf{m}_1 \cdot \hat{\mathcal{D}} \mathbf{m}_1 - \mathbf{m}_2 \cdot \hat{\mathcal{D}} \mathbf{m}_2) + B_{\text{AFM}} \partial_{zz} \mathbf{m}_1 \cdot \partial_{zz} \mathbf{m}_2 + \frac{1}{2} B_{ij} \partial_{ii} \mathbf{m}_v \cdot \partial_{jj} \mathbf{m}_v \right] d\mathbf{r}. \quad (\text{E1})$$

Here indices $\alpha \in \{x, y\}$ and $i, j \in \{x, y, z\}$ numerate coordinates, $v = 1, 2$ numerates the sublattices, and the summation over the repeating indices is assumed. We introduced the following parameters: $M_s = \mu_s/V_{\text{puc}}$ is the saturation magnetization of one sublattice, $A_{\text{AFM}} = J_{\text{AFM}} c_0 / (\sqrt{3} a_0^2)$ is antiferromagnetic stiffness, $A_{\text{FM}}^{\perp} = \sqrt{3}(J_{\text{FM}} - 3\tilde{J})/c_0$ is ferromagnetic stiffness in the xy plane, $A_{\text{FM}}^z = -8\sqrt{3}\tilde{J}c_0/a_0^2$ is stiffness in the z direction, $B_{\text{an}} = 2K/\mu_s$ is the anisotropy field, $B_{\text{AFM}} = J_{\text{AFM}} c_0^3 / (12\sqrt{3} a_0^2)$, and tensor B_{ij} has components $B_{xx} = B_{yy} = B_{xy} = B_{yx} = -\frac{\sqrt{3}}{16} \frac{a_0^2}{c_0} (J_{\text{FM}} - 9\tilde{J})$, $B_{xz} = B_{zx} = B_{yz} = B_{zy} = 3\sqrt{3}c_0\tilde{J}$, $B_{zz} = \frac{8}{\sqrt{3}} \frac{c_0^3}{a_0^2} \tilde{J}$.

The introduced constants determine typical length scales in the xy plane $\ell_{\perp} = \sqrt{A_{\text{FM}}^{\perp}/(M_s B_{\text{an}})}$ and in the z direction $\ell_z = \sqrt{(A_{\text{FM}}^z + A_{\text{AFM}})/(M_s B_{\text{an}})}$. Typical timescale is determined by the frequency of uniform antiferromagnetic resonance $\omega_{\text{AFMR}} = \gamma B_{\text{sf}}$ with $B_{\text{sf}} = \sqrt{B_{\text{ex}} B_{\text{an}}}$ being the spin-flop field. The parameters $\ell_{\perp}$, ℓ_z , ω_{AFMR} , and B_{sf} coincide with the same parameters in the main text determined via constants of the discrete model.

In continuum approximation, the DMI Hamiltonian (D1) gives the leading-order contribution in the form

$$\mathcal{H}_{\text{DMI}} \approx \frac{D_{\perp} \sqrt{3}}{2c_0} \int \{[\mathbf{m}_1 \times \hat{\mathcal{D}}_x \mathbf{m}_2]_x - [\mathbf{m}_1 \times \hat{\mathcal{D}}_y \mathbf{m}_2]_y\} d\mathbf{r}, \quad (\text{E2})$$

where $\hat{\mathcal{D}}_x = \frac{1}{2}(\partial_{xx} - \partial_{yy})$, and $\hat{\mathcal{D}}_y = \partial_{xy}$. Note that expression (E2) is invariant under the screw-symmetry operation, i.e., under the combined action of the rotation by $\pi/3$ and permutation of the sublattices. The DMI in form (E2) contributes to the magnetization (11) leading to the correction $\mathbf{m} \rightarrow \mathbf{m} + \delta \mathbf{m}_{\text{DMI}}$ with

$$\delta \mathbf{m}_{\text{DMI}} = \frac{3D_{\perp} a_0^2}{8J_{\text{AFM}}} \mathbf{n} \times [\mathcal{D}_x \mathbf{n} \times \mathbf{e}_x - \mathcal{D}_y \mathbf{n} \times \mathbf{e}_y] \times \mathbf{n}. \quad (\text{E3})$$

For the domain wall shown in Fig. 8, i.e., for $\varphi^* = \frac{\pi}{6}$ and $\phi_0 = 0$, the DMI-induced magnetization (E3) has components

$$\begin{aligned} (\delta m_{\text{DMI}})_z &= \frac{9\sqrt{3}}{128} \frac{a_0^2}{\ell_{\perp}^2} \frac{D_{\perp}}{J_{\text{AFM}}} \frac{1}{\cosh^3(\xi/x_{\text{DW}})}, \\ (\delta m_{\text{DMI}})_x &= \sinh(\xi/x_{\text{DW}}) (\delta m_{\text{DMI}})_z, \\ (\delta m_{\text{DMI}})_y &= \frac{2}{\sqrt{3}} \sinh(\xi/x_{\text{DW}}) (\delta m_{\text{DMI}})_z. \end{aligned} \quad (\text{E4})$$

Thus, the DMI-induced magnetization (E4) has the opposite parity as compared to the altermagnetically induced magnetization. Namely, $(\delta m_{\text{DMI}})_z$ is even while the in-plane components are odd. This property can be used to distinguish the altermagnetic and DMI contributions to the magnetization.

- [1] L. Šmejkal, R. González-Hernández, T. Jungwirth, and J. Sinova, Crystal time-reversal symmetry breaking and spontaneous Hall effect in collinear antiferromagnets, *Sci. Adv.* **6**, eaaz8809 (2020).
- [2] M. Naka, S. Hayami, H. Kusunose, Y. Yanagi, Y. Motome, and H. Seo, Spin current generation in organic antiferromagnets, *Nat. Commun.* **10**, 4305 (2019).
- [3] L.-D. Yuan, Z. Wang, J.-W. Luo, E. I. Rashba, and A. Zunger, Giant momentum-dependent spin splitting in centrosymmetric low- z antiferromagnets, *Phys. Rev. B* **102**, 014422 (2020).
- [4] M. Naka, Y. Motome, and H. Seo, Perovskite as a spin current generator, *Phys. Rev. B* **103**, 125114 (2021).
- [5] Y. Guo, H. Liu, O. Janson, I. C. Fulga, J. van den Brink, and J. I. Facio, Spin-split collinear antiferromagnets: A large-scale *ab-initio* study, *Mater. Today Phys.* **32**, 100991 (2023).
- [6] L. Šmejkal, J. Sinova, and T. Jungwirth, Emerging research landscape of altermagnetism, *Phys. Rev. X* **12**, 040501 (2022).
- [7] L. Šmejkal, J. Sinova, and T. Jungwirth, Beyond conventional ferromagnetism and antiferromagnetism: A phase with nonrelativistic spin and crystal rotation symmetry, *Phys. Rev. X* **12**, 031042 (2022).
- [8] L. Šmejkal, A. H. MacDonald, J. Sinova, S. Nakatsuji, and T. Jungwirth, Anomalous Hall antiferromagnets, *Nat. Rev. Mater.* **7**, 482 (2022).
- [9] T. Sato, S. Haddad, I. C. Fulga, F. F. Assaad, and J. van den Brink, Altermagnetic anomalous Hall effect emerging from electronic correlations, *Phys. Rev. Lett.* **133**, 086503 (2024).
- [10] O. Gomonay, V. P. Kravchuk, R. Jaeschke-Ubiergo, K. V. Yershov, T. Jungwirth, L. Šmejkal, J. v. d. Brink, and J. Sinova, Structure, control, and dynamics of altermagnetic textures, *npj Spintronics* **2**, 35 (2024).
- [11] P. M. Cõnsoli, M. Fornoville, and M. Vojta, Fluctuation-induced ferrimagnetism in sublattice-imbalanced antiferromagnets with application to $\text{SrCu}_2(\text{BO}_3)_2$ under pressure, *Phys. Rev. B* **104**, 064422 (2021).
- [12] K. V. Yershov, V. P. Kravchuk, M. Daghofer, and J. van den Brink, Fluctuation-induced piezomagnetism in local moment altermagnets, *Phys. Rev. B* **110**, 144421 (2024).
- [13] K. V. Yershov, O. Gomonay, J. Sinova, J. van den Brink, and V. P. Kravchuk, Curvature-induced magnetization of altermagnetic films, *Phys. Rev. Lett.* **134**, 116701 (2025).
- [14] S. Reimers, L. Odenbreit, L. Šmejkal, V. N. Strocov, P. Constantinou, A. B. Hellenes, R. Jaeschke Ubiergo, W. H. Campos, V. K. Bharadwaj, A. Chakraborty, T. Denneulin, W. Shi, R. E. Dunin-Borkowski, S. Das, M. Kläui, J. Sinova, and M. Jourdan, Direct observation of altermagnetic band splitting in CrSb thin films, *Nat. Commun.* **15**, 2116 (2024).
- [15] C. Li, M. Hu, Z. Li, Y. Wang, W. Chen, B. Thiagarajan, M. Leandersson, C. Polley, T. Kim, H. Liu, C. Fulga, M. G. Vergniory, O. Janson, O. Tjernberg, and J. van den Brink, Topological weyl altermagnetism in CrSb, *Commun. Phys.* **8**, 311 (2025).
- [16] G. Yang, Z. Li, S. Yang, J. Li, H. Zheng, W. Zhu, Z. Pan, Y. Xu, S. Cao, W. Zhao, A. Jana, J. Zhang, M. Ye, Y. Song, L.-H. Hu, L. Yang, J. Fujii, I. Vobornik, M. Shi, H. Yuan *et al.*, Three-dimensional mapping of the altermagnetic spin splitting in CrSb, *Nat. Commun.* **16**, 1442 (2025).
- [17] M. Zeng, M. Zhu, Y. Zhu, X. Liu, X. Ma, Y. Hao, P. Liu, G. Qu, Y. Yang, Z. Jiang, K. Yamagami, M. Arita, X. Zhang, T. Shao, Y. Dai, K. Shimada, Z. Liu, M. Ye, Y. Huang, Q. Liu *et al.*, Observation of spin splitting in room temperature metallic antiferromagnet crsb, *Adv. Sci.* **11**, 2406529 (2024).
- [18] J. Ding, Z. Jiang, X. Chen, Z. Tao, Z. Liu, T. Li, J. Liu, J. Sun, J. Cheng, J. Liu, Y. Yang, R. Zhang, L. Deng, W. Jing, Y. Huang, Y. Shi, M. Ye, S. Qiao, Y. Wang, Y. Guo *et al.*, Large band splitting in g -wave altermagnet crsb, *Phys. Rev. Lett.* **133**, 206401 (2024).
- [19] I. I. Mazin, Altermagnetism in mnte: Origin, predicted manifestations, and routes to detwinning, *Phys. Rev. B* **107**, L100418 (2023).
- [20] J. Krempaský, L. Šmejkal, S. W. D'Souza, M. Hajlaoui, G. Springholz, K. Uhlířová, F. Alarab, P. C. Constantinou, V. Strocov, D. Usanov, W. R. Pudelko, R. González-Hernández, A. Birk Hellenes, Z. Jansa, H. Reichlová, Z. Šobáň, R. D. Gonzalez Betancourt, P. Wadley, J. Sinova, D. Kriegner *et al.*, Altermagnetic lifting of kramers spin degeneracy, *Nature (London)* **626**, 517 (2024).
- [21] S. Lee, S. Lee, S. Jung, J. Jung, D. Kim, Y. Lee, B. Seok, J. Kim, B. G. Park, L. Šmejkal, C.-J. Kang, and C. Kim, Broken kramers degeneracy in altermagnetic mnte, *Phys. Rev. Lett.* **132**, 036702 (2024).
- [22] T. Osumi, S. Souma, T. Aoyama, K. Yamauchi, A. Honma, K. Nakayama, T. Takahashi, K. Ohgushi, and T. Sato, Observation of a giant band splitting in altermagnetic mnte, *Phys. Rev. B* **109**, 115102 (2024).
- [23] H.-H. Yang, L. Desplat, V. P. Kravchuk, M. Hervé, T. Balashov, S. Gerber, M. Garst, B. Dupé, and W. Wulfhekel, Observation of the sliding phason mode of the incommensurate magnetic texture in $\text{Fe/Ir}(111)$, *npj Quantum Mater.* **9**, 54 (2024).
- [24] Z. Liu, M. Ozeki, S. Asai, S. Itoh, and T. Masuda, Chiral split magnon in altermagnetic mnte, *Phys. Rev. Lett.* **133**, 156702 (2024).
- [25] S. Polesya, G. Kuhn, S. Mankovsky, H. Ebert, M. Regus, and W. Bensch, Structural and magnetic properties of CrSb compounds: Nias structure, *J. Phys.: Condens. Matter* **24**, 036004 (2012).
- [26] W. J. Takei, D. E. Cox, and G. Shirane, Magnetic structures in the MnSb-CrSb system, *Phys. Rev.* **129**, 2008 (1963).
- [27] I. I. Mazin and K. D. Belashchenko, Origin of the gossamer ferromagnetism in mnte, *Phys. Rev. B* **110**, 214436 (2024).
- [28] S. Greenwald, The antiferromagnetic structure deformations in CoO and MnTe, *Acta Crystallogr.* **6**, 396 (1953).
- [29] *Mnte Crystal Structure: Datasheet from "Pauling File Multinaries Edition, 2022"* (Springer, Berlin, 2023).
- [30] A. N. Bogdanov, A. V. Zhuravlev, and U. K. Rößler, Spin-flop transition in uniaxial antiferromagnets: Magnetic phases, reorientation effects, and multidomain states, *Phys. Rev. B* **75**, 094425 (2007).
- [31] S. V. Tyablikov, *Methods in the Quantum Theory of Magnetism*, 2nd ed. (Nauka, Moscow, 1975) [translation of 1st Russian edition (Plenum, New York, 1967)].
- [32] T. Holstein and H. Primakoff, Field dependence of the intrinsic domain magnetization of a ferromagnet, *Phys. Rev.* **58**, 1098 (1940).
- [33] S. M. Rezende, A. Azevedo, and R. L. Rodríguez-Suárez, Introduction to antiferromagnetic magnons, *J. Appl. Phys.* **126**, 151101 (2019).
- [34] N. W. Ashcroft and N. Mermin, *Solid State Physics* (Dover, New York, 1976).

- [35] Y. Nambu, J. Barker, Y. Okino, T. Kikkawa, Y. Shiomi, M. Enderle, T. Weber, B. Winn, M. Graves-Brook, J. M. Tranquada, T. Ziman, M. Fujita, G. Bauer, E. Saitoh, and K. Kakurai, Observation of magnon polarization, [Phys. Rev. Lett.](#) **125**, 027201 (2020).
- [36] O. J. Amin, A. Dal Din, E. Golias, Y. Niu, A. Zakharov, S. C. Fromage, C. J. B. Fields, S. L. Heywood, R. B. Cousins, F. Maccherozzi, J. Krempaský, J. H. Dil, D. Kriegner, B. Kiraly, R. P. Champion, A. W. Rushforth, K. W. Edmonds, S. S. Dhesi, L. Šmejkal, T. Jungwirth *et al.*, Nanoscale imaging and control of altermagnetism in mnte, [Nature \(London\)](#) **636**, 348 (2024).
- [37] J. P. Perdew, K. Burke, and M. Ernzerhof, Generalized gradient approximation made simple, [Phys. Rev. Lett.](#) **77**, 3865 (1996).
- [38] K. Koepnick and H. Eschrig, Full-potential nonorthogonal local-orbital minimum-basis band-structure scheme, [Phys. Rev. B](#) **59**, 1743 (1999).
- [39] S. P. Bommanaboyena, C. Müller, M. Jarošová, K. Wolk, S. Telkamp, P. Zeng, F. Krizek, T. Uchimura, A. Badura, K. Olejník, D. Scheffler, K. Beranová, S. Banerjee, M. Ledinský, H. Reichlová, T. Jungwirth, L. Horák, and D. Kriegner, Single-crystalline CrSb(0001) thin films grown by dc magnetron co-sputtering, [Phys. Rev. Mater.](#) **9**, 064402 (2025).
- [40] S. Abe, T. Kaneko, M. Ohashi, H. Yoshida, and K. Kamigaki, Magnetic Properties of CrSb, [J. Phys. Soc. Jpn.](#) **53**, 2703 (1984).
- [41] G. Kuhn, S. Polesya, S. Mankovsky, J. Minár, H. Ebert, M. Regus, and W. Bensch, Magnetic properties of CrSb compounds with zinc-blende and wurtzite structures, [J. Phys.: Condens. Matter](#) **24**, 306005 (2012).
- [42] Y.-F. Zhang, X.-S. Ni, K. Chen, and K. Cao, Chiral magnon splitting in altermagnetic CrSb from first principles, [Phys. Rev. B](#) **111**, 174451 (2025).