

Quantum geometry and elliptic optical dichroism in p -wave magnets

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(Received 23 April 2025; revised 10 June 2025; accepted 16 June 2025; published 1 July 2025)

The quantum geometric tensor is composed of the Berry curvature and the quantum metric, which is observable by means of optical absorption of elliptically polarized light. Especially, the quantum geometric tensor at the zero-momentum is observable by the optical absorption at the optical band edge. In this context, we study optical absorption of a p -wave magnet under irradiation of elliptically polarized light. The p -wave magnet has a band splitting along one axis, which we choose as the x axis. We obtain exact analytic formulas for the optical conductivity when the Néel vector is along the x , y , and z axis. The optical conductivity strongly depends on the degree of ellipticity of light, which is an elliptic dichroism. Especially, there is a perfect elliptic optical dichroism when the Néel vector is along the y axis. It is possible to determine the Néel vector by measuring the ellipticity of the perfect elliptic dichroism, which will be a key to future magnetic memory based on the p -wave magnet.

DOI: [10.1103/PhysRevB.112.045302](https://doi.org/10.1103/PhysRevB.112.045302)

I. INTRODUCTION

Quantum geometry attracts renewed attention in condensed-matter physics. Recent studies demonstrate that they appear in optical conductivity [1–12] and electric nonlinear conductivity [13–21]. The quantum geometry is characterized by the quantum geometric tensor, whose imaginary part is the Berry curvature and real part is the quantum metric. The Berry curvature is observable by the anomalous Hall effect. On the other hand, it is possible to observe the quantum metric by the optical conductivity under linearly polarized light [5–9, 11, 12].

Optical absorption experiments provide us with a powerful tool to observe material properties. For example, the band gap is determined by the edge of the optical absorption bands. Especially, circular optical dichroism of the gapped Dirac system can determine the chirality of the Dirac cones [22–25]. For example, there is no optical absorption at the band edge for the left-polarized light but there is a finite optical absorption for the right-polarized light. In a similar way, elliptic optical dichroism was studied in the anisotropic Dirac system [26]. In this context, the p -wave magnetic system with the Rashba interaction is interesting because the Rashba interaction forms a Dirac cone.

The band structure of the p -wave magnet has the p -wave symmetry. Because its net magnetization is zero [27], the p -wave magnet will be useful for future ultrafast and high-density memory. The electron coupled with the p -wave magnet feels a p -wave symmetric field. Recently, an experiment on the p -wave magnet $\text{Gd}_3\text{Ru}_4\text{Al}_{12}$ was reported [28].

Especially, the p -wave magnet at the interface is studied [29–31], where the Rashba interaction is present. In general, it is difficult to read out the Néel vector of the p -wave magnet due to the absence of the net magnetization. It was pointed out that this is possible by measuring linear and nonlinear conductivities. It is beneficial if the Néel vector of the p -wave magnet can be read out solely by optical absorption.

In this paper, we study optical absorption in a p -wave magnet coupled with the Rashba interaction. We derive the formulas for the quantum metric and the Berry curvature with the use of the optical conductivity under elliptically polarized light. The quantum metric and the Berry curvature at the zero momentum are especially observed by the optical absorption at the optical band edge. The p -wave magnet has a band splitting along one axis, which we choose as the x axis. We analytically calculate the optical conductivity under irradiation of elliptically polarized light. The optical conductivity is exactly calculated when the Néel is along the x , y , and z axis. The optical conductivity strongly depends on the degree of ellipticity of light, which is an elliptic dichroism. When the Néel vector is along the y axis, the perfect elliptic dichroism occurs, where there is no optical absorption at the optical band edge for a certain right-polarized light but there is a finite optical absorption for a corresponding left-polarized light.

This paper is composed as follows. In Sec. II, we introduce a model for a p -wave magnet and show the energy spectrum. In Sec. III, we review the optical absorption under elliptically polarized light. In Sec. IV, we relate the optical absorption under the elliptically polarized light and quantum geometric properties including the Berry curvature and the quantum metric. In Sec. V, we calculate the Berry curvature and the quantum metric for the p -wave magnet. In Sec. VI, we explicitly calculate the optical conductivity under the elliptically polarized light for p -wave magnets. We find a perfect elliptic dichroism occurs when the Néel is along the y direction.

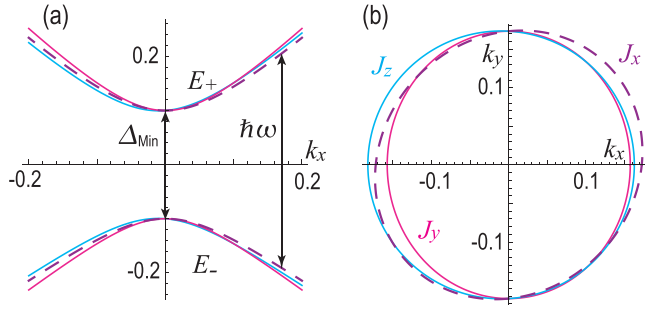


FIG. 1. (a) Band structure along the k_x axis. A gapped Dirac cone is formed with its center at $\mathbf{k} = \mathbf{0}$. The horizontal axis is $-0.2 < k_x/k_0 < 0.2$. The vertical axis is the energy in units of ε_0 . (b) The polar plot of $k_0(\phi)$ at $\omega = 0.4\varepsilon_0/\hbar$, where $k_0(\phi)$ is given by Eq. (26). The Néel vector is along the y axis ($\Theta = \pi/2$, $\Phi = \pi/2$) for magenta curves, along the x axis ($\Theta = \pi/2$, $\Phi = 0$) for dashed purple curves, and along the z axis ($\Theta = 0$) for cyan curves. We have set $J = 0.1\varepsilon_0/k_0$, $\lambda = \varepsilon_0/k_0$, $B = 0.1\varepsilon_0$ and $\hbar^2 k_0^2/(2m) = \varepsilon_0/2$, where ε_0 is a unit of the energy and k_0 is a unit of the momentum.

Sec. VII is devoted to discussions on possible applications to spintronic magnetic memories.

II. p -WAVE MAGNET

A. Hamiltonian

We consider a p -wave magnet on a substrate, whose minimal model is constructed as follows. First, the simplest Hamiltonian term [30,31] respecting the p -wave symmetry of the p -wave magnet is given by the term $k_x J \cdot \sigma$ with the p -wave Néel vector J and the Pauli matrix σ . Second, the circular optical dichroism is known to occur in the gapped Dirac system [22–25], which is induced by the Rashba interaction introduced by placing a sample on the substrate [21,30,32–38]. The minimal Hamiltonian is given by [28,30,31,39]

$$H(\mathbf{k}) = \frac{\hbar^2(k_x^2 + k_y^2)}{2m} \sigma_0 + \lambda(k_x \sigma_y - k_y \sigma_x) + k_x \mathbf{J} \cdot \sigma + B \sigma_z, \quad (1)$$

where m is the effective mass of free electrons, λ is the magnitude of the Rashba interaction, and

$$\mathbf{J} = J(\sin \Theta \cos \Phi, \sin \Theta \sin \Phi, \cos \Theta) \quad (2)$$

is the p -wave Néel vector with J its magnitude [27,29,30,39–41]. The $B\sigma_z$ term induces the band gap at the Dirac point, which is introduced by applying an external magnetic field or attaching a ferromagnet.

The energy of the Hamiltonian (1) is given by

$$E_{\pm}(\mathbf{k}) = \frac{\hbar^2(k_x^2 + k_y^2)}{2m} \pm \Delta(\mathbf{k}), \quad (3)$$

with

$$\Delta(\mathbf{k}) = \sqrt{(J_x k_x - \lambda k_y)^2 + (J_y k_x + \lambda k_x)^2 + (J_z k_x + B)^2}. \quad (4)$$

The band structure along the k_x axis is shown in Fig. 1(a). A gapped Dirac cone is formed with its center at $\mathbf{k} = \mathbf{0}$.

The free electron term $\hbar^2 k^2/(2m)$ does not play an important role compared with the interband transition $2\Delta(\mathbf{k})$

in optical absorption, as is understood from the formula (8) which we derive later.

B. Material parameters

We summarize material parameters in the Hamiltonian (1). The mass is given by $m = \hbar^2/(ta^2)$, where it is estimated [27,31,42] that the hopping amplitude $t = 1\text{eV}$ and the lattice constant $a = 8.1 \text{ \AA}$. The magnitude of the p -wave spin splitting is estimated [27,42] as 0.2eV . Then, the p -wave magnetization is estimated [31] as $J = 1.6\text{eV}\text{\AA}$. The strength of the Rashba interaction is $\lambda = 0.07\text{eV}\text{\AA}$ at the InGaAs/InAlAs surface [43]. The energy is of the order of 1meV . Nevertheless, all formulas are valid for any values of these parameters.

III. OPTICAL ABSORPTION

We study optical interband transitions from the state $|\psi_-(\mathbf{k})\rangle$ in the valence band to the state $|\psi_+(\mathbf{k})\rangle$ in the conduction band. We apply a beam of elliptical polarized light perpendicular onto the sample, where the corresponding electromagnetic potential is given by $\mathbf{A}(t) = (A_x \sin \omega t, A_y \cos \omega t)$. The electromagnetic potential is introduced into the Hamiltonian by way of the minimal substitution, that is, by replacing the momentum k_j with the covariant momentum $P_j \equiv \hbar k_j + eA_j$.

We start with the optical matrix element between the initial and final states in the photoemission process given by [22–26]

$$P_i(\mathbf{k}) \equiv \hbar \langle \psi_+(\mathbf{k}) | v_i | \psi_-(\mathbf{k}) \rangle, \quad (5)$$

with the velocity

$$v_\mu = \frac{\partial H(\mathbf{k})}{\hbar \partial k_\mu}. \quad (6)$$

The optical matrix element of the elliptic polarization is given by

$$P_\vartheta(\mathbf{k}) = P_x(\mathbf{k}) \cos \vartheta + iP_y(\mathbf{k}) \sin \vartheta, \quad (7)$$

where ϑ is the ellipticity of the injected beam, with $0 < \vartheta < \pi$ for the right polarization and $-\pi < \vartheta < 0$ for the left polarization. $P_{\pi/4}(\mathbf{k})$ corresponds to the right circularly polarized right and $P_{-\pi/4}(\mathbf{k})$ corresponds to the left circularly polarized right.

Optical absorption is described by the optical conductivity

$$\sigma(\omega; \vartheta) = \frac{\sigma_0}{\omega^2} \int d\mathbf{k} [f_-(\mathbf{k}) - f_+(\mathbf{k})] |P_\vartheta(\mathbf{k})|^2 \times \delta[E_+(\mathbf{k}) - E_-(\mathbf{k}) - \hbar\omega], \quad (8)$$

where $f_{\pm}(\mathbf{k}) = 1/(\exp[(E_{\pm}(\mathbf{k}) - \mu)/(k_B T) + 1])$ is the Fermi distribution function and we have set

$$\sigma_0 \equiv \frac{\pi e^2}{(2\pi)^2 \varepsilon_0}. \quad (9)$$

In the following, we assume that the temperature is absolutely zero.

IV. QUANTUM GEOMETRY AND ELLIPTIC DICHROISM

The quantum geometric tensor is defined by [44,45]

$$\mathcal{F}_{mn}^{\mu\nu}(\mathbf{k}) = \langle \partial_{k_\mu} \psi_m(\mathbf{k}) | 1 - \mathcal{P}(\mathbf{k}) | \partial_{k_\nu} \psi_n(\mathbf{k}) \rangle, \quad (10)$$

where

$$\mathcal{P}(\mathbf{k}) \equiv \sum_{n \in \text{Occupied}} |u_n(\mathbf{k})\rangle \langle u_n(\mathbf{k})| \quad (11)$$

is the projection operator to the occupied band. In the two-band model, its diagonal component is simply given by

$$\mathcal{F}_{--}^{\mu\nu}(\mathbf{k}) = \langle \partial_{k_\mu} \psi_-(\mathbf{k}) | \psi_+(\mathbf{k}) \rangle \langle \psi_+(\mathbf{k}) | \partial_{k_\nu} \psi_-(\mathbf{k}) \rangle. \quad (12)$$

$a = 8.1$ The optical matrix element for the elliptic polarized light is expanded as

$$|P_\vartheta(\mathbf{k})|^2 = |P_x(\mathbf{k})|^2 \cos^2 \vartheta + |P_y(\mathbf{k})|^2 \sin^2 \vartheta + i[P_x^*(\mathbf{k})P_y(\mathbf{k}) - P_y^*(\mathbf{k})P_x(\mathbf{k})] \sin \vartheta \cos \vartheta. \quad (13)$$

By using the Hellmann-Feynman theorem

$$\begin{aligned} \langle \psi_m(\mathbf{k}) | v_\mu | \psi_n(\mathbf{k}) \rangle \\ = \frac{1}{\hbar} (E_n(\mathbf{k}) - E_m(\mathbf{k})) \langle \psi_m(\mathbf{k}) | \partial_{k_\mu} | \psi_n(\mathbf{k}) \rangle \end{aligned} \quad (14)$$

for $m \neq n$, we have

$$\begin{aligned} |P_\mu(\mathbf{k})|^2 &= \hbar^2 \langle \psi_-(\mathbf{k}) | v_\mu | \psi_+(\mathbf{k}) \rangle \langle \psi_+(\mathbf{k}) | v_\mu | \psi_-(\mathbf{k}) \rangle \\ &= -\Delta^2(\mathbf{k}) \langle \partial_{k_\mu} \psi_-(\mathbf{k}) | \psi_+(\mathbf{k}) \rangle \langle \psi_+(\mathbf{k}) | \partial_{k_\mu} \psi_-(\mathbf{k}) \rangle \\ &= -\Delta^2(\mathbf{k}) g_{\mu\mu}(\mathbf{k}) \end{aligned} \quad (15)$$

with $\mu = x, y$, and

$$\begin{aligned} i[P_y^*(\mathbf{k})P_x(\mathbf{k}) - P_x^*(\mathbf{k})P_y(\mathbf{k})] \\ = i\Delta^2(\mathbf{k}) [\langle \psi_-(\mathbf{k}) | v_y | \psi_+(\mathbf{k}) \rangle \langle \psi_+(\mathbf{k}) | v_x | \psi_-(\mathbf{k}) \rangle \\ - \langle \psi_-(\mathbf{k}) | v_x | \psi_+(\mathbf{k}) \rangle \langle \psi_+(\mathbf{k}) | v_y | \psi_-(\mathbf{k}) \rangle] \\ = i\Delta^2(\mathbf{k}) [\mathcal{F}_{-+}^{xy}(\mathbf{k}) - \mathcal{F}_{+-}^{yx}(\mathbf{k})] = \Delta^2(\mathbf{k}) \Omega_{xy}(\mathbf{k}). \end{aligned} \quad (16)$$

Then, the optical conductivity is rewritten by using the quantum geometric information as

$$\sigma(\omega; \vartheta) = \hbar^2 \sigma_0 \int d\mathbf{k} f(\mathbf{k}) G(\mathbf{k}) \delta[E_+(\mathbf{k}) - E_-(\mathbf{k}) - \hbar\omega], \quad (17)$$

with

$$\begin{aligned} G(\mathbf{k}; \vartheta) &\equiv g_{xx}(\mathbf{k}) \cos^2 \vartheta + g_{yy}(\mathbf{k}) \sin^2 \vartheta \\ &\quad + \Omega_{xy}(\mathbf{k}) \sin \vartheta \cos \vartheta. \end{aligned} \quad (18)$$

V. QUANTUM GEOMETRY IN *p*-WAVE MAGNETS

The two-band Hamiltonian is expressed in general as

$$H = h_0 + \sum_{j=x,y,z} h_j \sigma_j, \quad (19)$$

with the use of the Pauli matrix σ_j . The energy spectrum is given by

$$E(\mathbf{k}) = \sqrt{\sum_{j=0}^N h_j^2(\mathbf{k})}. \quad (20)$$

The quantum metric for the two-band system is explicitly given by [9–12, 46, 47]

$$g_{\mu\nu}(\mathbf{k}) = \frac{1}{2} (\partial_{k_\mu} \mathbf{n}) \cdot (\partial_{k_\nu} \mathbf{n}), \quad (21)$$

where $n_j(\mathbf{k}) = h_j(\mathbf{k})/E(\mathbf{k})$ is the normalized Dirac vector with the energy. The Berry curvature for the two-band system is explicitly given by [48–50]

$$\Omega_{xy} = -\frac{1}{2} [\mathbf{n} \cdot (\partial_x \mathbf{n} \times \partial_y \mathbf{n})]. \quad (22)$$

The quantum metrics for the Hamiltonian (1) are given by

$$\begin{aligned} g_{xx}(\mathbf{k}) &= \frac{(\lambda k_y J_z + B J_x)^2 + (\lambda^2 k_y^2 + B^2)(\lambda + J_y)^2}{2E^4(\mathbf{k})}, \\ g_{yy}(\mathbf{k}) &= \lambda^2 \frac{(B + J_x k_x)^2 + k_x^2(\lambda + J_y)^2}{2E^4(\mathbf{k})}. \end{aligned} \quad (23)$$

The Berry curvature for the Hamiltonian (1) is given by

$$\Omega_{xy}(\mathbf{k}) = -\frac{B\lambda(\lambda + J_y)}{2E^3(\mathbf{k})}. \quad (24)$$

VI. OPTICAL CONDUCTIVITY IN *p*-WAVE MAGNETS

The optical conductivity (8) is rewritten as

$$\sigma(\omega; \vartheta) = \sigma_0 \int k dk d\phi \frac{G(\mathbf{k})}{2|\partial_k \Delta \mathbf{k}|} \delta(k - k_0(\phi)), \quad (25)$$

where $k_0(\phi)$ is the solution of the resonant condition $2\Delta(k_0(\phi)) = \hbar\omega$ and $k_x = k \cos \phi$ and $k_y = k \sin \phi$. It is explicitly given by

$$k_0(\phi) = \frac{-BJ_z \cos \phi + \sqrt{B^2 J_z^2 \cos^2 \phi - (B^2 - \frac{\omega^2}{4})F(\phi)}}{F(\phi)} \quad (26)$$

with

$$F(\phi) \equiv \lambda^2 + J^2 \cos^2 \phi - 2J\lambda \sin \Theta \cos \phi \sin(\phi - \Phi). \quad (27)$$

We show the polar plot of $k_0(\phi)$ in Fig. 1(b).

We have

$$\frac{\partial H}{\partial k_x} = \frac{\hbar^2 k_x}{m} + \lambda k_x \sigma_y + J \mathbf{n} \cdot \boldsymbol{\sigma}, \quad (28)$$

$$\frac{\partial H}{\partial k_y} = \frac{\hbar^2 k_y}{m} - \lambda \sigma_x. \quad (29)$$

In the optical absorption the energy difference of the conduction and valence bands $\Delta(\mathbf{k}) \equiv E_+(\mathbf{k}) - E_-(\mathbf{k})$ are important because the minimum of $\Delta(\mathbf{k})$ is

$$\Delta_{\text{Min}} = \frac{2|B(\lambda + J_y)|}{\sqrt{(\lambda + J_y)^2 + J_z^2}} \quad (30)$$

at

$$\begin{aligned} k_x^{\text{Min}} &= -\frac{BJ_z}{J_z^2 + (\lambda + J_y)^2}, \\ k_y^{\text{Min}} &= -\frac{BJ_z J_x}{\lambda(J_z^2 + (\lambda + J_y)^2)}. \end{aligned} \quad (31)$$

Especially, if $J_z = 0$, the band edge is taken at the Dirac point ($k_x = k_y = 0$), where the band gap is given by $2\Delta(0) = 2|B|$.

A. The case of $\mathbf{J} = (0, J_y, 0)$

We start with a specific case, where the Néel vector is along the y axis. The optical conductivity (25) is exactly calculated

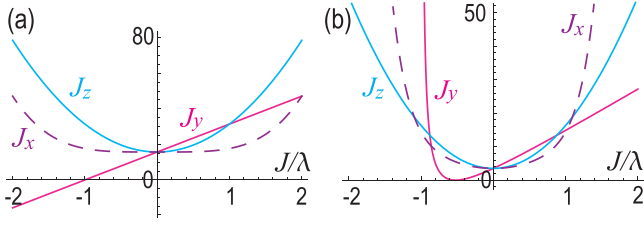


FIG. 2. Optical conductivity $\sigma(2|B|/\hbar; \vartheta)$ at the optical band edge as a function of J/λ under linear polarized light for (a) $\vartheta = 0$ and (b) $\vartheta = \pi/2$. See the caption of Fig. 1 for the colors. We have set $\lambda = \varepsilon_0/k_0$, $B = 0.1\varepsilon_0$ and $\hbar^2 k_0^2/(2m) = \varepsilon_0/2$, where ε_0 is a unit of the energy and k_0 is a unit of the momentum.

as

$$\frac{\sigma(\omega; \vartheta)}{\sigma_0} = \frac{\pi|\lambda + J_y|(\hbar^2\omega^2 + 4B^2)}{2\lambda\hbar\omega^3} \cos^2 \vartheta + \frac{\pi\lambda(\hbar^2\omega^2 + 4B^2)}{2|\lambda + J_y|\hbar\omega^3} \sin^2 \vartheta - \frac{4B\pi}{\omega^2} \cos \vartheta \sin \vartheta \quad (32)$$

for $\hbar\omega > 2|B|$, and $\sigma(\omega; \vartheta) = 0$ for $\hbar\omega < 2|B|$. At the optical band edge $\omega = 2|B|$, we have

$$G(\mathbf{0}; \vartheta) = \frac{(\lambda + J_y)^2}{32B^2} \cos^2 \vartheta + \frac{\lambda^2}{32B^2} \sin^2 \vartheta - \frac{\lambda(\lambda + J_y)}{16B^2} \sin \vartheta \cos \vartheta = \frac{\sqrt{\lambda^2 + (\lambda + J_y)^2}}{32B^2} \sin^2(\vartheta - \vartheta_0^y), \quad (33)$$

and the optical conductivity is simplified as

$$\frac{\sigma(2|B|/\hbar; \vartheta)}{\sigma_0} = \frac{\hbar^2 \sqrt{\lambda^2 + (\lambda + J_y)^2}}{4B^2(\lambda + J_y)} \sin^2(\vartheta - \vartheta_0^y), \quad (34)$$

because the band minimum exists at the Dirac point $k = 0$. It becomes zero at $\vartheta = \vartheta_0^y$ and $\vartheta_0^y + \pi$, which leads to a perfect elliptic optical dichroism. By searching the condition $\vartheta = \vartheta_0^y$, it is possible to determine the ratio $(\lambda + J_y)/\lambda$ experimentally. On the other hand, the optical conductivity takes the maximum value at $\vartheta = \vartheta_0^y \pm \pi/2$. The band gap $2|B|$ is determined by the optical band edge. By combining it with the result of the maximum optical conductivity $\sigma(2|B|; \vartheta_0^y \pm \pi/2)$, it is possible to determine B , λ and J_y experimentally. In addition, it is possible to differentiate two states J_y and $-J_y$ by the optical conductivity as shown in Figs. 2(a) and 2(b). This is one of the main results of this paper. It is interesting that it is impossible to differentiate J_x and J_z by the linear polarized light with $\Theta = 0$ but is possible by that with $\Theta = \pi/2$, as shown in Figs. 2(a) and 2(b).

The optical conductivity $\sigma(2|B|)$ at the optical band edge is shown as a function of ϑ in Fig. 3(a). It strongly depends on ϑ . Especially, it becomes exactly zero at $\vartheta = \vartheta_0^y$. The $\sigma(\omega)$ is shown in Fig. 3(b).

B. The case of $J = (J_x, 0, 0)$

We next study a specific case, where the Néel vector is along the x axis. The optical conductivity (25) is exactly

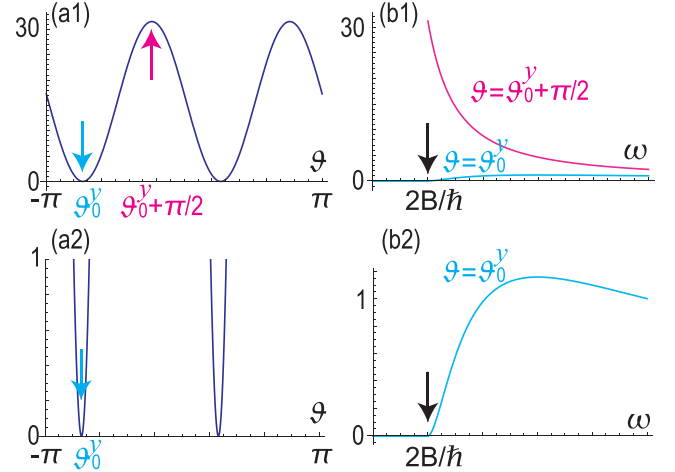


FIG. 3. Elliptic dichroism, where the Néel vector is taken along the y axis. (a1) and (a2) Optical conductivity as a function of ϑ . The horizontal axis is ϑ . The vertical axis is $\sigma(\omega)$ in units of σ_0 . (b1) and (b2) Optical conductivity as a function of ω . Magenta curves represents the $\sigma(\omega)$ with $\vartheta = \vartheta_0^y + \pi/2$, while cyan curves represents $\sigma(\omega)$ with $\vartheta = \vartheta_0^y$. (a2) and (b2) are the enlarged figures of (a1) and (b1) showing $\sigma(2|B|/\hbar; \vartheta_0^y) = 0$. The horizontal axis is $\hbar\omega$. We have set $J = 0.1\varepsilon_0/k_0$, $\lambda = \varepsilon_0/k_0$ and $B = 0.1\varepsilon_0$, where ε_0 is a unit of the energy and k_0 is a unit of the momentum.

calculated as

$$\frac{\sigma(\omega; \vartheta)}{\sigma_0} = \frac{\pi(\lambda^2 + J_x^2)(\hbar^2\omega^2 + 4B^2)}{2\lambda^2\hbar\omega^3} \cos^2 \vartheta + \frac{\pi(\hbar^2\omega^2 + 4B^2)}{2\hbar\omega^3} \sin^2 \vartheta - \frac{4B\pi}{\omega^2} \cos \vartheta \sin \vartheta \quad (35)$$

for $\hbar\omega > 2|B|$, and $\sigma(\omega; \vartheta) = 0$ for $\hbar\omega < 2|B|$. It is impossible to differentiate J_x and $-J_x$ because the optical conductivity is quadratic in J_x as shown in Figs. 2(a) and 2(b).

The optical conductivity $\sigma(2|B|)$ at the optical band edge is shown as a function of ϑ in Fig. 4(a1). The optical conductivity at the optical band edge is analytically calculated as

$$\sigma(2|B|/\hbar; \vartheta) = \frac{\hbar^2\pi J_x^2}{2B\lambda^2} \cos^2 \vartheta + \frac{\hbar^2\pi}{2B} (1 - \sin 2\vartheta), \quad (36)$$

where we have used the relation on the quantum geometry at the Dirac point,

$$G(\mathbf{0}; \vartheta) = \frac{J_x^2}{32B^2} \cos^2 \vartheta + \frac{\lambda^2}{32B^2} (1 - \sin 2\vartheta), \quad (37)$$

because the band minimum exists at the Dirac point $k = 0$. It strongly depends on ϑ . It takes the minimum at

$$\vartheta = \vartheta_0^x = -\arctan\left(\frac{2\lambda^2}{J_x^2}\right), \quad (38)$$

where the minimum value is

$$\sigma(2|B|/\hbar; \vartheta_0^x) = \frac{\pi\hbar^2}{2B} (2\lambda^2 + J_x^2 + \sqrt{4\lambda^4 + J_x^4}) > 0. \quad (39)$$

It does not become zero as shown in Fig. 4(a2). The $\sigma(\omega)$ is shown in Fig. 4(b1). The behavior is almost similar to that of the case where the Néel vector is along the y axis although

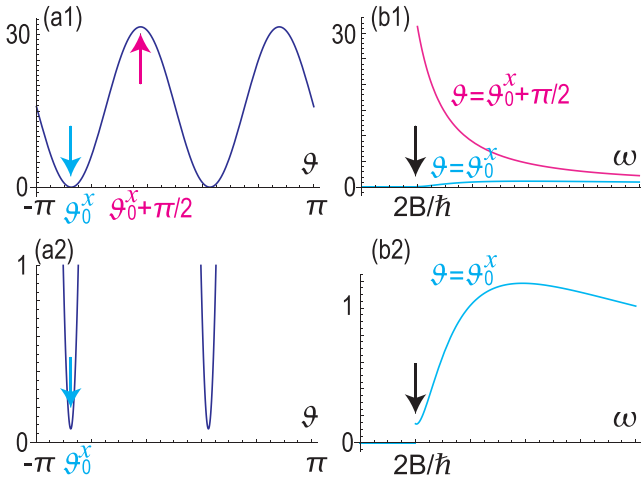


FIG. 4. Elliptic dichroism, where the Néel vector is taken along the x axis. (a2) and (b2) are the enlarged figures of (a1) and (b1) showing $\sigma(2|B|/\hbar; \vartheta_0^y) \neq 0$. See also the caption of Fig. 3.

there is a tiny nonzero contribution $\sigma(\omega; \vartheta_0 + \pi/2)$ at the optical band edge as shown in Fig. 4(b2).

C. In-plane case $\mathbf{J} = (J_x, J_y, 0)$

We then study a general in-plane case. In this case, it is difficult to derive an analytic result of $\sigma(\omega; \vartheta)$ but it is possible to derive an analytic result of $\sigma(2|B|; \vartheta)$. The optical conductivity (25) at the optical band edge is solely determined by the contribution at the Dirac point $k = 0$ as

$$\frac{\sigma(2|B|; \vartheta)}{\sigma_0} = \int_0^{2\pi} d\phi \frac{G(\mathbf{0}; \vartheta)}{2|\partial_k \Delta \mathbf{k}|_{\mathbf{k}=\mathbf{0}}} = \frac{G(\mathbf{0}; \vartheta)}{2|\lambda(\lambda + J_y)|}, \quad (40)$$

where we have used

$$\partial_k \Delta \mathbf{k} = \frac{k}{E(\mathbf{k})} [(J_x \cos \phi - \lambda \sin \phi)^2 + (\lambda + J_y)^2 \cos^2 \phi]. \quad (41)$$

The quantum metrics at the Dirac point are

$$g_{xx}(\mathbf{0}) = \frac{J_x^2 + (\lambda + J_y)^2}{32B^2}, \quad g_{yy}(\mathbf{0}) = \frac{\lambda^2}{32B^2}. \quad (42)$$

The Berry curvature at the Dirac point is

$$\Omega_{xy}(\mathbf{0}) = -\frac{\lambda(\lambda + J_y)}{16B^2}. \quad (43)$$

The optical conductivity (40) is exactly calculated by using

$$\begin{aligned} G(\mathbf{0}; \vartheta) &= \frac{J_x^2}{32B^2} \cos^2 \vartheta + \frac{(\lambda + J_y)^2}{32B^2} \cos^2 \vartheta + \frac{\lambda^2}{32B^2} \sin^2 \vartheta \\ &\quad - \frac{\lambda(\lambda + J_y)}{16B^2} \sin \vartheta \cos \vartheta \\ &= \frac{J_x^2}{32B^2} \cos^2 \vartheta + \frac{\sqrt{\lambda^2 + (\lambda + J_y)^2}}{32B^2} \sin^2 (\vartheta - \vartheta_0^y) \end{aligned} \quad (44)$$

with

$$\vartheta_0^y = -\arctan\left(\frac{\lambda + J_y}{\lambda}\right). \quad (45)$$

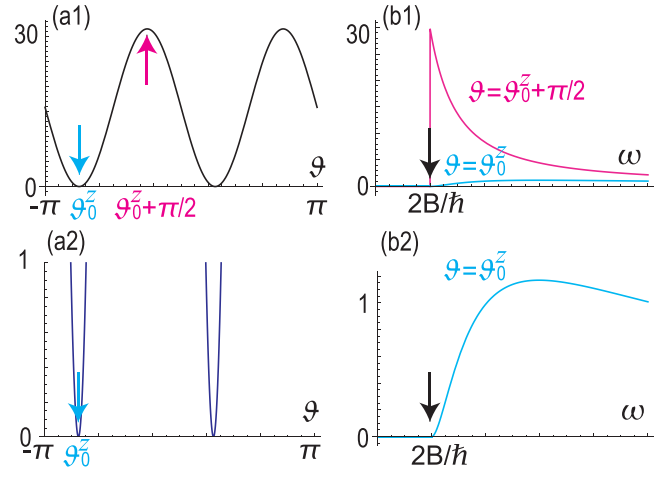


FIG. 5. Elliptic dichroism, where the Néel vector is taken along the z axis. See also the caption of Fig. 3.

It becomes zero for

$$\vartheta = \vartheta_0^y, \quad J_x = 0, \quad (46)$$

where a perfect elliptic dichroism occurs. On the other hand, the perfect elliptic dichroism does not occur for $J_x \neq 0$, but we will soon show that the optical conductivity depends strongly on ϑ .

D. The case of $\mathbf{J} = (0, 0, J_z)$

We proceed to study a specific case, where the Néel vector is along the z axis. The optical conductivity (25) is exactly calculated as

$$\begin{aligned} \frac{\sigma(\omega; \vartheta)}{\sigma_0} &= \left[\frac{\pi(\hbar^2 \omega^2 + 4B^2)}{2\hbar \omega^3} + \frac{\pi J_z^2(\hbar^2 \omega^2 - 4B^2)}{4\lambda^2 \hbar \omega^3} \right. \\ &\quad \left. - \frac{\pi J_z^4(\hbar^2 \omega^2 - 12B^2)}{16\lambda^4 \hbar \omega^3} \right] \cos^2 \vartheta \\ &\quad + \left[\frac{\pi(\hbar^2 \omega^2 + 4B^2)}{2\hbar \omega^3} - \frac{\pi J_z^2(\hbar^2 \omega^2 + 12B^2)}{4\lambda^2 \hbar \omega^3} \right. \\ &\quad \left. + \frac{3\pi J_z^4(\hbar^2 \omega^2 + 20B^2)}{16\lambda^4 \hbar \omega^3} \right] \sin^2 \vartheta \\ &\quad - \frac{B\pi}{\omega^2} \left(4 - \frac{2J_z^2}{\lambda^2} + \frac{3J_z^4}{2\lambda^4} \right) \cos \vartheta \sin \vartheta \end{aligned} \quad (47)$$

for $\hbar\omega > 2|B\lambda|/\sqrt{\lambda^2 + J_z^2}$, and $\sigma(\omega; \vartheta) = 0$ for $\hbar\omega < 2|B\lambda|/\sqrt{\lambda^2 + J_z^2}$. It is impossible to differentiate J_z and $-J_z$ because the optical conductivity is a function of J_z^2 as shown in Figs. 2(a) and 2(b).

The optical conductivity $\sigma(2|B|)$ at the optical band edge is shown as a function of ϑ in Fig. 5(a1). It strongly depends on ϑ . Especially, it becomes exactly zero at

$$\vartheta \simeq \vartheta_0^z = -\arctan\left(\frac{\lambda + J_z}{\lambda}\right). \quad (48)$$

The $\sigma(\omega)$ is shown in Fig. 5(b1).

E. General case

Finally, we study the general case, where all of J_x , J_y and J_z are nonzero. The optical conductivity is calculated as

$$\begin{aligned} & \frac{\sigma(\omega; \vartheta)}{\sigma_0} \\ &= \frac{\pi}{2\lambda^2\hbar\omega^3} \left[\left\{ \left(\lambda^2 + \lambda J_y + J_x^2 - \frac{J_z^2}{2} \right) (\hbar^2\omega^2 + 4B^2) \right. \right. \\ & \quad \left. \left. + 4\pi B^2 J_z^2 \right\} \cos^2 \vartheta \right. \\ & \quad \left. + \left\{ \left(\lambda^2 - \lambda J_y + J_y^2 - \frac{J_z^2}{2} \right) (\hbar^2\omega^2 + 4B^2) \right. \right. \\ & \quad \left. \left. - 4\pi B^2 J_z^2 \right\} \sin^2 \vartheta \right] \\ & \quad - \frac{2B\pi(2\lambda^2 + J_z^2)}{\lambda^2\omega^2} \cos \vartheta \sin \vartheta \end{aligned} \quad (49)$$

up to the second order in J . In general, it is possible to determine J_x , J_y , J_z , λ and B by extracting them from the band gap $2\Delta_{\text{Min}}$ given by Eq. (30) and from the optical conductivity $\sigma(2\Delta_{\text{Min}}; \vartheta)$ at the optical band edge. The optical conductivity is quadratic in J_x and J_z but not in J_y as shown in Figs. 2(a) and 2(b).

VII. DISCUSSIONS

The p -wave magnets have the zero net magnetization. It is expected to be useful for antiferromagnetic spintronics [51–57] with high density and fast switchable magnetic memories because the effect of the stray field is small compared with ferromagnets. In this context, it is important to differentiate the up and down states of the Néel vector. It was shown [30] that the Néel vector is detectable by measuring

the linear and nonlinear conductivities. We discuss a merit of optical detection of the Néel vector comparing conductivities. It is possible to observe the spatial dependence of the optical conductivity by a single shot experiment, where it is possible to observe the domain wall structure of the p -wave magnet. It is interesting to observe a domain-wall motion of the p -wave magnet by optical absorption.

By using the perfect elliptic dichroism condition (45), it is possible to determine J_y if λ is known. Hence, it is possible to differentiate the two states J_y and $-J_y$ by optical measurement. On the other hand, it is impossible to differentiate J_x and $-J_x$ because the optical conductivity is quadratic in J_x . This is also the case for J_z . In addition, it is hard to determine the direction of the Néel vector only by elliptic dichroism because the results are similar as shown in Figs. 3–5. In particular, the optical conductivity takes the minimum value at $\vartheta = \vartheta_0$ irrespective of the direction of the Néel vector. Nevertheless, our results will be useful for future spintronics memories based on the p -wave magnet because the direction of the Néel vector is fixed by the shape anisotropy of the sample and there are only two stable directions.

ACKNOWLEDGMENTS

The author is grateful to M. Hirschberger and S. Okumura for helpful discussions on the subject. This work is supported by CREST, JST (Grants No. JPMJCR20T2) and Grants-in-Aid for Scientific Research from MEXT KAKENHI (Grant No. 23H00171).

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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